熱場の量子論とその応用, Sep. 2-4, 2026 @KEK

System-environment entanglement transition in quantum critical states under continuous observation

Shunsuke Furukawa (Keio Univ.)

Y. Ashida, SF, & M. Oshikawa, Phys. Rev. B 110, 094404 (2024)

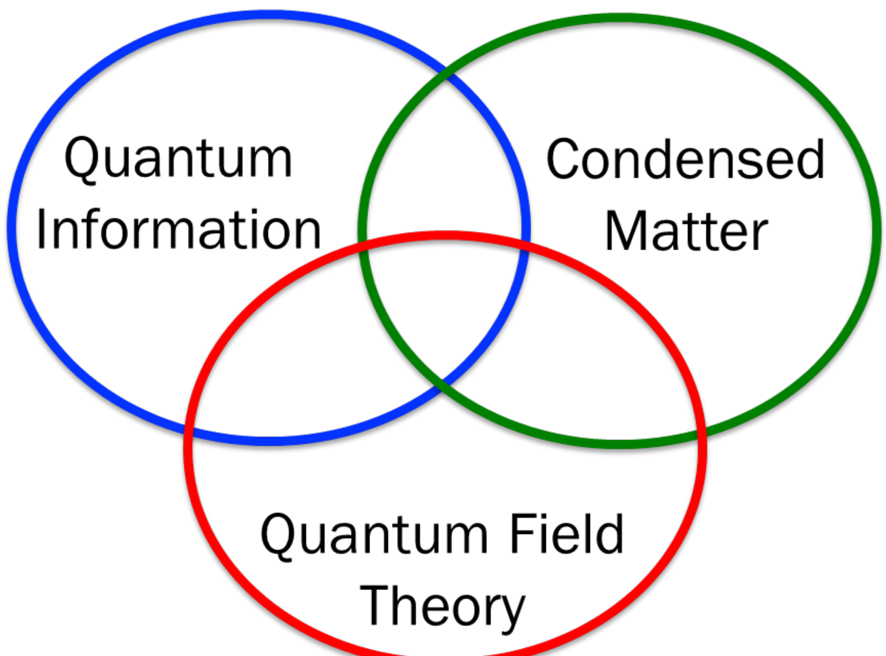
Y. Sui, Y. Ashida, and SF, in preparation

This talk

Quantum measurements provide a new control knob for quantum many-body systems.

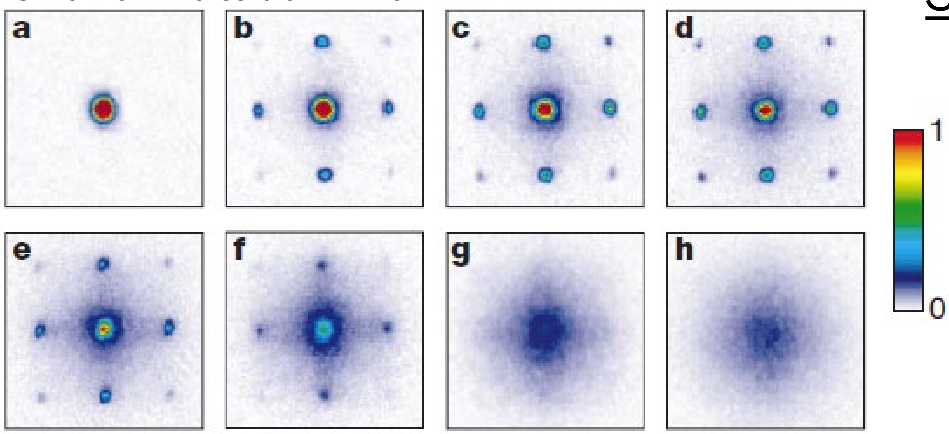
Apply quantum entanglement and boundary conformal field theory (CFT) to analyze phase transitions.

Find a behavior which is different from a naïve expectation from the g-theorem in CFT.

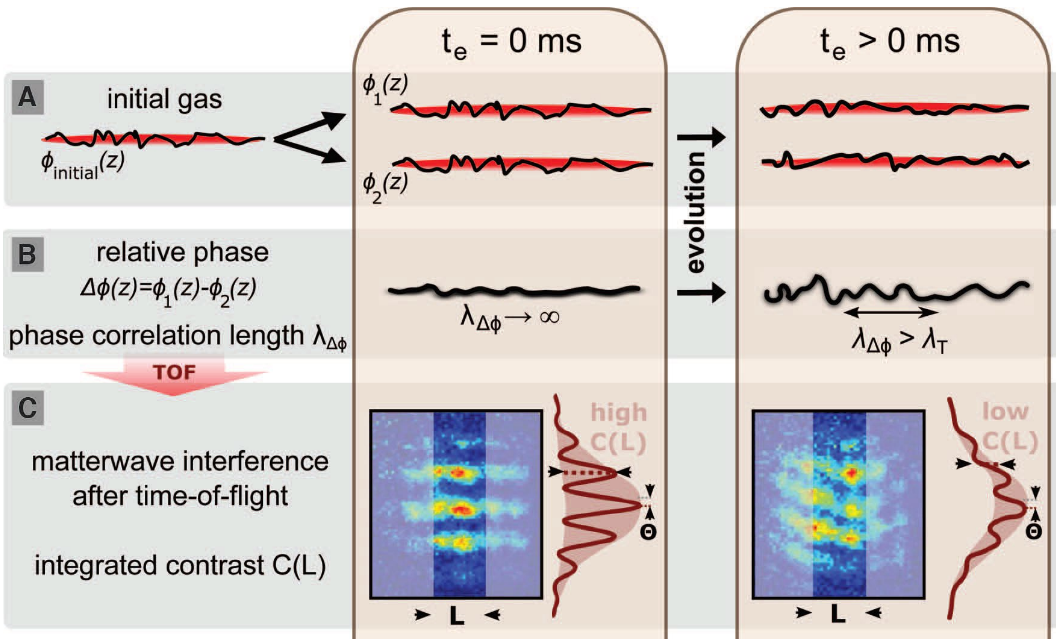


Quantum critical phenomena in ultracold atomic systems

Shallow lattice: BEC



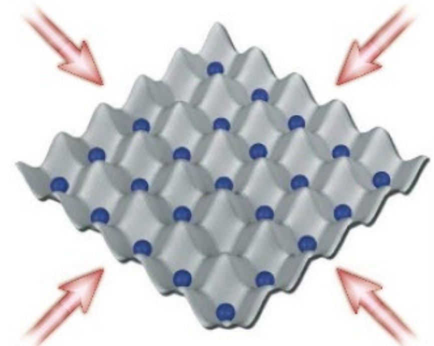
Deep lattice: Mott



<u>Superfluid-to-Mott-insulator transition</u>

Time-of-flight image: momentum distribution

M. Greiner et al., Nature **415**, 39 (2002)



Continuous observation

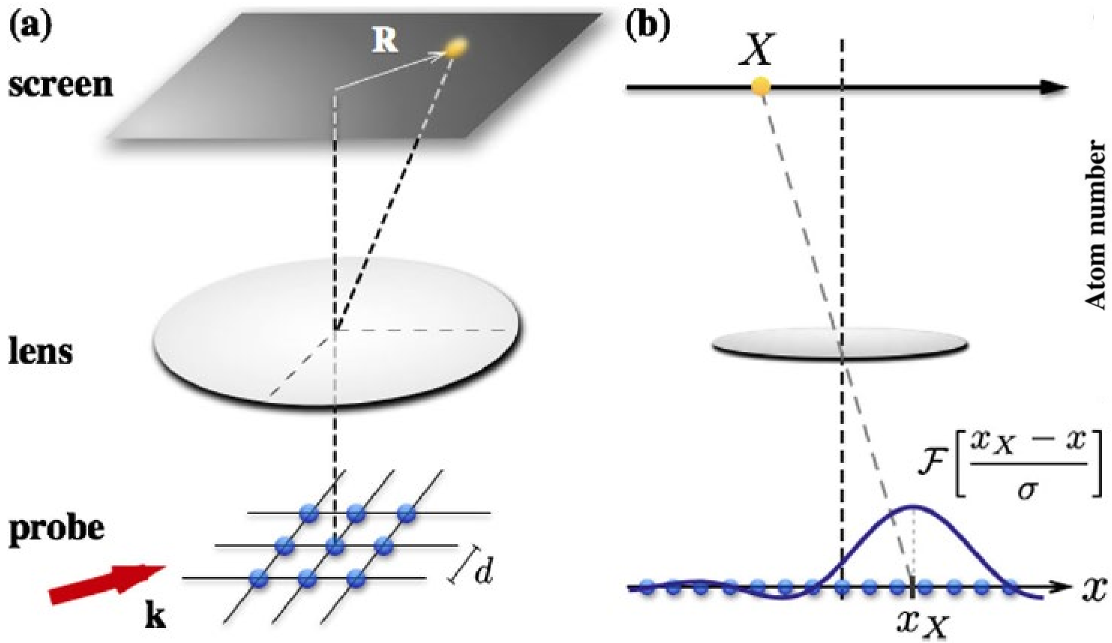
Non-unitary dynamics

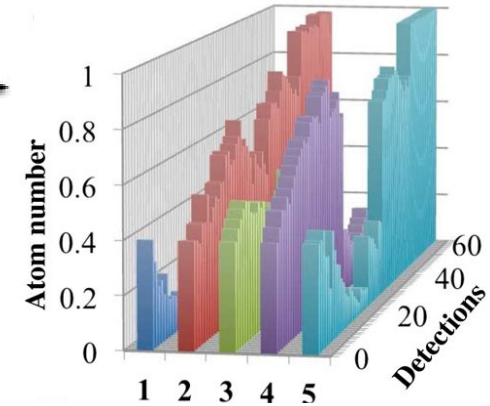
<u>Tomonaga-Luttinger liquid with spin-charge separation</u>

M. Gring et al., Science **337**, 1318 (2012)

Continuous monitoring of ultracold atoms in an optical lattice

Measurement backaction on the system





Y. Ashida and M. Ueda, PRL **115**, 095301 (2015); PRA **95**, 022124 (2017)

$$\alpha_3|00011\rangle + \alpha_5|00101\rangle + \cdots \longrightarrow \underline{|01001\rangle}$$

An "incomplete collapse" occurs in the process.

