汎函数繰り込み群を用いた2カラーQCD物質解析

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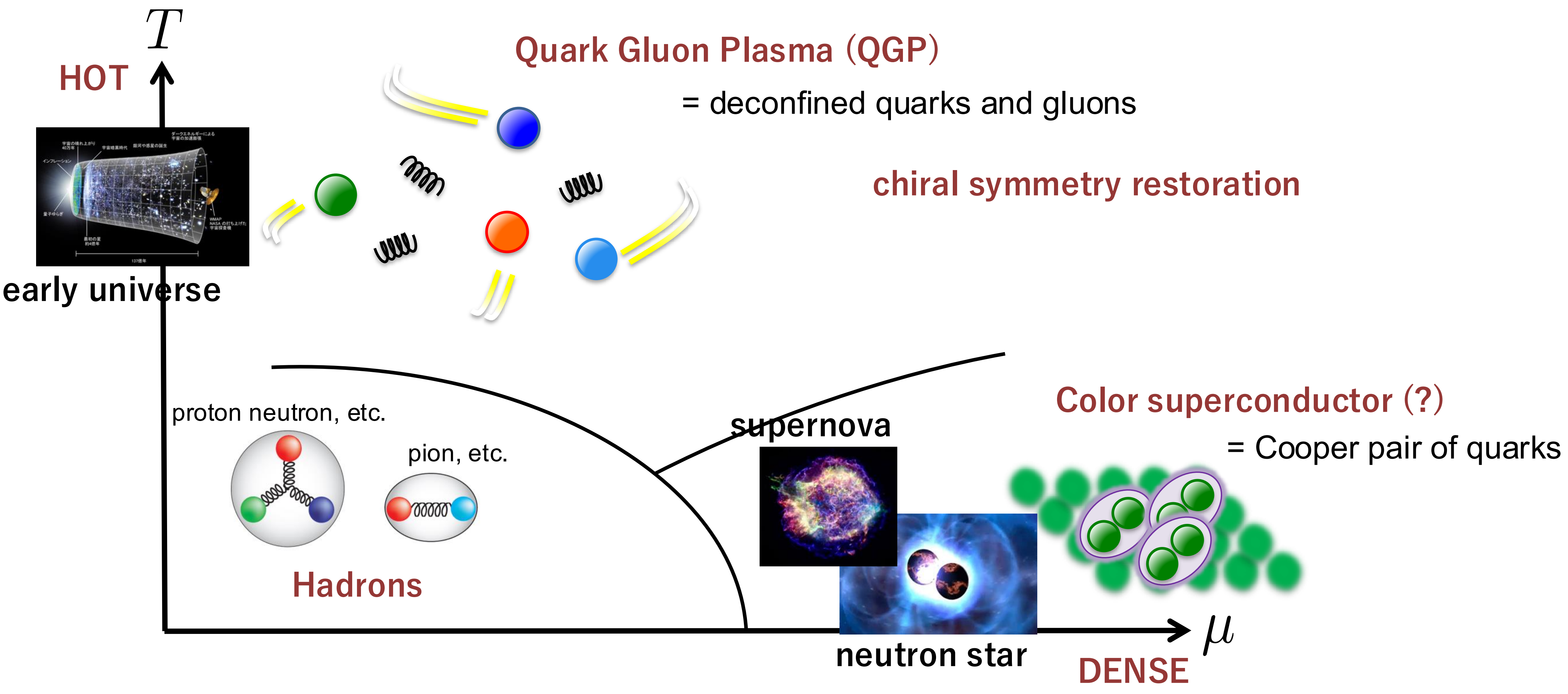
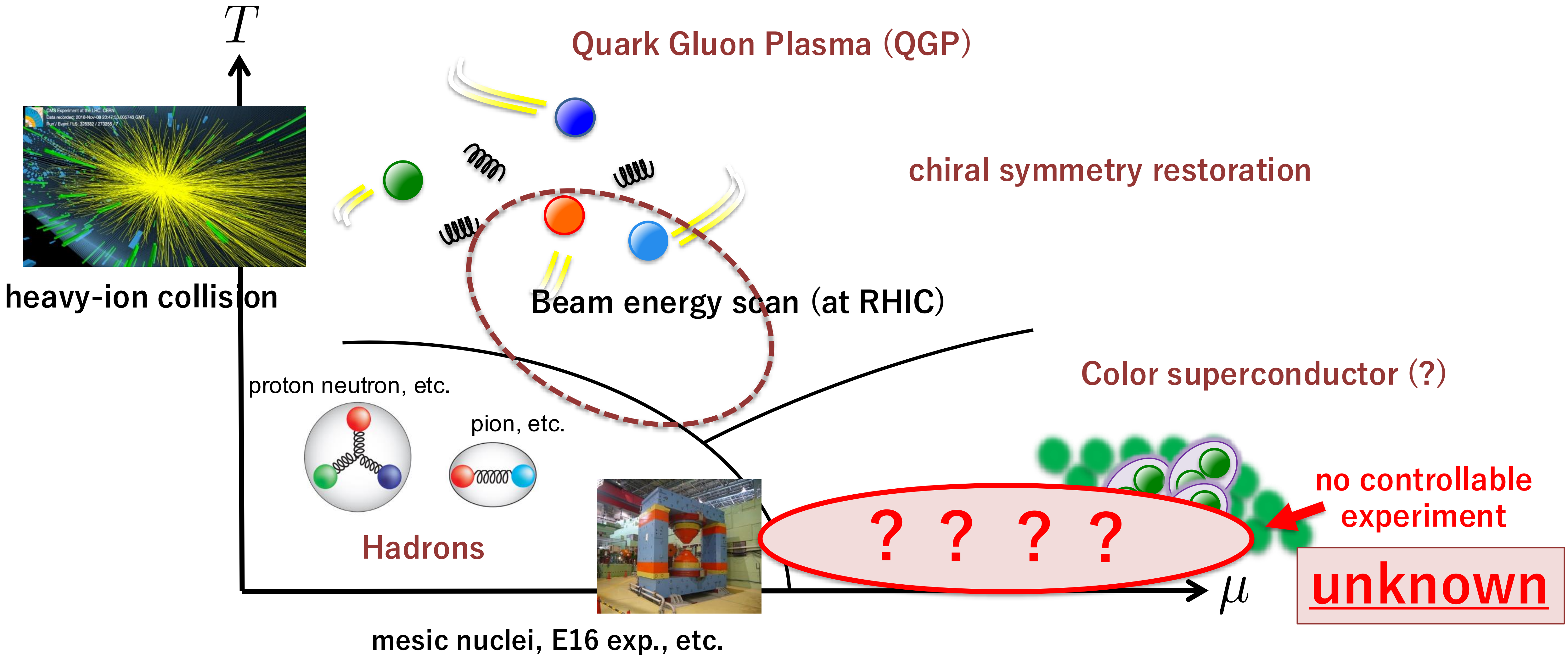
Introduction

- **QCD phase diagram**

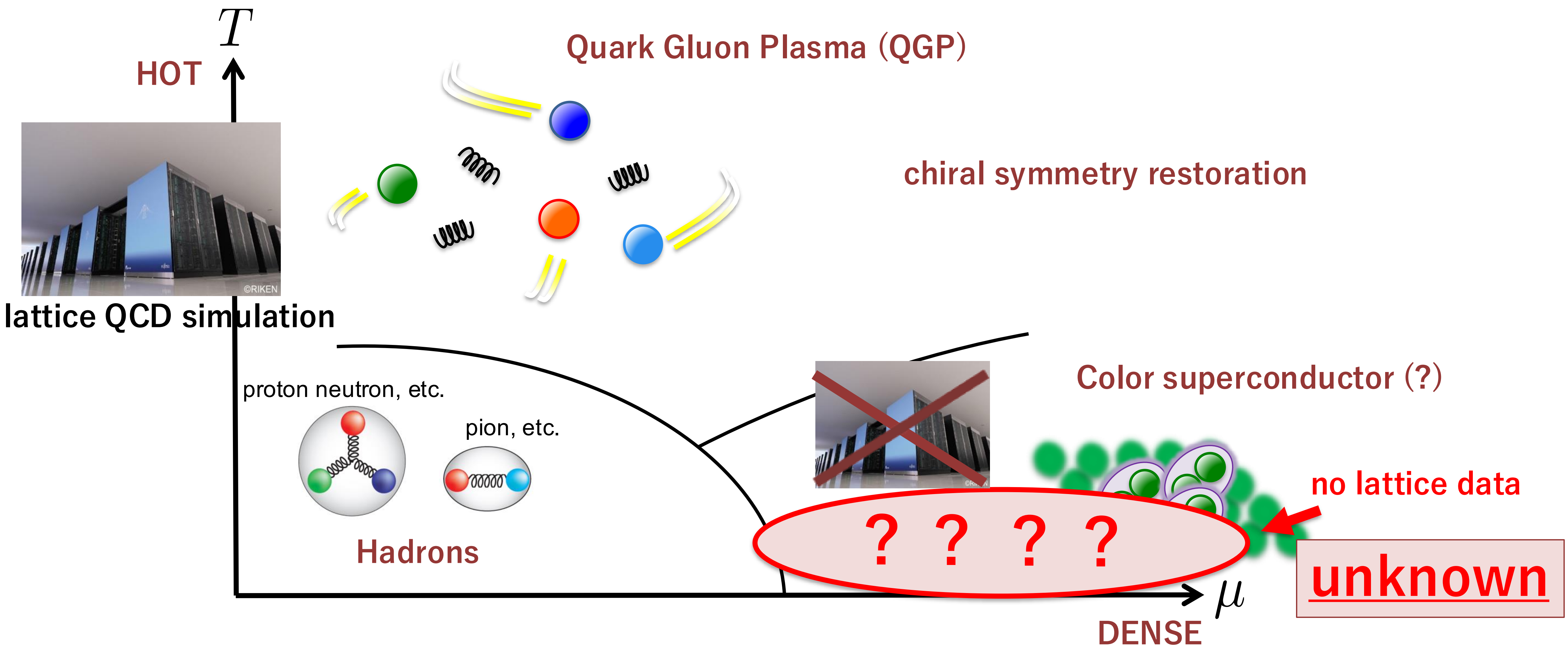


Introduction

- QCD phase diagram

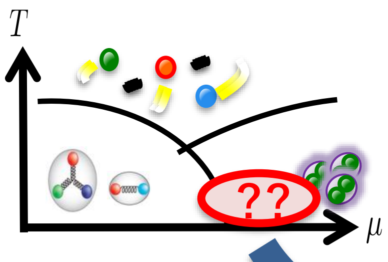


- **QCD phase diagram**



Introduction

- **Lattice study in dense QCD?**



three-color QCD (our world)

- Lattice sim. at density is not easy

$$\mathrm{Det}(\gamma \cdot D - \mu\gamma_4 + m) \in \mathbb{C} \quad \times$$

sign problem

with $N_f = (\text{even})$

two-color QCD (virtual world)

- Lattice sim. at density is **possible!**

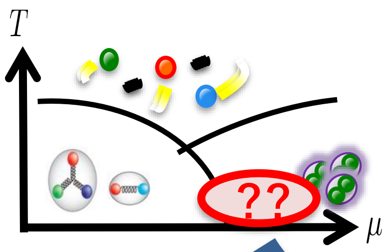
$$\mathrm{Det}(\gamma \cdot D - \mu\gamma_4 + m)^{N_f} \geq 0 \quad ◎$$

sign problem disappears thanks to $SU(2)_c$ pseudoreality

Introduction

- **Lattice study in dense QCD?**

with $N_f = (\text{even})$



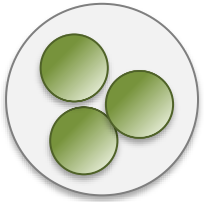
<u>three-color QCD (our world)</u>

- Lattice sim. at density is not easy

$$\mathrm{Det}(\gamma \cdot D - \mu\gamma_4 + m) \in \mathbb{C} \quad \times$$

sign problem

- Baryon is made of three quarks



nucleon

detail is differencnt

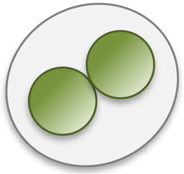
<u>two-color QCD (virtual world)</u>

- Lattice sim. at density is **possible!**

$$\mathrm{Det}(\gamma \cdot D - \mu\gamma_4 + m)^{N_f} \geq 0 \quad ◎$$

sign problem disappears
thanks to $SU(2)_c$ pseudoreality

- Baryon is made of two quarks



diquark baryon

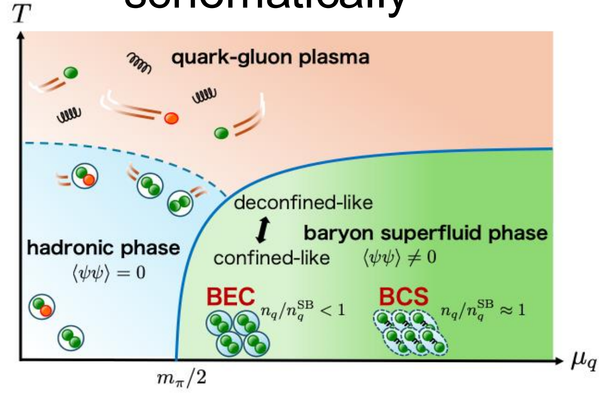
- **Two-color QCD** is a useful laboratory to explore cold-dense medium with lattice simulations

