電弱バリオン数生成の輸送理論：
場の理論からボルツマン方程式へ

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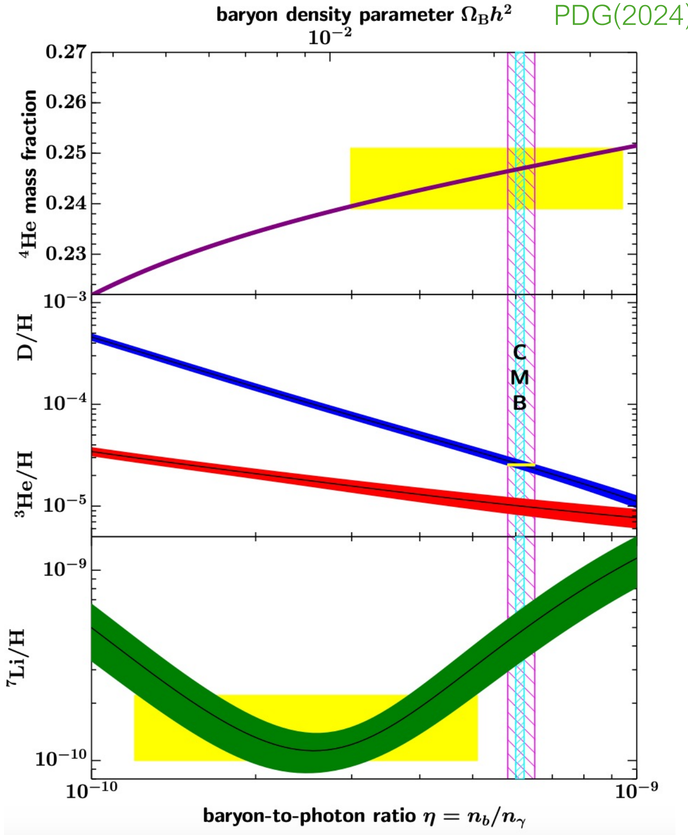
Baryon asymmetry of the Universe



PDG(2024)

- Success of the standard model and physics beyond the SM

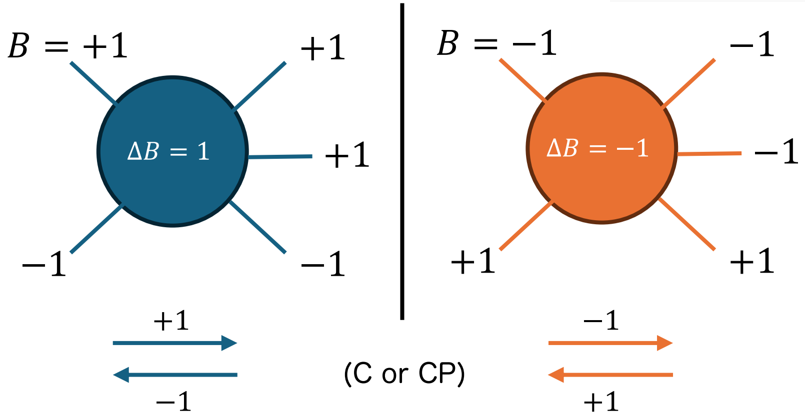
- Baryon asymmetry of the Universe

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} \qquad \eta_B = (6.12 \pm 0.04) \times 10^{-10}$$ Planck (2018)

- Sakharov's three conditions for baryogenesis Sakharov (1967)

① Baryon # violation
② C and CP violation
③ Departure from thermal equilibrium

No baryon asymmetry remains
if the lengths of the arrows are same.

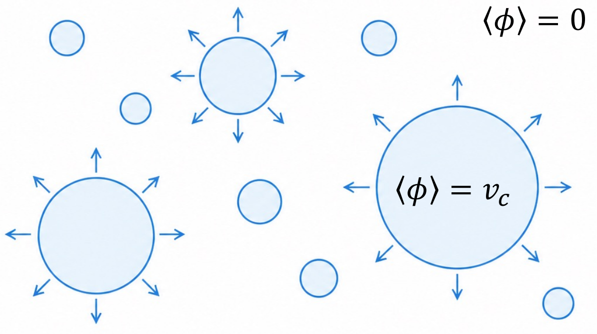


Electroweak baryogenesis

- **Electroweak baryogenesis** Kuzmin, Rubakov, and Shaposhnikov (1985)

  ① EW sphaleron (baryon # violation)
  ② EW theory with CP violation (C and CP violation)
  ③ EW first order phase transition (departure from thermal equilibrium)