Occupation numbers at the sites

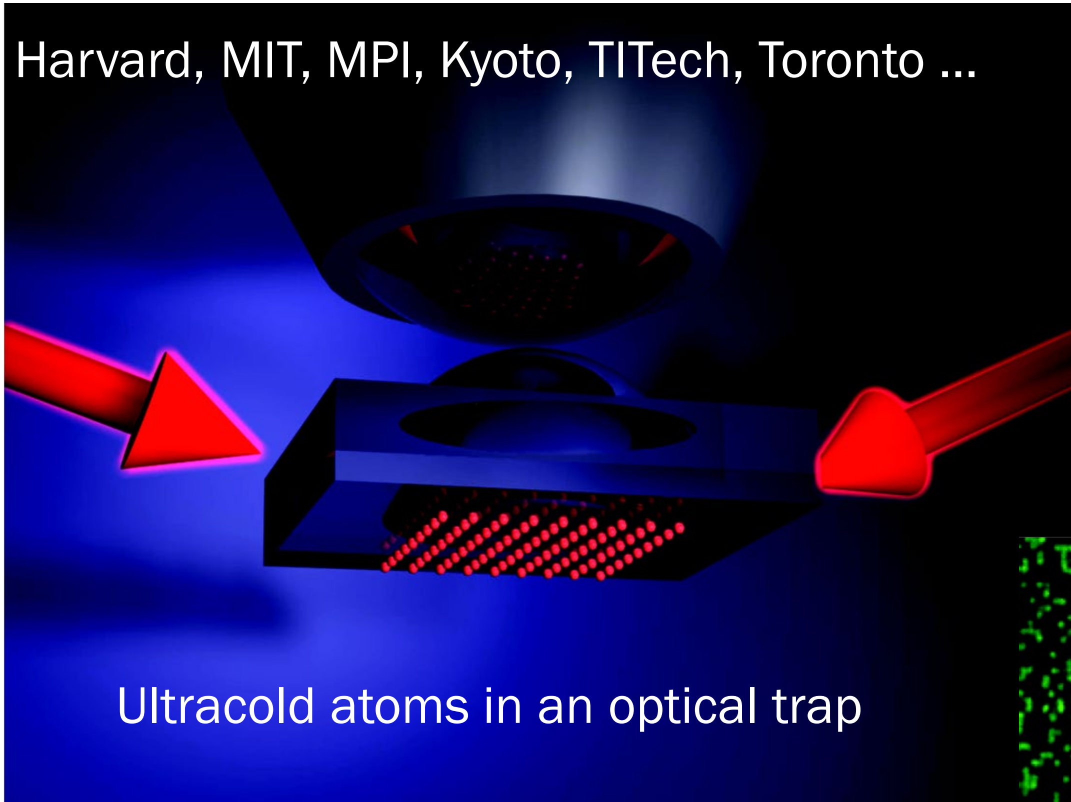
Limit of strong measurement (projection measurement)

$$|\Psi\rangle\langle\Psi| = \sum_{c,c'} \alpha_c \alpha_{c'}^* |c\rangle\langle c'| \longrightarrow \sum_c |\alpha_c|^2 |c\rangle\langle c| \quad \text{(averaged)}$$

Pure

Mixed (or entangled with env.)

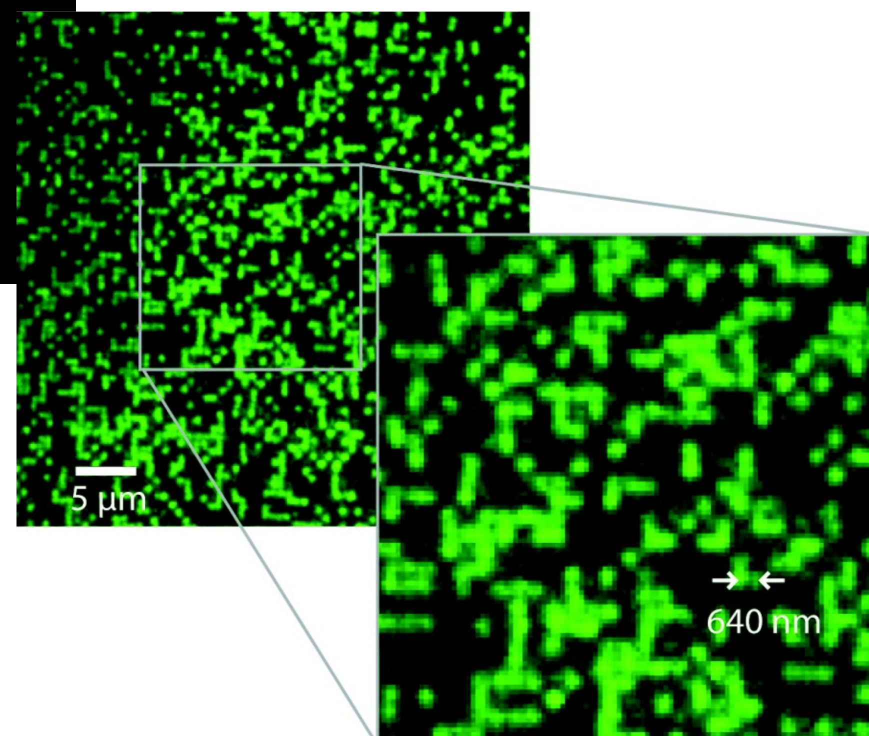
Quantum Gas Microscopy (QGM)



W. S. Bakr et al., Science **329**, 547 (2010).

Measuring quantum many-body systems at the **single-particle** and **single-site** level.

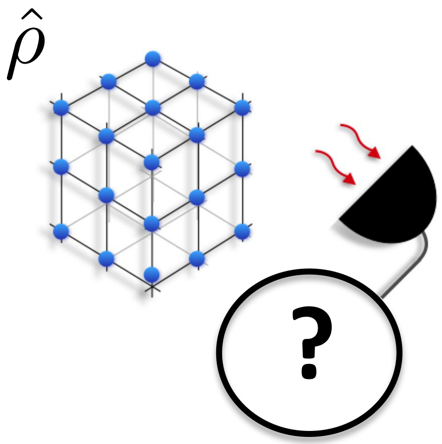
Measurement backaction should be significant.



So far, only strong single-shot measurements using a resonant probe light.

However, the technique can be extended for a weak continuous measurement using an off-resonant probe light.

Theoretical description of continuously monitored systems



Non-unitary dynamics due to the measurement backaction.

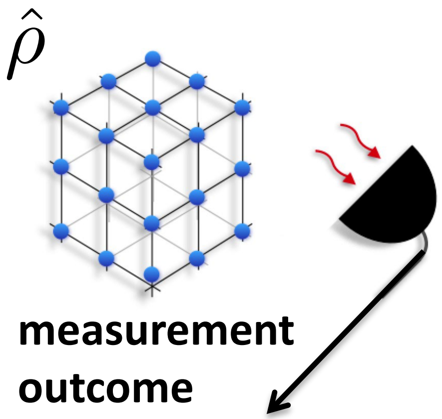
If one does not know measurement outcomes,

→ GKSL (Lindblad) master equation (**dissipative dynamics**)

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}, \hat{\rho}] - \sum_a \left(\frac{1}{2}\hat{L}_a^\dagger \hat{L}_a \hat{\rho} + \frac{1}{2}\hat{\rho}\hat{L}_a^\dagger \hat{L}_a - \hat{L}_a \hat{\rho} \hat{L}_a^\dagger \right)$$

$\hat{L}_a$: jump operator

Theoretical description of continuously monitored systems



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$\hat{L}_a$: jump operator

Continuous observation

If one can **access** the information about measurement outcomes,

→ **Quantum trajectory dynamics** (keeping the **purity** of the state)

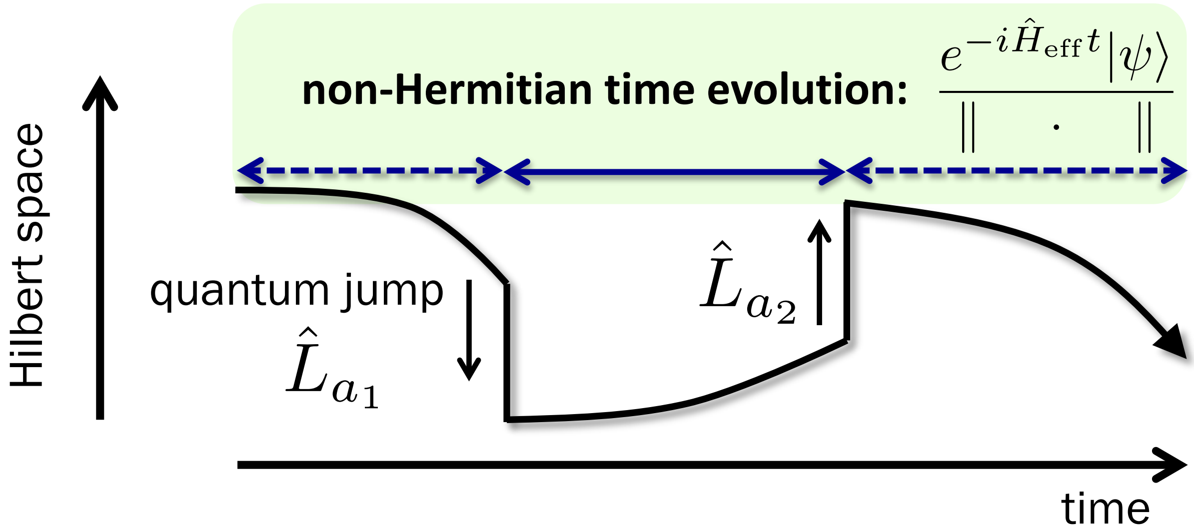
(norm preservation)

$$d|\psi\rangle = \underbrace{-i\hat{H}|\psi\rangle dt}_{\text{unitary evolution}} \underbrace{- \sum_a \frac{1}{2}\left(\hat{L}_a^\dagger \hat{L}_a - \langle \hat{L}_a^\dagger \hat{L}_a \rangle\right)|\psi\rangle dt}_{\text{non-unitary evolution without quantum jumps}} + \underbrace{\sum_a \left(\frac{\hat{L}_a |\psi\rangle}{\sqrt{\langle \hat{L}_a^\dagger \hat{L}_a \rangle}} - |\psi\rangle \right) dN_a}_{\text{stochastic quantum jumps}}$$

*Taking ensemble average E over dN_a, $\hat{\rho} = E[|\psi\rangle\langle\psi|]$ obeys Lindblad master eq.

