Deconfinement-Higgs continuity in SU(2) adjoint Higgs model at finite temperature

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Joint work with

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- Introduction
- Global symmetry
- Monte Carlo simulation
- Summary

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今回考える系

$$\begin{cases} SU(2)\text{ゲージ場} & f_{\mu\nu} = f_{\mu\nu}^a \sigma^a \\ \text{随伴Higgs場} & \phi = \phi^a \sigma^a \end{cases}$$

作用

$$S = \int_0^\beta \mathrm{d}t \int \mathrm{d}^3x \left[\mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{Higgs}}\right]$$

$$\mathcal{L}_{\mathrm{gauge}} = \frac{1}{2g^2}\mathrm{tr}\left(f^{\mu\nu} f_{\mu\nu}\right)$$

$$\mathcal{L}_{\mathrm{Higgs}} = \mathrm{tr}\left(\left(D_\mu \phi\right)^2 + m^2\phi^2 + \lambda\phi^4\right)$$

動機　　大統一理論のトイモデル

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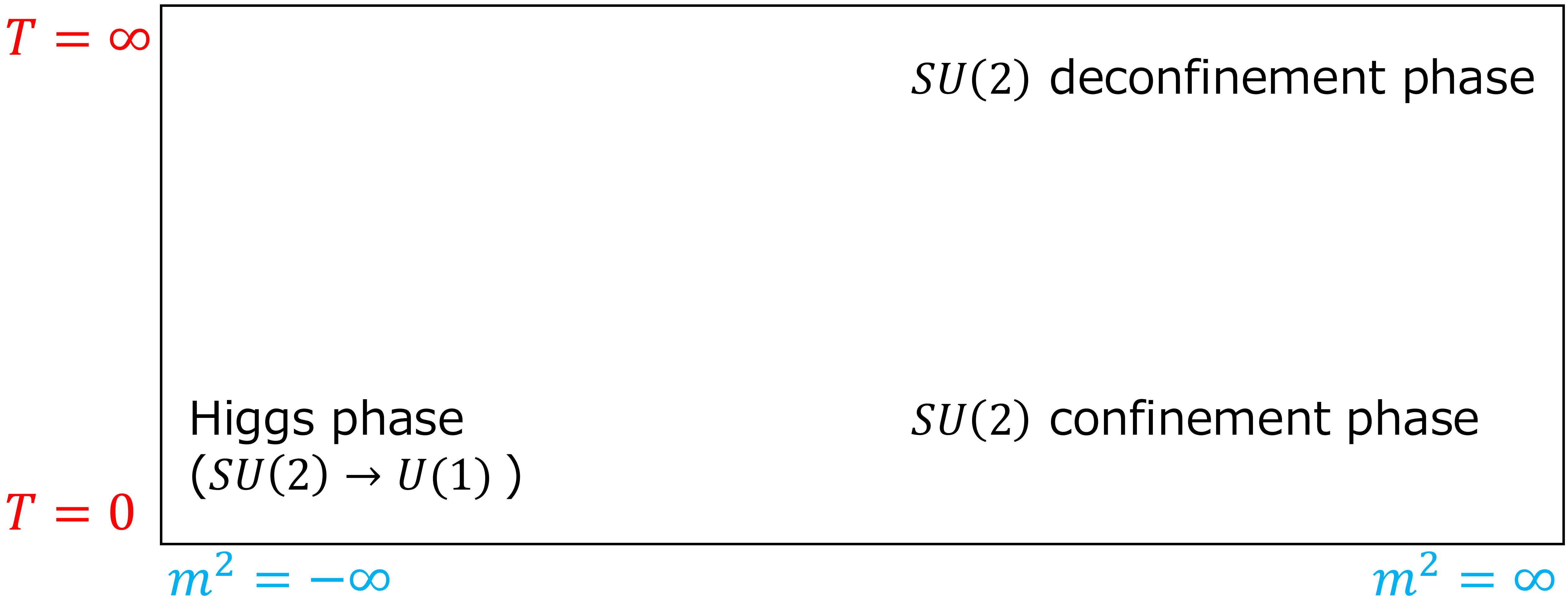

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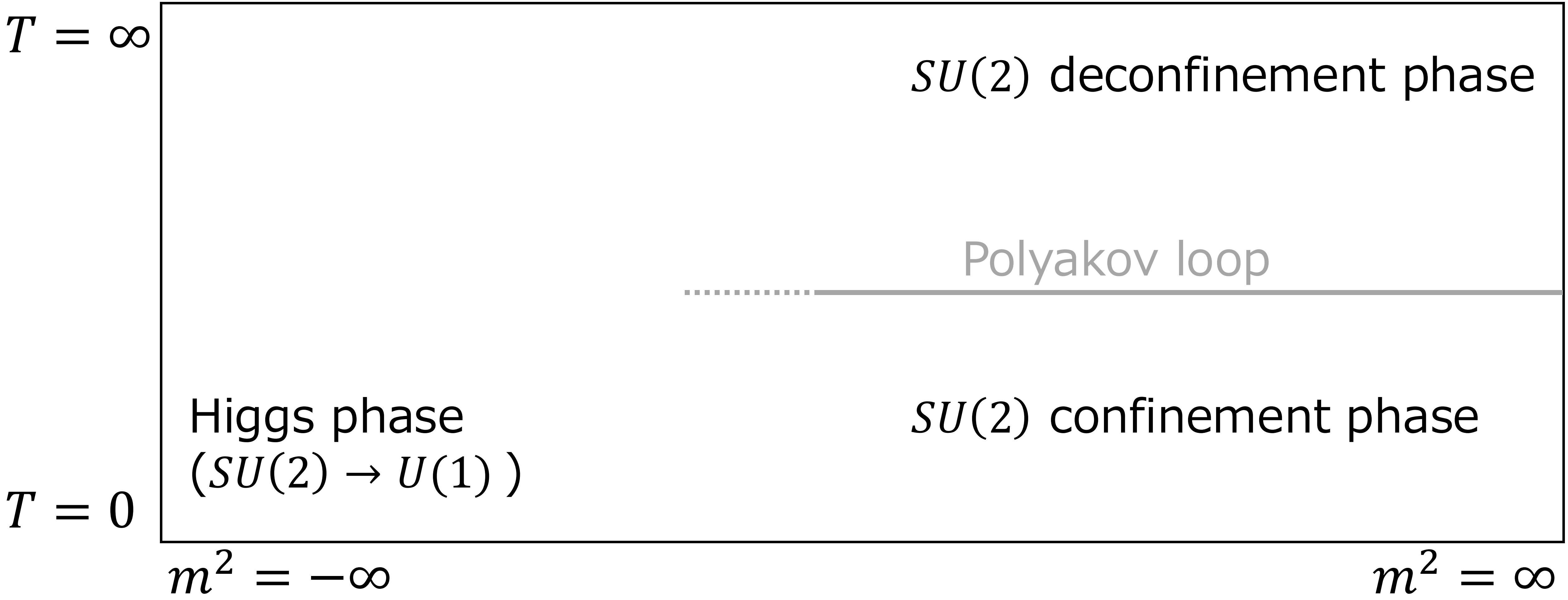

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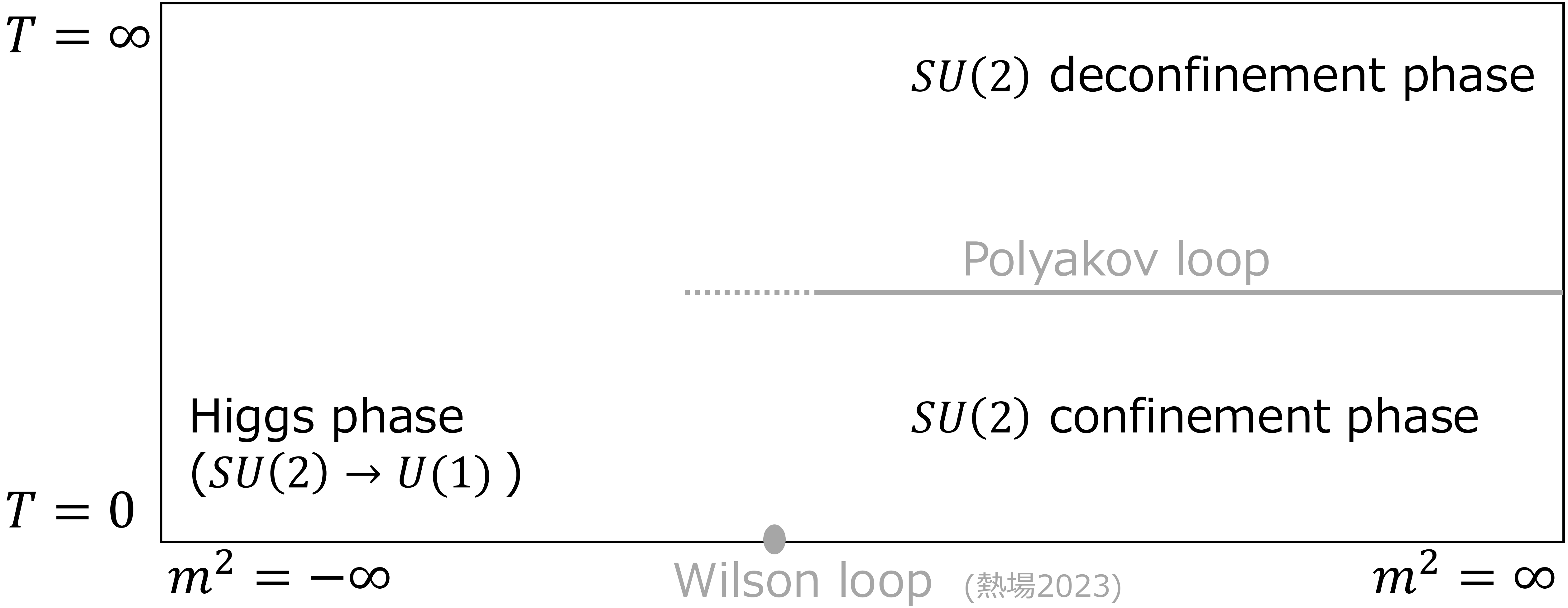
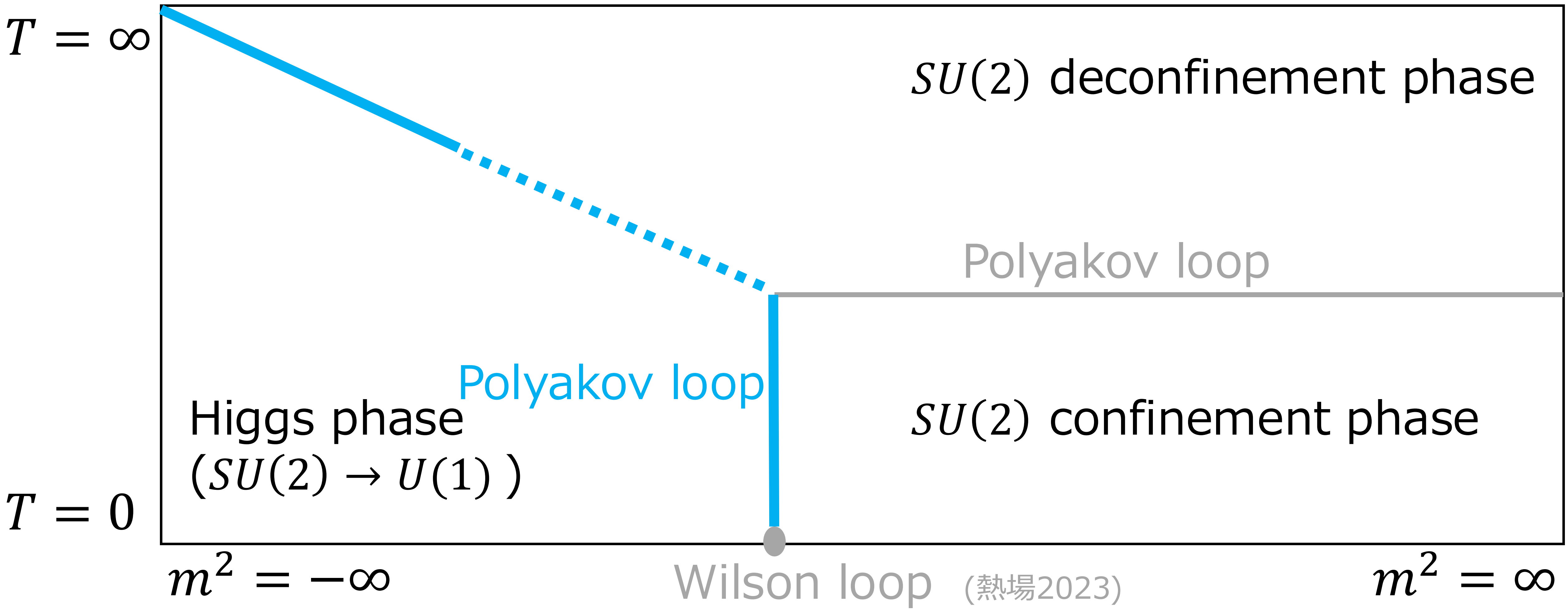


