膨張する量子ホールエッジ系のバルクカレント

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arXiv:2506.20338

TQFT@ISSP, 2025/9/3

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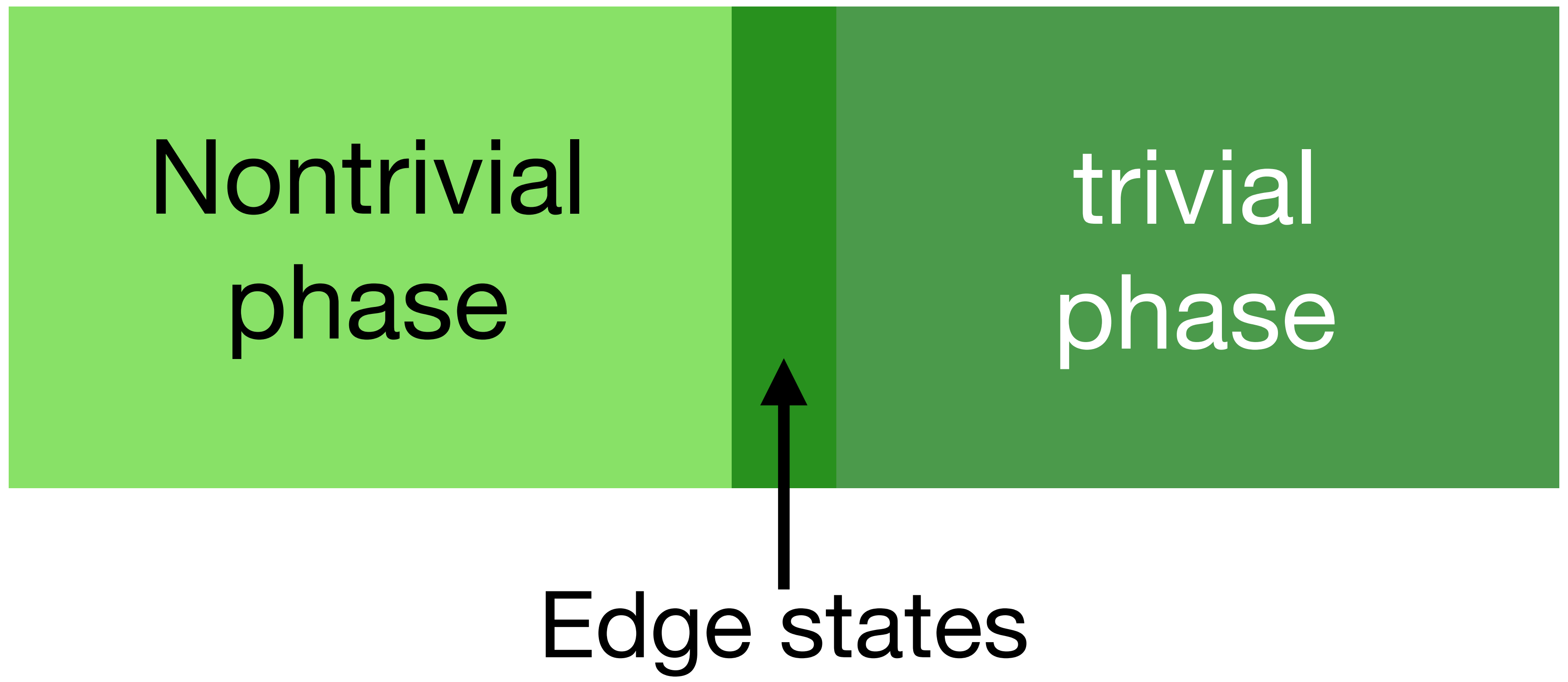
Introduction

Extending our understanding of topological matter including edge dynamics



- characterized by **topological invariants**

  ➡ existence of non-trivial phase

  ➡ boundary of trivial/non-trivial phase: edge states

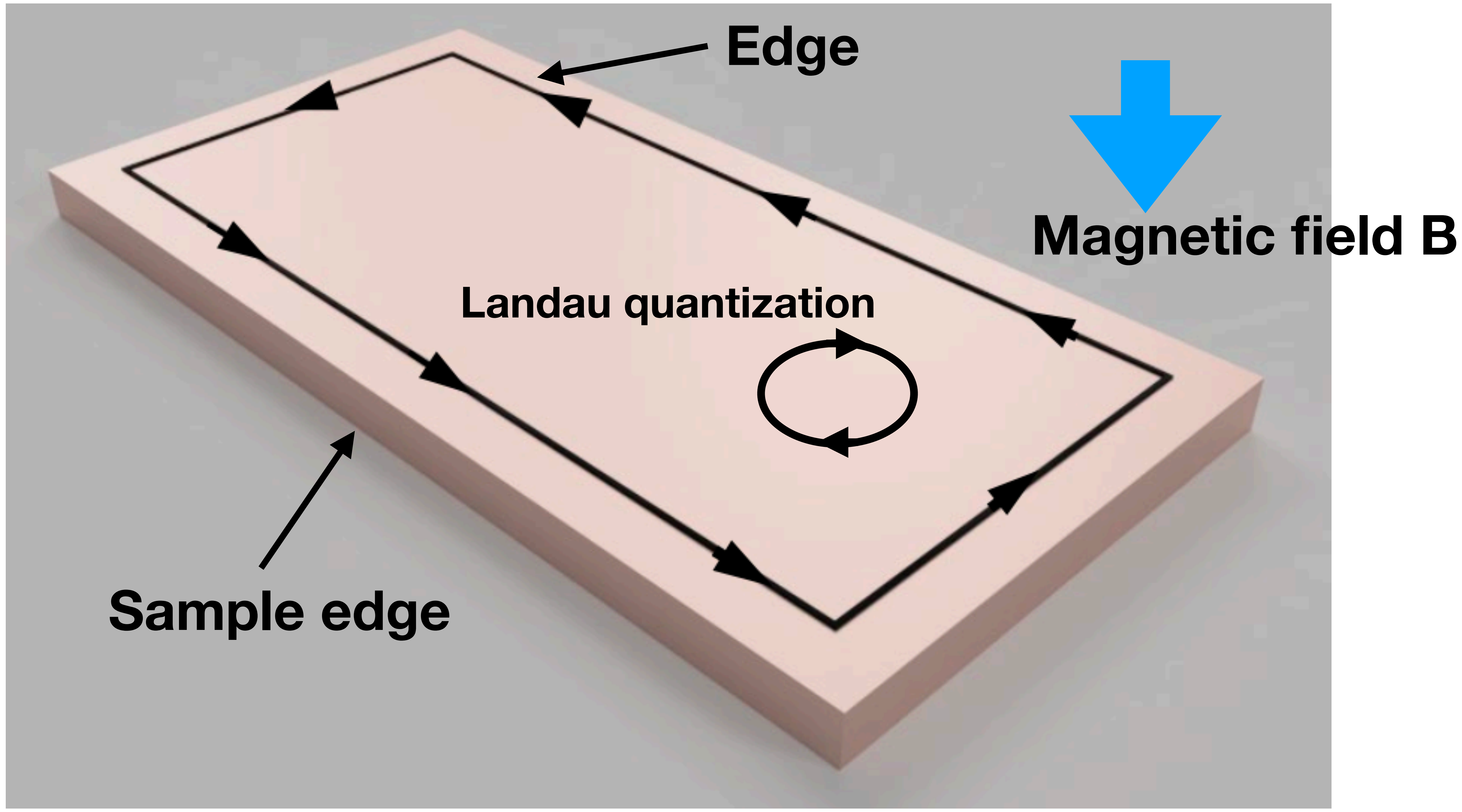
- expected to be the basis for state-of-the-art tech.

  No backscattering ➡ robust to impurities ➡ **suppress decoherence**

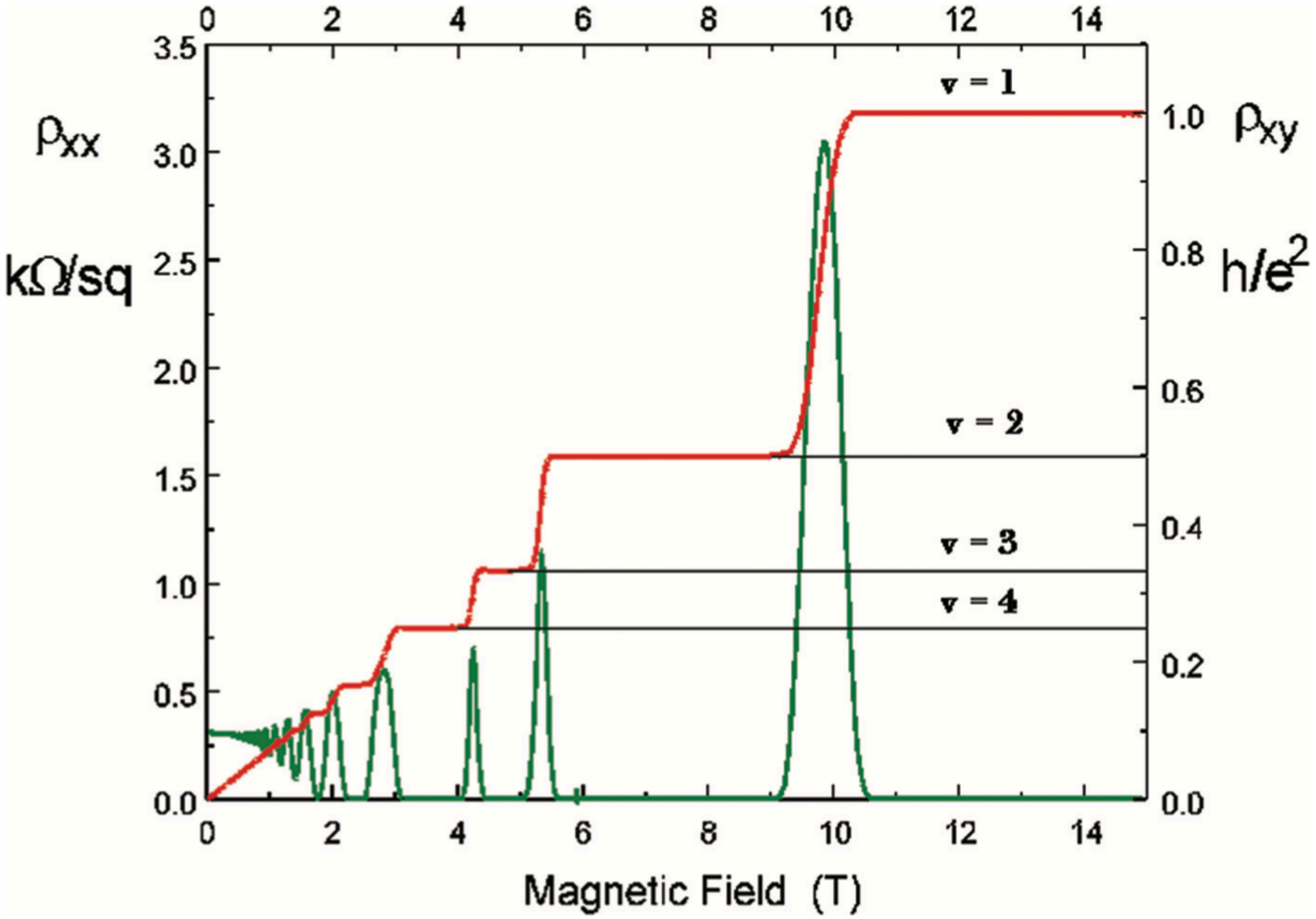
Topological matter — a central thema in Condensed Matter Physics

Quantum Hall Systems (QHS)

2DEG system with strong magnetic field and low temperature



[Figure taken from G. Yusa's slide]



[Gracia and Pimental, Rev. Brasileira de Ensino de Física, 2023]

Hall resistivity (and conductance) is quantized $\rho_{xy} = \frac{1}{\nu}\frac{h}{e^2}$ $\rho_{xx} = 0$

Quantum Hall Systems (QHS)