Dalibard, Castin, Mølmer; Carmichael (1993)

Theoretical description of continuously monitored systems



(effective non-Hermitian Hamiltonian)

$$\hat{H}_{\rm eff} = \hat{H} - \frac{i}{2}\sum_a \hat{L}_a^\dagger \hat{L}_a$$

Continuous observation

If one can **access** the information about measurement outcomes,

$\longrightarrow$ **Quantum trajectory dynamics** (keeping the **purity** of the state)

(norm preservation)

$$d|\psi\rangle = \underbrace{-i\hat{H}|\psi\rangle dt}_{\text{unitary evolution}} \underbrace{- \sum_a \frac{1}{2}\left(\hat{L}_a^\dagger \hat{L}_a - \langle \hat{L}_a^\dagger \hat{L}_a\rangle\right)|\psi\rangle dt}_{\text{non-unitary evolution without quantum jumps}} + \underbrace{\sum_a \left(\frac{\hat{L}_a|\psi\rangle}{\sqrt{\langle \hat{L}_a^\dagger \hat{L}_a\rangle}} - |\psi\rangle\right) dN_a}_{\text{stochastic quantum jumps}}$$

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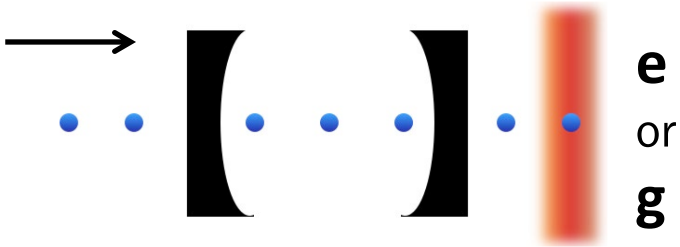
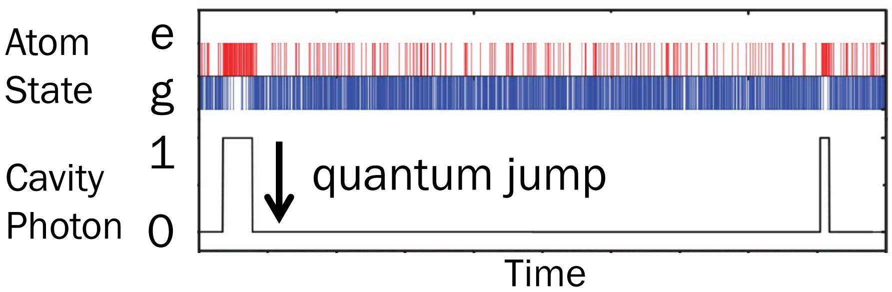
Dalibard, Castin, Mølmer; Carmichael (1993)

Experiments: Continuous observation of small quantum systems

Microwave cavity photon



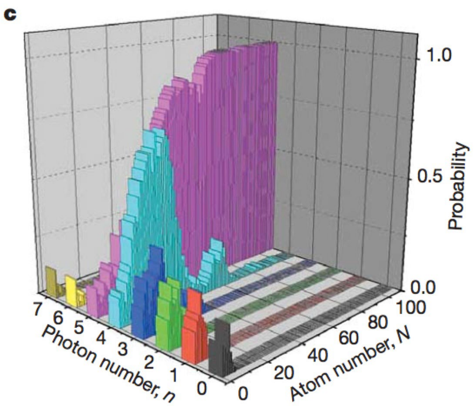
observation of quantum jumps



Gleyzes et al., Nature 446, 297 (2007)

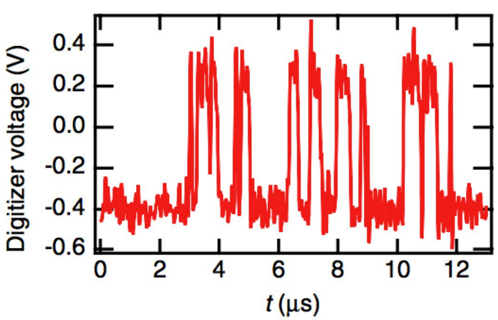
Serge Haroche
2012 Nobel Prize

observation of wavefunction collapse



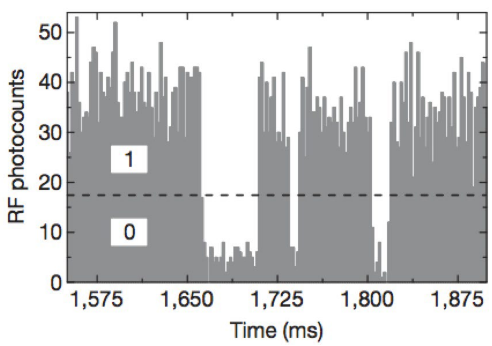
Guerlin et al., Nature 448, 889 (2007)

Superconducting qubit



Vijay et al., PRL 106, 110502 (2011)

Quantum dots



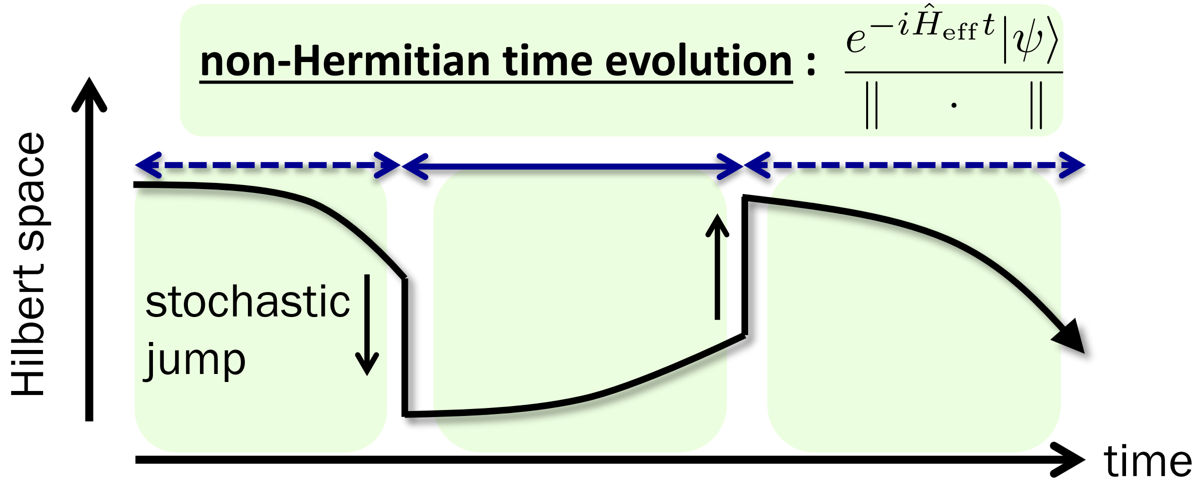
Vamivakas et al., Nature 467, 297 (2010)

Quantum gas microscopy for ultracold atoms

Continuously monitored many-body systems

How do we observe quantum criticality in continuous monitoring?

Quantum Trajectory Dynamics (Dalibard, Castin, Mølmer; Carmichael (1993))



□Average over stochastic jumps

J. Schachenmayer et al., PRA 89, 011601 (2014).
Y. Yanay and E. J. Mueller, PRA 90, 023611 (2014).

- Disspative dynamics described by Lindblad equation
- Quantum criticality is destroyed after long time.

□Postselection of data without jumps (typically, events with no loss of atoms)

□Gradually include non-Hemiticity to keep track of the ground state

→ Quantum critical behavior of non-Hermitian H_{eff} !!

Y. Ashida, SF, & M. Ueda, PRA **94**, 053615 (2016); Nat. Commun. 8, 15791 (2017)

<u>Problem</u>: Success prob. of postselection becomes exp. smaller for larger systems.

Alternative strategy: <u>Unconditioned</u> evolution over finite time

without postselection

- Start from a ground state at quantum criticality (e.g., Tomonaga-Luttinger liquid phase of lattice bosons)

$$\hat{\rho}_S = |\Psi_0\rangle\langle\Psi_0| \quad \text{(pure state)}$$

- Continuously monitor it over a finite time t while discarding the outcomes (Lindblad dynamics)

$$\hat{\rho}_{\mathcal{E}} = \mathcal{E}(\hat{\rho}_S), \quad \mathcal{E} = e^{\mathcal{L}t} \quad \text{(mixed state)}$$

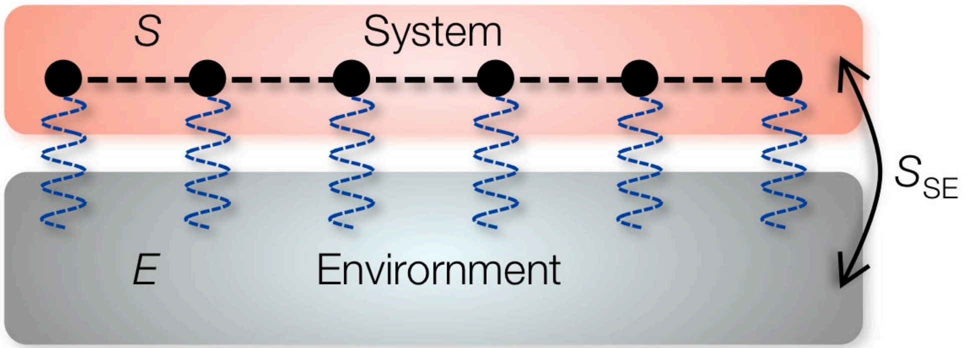
Notes: Stinespring rep. $\mathcal{E}(\hat{\rho}_S) = \mathrm{tr}_E\left[\hat{U}(\hat{\rho}_S \times \underbrace{\hat{\rho}_E}_{\text{pure}})\hat{U}^\dagger\right]$

- Study the system-environment entanglement

$$S_{SE}^{(n)} = \frac{1}{1-n}\ln \mathrm{tr}_S\left[\rho_{\mathcal{E}}^n\right]$$

Especially, n=2 case (purity of the system)

$$S_{SE} = -\log \mathrm{tr}\left[\hat{\rho}_{\mathcal{E}}^2\right]$$



Y. Ashida, SF, & M. Oshikawa, [PRB (2024)]

Equivalent CPTP (completely positive and trace preserving) map

- Kraus representation (generalized measurement)

$$\hat{\rho}_{\mathcal{E}} = \mathcal{E}(\hat{\rho}_S) \qquad \mathcal{E} = \prod_j \mathcal{E}_j, \quad \mathcal{E}_j(\cdot) = \sum_m \hat{K}_{m,j}(\cdot)\hat{K}^\dagger_{m,j}.$$

j site m measurement outcome

- Kraus operators for local density or phase measurements

$$\hat{K}_{0,j} = e^{-\gamma t}\hat{I}_j, \quad \hat{K}_{\pm,j} = \sqrt{1-e^{-2\gamma t}}\underbrace{\frac{1 \pm \hat{\sigma}_j^\alpha}{2}}_{\text{proj. op.}} \qquad 0 \le \gamma t < \infty, \quad \alpha \in \{x, y, z\}$$

$$\text{Note: } L_j = \sqrt{\gamma}\hat{\sigma}_j^\alpha$$

Site-resolved proj. measurement with prob. $1 - e^{-2\gamma t}$ and doing nothing otherwise

