The Infrared Physics in QED and cosmology

Tadashi Kuramoto[1], Takahiro Tanaka[2], and Yuko Urakawa[1,3]
[1]The graduate University for Advanced Studies (SOKENDAI), Tsukuba, Japan
[2]Kyoto University, Kyoto, Japan
[3]Institute of Particle and Nuclear physics, High Energy Accelerator Research Organization (KEK), Tsukuba, Japan

Introduction: infrared triangle

Infrared triangle

Strominger (2013, 2017)
Strominger, and Zhiboedov (2014)

- Infrared dynamics is represented by the three equivalent subjects.
- This equivalence is expected to be valid in various theories with massless particles (e.g. linearized GR, QED, YM theory, cosmology,…).

Mostly, gauge theory in asymptotically flat spacetime

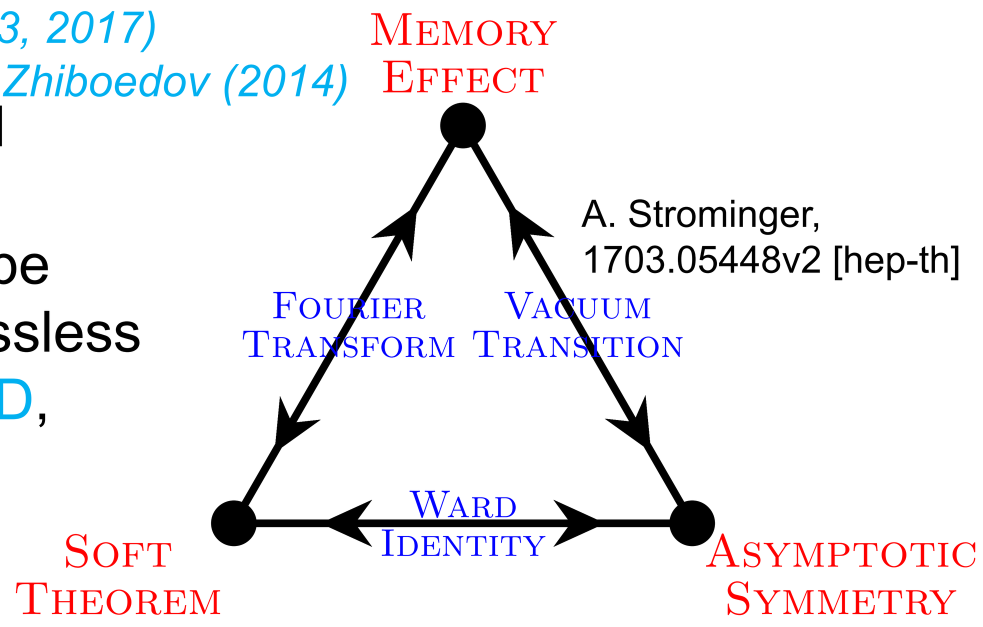

Fig. 1 The infrared triangle

Special features of cosmology (inflation)

- nearly de Sitter spacetime (not asymptotically flat)
- extended to $\boldsymbol{k} \neq 0$ modes

Goals of this work

- to understand the locality condition, which ensure the effective description of primordial perturbation.
- to extend IR triangle to $\omega \neq 0$ modes

Weinberg's soft theorem and IR cancellation in QED

Weinberg (1965)