First order phase transition at $T \simeq T_c$

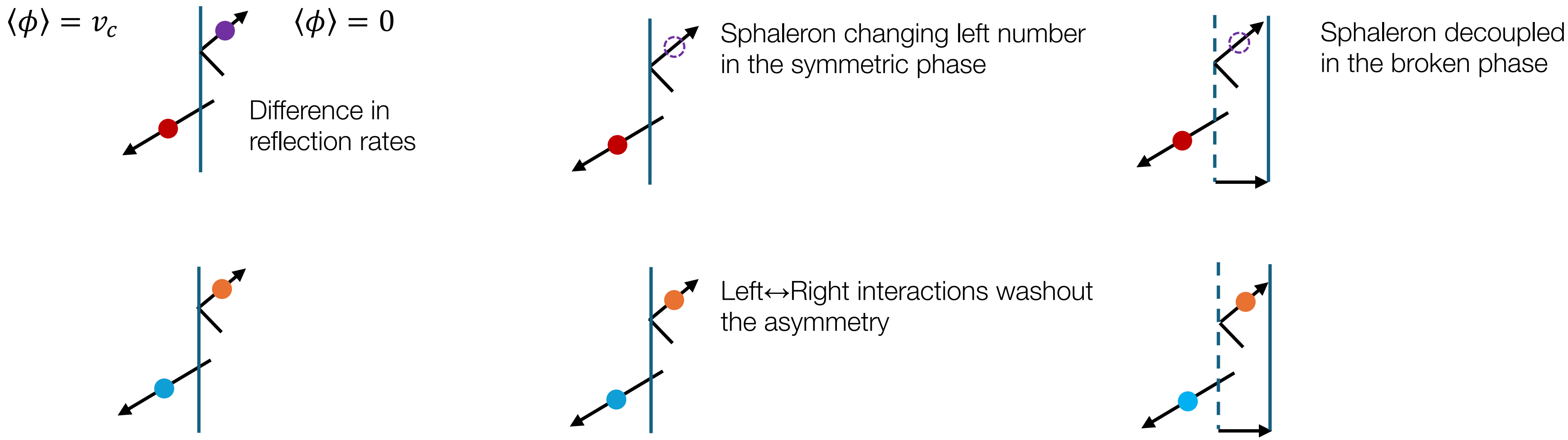


- **Baryon number generation around expanding bubble walls**

1. CP violation **2. C violation & B violation** **3. Freeze out**

Chirality: Left ● Anti-Left ●
Charge: +1 -1

Chirality: Right ● Anti-Right ●
Charge: +1 -1

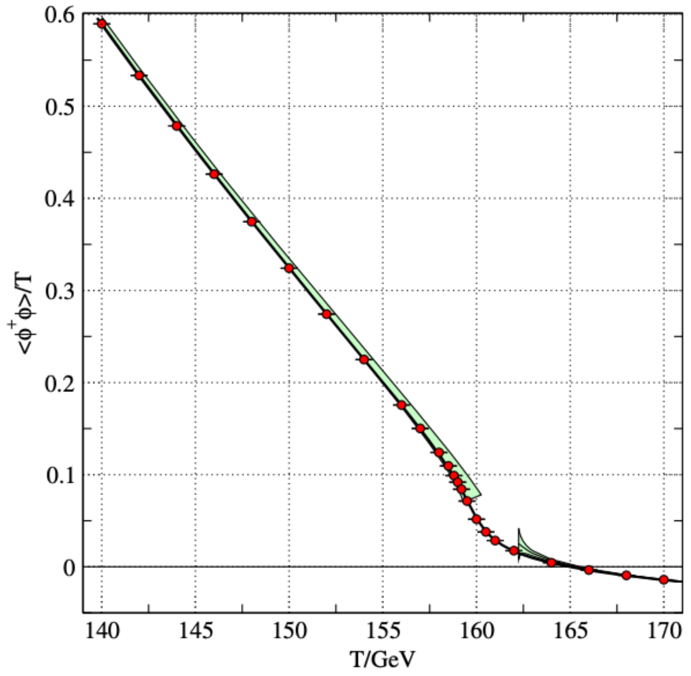


Total charge = 0 (CPT theorem) Total charge = +1 Total charge = +1

Failure in the SM

- **EW phase transition is a cross-over in the SM**

Kajantie et al., PRL (1996); D'Onofrio and Rummukainen, PRD (2016);



- **CP violation in the CKM is insufficient**

Gavela et al. (1994);

Huet and Sather (1995);

