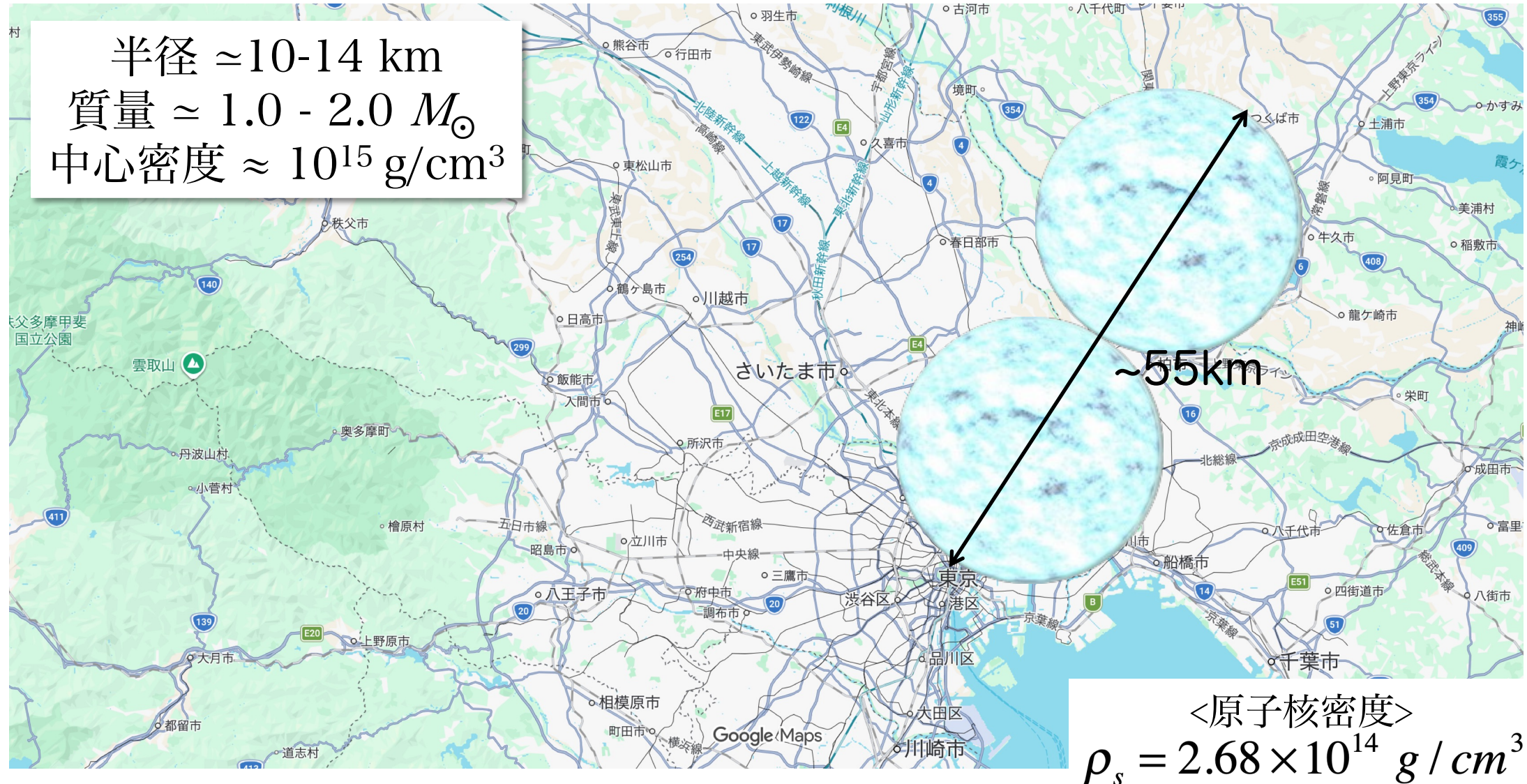


星震学で探る中性子星の物理

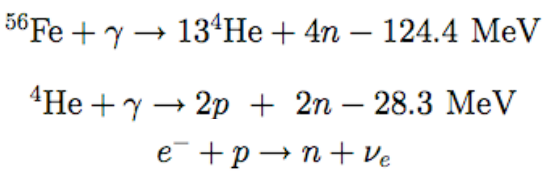
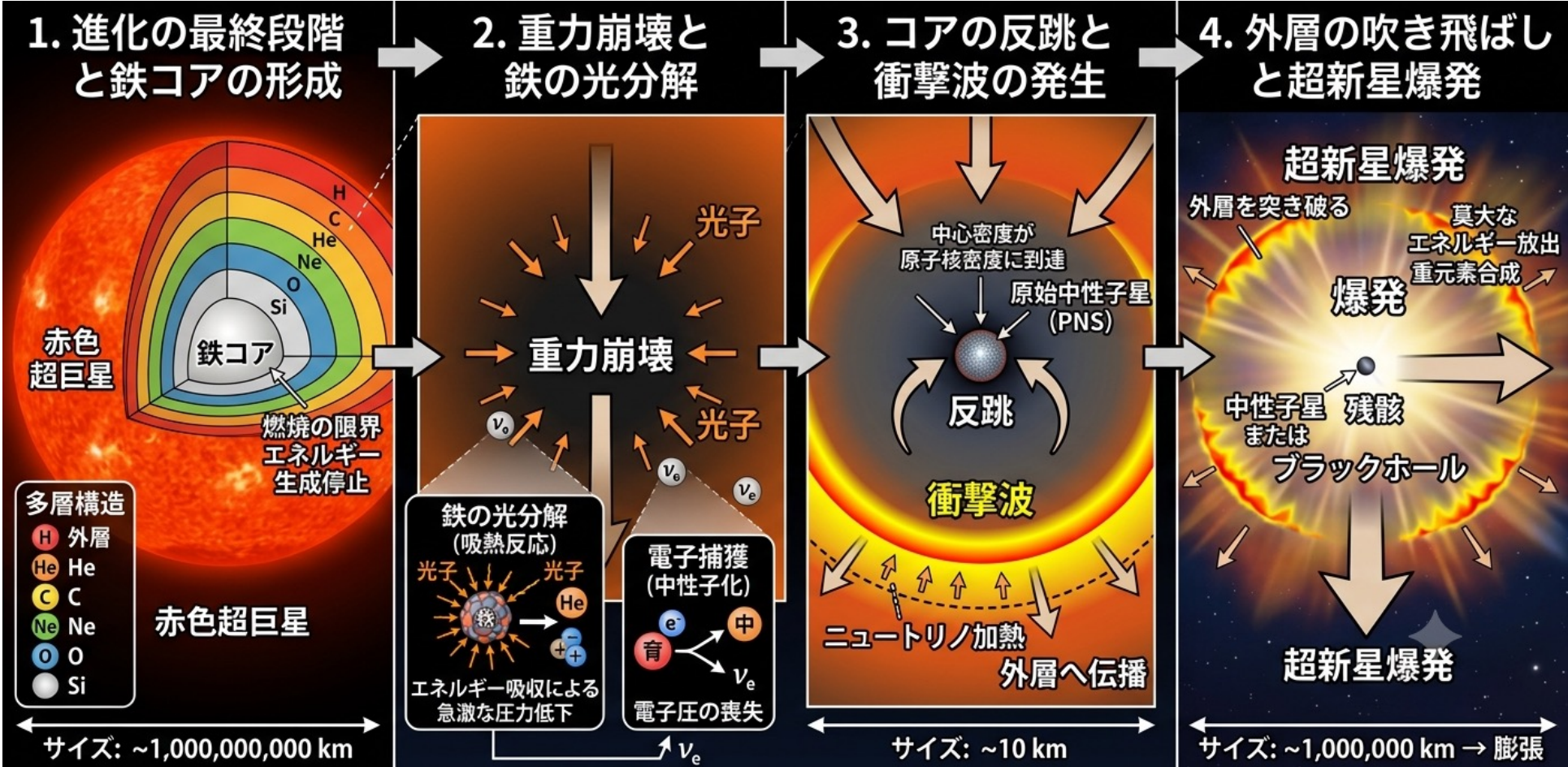
祖谷 元（高知大学）



中性子星のサイズ



重い星の最期 = 超新星爆発



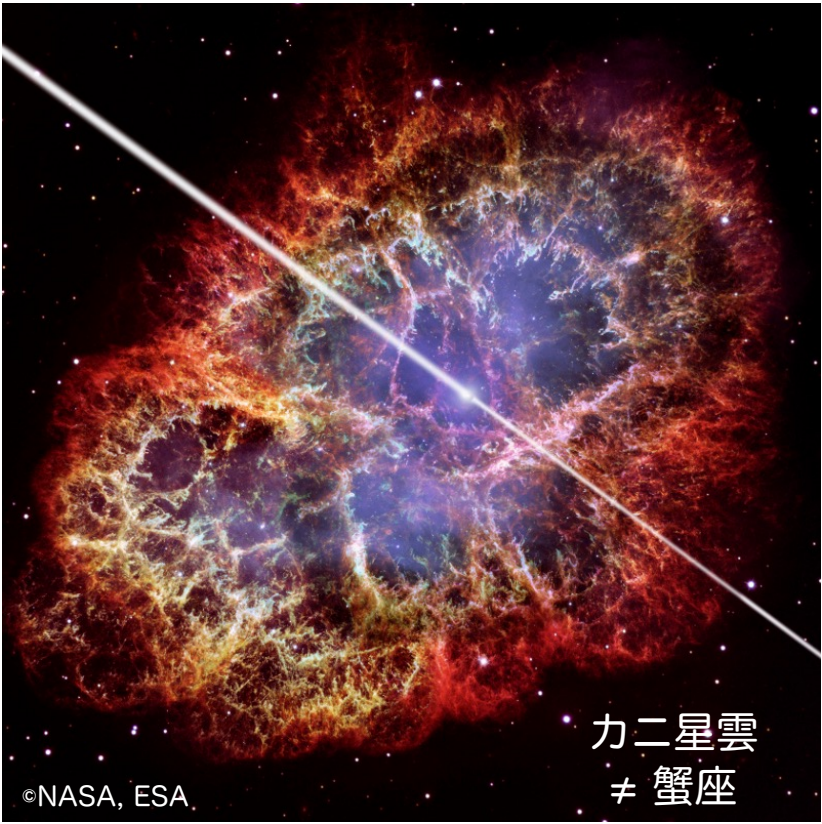
- 10倍の太陽質量より重い星の最期の瞬間
- 中心密度が原子核密度に達すると“コアバウンス”が起こる
- 衝撃波が星表面まで伝わると爆発！

超新星爆発と中性子星

- 重力崩壊の結果中性子星ができたという直接的な観測はまだない（1987Aが最初の例？）
- 間接的な証拠はある 例) カニ星雲の中のカニパルサー



明月記
「客星出現例」



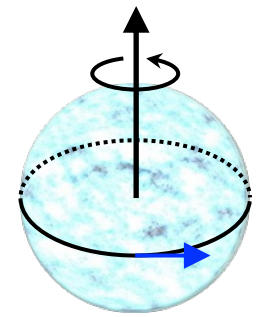
カニ星雲
≠ 蟹座



藤原定家
(1162-1241)

1180-1235 (56年間)

中性子星の発見

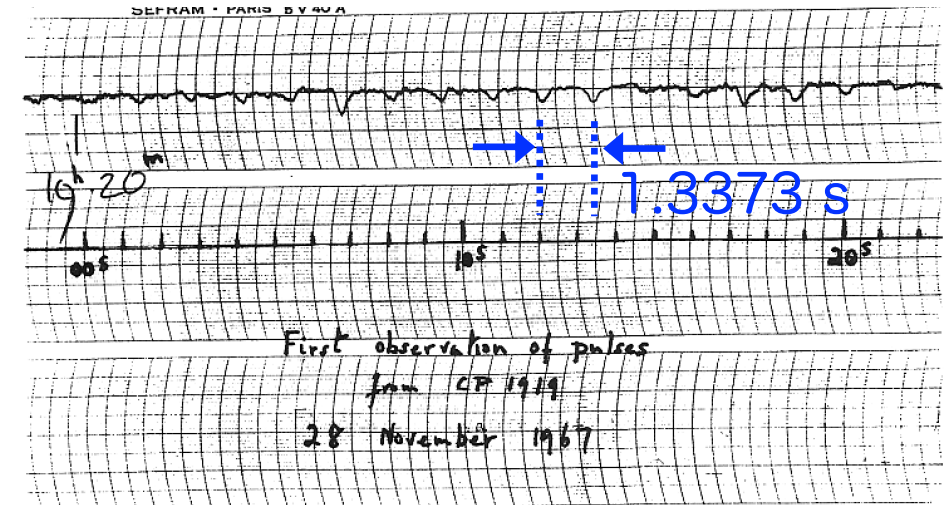


- 1932: 中性子の発見 (J. Chadwick)
- 1934: 超新星爆発後に中性子星ができることを予言(W. Baade, F. Zwicky)
- 1939: 理論モデルの提唱 (J.R. Oppenheimer, G.M. Volkoff)
- 1967: パルサーの発見 (S.J. Bell Burnell, A. Hewish)

- ケプラー回転の周期

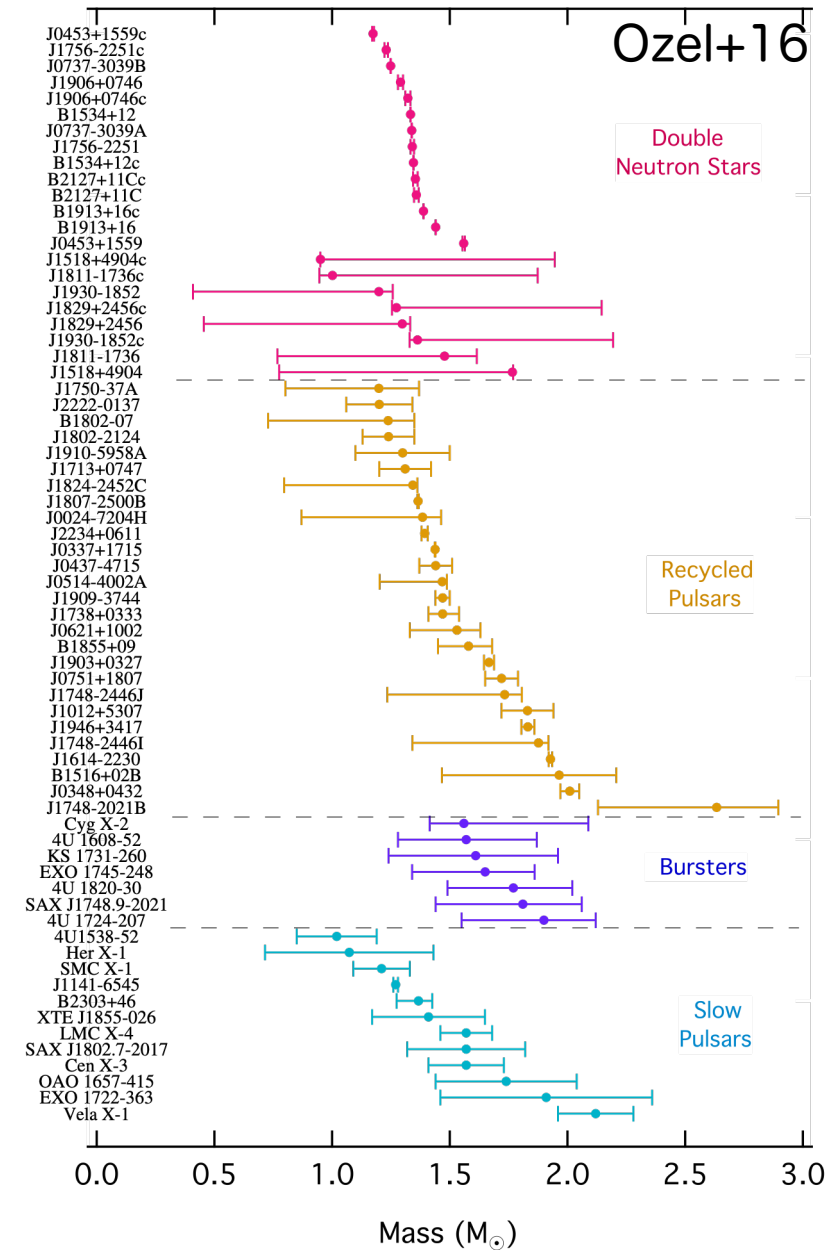
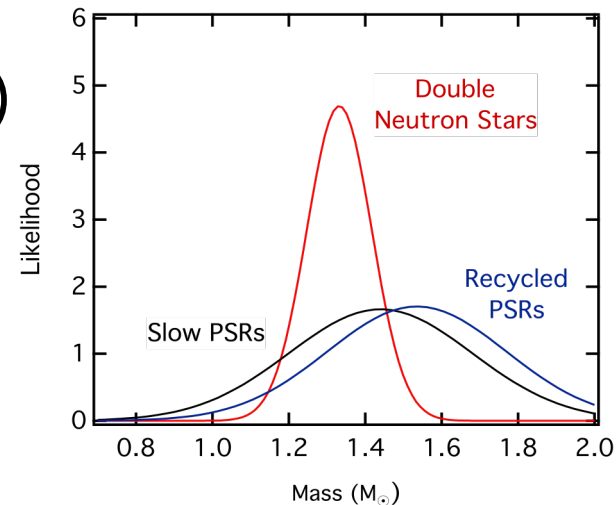
$$T = 2\pi\sqrt{\frac{R^3}{GM}} = 0.46 \text{ ms} \left(\frac{M}{1.4M_{\odot}}\right)^{-1/2} \left(\frac{R}{10 \text{ km}}\right)^{3/2}$$
$$= 14.56 \text{ s} \left(\frac{M}{1.4M_{\odot}}\right)^{-1/2} \left(\frac{R}{10000 \text{ km}}\right)^{3/2}$$

- ✓ 高速で回転する星の半径は小さくならない
- ✓ PSR B1919+21のパルサー周期は 1.3373 秒
- ✓ 発見当時、LGM “Little Green Men”-1と呼ばれる
- ✓ 1974年、A. Hewishはノーベル賞を受賞



中性子星の観測例

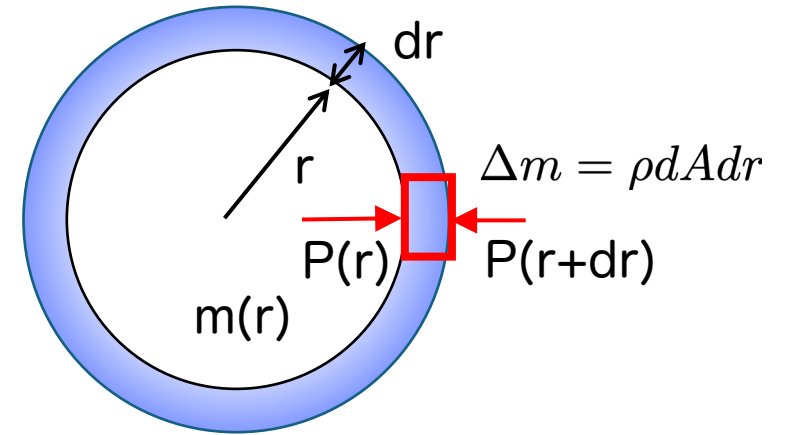
- 一番重い中性子星
 - ✓ $2.08 \pm 0.07 M_{\odot}$ (PSR J0740+6620)
 - ✓ $2.35 \pm 0.17 M_{\odot}$ (PSR J0952-0607)
- 一番軽い中性子星
 - ✓ $1.174 \pm 0.004 M_{\odot}$ (J0453+1559)
 - ✓ $0.7_{-0.17}^{+0.20} M_{\odot}$ (HESS J1731-347)
- 一番早く回っている中性子星
 - ✓ 716Hz (PSR J1748-2446ad)
- 一番早く動いている中性子星
 - ✓ 1500km/s (RX J0822-4300)



Tolman-Oppenheimer-Volkoff (TOV) eq.

- 静的球対称な天体（磁場や自転がない）
- 静水圧平衡（ニュートン重力）

$$\frac{Gm\Delta m}{r^2} = [-P(r+dr) + P(r)] dA \quad \Rightarrow \quad \frac{dP}{dr} = -\rho \frac{Gm}{r^2}$$



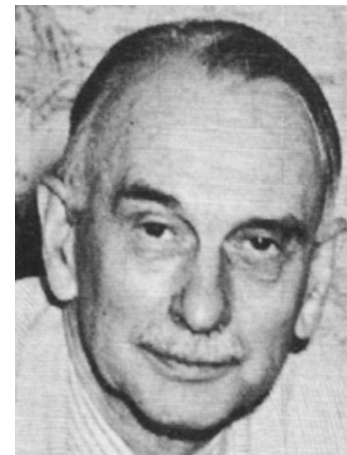
- TOV方程式

$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2} \underbrace{\left(1 + \frac{4\pi r^3 P}{mc^2}\right) \left(1 + \frac{P}{\rho c^2}\right) \left(1 - \frac{2Gm}{c^2 r}\right)^{-1}}_{\text{相対論的な補正}}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

- 状態方程式 (EOS)

$$P = P(\rho)$$



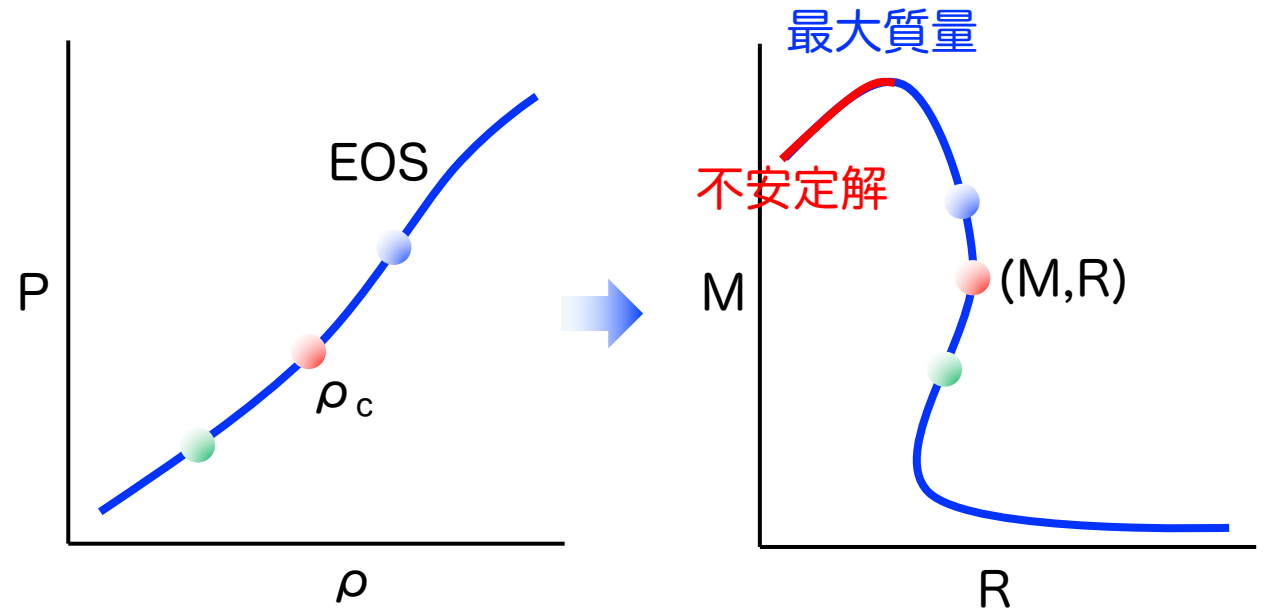
TOV eq. & M-R relation

- 境界条件:

$$\rho = \rho_c; m = 0 @ r = 0$$

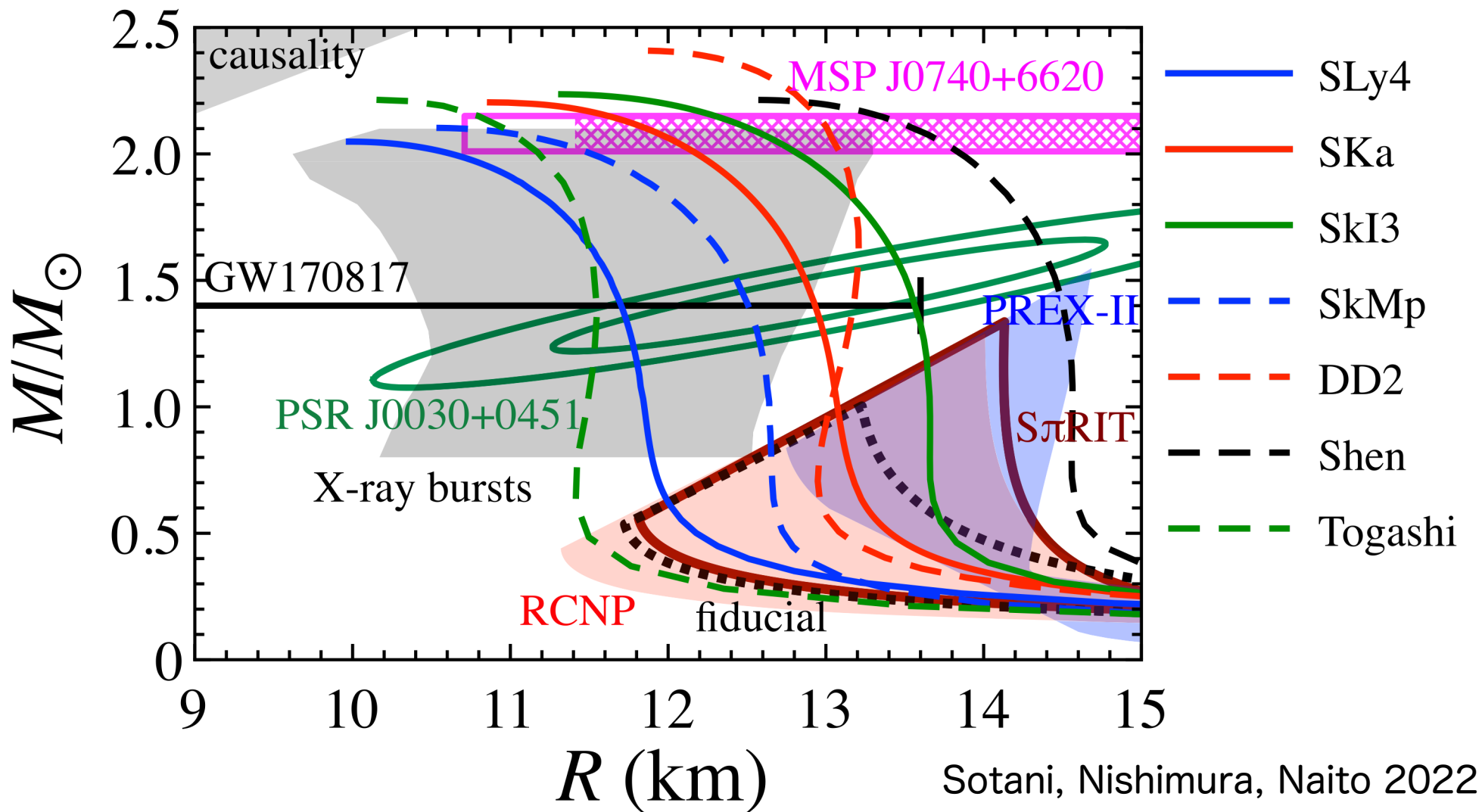
$$P = 0 @ r = R;$$

$$M = m(R)$$



- 最大質量の存在 (フェルミガスだと思つと $\sim 0.7M_{\odot}$)
- 中性子星モデルを構築する上で、状態方程式は不可欠ではあるが、よく理解されていない ← 中性子過剰/飽和性