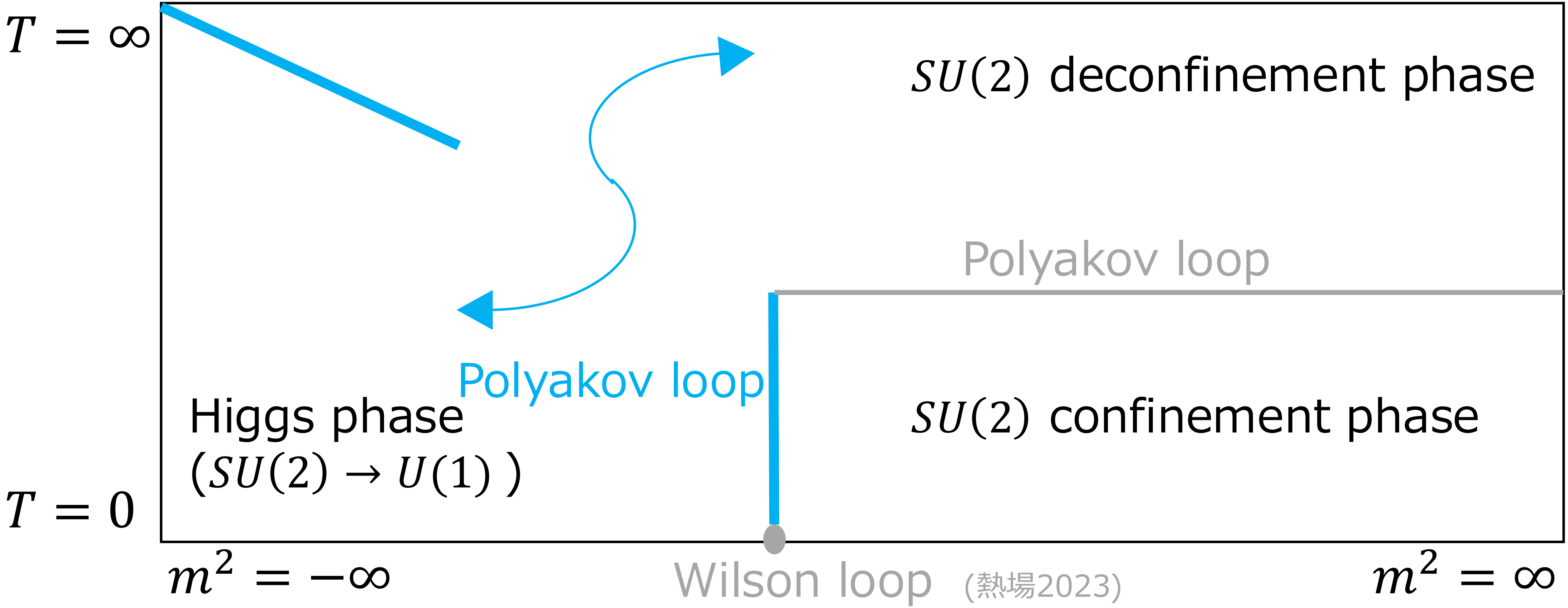
Today's talk



$T=\infty$
$SU(2)$ deconfinement phase
Polyakov loop
Polyakov loop
Higgs phase
$(SU(2) \to U(1))$
$SU(2)$ confinement phase
$T=0$
$m^2=-\infty$
Wilson loop
(熱場2023)
$m^2=\infty$

Today's talk

Deconfinement-Higgs continuity



We have analyzed the phase structure with

(1) Global symmetry

(2) Center destabilization

(3) Monte Carlo simulation

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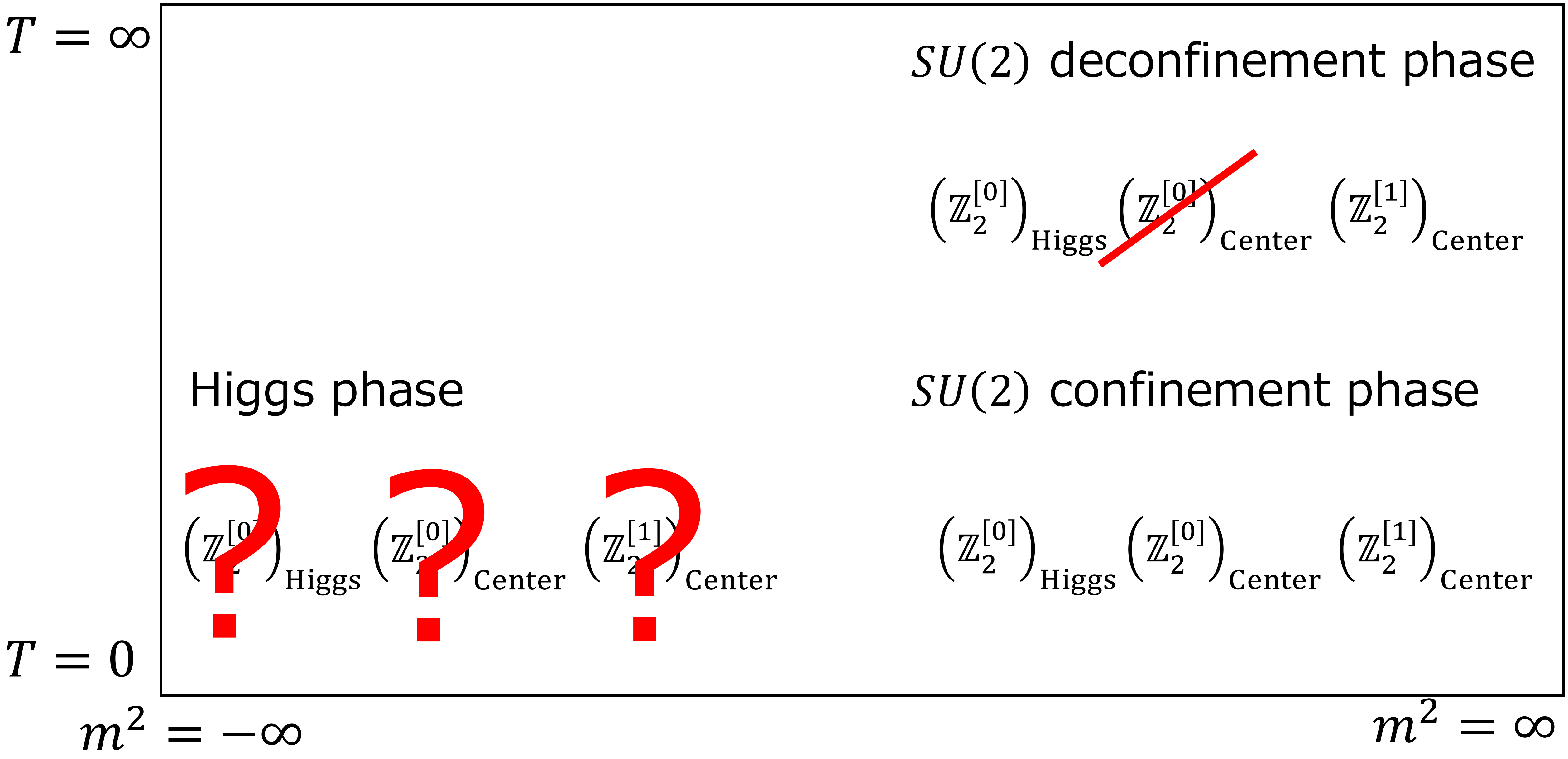
Global symmetries

$$\left(\mathbb{Z}_2^{[0]}\right)_{\text{Higgs}} \qquad \phi \mapsto -\phi$$

$$\left(\mathbb{Z}_2^{[0]}\right)_{\text{Center}} \qquad \mathrm{tr}\,(P) \mapsto -\mathrm{tr}\,(P) \qquad P = \exp\left[\int_0^\beta \mathrm{d}t\;\, A_0^a \sigma^a\right]$$

$$\left(\mathbb{Z}_2^{[1]}\right)_{\text{Center}} \qquad \mathrm{tr}\,(W) \mapsto -\mathrm{tr}\,(W) \qquad W = \exp\left[\oint_{\text{spatial}} \mathrm{d}x^\mu\, A_\mu^a \sigma^a\right]$$