Jarlskog invariant $J_{CP} = \mathrm{Im}[V_{ud}V_{us}^* V_{cs} V_{cd}^*] = 3.12\times10^{-5}$ PDG (2024)

CP violating observables need to be proportional to

$$\Delta_{CP} = J_{CP} \times (m_t^2 - m_u^2)(m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_b^2 - m_d^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2).$$

Need a scale $T_c \sim 100$ GeV, Δ_{CP} to be dimensionless: $\eta_B \sim \frac{\Delta_{CP}}{T_c^{12}} \sim 10^{-20}$ → much smaller than the observed value $\eta_B \sim 10^{-10}$

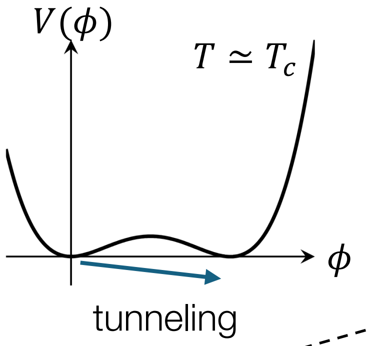
- **Various extensions of the SM:**

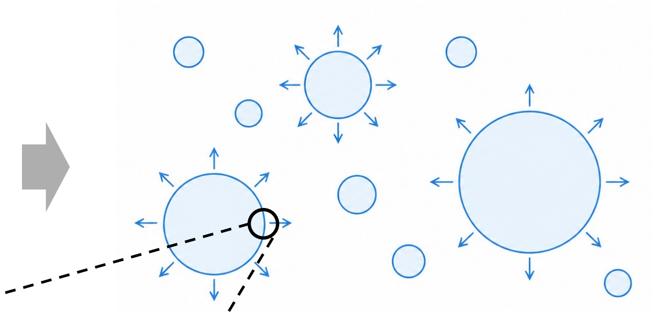
SM + $SU(2)_L$ doublet scalar, SM + $SU(2)_L$ singlet scalar, …

New physics near the EW scale → many observables in collider, flavor, EDMs, GWs,…

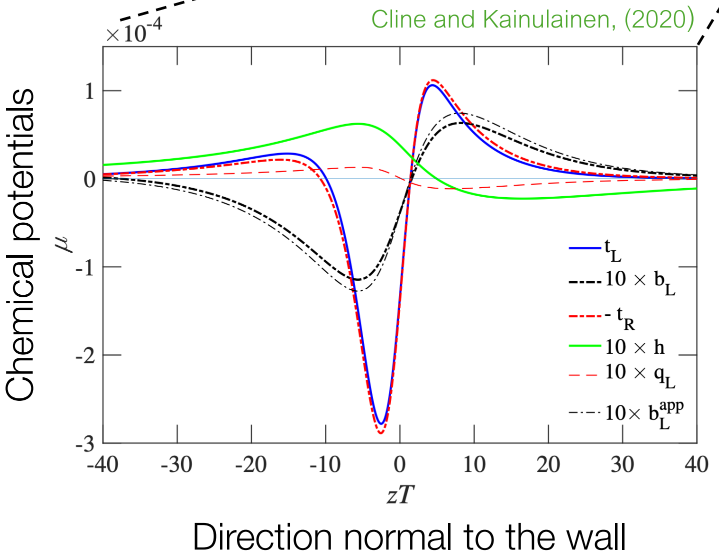
How to calculate baryon asymmetry?

1. Calculate the EW first order phase transition, the bubble profile, and its dynamics

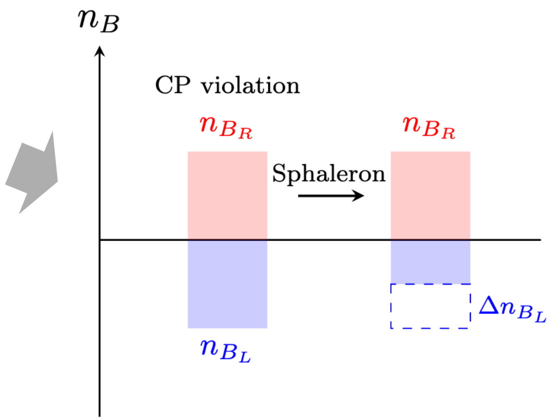




2. Evaluate the chiral charge asymmetry around the wall generated by the CP violation



3. Calculate the total baryon charge generated by the EW sphaleron



How to calculate baryon asymmetry?