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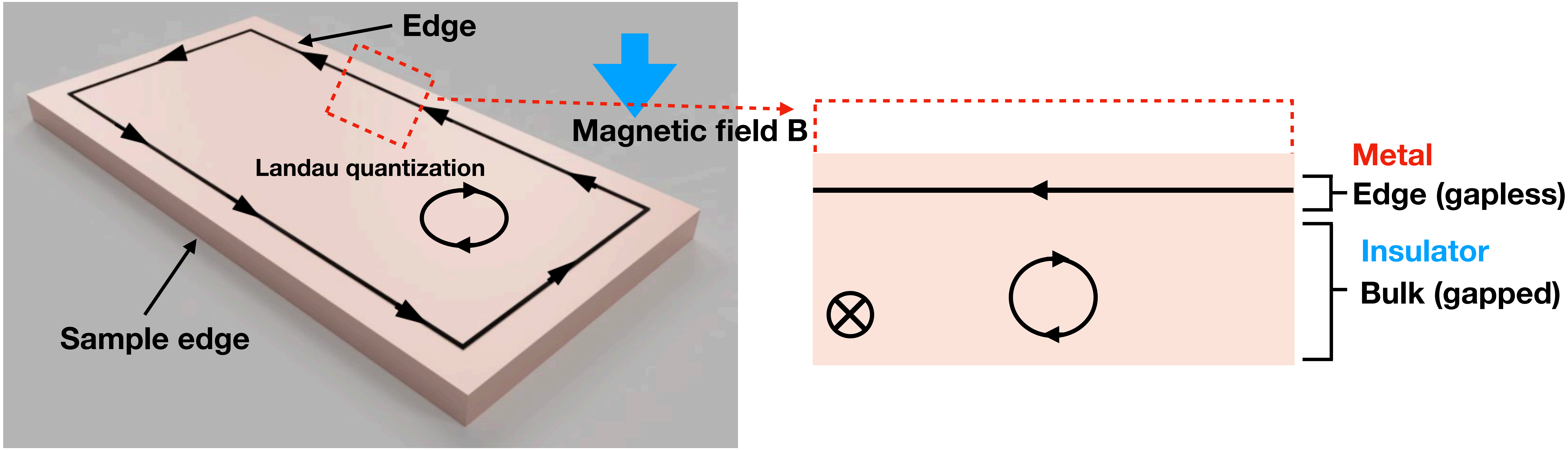


[Figure taken from G. Yusa's slide]

Edge states are described by 1+1 dim. chiral gapless boson

[X. G. Wen PRB 1990]

Conformal field theory in 1+1 dim.

QHS with an expanding edge

[M. Hotta, Y. Nambu, YS, K. Yamamoto, G. Yusa, PRD 105. 105009 (2022)]

L : length of the expanding region

[experiment: in progress@ Tohoku U.]

A region of the edge state is experimentally expanded

QHS with an expanding edge

[M. Hotta, Y. Nambu, YS, K. Yamamoto, G. Yusa, PRD 105. 105009 (2022)]

Theoretically, we smoothly connetcted regions I, II, and III

$$ds^2 = \omega^2(t,x)\eta_{ij}dx^i dx^j$$
$$= \begin{cases} -dx_{\mathrm{I}}^+ dx_{\mathrm{I}}^- & \text{Regions I} \\ -e^{2\Theta(t)}dx^+dx^- & \text{Regions II} \\ -dx_{\mathrm{III}}^+ dx_{\mathrm{III}}^- & \text{Regions III} \end{cases}$$

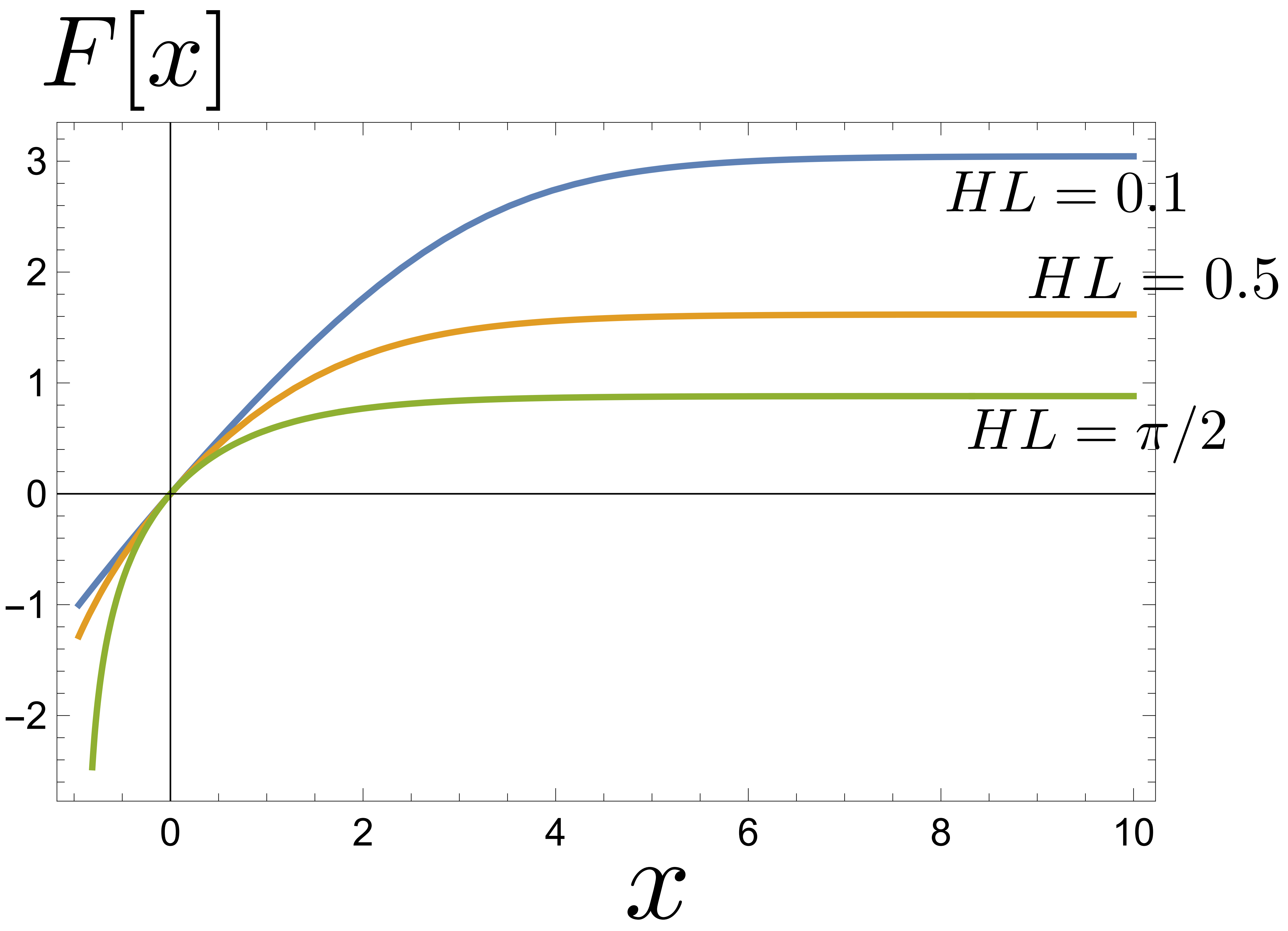
Relationship between regions I and III

$$x_{\mathrm{I}}^+ = \Phi[-L + \Phi^{-1}[x_{\mathrm{III}}^+ + \Phi[L/2]]] - \Phi[-L/2]$$

$$\Phi[x] = \int_0^x dy e^{\Theta(y)}$$

Considering expanding scale factor

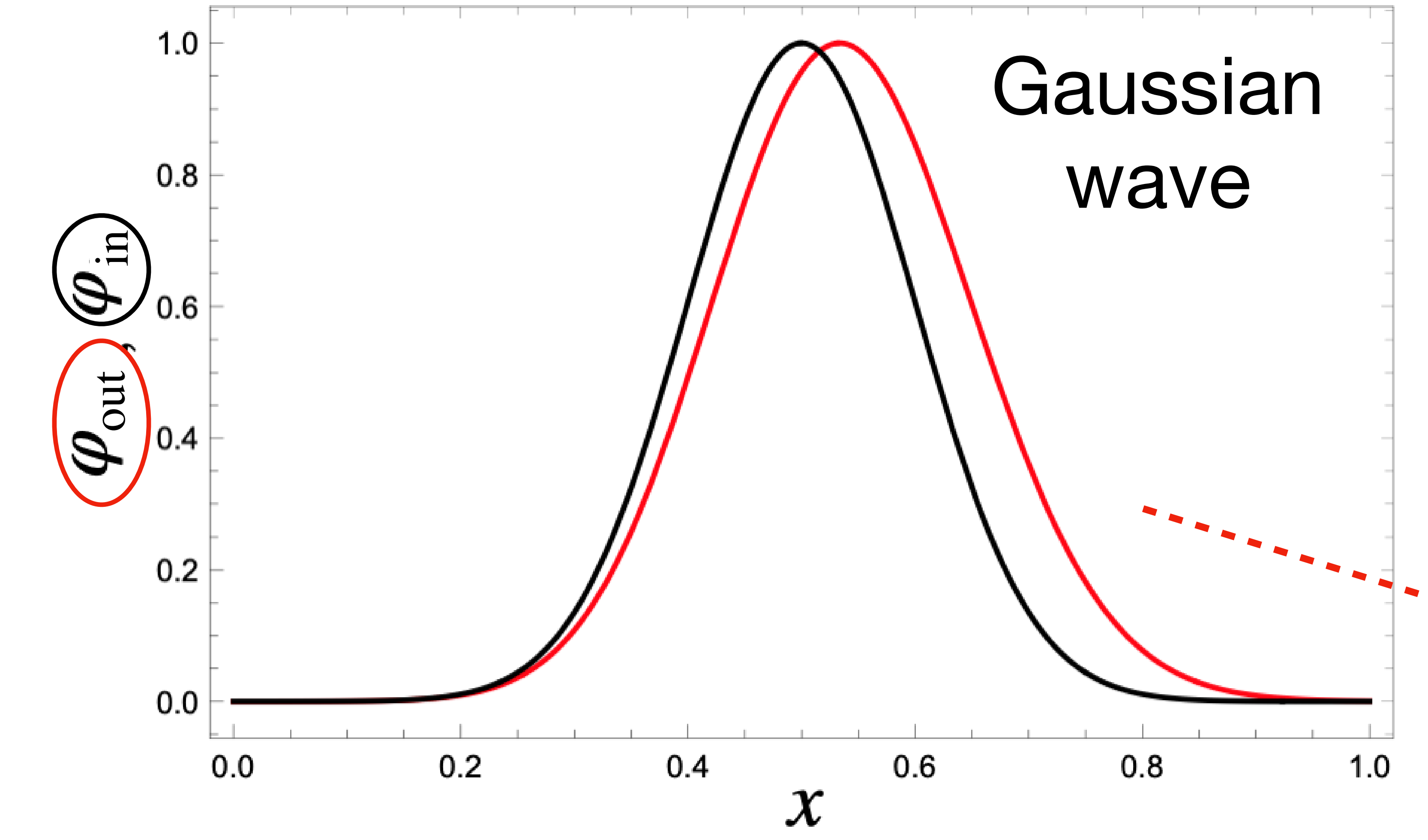
$$e^{\Theta(t)} = \frac{1}{\cos(Ht)} \Rightarrow x_{\mathrm{I}}^+ = F[x_{\mathrm{III}}^+]$$