$T = \infty$
$SU(2)$ deconfinement phase
$\left(\mathbb{Z}_2^{[0]}\right)_{\mathrm{Higgs}}$ $\cancel{\left(\mathbb{Z}_2^{[0]}\right)}_{\mathrm{Center}}$ $\left(\mathbb{Z}_2^{[1]}\right)_{\mathrm{Center}}$
Higgs phase
$SU(2)$ confinement phase
? ? ?
$\left(\mathbb{Z}_2^{[0]}\right)_{\mathrm{Higgs}}$ $\left(\mathbb{Z}_2^{[0]}\right)_{\mathrm{Center}}$ $\left(\mathbb{Z}_2^{[1]}\right)_{\mathrm{Center}}$
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$T = 0$
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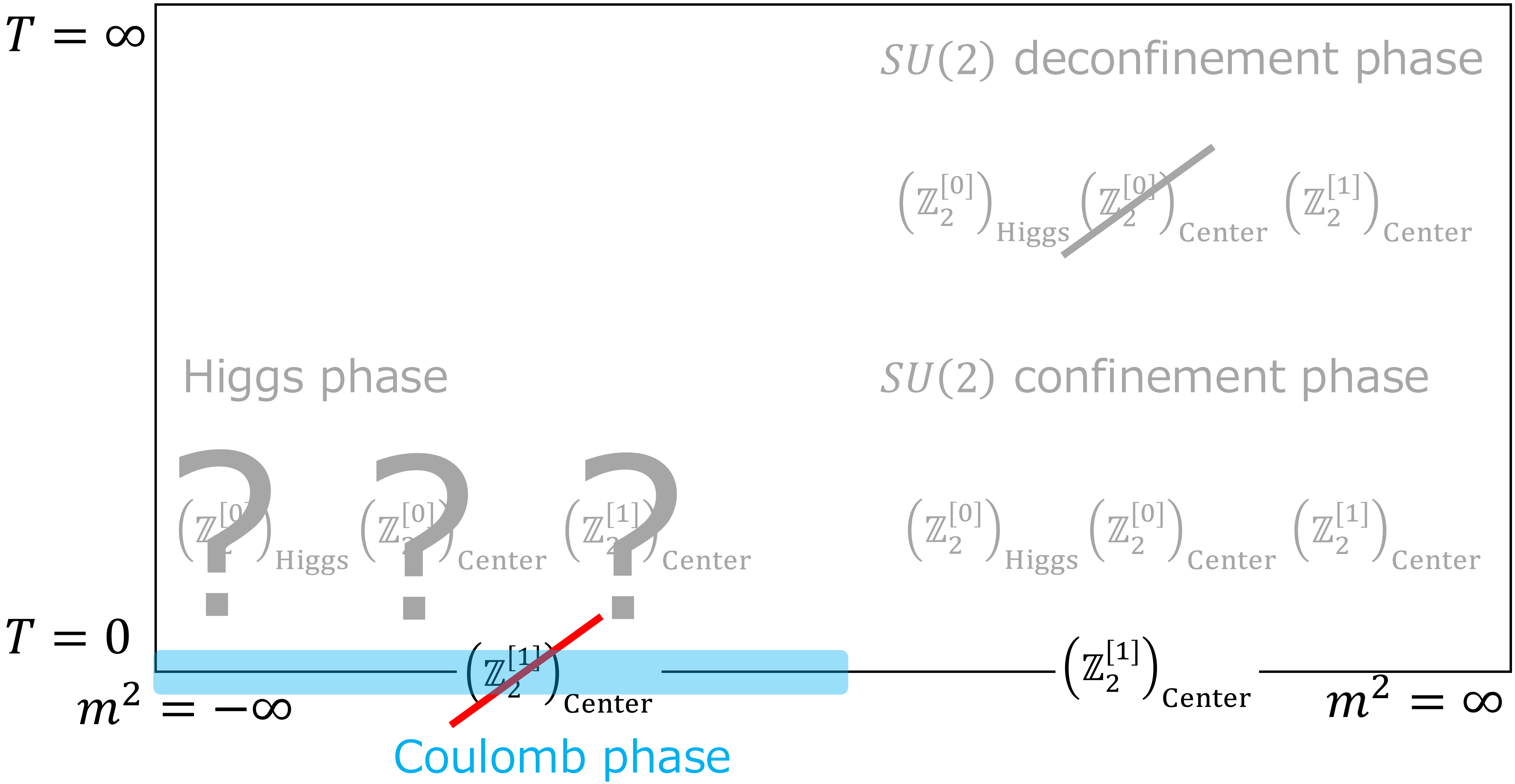
$m^2 = \infty$



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$SU(2)$ deconfinement phase
$(\mathbb{Z}_2^{[0]})_{\text{Higgs}}$ $\cancel{(\mathbb{Z}_2^{[0]})}_{\text{Center}}$ $(\mathbb{Z}_2^{[1]})_{\text{Center}}$
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$T = 0$
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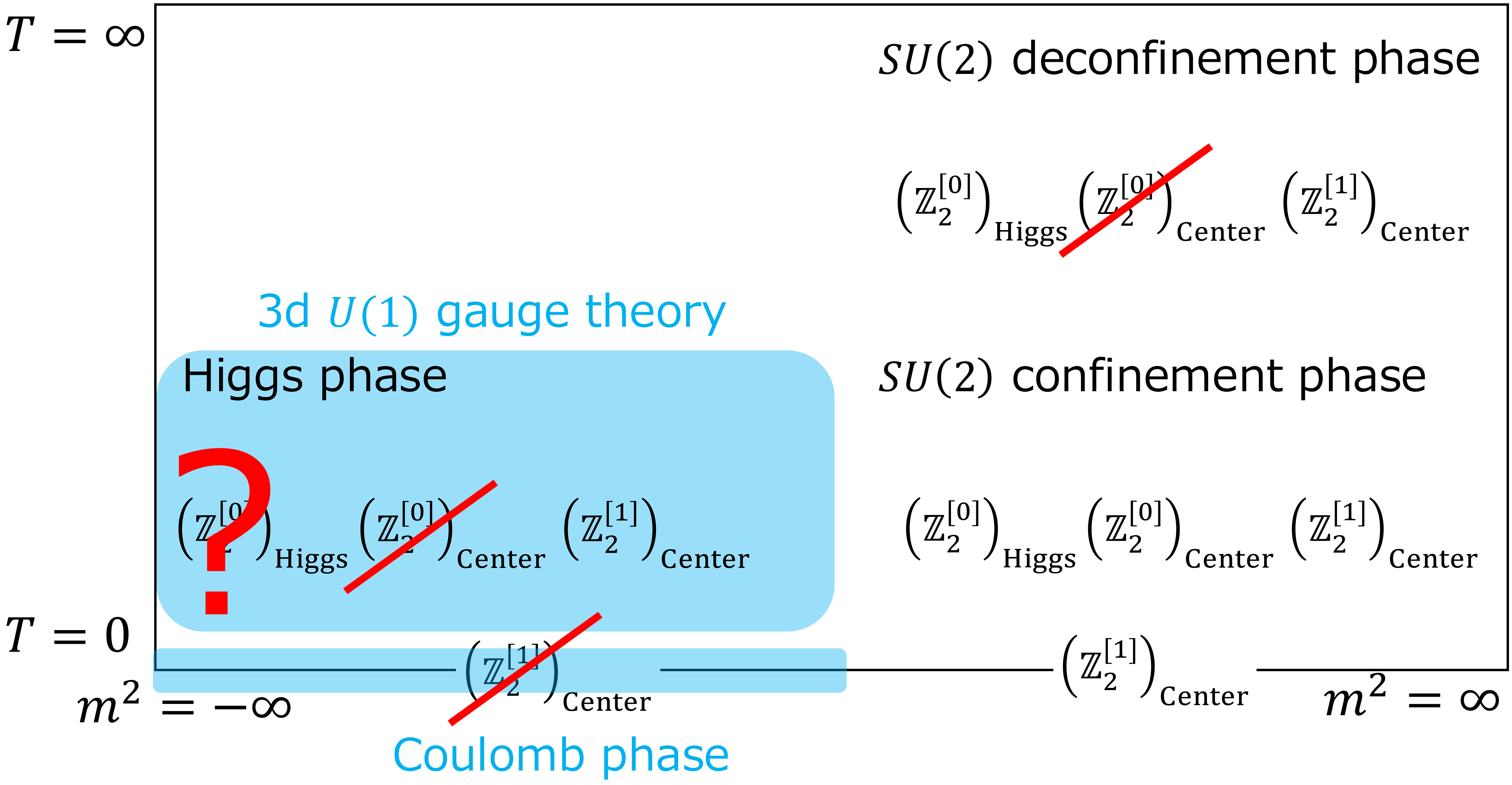
$(\mathbb{Z}_2^{[1]})_{\text{Center}}$
$m^2 = -\infty$
$m^2 = \infty$
Coulomb phase



$T = \infty$
$SU(2)$ deconfinement phase
$(\mathbb{Z}_2^{[0]})_{\text{Higgs}}$ $\cancel{(\mathbb{Z}_2^{[0]})}_{\text{Center}}$ $(\mathbb{Z}_2^{[1]})_{\text{Center}}$
3d $U(1)$ gauge theory
Higgs phase
?
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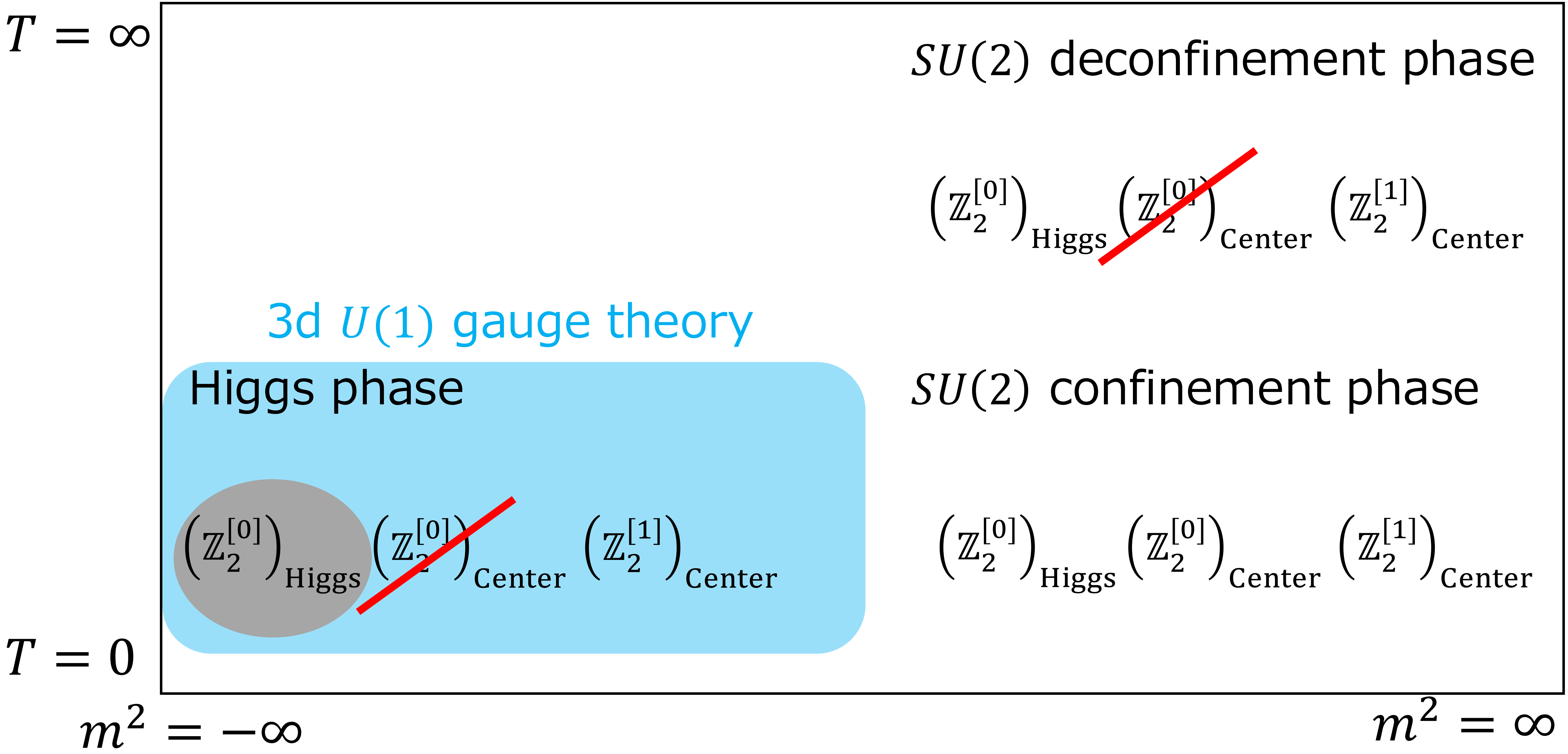
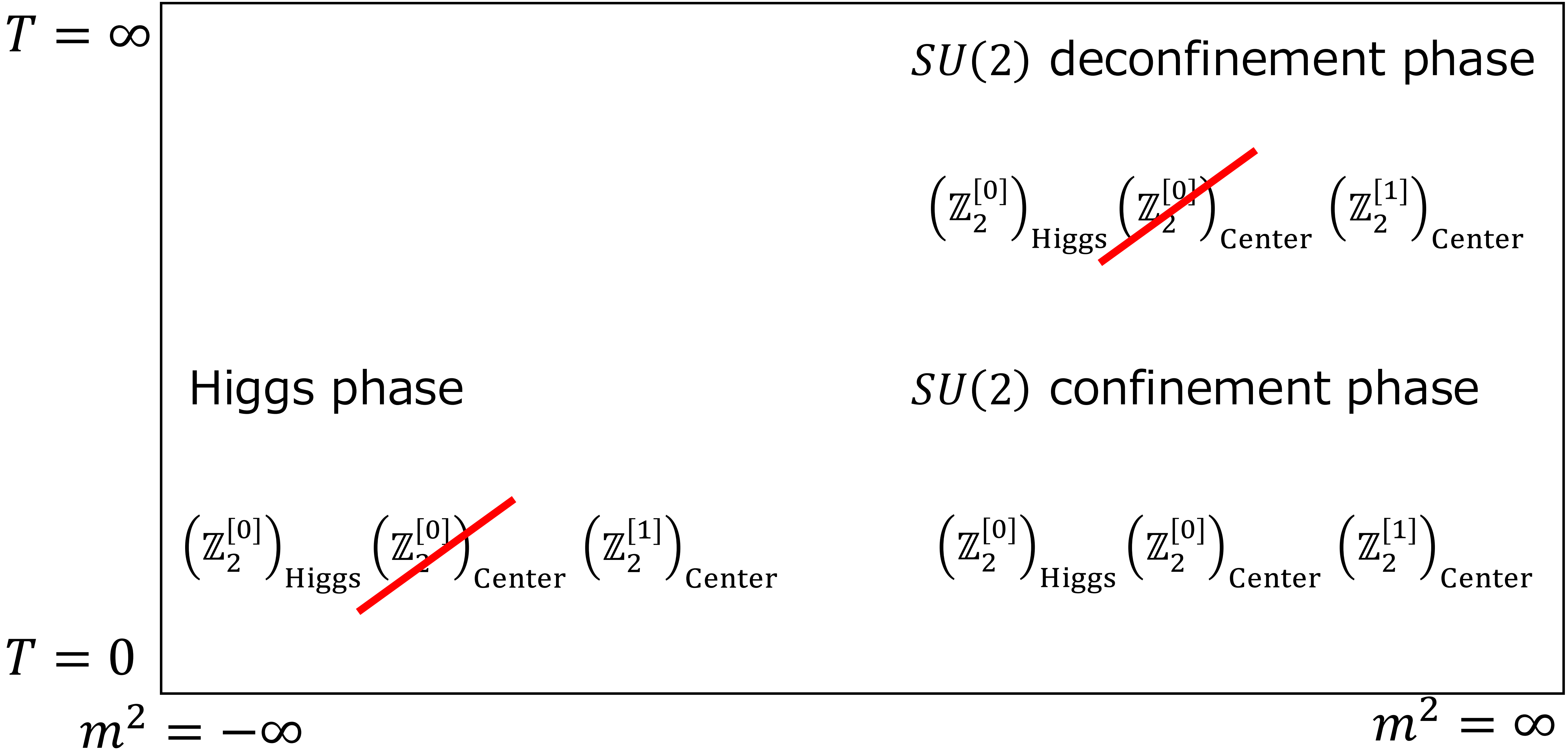