M-R relation



Zero-temperature

- Typical core temperature is of order 10^8 K
 - ✓ far below the Fermi temperature

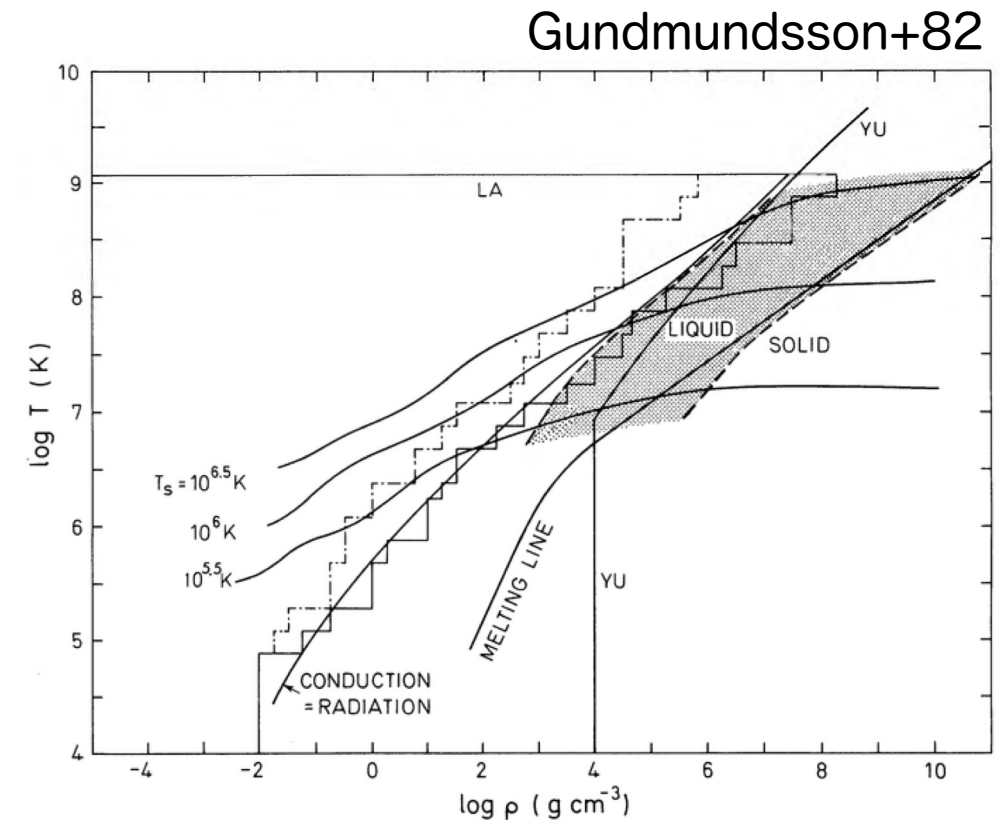
$$T_F = \frac{E_F}{k_B} \approx 10^{12} \left(\frac{n_b}{n_0} \right)^{2/3} \text{ K}$$

- ✓ this makes the formation of a superfluid/superconducting phase
- In the lower density region (envelope), the thermal effects may become important

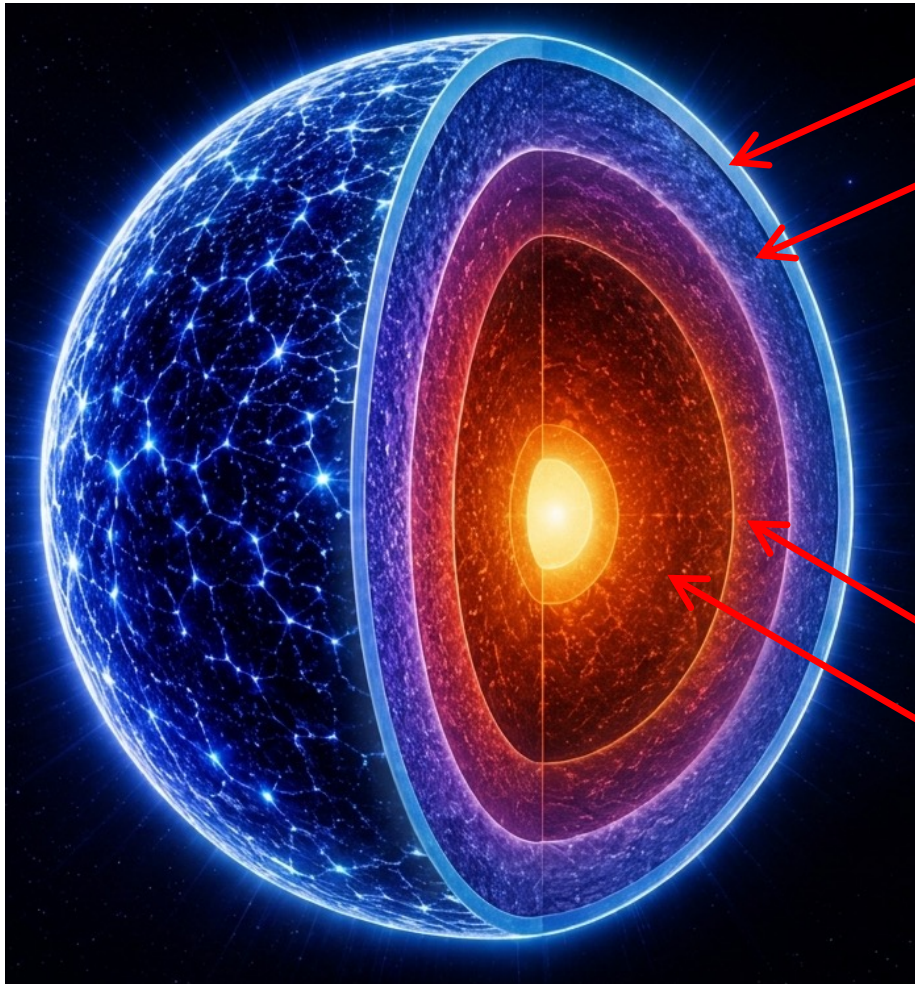
Coulomb coupling parameter $\Gamma \simeq \frac{Z^2 e^2}{r_i k_B T}$

$$\Gamma > \Gamma_m = 158 : \text{solid}$$

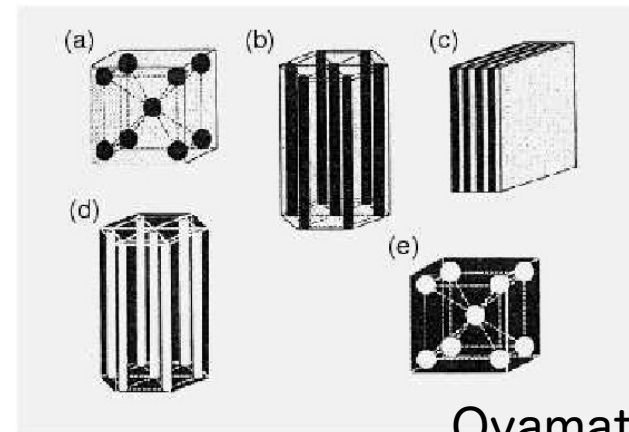
$$\Gamma < \Gamma_m = 158 : \text{liquid}$$



NS structure



Outer crust
Inner crust } Coulomb lattice



pasta structure

Oyamatsu 93

..... $\rho_t \simeq (1/3 \sim 1/2)\rho_s$

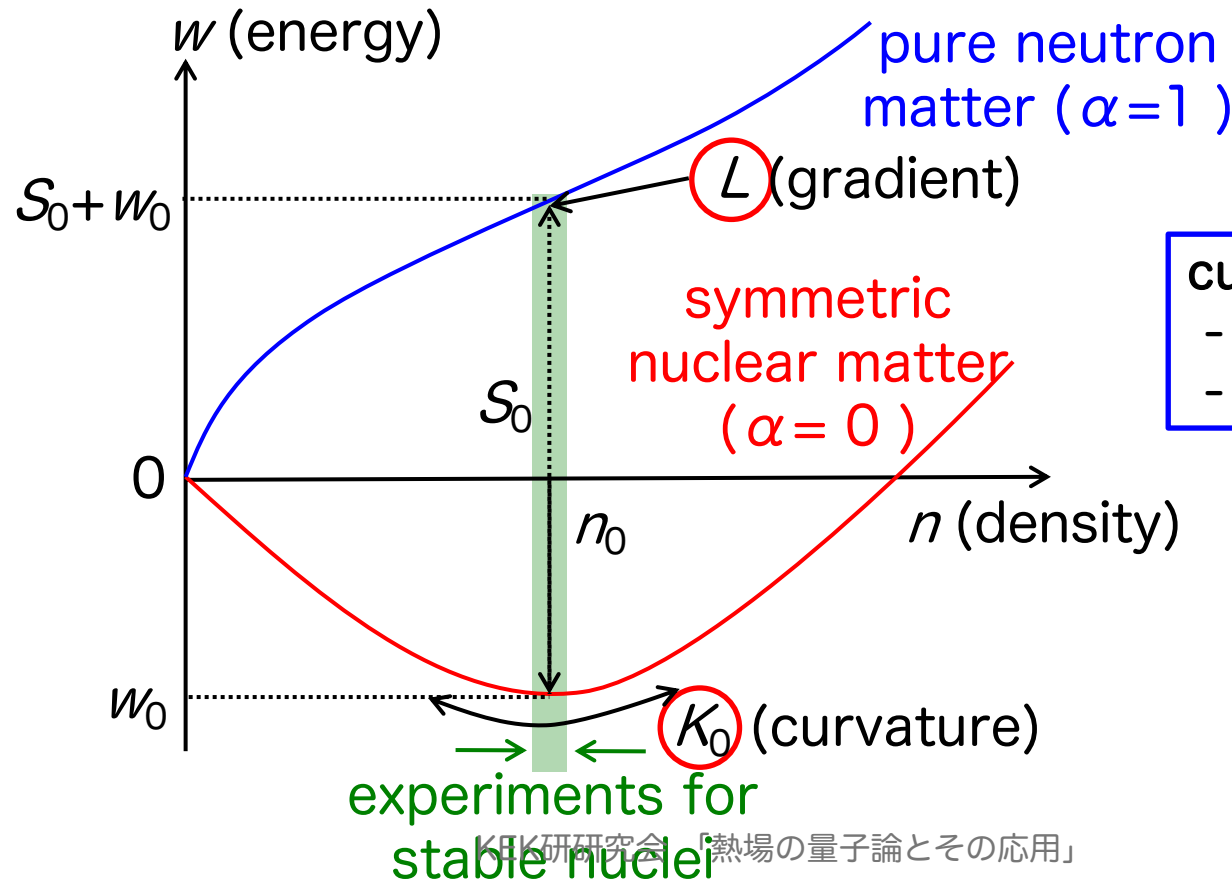
Outer core
Inner core } Uniform matter

EOS near the saturation point

- Bulk energy per nucleon near the saturation point of symmetric nuclear matter at zero temperature;

$$w = w_0 + \frac{\overset{\text{incompressibility}}{K_0}}{18n_0^2}(n - n_0)^2 + \left[S_0 + \frac{\overset{\text{symmetry parameter}}{L}}{3n_0}(n - n_0) \right] \alpha^2$$

$$p = n^2 \frac{dw}{dn}$$

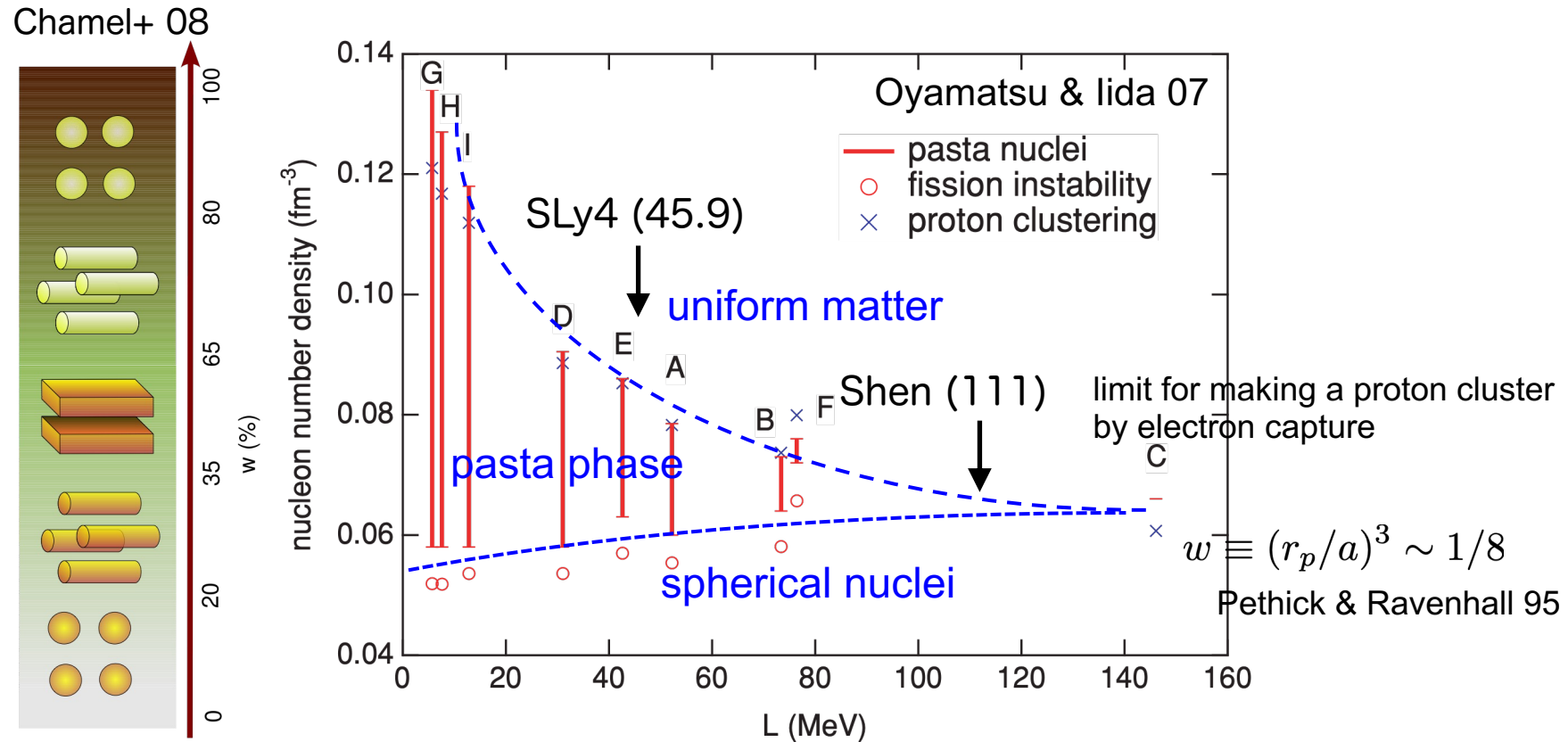


current constraints on K_0 & L

- $K_0 = 230 \pm 40$ MeV (Khan+13)
- $40 \leq L \leq 80$ MeV (Li+ 13)

Any EOSs can be characterized by these saturation parameters

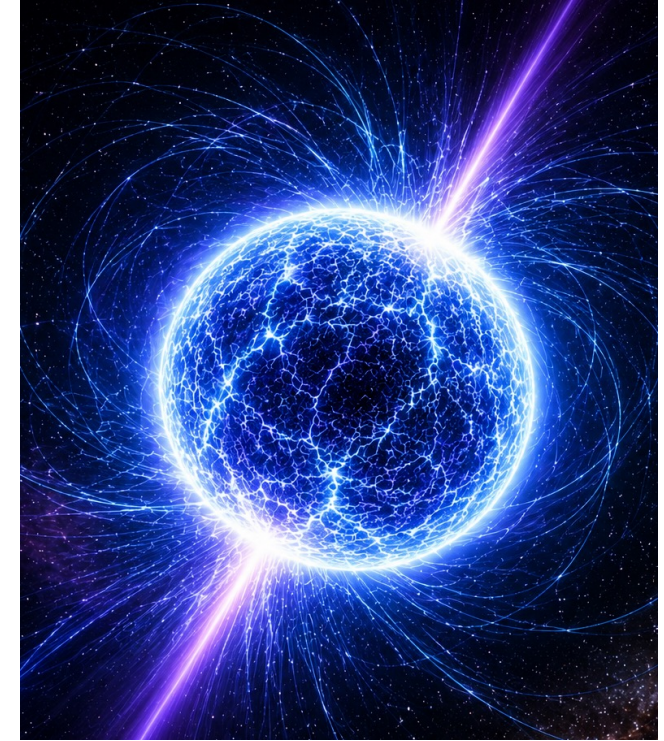
Pasta phases



- Whether pasta phase exists or not depends strongly on L .
- For $L \gtrsim 100\text{MeV}$, pasta structure almost disappears.

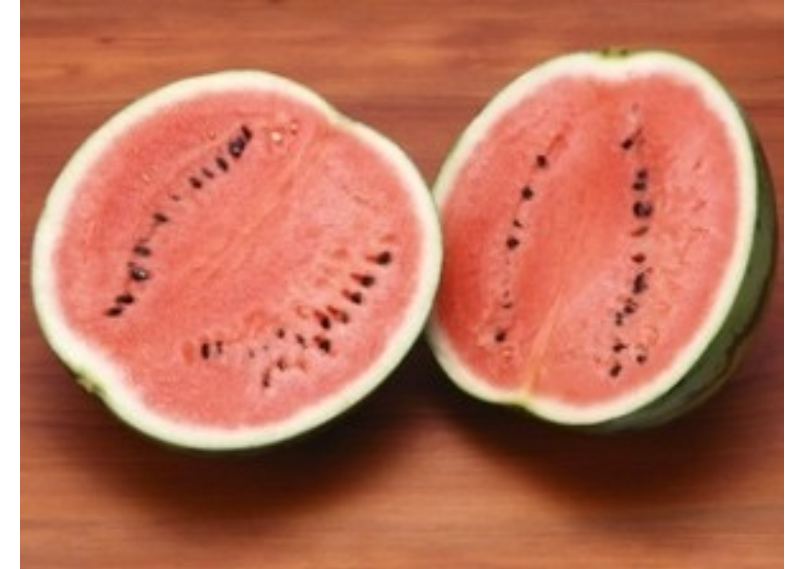
NS as a laboratory for testing physics

- Density becomes extremely high
 - central density $\approx 10^{15} \text{ g/cm}^3$
c.f. nuclear saturation density = $2.7 \times 10^{14} \text{ g/cm}^3$
- Strong magnetic field: $10^{12} \sim 10^{13} \text{ G}$
 - strongly magnetized NS (magnetar): $\gtrsim 10^{14} \text{ G}$
c.f. Earth's magnetic field $\sim 10^{-5} \text{ G}$
- Strong gravitational field (relativistic objects)
 - $g_E \approx 10 \text{ m/s}^2$, $g_S \approx 270 \text{ m/s}^2$, $g_{NS} \approx 2.4 \times 10^{12} \text{ m/s}^2$
- Neutron superfluid / proton superconductor
- An ideal “laboratory” for testing physics in extreme states



How to know the NS properties

- observing M and R
- observing the frequencies
- how to know the best time to eat a watermelon ?
 - The inside can not be seen before cutting
- “empirical rule”
 - to check the best time, knock a watermelon
 - high frequency “KIN-KIN” ; too young
 - “BAN-BAN” ; best time !
 - low frequency “BON-BON” ; too old
 - need many years to get this ability
- One could see the interior with specific sound from object.
 - **asteroseismology !!**



Watermelon

Non-radial Oscillations in NSs

- axial-type oscillations

- no stellar deformation, no density variation
 - **t-modes (torsional) : elasticity $\sim vs/R$**
 - **Alfven modes**
 - w-modes (spacetime) : oscillations of spacetime itself $\sim M/R$
 - **r-modes (rotational) $\sim m\Omega$**

- polar-type oscillations

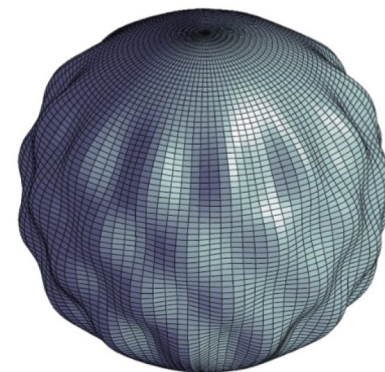
- with density variation & stellar deformation
- important for considering the GWs emission
 - f-mode (fundamental) $\sim (M/R^3)^{1/2}$
 - p-modes (pressure) : sound speed crossing $\sim (M/R^3)^{1/2}$
 - g-modes (gravity) : thermal/composition gradients $\sim$ B-V frequency
 - **s-/i-modes (shear/interface) : elasticity**
 - **Alfven modes**
 - w-modes (spacetime) : oscillations of spacetime itself $\sim M/R$
 - **inertial modes (effect of rotation)**

Parity transformations on a sphere

$$(\theta, \phi) \rightarrow (\pi - \theta, \phi + \pi)$$

$$\text{axial: } (-1)^{\ell+1}$$

$$\text{polar: } (-1)^\ell$$



specific mode



physics

fundamental (f-) mode (Lord Kelvin 1863)

- considering a self-gravitating fluid with **constant** ρ

✓ constant ρ : $\delta\rho = 0$

✓ continuity equation ($\vec{\nabla} \cdot \vec{v} = 0$) $\rightarrow \vec{\nabla} \cdot \delta\vec{v} = 0$ $\xrightarrow{\quad}$

- if $\vec{\nabla} \times \delta\vec{v} = 0$, $\delta\vec{v} = \vec{\nabla}\chi$, where χ is “stream function” $\rightarrow \underline{\nabla^2\chi = 0}$

- perturbed Poisson equation $\rightarrow \underline{\nabla^2\delta\Phi = 4\pi G\delta\rho = 0}$

- perturbed Euler equation:

$$\partial_t \delta\vec{v} + \frac{1}{\rho} \vec{\nabla} \delta P + \vec{\nabla} \delta\Phi = 0 \rightarrow \partial_t \vec{\nabla}\chi + \frac{1}{\rho} \vec{\nabla} \delta P + \vec{\nabla} \delta\Phi = 0 \rightarrow \partial_t \chi + \frac{1}{\rho} \delta P + \delta\Phi = \text{const.}$$

$$\xrightarrow{\text{div}} \partial_t \nabla^2 \chi + \frac{1}{\rho} \nabla^2 \delta P + \nabla^2 \delta\Phi = 0 \rightarrow \underline{\nabla^2 \delta P = 0}$$

- χ , δP , and $\delta\Phi$ satisfy Laplace's equation

$$\nabla^2 X = \frac{1}{r} \frac{\partial^2}{\partial r^2} (rX) + \frac{1}{r^2} \nabla_\theta^2 X = 0 \text{ where } \nabla_\theta^2 X = \frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial X}{\partial\theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2 X}{\partial\phi^2}$$

- ✓ angular part can be expressed in terms of the spherical harmonics as $X(r, \theta, \phi) = \sum_{l,m} X_l(r) Y_{lm}$,
then Laplace's equation becomes

$$\frac{1}{r} \frac{d^2}{dr^2} (rX_l) - \frac{l(l+1)}{r^2} X_l = 0 \text{ where we use } \nabla_\theta^2 Y_{lm} = -l(l+1) Y_{lm}$$

- ✓ general solution is $X_l = Ar^l + Br^{-(l+1)}$ with the regularity condition at the stellar center, $B=0$ in the star's interior

Basic eqs.