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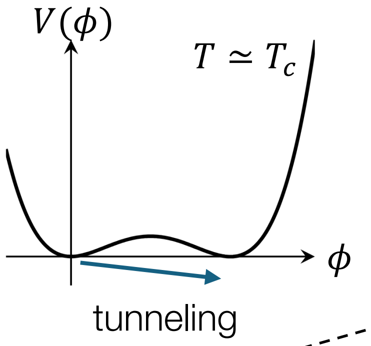
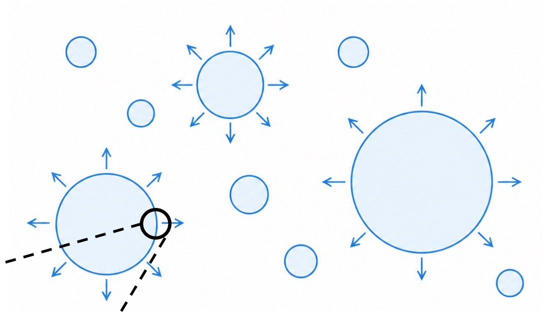
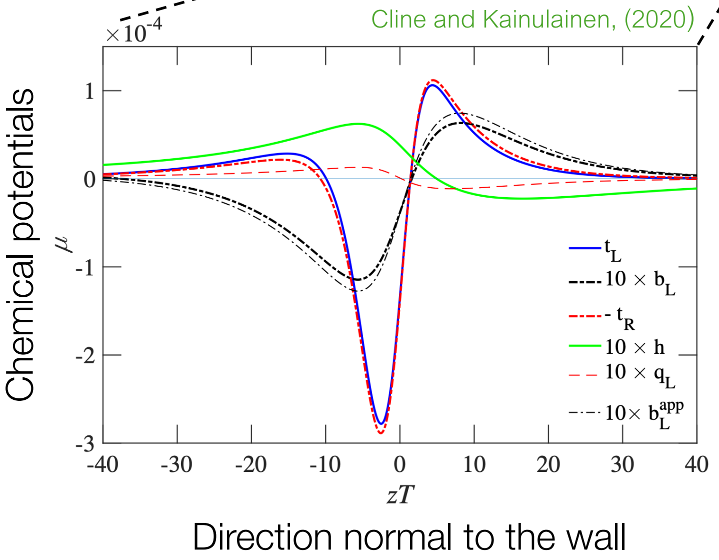




Today's topic: particle transport in the plasma

2. Evaluate the chiral charge asymmetry around the wall generated by the CP violation



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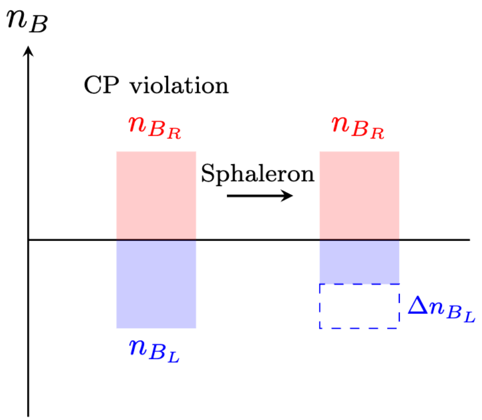


The Boltzmann equation

- **Particle transport in a non-equilibrium system (flat spacetime) can be described by the Boltzmann equation:**

$$\boldsymbol{L}[f_i] = \boldsymbol{C}[f_i, f_a, \dots]$$

Liouville term   Collision term

$f_i(k,x)$: distribution function of particle i

Liouville term: $\boldsymbol{L}[f_i] = (\partial_t + v\, \partial_x + F\, \partial_k)\, f_i$ describing the evolution of the distribution.

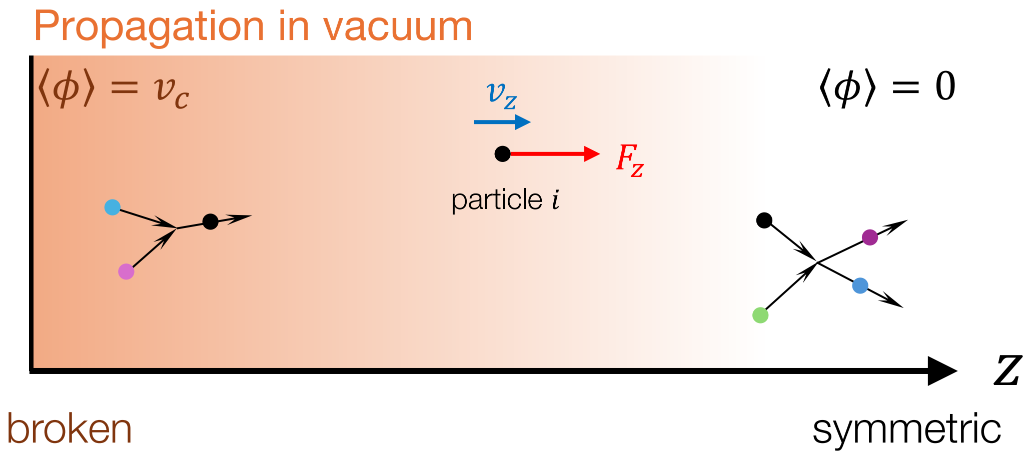
velocity (v)   force (F)