Charge conjugation symmetry should not be broken.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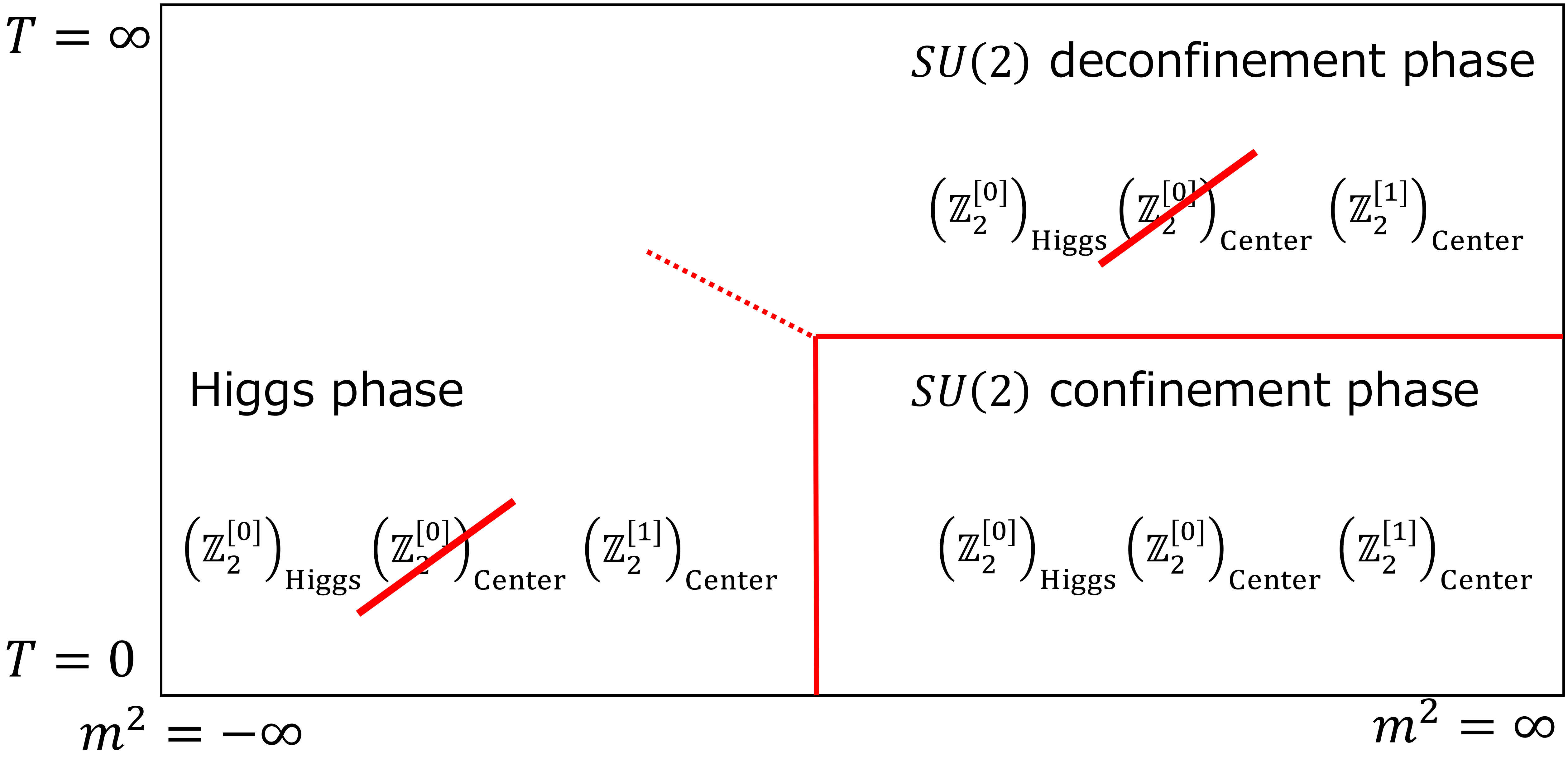

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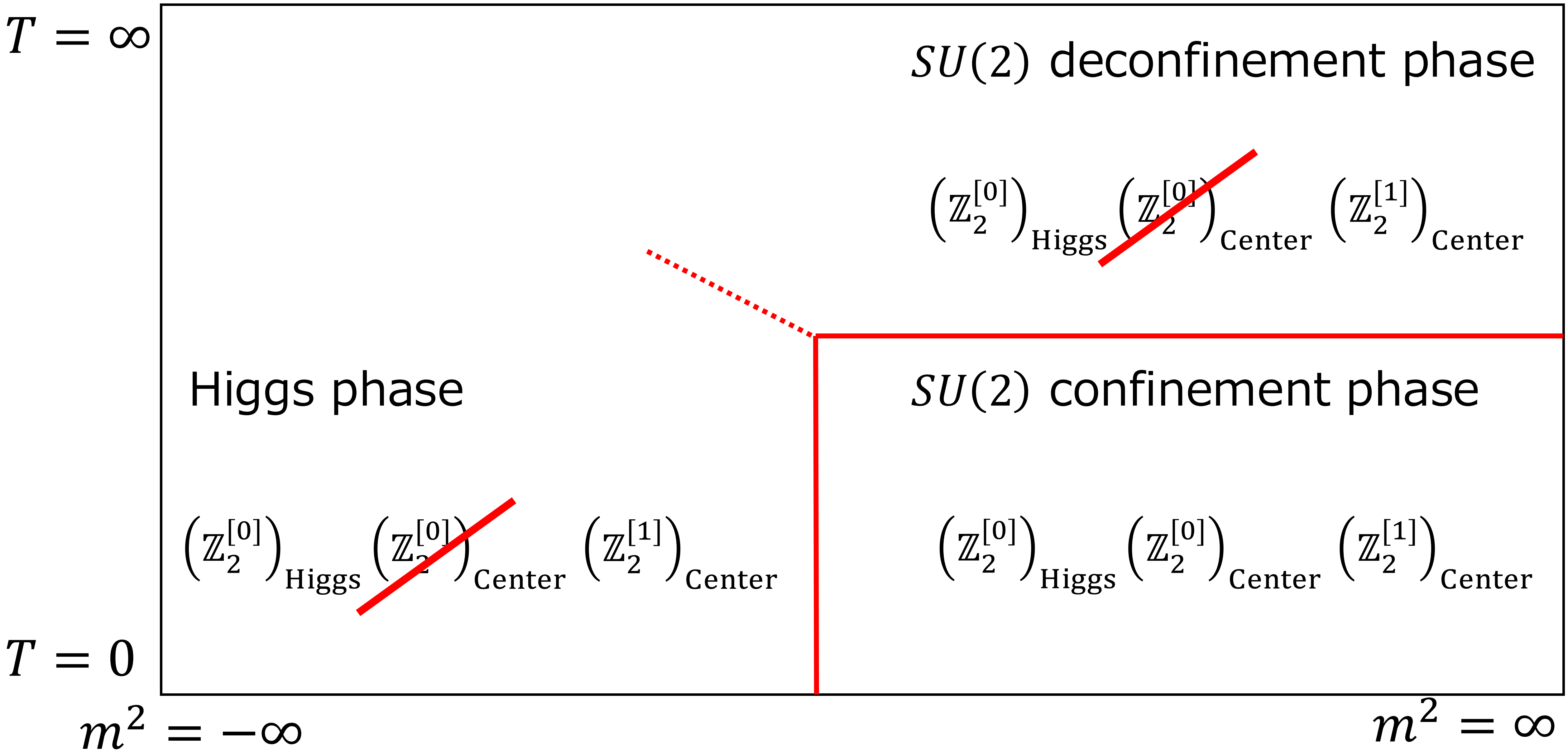

Consistent with center destabilization analysis.