① continuity equation

② Euler equation

③ Poisson equation

fundamental (f-) mode (Lord Kelvin 1863)

- the perturbed variables are expressed as $\chi = a_l r^l Y_{lm}$; $\delta\Phi = b_l r^l Y_{lm}$; $\delta P = c_l r^l Y_{lm}$
- if we further assume that the fluid motion has a harmonic dependence on time $\propto e^{i\omega t}$

$$\partial_t \chi + \frac{1}{\rho} \delta P + \delta\Phi = \text{const.} \longrightarrow \underline{i\omega a_l + \frac{c_l}{\rho} + b_l = 0} \quad \text{we set const} = 0, \text{ because all functions on the LHS vanished at } r=0$$

← perturbed Euler equation

- To complete our solution, we need to consider the boundary conditions at $r=R$

- ✓ Lagrangian perturbation of the pressure must vanish: $\Delta P = \delta P + \xi^r P' = 0$
 - radial component of the displacement vector: $i\omega \xi^r = \delta v^r = \chi' = a_l l r^{l-1} Y_{lm}$
 - balance equation for background: $P' = -\rho \Phi' = -4\pi G \rho^2 r / 3$
- $$\left. \begin{array}{l} \Delta P = 0 \longrightarrow c_l = \frac{4\pi G \rho^2 l}{3i\omega} a_l \end{array} \right\}$$

- if we adopt the **Cowling approximation**, i.e., $b_l = 0$, we arrive at $\omega_{\text{Cow}}^2 = \frac{4\pi G \rho l}{3}$
- These modes are known as the **fundamental (f-) modes**, which scale with the average density of the star, and increase with the multipole ℓ

$$\Phi_{\text{in}} = \frac{2\pi G \rho}{3} (r^2 - 3R^2) \quad \Phi_{\text{out}} = -\frac{GM}{r}$$

$$\Phi_{\text{in}}(R + \xi^r) = \Phi_{\text{out}}(R + \xi^r)$$

$$\Phi'_{\text{in}}(R + \xi^r) = \Phi'_{\text{out}}(R + \xi^r)$$

- Without the Cowling approximation, the appropriate matching condition for $\delta\Phi$ at $r = R$

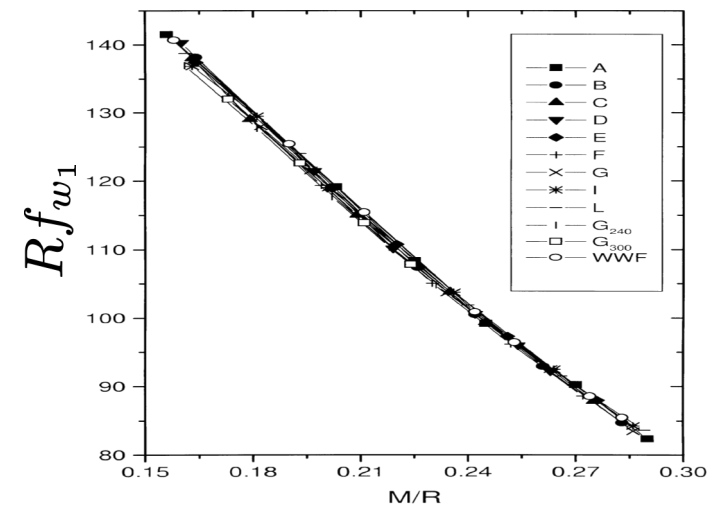
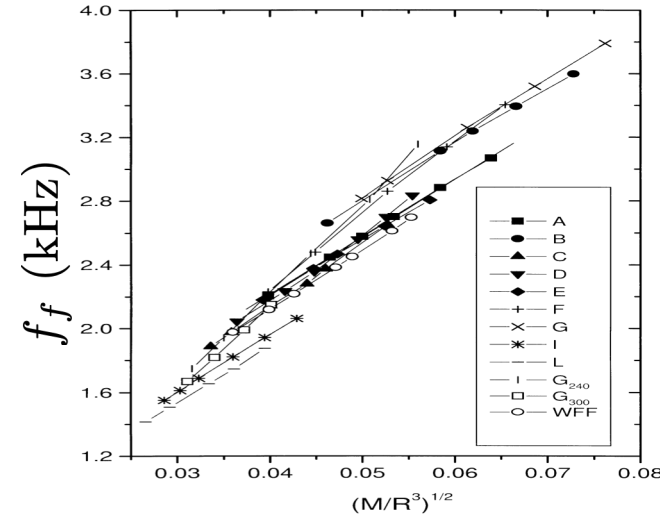
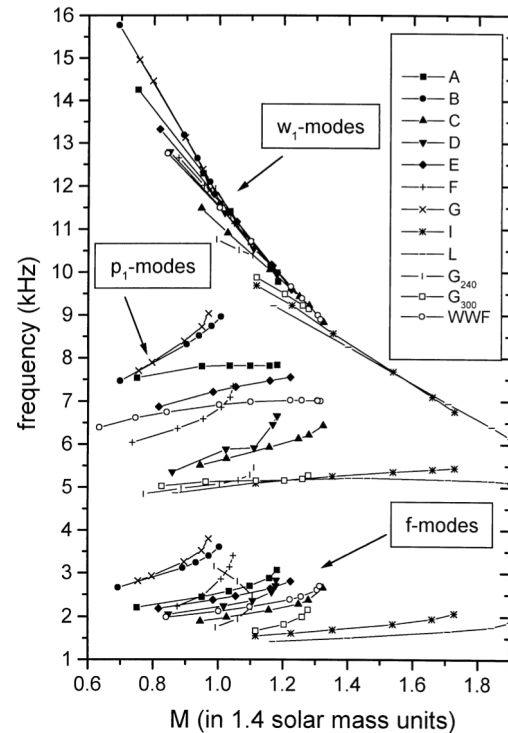
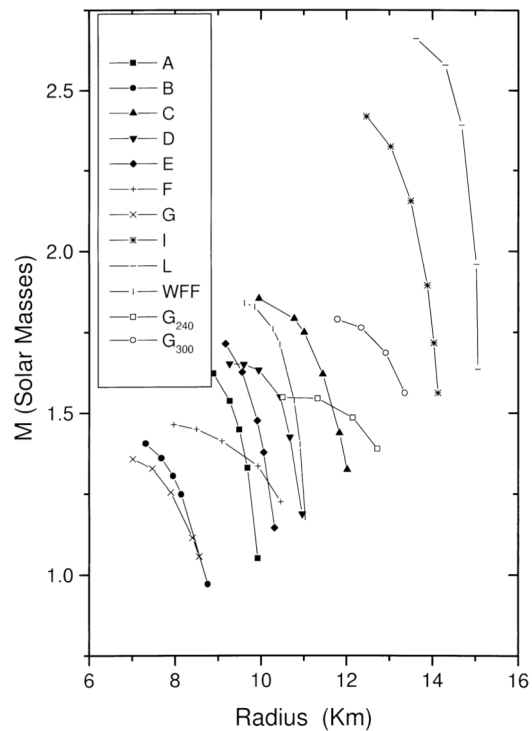
$$\delta\Phi' + \frac{l+1}{R} \delta\Phi = -4\pi G \rho \xi^r \longrightarrow b_l = -\frac{4\pi G \rho l}{(2l+1)i\omega} a_l \longrightarrow \omega^2 = \frac{8\pi G \rho l(l-1)}{3(2l+1)}$$

- by taking the ratio $\left(\frac{\omega_{\text{Cow}}}{\omega}\right)^2 = \frac{2l+1}{2(l-1)}$, the error is fairly large for the $l=2$ modes, but decreases rapidly with ℓ

GW frequencies from various NS models

(Andersson & Kokkotas 98)

- NS properties strongly depend on the EOSs
 - ✓ corresponding GW frequencies also depend on the EOSs
 - ✓ the relations expressing the GW frequencies independently of EOS are quite useful, if any, to explore the inverse problem for GWs from oscillating NSs (“**universal relation(s)**”)



$$f_f \text{ (kHz)} \simeq 0.22 + 2.16 \left(\frac{M}{1.4M_\odot} \right)^{1/2} \left(\frac{R}{10 \text{ km}} \right)^{-3/2}$$

$$f_{w_1} \text{ (kHz)} \simeq \left(\frac{R}{10 \text{ km}} \right)^{-1} \left[20.92 - 9.14 \left(\frac{M}{1.4M_\odot} \right) \left(\frac{R}{10 \text{ km}} \right)^{-1} \right]$$

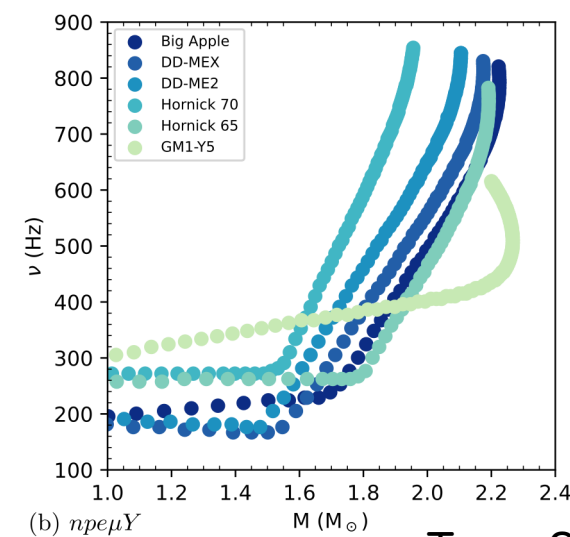
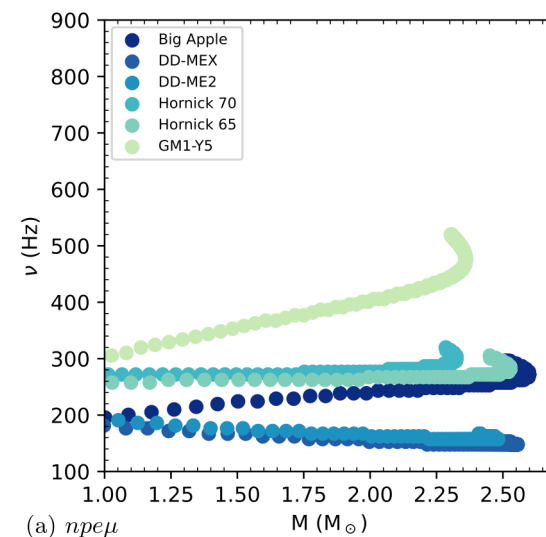
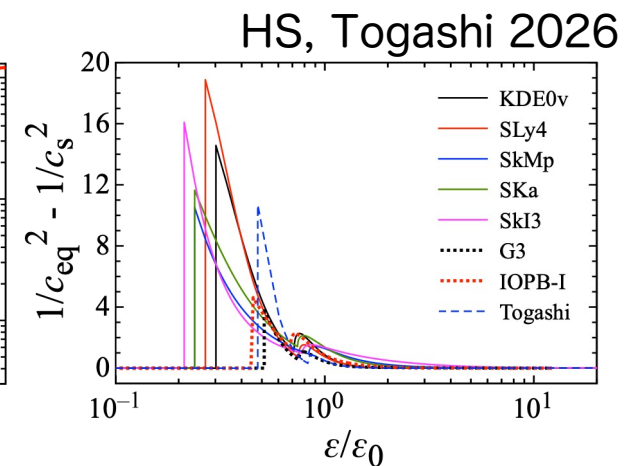
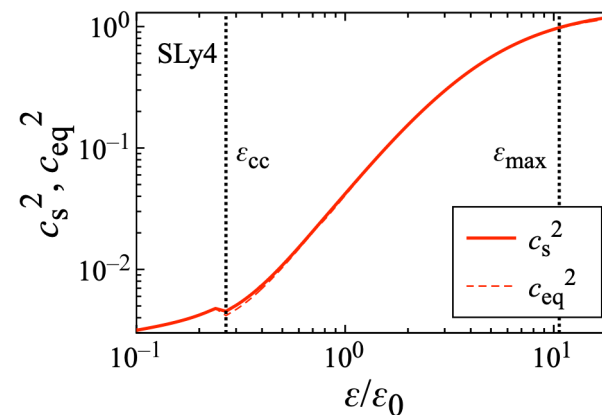
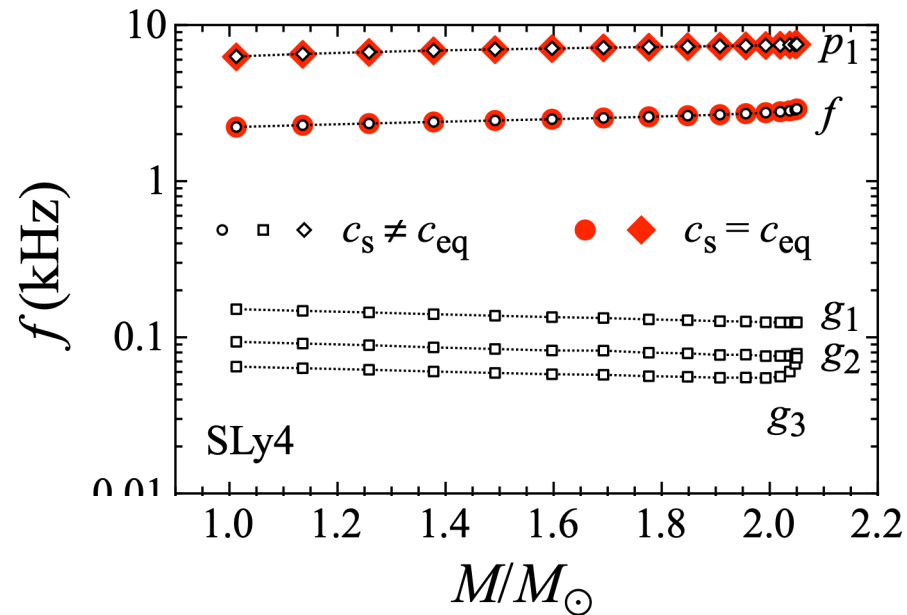
$\left. \vphantom{\begin{matrix} f_f \\ f_{w_1} \end{matrix}} \right\} \longrightarrow \begin{matrix} 5\% \text{ in } R \\ 2\% \text{ in } M \end{matrix}$

gravity (g-) mode

- g-modes are excited by the thermal/composition gradient

$$c_s^2 \equiv \left(\frac{\partial p}{\partial \varepsilon} \right)_{s, Y_i} \simeq \frac{dp}{d\varepsilon} \equiv c_{eq}^2$$

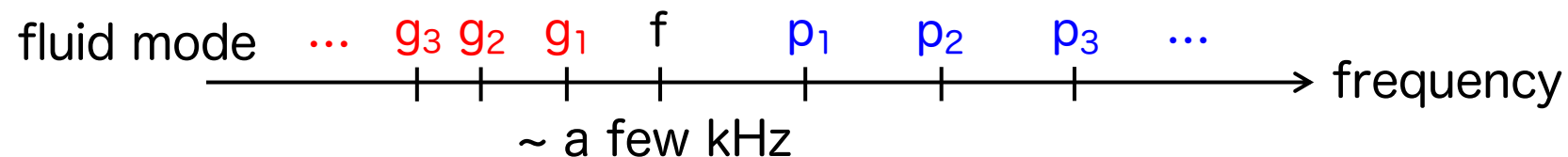
$$\mathcal{N}^2 = -e^{2\Phi-2\Lambda} \frac{\Phi' p'}{\varepsilon + p} \left(\frac{1}{c_{eq}^2} - \frac{1}{c_s^2} \right)$$



Tran+23

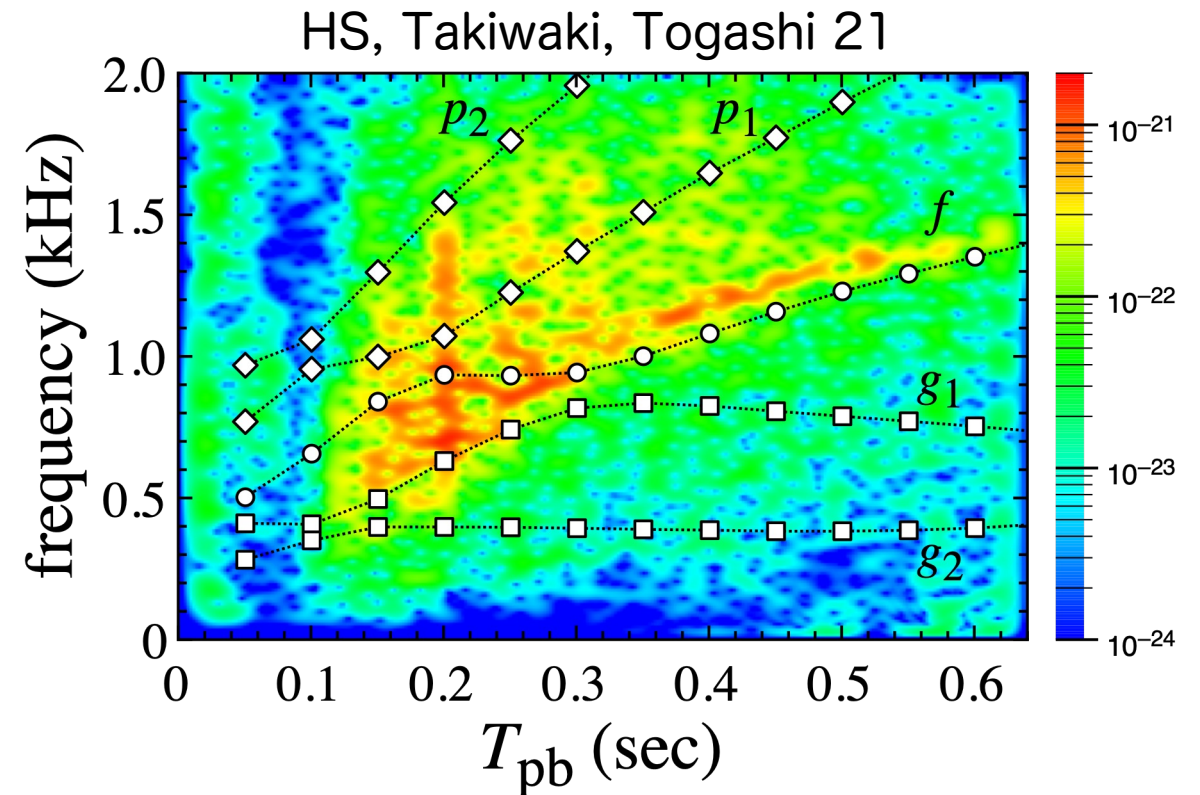
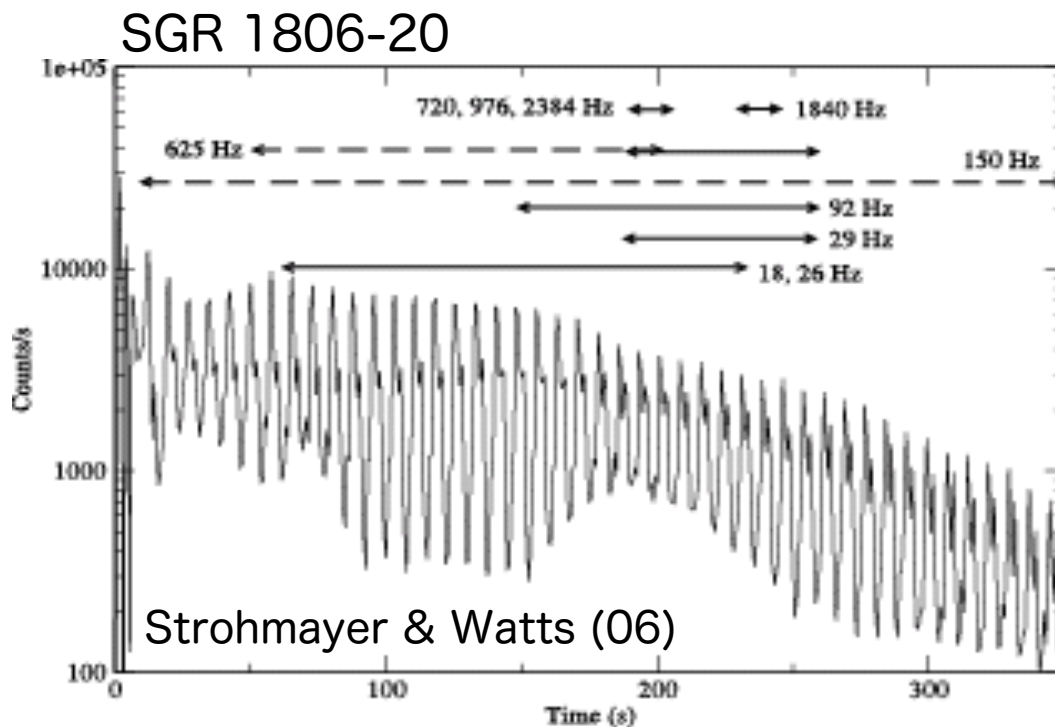
Asteroseismology

- Linear analysis
- variable = background + perturbation: $f = f_0 + \delta f$
- decompose the perturbed variables: $\delta f(t, r, \theta, \phi) = \delta f(r) e^{i\omega t} Y_{lm}(\theta, \phi)$
- Here, the background is assumed to be a spherically symmetric equilibrium
 - ✓ polar-type oscillations are completely decoupled from axial-type oscillations
 - ✓ modes with $k \neq 0$ degenerate into the modes with $k = 0$, using spherical harmonics $Y_{\ell k}$
 - ✓ ℓ -th modes are decoupled from modes with $\ell' \neq \ell$
- One cannot discuss the oscillation energy due to the nature of linear analysis



Applications

- Crustal torsional oscillations & magnetar QPOs
- Supernova gravitational waves & PNS oscillations



Crustal torsional oscillations

- Background: Spherically symmetric, non-stressed NS

- metric: $ds^2 = -e^{2\Phi} dt^2 + e^{2\Lambda} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$

- consider axial perturbations

- $\mathcal{Y}$: the radial dependence of the angular displacement
 - $\tilde{H}$: the effective enthalpy density

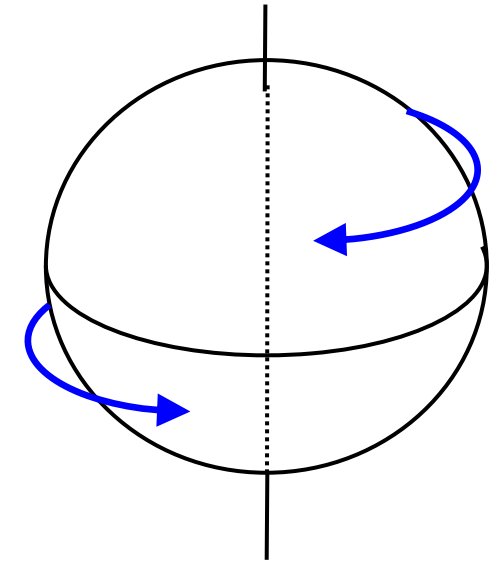
$$\mathcal{Y}'' + \left[\left(\frac{4}{r} + \Phi' - \Lambda' \right) + \frac{\mu'}{\mu} \right] \mathcal{Y}' + \left[\frac{\tilde{H}}{\mu} \omega^2 e^{-2\Phi} - \frac{(\ell+2)(\ell-1)}{r^2} \right] e^{2\Lambda} \mathcal{Y} = 0,$$