- Limit of strong measurement: $\mu \equiv \gamma t \to \infty$

$$\hat{\rho}_{\mathcal{E}} = \sum_c p_c |c\rangle\langle c|$$ (diagonal ensemble) c: spin configuration

$$p_c = |\langle c|\Psi_0\rangle|^2$$

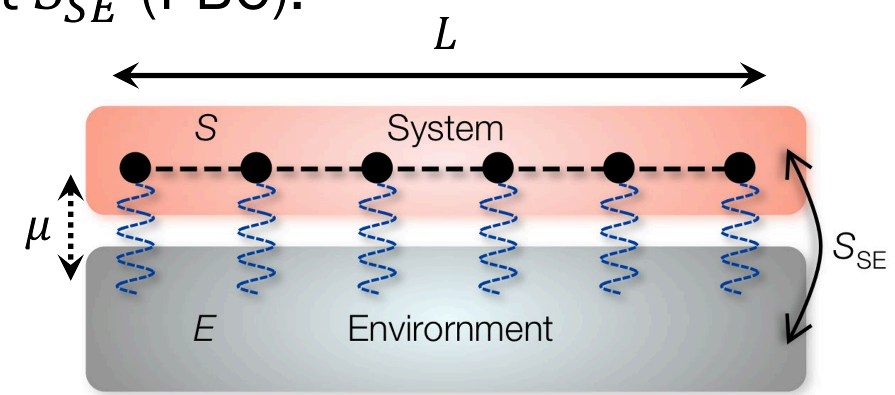
$$S_{\mathrm{SE}}^{(n)} = \frac{1}{1-n}\ln\left(\sum_c p_c^n\right)$$

<u>Field-theoretical and numerical studies on "participation entropy":</u>
Stephan, SF, Misguich, & Pasquier, PRB 80, 184421 (2009);
Oshikawa, arXiv:1007.3739 (2010);
Hsu & Fradkin, J. Stat. Mech. 2010, P09004

Main results: Universality and transitions in sys.-env. entanglement

✓ Scaling of sys.-env. entanglement S_{SE} (PBC):

$$S_{SE} = s_1 L - s_0 + \mathrm{o}(1)$$



s_1: nonuniversal coefficient (depending on UV cutoff)

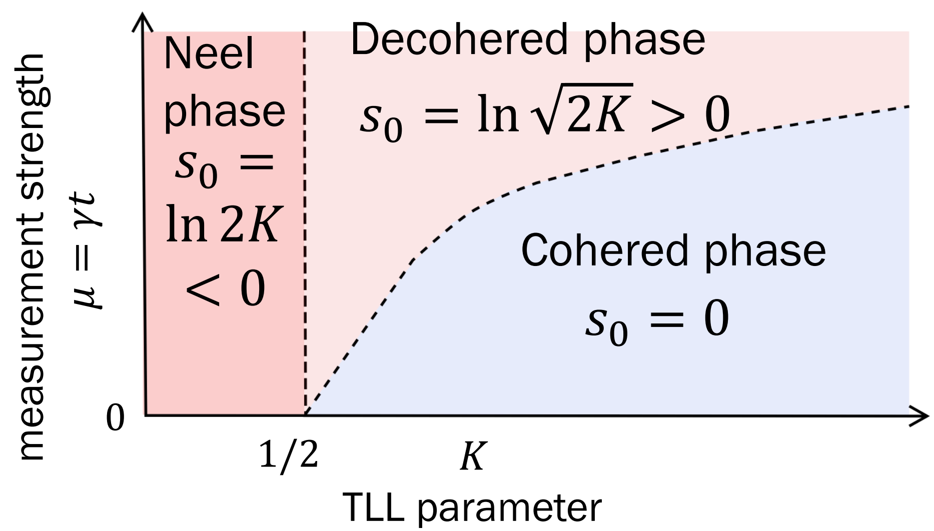
s_0: universal term (characterized by K)

$s_0 \neq 0 \Leftrightarrow$ CPTP map $\mathcal{E}$ is relevant in the RG sense.

$$s_0 = \ln \frac{g_{\mathcal{E}}}{g_{\mathcal{I}}}$$

Always non-positive due to the g-theorem?

Phase diagram (local density measurement)



✓ Entanglement transition (singular change of s_0)

Y. Ashida, SF, & M. Oshikawa, [Phys. Rev. B 110, 094404 (2024)]()

See also: Zou, Sang, Hsieh, PRL 130, 250403 (2023)

Doubled Hilbert space formulation

- Choi-Jamiolkowski representation

$$\hat{\rho}_S = |\Psi_0\rangle\langle\Psi_0| \longrightarrow |\rho_S) = |\Psi_0\rangle \otimes |\Psi_0^*\rangle$$

"ket" space "bra" space

- CPTP map

$$|\rho_\mathcal{E}) = \prod_j \Big(\sum_m \hat{K}_{m,j} \otimes \hat{K}^*_{m,j} \Big) |\rho_S) = \exp\Big\{ -\mu \Big[\sum_j (1 - \hat{\sigma}_j^\alpha \otimes \hat{\sigma}_j^\alpha) \Big] \Big\} |\rho_S)$$

site measurement outcome for local density/phase measurements

- System-environment entanglement

$$S_{\mathrm{SE}} = -\ln(\rho_\mathcal{E}|\rho_\mathcal{E})$$

$$(\rho_\mathcal{E}|\rho_\mathcal{E}) = \mathrm{tr}\left[\exp\left\{ -2\mu \sum_j (1 - \hat{\sigma}_j^\alpha \otimes \hat{\sigma}_j^\alpha) \right\} |\rho_S)(\rho_S| \right]$$

Effective field theory in a doubled Hilbert space

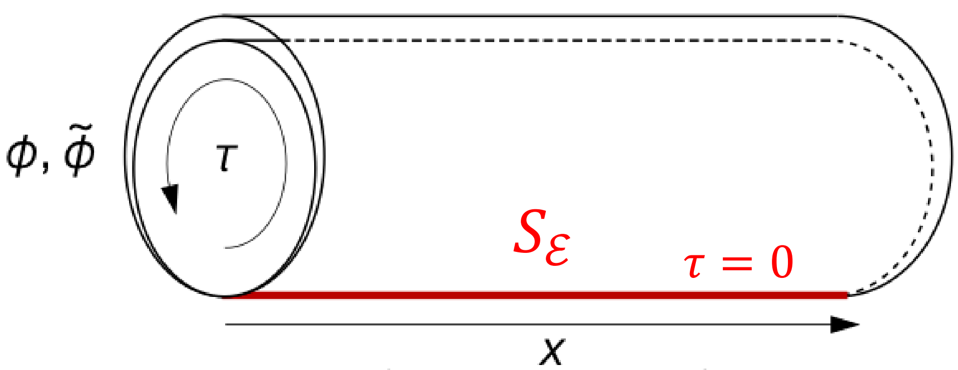
$$(\rho_{\mathcal{E}}|\rho_{\mathcal{E}}) = \mathrm{tr}\left[\exp\left\{-2\mu\sum_j(1-\hat{\sigma}_j^{\alpha}\otimes\hat{\sigma}_j^{\alpha})\right\}|\rho_S)(\rho_S|\right]$$

$$\begin{pmatrix} \hat{\sigma}_j^z & \simeq & \frac{2a}{\pi}\partial_x\hat{\phi} + c_1(-1)^j\cos(2\hat{\phi}), \\ \hat{\sigma}_j^+ & \simeq & e^{i\hat{\theta}}\left[c_2(-1)^j + c_3\cos(2\hat{\phi})\right], \end{pmatrix}$$

Path-integral representation: (1+1)D two-component scalar fields $(\phi, \tilde{\phi})$

$$(\rho_{\mathcal{E}}|\rho_{\mathcal{E}}) = \frac{Z_{\mathcal{E}}}{Z_{\mathcal{I}}} \qquad Z_{\mathcal{E}} = \int \mathcal{D}\phi\mathcal{D}\tilde{\phi}\, e^{-\mathcal{S}^{\mathcal{E}}_{\mathrm{tot}}[\phi,\tilde{\phi}]}$$

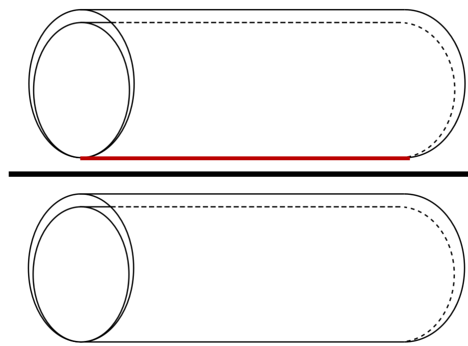
$$\mathcal{S}^{\mathcal{E}}_{\mathrm{tot}}[\phi,\tilde{\phi}] \equiv \underbrace{\mathcal{S}_0[\phi] + \mathcal{S}_0[\tilde{\phi}]}_{\text{Bulk CFT for the doubled space}} + \underbrace{\mathcal{S}_{\mathcal{E}}[\phi,\tilde{\phi}]}_{\text{Boundary interaction induced by CPTP map } \mathcal{E}}$$

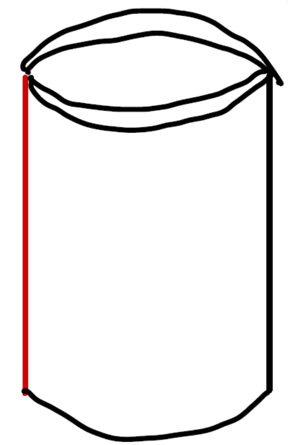


$Z_{\mathcal{I}}$: partition function w/o boundary int.

see also: Lee, Jian, Xu, PRX Quantum 4, 030317 (2023); Bao, Fan, Vishwanath, Altman, arXiv:2301.05687 (2023)