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Lattice regularization

$$S[U,\phi] = \frac{\beta}{2} S_{\square}[U] - \frac{\beta_{\mathrm{H}}}{2} \sum_{x,\mu} \mathrm{tr}\left[U_\mu(x)\phi(x)U_\mu^\dagger(x)\phi\left(x+\hat{e}_\mu\right)\right]$$

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Impose, for $\phi = \phi^a \sigma^a$

$$\sum_a (\phi^a)^2 = 1$$

which roughly means that $\lambda \to \infty$.

Lattice regularization

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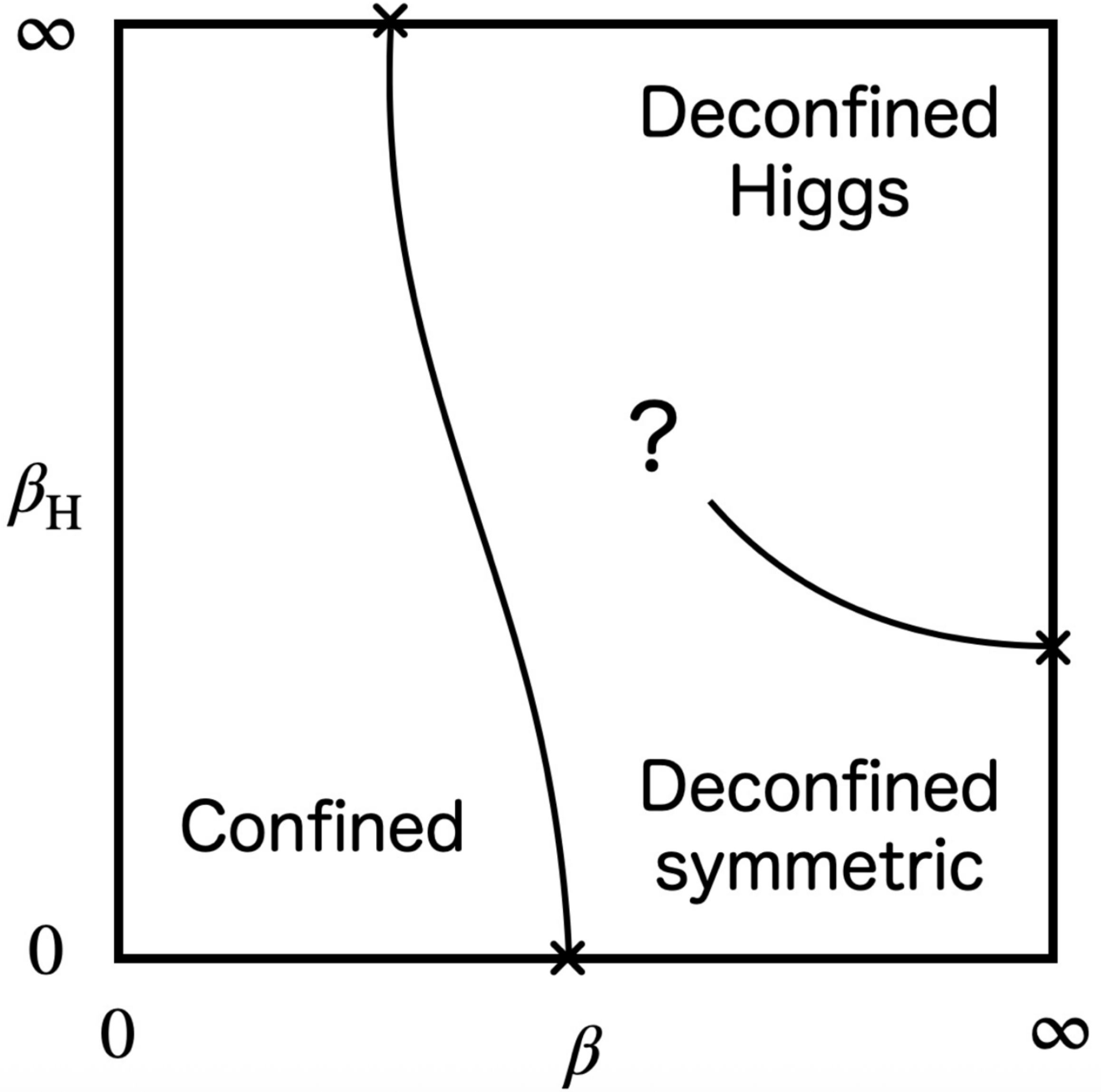
Taking unitary gauge

$$(\text{Blue Part}) = \mathrm{tr}\left[U_\mu(x)\sigma^y U_\mu^\dagger(x)\sigma^y\right]$$

Phase diagram

[Karsch, Seiler, Stamatescu, ('83)]



$N_t \times N_s^3$: fix

Order parameters

[Nishimura, Ogilvie, ('12)]

$$\mathrm{tr}(P) \quad \cdots \quad \left(\mathbb{Z}_2^{[0]}\right)_{\mathrm{Center}}\text{-charge}$$

$$\mathrm{tr}(\phi P) \quad \cdots \quad \left(\mathbb{Z}_2^{[0]}\right)_{\mathrm{Center}} \times \left(\mathbb{Z}_2^{[0]}\right)_{\mathrm{Higgs}}\text{-charge}$$

$$\mathrm{tr}(\phi P^2) \quad \cdots \quad \left(\mathbb{Z}_2^{[0]}\right)_{\mathrm{Higgs}}\text{-charge}$$

They are gauge-invariant.

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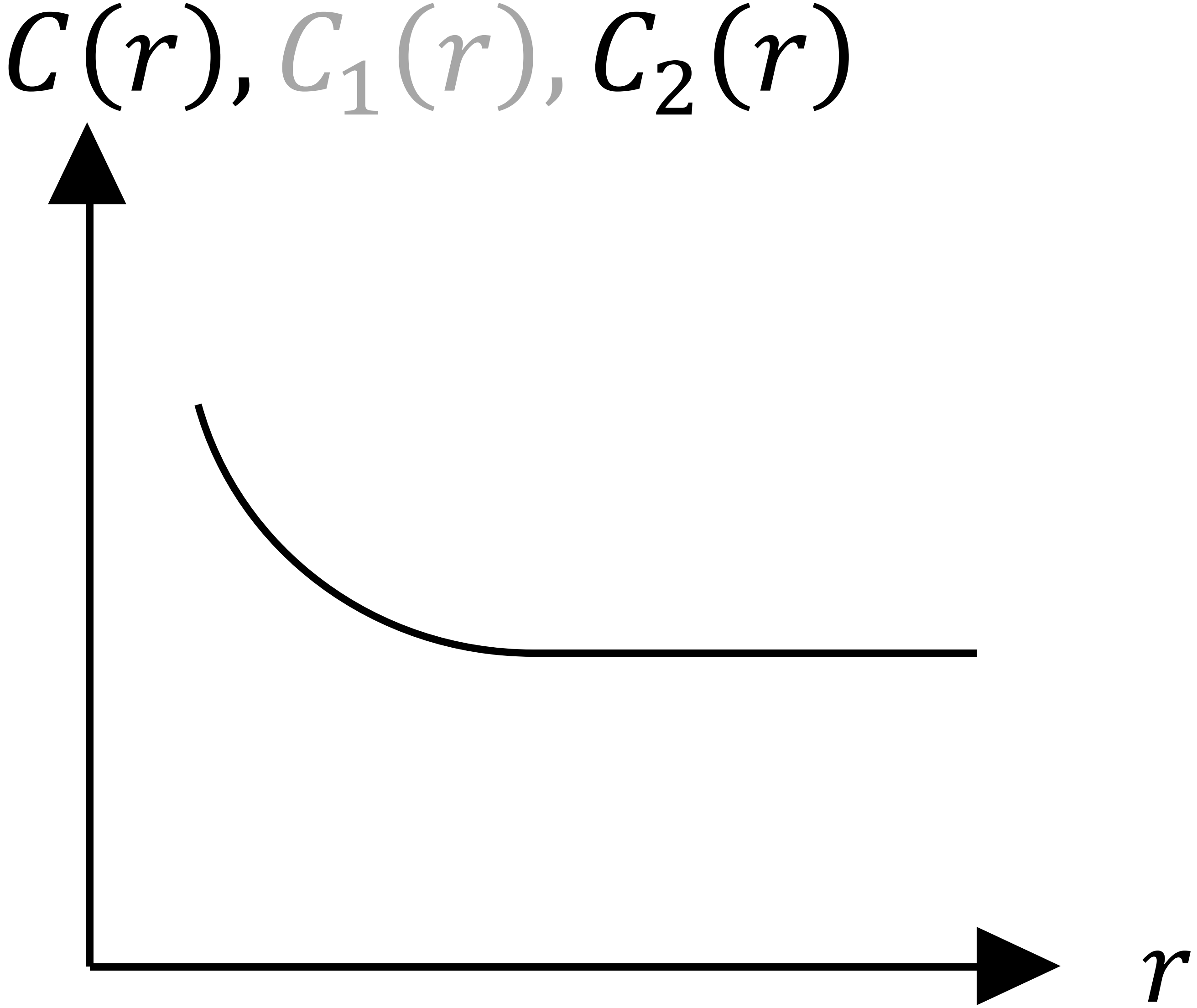
Correlation functions

$$C(r) := \langle \mathrm{tr}(P)(\vec{x})\mathrm{tr}(P)(\vec{y})\rangle$$

$$C_1(r) := \langle \mathrm{tr}(\phi P)(\vec{x})\mathrm{tr}(\phi P)(\vec{y})\rangle$$