$\tilde{H} = (1 - N_s/A)H$

μ (circled in red) shear modulus

- boundary conditions

- the vanishing of the traction force @ $r = R_c$ and $r = R$
 - the problem to solve becomes the eigenvalue problem



$$\delta T^{\mu\nu}_{;\mu} = 0$$

$$\delta u^\phi = e^{-\Phi} \partial_t \mathcal{Y}(t, r) \partial_\theta P_\ell(\cos \theta) / \sin \theta$$

$$\mathcal{Y}(t, r) = e^{i\omega t} \mathcal{Y}(r)$$

Superfluidity

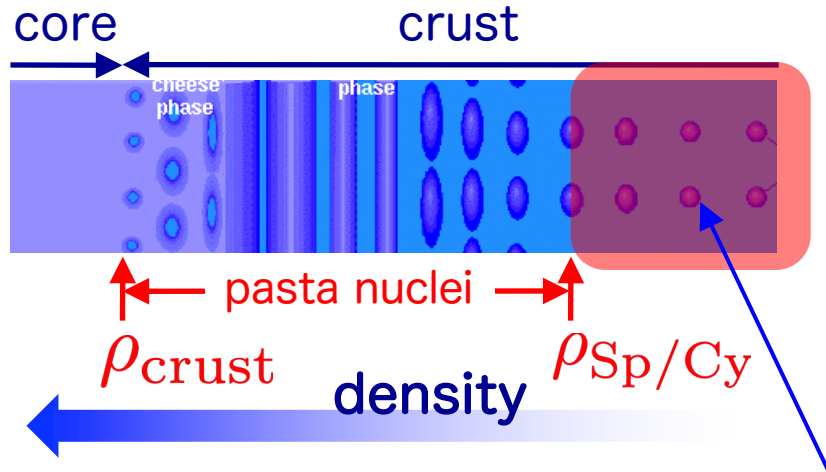
- some of the dripped neutrons behave as superfluidity
 - do not contribute to the inertia for the torsional oscillations

$$\mathcal{Y}'' + \left[\left(\frac{4}{r} + \Phi' - \Lambda' \right) + \frac{\mu'}{\mu} \right] \mathcal{Y}' + \left[\frac{\tilde{H}}{\mu} \omega^2 e^{-2\Phi} - \frac{(\ell + 2)(\ell - 1)}{r^2} \right] e^{2\Lambda} \mathcal{Y} = 0,$$

$$\tilde{H} = (1 - N_s/A)H$$

- in the phase of spherical nuclei: suggested by Chamel (2012)
 - we adopt his result
- in the cylindrical nuclei: poorly understood
 - we consider the extreme limit
 - $N_s/N_d = 0$: all dripped N comove with the nuclei
 - $N_s/N_d = 1$: all dripped N behave as superfluid

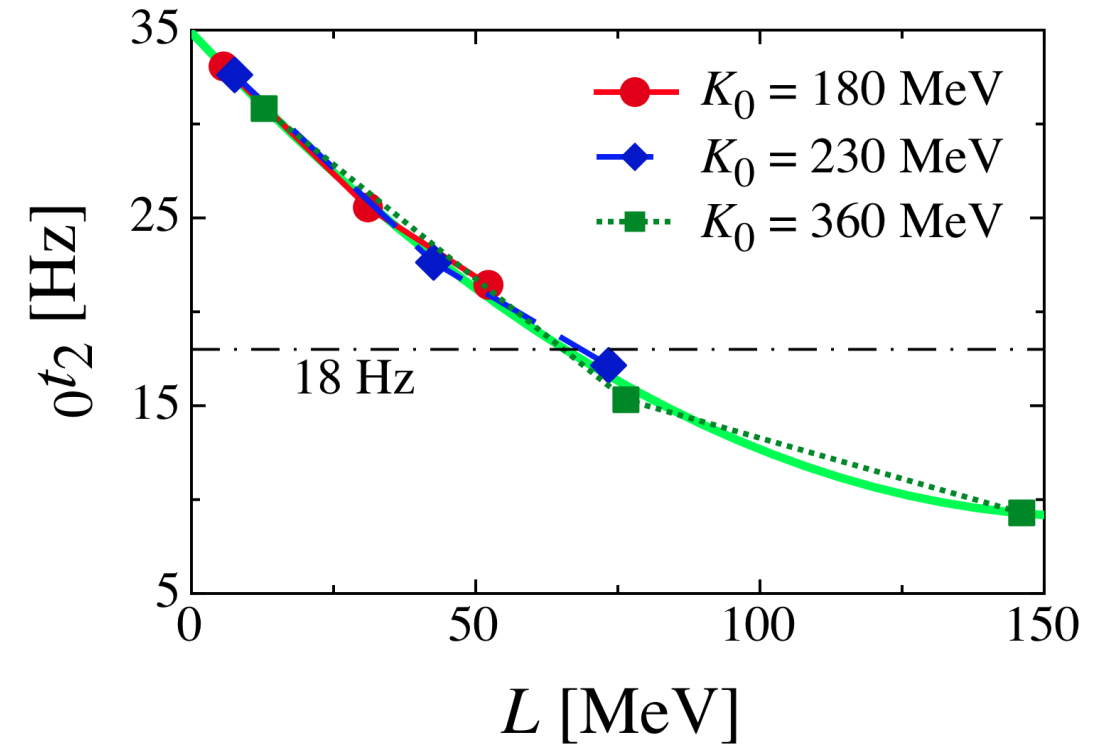
Shear modulus in spherical nuclei



for bcc lattice (Strohmayer+ 91)

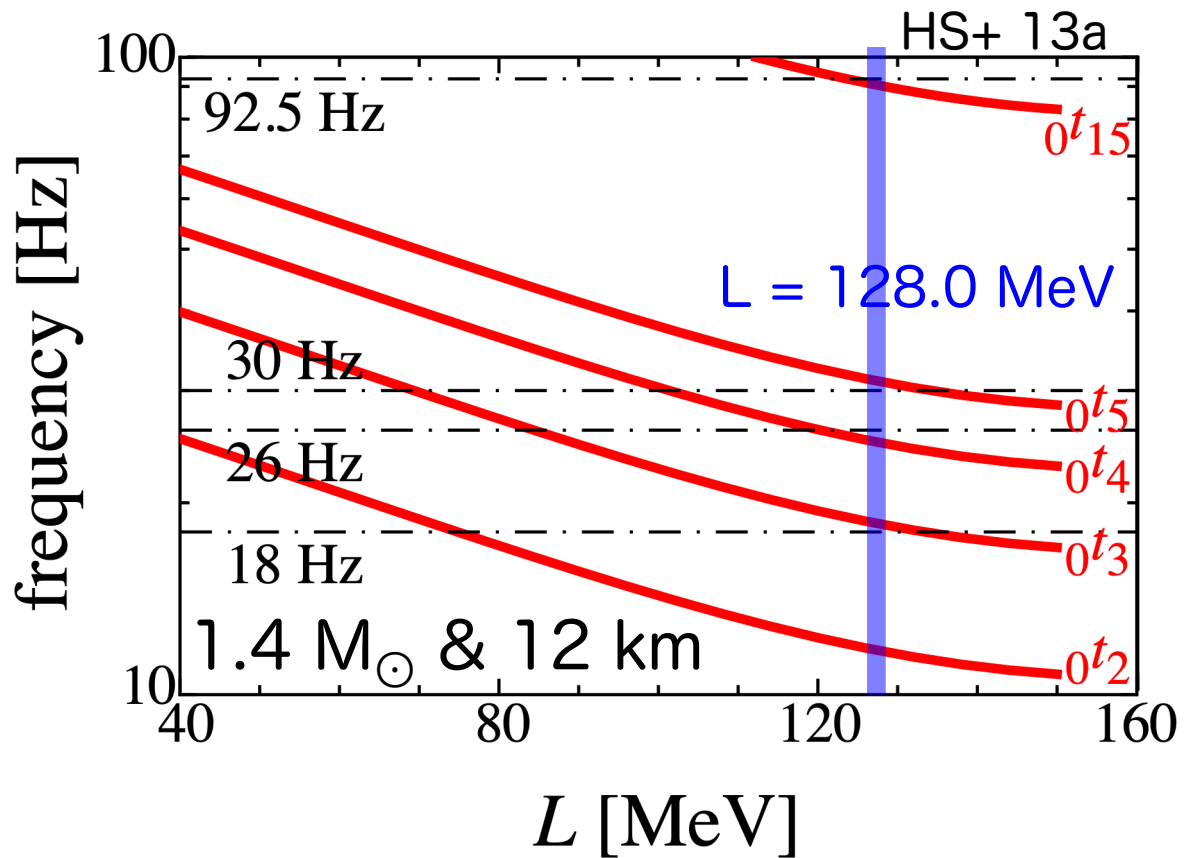
$$\mu = 0.1194 \frac{n_i (Ze)^2}{a}$$

n_i : ion number density
 Z : charge of nuclei
 a : Wigner-Seitz radius

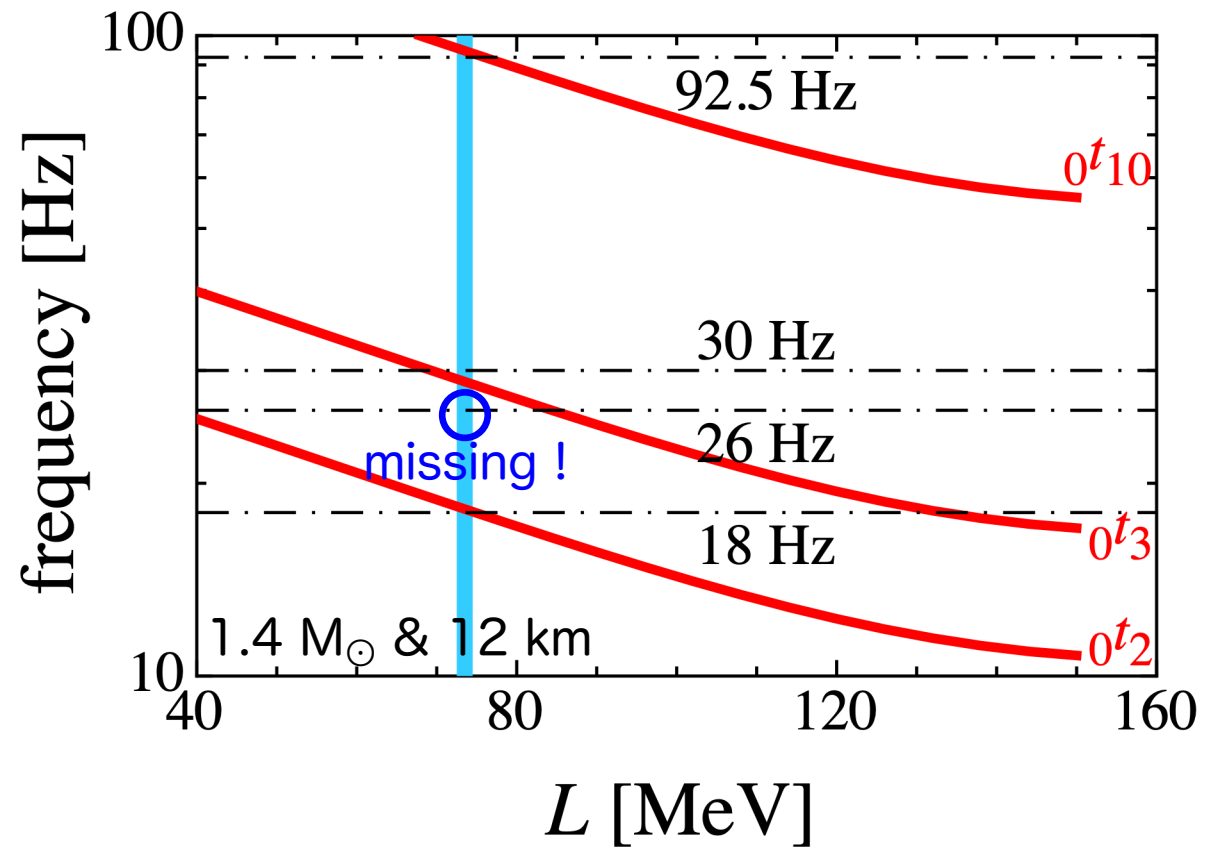


$${}_{\ell}t_0 \text{ (Hz)} = c_{\ell}^{(0)} + c_{\ell}^{(1)} L_{100}^{1/2} + c_{\ell}^{(2)} L_{100} + c_{\ell}^{(3)} L_{100}^2,$$

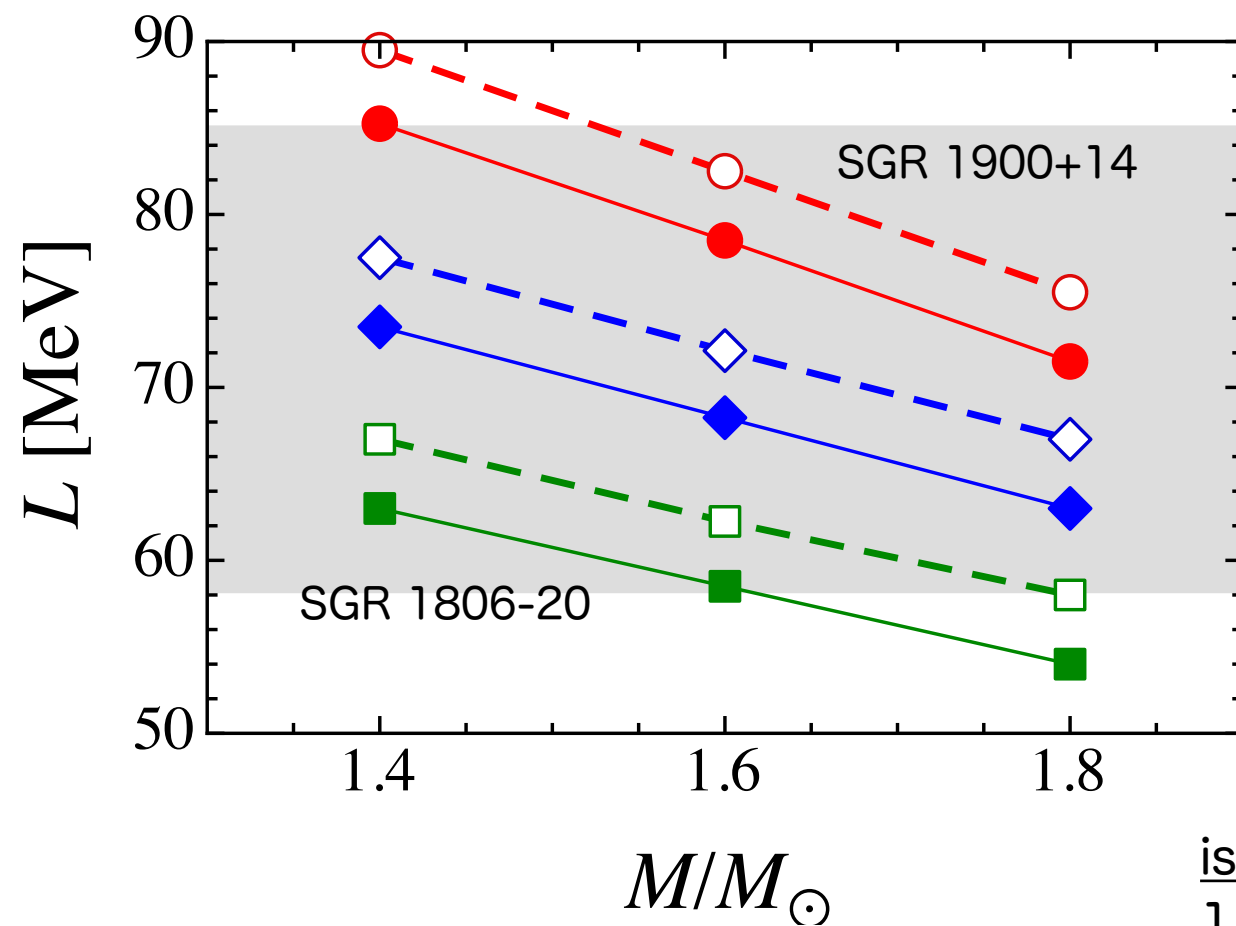
Identification of SGR 1806-20



cf. $40 \lesssim L \lesssim 80$ MeV (Li+ 13)



Alternative allowed region for L



→ $58.0 \text{ MeV} \leq L \leq 85.3 \text{ MeV}$

issues

1. neglecting the 26 Hz QPO in SGR 1806-20
2. considering only the phase of spherical nuclei

Elastic properties in pasta

- cylindrical nuclei

$$E_d = \underbrace{\frac{B}{2} \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right)^2}_{\text{uniform transverse compression}} + \underbrace{\frac{C}{2} \left[\left(\frac{\partial u_x}{\partial x} - \frac{\partial u_y}{\partial y} \right)^2 + \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right)^2 \right]}_{\text{transverse shear}} + \underbrace{\frac{K_3}{2} \left(\frac{\partial^2 \mathbf{u}}{\partial z^2} \right)^2}_{\text{bending}} + \underbrace{\left[B' \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right) \left(\frac{\partial \mathbf{u}}{\partial z} \right)^2 + \frac{B''}{2} \left(\frac{\partial \mathbf{u}}{\partial z} \right)^4 \right]}_{\text{higher order term}}.$$

- slab-like nuclei

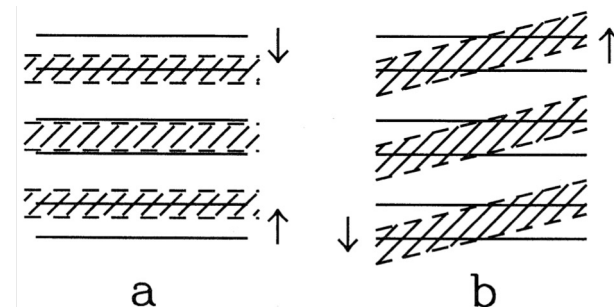
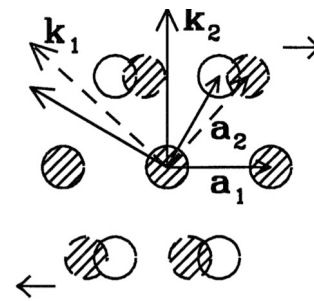
$$E_d = \underbrace{\frac{B}{2} \left[\frac{\partial u}{\partial z} - \frac{1}{2} (\nabla_{\perp} u)^2 \right]^2}_{\text{compression}} + \underbrace{\frac{K_1}{2} (\nabla_{\perp}^2 u)^2}_{\text{splay deformation}}.$$

shear property in slab-like nuclei
becomes higher order effect!
→ linear response behaves as fluid

Pethick & Potekhin 98

$$\log_{10}(C/E_{C0}) \approx 2.1(w - 0.3)$$

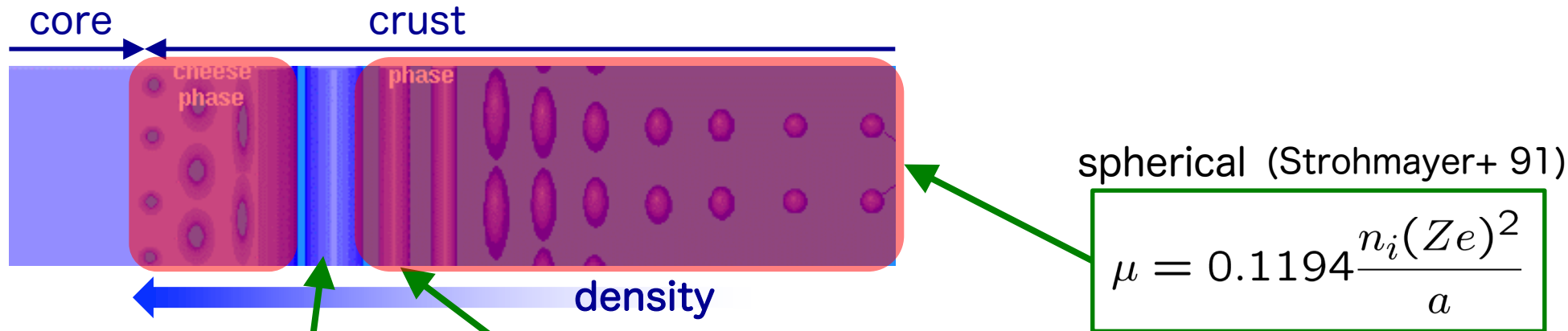
E_{C0} : Coulomb energy per unit volume
 w : volume fraction



compression

shear

Double-layer model (lasagna sandwich) HS+19



spherical (Strohmayer+ 91)

$$\mu = 0.1194 \frac{n_i (Ze)^2}{a}$$

slab-like

linear response : fluid
(Landau)

cylindrical (Potekhin+98)

$$\mu_{cy} = \frac{2}{3} E_{Coul} \times 10^{2.1(w_2-0.3)}$$

E_{Coul} : Coulomb energy per unit volume
 w_2 : volume fraction

- two elastic regions
 - (i) spherical + cylindrical (sp+cy)
 - (ii) tube + bubble (tu+bu)

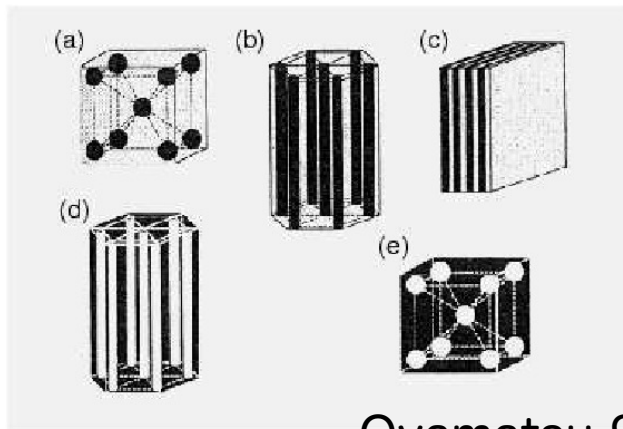
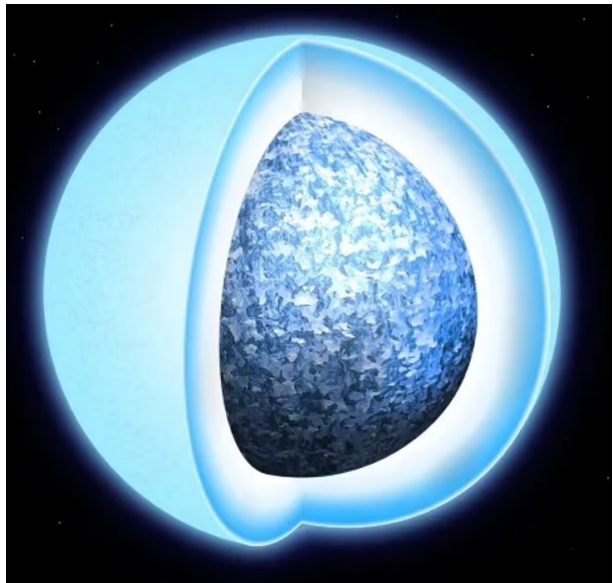
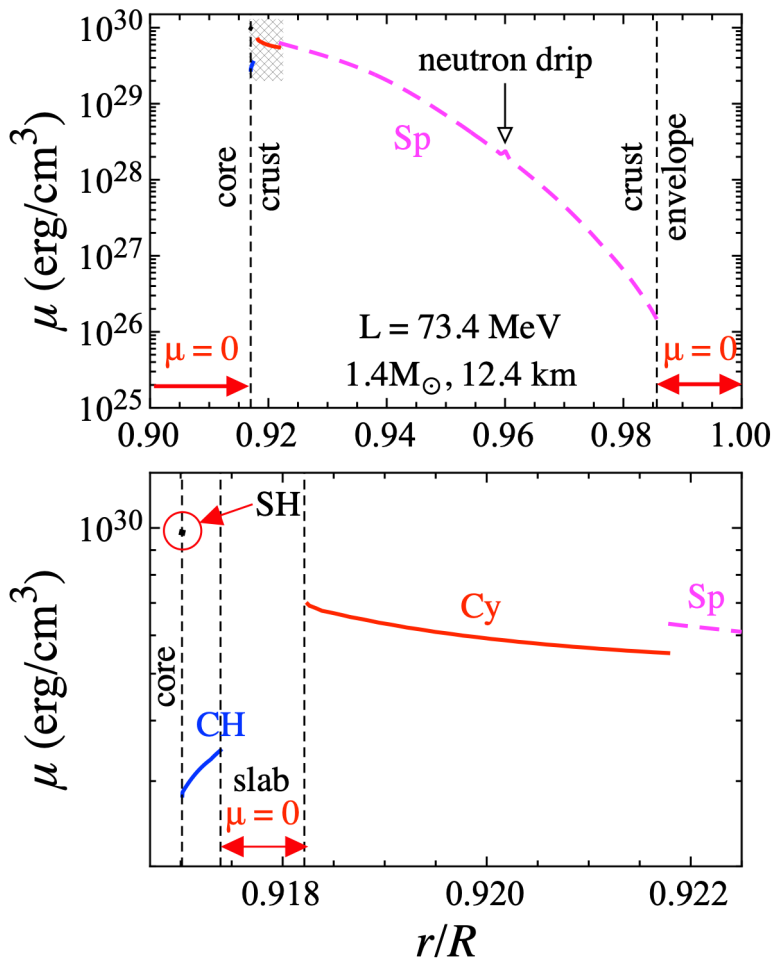
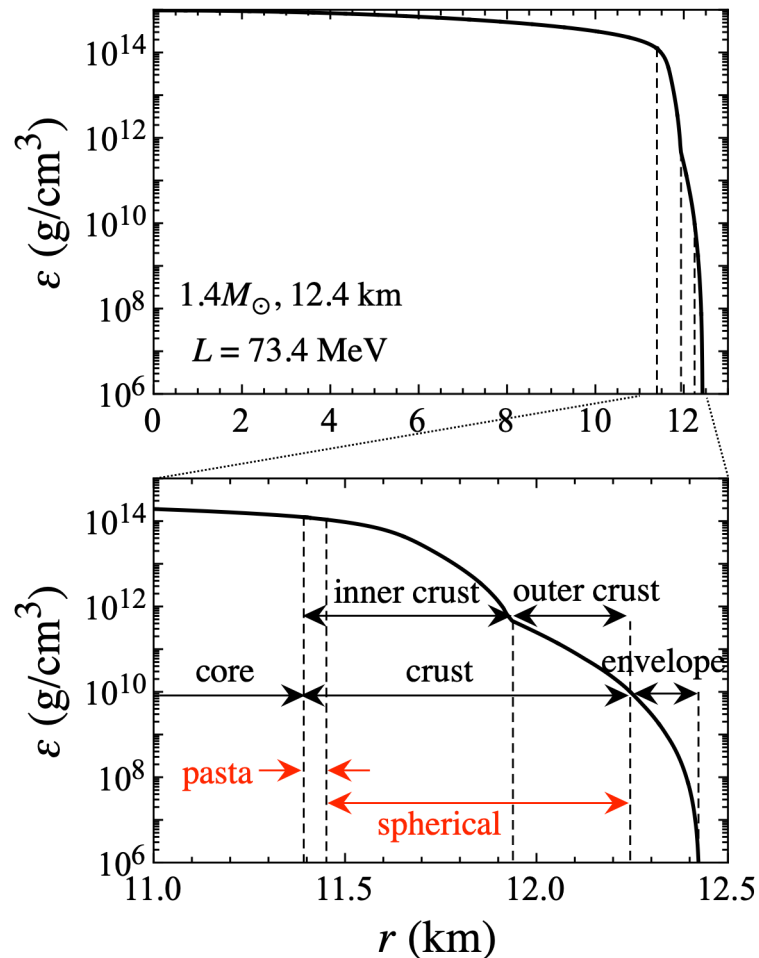
- bubble ~ spherical
- tube ~ cylindrical