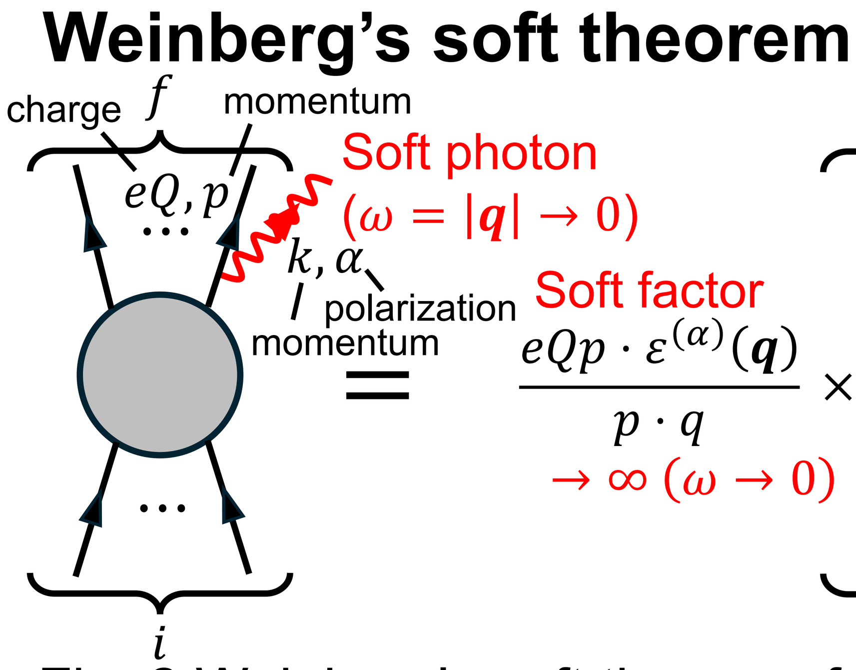

Fig. 2 Weinberg's soft theorem for photon

IR divergence and cancellation

$$S_{fi} = \langle f|S|i\rangle \to e^{-\infty} = 0$$

(IR divergence by radiative soft photons)

$$\overline{|S_{fi}|^2} := \sum_{\gamma^S} |\langle f, \gamma^S|S|i\rangle|^2 \neq 0$$

Sum w.r.t. invisibly soft photons

Asymptotic symmetry in QED

Strominger (2013, 2017)
He, Mitra, Porfyriadis, Strominger (2014)

Large gauge transformation (LGT) (= asymptotic symmetry in QED)

$$\begin{cases} A_\mu(x) \to A_\mu(x) + \partial_\mu \lambda(x) \\ \psi(x) \to e^{iQ\lambda(x)}\psi(x) \end{cases} \quad \text{with} \quad \lambda(x) \xrightarrow[r \to \infty\ (t \pm r\text{: fixed})]{\text{asymptotic limit}} \varepsilon(\hat{x}) \neq 0$$

Generator of LGT (*out* region) ("(0)" means the leading order in $1/r$ expansion.)

$$Q_\varepsilon^+ := \frac{1}{e^2}\int du d^2\hat{x}\left[-\partial_A \varepsilon(\hat{x})\, F_u^{(0)A}(u,\hat{x}) + e^2 \varepsilon(\hat{x}) j_u^{(0)}(u,\hat{x})\right]$$

$$(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, u = t - r)$$

Q_ε^+ includes only $\omega = 0$ photons by integral with respect to u.

Ward identity (conservation of the LGT charge)

$$\langle f|(Q_\varepsilon^+ S - S Q_\varepsilon^-)|i\rangle = 0$$

$$\varepsilon(z,\bar{z}) = \text{const.} \quad \Longrightarrow \quad \text{total electric charge consevation}$$

$$\varepsilon(z,\bar{z}) = \frac{1}{z - w} \quad \Longrightarrow \quad \text{Weinberg's soft photon theorem}$$

($z, w \in \mathbb{C}$ are coordinates on S^2 (stereographic projection))

Faddeev-Kulish's dressed state and IR cancellation

Chung (1965), Faddeev & Kulish (1970) Kapec, Perry, Raclariu, & Strominger (2017)

$$|\alpha; \mathrm{FK}\rangle := e^{iR}|\alpha\rangle, \qquad (\alpha = i, f)$$

R generates cloud of soft photons

$$Q_\varepsilon^-|i;\mathrm{FK}\rangle = 0, \qquad \langle f;\mathrm{FK}|Q_\varepsilon^+ = 0$$

asymptotic symmetry

$$S_{fi} = \langle f|S|i\rangle \propto e^{-\infty} = 0 \quad ▶ \quad S_{fi}^{\mathrm{FK}} = \langle f;\mathrm{FK}|S|i;\mathrm{FK}\rangle \neq 0$$

(IR divergence by radiative soft photons) (IR cancellation)

Cosmology and extension of infrared physics in QED

Infrared physics in cosmology (inflation)

Tanaka & Urakawa (2017,2026)