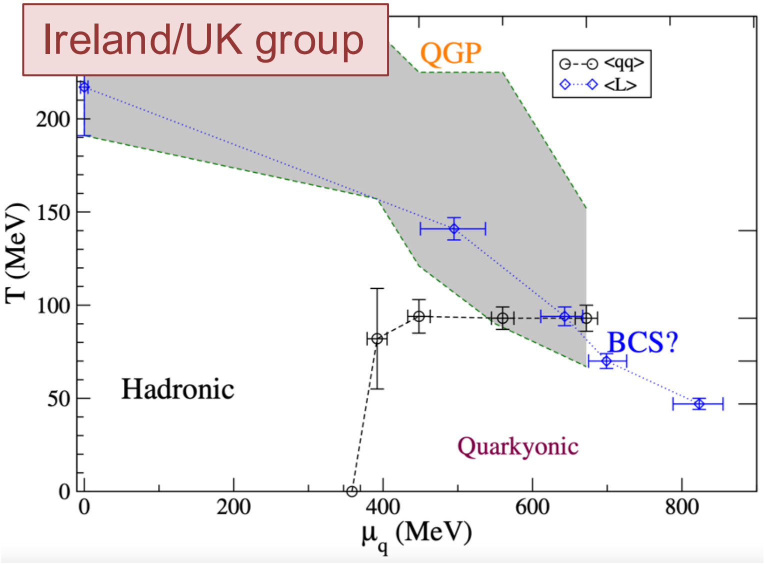
Introduction

- **Phase diagram in two-color QCD (QC_2D)**

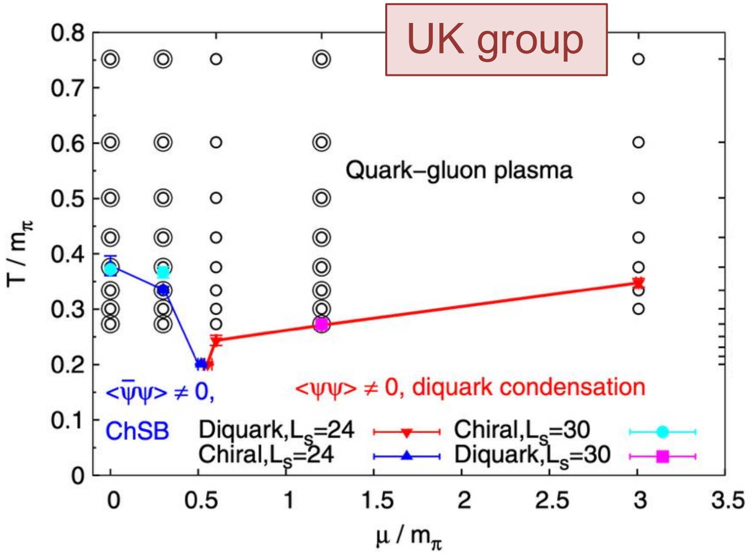
  - Examples of simulated phase diagram in QC_2D

schematically





Boz-Cotter-Fister-Mehta-Skullerud (2013)

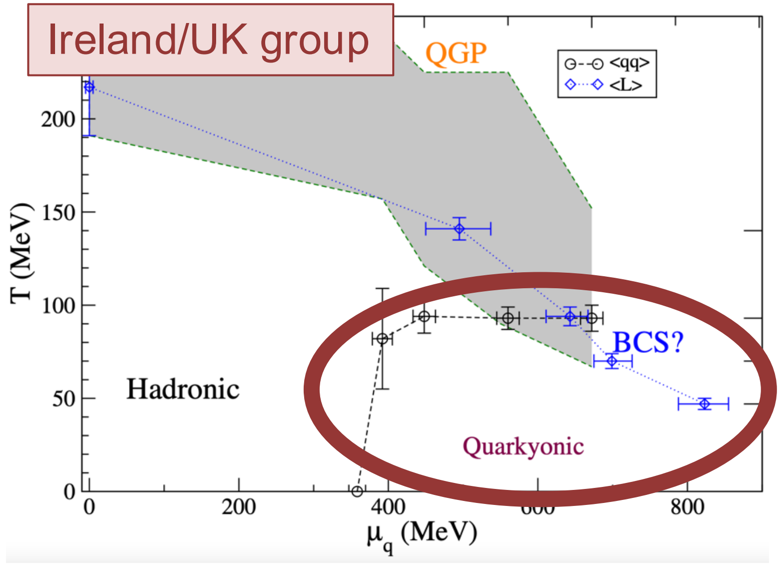


Buividovich-Smith-Smekal (2020)

...

- Currently at least four lattice simulation groups are active
  - Ireland/UK group (Hands, Skullerud, …)
  - UK group (Buividovich, ...)
  - Russian group (Bornyakov, ...)
  - Japanese group (Iida-san, Itou-san, …)

- **Phase diagram in two-color QCD (QC_2D)**
  - Examples of simulated phase diagram in QC_2D



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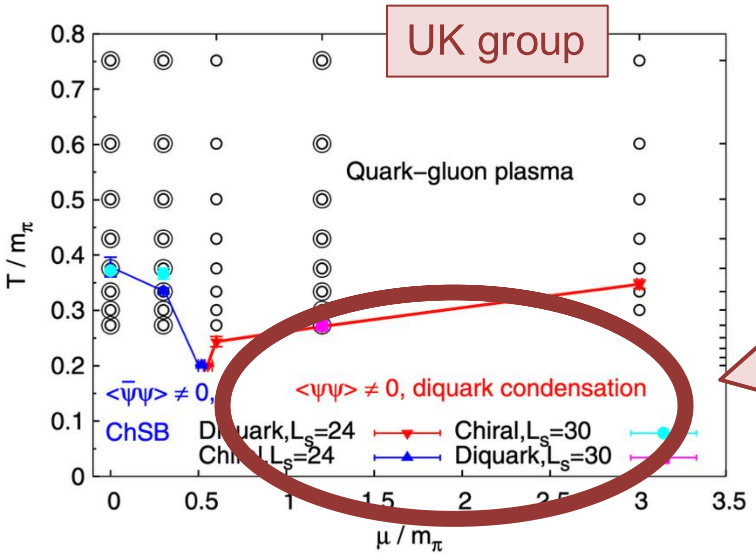
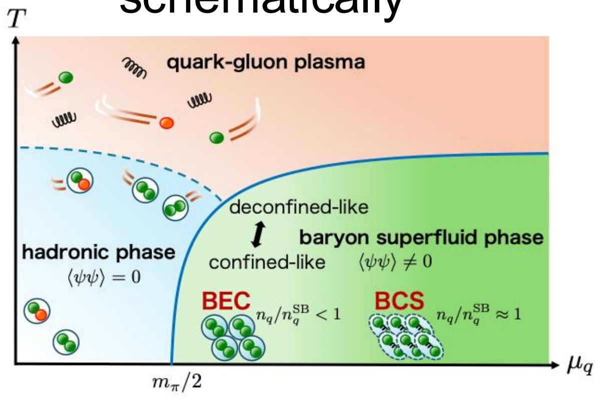


Buividovich-Smith-Smekal (2020)

schematically



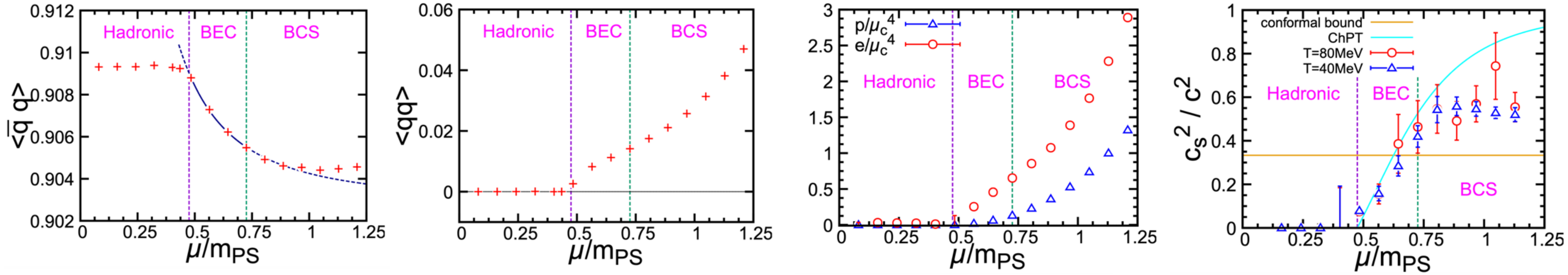
…

diquark condensed phase
(baryon superfluid phase)
$\langle \psi\psi \rangle \neq 0$

- Currently at least four lattice simulation groups are active
  - Ireland/UK group (Hands, Skullerud, …)
  - Russian group (Bornyakov, ...)
  - UK group (Buividovich, ...)
  - Japanese group (Iida-san, Itou-san, …)

- **Lattice results**
  - There are also, hadron mass spectrum, gluon propagator, transport coefficient, EoS, sound velocity, $\langle\bar{\psi}\psi\rangle, \langle\psi\psi\rangle, \langle L\rangle$, etc.

Japanese group



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My approach

– (i) Regard QC_2D lattice simulations as useful "numerical experiments" of cold and dense QCD, and (ii) give interpretation from symmetry viewpoints based on effective models

Suenaga (2025) (Review paper)

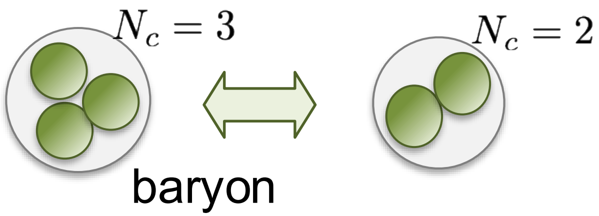
My publications on QC_2D

Gluon propagator: **Suenaga**-Kojo(2019), Kojo-**Suenaga**(2021), CSE effect: **Suenaga**-Kojo(2021), Sound velocity: Kojo-**Suenaga**(2022), Kawaguchi-**Suenaga**(2024), Topological susceptibility: Kawaguchi-**Suenaga**(2023), Hadron mass: **Suenaga**-Murakami-Itou-Iida (2023, 2024), FRG analysis: Fejos-**Suenaga**(2025, 2025, 2026), LSM with Nf=2+2: Sakai-**Suenaga** (2025), in preparations

Q: What can we learn from QC_2D?

- It is too naïve to bring all obtained information to Nc=3 world

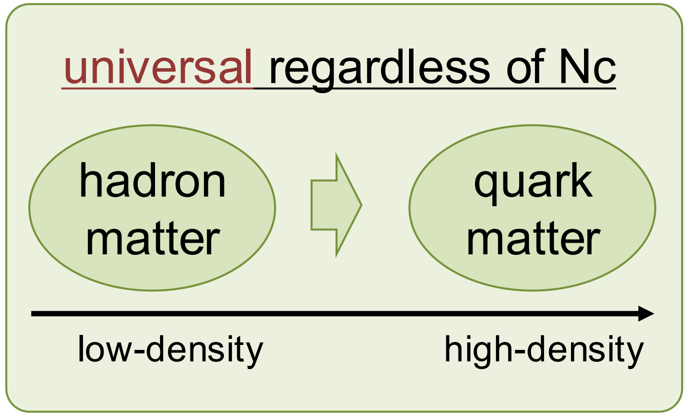
  → detailed dynamics is different   eg



- But we can *qualitatively* get information on

  - to what extent hadron model description can apply
  - how to incorporate quark dof into hadron model → "unified model"
  - deep understanding of quark matter in high-dense regime

  ⋮



Model

- **My approach: Linear sigma model (LSM)**
  - I constructed the LSM as a useful hadron effective model to study dense QC$_2$D property

chemical potential — quark mass — $U(1)_A$ anomaly

$$\mathcal{L}_{\rm LSM} = {\rm tr}[D_\mu\Sigma^\dagger D^\mu\Sigma] - m_0^2{\rm tr}[\Sigma^\dagger\Sigma] - \lambda_1\left({\rm tr}[\Sigma^\dagger\Sigma]\right)^2 - \lambda_2{\rm tr}[(\Sigma^\dagger\Sigma)^2] + {\rm tr}[H^\dagger\Sigma + \Sigma^\dagger H] + c({\rm det}\Sigma + {\rm det}\Sigma^\dagger)$$

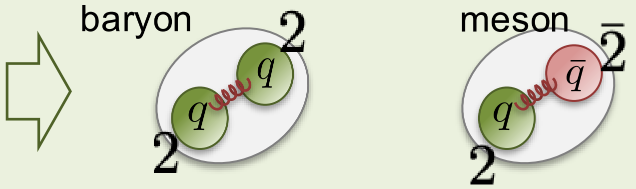
based on Pauli-Gursey SU(4) symmetry

with $$\Sigma = \frac{1}{2}\begin{pmatrix} 0 & -\frac{B'-iB}{2\sqrt{2}} & \frac{\sigma-i\eta+a_0^0-i\pi^0}{4} & \frac{a_0^+-i\pi^+}{2\sqrt{2}} \\ \frac{B'-iB}{2\sqrt{2}} & 0 & \frac{a_0^--i\pi^-}{2\sqrt{2}} & \frac{\sigma-i\eta-a_0^0+i\pi^0}{4} \\ -\frac{\sigma-i\eta+a_0^0-i\pi^0}{4} & -\frac{a_0^--i\pi^-}{2\sqrt{2}} & 0 & -\frac{\bar{B}'-i\bar{B}}{2\sqrt{2}} \\ -\frac{a_0^+-i\pi^+}{2\sqrt{2}} & -\frac{\sigma-i\eta-a_0^0+i\pi^0}{4} & \frac{\bar{B}'-i\bar{B}}{2\sqrt{2}} & 0 \end{pmatrix}$$

Hadron	J^P	Quark number	Isospin
σ	0^+	0	0
a_0	0^+	0	1
η	0^-	0	0
π	0^-	0	1
B ($\bar{B}$)	0^+	$+2(-2)$	0
B' ($\bar{B}'$)	0^-	$+2(-2)$	0

0^+ and 0^- hadrons
$=$ **parity partner**

- Pseudoreality of SU(2) color group: $2 \simeq \bar{2}$



-Baryon and meson are treated in a unified way

Model

- **My approach: Linear sigma model (LSM)**

  - I constructed the LSM as a useful hadron effective model to study dense QC_2D property

$U(1)_A$ anomaly

chemical potential

quark mass

$$\mathcal{L}_{\rm LSM} = {\rm tr}[D_\mu\Sigma^\dagger D^\mu\Sigma] - m_0^2{\rm tr}[\Sigma^\dagger\Sigma] - \lambda_1\left({\rm tr}[\Sigma^\dagger\Sigma]\right)^2 - \lambda_2{\rm tr}[(\Sigma^\dagger\Sigma)^2] + {\rm tr}[H^\dagger\Sigma + \Sigma^\dagger H] + c({\rm det}\Sigma + {\rm det}\Sigma^\dagger)$$

based on Pauli-Gursey SU(4) symmetry

- At T=0, my LSM at mean-field
  - can *qualitatively* reproduce lattice results on hadron mass spectrum and sound velocity
  - *may* fail in reproducing lattice results on topological susceptibility

Suenaga et al, PRD 107, 054001 (2023); Kawaguchi-Suenaga, PRD 109, 096034 (2024)
Kawaguchi-Suenaga, JHEP 08, 189 (2023)

<u>This work</u>

- What happened when including hadron fluctuations beyond mean field? Quantitatively improved?
- How about finite T?