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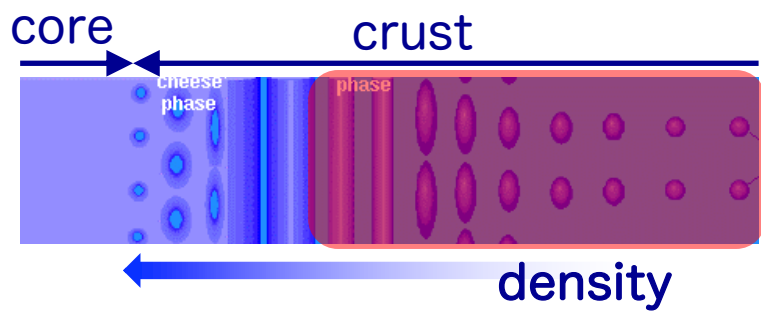
EOSs & NS properties

- crust thickness strongly depends on L and M/R
 - as L and M/R increase, crust thickness decreases

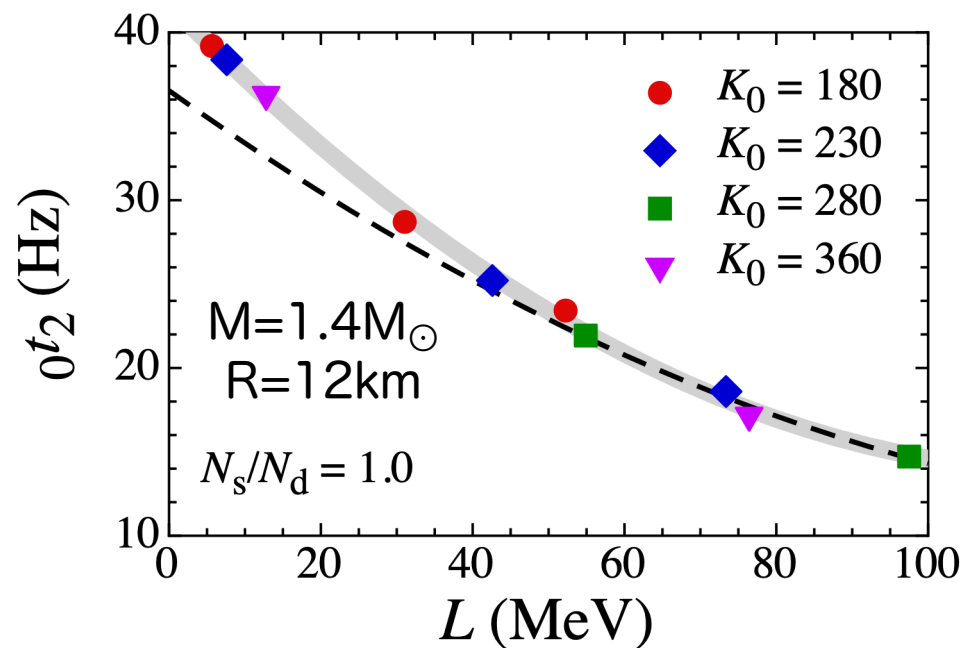


Oyamatsu 93

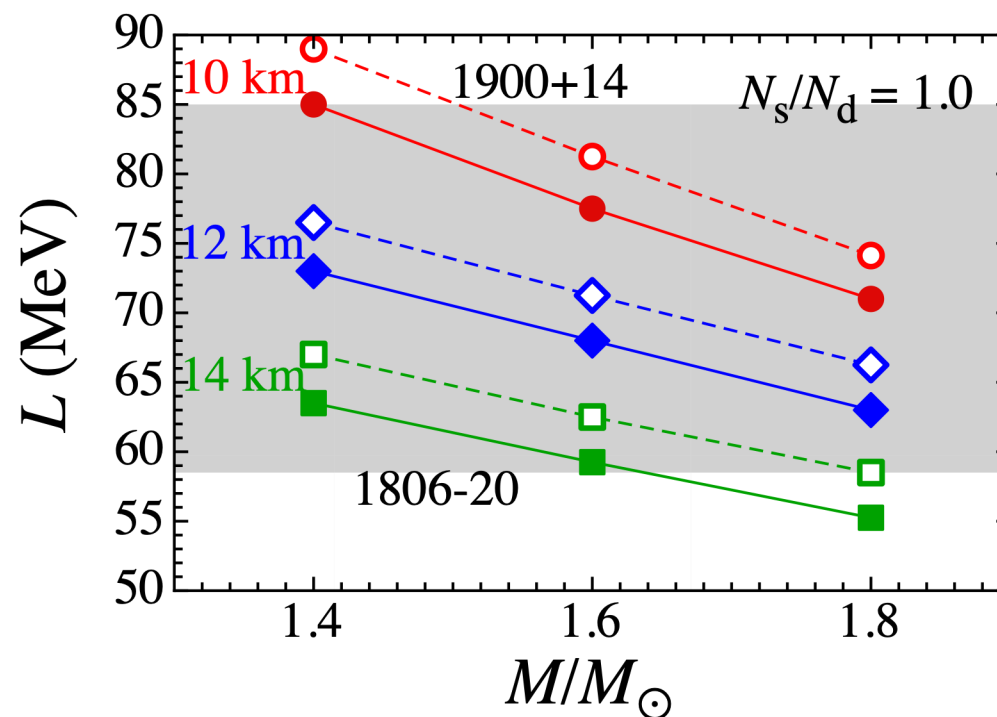
Effect of cylindrical nuclei



- fundamental oscillations weakly depend on the existence of a cylindrical phase.
- all the QPOs except for 26Hz can be identified as the previous calculations.



$$\omega_{t0} \text{ (Hz)} = c_\ell^{(0)} + c_\ell^{(1)} L_{100}^{1/2} + c_\ell^{(2)} L_{100} + c_\ell^{(3)} L_{100}^2,$$

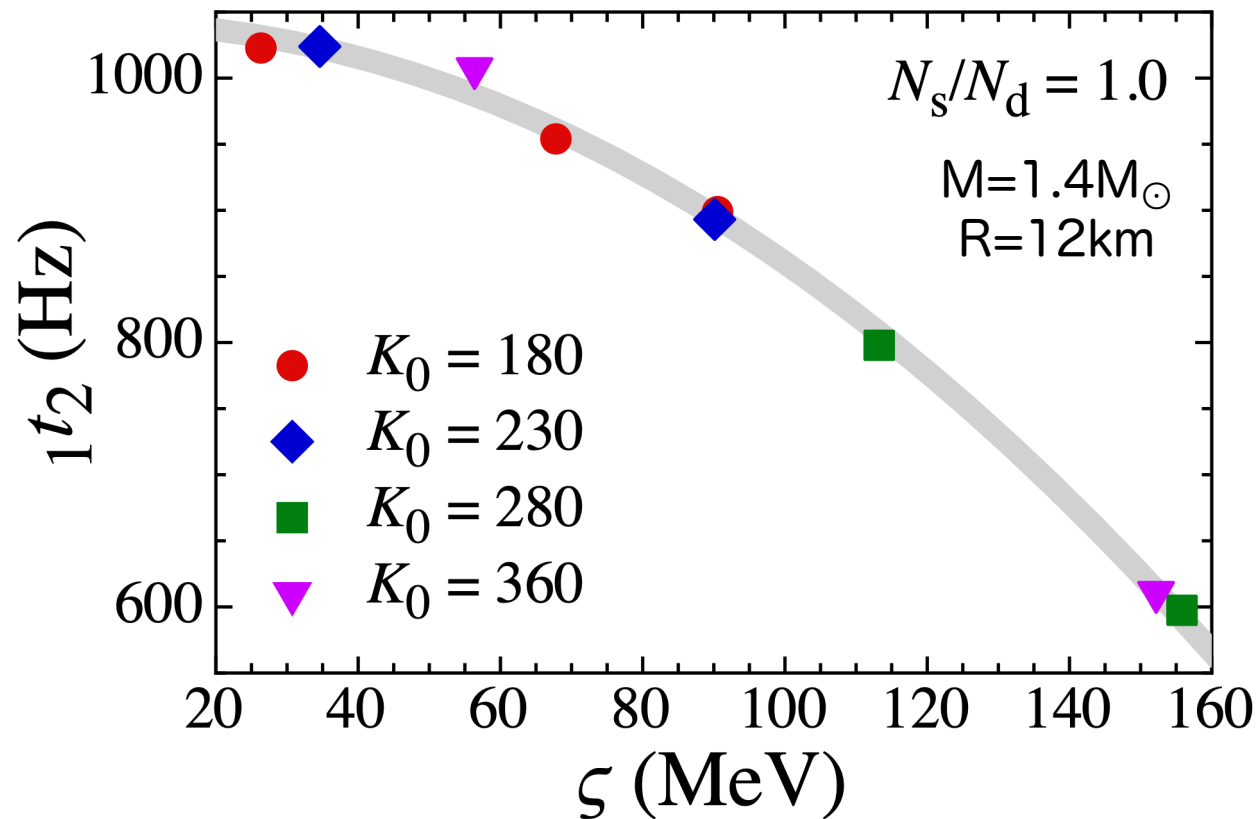


$L = 58 - 85 \text{ MeV (HS+18)}$

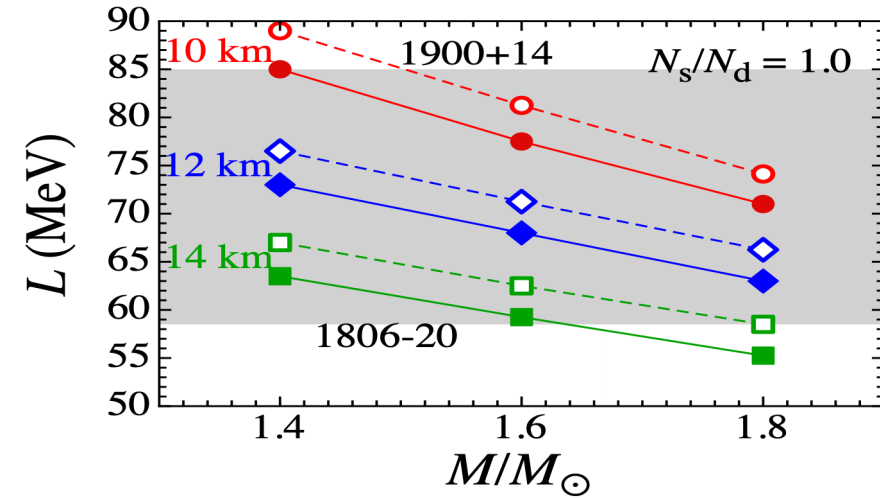
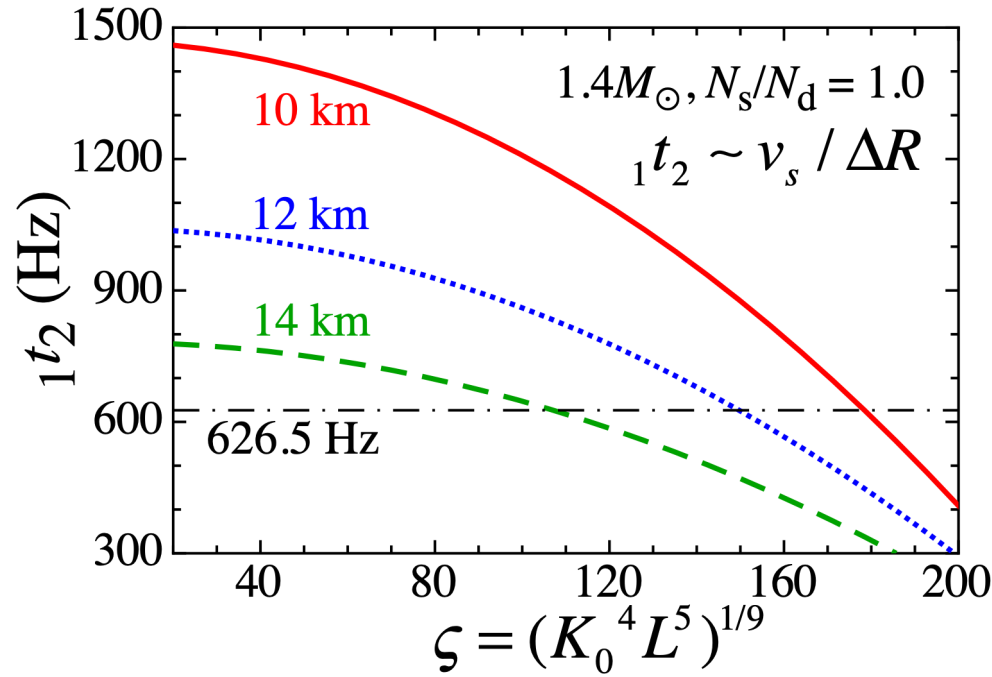
1st overtone

- frequencies of 1st overtone depend on K_0 & L
- we find a good combination of K_0 & L for expressing the 1st overtone frequencies.

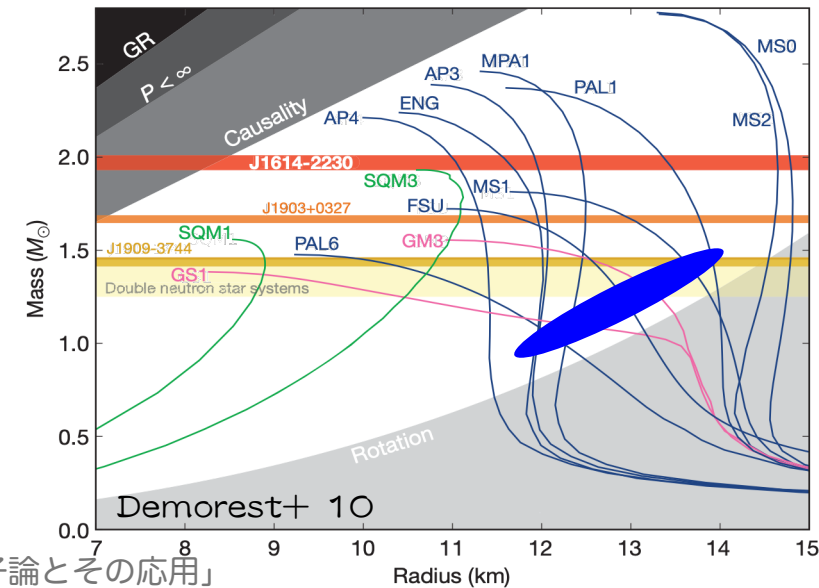
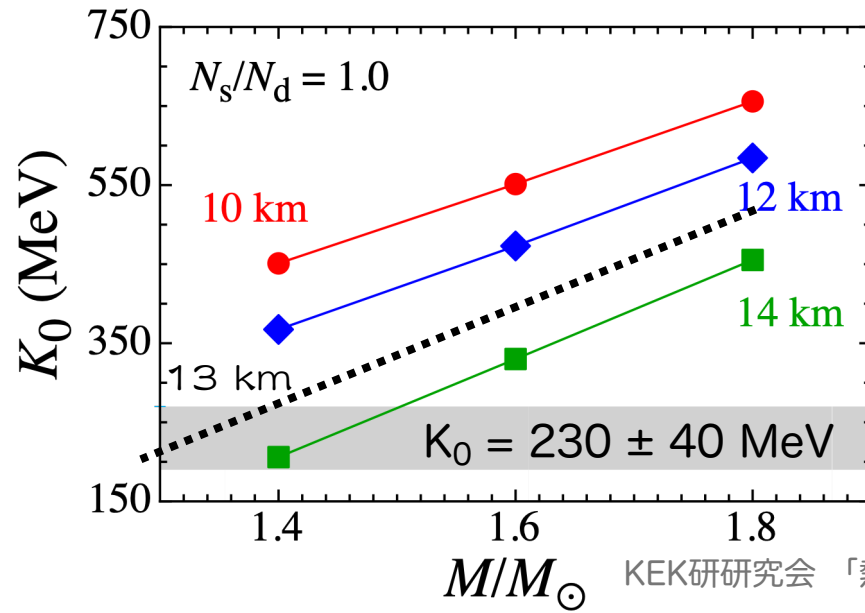
$$\zeta = (K_0^4 L^5)^{1/9}$$



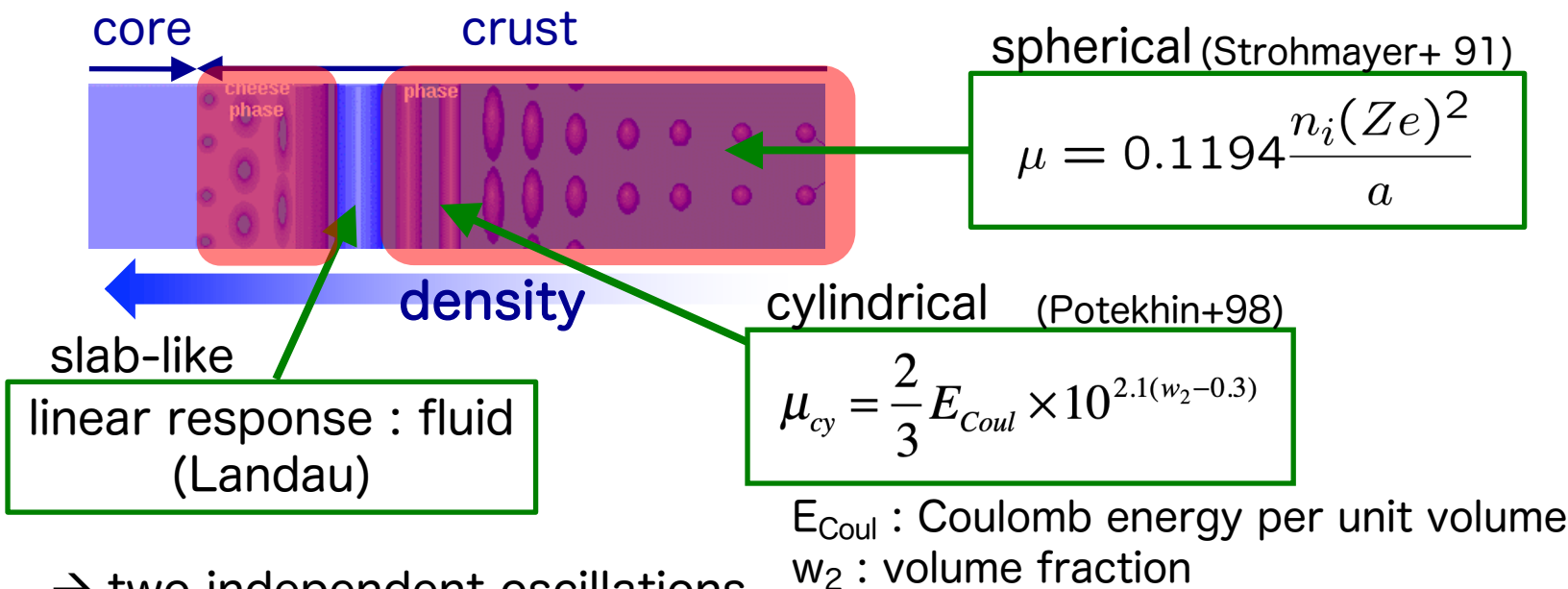
1st overtone & fundamental modes



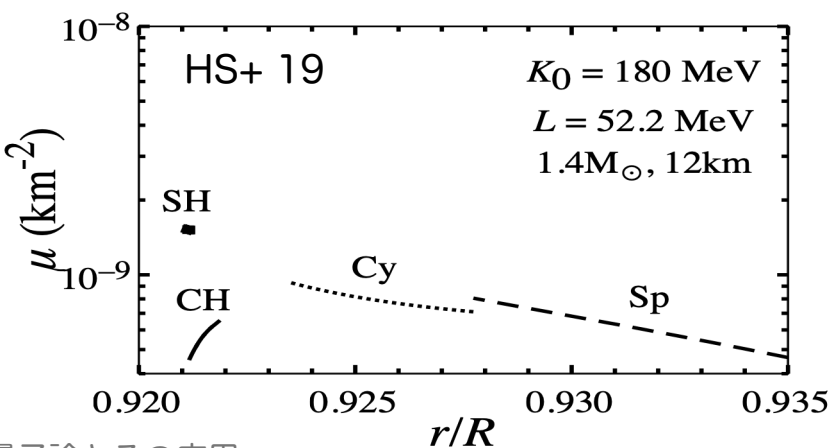
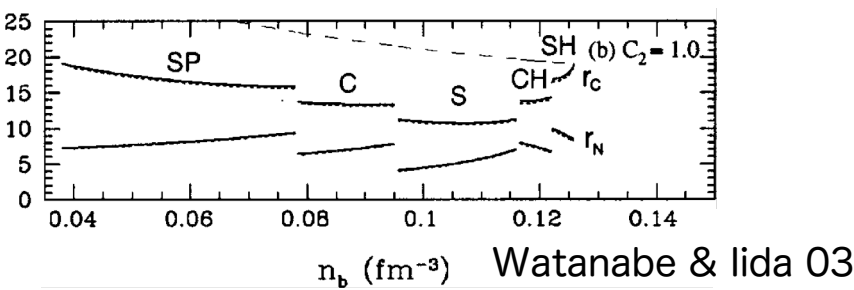
SGR 1806-20 : 13km & $1.3-1.4M_{\odot}$
 $\rightarrow L \simeq 58-73 \text{ MeV}$