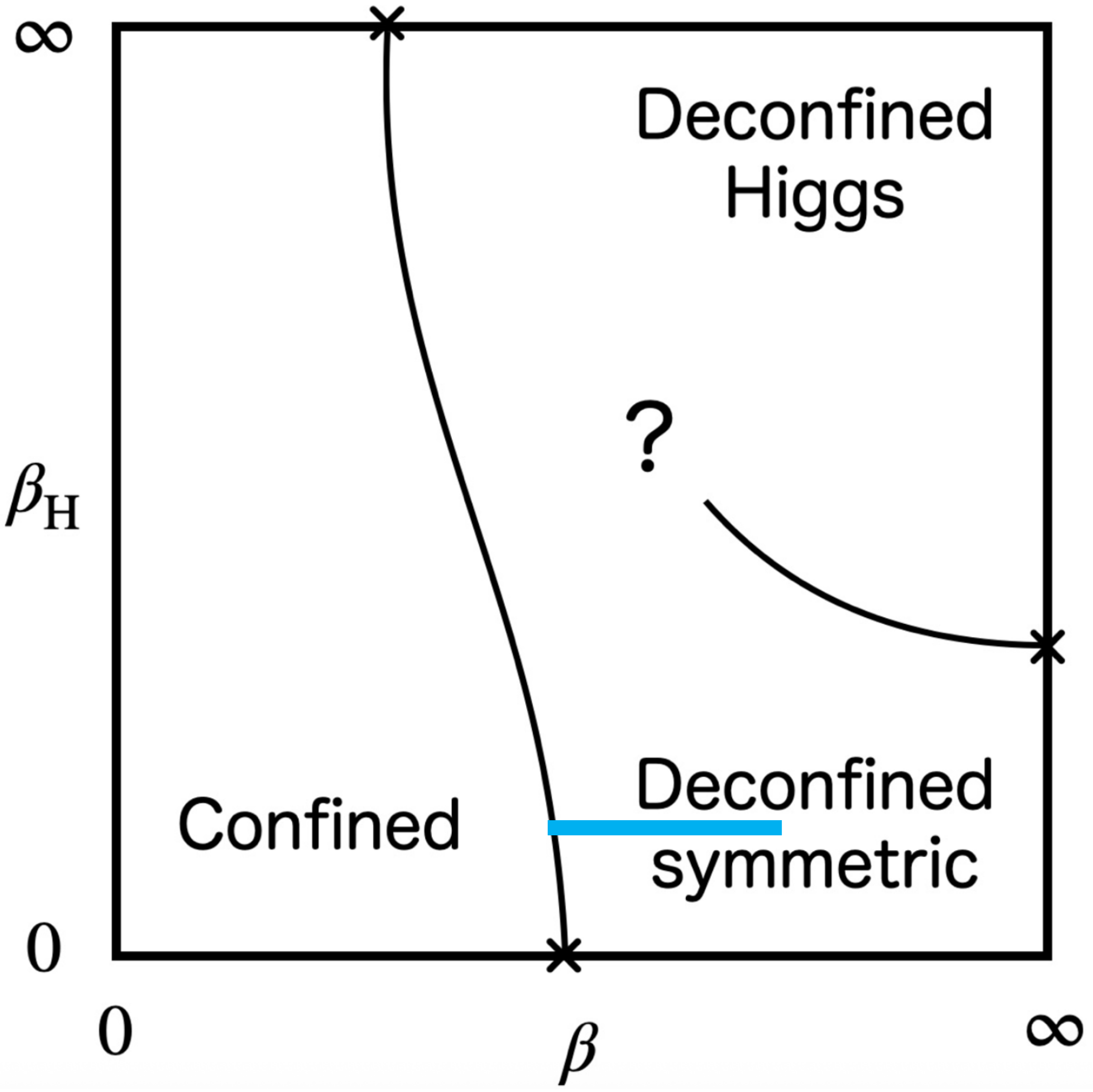
$$C_2(r) := \langle \mathrm{tr}(\phi P^2)(\vec{x})\mathrm{tr}(\phi P^2)(\vec{y})\rangle$$

$$|\vec{x} - \vec{y}| = r$$



$\Rightarrow$ SSB

Analysis (1)

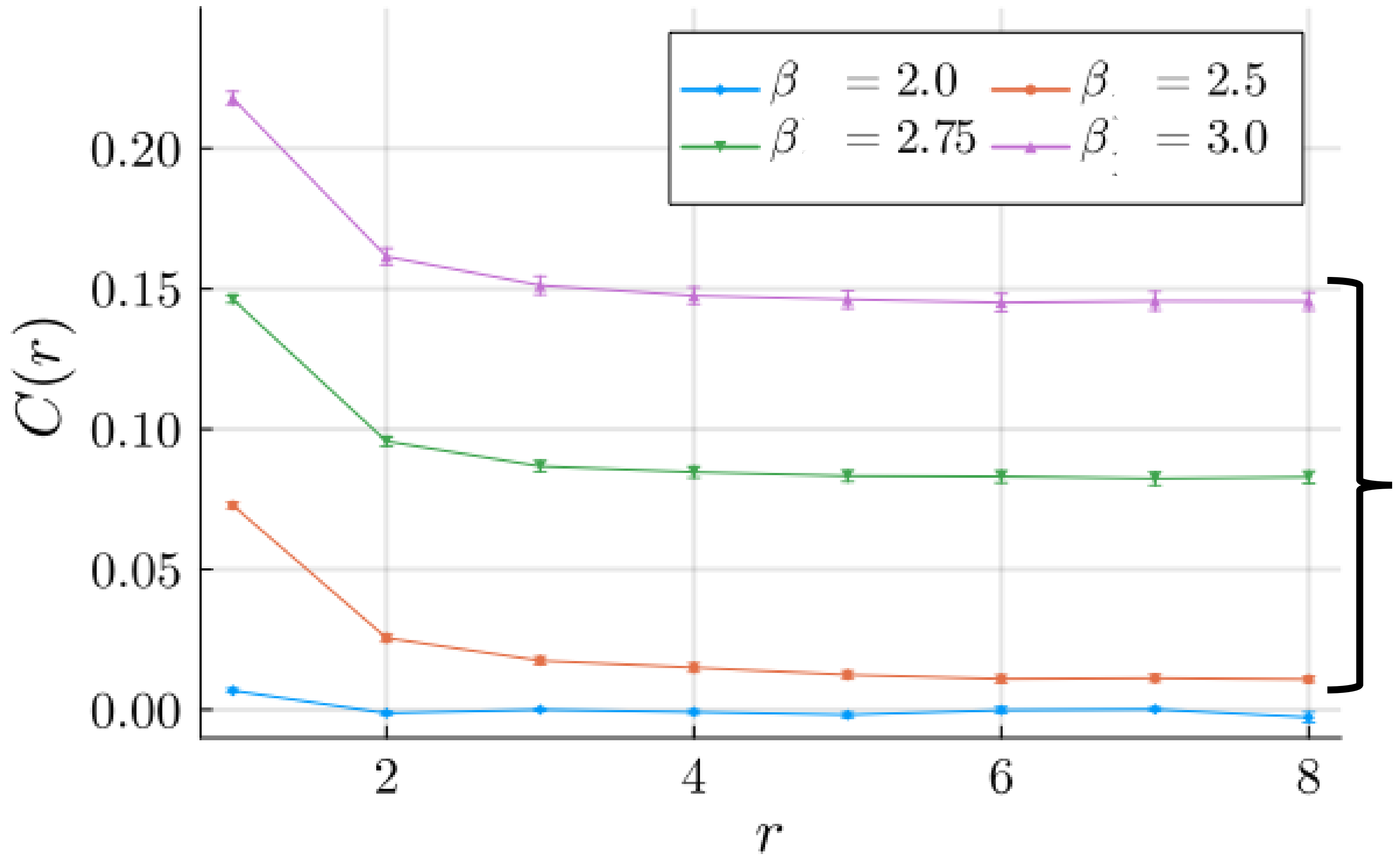


$$N_t \times N_s^3 = 8 \times 16^3$$

$$\beta_{\mathrm{H}} = 0.5$$

$$\beta = 2.0, 2.5, 2.75, 3.0$$

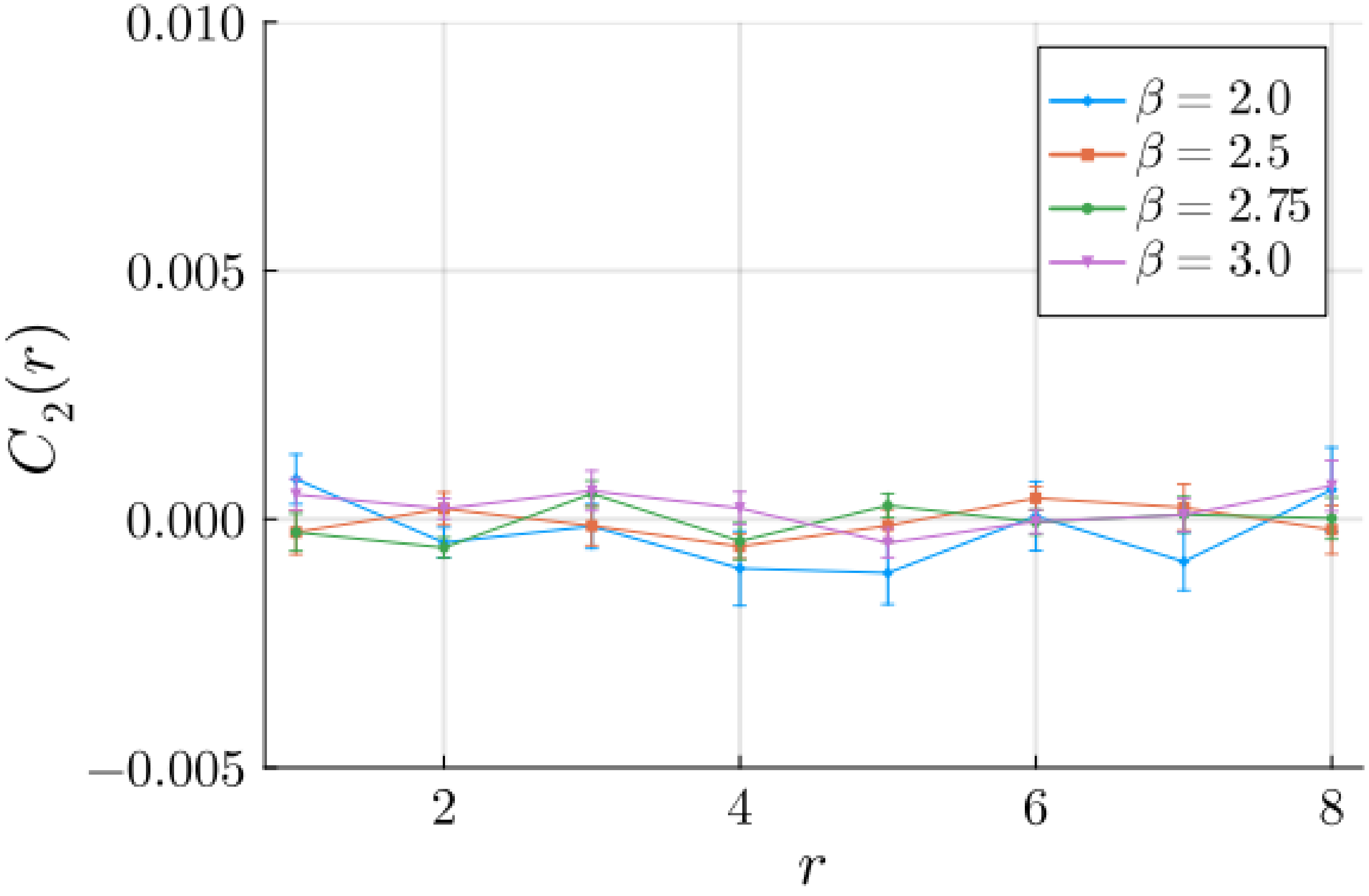
$$C(r) \coloneqq \langle \mathrm{tr}(P)(\vec{x})\mathrm{tr}(P)(\vec{y})\rangle$$