FRG analysis

Model

- **What is functional renormalization group (FRG)?**

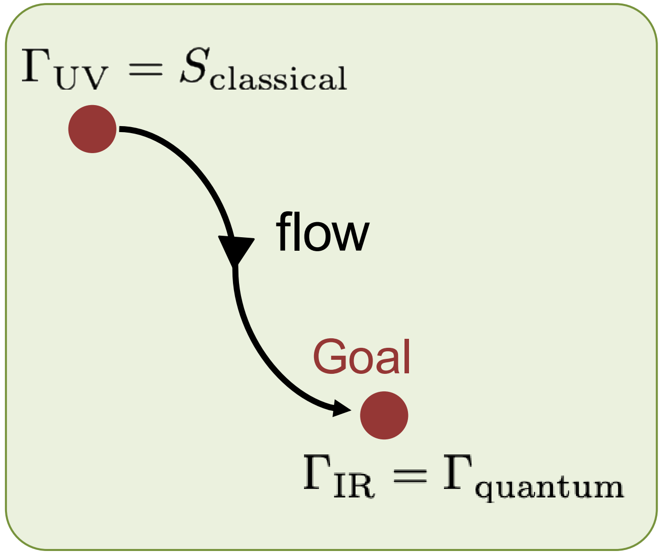
- The flow equation is $\partial_k \Gamma_k = \frac{1}{2}\tilde{\partial}_k \mathrm{Tr}\,\mathrm{Log}\,[\Gamma_k'' + R_k]$, with $\tilde{\partial}_k \equiv \partial_k R_k \frac{\partial}{\partial R_k}$

Wetterich PLB(1993)

- The regulator R_k allows to include fluctuations of $p \gtrsim k$

For $k \to \Lambda_{\mathrm{UV}} (\approx \infty)$, no fluctuation (p) is integrated: $\Gamma_{\mathrm{UV}} = S_{\mathrm{classical}}$

For $k \to \Lambda_{\mathrm{IR}} (\approx 0)$, all fluctuations (p) are integrated: $\Gamma_{\mathrm{IR}} = \Gamma_{\mathrm{quantum}}$

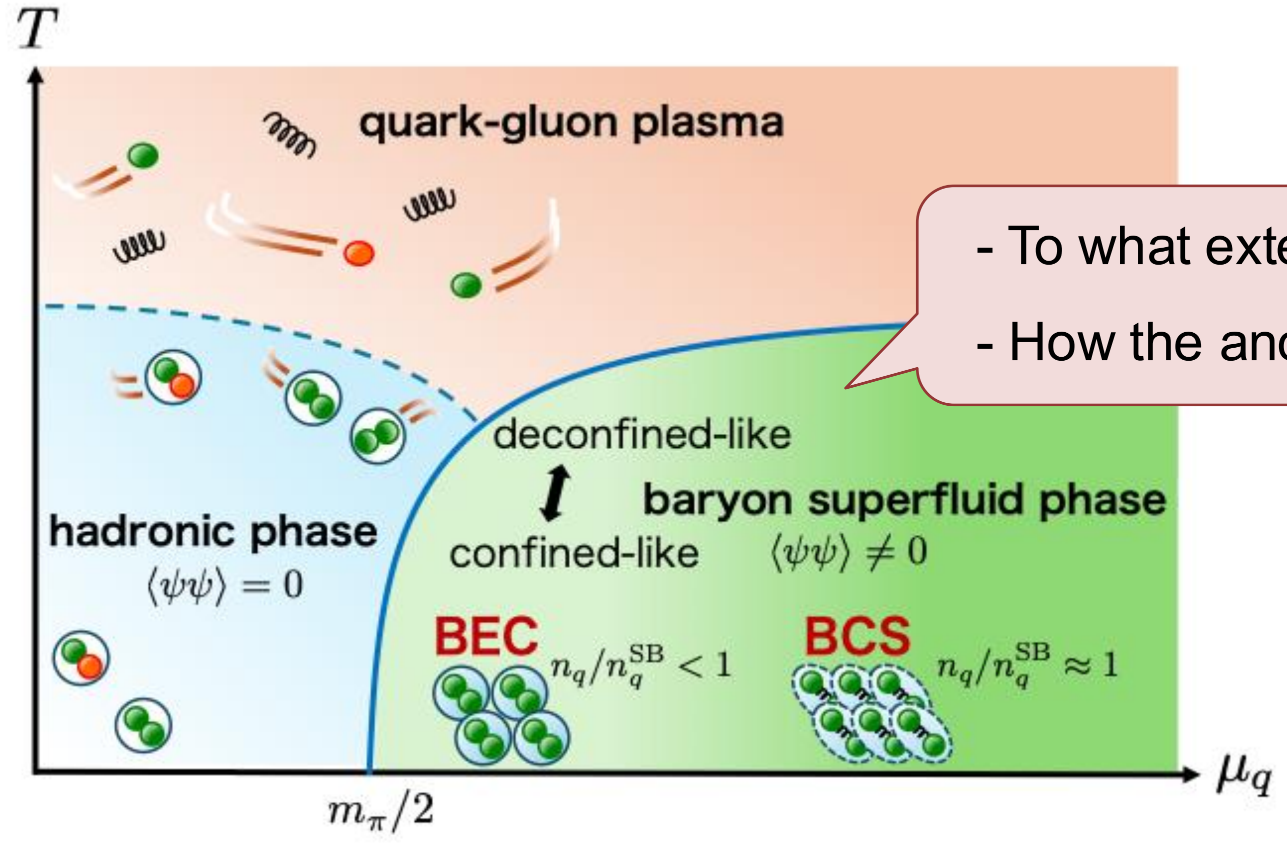


- Quantum fluctuations are incorporated by integrating the flow equation from $k = \Lambda_{\mathrm{UV}}$ to $k = \Lambda_{\mathrm{IR}}$

- We employ 3D Litim regulator $R_k(\boldsymbol{p}) = (k^2 - \boldsymbol{p}^2)\theta(k^2 - \boldsymbol{p}^2)$

Model

- **FRG analysis with the LSM in QC$_2$D medium** Fejos-**Suenaga** (2025, 2025, 2026)



Fluctuations are hadronic (no quarks)

- To what extent can we adopt the hadronic model in medium ?
- How the anomaly couplings are modified in medium ?

anomaly enhancement in hadronic medium?
[eg, Indication in QC$_2$D: Suenaga et al (2023)
FRG in three-color QCD: Fejos-Hosaka(2015)]

- **Ansatz of effective action in medium**

$$\Gamma = \int d^4x \Big(\mathrm{Tr}[D_\mu \Sigma^\dagger D^\mu \Sigma] + \bar{c} m_q \mathrm{tr}[E^T \Sigma + \Sigma^\dagger E] - (V_{\mathrm{AF}} + V_{\mathrm{A}}) \Big)$$

kinetic term with μ_q quark mass effect

Symmetry with μ_q [Not SU(4)]

$SU(2)_L \times SU(2)_R \times U(1)_B$

parity and time-reversal

with

non-anomalous

$$\begin{aligned} V_{\mathrm{AF}} &= m_M^2 \mathrm{tr}\left[\Sigma_M^\dagger \Sigma_M\right] + m_B^2 \mathrm{tr}\left[\Sigma_{B_R}^\dagger \Sigma_{B_R} + \Sigma_{B_L}^\dagger \Sigma_{B_L}\right] + \lambda_{M1}\left(\mathrm{tr}\left[\Sigma_M^\dagger \Sigma_M\right]\right)^2 \\ &+ \lambda_{M2} \mathrm{tr}\left[\left(\Sigma_M^\dagger \Sigma_M - \frac{1}{2}\mathrm{tr}[\Sigma_M^\dagger \Sigma_M]\right)^2\right] + \lambda_{B1}\left(\mathrm{tr}\left[\Sigma_{B_R}^\dagger \Sigma_{B_R} + \Sigma_{B_L}^\dagger \Sigma_{B_L}\right]\right)^2 \\ &+ \lambda_{B2}\left(\mathrm{tr}\left[\Sigma_{B_R}^\dagger \Sigma_{B_R} - \Sigma_{B_L}^\dagger \Sigma_{B_L}\right]\right)^2 + \gamma_1 \mathrm{tr}\left[\Sigma_M^\dagger \Sigma_M\right] \mathrm{tr}\left[\Sigma_{B_R}^\dagger \Sigma_{B_R} + \Sigma_{B_L}^\dagger \Sigma_{B_L}\right] \\ &+ \gamma_2 \left(\mathrm{tr}\left[\Sigma_{B_R}^\dagger \Sigma_{B_L}\right] \det\Sigma_M^\dagger + \mathrm{tr}\left[\Sigma_{B_L}^\dagger \Sigma_{B_R}\right] \det\Sigma_M\right) , \end{aligned}$$

anomalous

$$\begin{aligned} V_{\mathrm{A}} &= a_M\left(\det\Sigma_M + \det\Sigma_M^\dagger\right) + a_B \mathrm{tr}\left[\Sigma_{B_R}^\dagger \Sigma_{B_L} + \Sigma_{B_L}^\dagger \Sigma_{B_R}\right] + c_{M1}\left((\det\Sigma_M)^2 + (\det\Sigma_M^\dagger)^2\right) \\ &+ c_{M2} \mathrm{tr}\left[\Sigma_M^\dagger \Sigma_M\right]\left(\det\Sigma_M + \det\Sigma_M^\dagger\right) + c_{B1}\left((\mathrm{tr}[\Sigma_{B_R}^\dagger \Sigma_{B_L}])^2 + (\mathrm{tr}[\Sigma_{B_L}^\dagger \Sigma_{B_R}])^2\right) , \\ &+ c_{B2} \mathrm{tr}\left[\Sigma_{B_R}^\dagger \Sigma_{B_R} + \Sigma_{B_L}^\dagger \Sigma_{B_L}\right] \mathrm{tr}\left[\Sigma_{B_R}^\dagger \Sigma_{B_L} + \Sigma_{B_L}^\dagger \Sigma_{B_R}\right] + d_1 \mathrm{tr}\left[\Sigma_M^\dagger \Sigma_M\right] \mathrm{tr}\left[\Sigma_{B_R}^\dagger \Sigma_{B_L} + \Sigma_{B_L}^\dagger \Sigma_{B_R}\right] \\ &+ d_2 \mathrm{tr}\left[\Sigma_{B_R}^\dagger \Sigma_{B_R} + \Sigma_{B_L}^\dagger \Sigma_{B_L}\right]\left(\det\Sigma_M + \det\Sigma_M^\dagger\right) + d_3\left(\mathrm{tr}\left[\Sigma_{B_R}^\dagger \Sigma_{B_L}\right]\det\Sigma_M + \mathrm{tr}\left[\Sigma_{B_L}^\dagger \Sigma_{B_R}\right]\det\Sigma_M^\dagger\right) \end{aligned}$$

$$\begin{aligned} \Sigma &= \frac{1}{2}\begin{pmatrix} 0 & -\frac{B'-iB}{2\sqrt{2}} & \frac{\sigma-i\eta+a_0^0-i\pi^0}{4} & \frac{a_0^+-i\pi^+}{2\sqrt{2}} \\ \frac{B'-iB}{2\sqrt{2}} & 0 & \frac{a_0^- -i\pi^-}{2\sqrt{2}} & \frac{\sigma-i\eta-a_0^0+i\pi^0}{4} \\ -\frac{\sigma-i\eta+a_0^0-i\pi^0}{4} & -\frac{a_0^- -i\pi^-}{2\sqrt{2}} & 0 & -\frac{\bar{B}'-i\bar{B}}{2\sqrt{2}} \\ -\frac{a_0^+-i\pi^+}{2\sqrt{2}} & -\frac{\sigma-i\eta-a_0^0+i\pi^0}{4} & \frac{\bar{B}'-i\bar{B}}{2\sqrt{2}} & 0 \end{pmatrix} \\ &= \frac{1}{2\sqrt{2}}\begin{pmatrix} i\Sigma_{B_R} & \Sigma_M^\dagger \\ -\Sigma_M^* & -i\Sigma_{B_L}^\dagger \end{pmatrix} , \end{aligned}$$