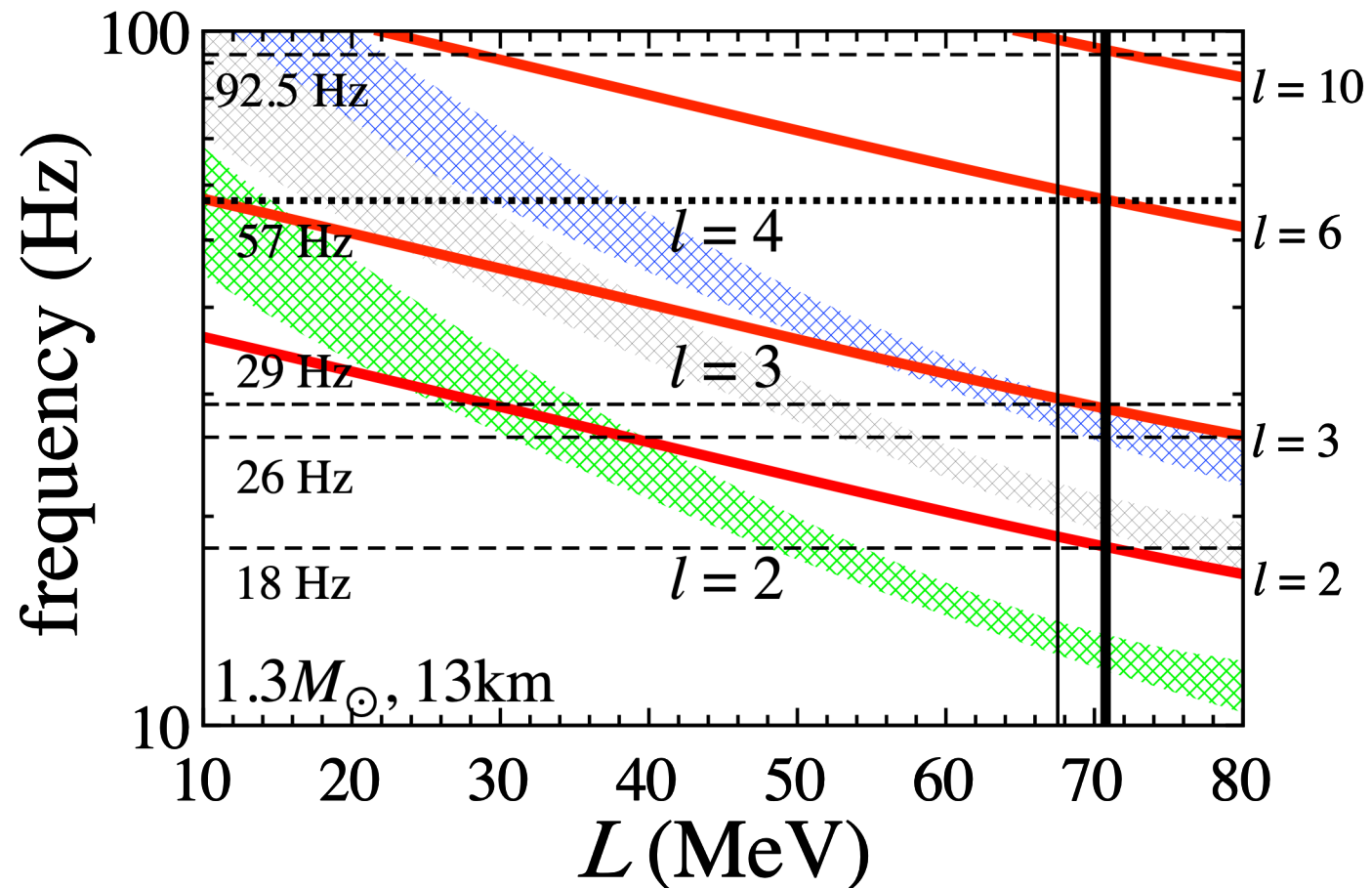
Can we identify the 26Hz QPO?



- two independent oscillations
- (i) spherical + cylindrical (sp+cy) → except for 26Hz (OK)
- (ii) **tube + bubble (tu+bu) → 26Hz? consistently with the L constraint?**
- bubble ~ spherical
- tube ~ cylindrical



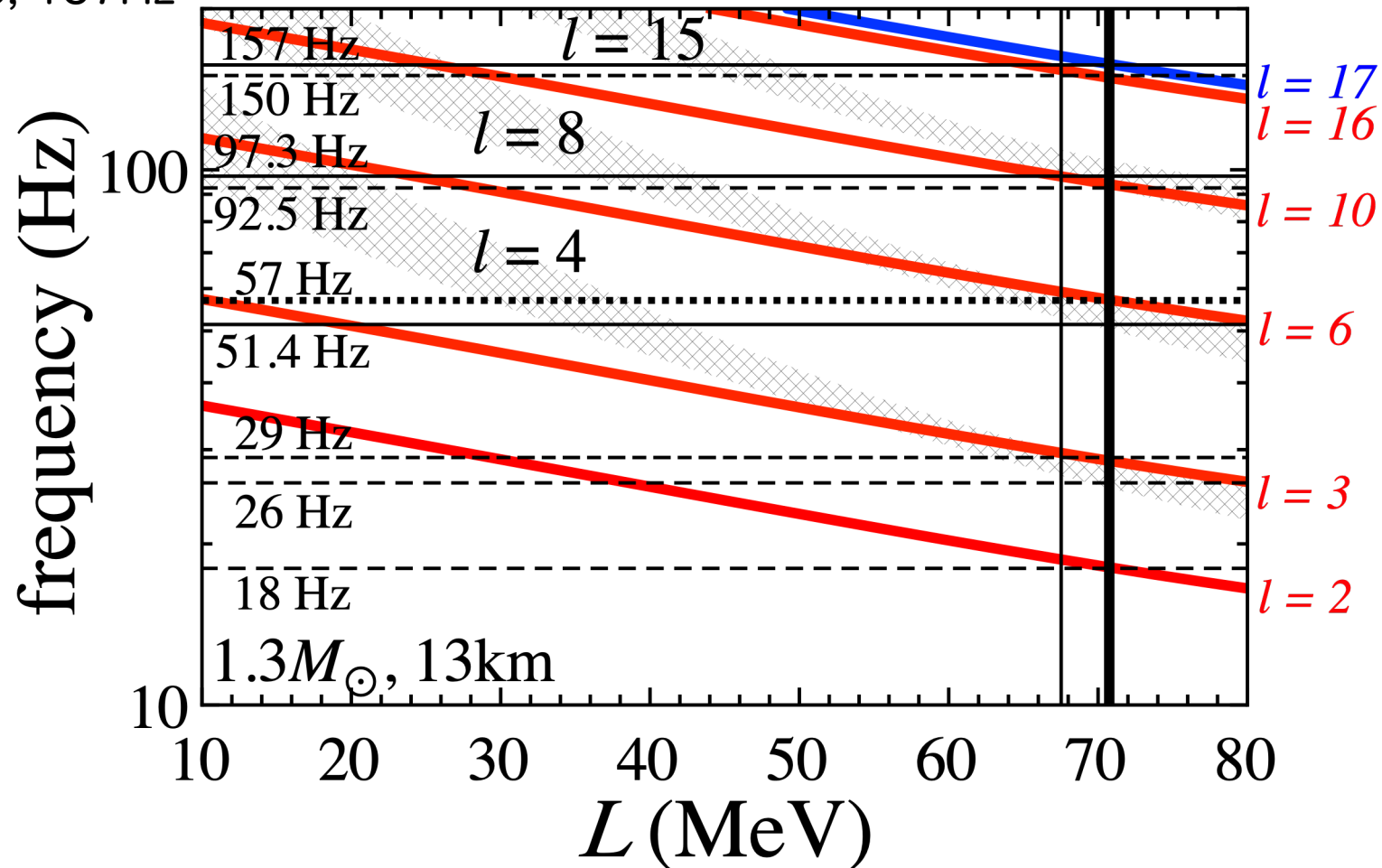
Oscillations in tu+bu & in Sp+Cy_{HS+ 19}



- one can explain the 26Hz QPO with the torsional oscillations in tu+bu phase

Additional QPOs

- recently, the existence of additional QPOs is suggested by Miller et al.
 - 51.4, 97.3, 157Hz



Other objects

Article

Very-high-frequency oscillations in the main peak of a magnetar giant flare

<https://doi.org/10.1038/s41586-021-04101-1>

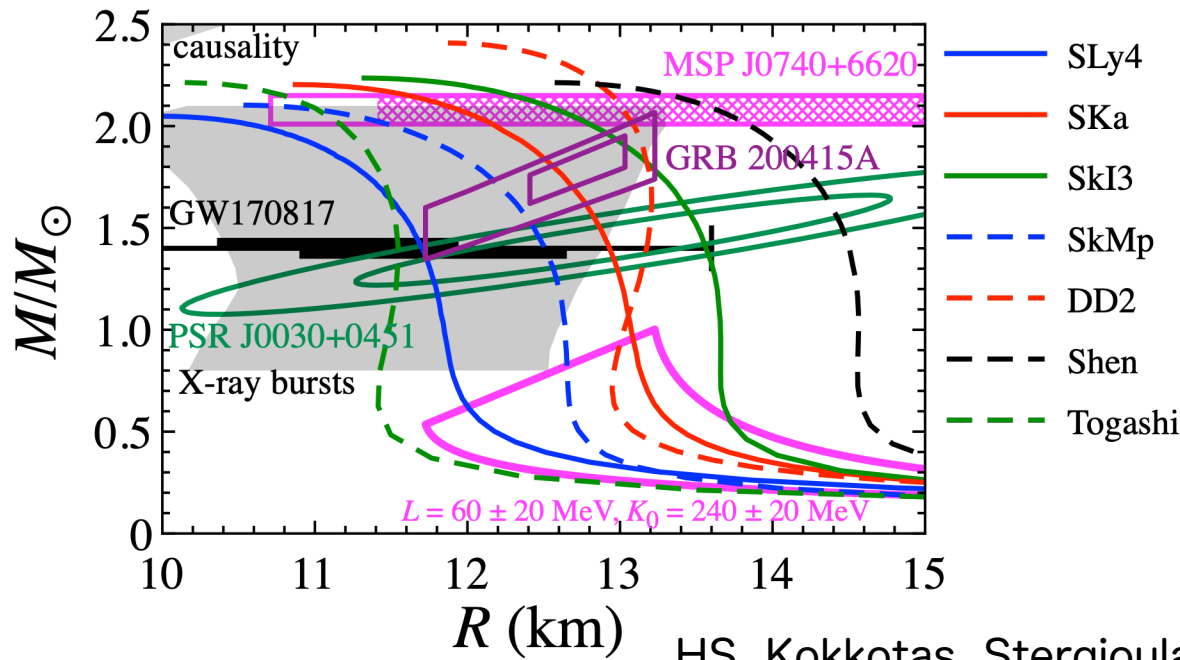
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Check for updates

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HS, Kokkotas, Stergioulas 23

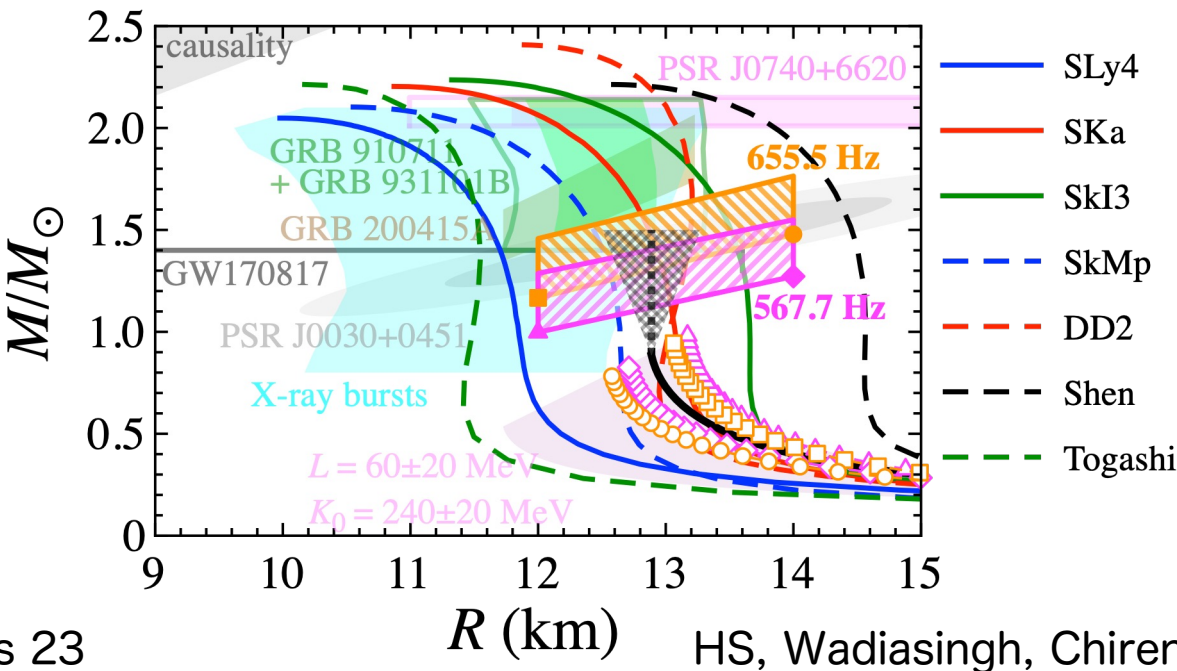
SCIENCE CHINA
Physics, Mechanics & Astronomy



• Article •

April 2026 Vol. 69 No. 4: 249512
<https://doi.org/10.1007/s11433-025-2866-2>

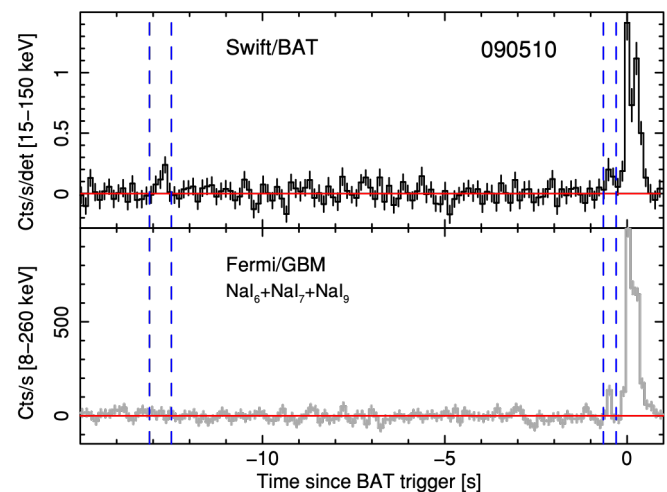
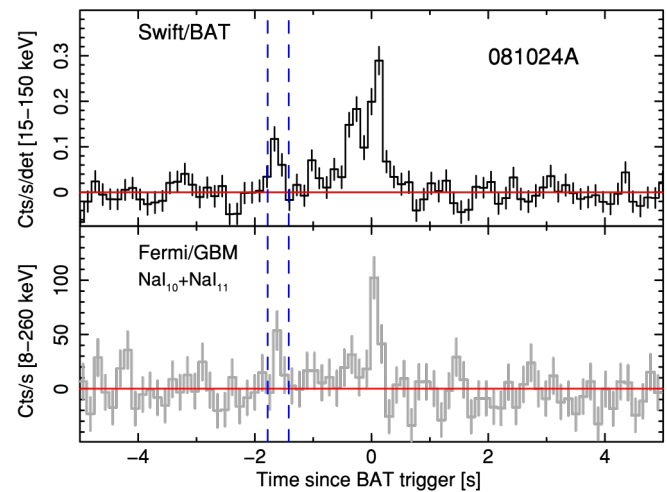
A comprehensive search for long and short periodic features from an extremely active cycle of FRB 20240114A



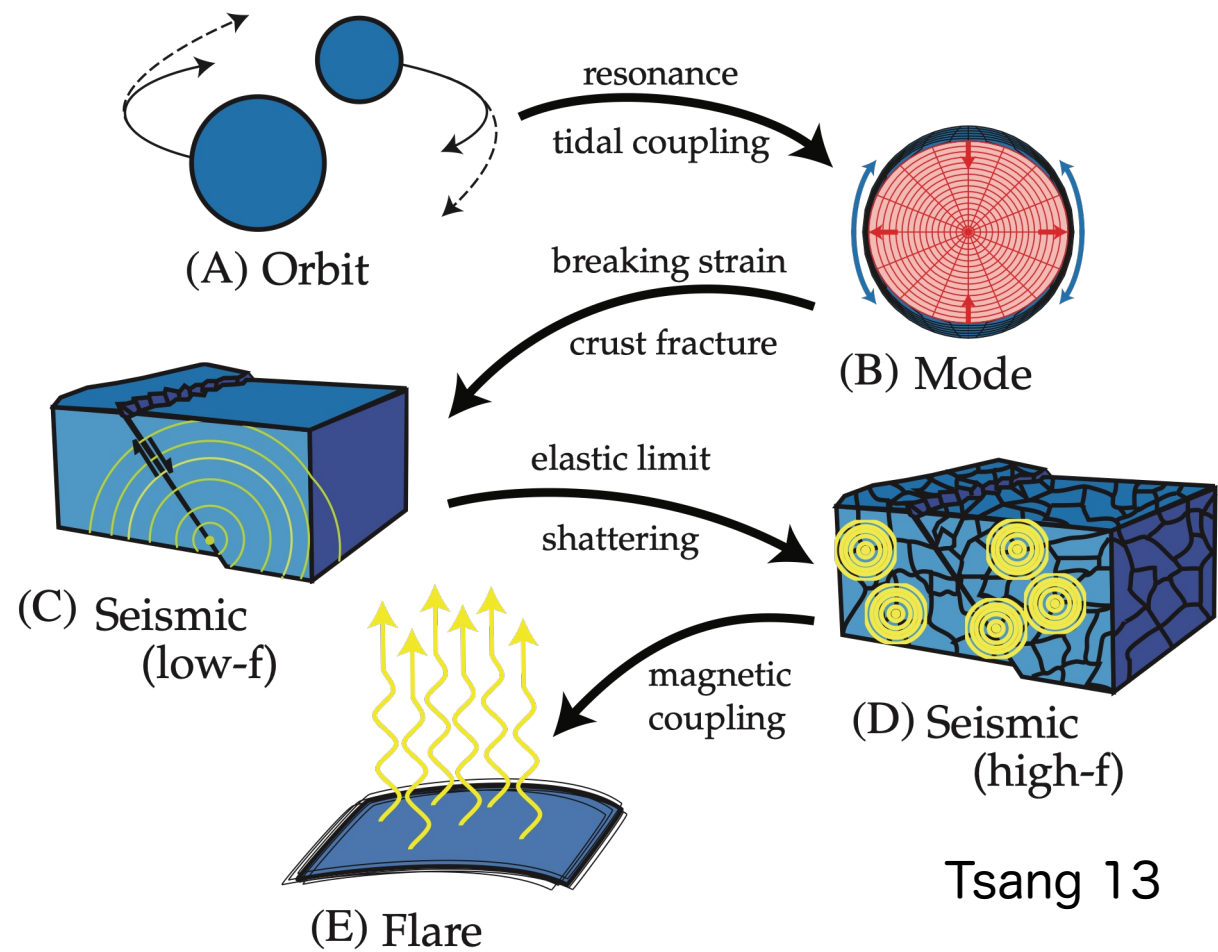
HS, Wadiasingh, Chirenti 26

Resonant shattering

- Precursors 1–10 s prior to the main flare were detected with high significance for three SGRBs out of the 49 (Troja+10)



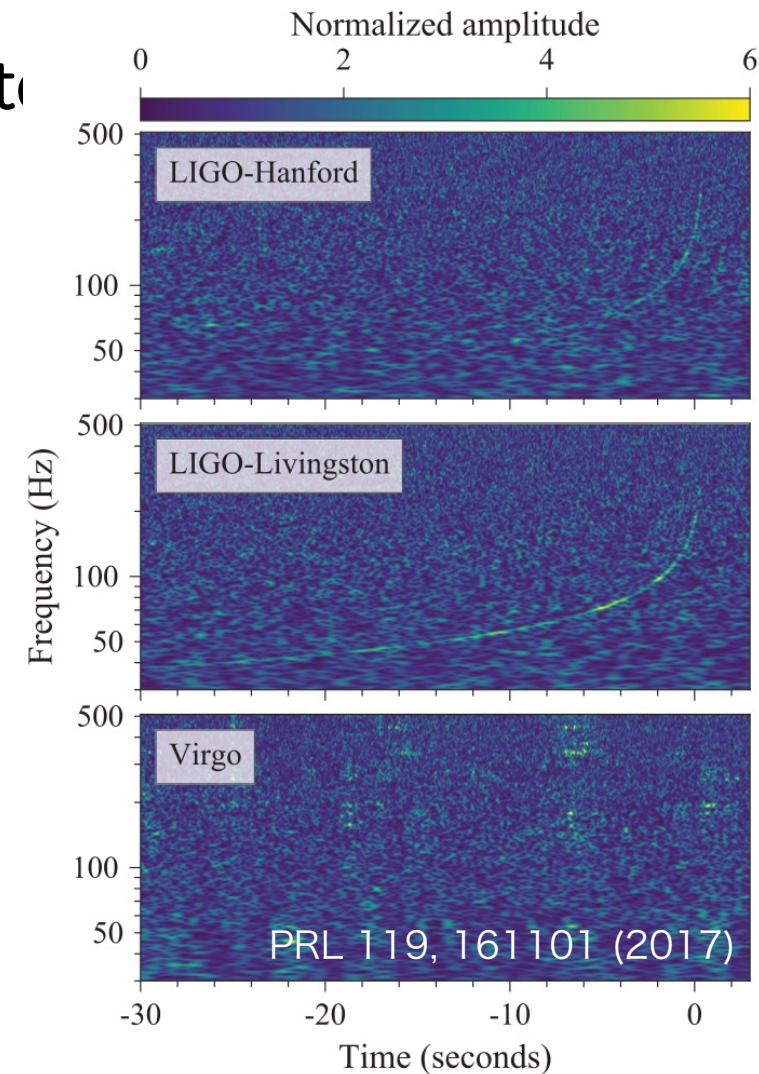
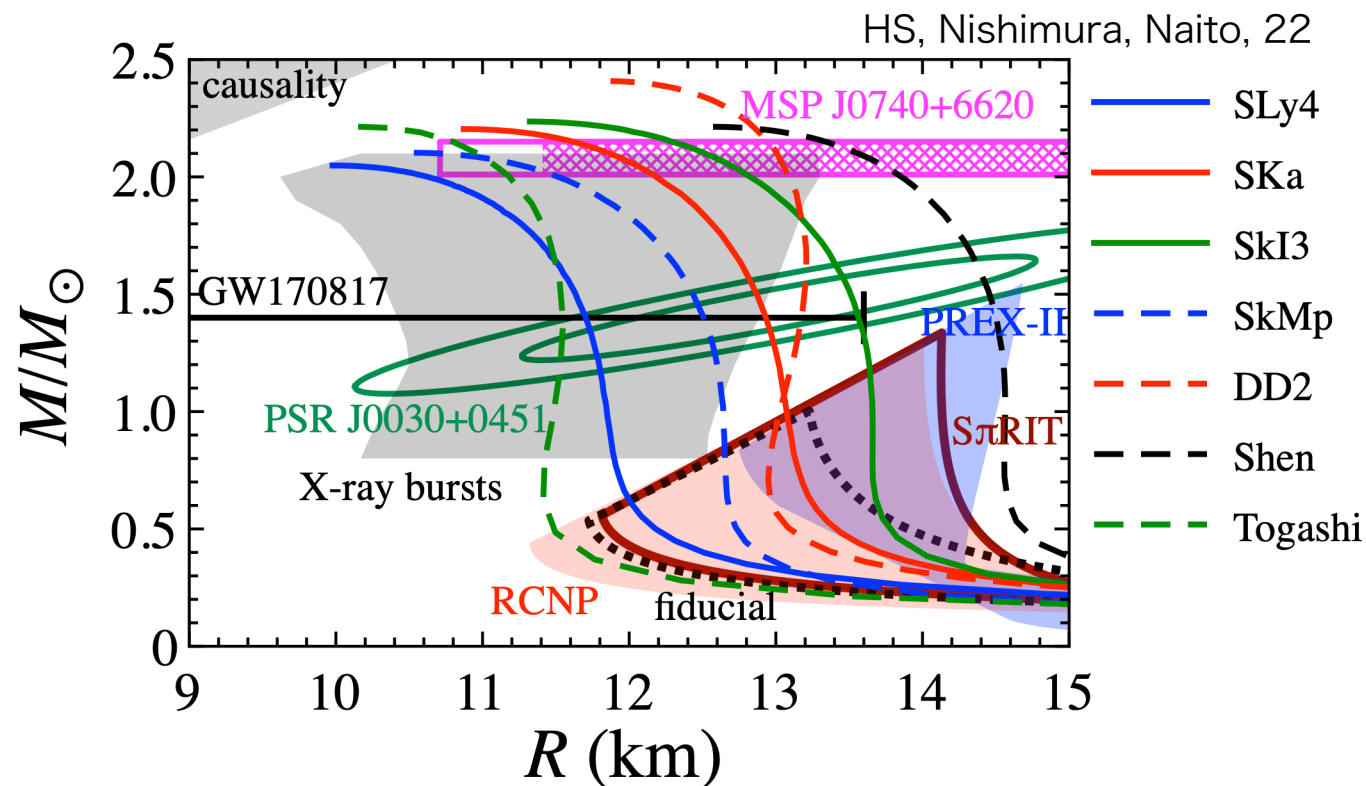
Troja+10