CP violating effects from spacetime-dependent VEV are encoded in velocity & force terms.

Collision term: $\boldsymbol{C}[f_i, f_a, \dots] = \int d\Pi_i \dots |M|^2 (f_b f_c - f_i f_a) + \cdots$

describing various particle scatterings.

Processes: top Yukawa, weak current, Higgs interactions, etc.



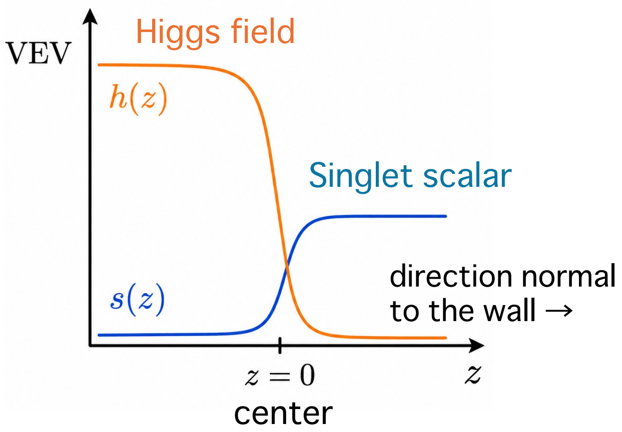
Conventional derivation for velocity and force

Kainulainen, Prokopec, Schmidt, and Weinstock, (2001), (2022);
Prokopec, Schmidt, and Weinstock, (2004);

- **CP violating interaction included in the complex mass**

(e.g.) Singlet extension of the SM Cline, Kainulainen, and Smith, (2017);

Effective top quark mass $$m_t(z) = y_t\, h(z) \left(1 + i\,\frac{s(z)}{\Lambda}\right)$$ CPV



- **Solving the homogeneous equation for the Green function in slowly varying VEV $(\partial_x \ll T)$**

For the fermionic case, it is schematically written by $$\left(k_\mu\gamma^\mu - m_R - i\, m_I\, \gamma^5 + \mathcal{O}(\partial_x m)\right) G = 0.$$

Dirac operator ∂_x correction w/ CPV

→ $G \sim n \times \delta(\Omega^2 + \delta\Omega^2)$ where $\Omega^2 = k^2 - m^2, \quad \delta\Omega^2 = \mathcal{O}(\partial_x m)$.

→ modifying dispersion relation $k^0 = \omega_0 \pm \delta\omega_0$ +: particle
- : anti-particle

→ on the mass-shell, n relates to the distribution f : $G \sim \langle a^\dagger a\rangle \sim f\,\delta(k^0 - \omega)$

Conventional derivation for velocity and force

Kainulainen, Prokopec, Schmidt, and Weinstock, (2001), (2022);
Prokopec, Schmidt, and Weinstock, (2004);

- **To determine the evolution of $n \sim f$, substitute the solution into a kinetic equation:**

$$\int_0^\infty dk^0\, i\, \not{\partial}_x\, G \;=\; \boldsymbol{C}[f_i, f_a, \dots] \quad\to\quad (v\partial_x + F\partial_k)\, f = \boldsymbol{C}[f, f_a, \dots]$$

cf.) Derivation from QM

Cline, Joyce, and Kainulainen, (1998), (2000);
Fromme and Huber, (2007);

Due to the gradient correction to the mass shell, a CP-violating difference appears: $v = v_0 \pm \delta v\,, \quad F = F_0 \pm \delta F$