-We assume this “classical form” at any flow momentum k

- **Phase diagram**

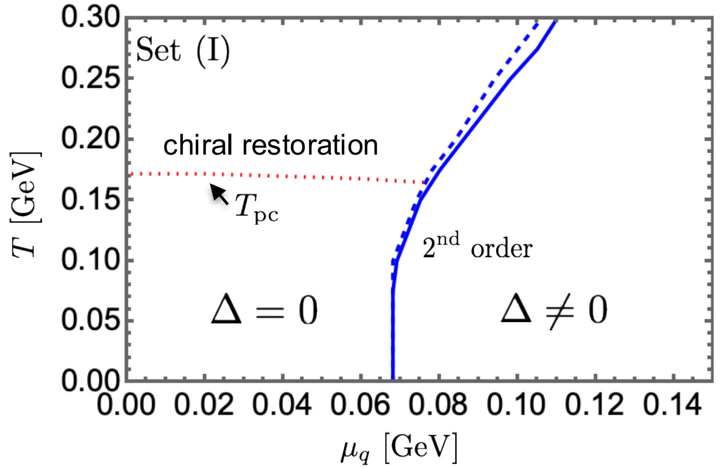
input/initial condition — Fejos-**Suenaga** (2025)

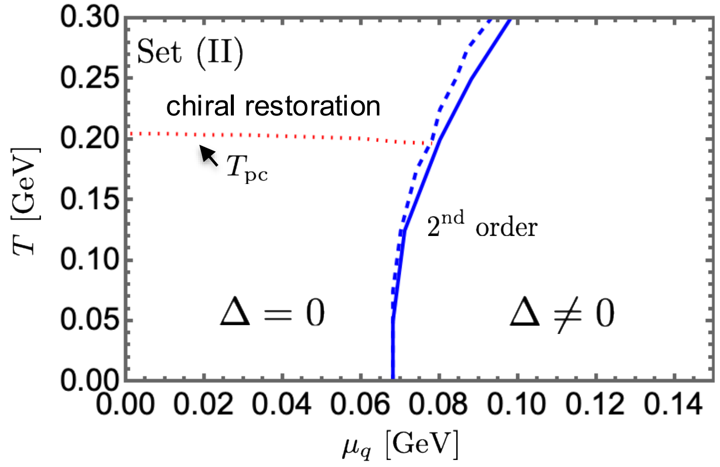
$M_\pi \approx 0.14\,\mathrm{GeV}$
$f_\pi = 0.093\,\mathrm{GeV}$
large-Nc sup. at UV
SU(4) relation at UV

$+$

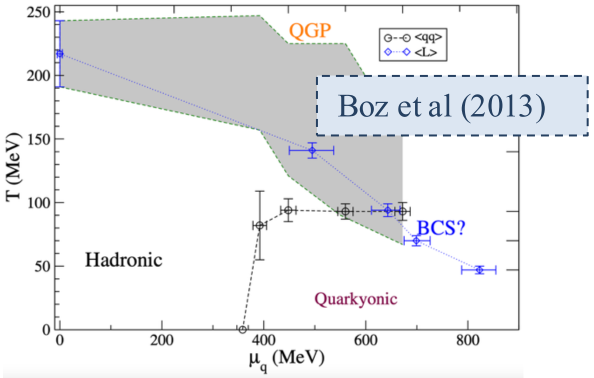
$M_\eta \approx 0.3\,\mathrm{GeV}$ → Set (I) (small anomaly)
$M_\eta \approx 0.95\,\mathrm{GeV}$ → Set (II) (large anomaly)

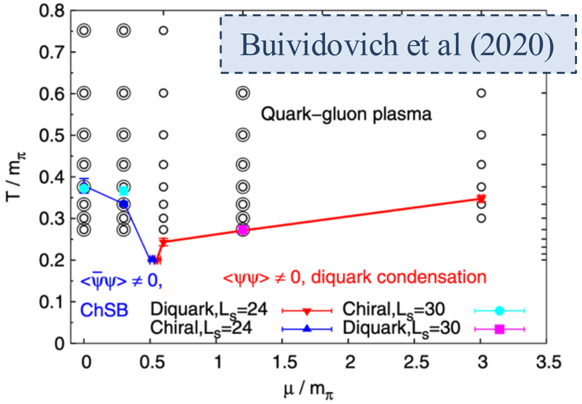
$(k_{\mathrm{UV}} = 1\,\mathrm{GeV}\,,\ k_{\mathrm{IR}} = 0.3\,\mathrm{GeV})$





lattice data





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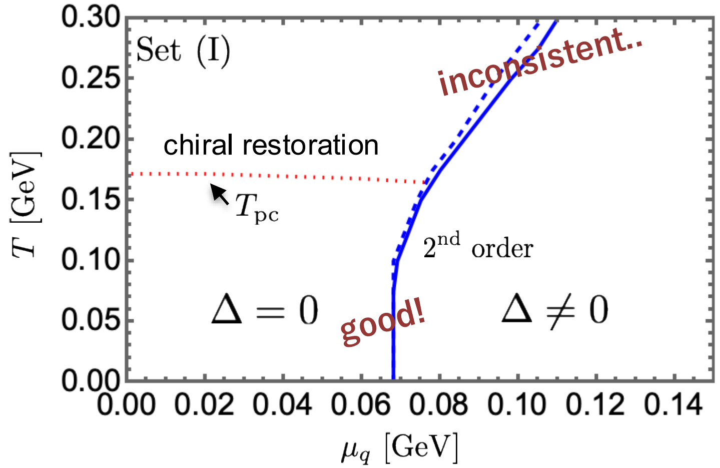

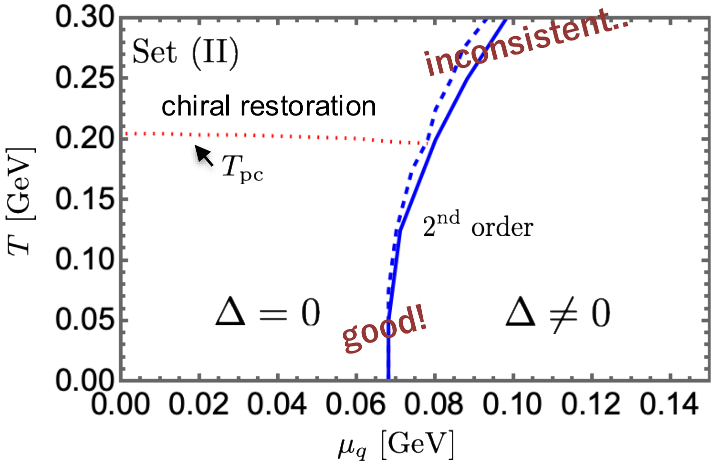

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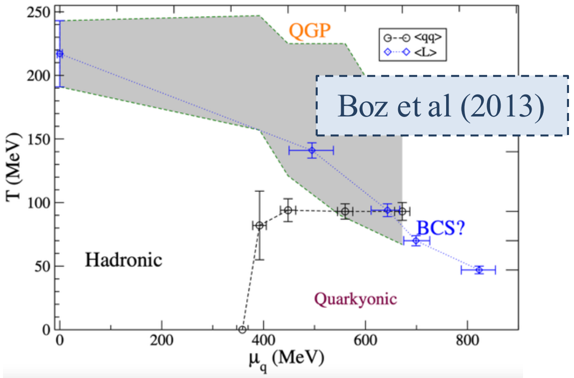
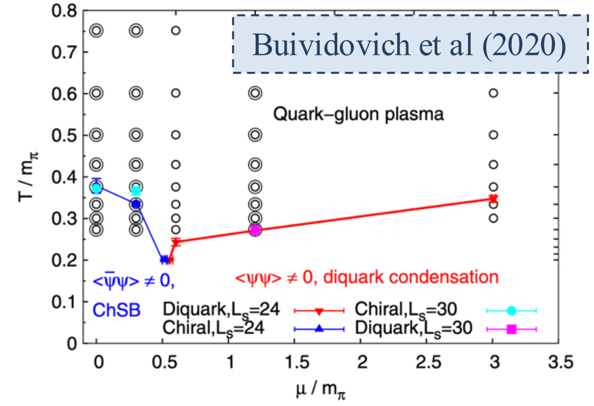
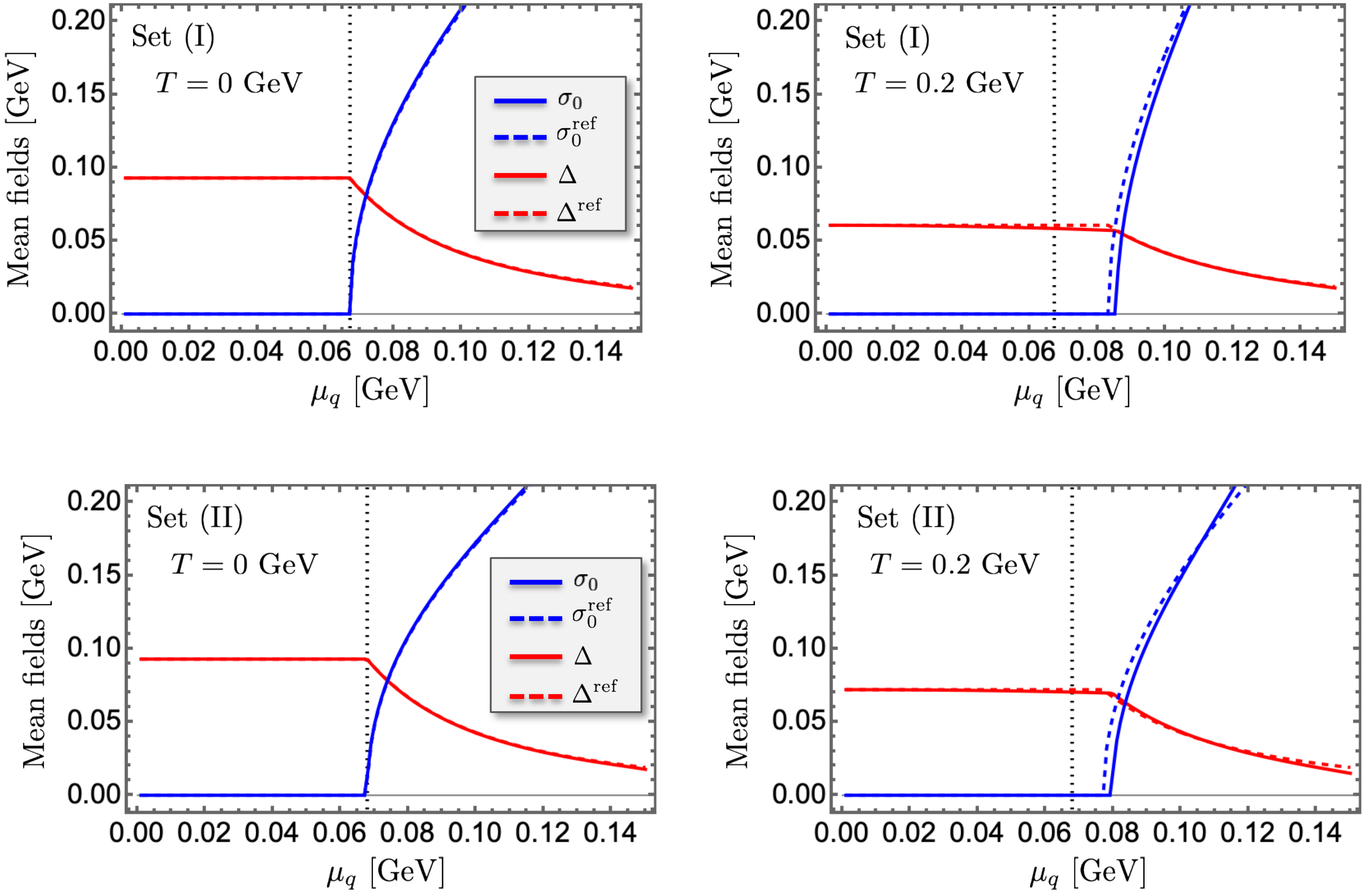




- For $T \lesssim T_{\rm pc}$, our treatment is reasonable
- Phase boundary at $T \gtrsim T_{\rm pc}$ is inconsistent with lattice data → explicit quark d.o.f. is necessary

Results

- **Mean fields**



Set (I): small anomaly $M_\eta \approx 0.3\,\mathrm{GeV}$

Set (II): large anomaly $M_\eta \approx 0.95\,\mathrm{GeV}$

- Phase transition is always second order

Results

- **Anomaly enhancement**
  - (Normalized) anomaly coefficients for meson and baryon: a_M and a_B