Dilatation (LGT in cosmology)

$$\begin{cases} x^i \to x_\lambda^i = e^\lambda x^i \\ \zeta(t,\boldsymbol{x}) \to \zeta_\lambda(t,\boldsymbol{x}) = \zeta(t, e^{-\lambda}\boldsymbol{x}) - \lambda \end{cases}$$

(ζ: curvature perturbation ($ds^2 \supset e^{2\zeta} d\boldsymbol{x}^2$))

Generator

$$Q[\lambda] = a^3(t)\int d^3x\, \Delta_\lambda \zeta(t,\boldsymbol{x}) \pi_\zeta(t,\boldsymbol{x})$$

($a(t)$: scale factor, $\Delta_\lambda\zeta$: linear-order deviation)

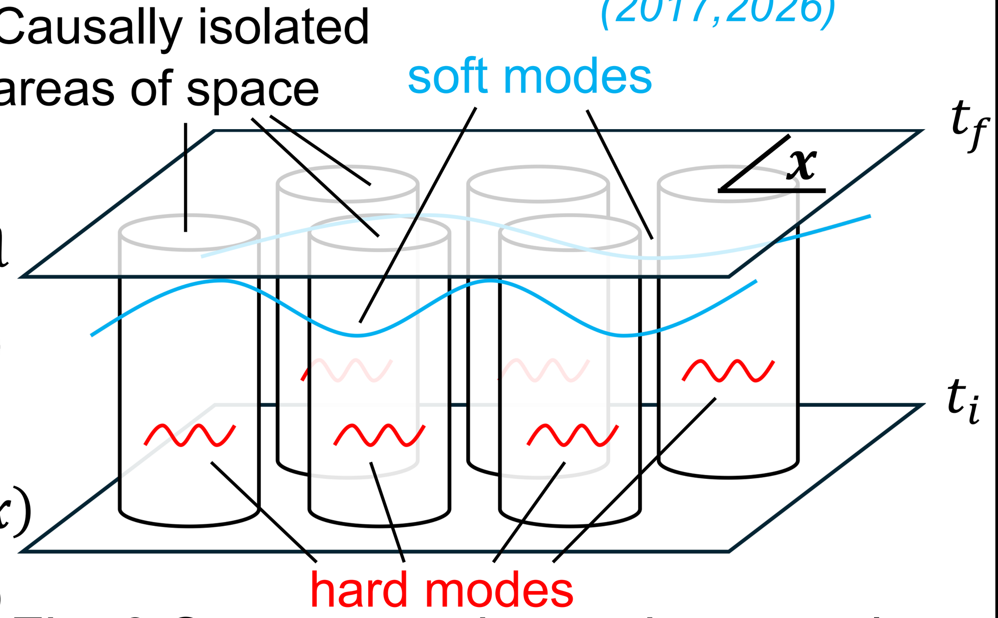

Fig. 3 Separate universe in cosmology

Dilatation invariance (large gauge symmetry)

$$Q[\lambda]|\Psi\rangle = 0$$

($|\Psi\rangle$: state of universe)

&

Locality condition (inflation)

$$\langle \zeta^c|\Psi\rangle = |\psi(\zeta^c)| \prod_a |\Psi_a; \zeta_a^c\rangle$$

($\langle\zeta_a|\zeta_a^c\rangle = \zeta_a^c|\zeta_a^c\rangle$, $|\zeta^c\rangle = \prod_a |\zeta_a^c\rangle$, ζ_a: average of ζ over the patch a)

LGT invariance in ζ-representation

$$\begin{cases} \dfrac{\partial|\psi(\zeta^c)|}{\partial \bar{\zeta}^c} = 0 \\ \left(iQ[\lambda] - \lambda \dfrac{\partial}{\partial \zeta_a}\right)|\Psi_a; \zeta_a^c\rangle = 0 \end{cases}$$

Soft theorem for $\boldsymbol{k} \neq \boldsymbol{0}$ (generalized consistency relation)

$$\langle \Psi|\zeta_{\boldsymbol{k}}^S \mathcal{O}_{\boldsymbol{k}'}|\Psi\rangle \propto \langle \Psi|\zeta_{\boldsymbol{k}}^S \zeta_{\boldsymbol{k}'}^S|\Psi\rangle\, \langle \Psi|\mathcal{O}_{-\boldsymbol{k}-\boldsymbol{k}'}|\Psi\rangle$$