Tsang 13

Dawn of GW astronomy

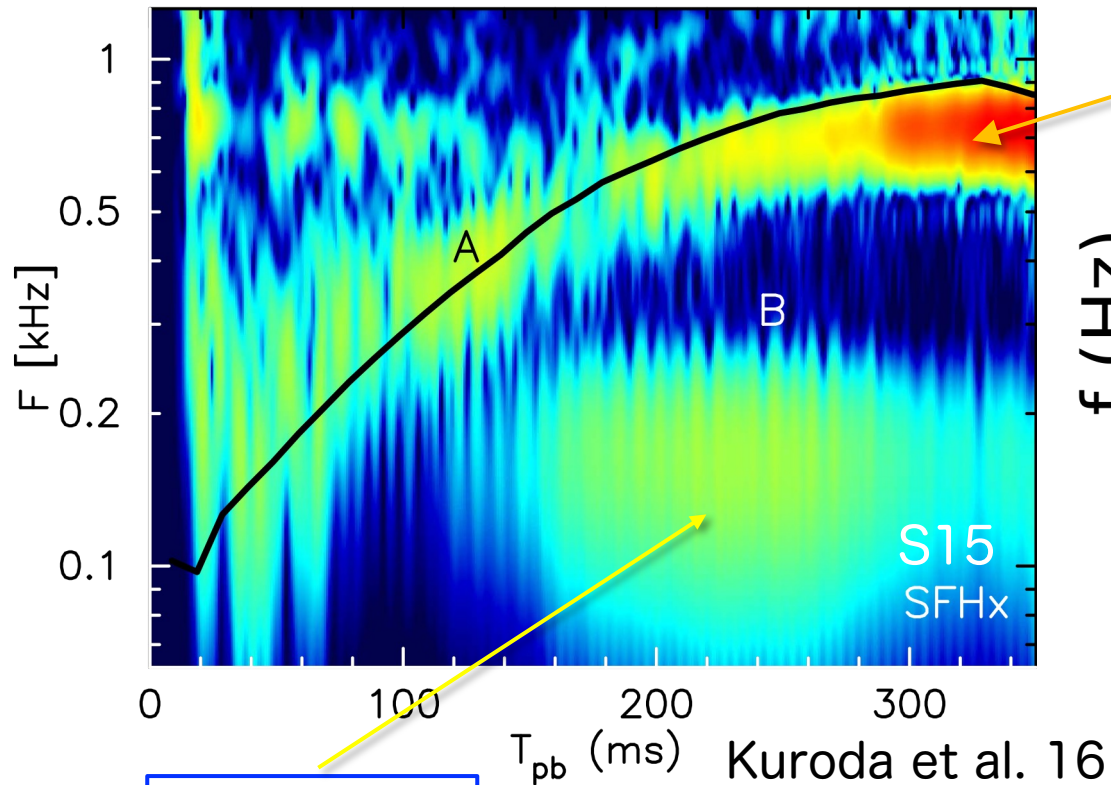
- GWs from compact binary mergers have been detected
 - ✓ GWs become a new tool for extracting astronomical information.
- The next candidate must be a supernova explosion.



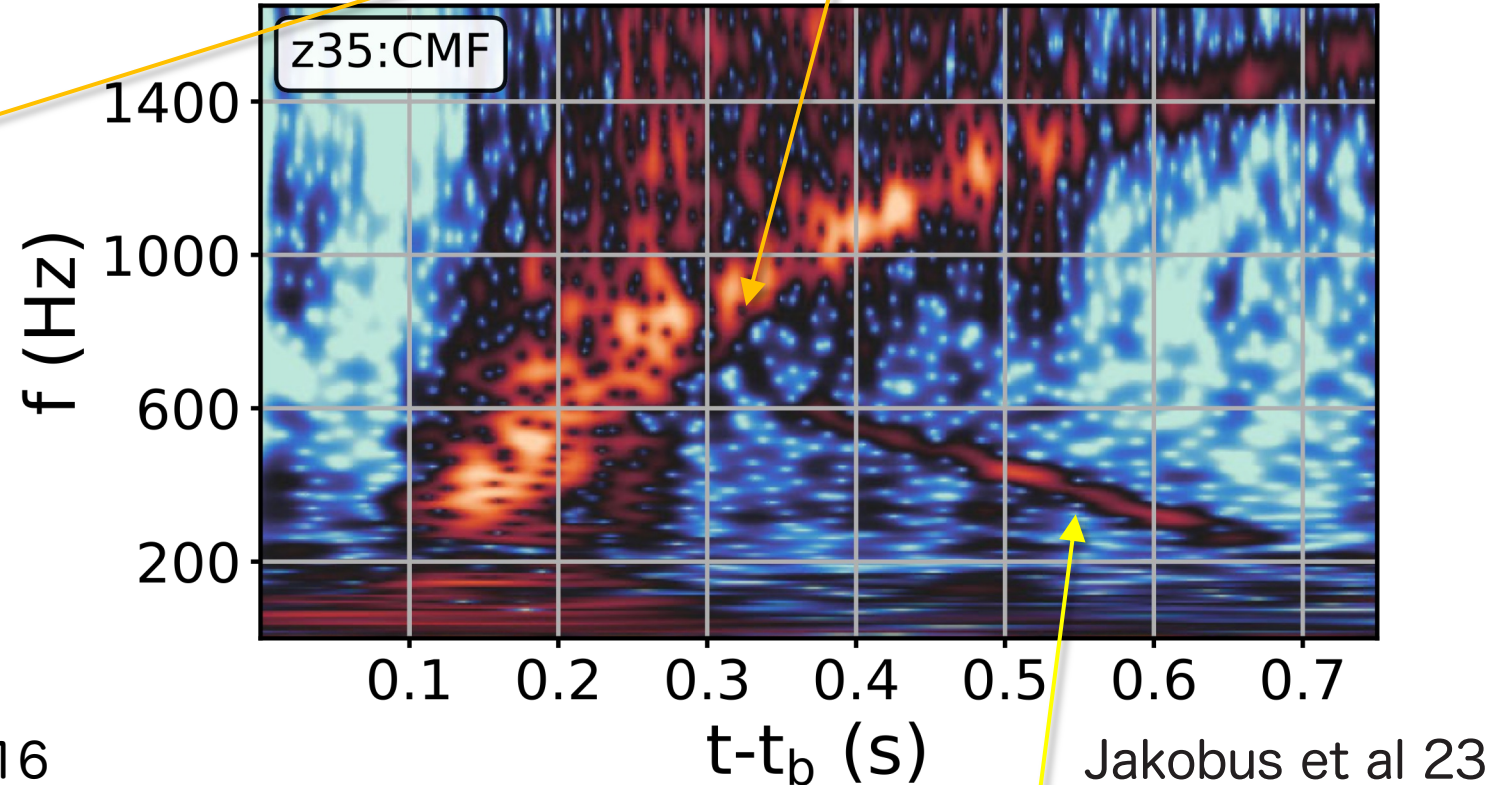
Next candidate of GW sources

- core-collapse supernovae
 - ✓ compared to the binary merger, the system is almost spherically symmetric
 - weak gravitational waves
 - we may be able to detect only the event happened in our galaxy
 - ✓ many numerical simulations show the existence of GW signals
 - SN GWs depend on the SN models, such as progenitor mass, EOS, and gravity
 - it may be difficult to extract physics of PNS from the GW signals.
- We adopt the **perturbation approach, the so-called asteroseismology**, to see the physic behind the GWs by identifying them with the specific frequency of PNS.

Supernova gravitational waves



association
with SASI

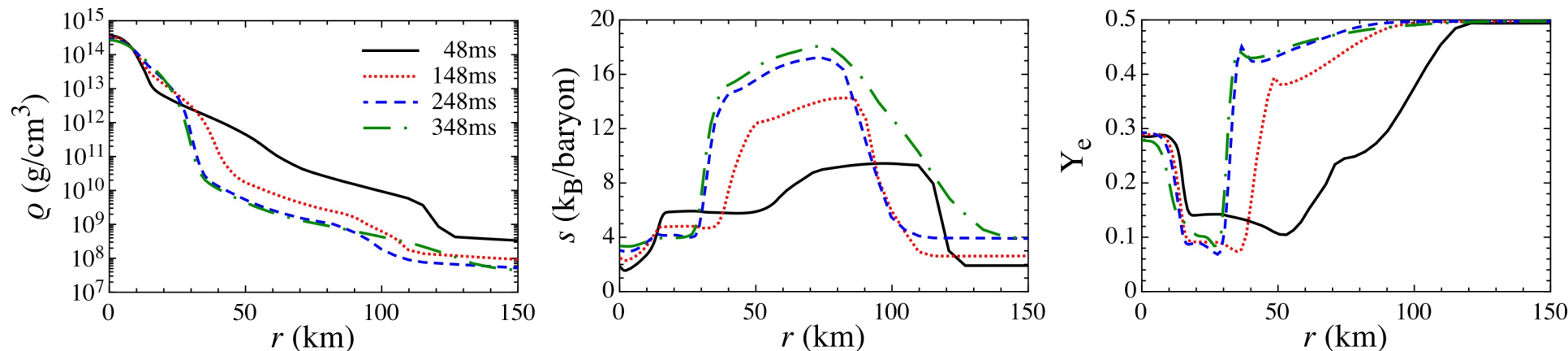


ramp-up signals
a few 100Hz to kHz

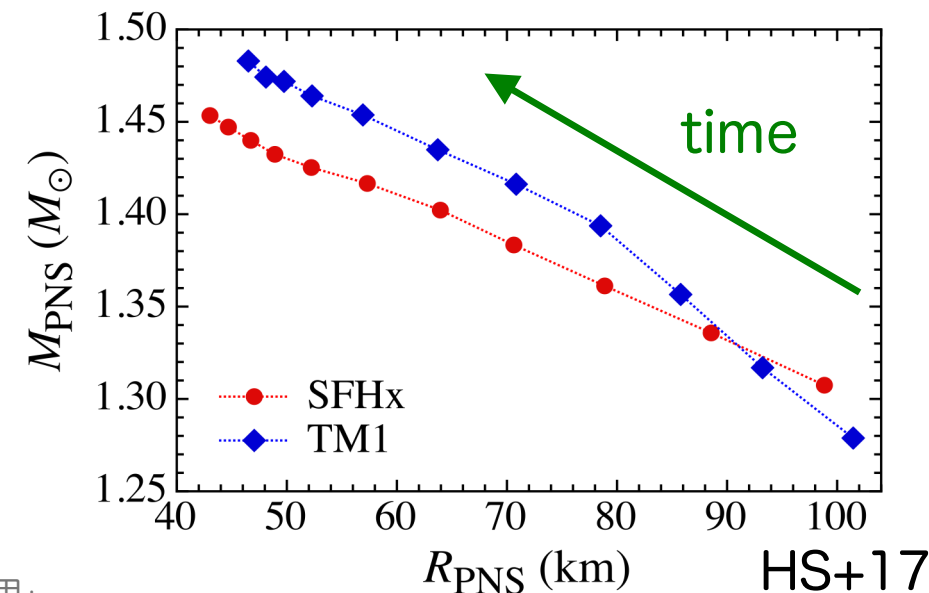
another distinct frequency band

Asteroseismology on SN GWs

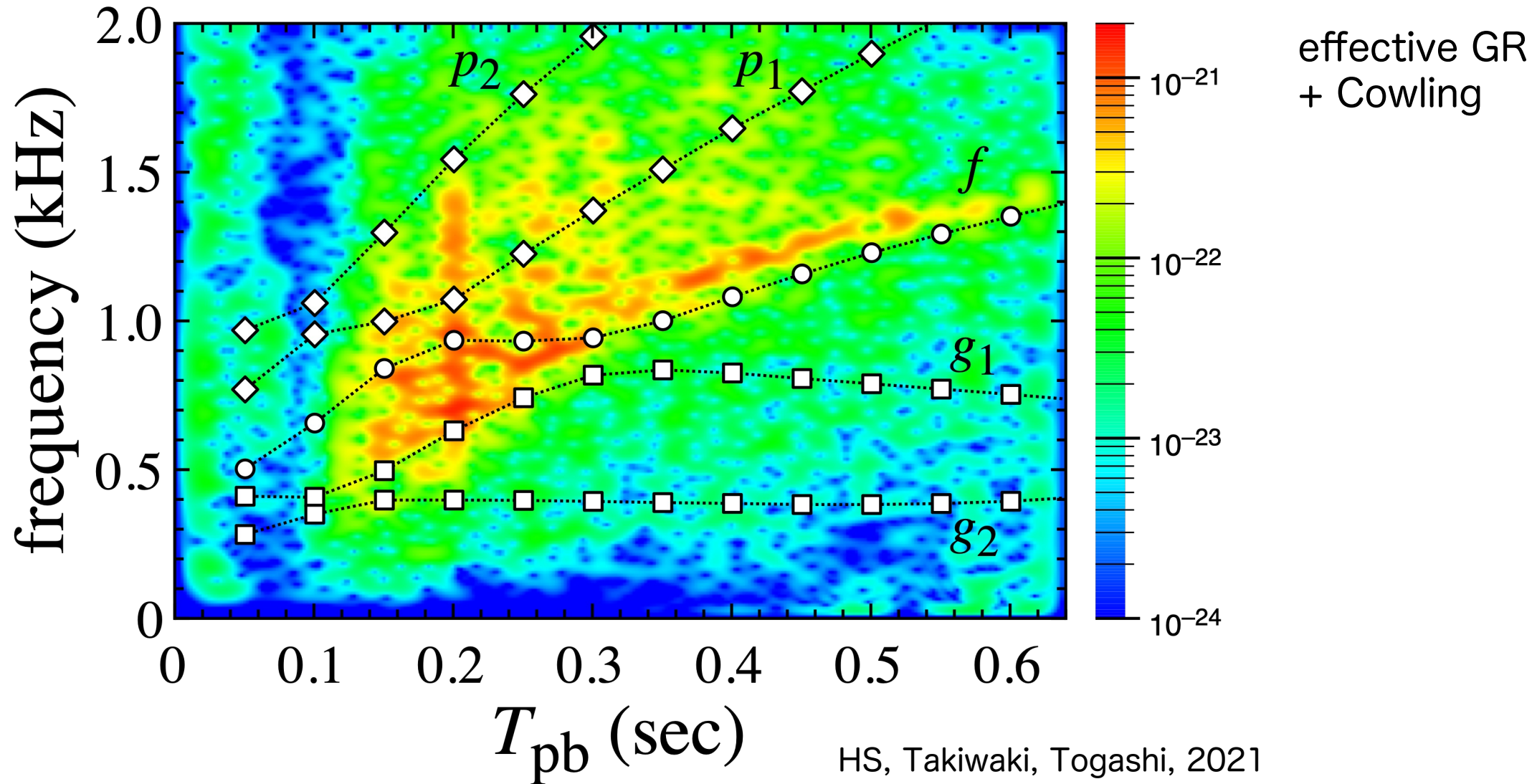
- PNS structure depends not only on the density and pressure profiles but also on the distribution of electron fraction and entropy (or temperature)



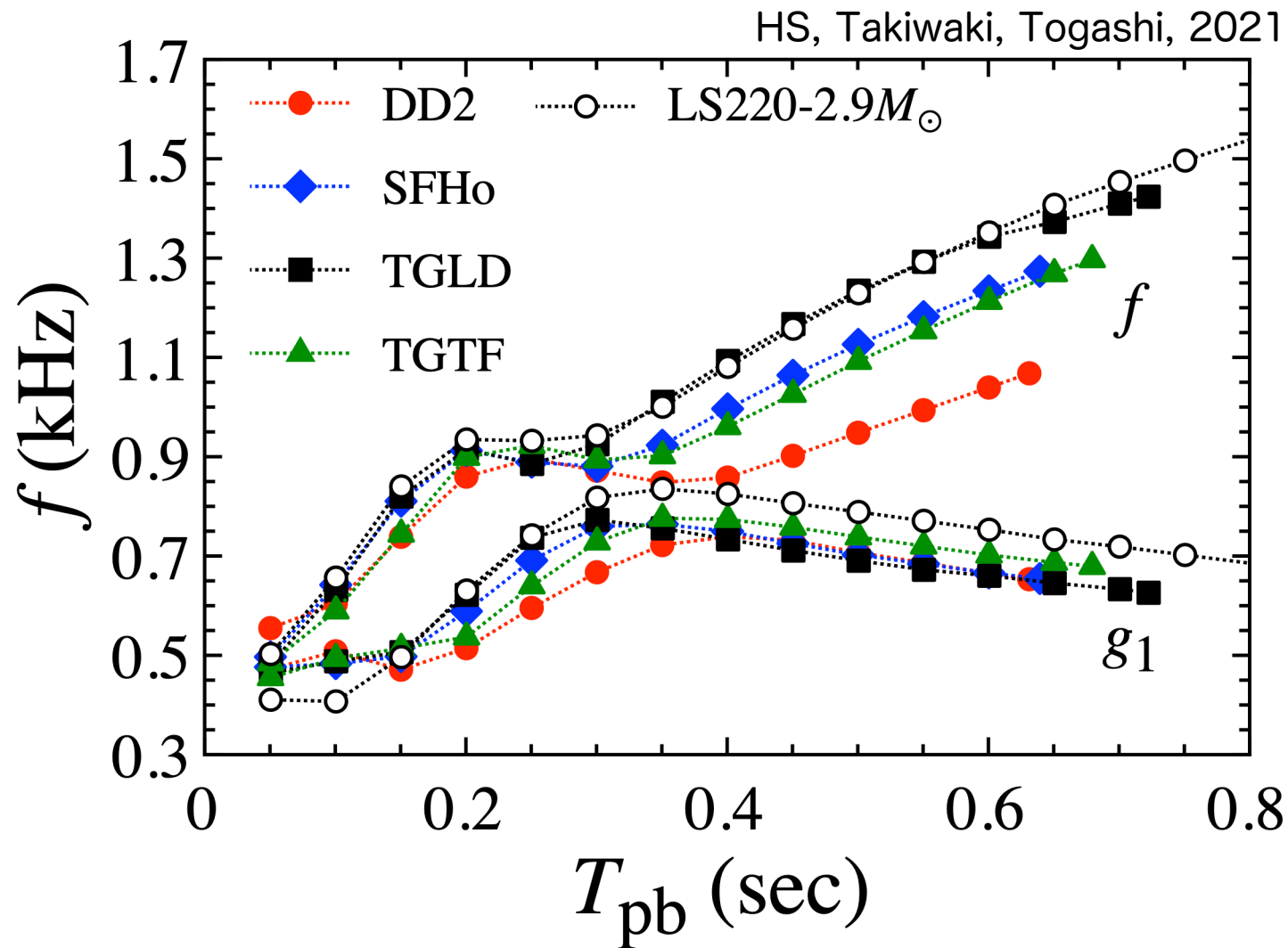
- Using the numerical data for core-collapse SNe, first one has to prepare background models, on which the linear perturbations are considered.
 - ✓ spherically symmetric background models are prepared by averaging in the angular direction
- GW frequencies are determined by solving the eigenvalue problem



Comparison with GW signals in simulation



Dep. of GW signals on PNS models



time evolution of GW frequencies strongly depend on the supernova models

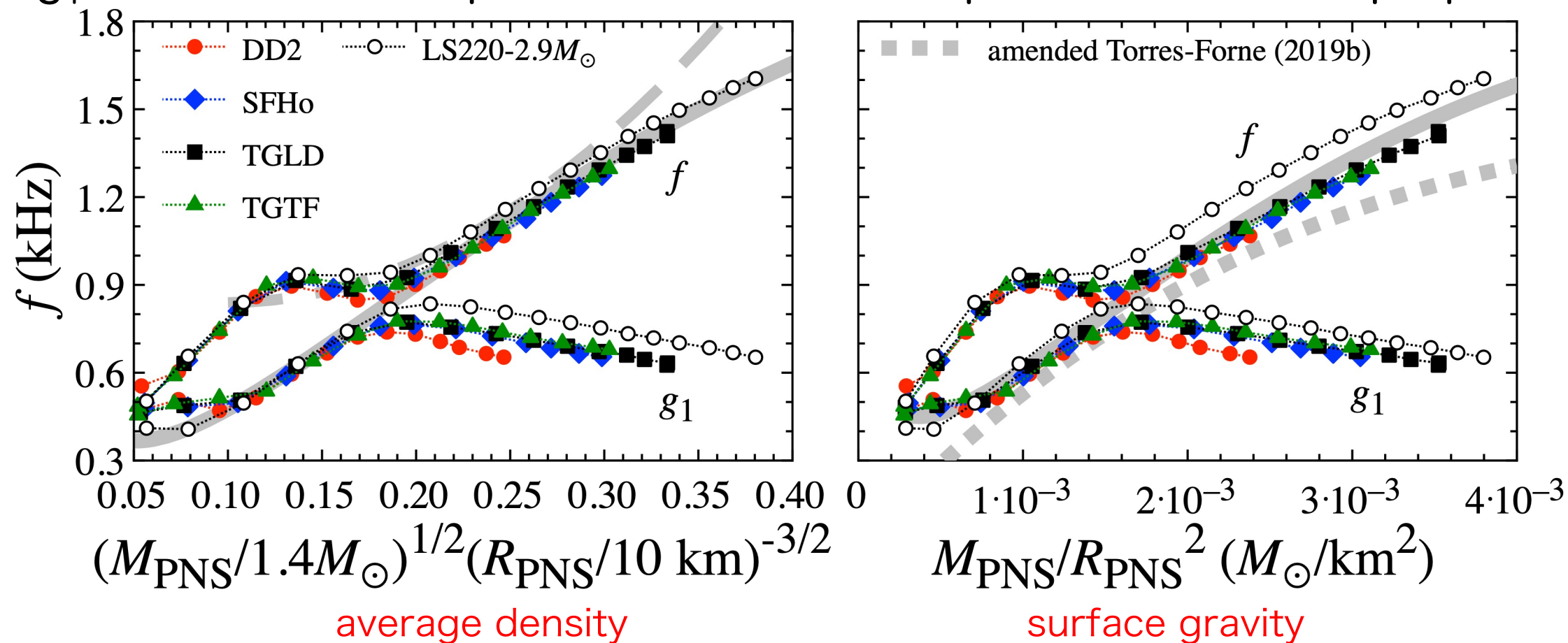
TGTF : single nucleus approximation
TGLD : nuclear statistical equilibrium