- **The conventional derivation may rely on several simplifications.**

Considering the particle propagation in the vacuum

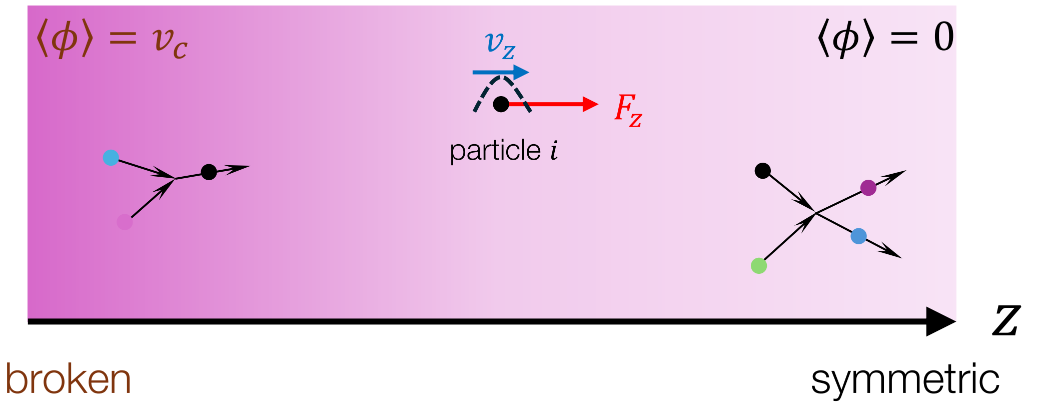
Thermal correction or damping?
How can we take particle picture?

Thermal corrections
(single fermion): Kainulainen, (2021)

It is not transparent which approximations are required to reduce the full Kadanoff-Baym equations to the Boltzmann equation for EWBG

What approximations are indeed needed?
Bosons? Fermions? Multi-flavor cases?

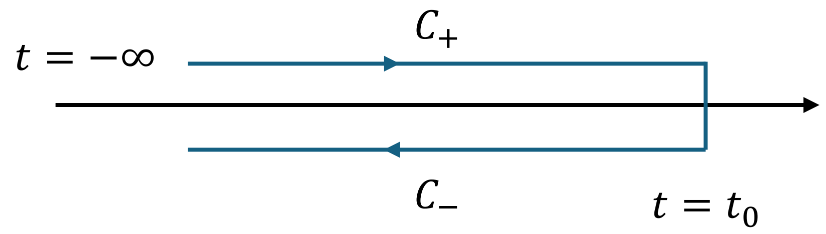
Propagation in thermal plasma & vacuum



Closed-time-path formalism

Schwinger (1961); Keldysh (1965);
Kadanoff and Baym (1962); and more

- **Time ordered 2×2 propagators on the closed-time-path**



(e.g.) bosonic propagators

$$iG^{++}(u,v) \equiv iG^{t}(u,v) = \langle \mathrm{T}[\phi(u)\phi^\dagger(v)]\rangle, \quad \text{: time ordered propagator on } C_+$$

$$iG^{+-}(u,v) \equiv iG^{<}(u,v) = \langle \phi^\dagger(v)\phi(u)\rangle,$$

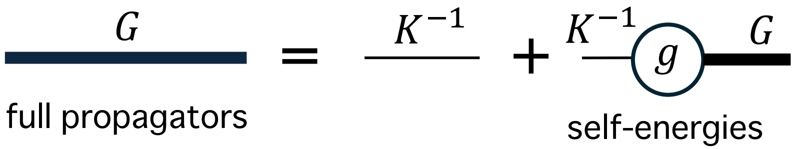
$$iG^{-+}(u,v) \equiv iG^{>}(u,v) = \langle \phi(u)\phi^\dagger(v)\rangle,$$

: Wightman functions G^λ $(\lambda = <,>)$

$$iG^{--}(u,v) \equiv iG^{\bar{t}}(u,v) = \langle \overline{\mathrm{T}}[\phi(u)\phi^\dagger(v)]\rangle. \quad \text{: time ordered propagator on } C_-$$

- **Schwinger-Dyson equation on the path**

$$K(u)\, iG^{ab}(u,v) = ia\, \delta^{ab}\delta^4(u-v) + \sum_c c\int d^4w\ g^{ac}(u,w) iG^{cb}(w,v), \qquad (a,b,c=\pm)$$



The Kadanoff-Baym equations

- **The Kadanoff-Baym equations** $(u, v) \to (u - v, (u + v)/2)$ (the relative coordinate is Fourier transformed)

relative: r average: x → momentum: k

$$\mathcal{D}(k, x)\, G^{\lambda} - e^{-i\diamond}\left(\mathcal{M} + g_{\mathcal{H}}, G^{\lambda}\right) - e^{-i\diamond}\left(g^{\lambda}, G_{\mathcal{H}}\right) = \frac{1}{2}\left(e^{-i\diamond}(g^{>}, G^{<}) - e^{-i\diamond}(g^{<}, G^{>})\right)$$

kinetic operator mass (VEV)

self-energies (plasma)