Conjugate coherent state in QED

TK, Tanaka, & Urakawa (in progress)

$$N_z^+(z,\bar{z})|f;\mathrm{FK}\rangle := N_z^{\mathrm{out}}(z,\bar{z})|f;\mathrm{FK}\rangle,$$

$$N_z^+(z,\bar{z}) = \int du \underbrace{F_{uz}^{(0)}(u,z,\bar{z})}_{\text{gauge invariant}}$$

Table. 1 Corresponding quantities

QED	Cosmology
$A_z \to A_z + \partial_z \varepsilon$	$\zeta(t,\boldsymbol{x}) \to \zeta_\lambda(t,\boldsymbol{x})$
$Q_\varepsilon^\pm$	$Q[\lambda]$
$\lvert\alpha;\mathrm{FK}\rangle$	$\lvert\Psi\rangle$
$\partial_z\phi(z,z)$	$\bar{\zeta}(t)$
$\mathcal{N}$	$\lvert\psi(\zeta^c)\rvert$
$u \in \mathcal{I}^+$	$\boldsymbol{x} \in \mathbb{R}^3$

ζ is large gauge dependent.

$$▶ \quad \zeta(t,\boldsymbol{x}) \iff A_z^{(0)}(u,z,\bar{z}) \quad (\text{not } N_z^+)$$

For unseparated case,

$$\bar{\zeta}(t) \ \ (\text{spatial average}) \iff \partial_z\phi(z,\bar{z}) \ \ (\text{average w.r.t. } u)$$

$\partial_{\bar{z}}\phi$ is conjugate to $N_{\bar{z}}^+$: $\quad [N_{\bar{z}}^+(z,\bar{z}), \partial_z\phi(z',\bar{z}')] := -\dfrac{ie^2}{2}\delta^2(z - z')$

▶ introduce conjugate coherent state: $\partial_z\phi|\partial_z\phi^c\rangle = \partial_z\phi^c|\partial_z\phi^c\rangle$

$$|f;\mathrm{FK}\rangle = \int \mathcal{D}\partial_z\phi^c|\partial_z\phi^c\rangle\langle\partial_z\phi^c|f;\mathrm{FK}\rangle,$$

$$\langle\partial_z\phi^c|f;\mathrm{FK}\rangle = \mathcal{N}\underline{e^{2ie^{-2}N^{\mathrm{out}\,A}\cdot\partial_A\phi^c}|f\rangle}$$

$$[\hat{p},\hat{x}] = -i$$
$$\hat{x}|x\rangle = x|x\rangle, \qquad \hat{p}|p\rangle = p|p\rangle$$
$$\langle x|p\rangle = Ce^{ipx}$$

LGT invariance in $\partial_z\phi$ -representation (Q_H^+: fermion part of the LGT generator)

$$\frac{\partial\mathcal{N}}{\partial(\partial_A\phi^c)} = 0, \qquad \left(iQ_H^+ - \partial_A\varepsilon\cdot\frac{\partial}{\partial(\partial_A\phi^c)}\right)\underline{|f;\partial_z\phi^c\rangle} = 0$$

LGT invariance is rewritten in the similar form as that in cosmology.