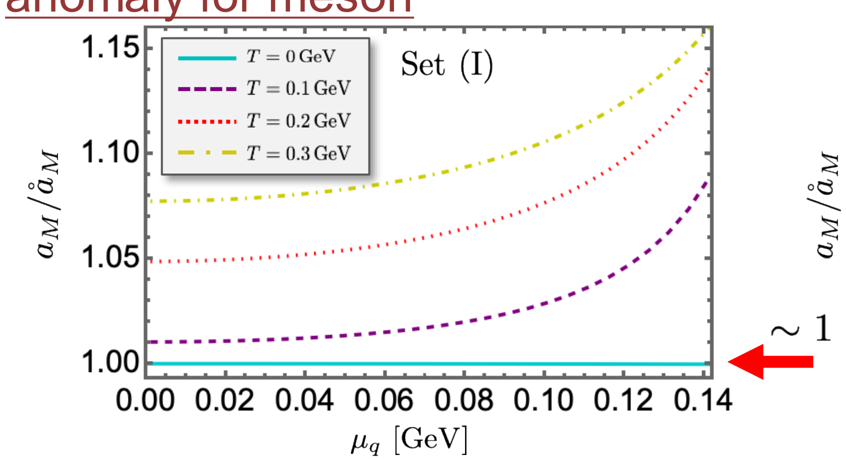
$$V_A = a_M \left(\det\Sigma_M + \det\Sigma_M^\dagger\right) + a_B \mathrm{tr}\left[\Sigma_{B_R}^\dagger \Sigma_{B_L} + \Sigma_{B_L}^\dagger \Sigma_{B_R}\right] + \cdots$$

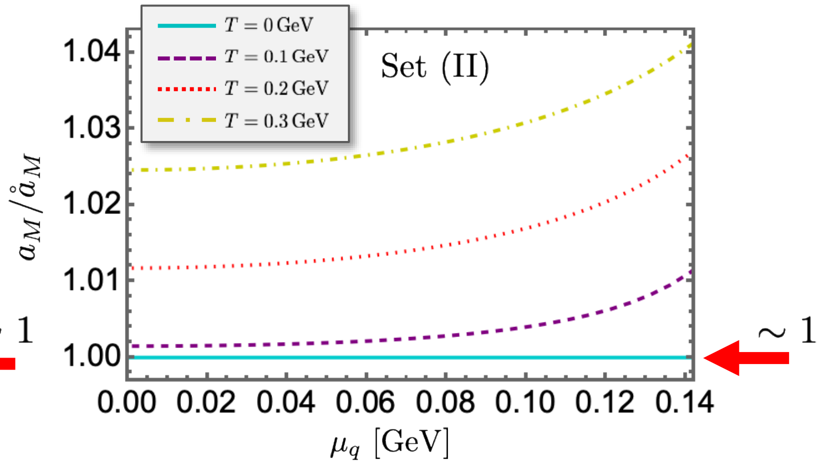
$\mathring{a}_M, \mathring{a}_B$: vacuum values

Set (I): small anomaly $M_\eta \approx 0.3\,\mathrm{GeV}$

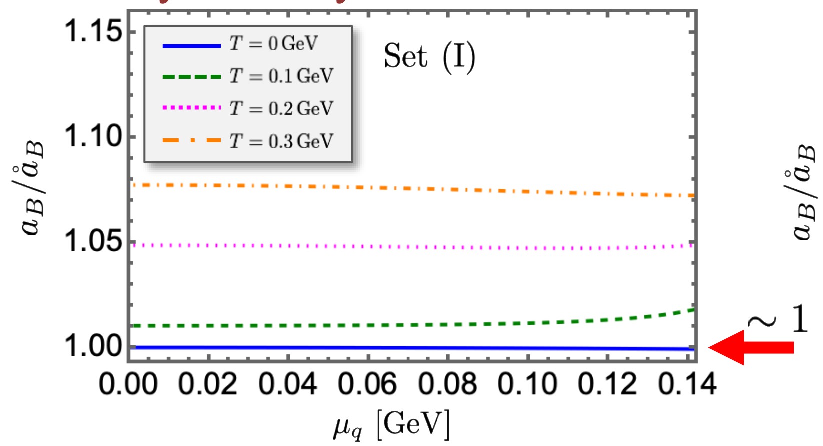
Set (II): large anomaly $M_\eta \approx 0.95\,\mathrm{GeV}$

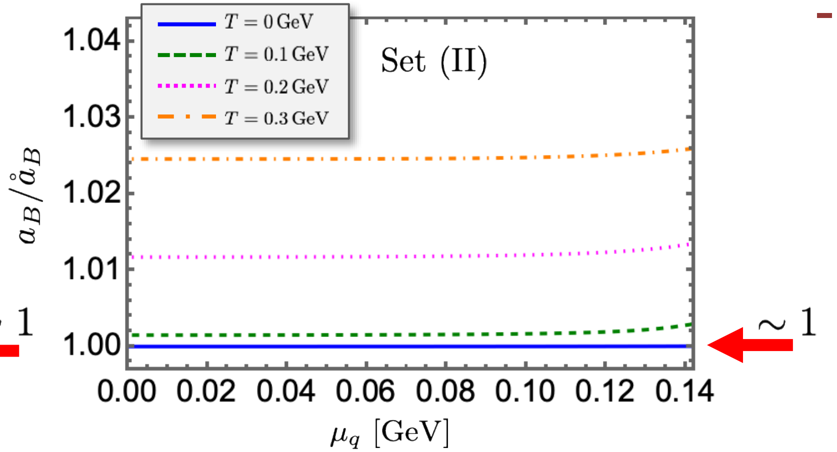
anomaly for meson





anomaly for baryon





- At T=0, no anomaly enhancement in finite μ_q

Results

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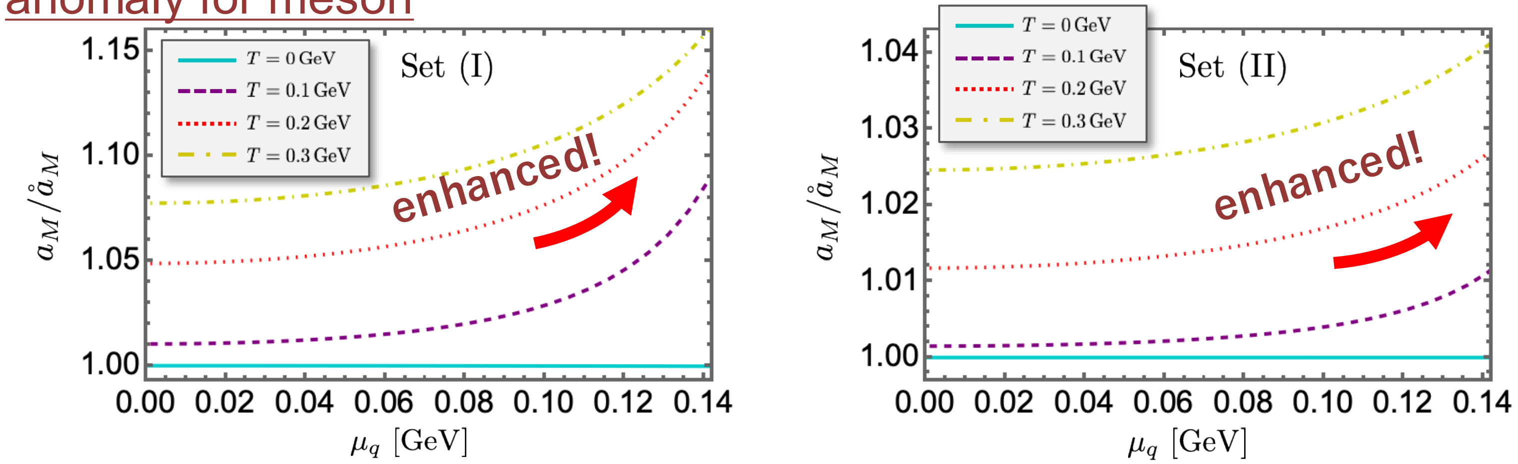
$$V_{\rm A} = a_M\left(\det\Sigma_M + \det\Sigma_M^\dagger\right) + a_B \mathrm{tr}\left[\Sigma_{B_R}^\dagger \Sigma_{B_L} + \Sigma_{B_L}^\dagger \Sigma_{B_R}\right] + \cdots$$

$\mathring{a}_M, \mathring{a}_B$: vacuum values

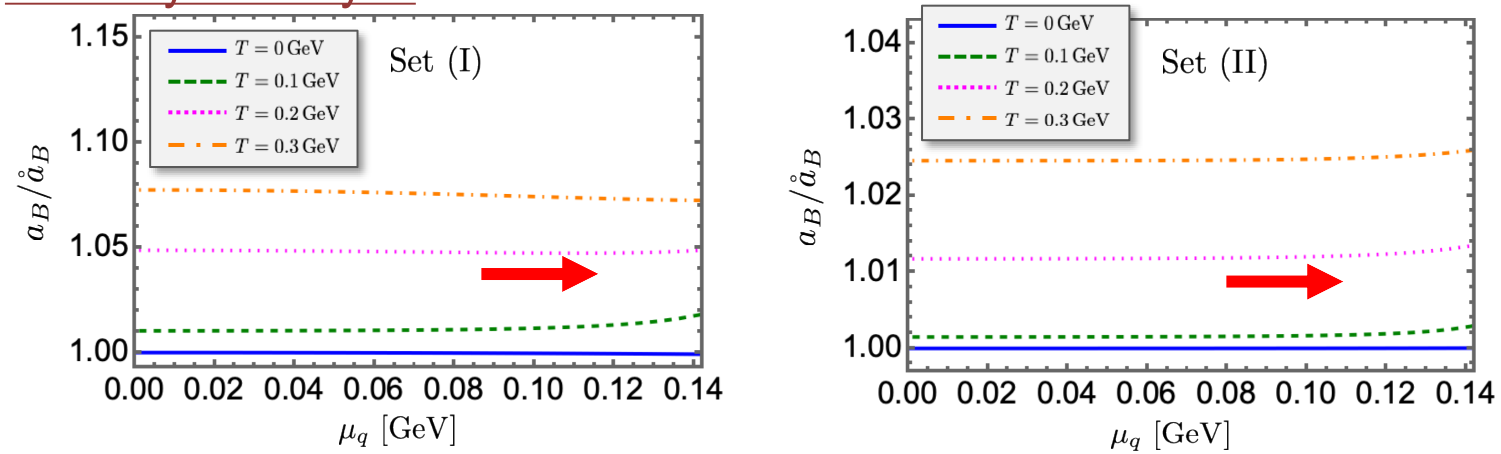
Set (I): small anomaly $M_\eta \approx 0.3\,\mathrm{GeV}$

Set (II): large anomaly $M_\eta \approx 0.95\,\mathrm{GeV}$

anomaly for meson



anomaly for baryon



- At T=0, no anomaly enhancement in finite μ_q

- At finite T

$$\begin{cases} a_M \text{ is enhanced} \\ a_B \text{ is not enhanced} \end{cases}$$

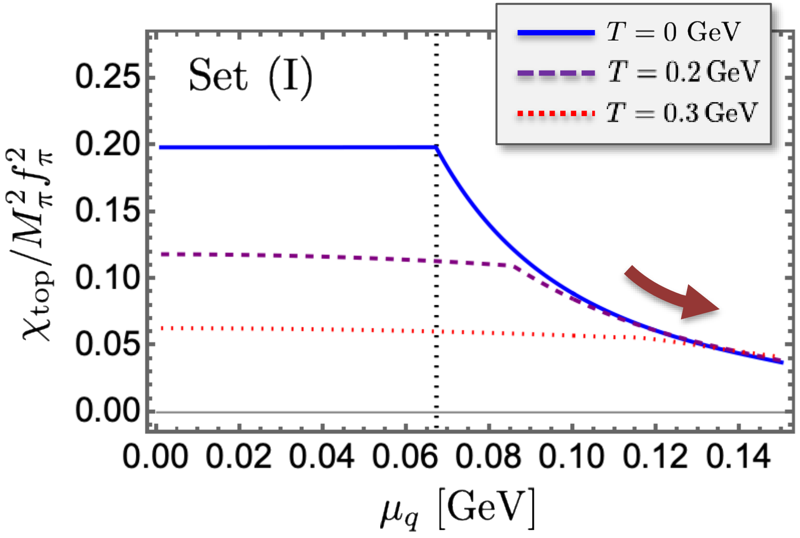
Results

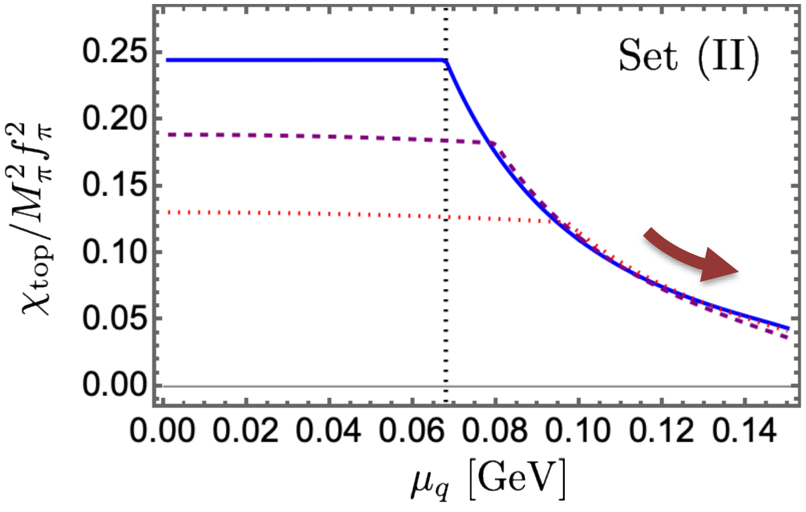
- **Topological susceptibility**

propagator at $p = 0$

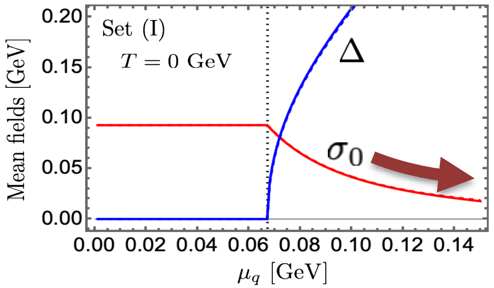
- Within our model, $\chi_{\rm top}/M_\pi^2 f_\pi^2 = (M_\pi^2/4)\left[-D_{\pi\pi}(0) + D_{\eta\eta}(0)\right]$