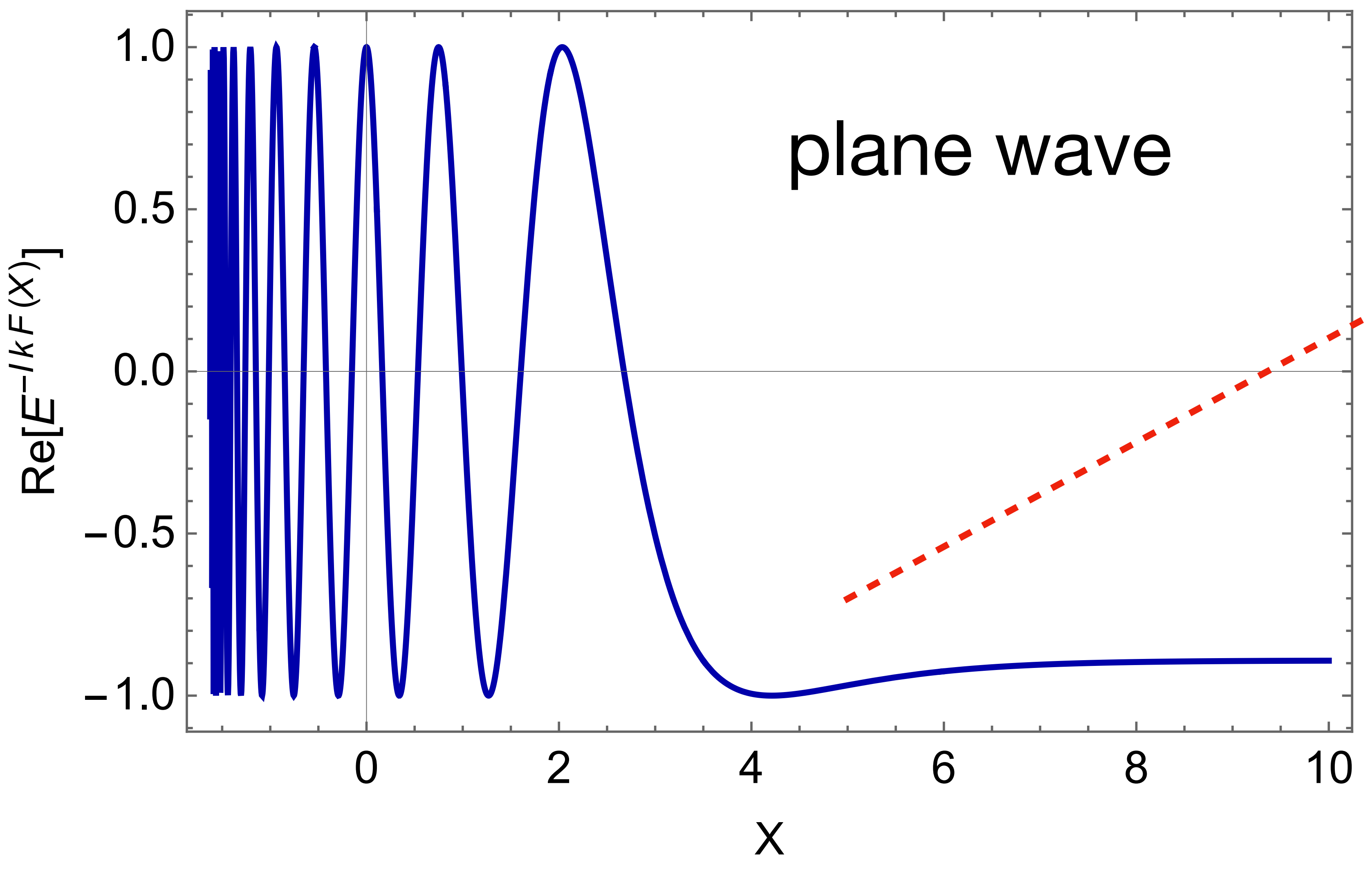


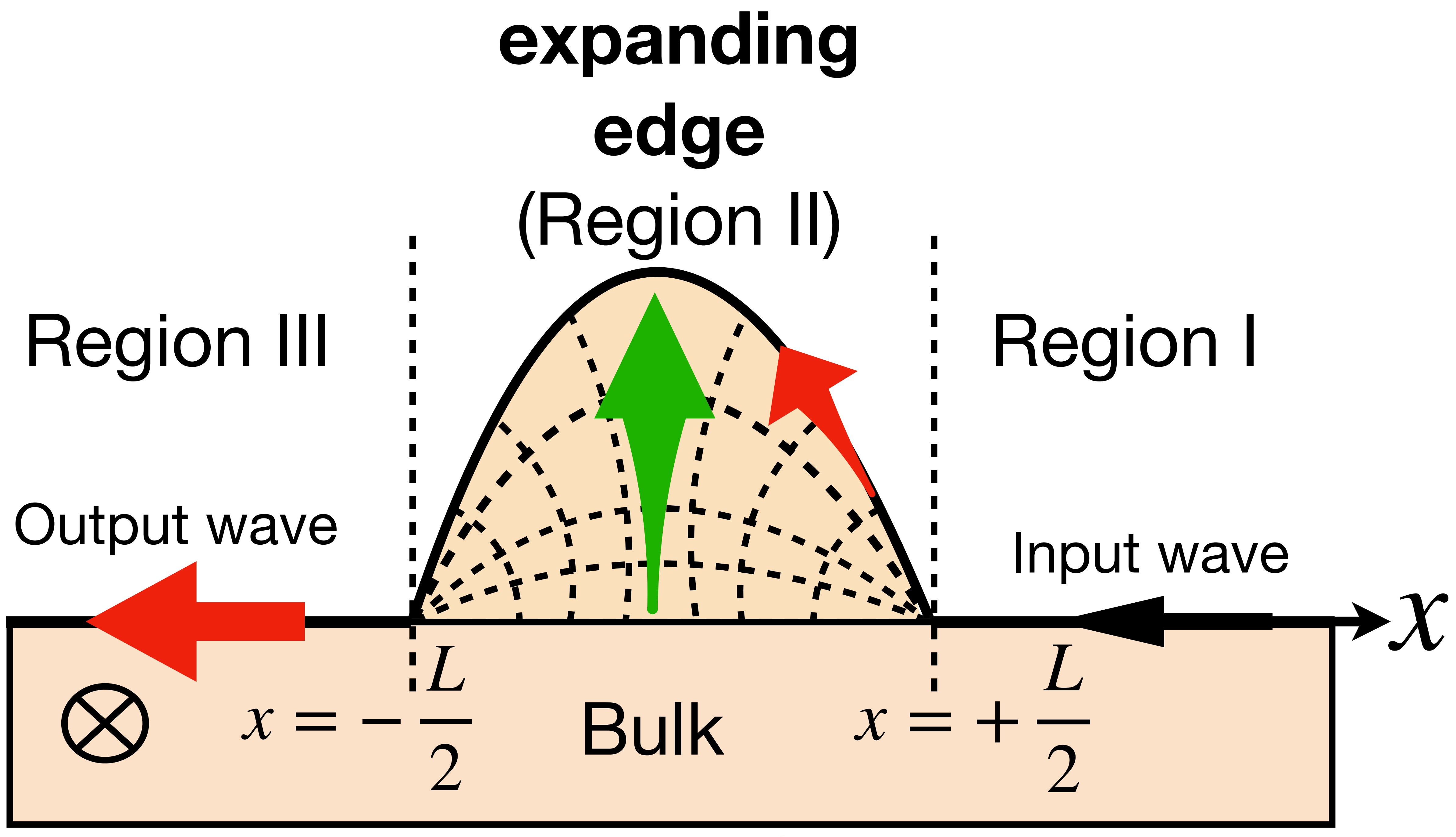
QHS with an expanding edge

[M. Hotta, Y. Nambu, YS, K. Yamamoto, G. Yusa, PRD 105. 105009 (2022)]

Ex.) redshift (stretching the wave length)







L : length of the expanding region

Plane wave: $u_k(x_{\rm I}^+) = \dfrac{e^{-ikx_{\rm I}^+}}{\sqrt{4\pi k}} = \dfrac{e^{-ikF[x_{\rm III}^+]}}{\sqrt{4\pi k}}$

QHS with an expanding edge

[M. Hotta, Y. Nambu, YS, K. Yamamoto, G. Yusa, PRD 105. 105009 (2022)]

A thermal distribution: (Gibbons-)Hawking radiation

- gapless chiral boson $\partial_{-}\hat{\phi} = 0$

$$\hat{\phi}(x^+) = \int_0^\infty dk \left(u_k(x^+)\hat{a}_k + u_k^*(x^+)\hat{a}_k^\dagger \right)$$

- Bogoliubov transformation between region I and III

[Nambu, Hotta, PRD (2023)]

$$u_k^{(\mathrm{I})} = \int_0^\infty dk' \left[\alpha\left(k, k'\right) u_{k'}^{(\mathrm{III})} + \beta\left(k, k'\right) u_{k'}^{(\mathrm{III})*} \right]$$

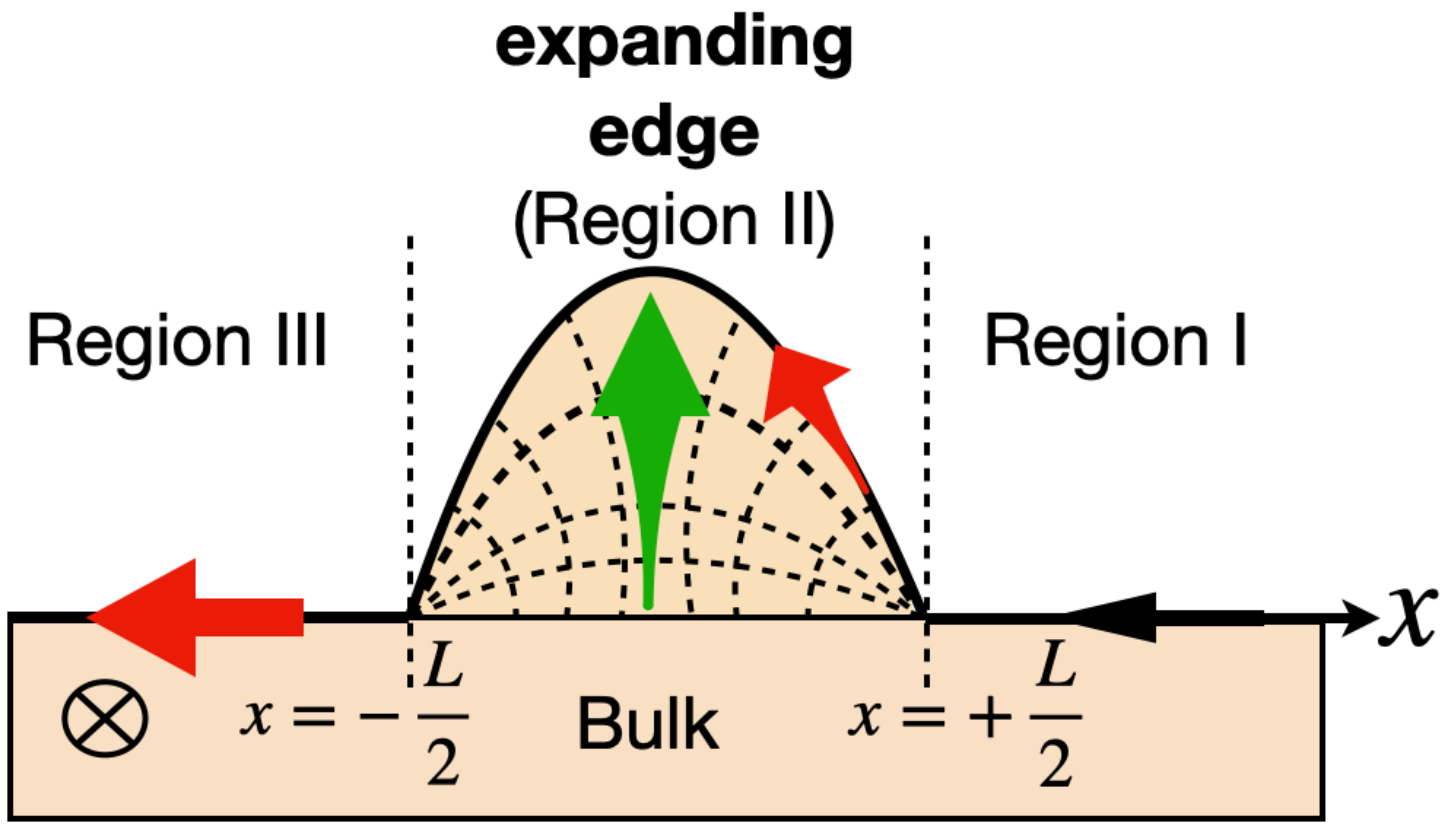
- At $x_{\mathrm{III}}^+ \to \infty$

$$\left|\beta\left(k, k'\right)\right|^2 \sim \frac{1}{2\pi H k'} \frac{1}{\exp(2\pi k/H) - 1}$$

GH temperature

$$T = \frac{H}{2\pi}$$

Planck distribution !