Separation of asymptotic region in QED

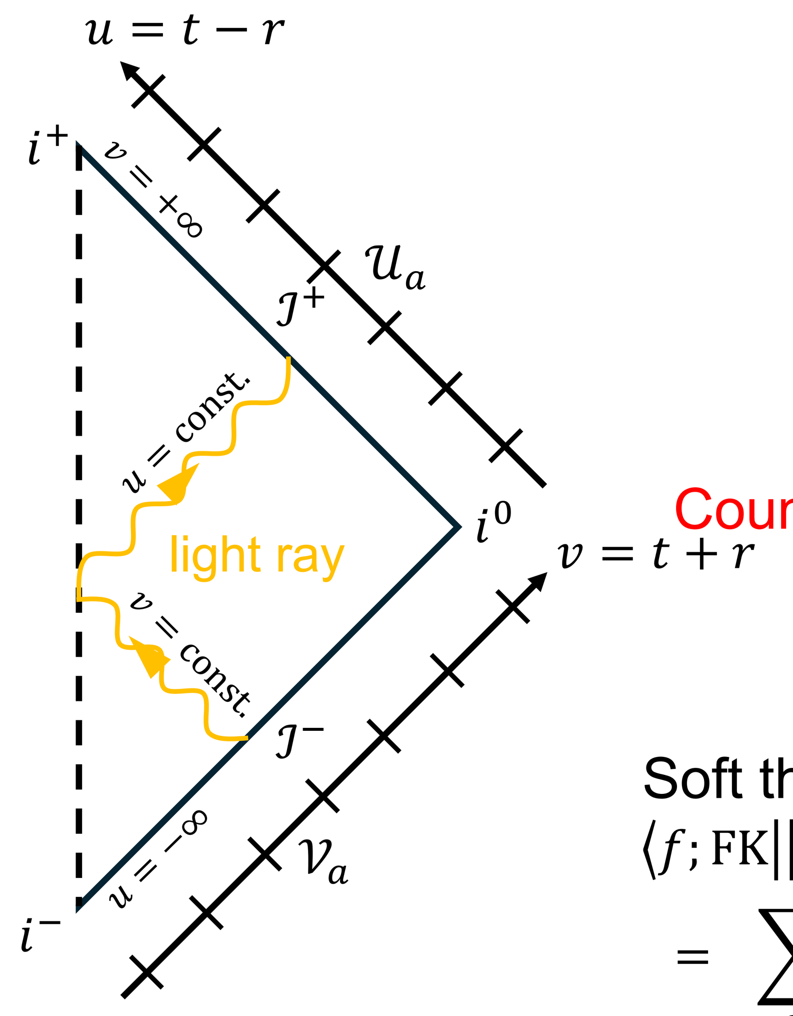

Fig. 4 Penrose diagram of Minkowski spacetime ($r \geq 0$)

Finite-time(u) extension

$$Q_\varepsilon^+ \to Q_{\varepsilon,a}^+, \qquad \left(\int_{-\infty}^{+\infty} du \to \int_{u_a} du\right)$$

$Q_{\varepsilon,a}^+$ includes $\omega \neq 0$ photons

$$|\alpha;\mathrm{FK}\rangle = e^{iR}|\alpha\rangle = \prod_a e^{iR_a}|\alpha_a\rangle$$

Counterpart of locality condition is satisfied.

The FK state with photons with $\omega \lesssim (\text{width of segments})^{-1}$ can be constructed from invariance under $Q_{\varepsilon,a}^+$

Soft theorem with finite-energy($\omega \neq 0$) photon

$$\langle f;\mathrm{FK}|[a^{(+)}(\boldsymbol{q}) + a^{(-)\dagger}(\boldsymbol{q})]S|i;\mathrm{FK}\rangle$$

$$= \sum_{u_m \in \mathcal{U}_a} \frac{eQ_m p_m \cdot \varepsilon^{(+)}(\boldsymbol{q})}{p_m \cdot q}\cos(\omega u_m)\langle f;\mathrm{FK}|S|i;\mathrm{FK}\rangle$$

$$(\omega \lesssim (\text{width of segments})^{-1})$$

Similarities and differences

Similar points

- The correspondence of quantities is made (Table. 1).
- LGT invariance for hard state can be written in the similar form.

Different points

- The magnitude of the wave function is constant or flat in one direction,
- and so on.

Summary and prospects

Summary

- LGT invariance is rewritten in the similar form to that in cosmology.
- The magnitude of wave function is constant in QED.
- Soft theorem with $\omega \neq 0$ is obtained by separating $\mathcal{I}^\pm$.
- Separation of Q_ε^+ is needed to obtain the soft theorem for $\omega \neq 0$.
- The counterpart of the locality condition is satisfied for FK state in QED.

Prospects

- electromagnetic memory in finite time? How can it be detected?
- physical meaning of the finite-energy formulation in QED?