↓
different treatment in EOS for lower density region can change the GW spectra

Universal relations

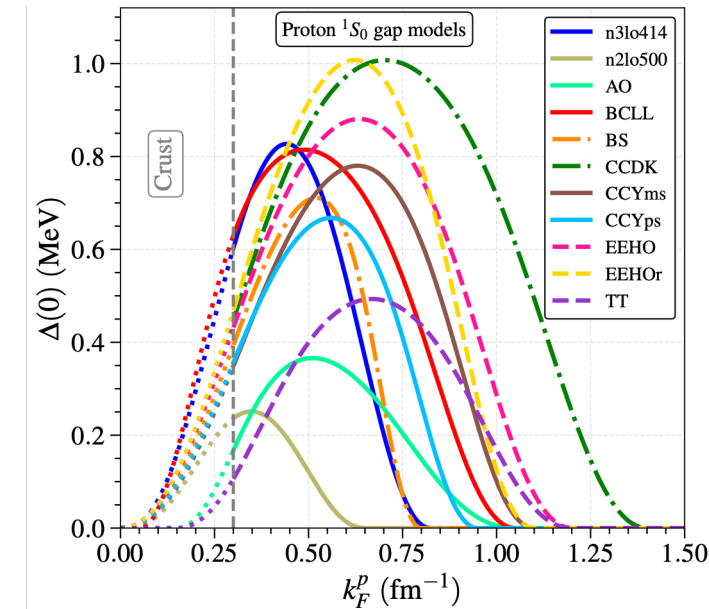
HS, Takiwaki, Togashi, 2021

- The g_1 - and f -mode frequencies can be well expressed as a PNS properties



$$f \text{ (kHz)} = -1.410 - 0.443 \ln(x) + 9.337x - 6.714x^2$$

Proton superconductivity inside NS core

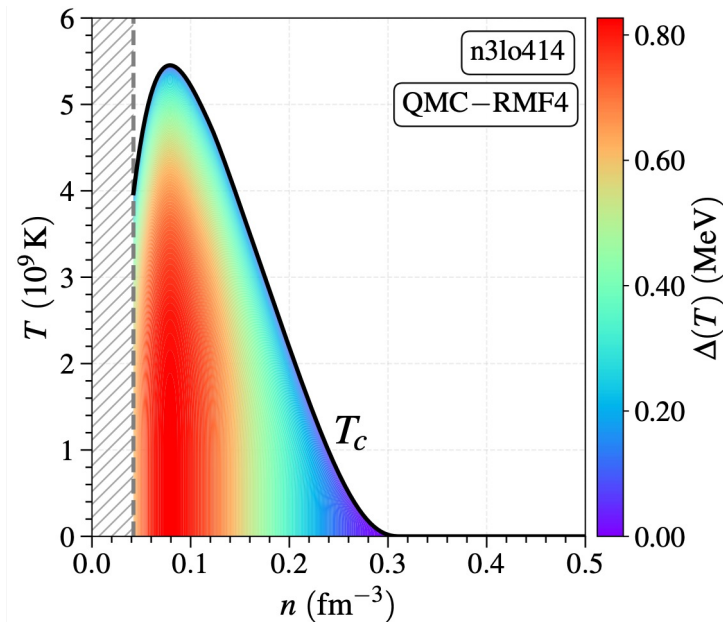


zero-temperature gap

→ critical temperature

$$T_c(n) = \frac{\Delta(0, n)}{1.76 k_B}$$

from the standard weak-coupling BCS relation



finite-temperature gap

$$\Delta(T, n) = \Delta(0, n) R(\tau(T, n))$$

$$\tau \equiv T/T_c(n)$$

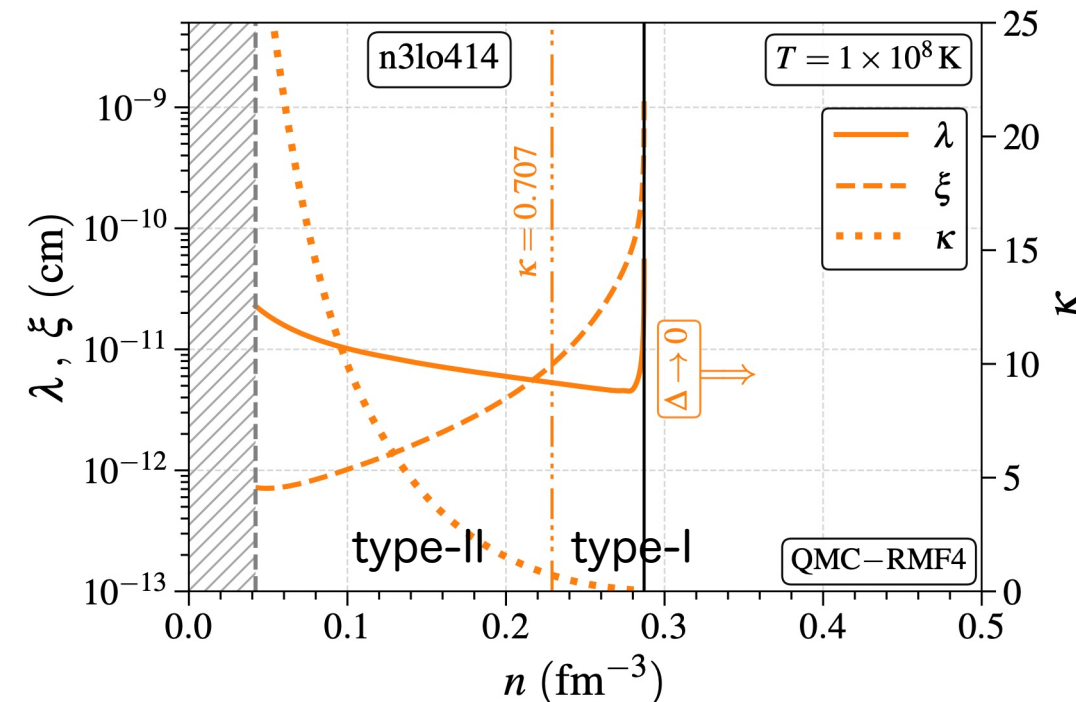
$R(\tau)$ is taken from the analytic approximations of Ferrell for the BCS temperature dependence of the superconducting gap

- the London penetration depth: $\lambda(T, n)$
- the coherence length: $\xi(T, n)$
- the Ginzburg-Landau parameter:

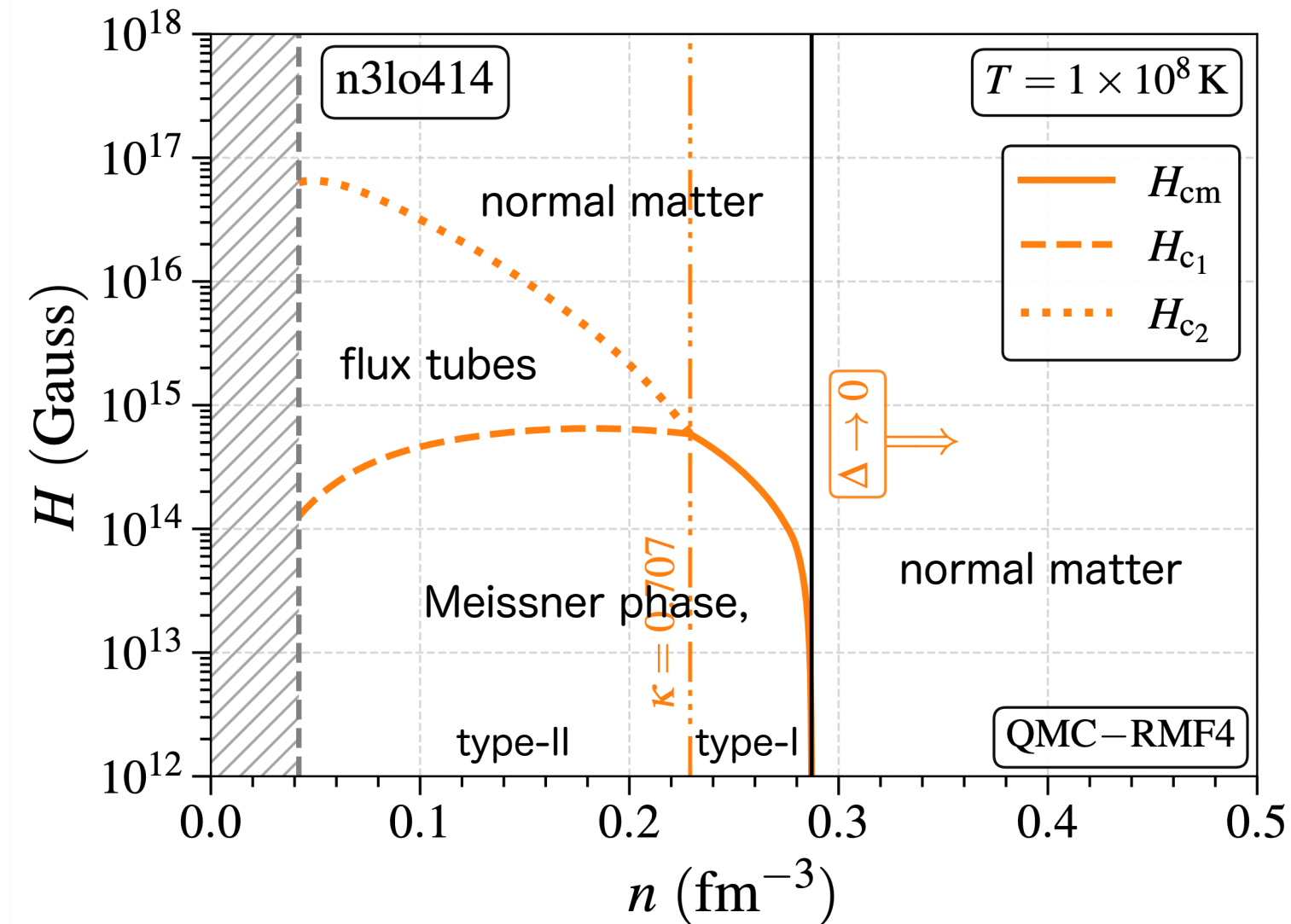
$$\kappa(T, n) \equiv \lambda/\xi$$

$\kappa(T, n) < 1/\sqrt{2}$: type-I superconductor

$\kappa(T, n) > 1/\sqrt{2}$: type-II superconductor

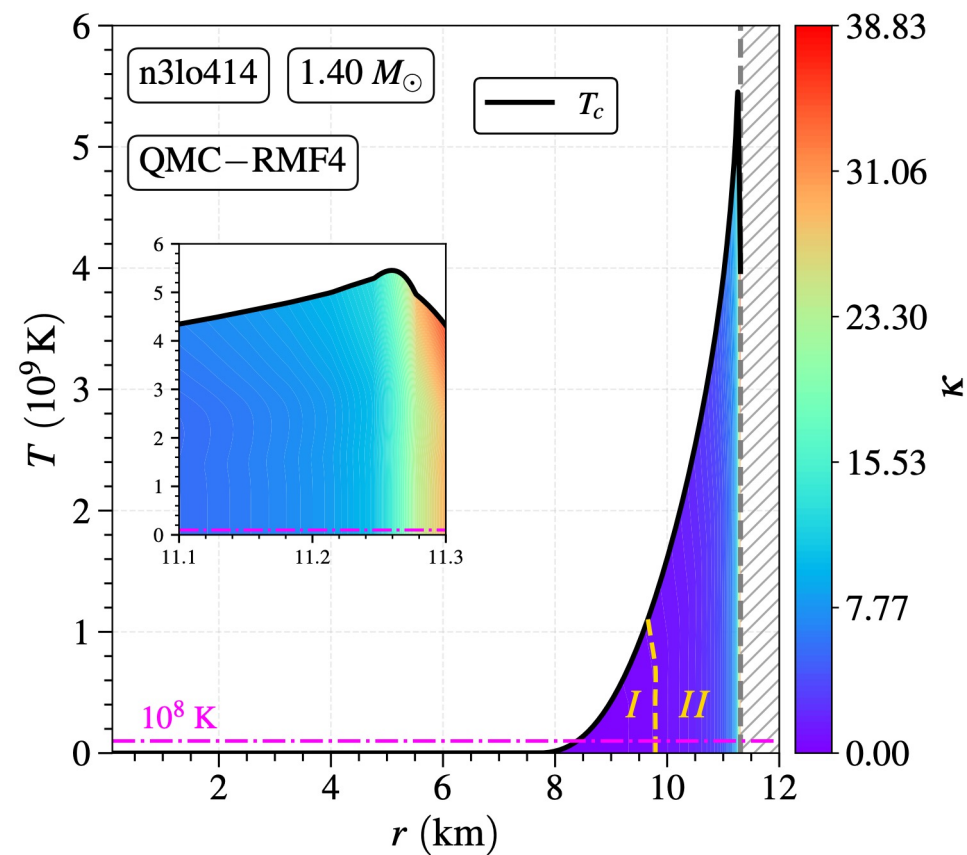


Magnetic suppression

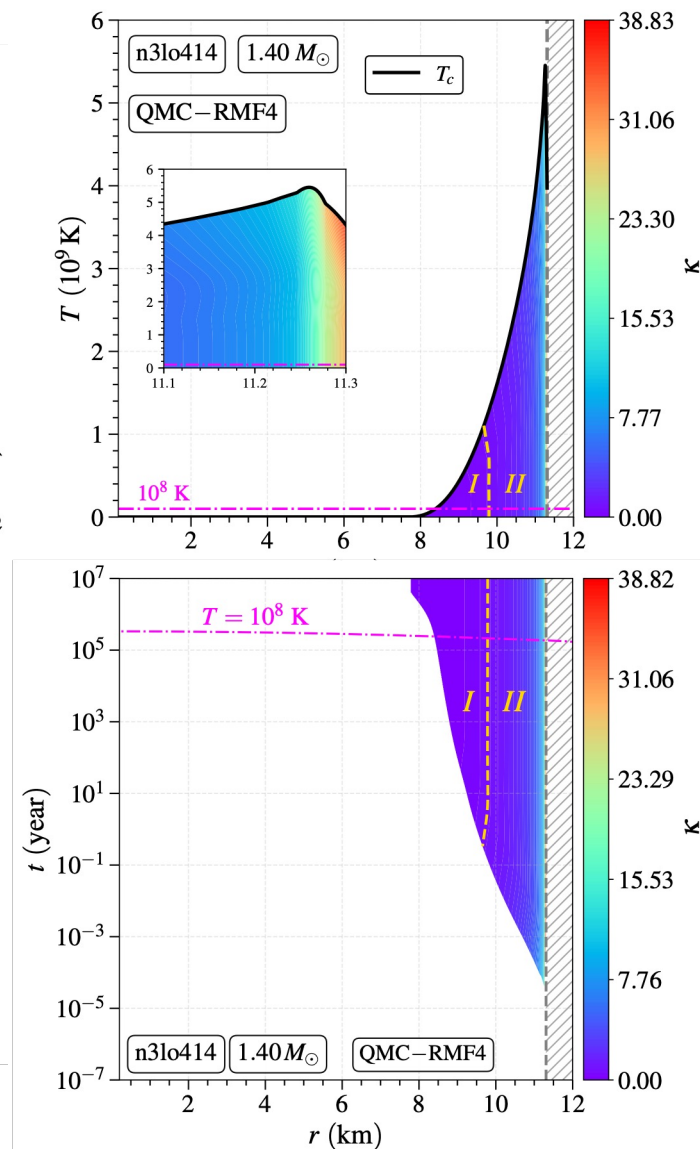
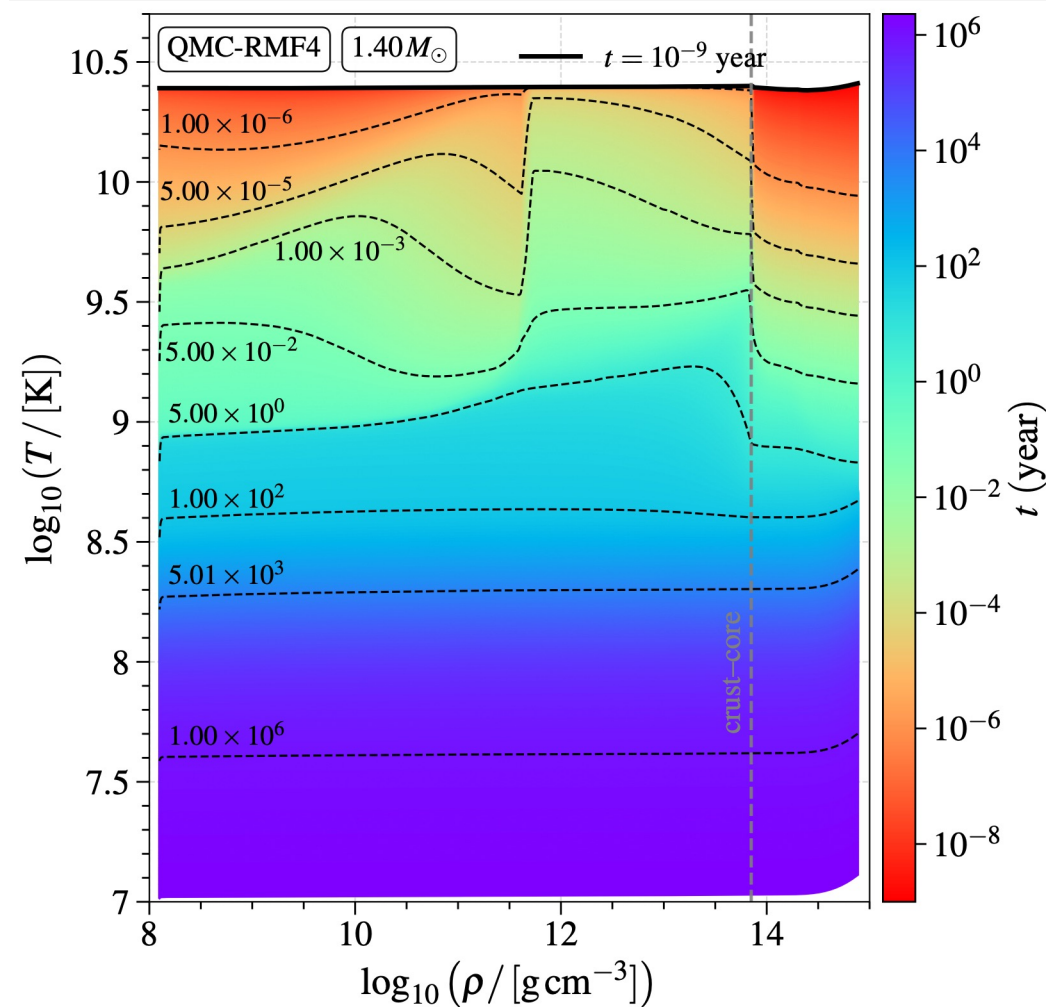


$$H_{cm} = H_{cm}(\lambda, \kappa)$$

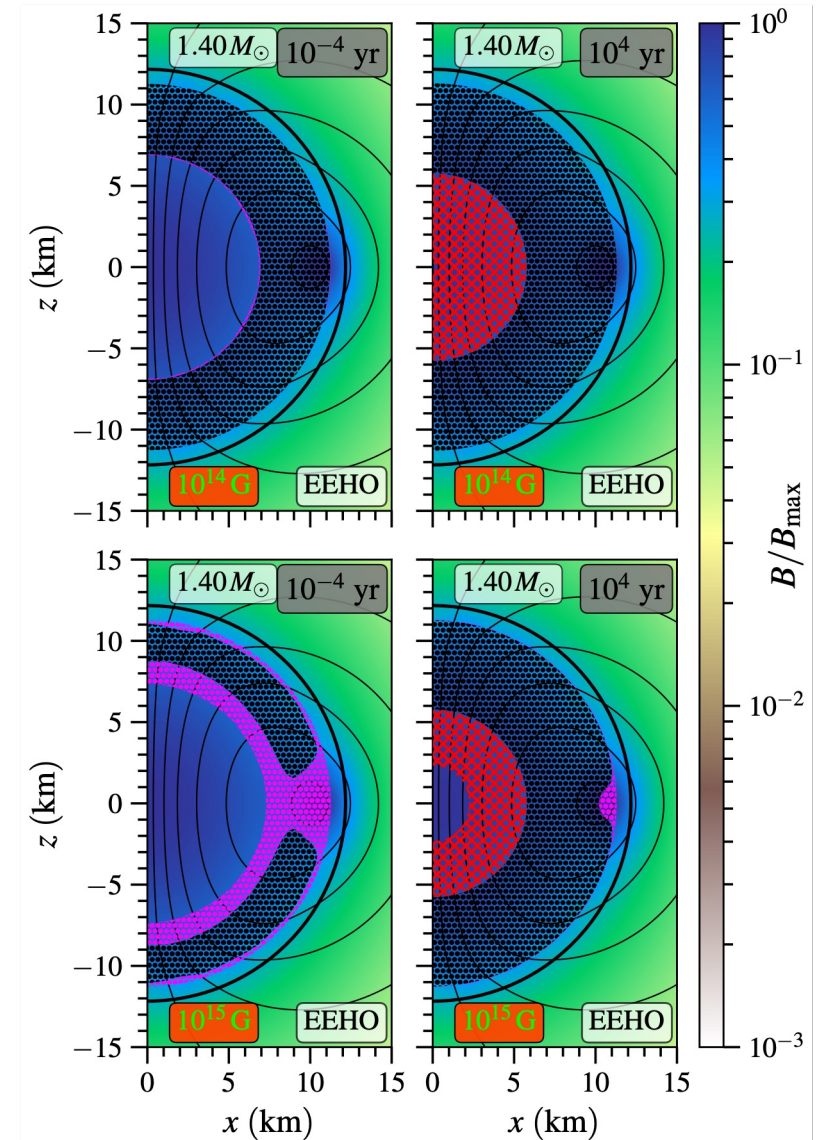
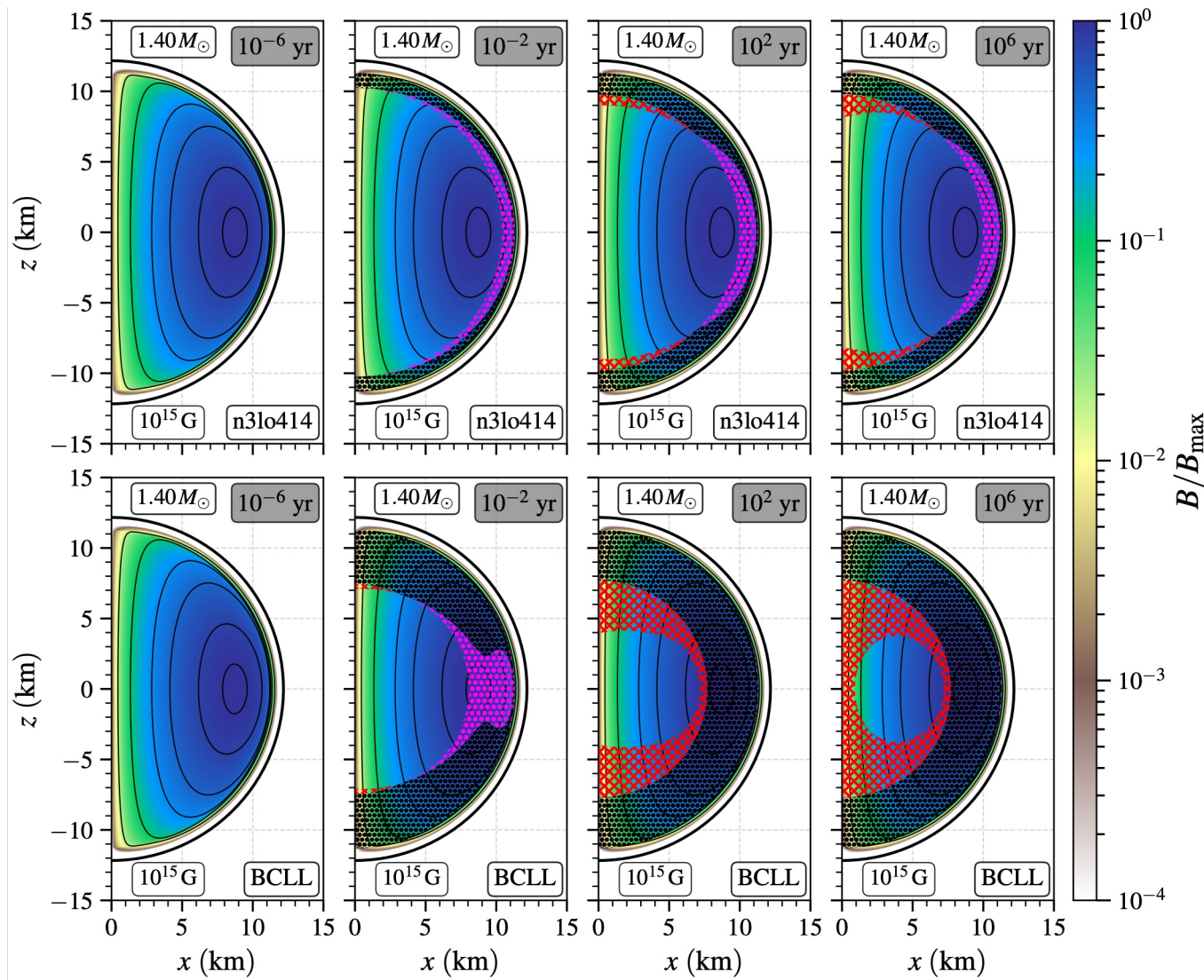
$$H_{c1} = H_{c1}(\lambda, \kappa); \quad H_{c2} = H_{c2}(\lambda, \kappa)$$



Thermal (cooling) evolution



Evolution of superconducting region



Summary

- NS is a suitable laboratory for testing physics in extreme states
- Asteroseismology is one of the powerful techniques to extract NS properties
 - By identifying the observed frequencies with specific modes, one can extract the corresponding physics
- Specifically, I introduced two applications
 - crustal torsional oscillations and magnetar QPOs
 - supernova gravitational waves and PNS oscillations
- There are still few research examples of asteroseismology in more complex systems, with rotation, magnetic field, superfluidity, dark matter, ..., and/or in modified theory...