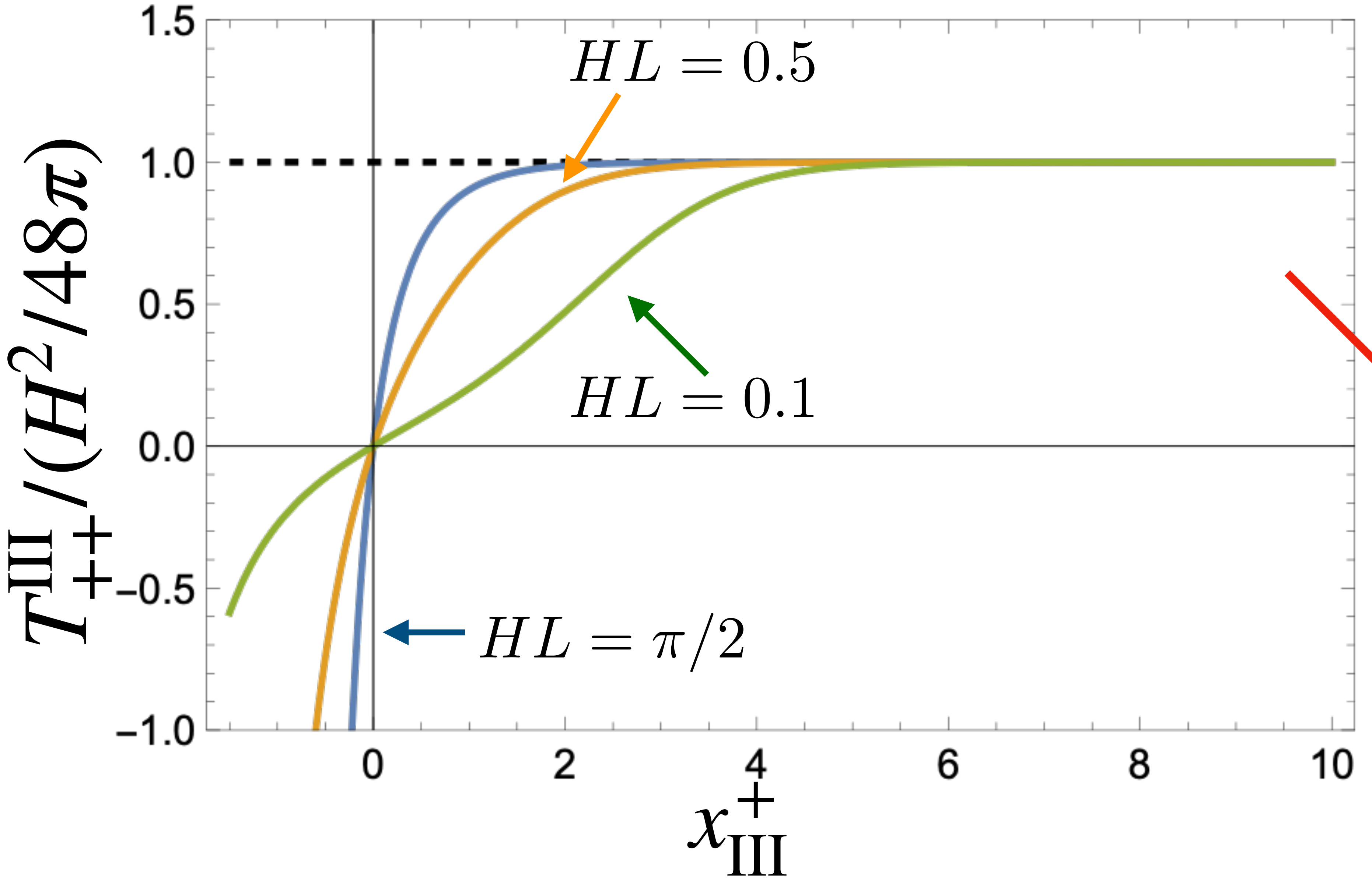


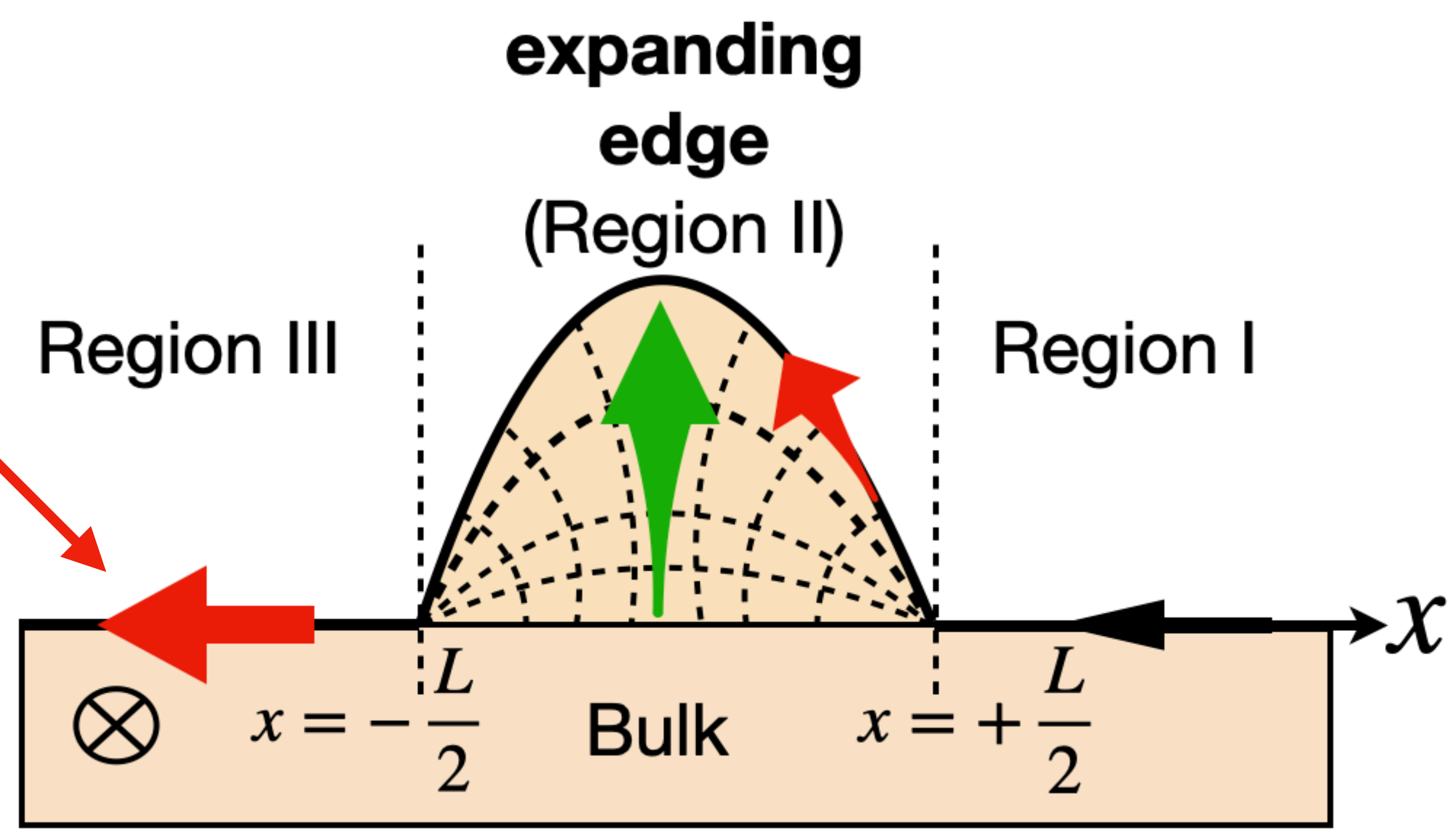
L : length of the expanding region

QHS with an expanding edge: Anomaly method

Cancellation of the gravitational anomaly on the edge [Yoshimoto, Nambu (2025)]

- Gravitational anomaly eq. $\nabla_i T^{ij}_{\mathrm{edge}} = \frac{1}{96\pi}\frac{\epsilon^{ij}}{\sqrt{-h}}\nabla_i R \Rightarrow T^{\mathrm{III}}_{++} = \frac{H^2}{48\pi}\left[1-\left(\frac{\partial}{\partial x^+_{\mathrm{III}}}F[x^+_{\mathrm{III}}]\right)^2\right]$





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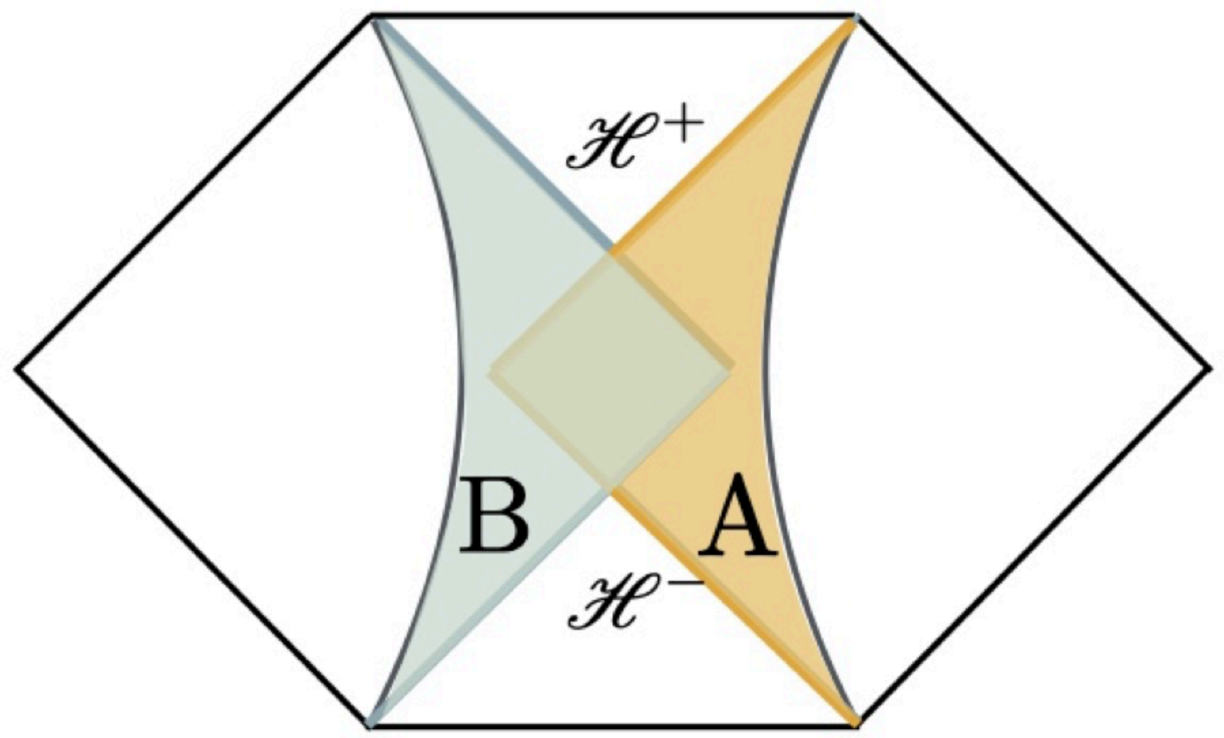

[taken from Yoshimoto, Nambu (2025)]

At $x^+_{\mathrm{III}} \to \infty$, $T^{\mathrm{III}}_{++} = \frac{H^2}{48\pi} = \frac{\pi}{12}T^2$ with $T = \frac{H}{2\pi}$

[Cappelli et al., (2002)]

Energy flux formula carried by a chiral edge mode appeared

Motivation

The role of bulk in this quantum Hall system is unclear

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[Callan and Harvey, 1985]

Existence the of bulk ensures conservation of energy and momentum and cancells the gravitational anomly in the whole system

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- Chiral edge modes have a gravitational anomaly [Alvarez-Gaumé, Witten (1984)]

Existence of bulk current due to the conservation of energy and momentum?

Motivation

The role of bulk in this quantum Hall system is unclear

[Callan and Harvey, 1985]

➡ **Existence the of bulk ensures conservation of energy and momentum and cancells the gravitational anomly in the whole system**

- Chiral edge modes have a gravitational anomaly [Alvarez-Gaumé, Witten (1984)]

Existence of bulk current due to the conservation of energy and momentum?