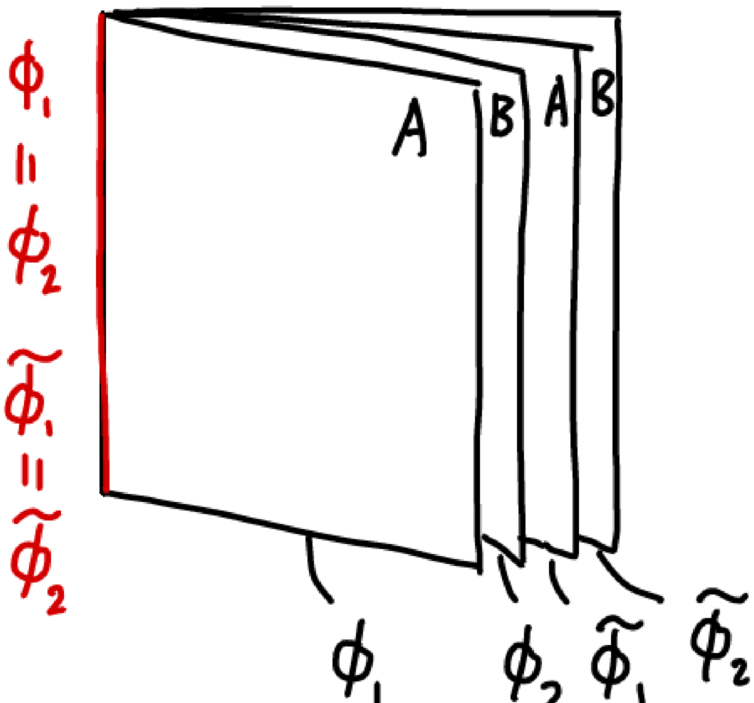
Effective field theory in a doubled Hilbert space

$S_{SE} = -\log$ 



Folding

cf. Book boundary condition



$\mathcal{E}$: relevant

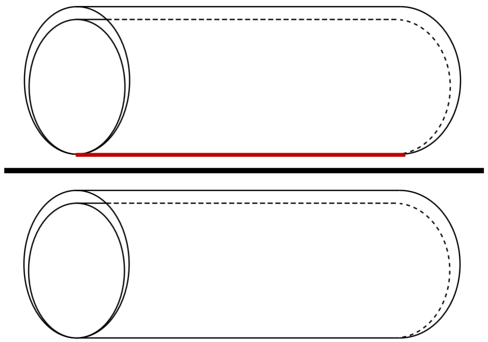
$\phi_1 = \tilde{\phi}_1$

$\mathcal{E}$: irrelevant ($\to \mathcal{I}$)

$\phi_{1,2}$ & $\tilde{\phi}_{1,2}$ are independent

Effective field theory in a doubled Hilbert space

$S_{SE} = -\log$



Each partition function can be evaluated by boundary CFT:

$$\log Z_\xi = b_\xi L + \underbrace{\log g_\xi}_{} + \mathrm{o}(1), \quad \xi \in \{\mathcal{I}, \mathcal{E}\}$$

$\boldsymbol{g}$ function: boundary "ground-state degeneracy"

Affleck and Ludwig, PRL 67, 161 (1991)

$$S_{SE} = s_1 L - s_0 + \mathrm{o}(1) \qquad s_0 = \ln \frac{g_{\mathcal{E}}}{g_{\mathcal{I}}}$$

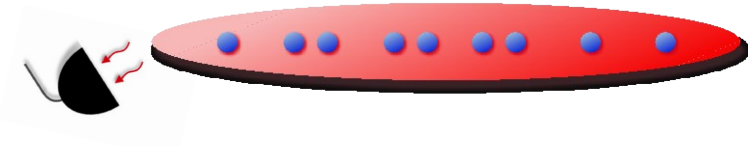
- Universality of $\boldsymbol{S_{SE}}$ ⇔ Universality of $\boldsymbol{g}$ function
- $\boldsymbol{g}$-theorem: $\boldsymbol{g}$ monotonically decreases under the boundary RG flow

→ s_0 is always non-positive?

Friedan & Konechny (2004);
Casini, Landea, & Torroba (2016)

Application to Tomonaga-Luttinger liquid under local measurement

Consider TLL under local density measurement



Effective field theory

$$\mathcal{S}_{\rm tot}[\phi_+,\phi_-] = \mathcal{S}_0[\phi_+] + \mathcal{S}_0[\phi_-] + \mathcal{S}_{\mathcal{E}}[\phi_+,\phi_-]$$

$$\phi_+ = 2\left(\phi + \tilde{\phi}\right)$$

$$\phi_- = 2\left(\phi - \tilde{\phi}\right)$$

bulk action (Gaussian): $\mathcal{S}_0[\phi_\pm] = \int dx d\tau \frac{1}{16\pi K}\left[(\partial_x\phi_\pm)^2 + (\partial_\tau\phi_\pm)^2\right]$

boundary interaction: $\mathcal{S}_{\mathcal{E}} \simeq \int dx d\tau\, \delta(\tau)\left[\boxed{\frac{\gamma}{\Lambda_0}(\partial_x\phi_-)^2} - u_-\Lambda_0 \cos(\phi_-)\right]$

$\gamma, u_- \propto \mu$ at UV scale Λ_0: UV cutoff

Careful treatment necessary for γ :

it can be relevant in nonperturbative regimes despite being perturbatively irrelevant (i.e., it is dangerously irrelevant)

cf. Masuki, Sudo, Oshikawa, Ashida., PRL 129, 087001 (2022);
Daviet and Dupuis, PRB 108, 184514 (2023)

Call for nonperturbative RG analysis

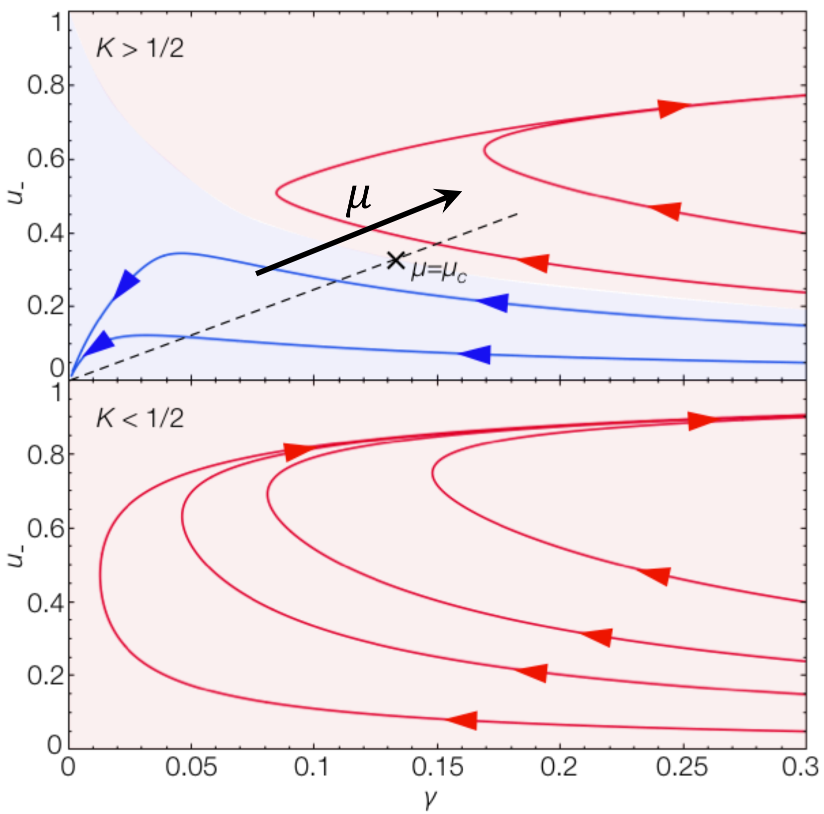
see also: Garratt, Weinstein, Altman, PRX 13, 021026 (2023) for conditioned evolution of TLL under local meas.

Tomonaga-Luttinger liquid under local measurement

$$\mathcal{S}_{\mathcal{E}} \simeq \int dx d\tau\, \delta(\tau) \left[\frac{\gamma}{\Lambda_0} (\partial_x \phi_-)^2 - u_- \Lambda_0 \cos(\phi_-) \right] - u_+ \Lambda_0 \cos \phi_+$$

$\gamma, u_- \propto \mu$ at UV scale

Functional RG results



<u>$K > 1/2$</u>

$\mu > \mu_c$: u_- diverges in IR limit

$\rightarrow \phi_-$ obeys Dirichlet b.c.

$\mu < \mu_c$: u_- is irrelevant

$\rightarrow \phi_-$ obeys Neumann b.c.

Entanglement transition induced by environment

<u>$K < 1/2$</u>

$u_\pm$ diverge at any $\mu > 0$

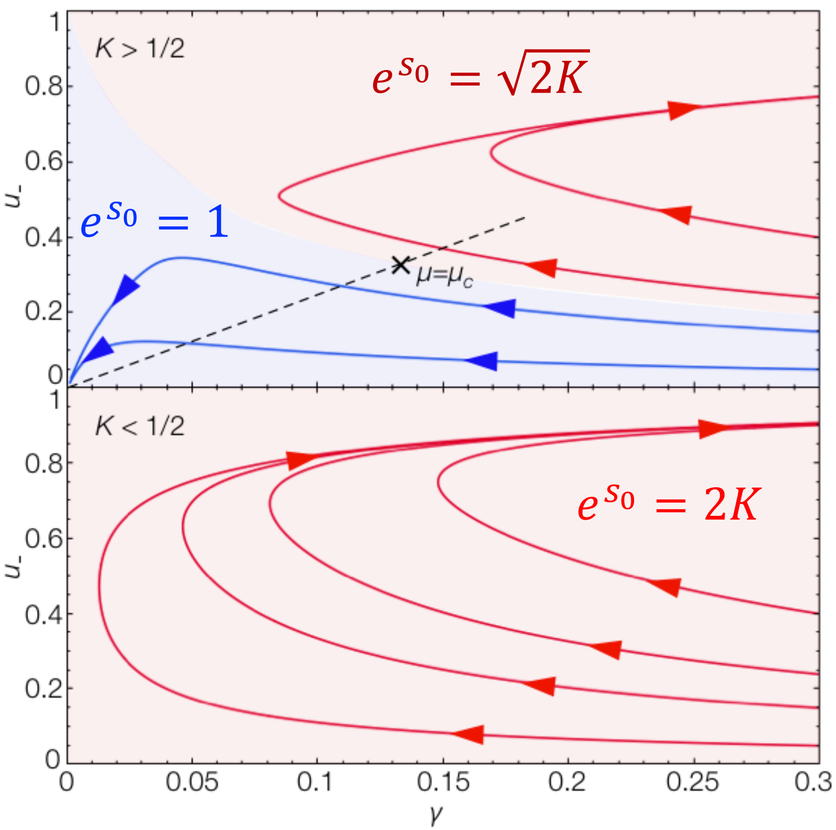
$\rightarrow \phi_\pm$ obey Dirichlet b.c.'s

Arbitrarily weak coupling can affect entanglement in IR limit

In either case, we have <u>a mixed Dirichlet-Neumann boundary condition</u> in the IR limit.