$$\cancel{\left(\mathbb{Z}_2^{[0]}\right)}_{\text{Center}}$$

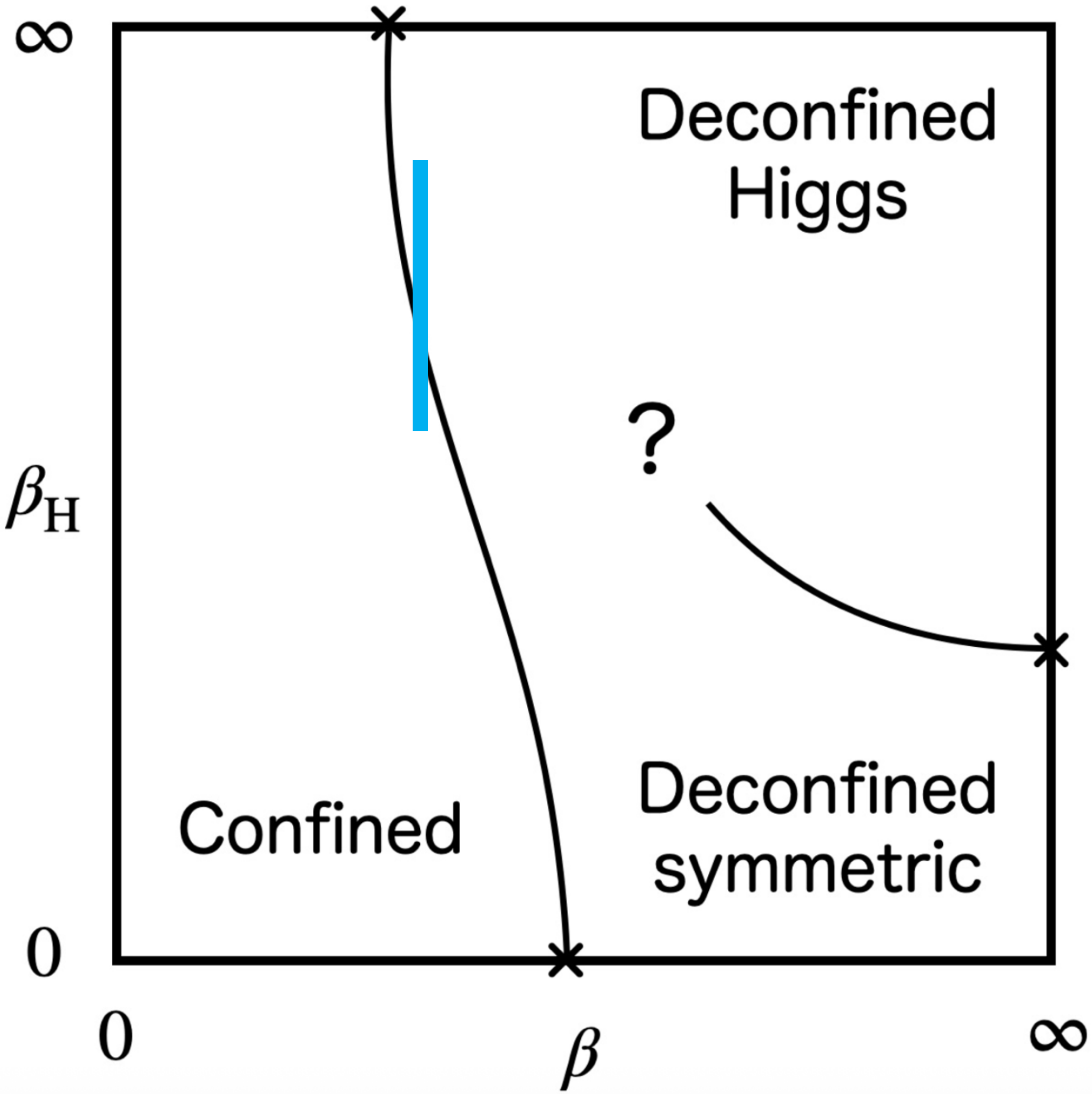
$$\left(\mathbb{Z}_2^{[0]}\right)_{\text{Center}}$$

$$C_2(r) \coloneqq \langle \mathrm{tr}(\phi P^2)(\vec{x})\,\mathrm{tr}(\phi P^2)(\vec{y})\rangle$$



$$\left(\mathbb{Z}_2^{[0]}\right)_{\mathrm{Higgs}}$$

Analysis (2)

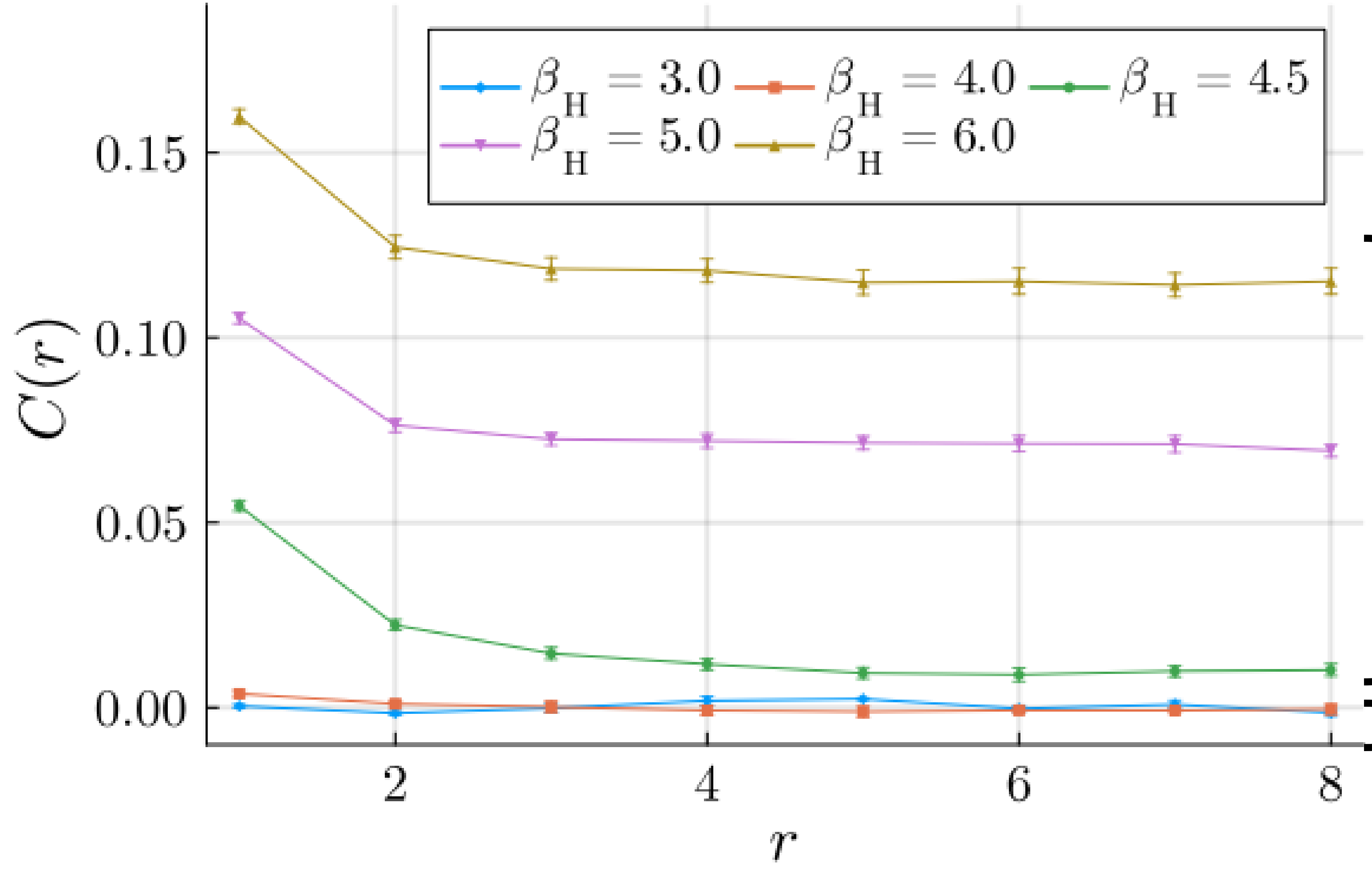


$$N_t \times N_s^3 = 8 \times 16^3$$

$$\beta = 1.5$$

$$\beta_{\mathrm{H}} = 3.0, 4.0, 4.5, 5.0, 6.0$$

$$C(r) := \langle \mathrm{tr}(P)(\vec{x})\mathrm{tr}(P)(\vec{y}) \rangle$$



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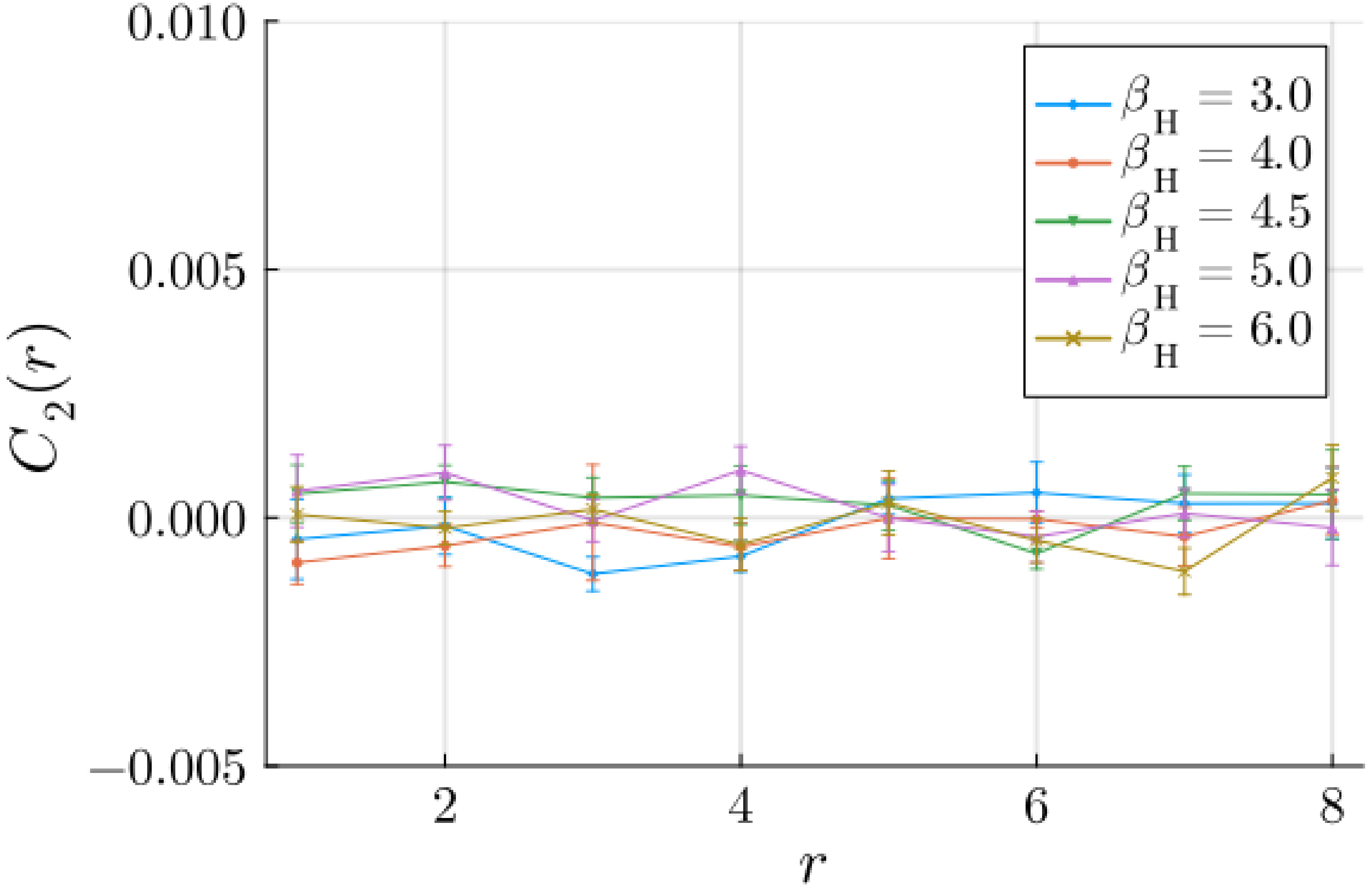
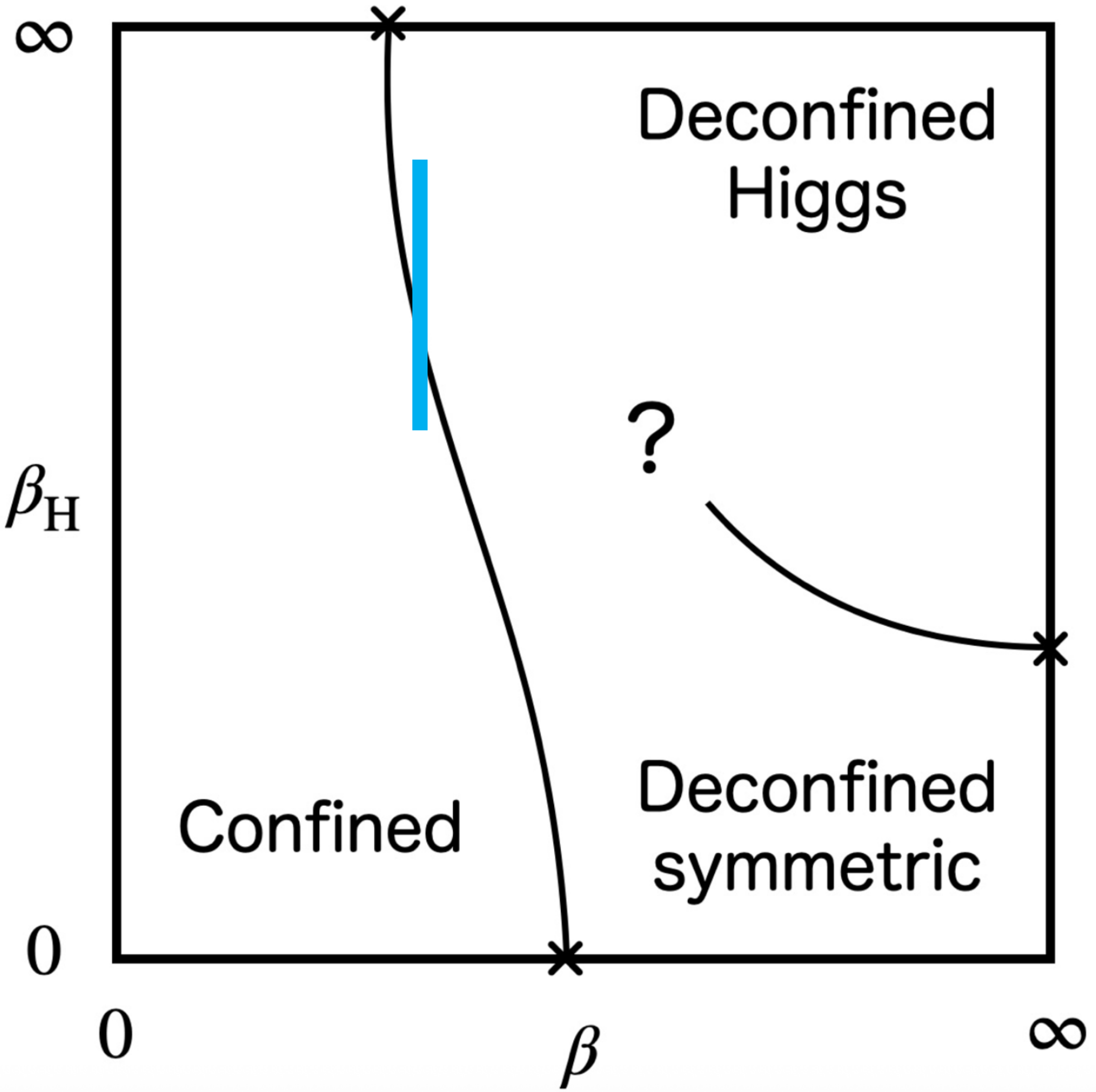