Set (I): small anomaly $M_\eta \approx 0.3\,\mathrm{GeV}$
Set (II): large anomaly $M_\eta \approx 0.95\,\mathrm{GeV}$





Mean fields



- The topological susceptibility is suppressed simply following chiral restoration of $\sigma_0 \propto \mu_q^{-2}$
- Anomaly enhancement is not seen; it is hidden by chiral restoration effect

Results

- **Topological susceptibility**

propagator at $p = 0$

  - Within our model, $\chi_{\rm top}/M_\pi^2 f_\pi^2 = (M_\pi^2/4)\,[-D_{\pi\pi}(0) + D_{\eta\eta}(0)]$

Set (I): small anomaly $M_\eta \approx 0.3\,\mathrm{GeV}$

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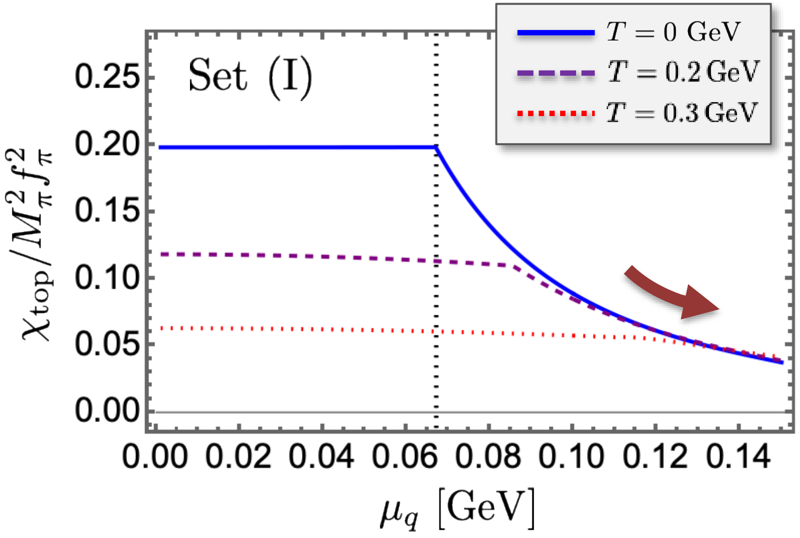
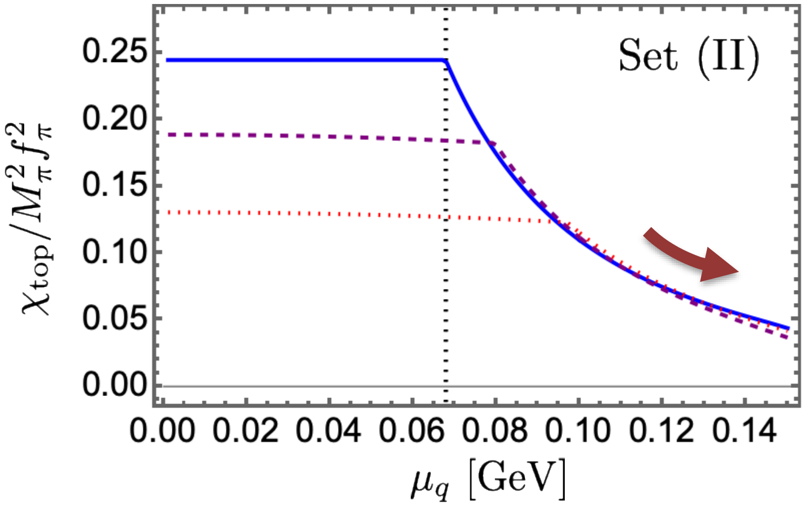
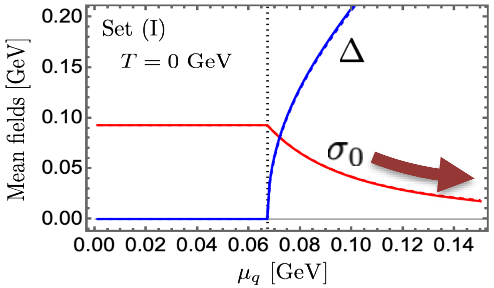
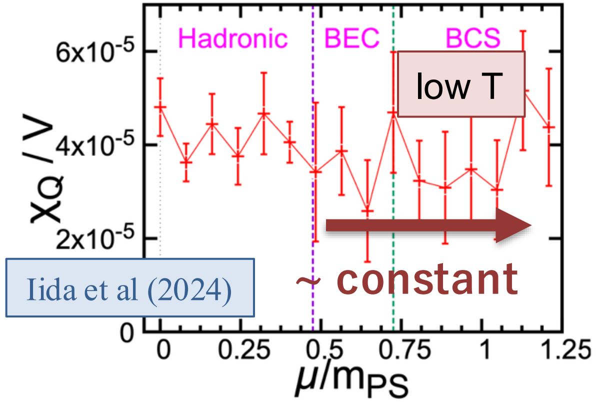


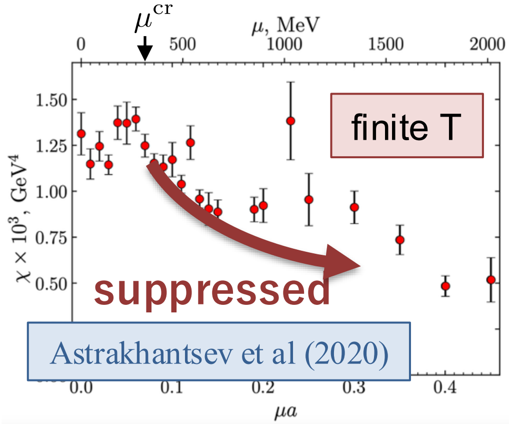


Mean fields



lattice





- Inconsistent with lattice at low T?
  → Needs explicit quark d.o.f.?
- More lattice results are welcome

Conclusions

- Constructed a hadron effective model, Linear sigma model (LSM) in dense QC_2D

- I employed FRG in LSM to see to what extent hadron model can apply in medium

- For $T \gtrsim T_{\rm pc}$ my FRG result is inconsistent with the lattice data

  → Inclusion of quark d.o.f. is necessary

  +

  Enhancement of anomaly effect to mesons in (hadronic) medium

  Top. susceptibility is suppressed in medium with chiral restoration

QC_2D lattice sim.

"numerical experiments"

From QC_2D study we can qualitatively learn

- to what extent hadron model description can apply

- how to incorporate quark dof into hadron model → "unified model"

- deep understanding of quark matter in high-dense regime

⋮

universal regardless of Nc

hadron matter → quark matter

low-density → high-density

Thank you!

Mean fields

• Mean field

- The mean fields are $\sigma_0 \equiv \langle \sigma \rangle$ and $\Delta \equiv \left\langle \frac{B+\bar{B}}{\sqrt{2}} \right\rangle$

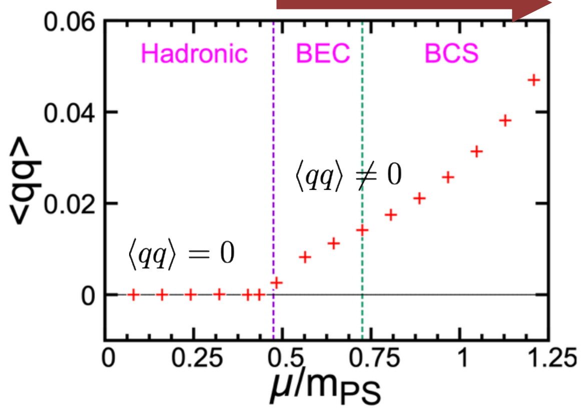
$\sigma_0 \sim \langle \bar{\psi}\psi \rangle$: chiral condensate

$\Delta \sim -\frac{i}{2}\langle \psi^T C\gamma_5 \tau_c^2 \tau_f^2 \psi \rangle + \mathrm{h.c.}$: diquark condensate

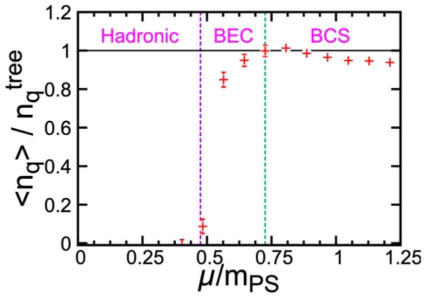
diquark condensate by lattice

Iida et al (2024)

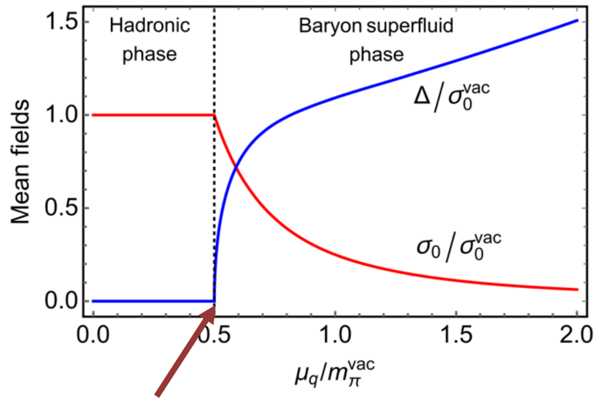
baryon superfluidity



$\begin{cases} \mathrm{BEC}: \langle n_q \rangle / n_q^{\mathrm{tree}} \lesssim 1 \\ \mathrm{BCS}: \langle n_q \rangle / n_q^{\mathrm{tree}} \approx 1 \end{cases}$



σ_0 and Δ vs μ_q



Input here

$\sigma_0^{\mathrm{vac}} = 250\,\mathrm{MeV}$ (put by hand)

$\lambda_1 = c = 0$ (large Nc)

$m_\pi^{\mathrm{vac}} = 738\,\mathrm{MeV}$

$m_{a_0}^{\mathrm{vac}}/m_\pi^{\mathrm{vac}} = 2.18$

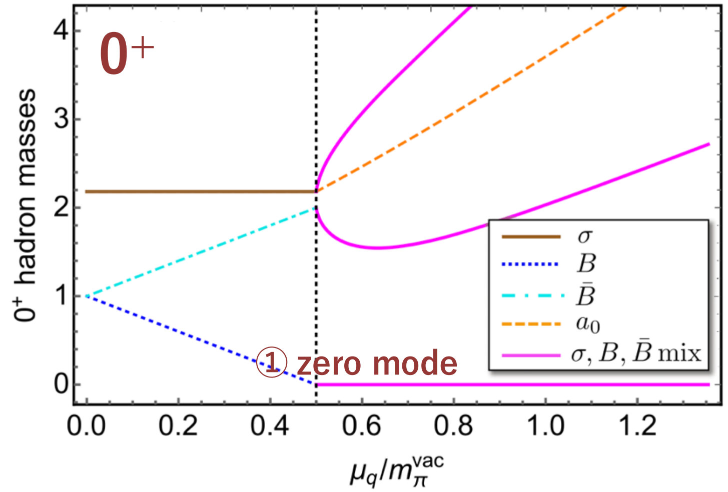
lattice Murakami et al

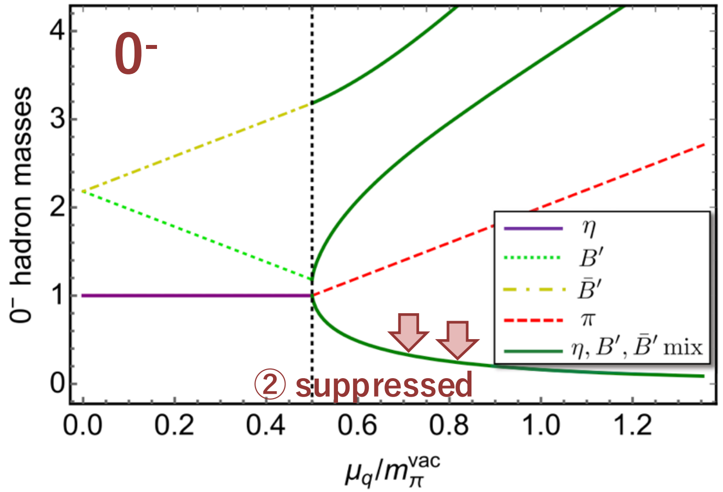
2nd order phase transition at $\mu_q = m_\pi^{\mathrm{vac}}/2$

Hadron masses

- **Mass spectrum from my model and lattice result at T~0**

My model





Input here

$\sigma_0^{\rm vac} = 250\,{\rm MeV}$ (put by hand)