- **Dependence on x is expressed by the diamond operator :** $\diamond\, (A, B) = \frac{1}{2}\left(\partial_x A \cdot \partial_k B - \partial_k A \cdot \partial_x B\right)$

- **Spectral function** $\mathcal{A} \equiv \frac{i}{2}(G^{>} - G^{<})$ **describes spectral information of the system.**

cf.) free field theory (on-shell) $\mathcal{A} \propto \delta(\Omega^2), \quad \Omega^2 = k^2 - m^2$

- **Effects of interactions in plasma are encoded in the self energy g.**

$g_{\mathcal{H}}$: Hermitian part ~ thermal mass
$g_{\mathcal{A}}$: Anti-Hermitian part ~ thermal width

Conventional derivation: $g = 0$

Our derivation: $g \neq 0$

Our approach and results

- **Systematic derivation of the Boltzmann equation**

    ① Algebraically solve the “general” KB equations (bosons, fermions, and multi-flavor cases w/ self-energies)

    ② Apply approximations commonly or implicitly used in the EWBG community

    ③ By using the quasiparticle approximation, identify the on-shell part of the solutions

    ④ Substitute the on-shell solution into the kinetic equation

- **Results:**

    We clarify what approximations are needed to derive the Boltzmann equation.

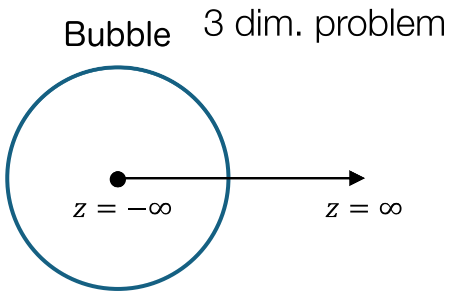
    We find a Boltzmann equation including self-energy corrections for both bosons and fermions.

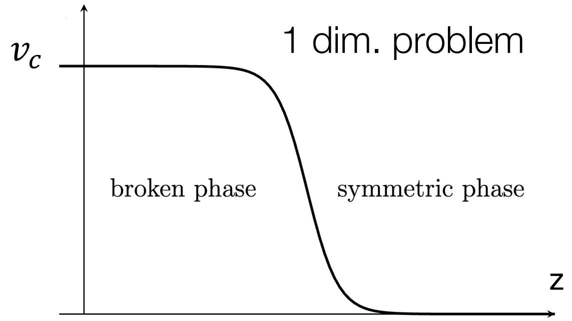
Various approximations

- **Assume the bubble wall is static and planar in the wall-comoving frame**

Spin for z axis is almost conserved in EW plasma

→ (spinors) = (spin) × (chiral)





- **Assume the wall is thick ($\partial_z \ll T$)** (typically, $L_w T \sim O(10)$, but model-dependent)

Derivative expansion: $e^{-i\diamond} = 1 + (-i\diamond) + \frac{1}{2}(-i\diamond)^2 + \cdots$

By using derivative expansion, the KB equations are algebraically solved order by order: $G = \sum_0^\infty G^{[n]}$

- **Assume flavor-universal self energies:** $g = \bar{g} + \cancel{\delta g}$ (dropping quantum coherent effects among flavors)

flavor dependent

Various approximations

- **Spectral function under the quasiparticle approximation: Γ (width) << energy**

0th order in derivative

$$\mathcal{A}^{[0]} \sim \frac{\Gamma}{\Omega_0^4 + \Gamma^2} \;\rightarrow\; \delta(\Omega_0^2)$$

Breit-Wigner distribution

1st order in derivative (non-trivial correction arises only for fermionic case)

$$\mathcal{A}^{[1]} \sim \Omega_1^2 \times \frac{2\Gamma\,\Omega_0^2}{(\Omega_0^4 + \Gamma^2)^2} \;\rightarrow\; \Omega_1^2 \times \delta'(\Omega_0^2)$$

(For exact formulae, please see our paper.)

- **Modification of dispersion relation from ∂_z can be understood by** $\delta(x) + \delta x\,\delta'(x) = \delta(x+\delta x) + \mathcal{O}(\partial_z^2)$.