Boundary CFT (BCFT) calculation of the g function

RG phase diagram



- ***g*** function for the mixed Dirichlet-Neumann b.c.

$$g_\Gamma = \sqrt{v_0(\mathcal{T}_\Gamma)\, v_0(\mathcal{T}_\Gamma^*)}.$$

Unit-cell volume of the compactification lattice consistent with b.c.

Oshikawa, arXiv:1007.3739 (2010);
Hsu & Fradkin, J. Stat. Mech. 2010, P09004;
Furukawa & Kim, PRB 83, 085112 (2011)

- BCFT results of TLL under density measurement:

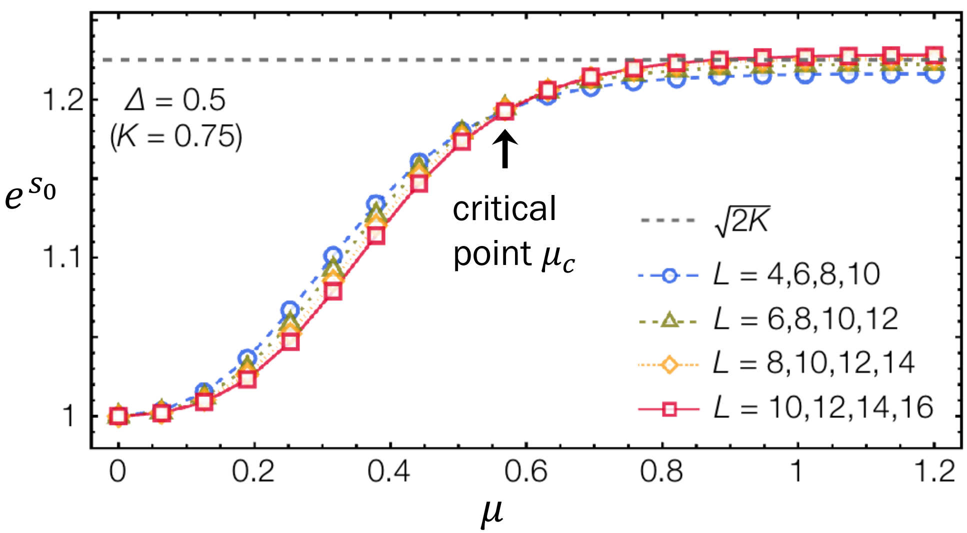
$$e^{s_0} = \frac{Z_\mathcal{E}}{Z_\mathcal{I}} \approx \frac{g_\mathcal{E}}{g_\mathcal{I}} = \begin{cases} \sqrt{2K} & \mu > \mu_c,\ K > 1/2 \\ 1 & \mu < \mu_c,\ K > 1/2 \\ 2K & \forall \mu > 0,\ K < 1/2 \end{cases}$$

Numerical check in the XXZ chain

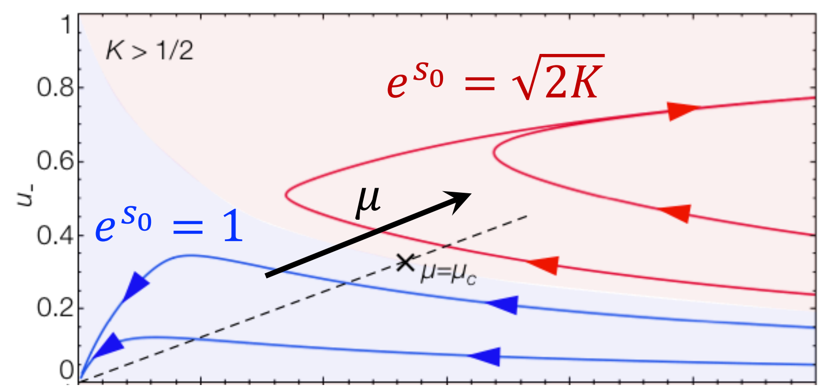
Exact diagonalization of $\hat{H}_{\mathrm{XXZ}} = J\sum_{i=1}^{L}\left(\hat{\sigma}_i^x\hat{\sigma}_{i+1}^x + \hat{\sigma}_i^y\hat{\sigma}_{i+1}^y + \Delta\hat{\sigma}_i^z\hat{\sigma}_{i+1}^z\right)$ $\quad -1 < \Delta \leq 1$ $\quad (K \to \infty, \; K = \frac{1}{2})$

→ directly calculate $|\rho_{\mathcal{E}}) = \exp\left\{-\mu\left[\sum_j (1 - \hat{\sigma}_j^z \otimes \hat{\sigma}_j^z)\right]\right\}|\mathrm{GS}\rangle\otimes|\mathrm{GS}\rangle$ and $S_{SE} = -\ln(\rho_{\mathcal{E}}|\rho_{\mathcal{E}})$

→ determine s_0 numerically by fitting S_{SE} to $S_{SE} = s_1 L - s_0 + \frac{s_{-1}}{L}$



cf. RG + BCFT analysis:



✓ transition at critical point $\mu = \mu_c$

✓ convergence to $\sqrt{2K}$ at $\mu > \mu_c$

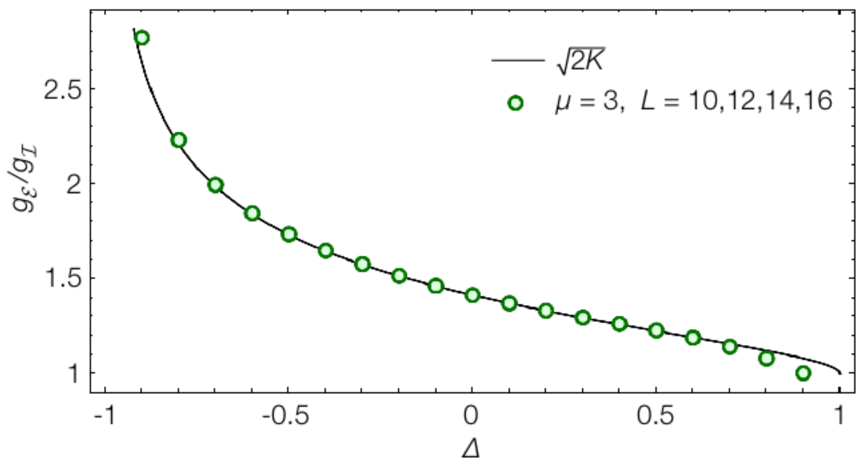
Consistent with fRG and BCFT results

Numerical check in the XXZ chain

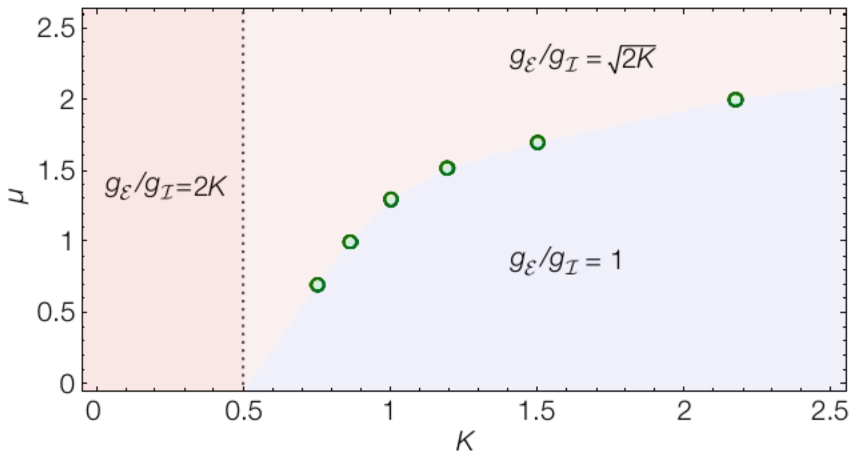
Exact diagonalization of $\hat{H}_{\mathrm{XXZ}} = J \sum_{i=1}^{L} \left(\hat{\sigma}_i^x \hat{\sigma}_{i+1}^x + \hat{\sigma}_i^y \hat{\sigma}_{i+1}^y + \Delta \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z \right)$ $\quad -1 < \Delta \leq 1$

$K \to \infty \qquad K = \frac{1}{2}$

Converged values of e^{s_0} with a varying Δ



Numerical phase diagram



✓ consistent with Bethe-ansatz result:

$$K = \frac{\pi}{2\left(\pi - \cos^{-1}\Delta\right)}$$

✓ qualitatively consistent with fRG analysis

Numerical results agree with the field-theoretical analysis despite small system sizes.

Case of open boundary conditions

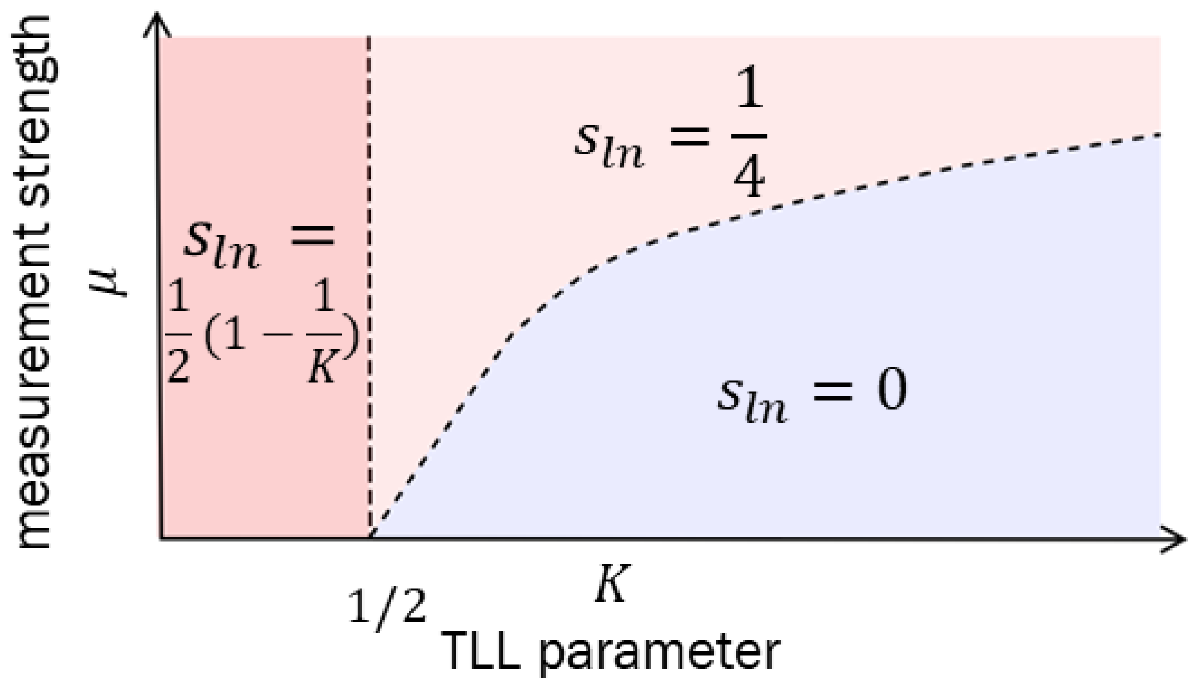
$$S_{SE} = s_1' L - s_{\ln} \ln(L)$$

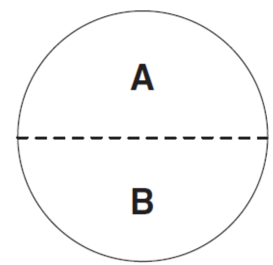
Case of participation entropy:

J.M. Stéphan, G. Misguich, and V. Pasquier, Phys. Rev. B 84, 195128 (2011)

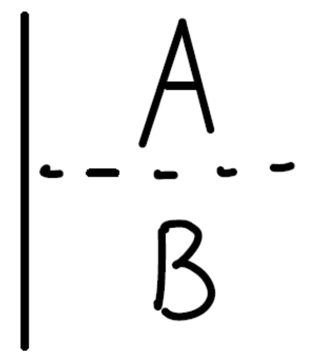
cf. Cardy & Peschel, Nucl. Phys. (1988)
Fradkin & Moore, PRL (2006)

$$F_A = f_b|A| + f_s L - \frac{c\chi}{6}\log L + O(1),$$

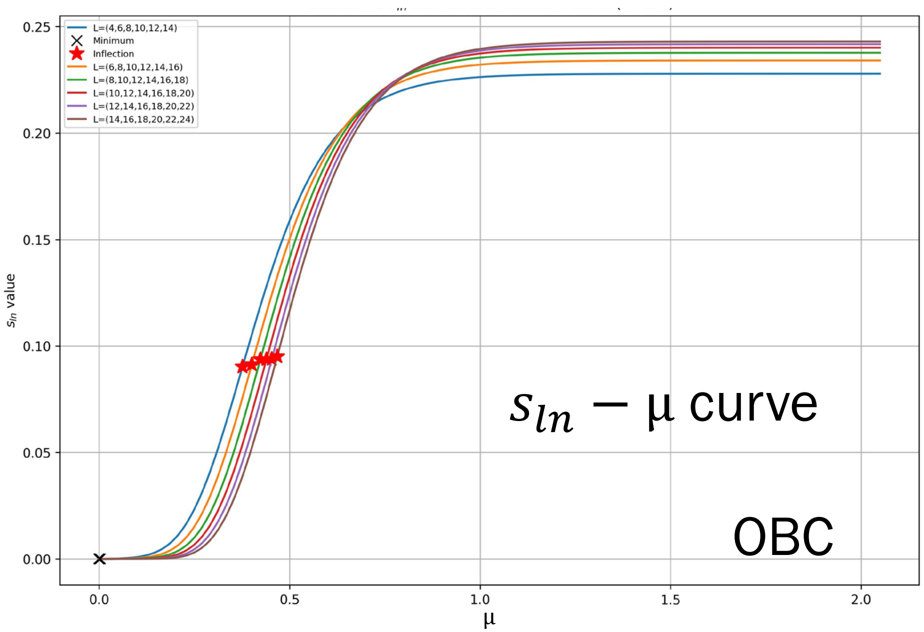


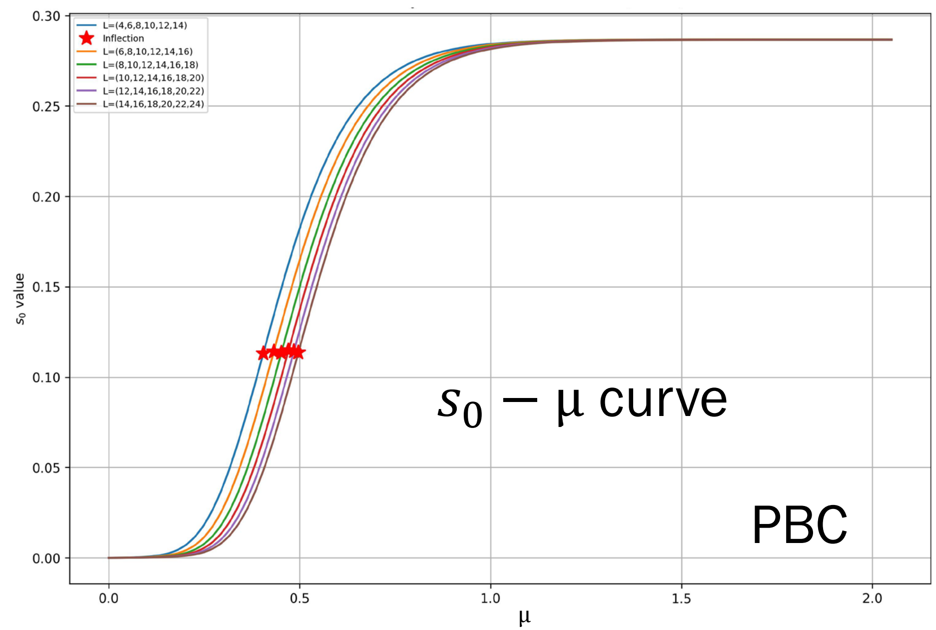


$$S_{\log} = -\frac{c\log R}{4}$$



Numerical results: OBC vs. PBC

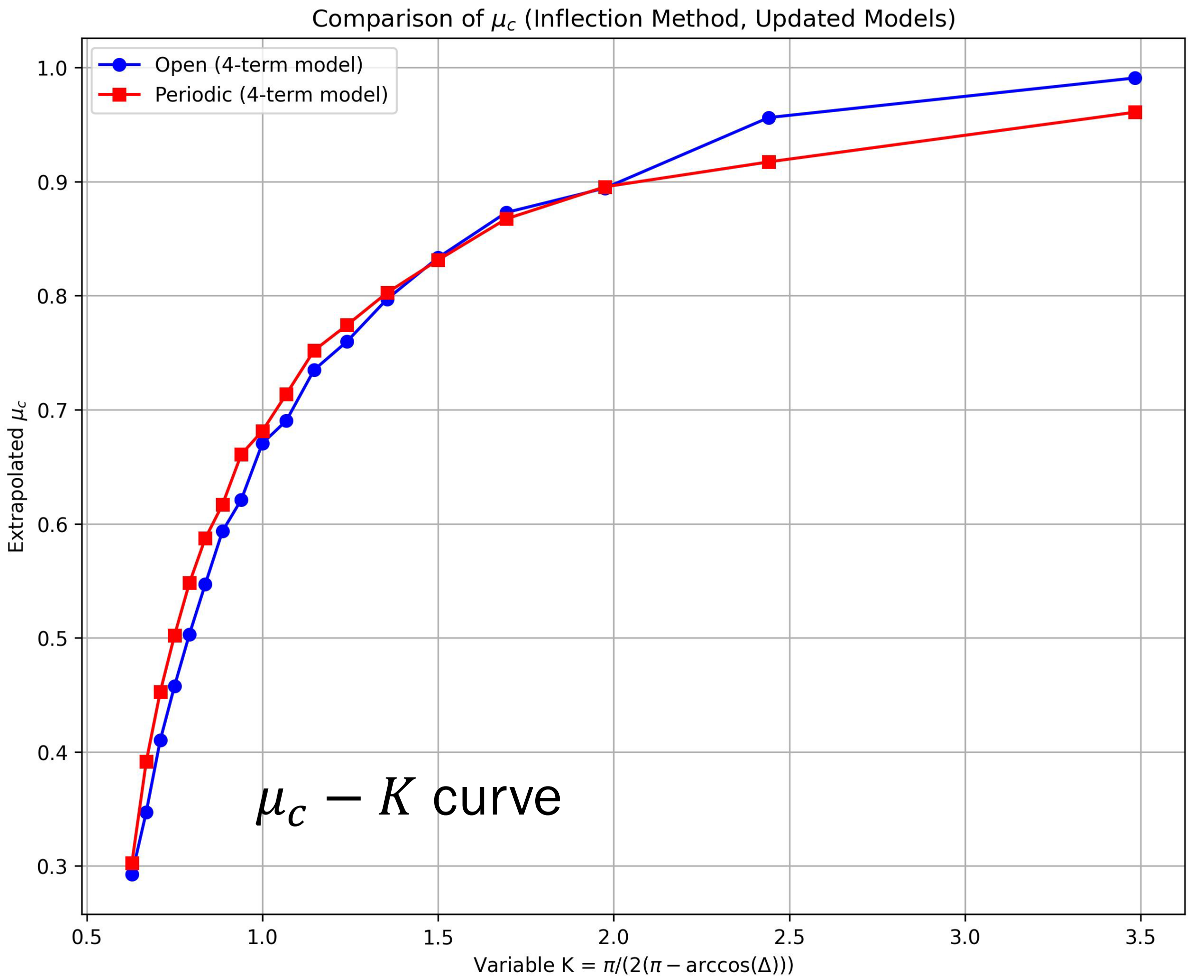




$$S_{SE}(L) = s_1 L - \mathrm{s_{ln}} \ln(\mathrm{L}) - \mathrm{s}_0 + \mathrm{s}_{-1}\frac{1}{L}$$

$$S_{SE}(L) = s_1' L - \mathrm{s}_0 + \mathrm{s}_{-1}\frac{1}{L} + s_{-2}\frac{1}{L^2}$$