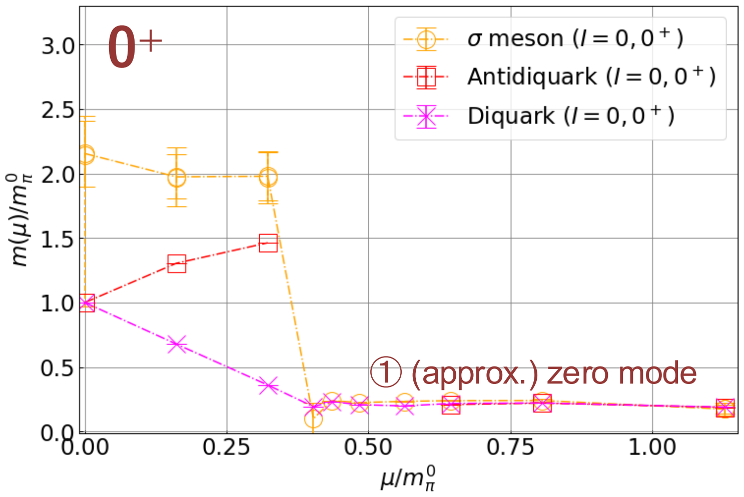
$\lambda_1 = c = 0$ (large Nc)

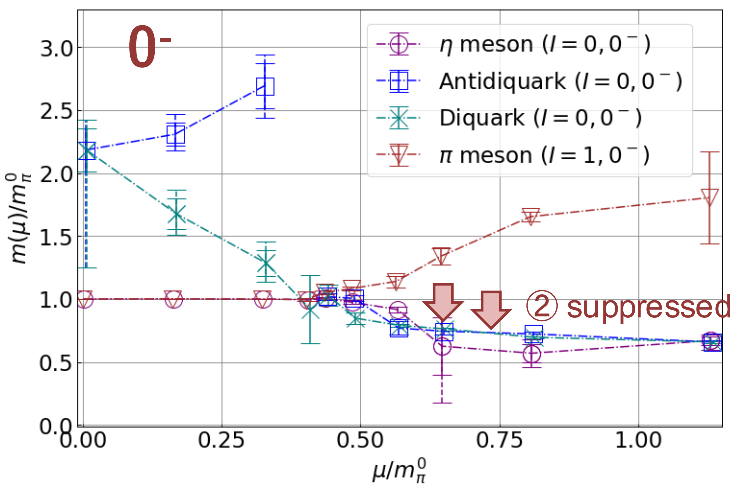
$m_\pi^{\rm vac} = 738\,{\rm MeV}$

$m_{a_0}^{\rm vac}/m_\pi^{\rm vac} = 2.18$

lattice
Murakami et al

Lattice (Murakami et al)





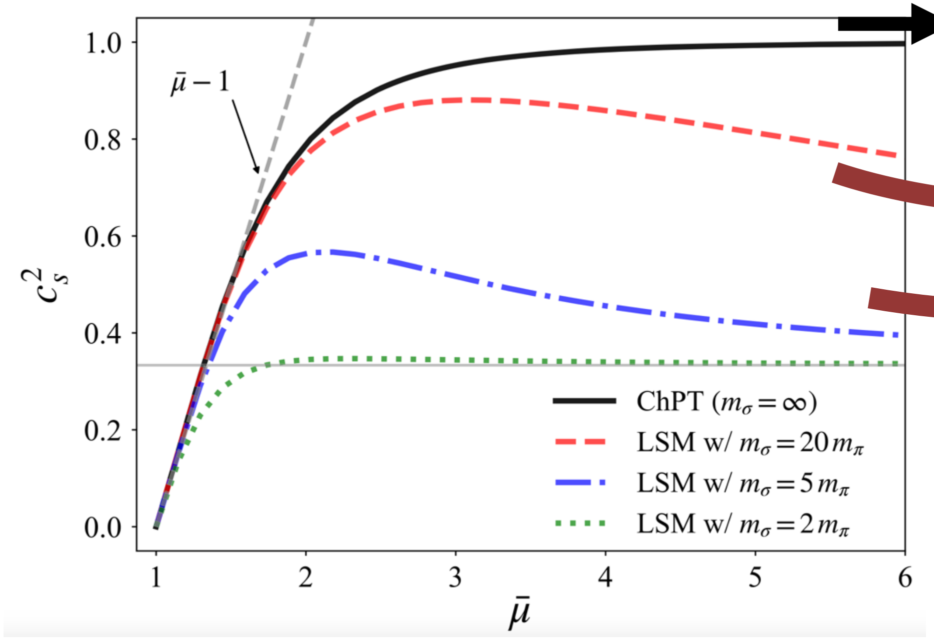
- qualitatively good agreement for low-lying hadrons!

Sound velocity

- **Sound velocity peak**

Kawaguchi-**Suenaga** (2024)

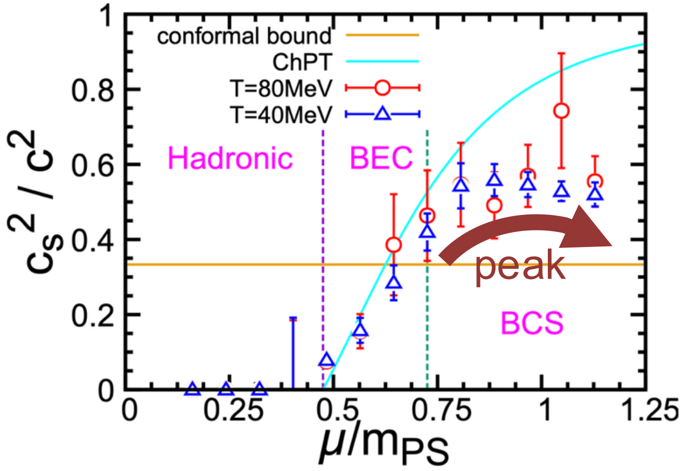
$$\bar{\mu} = \mu_q/\mu_{\rm cr} \ (\mu_{\rm cr} = m_\pi/2)$$

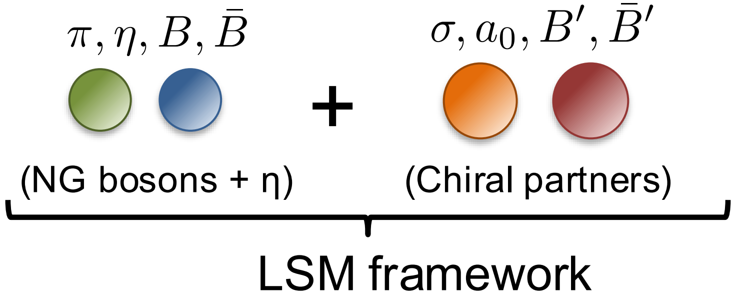


ChPT result $(c_s^2 \to 1)$: no peak

peak when $m_\sigma < \infty$ $(c_s^2 \to 1/3)$

Lattice: Iida et al (2024)





- The peak structure is driven by contributions from parity partners

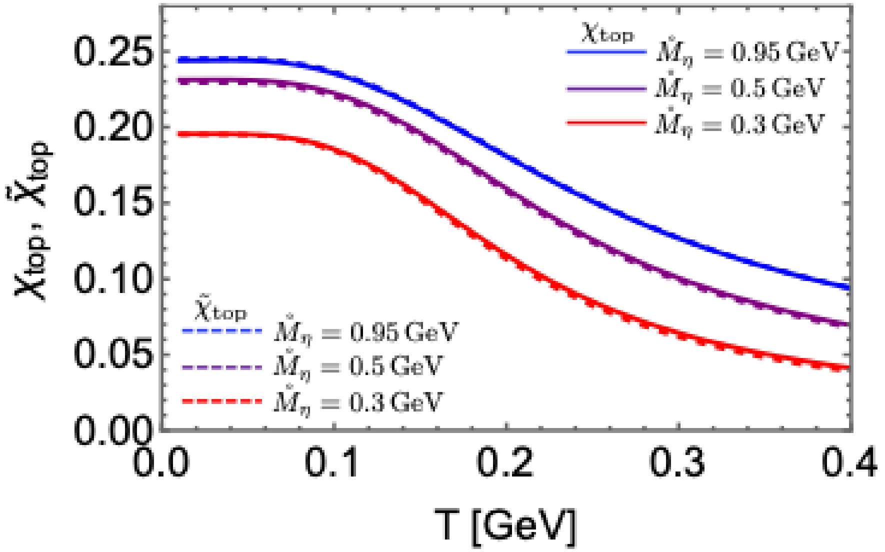
- Any connection with crossover to quark matter?

FRG for anomaly effect

- **Topological susceptibility at finite T**

Definition: $\bar{\chi}_{\rm top} \equiv -i\int d^4x \langle 0|\mathrm{T}Q(x)Q(0)|0\rangle = \frac{\mathring{M}_\pi^4 \mathring{\sigma}_0^2}{4}\left(\frac{1}{M_\pi^2} - \frac{1}{M_\eta^2}\right)$ with $Q = (g_s^2/64\pi^2)\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}$

$\bar{\chi}_{\rm top} = -\frac{m_l\langle\bar\psi\psi\rangle}{4}\delta_m$ with $\delta_m = 1 - \frac{\chi_\eta}{\chi_\pi}$



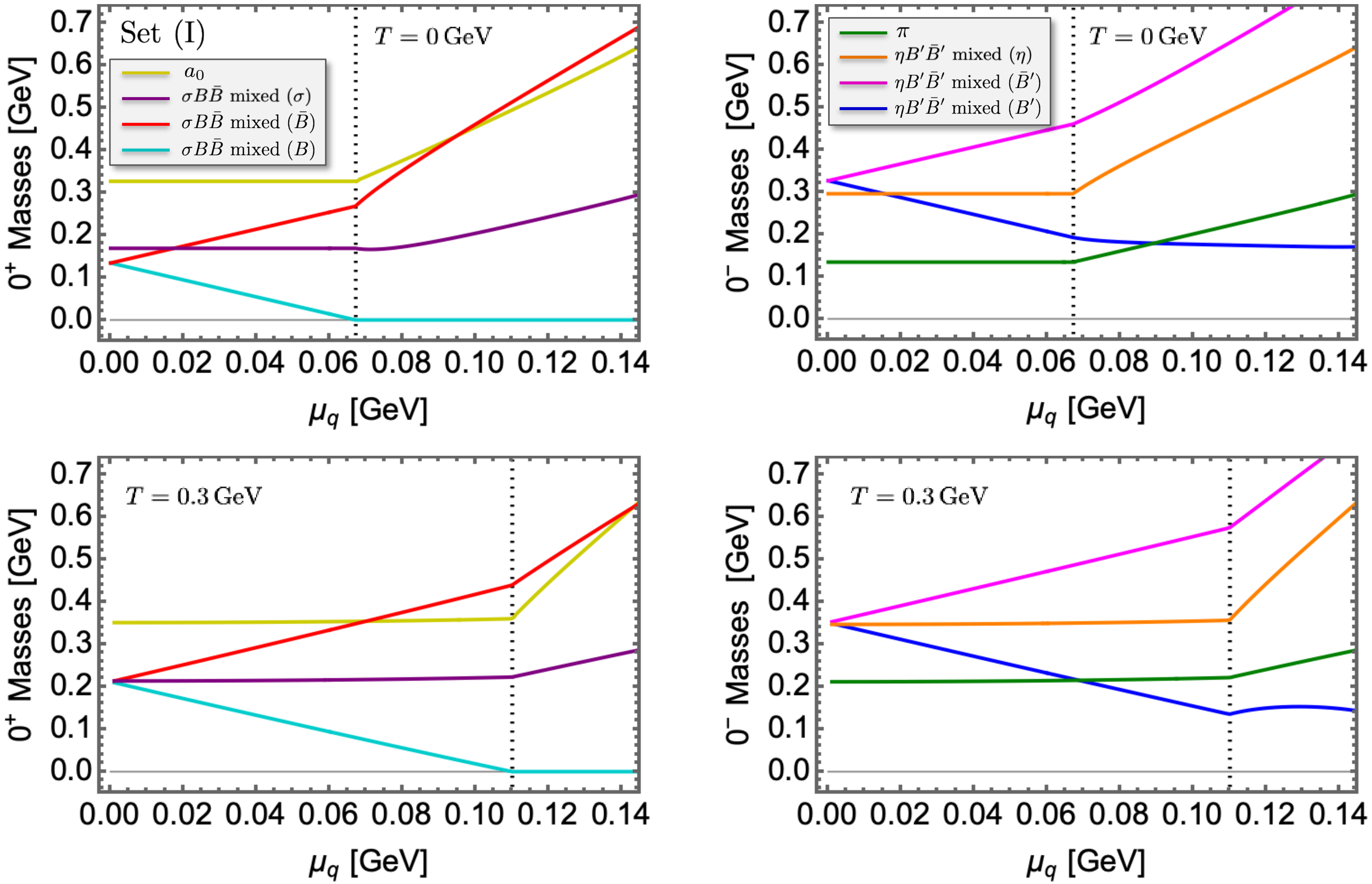
$$\chi_{\rm top} \equiv \frac{\bar{\chi}_{\rm top}}{\mathring{M}_\pi^2\mathring{\sigma}_0^2} = \frac{\mathring{M}_\pi^2}{4}\left(\frac{1}{M_\pi^2} - \frac{1}{M_\pi^2 + (M_\eta^2 - M_\pi^2)}\right) \quad (\leftarrow \text{full result})$$

$$\tilde{\chi}_{\rm top} \equiv \frac{\mathring{M}_\pi^2}{4}\left(\frac{1}{M_\pi^2} - \frac{1}{M_\pi^2 + (\mathring{M}_\eta^2 - \mathring{M}_\pi^2)}\right) \quad (\leftarrow \text{reference})$$

NOTE: $M_\eta^2 - M_\pi^2 = -2a - (2c_1 + c_2)\sigma_0^2 \propto$ (anomaly effect)