- Recent experiment and numerical simulation suggest a bulk-edge interaction

[France, et al., PRL (2025)]

➡ **Excitation in the edge region leading to excitation in the bulk**

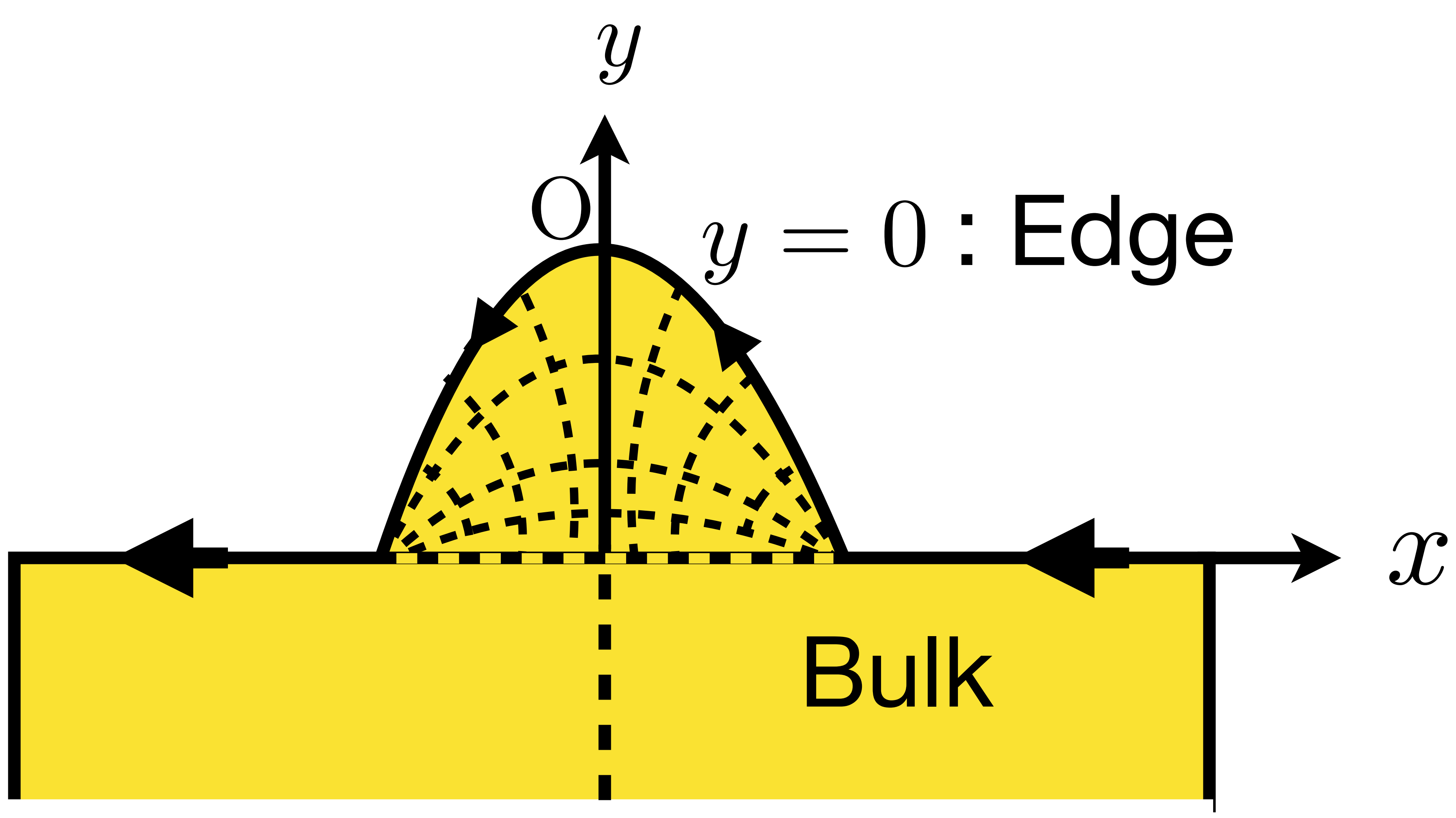
Contents

Expanding edge and bulk in the QHS

- Metric (Gaussian normal coordinates)

[Kraus and Larsen (2006)]

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$
$$= a^2(t,x,y)\eta_{ij}dx^i dx^j + dy^2$$



- Scale factor

$$a^2(t,x,y) = \underbrace{\left(\omega^2(t,x)-1\right)}_{\text{curved region}} f(y) + \underbrace{1}_{\text{flat region}}$$

curved region **flat region**

$f(y)$: Spatial profile function

➡ smoothly connect edge region and bulk region

$$f(0)=1 \qquad f'(y)|_{y=0}=0$$

- On the edge

$$ds^2 = h_{ij}dx^i dx^j + dy^2 = \omega^2(t,x)\eta_{ij}dx^i dx^j + dy^2$$

Gravitational Chern-Simons action

- Effective model of QHS $W_{\rm tot} = W_{\rm edge} + W_{\rm CS}$, $\delta W_{\rm edge} = \frac{1}{2}\int d^2x\sqrt{-h}T^{ij}_{\rm edge}\delta h_{ij}$

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- Bulk effective action

$$W_{\text{CS}} = \underbrace{\frac{c_-}{96\pi}}_{\text{chiral central charge}} \int d^3x \underbrace{\epsilon^{\mu\nu\rho}}_{\text{Levi-Civita}} \mathrm{Tr}\left[\Gamma_\mu \partial_\nu \Gamma_\rho + \frac{2}{3}\Gamma_\mu\Gamma_\nu\Gamma_\rho\right] , \mathrm{Tr}[(A)^\alpha_\beta (B)^\gamma_\delta] = A^\alpha_\beta B^\beta_\alpha$$

➡ TRS broken

Gravitational Chern-Simons action

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chiral central charge

Levi-Civita

➡ TRS broken

The energy and momentum should conserve in the entire system

➡ $\delta_\xi W_{\mathrm{tot}} = 0$ with $x^i \to x^i - \xi^i(x)$ and $\delta_\xi h_{ij} = \nabla_i\xi_j + \nabla_j\xi_i$

Gravitational Chern-Simons action

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➡ $$\delta_\xi W_{\text{edge}} = -\delta_\xi W_{\text{CS}} = \frac{c_-}{96\pi}\int d^2x h^{jk}\epsilon^{lm}\partial_i\partial_l\Gamma^i_{mk}$$

➡ $$\nabla_i T^{ij}_{\text{edge}} = -\frac{c_-}{96\pi}h^{jk}\frac{\epsilon^{lm}}{\sqrt{-h}}\partial_i\partial_l\Gamma^i_{mk}$$

A naive coordinate transformation gives anomaly eq. which is not covariant form

Gravitational anomaly

- General variation

[Jensen et al., 2013]

$$\delta W_{\rm tot} = \delta W_{\rm edge} + \delta W_{\rm CS}$$
$$= \frac{1}{2}\int d^2x\sqrt{-h}T^{ij}_{\rm edge}\delta h_{ij} + \frac{1}{2}\int d^3x\sqrt{-g}T^{\mu\nu}_{\rm bulk}\delta g_{\mu\nu} + \frac{1}{2}\int d^2x\sqrt{-h}P^{ij}\delta h_{ij}$$

boundary term of CS

$$= \frac{1}{2}\int d^2x\sqrt{-h}T^{ij}_{\rm cov}\delta h_{ij} + \frac{1}{2}\int d^3x\sqrt{-g}T^{\mu\nu}_{\rm bulk}\delta g_{\mu\nu}$$

$$T^{ij}_{\rm cov} = T^{ij}_{\rm edge} + P^{ij} \Rightarrow \nabla_i T^{ij}_{\rm cov} = \nabla_i T^{ij}_{\rm edge} + \nabla_i P^{ij} = -\frac{c_-}{96\pi}\frac{\epsilon^{ij}}{\sqrt{-h}}\nabla_i R$$

☑ Bardeen-Zumino polynomial [K. Hotta et al., 2009]

$$P^{ij} = -\frac{c_-}{192\pi\sqrt{-h}}(\delta^i_p\delta^j_q + \delta^j_p\delta^i_q)\left(\epsilon^{rl}g^{mq}\nabla_r\Gamma^p_{ml} + \epsilon^{pl}g^{mq}\nabla_s\Gamma^s_{ml} - \epsilon^{pl}g^{mr}\nabla_r\Gamma^q_{ml}\right)$$

☑ Bulk energy-momentum tensor

$$T^{\mu\nu}_{\rm bulk} = -\frac{c_-}{48\pi}\frac{1}{\sqrt{-g}}\left(\epsilon^{\gamma\rho\mu}\nabla_\gamma R^\nu_\rho + \epsilon^{\gamma\rho\nu}\nabla_\gamma R^\mu_\rho\right)$$

The existence of bulk modifies the anomaly eq. as covariant form

Contents

Bulk energy-momentum tensor

We consider behaviors of bulk energy-momentum tensor

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = a^2(t,x,y)\eta_{ij}dx^i dx^j + dy^2$$

$$a^2(t,x,y) = \left(\omega^2(t,x) - 1\right)f(y) + 1 \qquad , \omega(t,x) = \frac{1}{\cos(Ht)}$$

$$T^{\mu\nu}_{\text{bulk}} = -\frac{c_-}{48\pi}\frac{1}{\sqrt{-g}}\left(\epsilon^{\gamma\rho\mu}\nabla_\gamma R^\nu_\rho + \epsilon^{\gamma\rho\nu}\nabla_\gamma R^\mu_\rho\right)$$

The system is invariant under the combination of the reflection and sign flip

$$(t,x,y) \to (t,-x,y) \qquad c_- \to -c_-$$

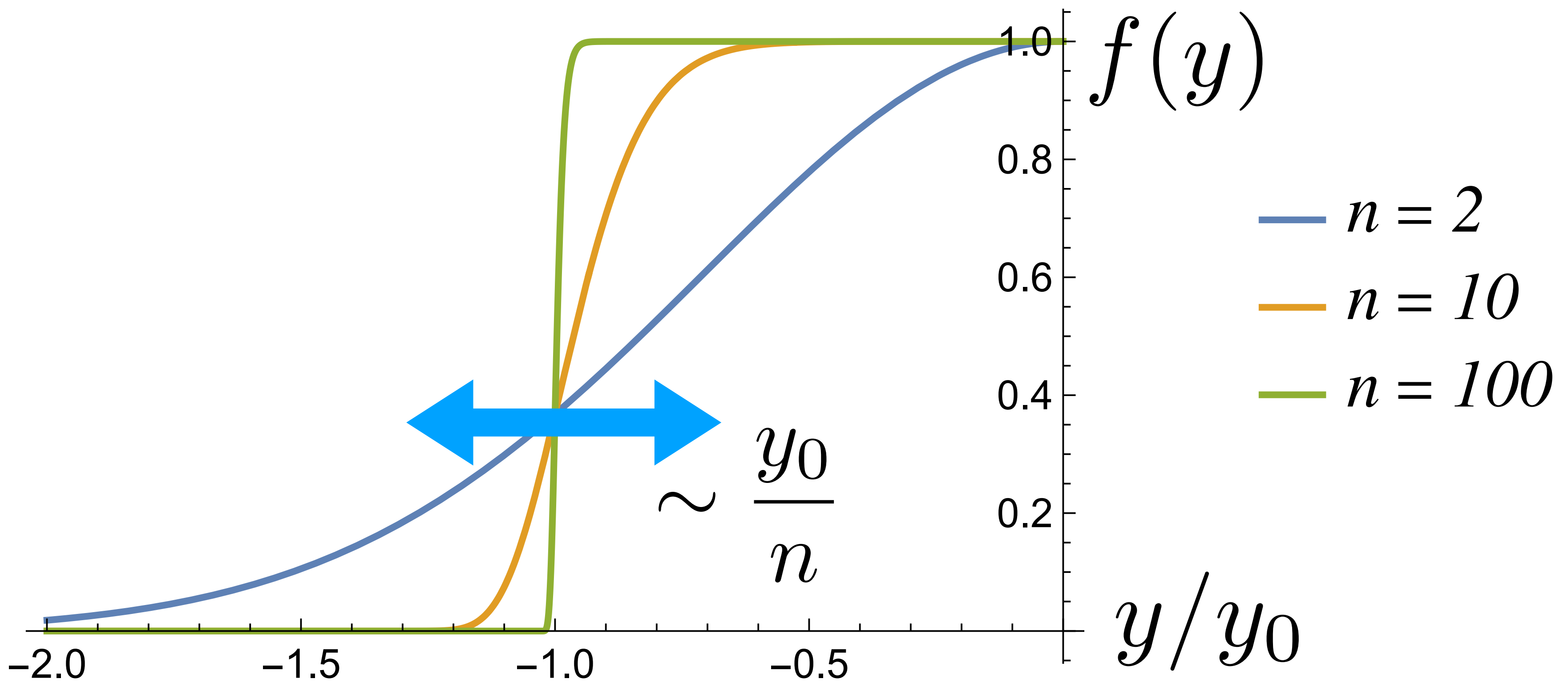
$$T^{\text{bulk}}_{\mu\nu} = \begin{pmatrix} T^{\text{bulk}}_{00} & T^{\text{bulk}}_{01} & T^{\text{bulk}}_{02} \\ T^{\text{bulk}}_{10} & T^{\text{bulk}}_{11} & T^{\text{bulk}}_{12} \\ T^{\text{bulk}}_{20} & T^{\text{bulk}}_{21} & T^{\text{bulk}}_{22} \end{pmatrix} \to \begin{pmatrix} -T^{\text{bulk}}_{00} & T^{\text{bulk}}_{01} & -T^{\text{bulk}}_{02} \\ T^{\text{bulk}}_{10} & -T^{\text{bulk}}_{11} & T^{\text{bulk}}_{12} \\ -T^{\text{bulk}}_{20} & T^{\text{bulk}}_{21} & -T^{\text{bulk}}_{22} \end{pmatrix}$$