Comparision of μ_c in OBC and PBC



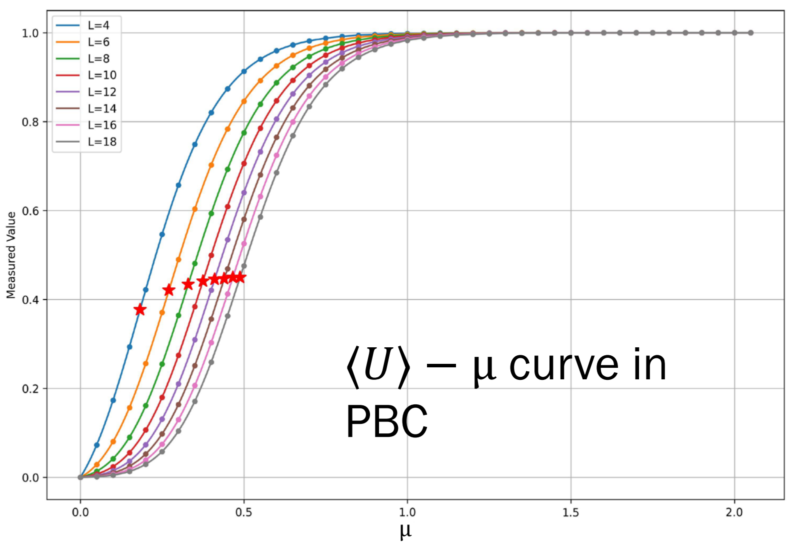
Another method: Twist operator

Twist operator $U = e^{\frac{i2\pi}{L}\sum_j j\left(\sigma_j^z \otimes I_j - I_j \otimes \sigma_j^z\right)}$ can detect the phase transition in PBC in double Hilbert space

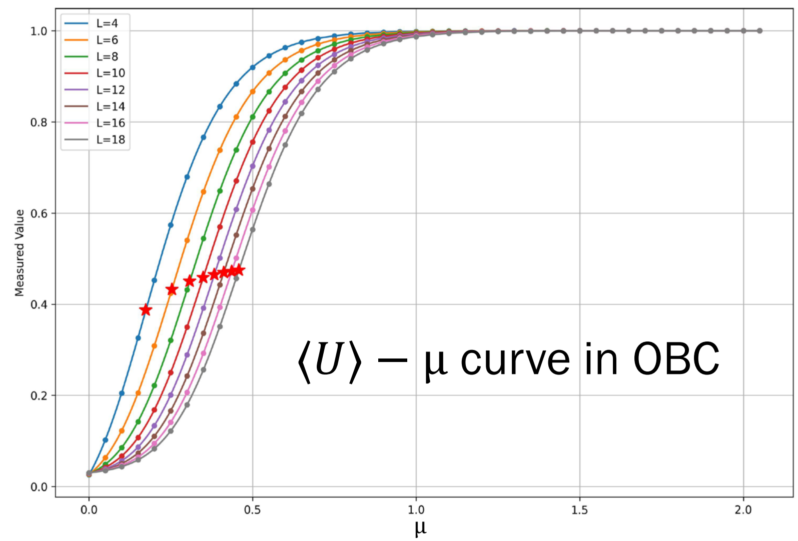
$$\langle U \rangle \equiv \frac{\langle \rho_\varepsilon | U | \rho_\varepsilon \rangle}{\langle \rho_\varepsilon | \rho_\varepsilon \rangle} \qquad U \sim \mathrm{e}^{i\phi_-}$$

cf. Nakamura & Todo, PRL (2002)

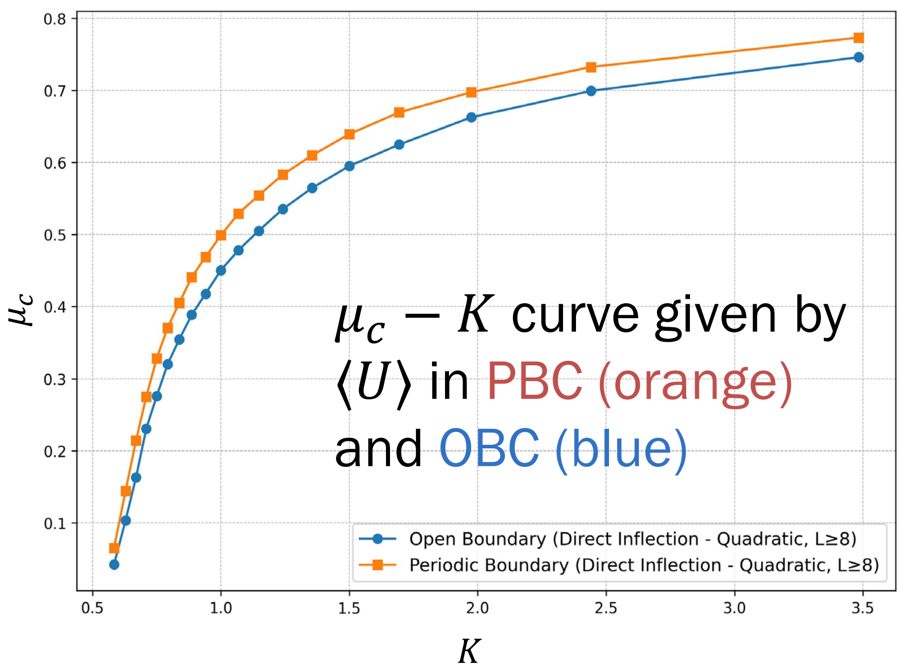
$\Delta = -0.7$

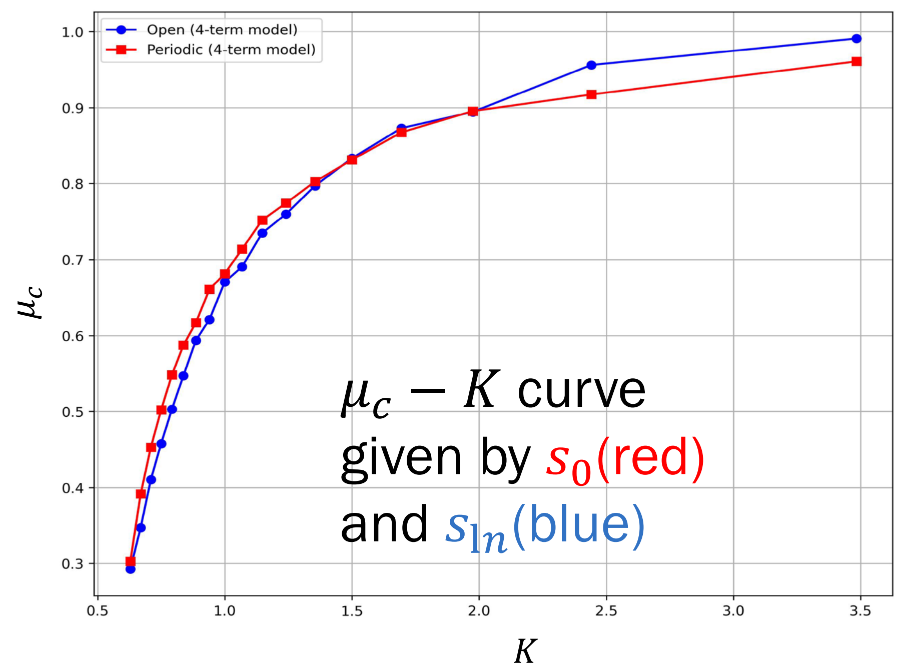


$\Delta = -0.7$



Comparision of μ_c in different methods

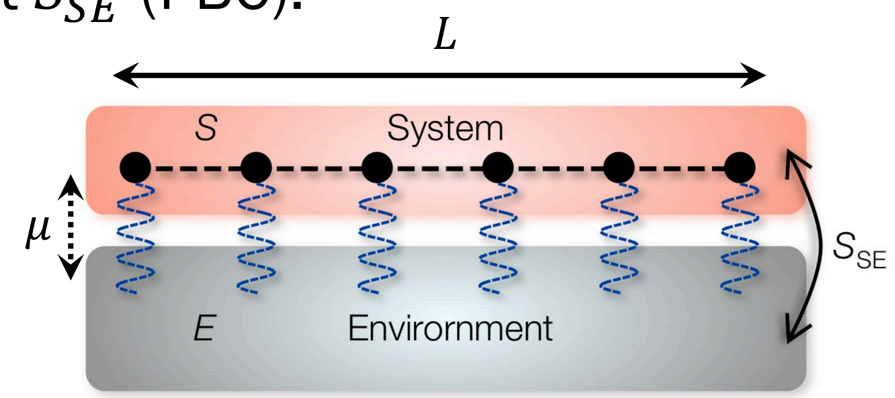




Main results: Universality and transitions in sys.-env. entanglement

✓ Scaling of sys.-env. entanglement S_{SE} (PBC):

$$S_{SE} = s_1 L - s_0 + \mathrm{o}(1)$$



s_1: nonuniversal coefficient (depending on UV cutoff)

s_0 : universal term (characterized by K)

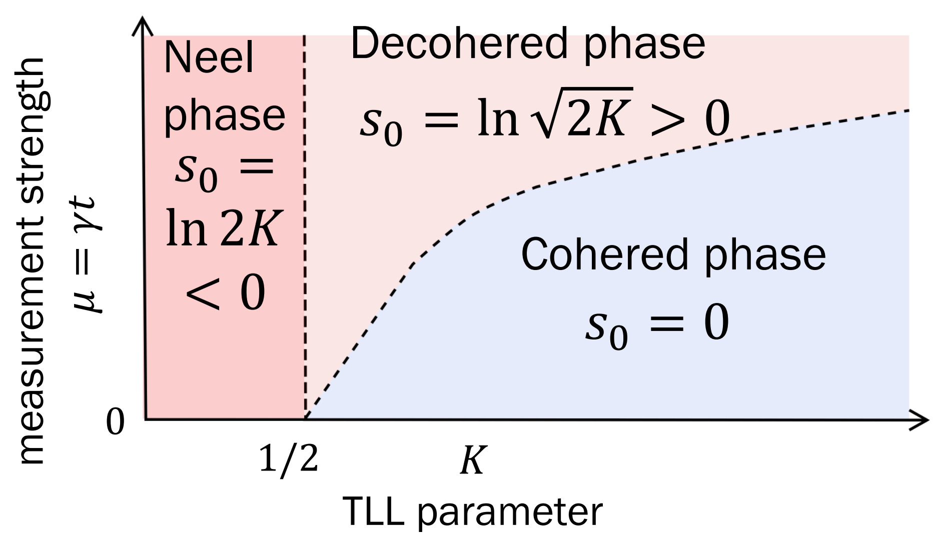
$s_0 \neq 0 \Leftrightarrow$ CPTP map $\mathcal{E}$ is relevant in the RG sense.

$$s_0 = \ln \frac{g_{\mathcal{E}}}{g_{\mathcal{I}}}$$

Always non-positive due to the g-theorem?

✓ Entanglement transition (singular change of s_0)

Phase diagram (local density measurement)



Y. Ashida, SF, & M. Oshikawa, Phys. Rev. B 110, 094404 (2024)

See also: Zou, Sang, Hsieh, PRL 130, 250403 (2023)