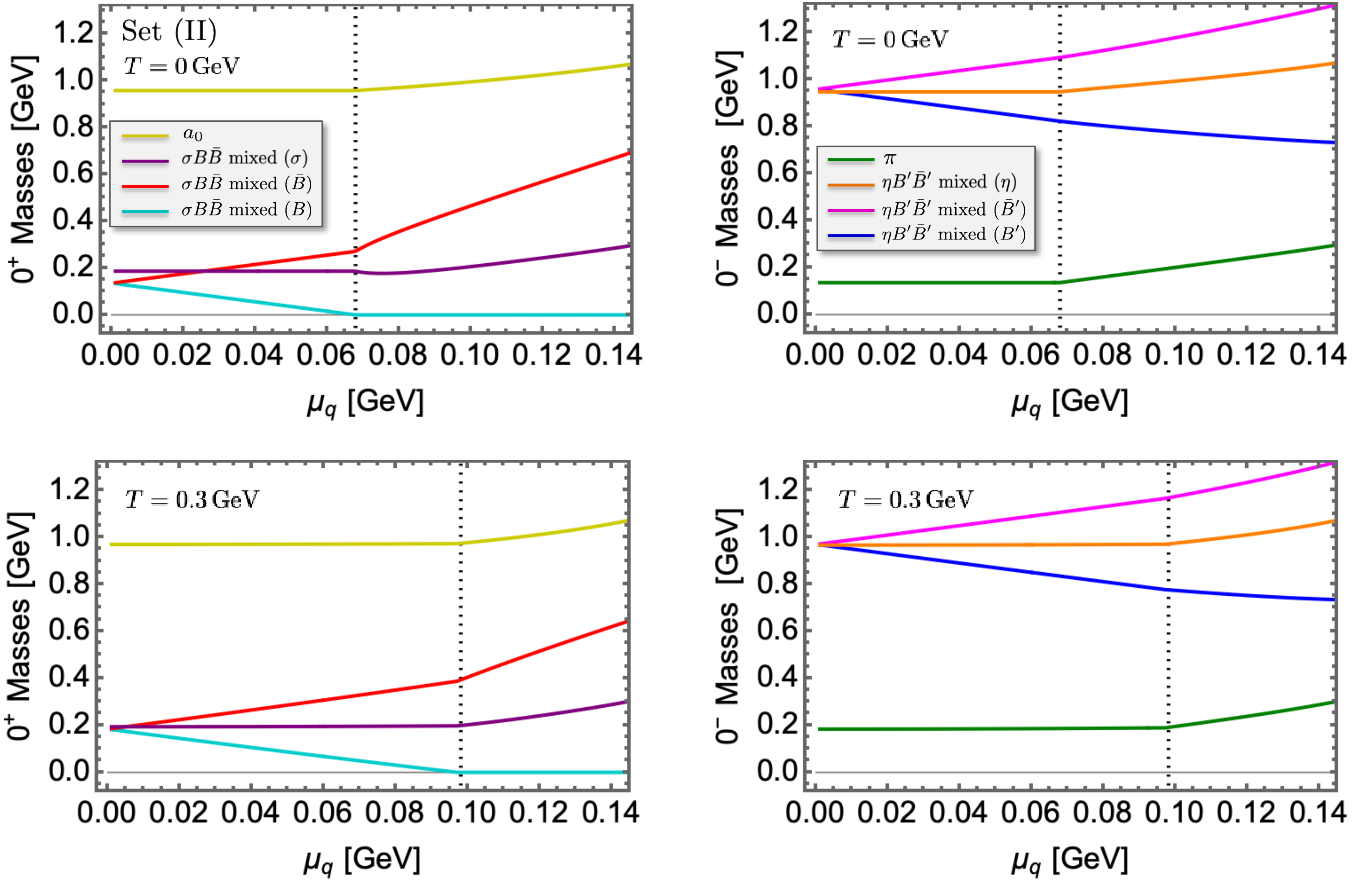
- $\chi_{\rm top}$ is suppressed at higher temperature followed by chiral restoration: $\chi_{\rm top} \sim \frac{\mathring{M}_\pi^2}{4}\frac{1}{M_\pi^2} \propto \sigma_0$
- No difference between $\chi_{\rm top}$ and $\tilde{\chi}_{\rm top}$
  $\rightarrow$ topological susceptibility is not suitable probe to see corrections of anomaly effects

Hadron mass

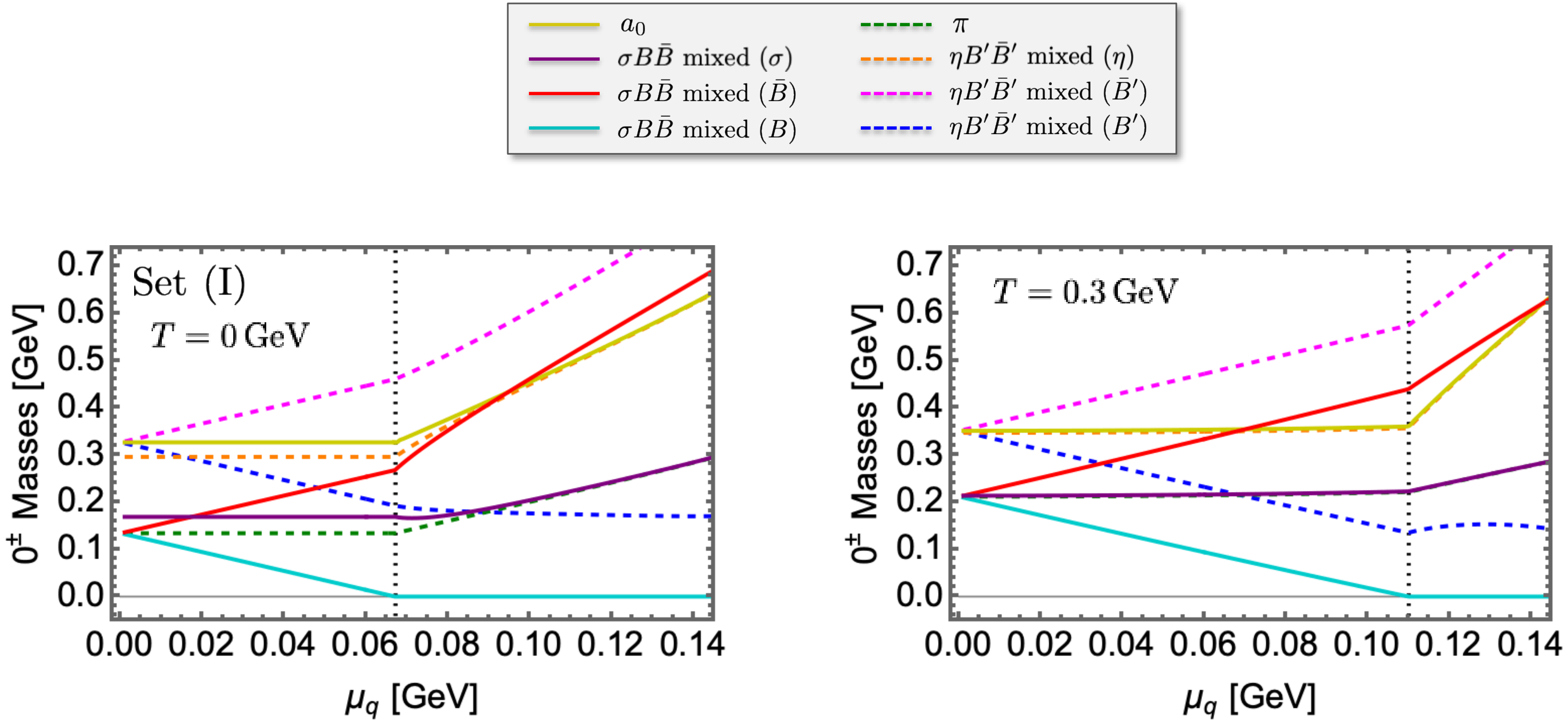


Set (I)
$T = 0\,\mathrm{GeV}$
a_0
$\sigma B\bar{B}$ mixed (σ)
$\sigma B\bar{B}$ mixed ($\bar{B}$)
$\sigma B\bar{B}$ mixed (B)
0^+ Masses [GeV]
μ_q [GeV]
π
$\eta B'\bar{B}'$ mixed (η)
$\eta B'\bar{B}'$ mixed ($\bar{B}'$)
$\eta B'\bar{B}'$ mixed (B')
0^- Masses [GeV]
$T = 0.3\,\mathrm{GeV}$
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
0.00 0.02 0.04 0.06 0.08 0.10 0.12 0.14

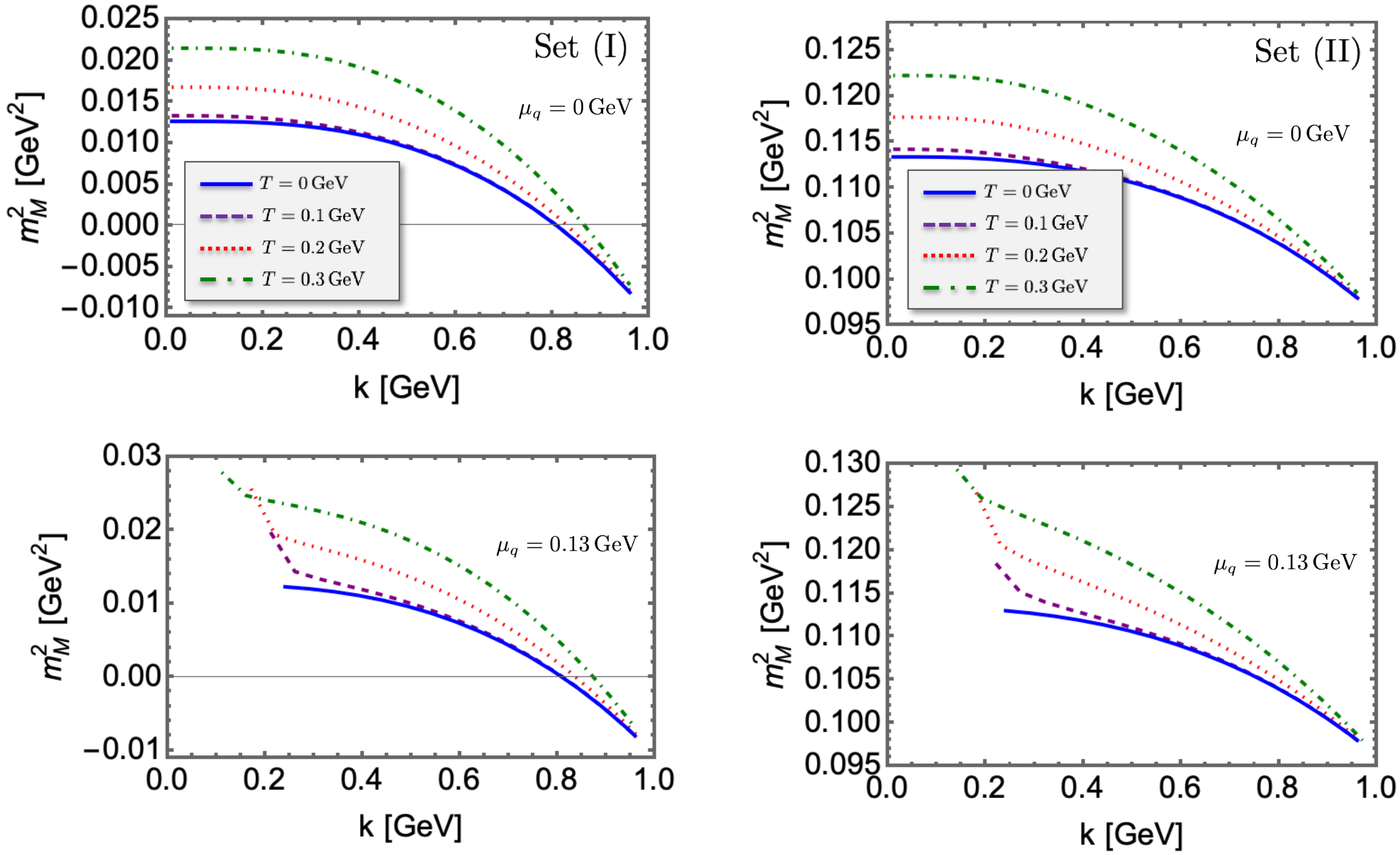
Hadron mass


Set (II)
$T = 0\,\mathrm{GeV}$
a_0
$\sigma B\bar{B}$ mixed (σ)
$\sigma B\bar{B}$ mixed ($\bar{B}$)
$\sigma B\bar{B}$ mixed (B)
0^+ Masses [GeV]
μ_q [GeV]
$T = 0\,\mathrm{GeV}$
π
$\eta B'\bar{B}'$ mixed (η)
$\eta B'\bar{B}'$ mixed ($\bar{B}'$)
$\eta B'\bar{B}'$ mixed (B')
0^- Masses [GeV]
μ_q [GeV]
$T = 0.3\,\mathrm{GeV}$
0^+ Masses [GeV]
μ_q [GeV]
$T = 0.3\,\mathrm{GeV}$
0^- Masses [GeV]
μ_q [GeV]
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.00 0.02 0.04 0.06 0.08 0.10 0.12 0.14

- **Mass degeneracies of chiral partners**



Determination of IR cutoff



$\Rightarrow$ $k_{\mathrm{IR}} = 0.3\,\mathrm{GeV}$