- **The corrections from the Hermitian self-energy $g_{\mathcal{H}}$ are included in Ω_0^2 and Ω_1^2**

For example, $\Omega_0^2 = \left(k^0 - g_{\mathcal{H}}^0\right)^2 - \left(k_z + g_{\mathcal{H}}^3\right)^2 - \left(m_R + g_{\mathcal{H}}^1\right)^2 - \left(m_I + g_{\mathcal{H}}^2\right)^2$ (fermionic case)

Derivative corrections included in Ω_1^2 are CP-odd, $\Omega_{1,\mathrm{CP}}^2 = -\Omega_1^2$.

- **With a decomposition of g^λ, the Wightman functions also have the on-shell part: $G^< \sim n^< \mathcal{A}$.**

Deriving Boltzmann equation

- **Substituting the on-shell solutions up to n=1 into the kinetic equation (part of the KB equations), we finally obtain the self-energy corrected (fermionic) Boltzmann equation for spin-s and flavor-i.**

For the effective shell function $\Omega^2_{s,i,\mathrm{eff}} = \Omega_0^2 + \underbrace{\delta\Omega^2}_{\text{CP-odd}}$, we find

Liouville term: $$\boldsymbol{L}_{s,i}\left[f_{s,i}\right] = \int_0^\infty dk^0 \underbrace{\{\Omega^2_{s,i,\mathrm{eff}}, n^<_{s,i}\}_{z,k_z}}_{\text{Poisson bracket}} \delta\left(\Omega^2_{s,i,\mathrm{eff}}\right) = \frac{1}{\left|\partial_{k^0}\Omega^2_{s,i,\mathrm{eff}}\right|}\{\Omega^2_{s,i,\mathrm{eff}}, n^<_{s,i}\}_{z,k_z}\Big|_{k^0=\omega_{s,i}}$$

$$= \underbrace{\partial_{k_z}\omega_{s,i}}_{\text{velocity}}\, \partial_z f_{s,i} - \underbrace{\partial_z\omega_{s,i}}_{\text{force}}\, \partial_{k_z} f_{s,i},$$

where $k^0 = \omega_{s,i}$ is a positive solution of $\Omega^2_{s,i,\mathrm{eff}}\left(k^0\right) = 0$.

Collision term: $$\boldsymbol{C}_{s,i}\left[f_{s,i}, \ldots\right] = \int_0^\infty \frac{dk^0}{2\pi} \mathrm{Tr}[P_s P_i (g^> G^< - g^< G^>)]$$ $(P_s, P_i$: projection matrices).

Summary and Outlook

- **We gave a systematic way to obtain the Boltzmann equation for EWBG.**
- **We algebraically solved the KB equation order by order in the derivative expansion and applying approximations to the solutions.**

  Static- and planar-wall approximation, flavor universal approximation, quasiparticle approximation, etc.

- **We obtained the Boltzmann equation with self-energy corrections.**

Future works

- Including flavor coherent effects
- Phenomenological applications
- …

Supplement

Restoring on-shell structure

- **Quasiparticle approximation with narrow width limit**

e.g., for bosonic case, at the zeroth order of derivative expansion, the spectral function takes Breit-Wigner form as given by

(cf.) spectral function

$$G_{\mathcal{A}} \equiv \frac{1}{2i}(G^a - G^r) = \frac{i}{2}(G^> - G^<) \equiv \mathscr{A},$$

$$\hat{\mathscr{A}}_\phi^{[0]} = \sum_i \frac{\overline{\Pi}_{\mathcal{A}}}{\Omega_i^4 + \overline{\Pi}_{\mathcal{A}}^2} P_i \qquad P_i \hat{\mathscr{A}}_\phi^{[0]} \to \pi\, \mathrm{sign}(k^0)\, \delta(\Omega_i^2)$$

P_i : projection to i-th flavor

- **Prescription such that the Wightman functions have the on-shell structure**

In the absence of the wall-gradient, the system should be in local thermal equilibrium ($\eta = \pm 1$)

$$g^\lambda = \frac{2\eta}{i} n_{\mathrm{leq}}^\lambda g_{\mathcal{A}}$$

$$n_{\mathrm{leq}}^< = \frac{1}{e^{\beta k \cdot u} - \eta}$$

$$n_{\mathrm{leq}}^> = n_{\mathrm{leq}}^< + \eta$$

Distribution will be modified by the wall-gradient $\quad n_{\mathrm{leq}}^\lambda \to n^\lambda(k, x)$

e.g., for bosonic case, the zeroth order Wightman functions are $\quad P_i i\hat{\Delta}_{\mathrm{shell}}^\lambda = 2\pi n_i^\lambda \mathrm{sign}(k^0)\delta(\Omega_i^2)$.