- non-trivial components: T^{bulk}_{01} (parallel to edge) , T^{bulk}_{12} (shear stress)

Results

- Gaussian function case

$$f(y) = e^{-(y/y_0)^n}$$

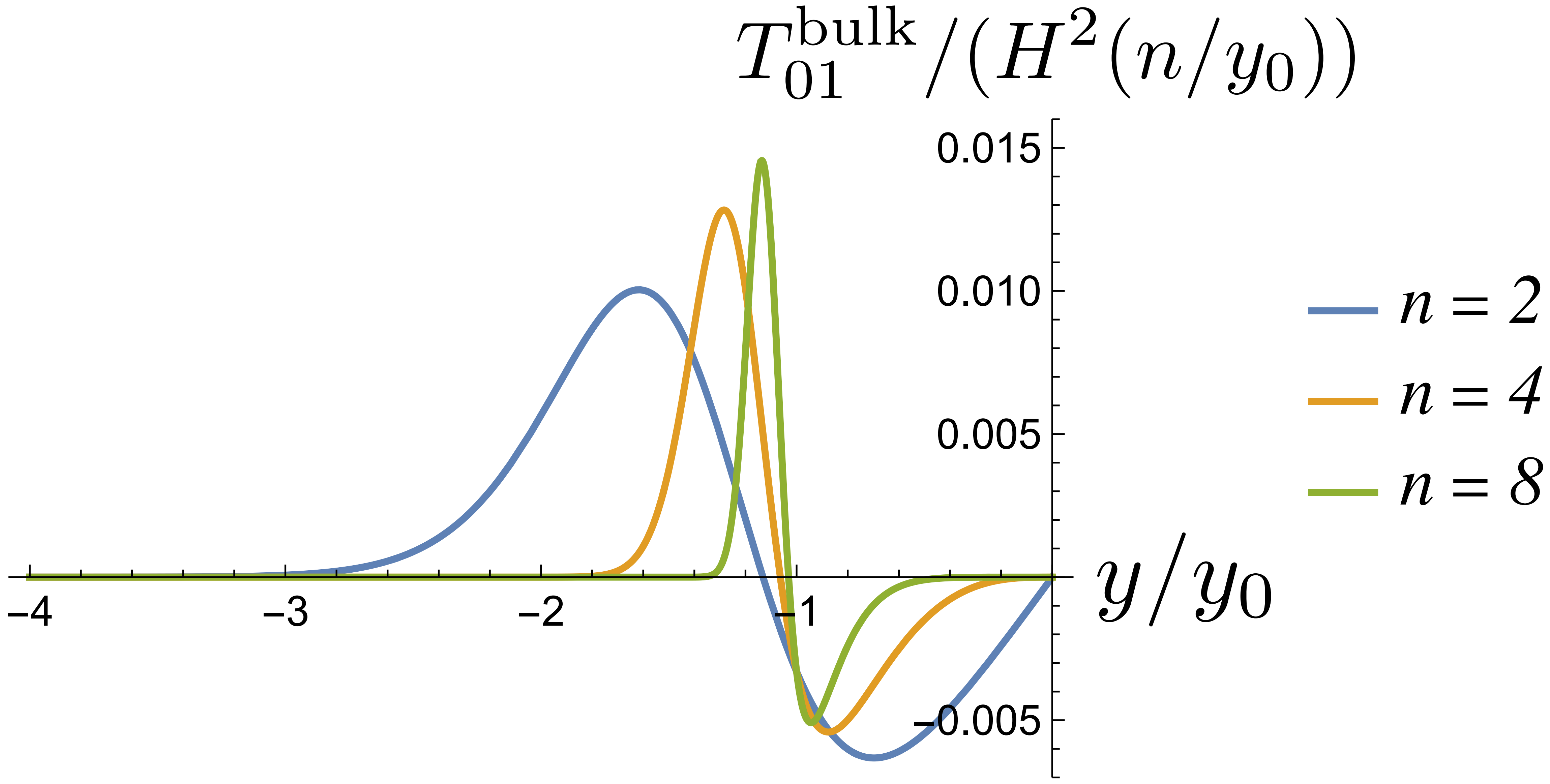


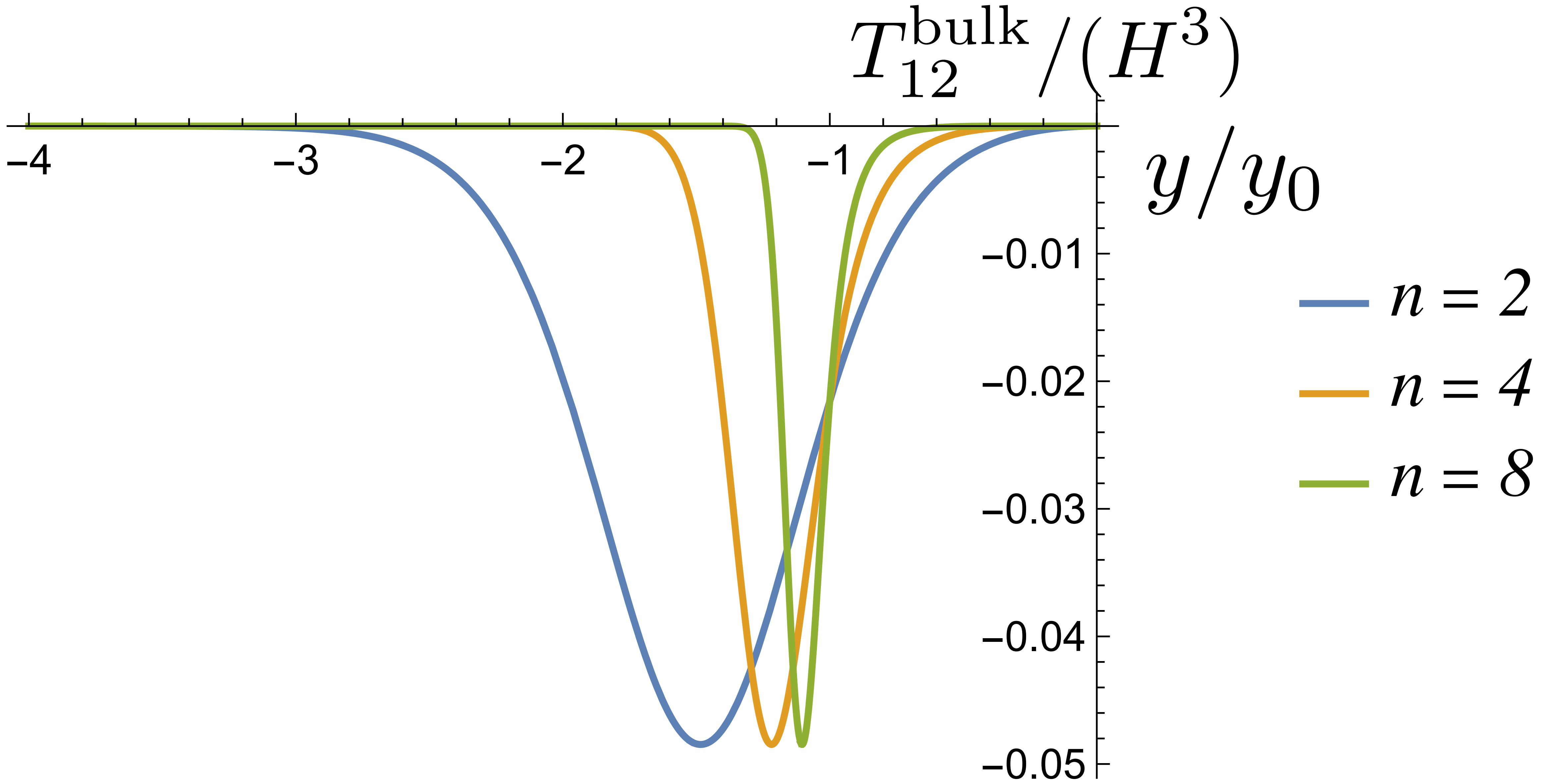
$$f(0) = 1$$

$$f'(y)|_{y=0} = 0$$

satisfied

$$n \geq 2$$



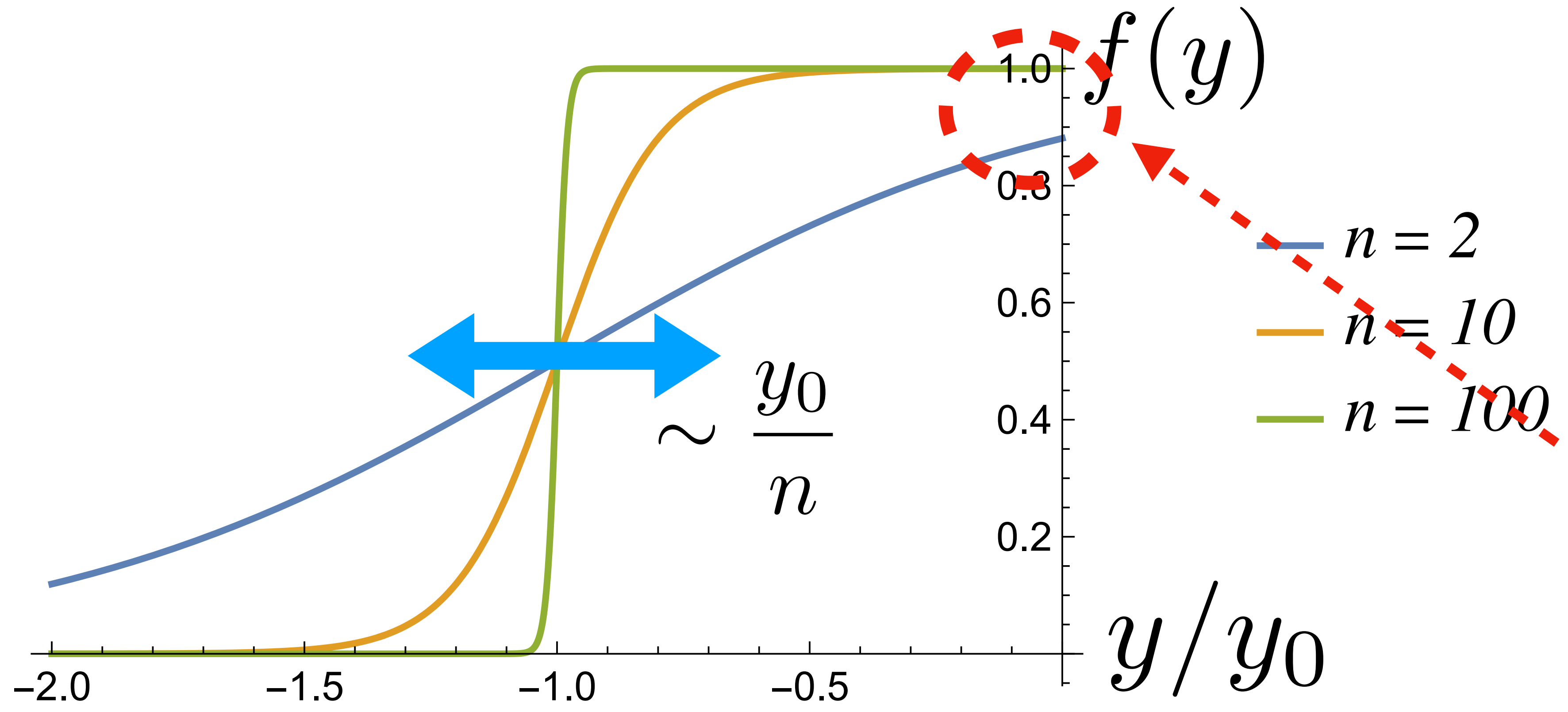


Existing non-trivial bulk current being parallel and perpendicular to the edge?

Results

- Fermi-Dirac function case

$$f(y) = \frac{1}{e^{n(y/y_0-1)}+1}$$

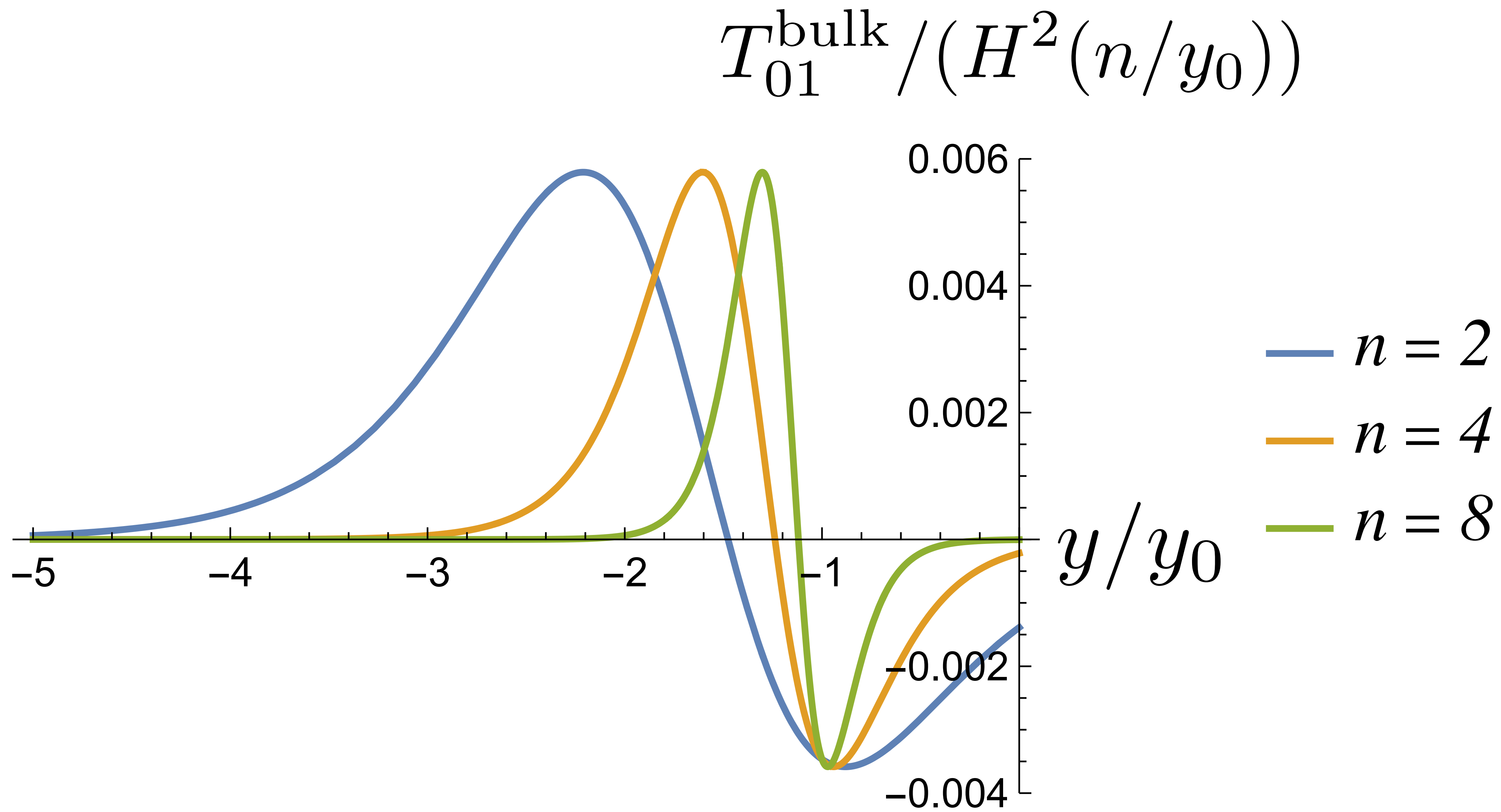


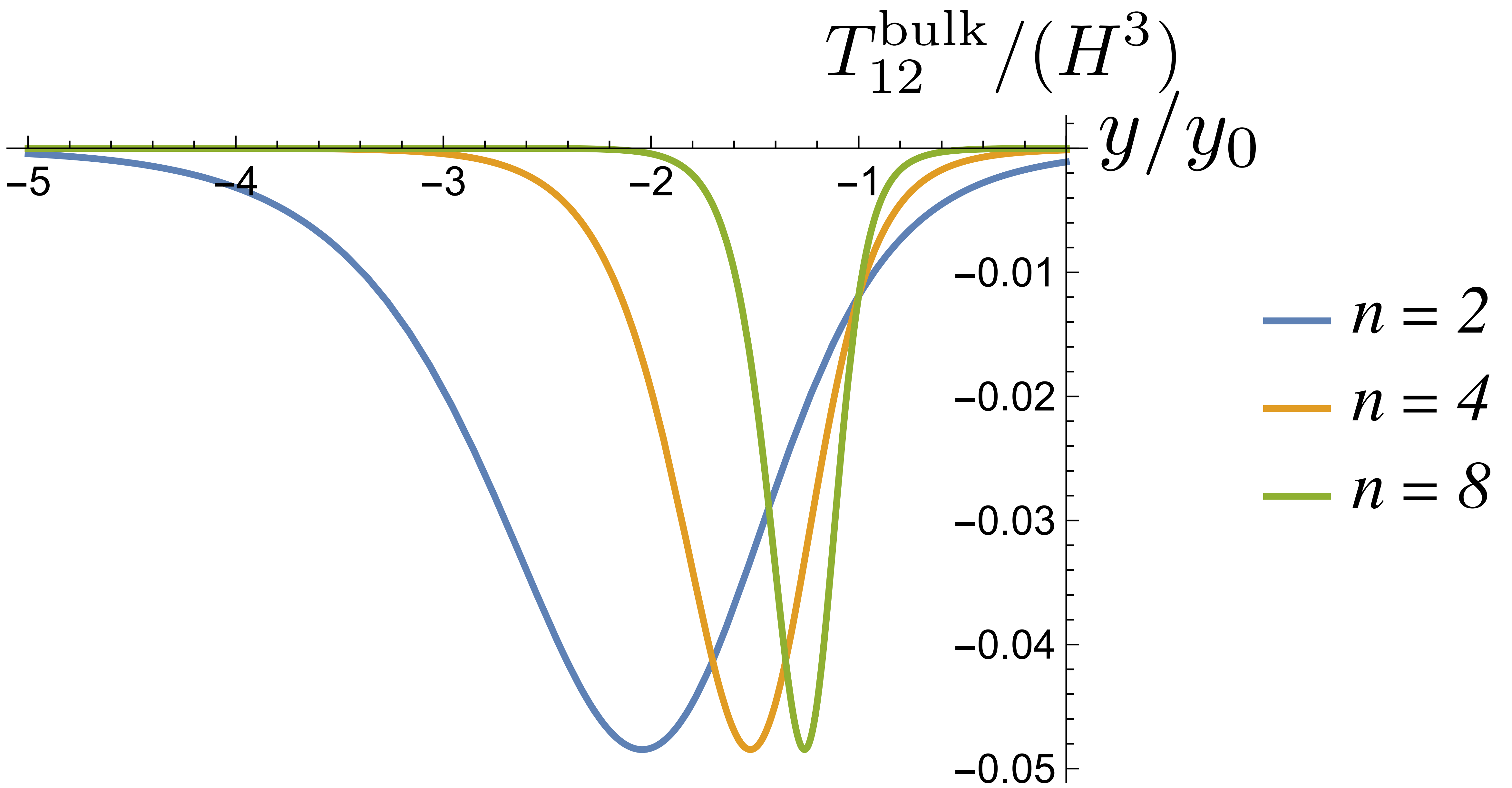
$f(0) = 1$

$f'(y)|_{y=0} = 0$

satisfied

$n \to \infty$

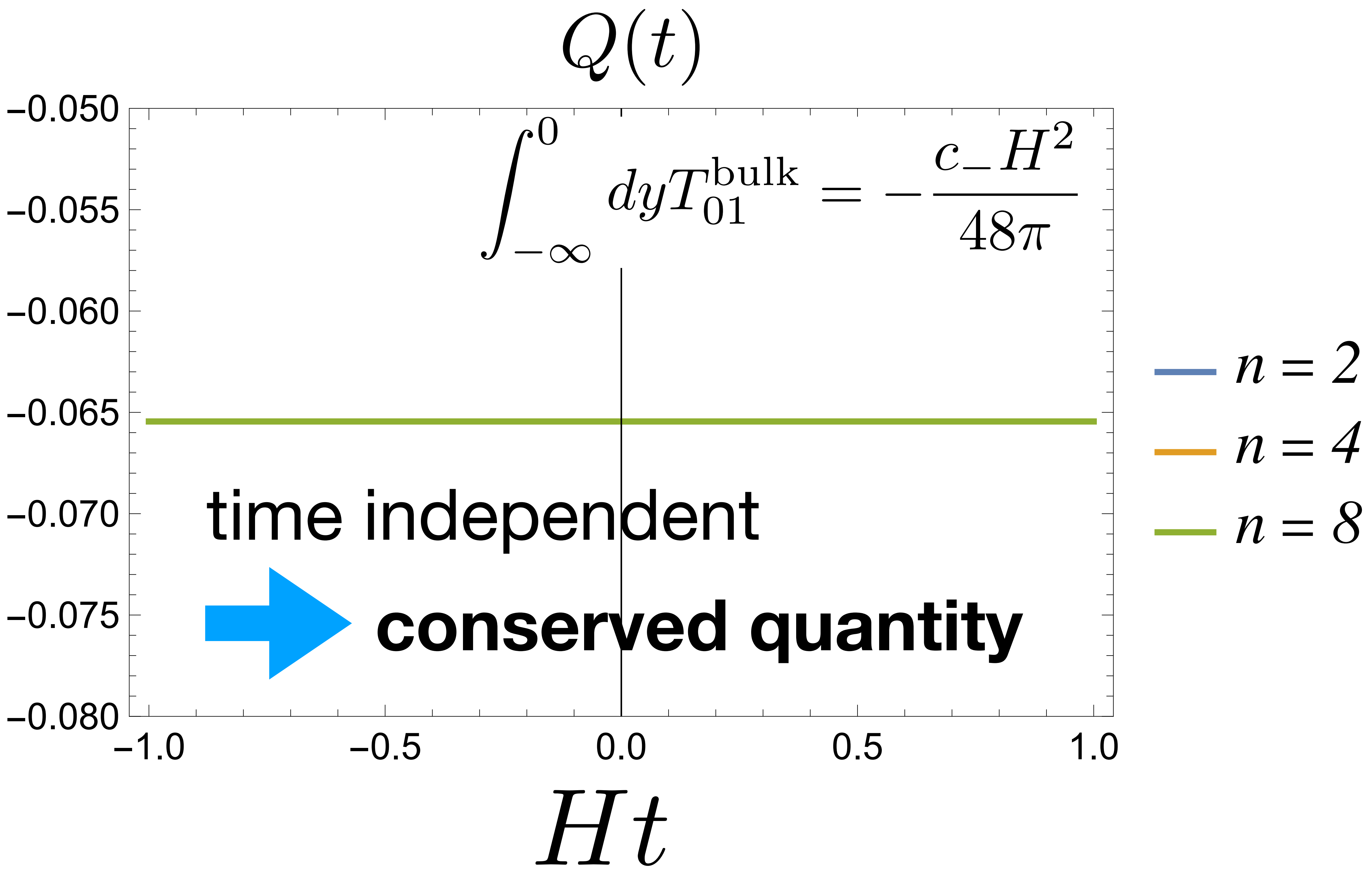


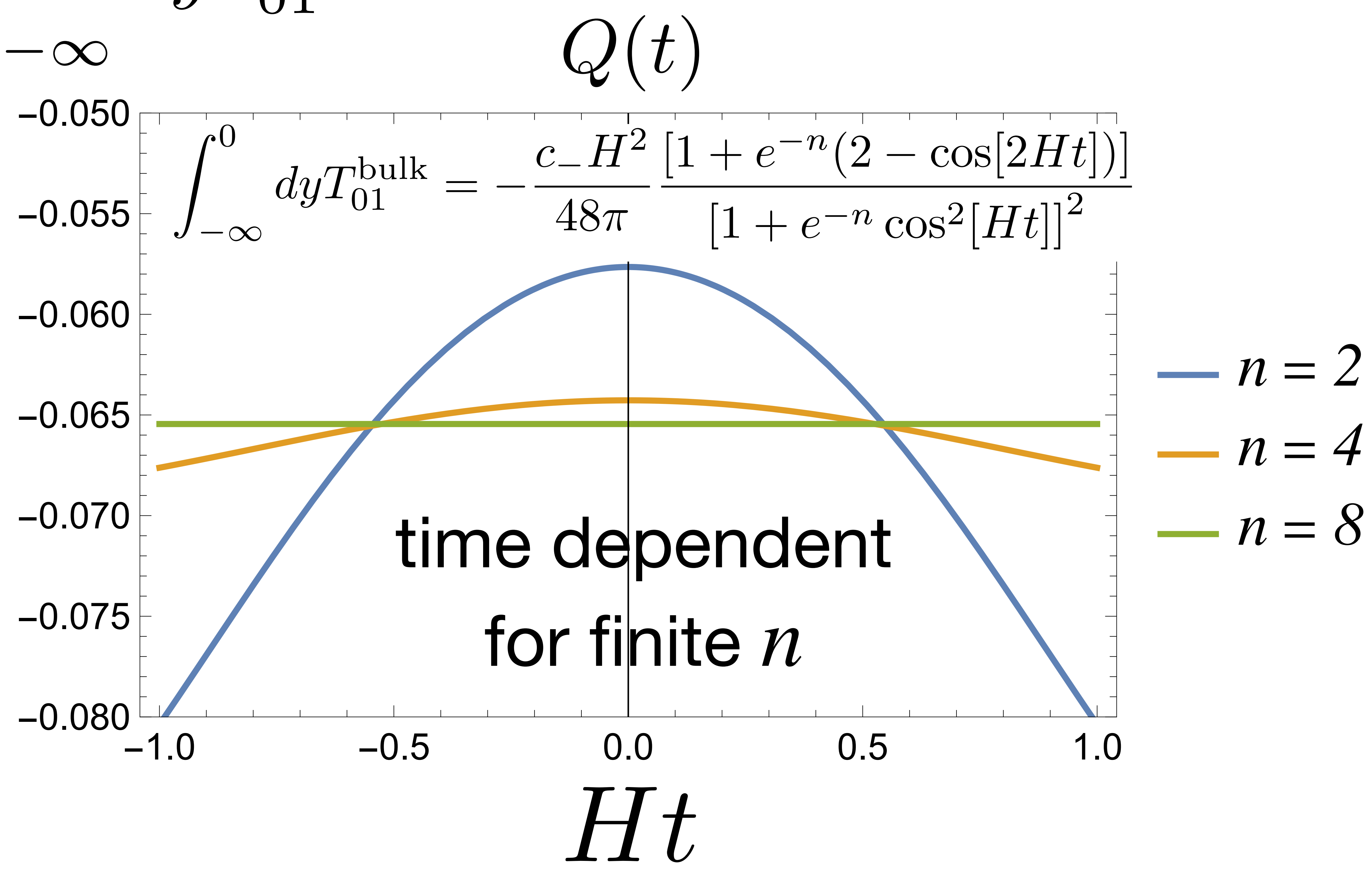


Similar behaviors appear except for $y = 0$

Results

$$Q(t) = H^{-2}\int_{-\infty}^{0} dy T_{01}^{\mathrm{bulk}}$$





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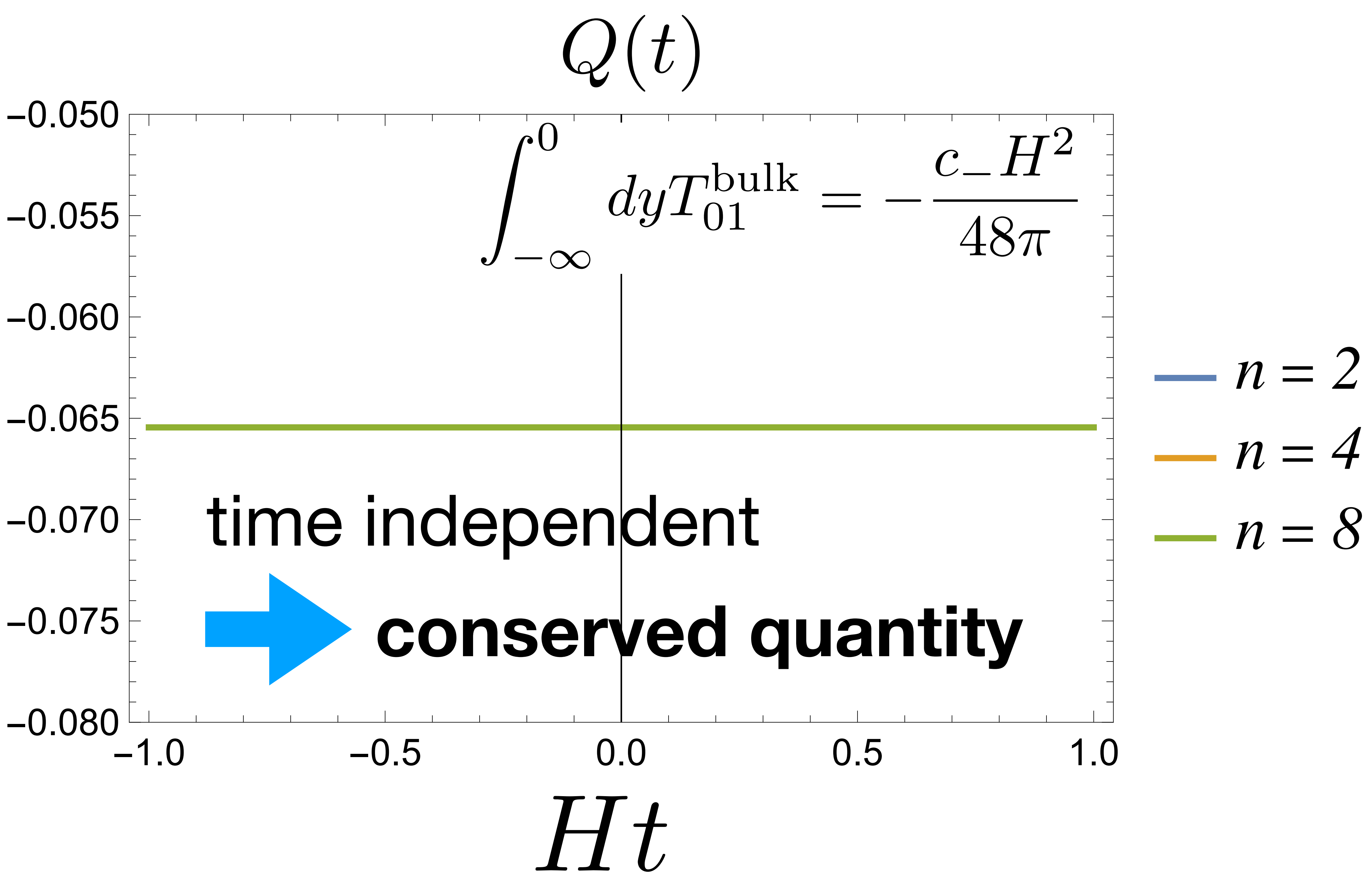

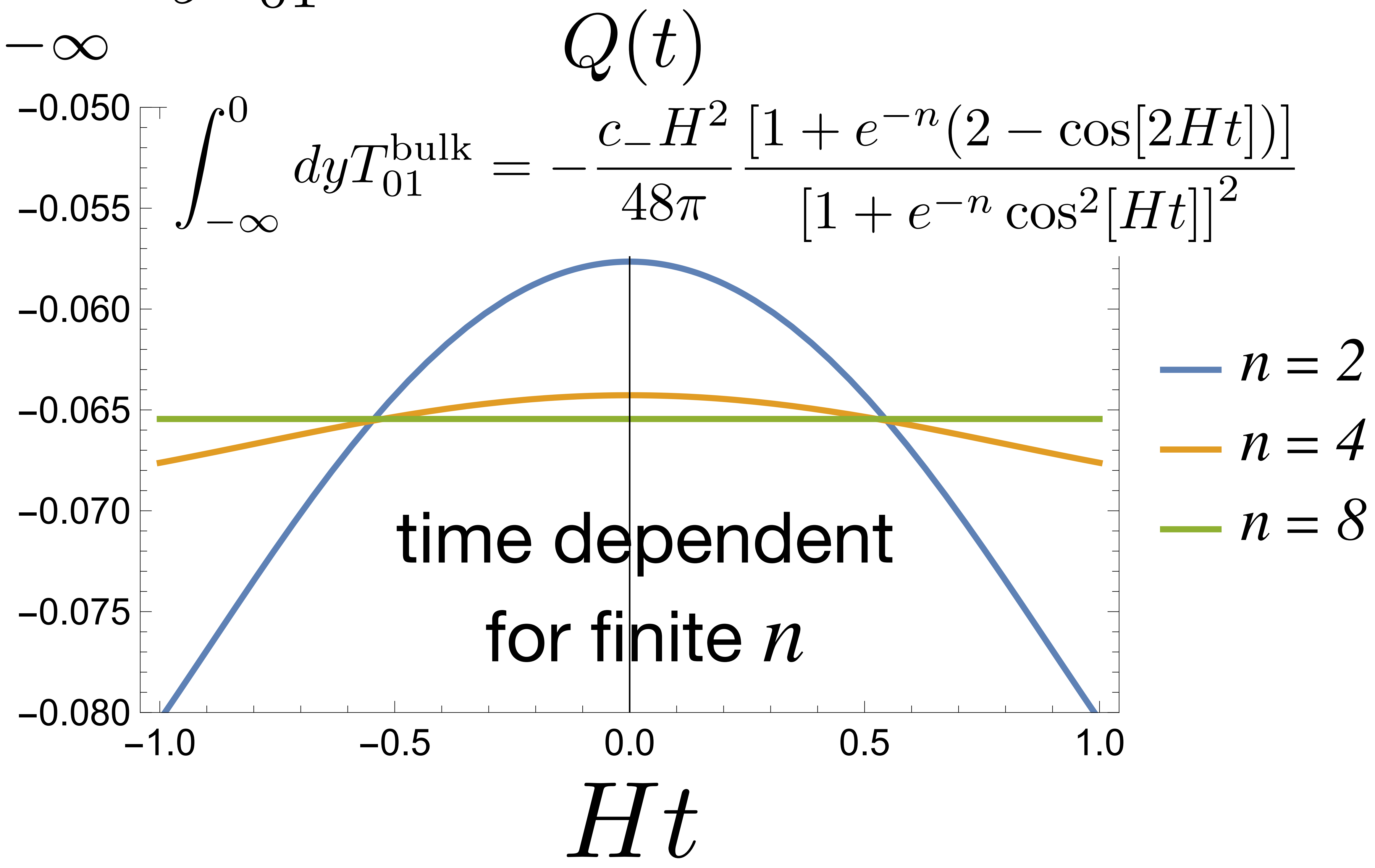

$\int_{-\infty}^{0} dy T_{01}^{\mathrm{bulk}} = -\frac{c_- H^2}{48\pi}$ has the opposite sign compated to $T_{++}^{\mathrm{III}} = \frac{H^2}{48\pi}$

→ T_{01}^{bulk} is energy flux density and cancels the gravitational anomaly

Contents

Conclusion

We considered QHS with the expanding edge and the bulk

☑ We derived covariant anomaly equation by considering bulk Chern-Simons action

☑ Non-trivial energy flux in the bulk appeared due to cancellation of gravitational anomaly

Future works

- Deepen understanding T_{12}^{bulk} component
- Effects of gravitational anomaly from microscopic models
- Entanglement entropy