$$\left(\mathbb{Z}_2^{[0]}\right)_{\mathrm{Higgs}}$$

Analysis (2′)

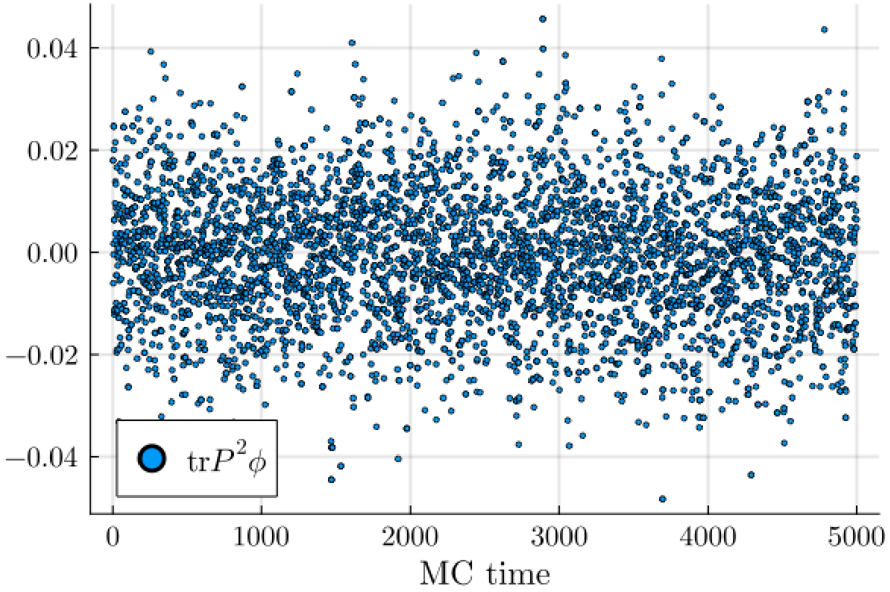


$$N_t \times N_s^3 = 8 \times 16^3$$

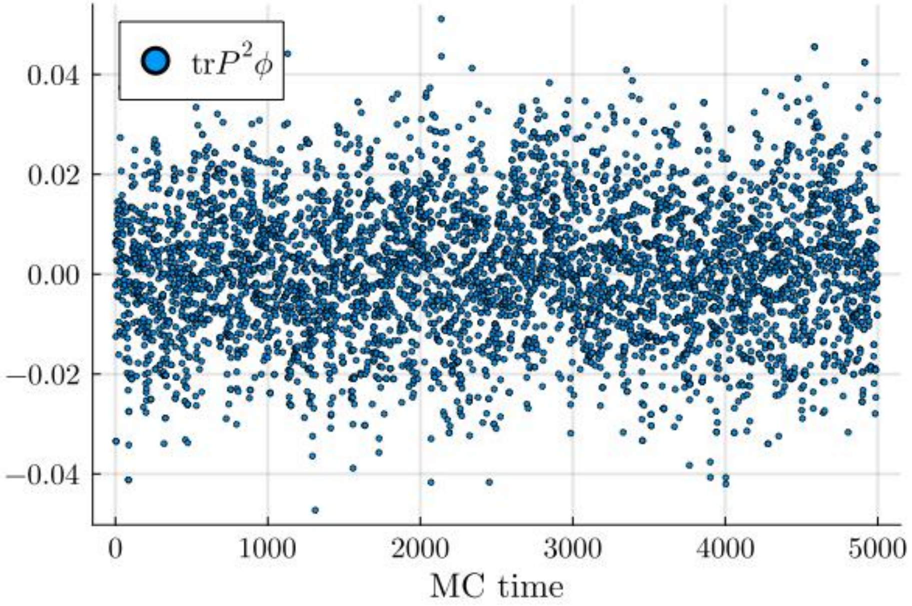
$\beta = 1.5$

$\beta_{\mathrm{H}} = 3.0, 4.0, 4.5, 5.0, 6.0$

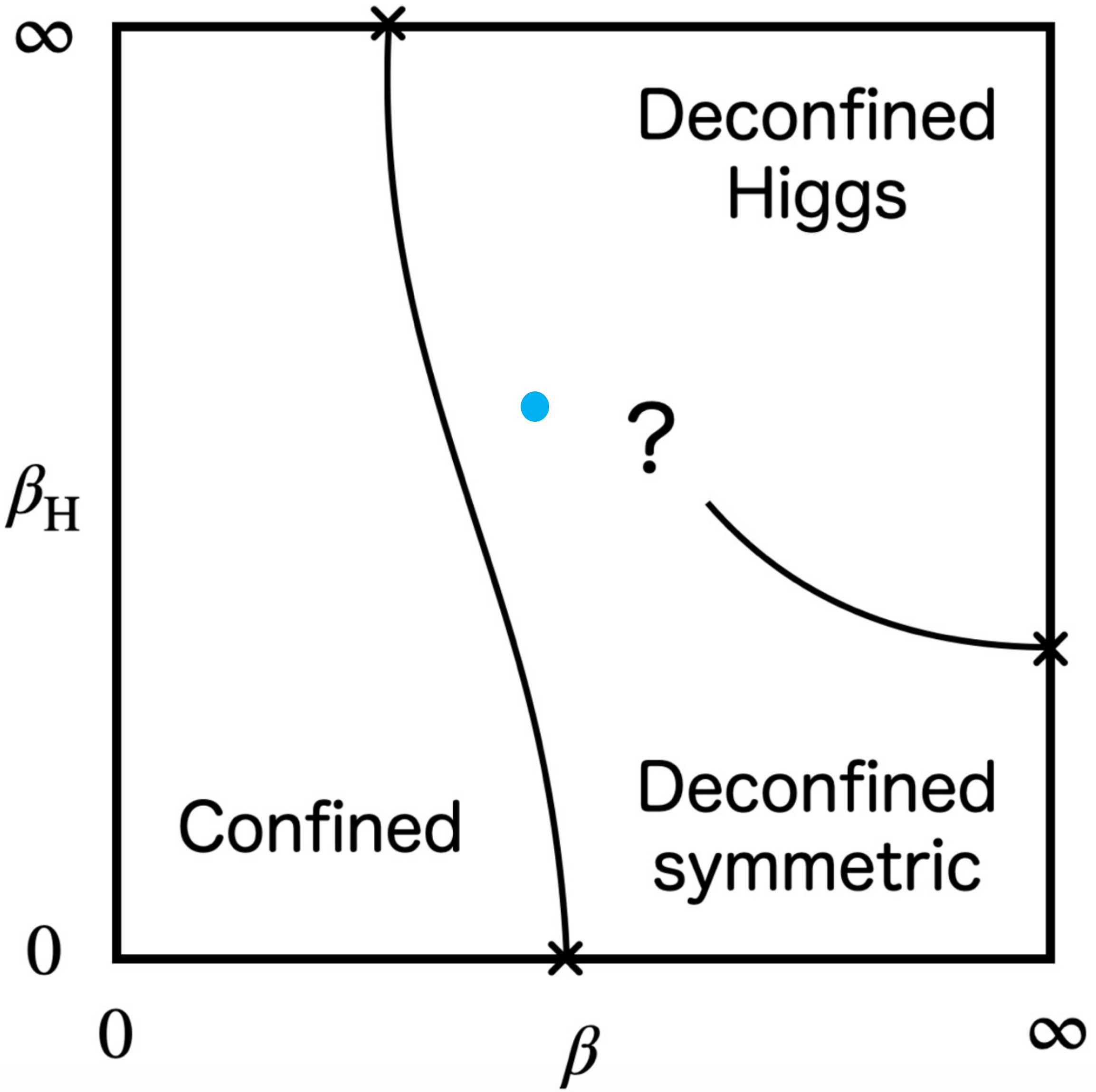
$\beta_{\mathrm{H}} = 4.0$



$\beta_{\mathrm{H}} = 5.0$

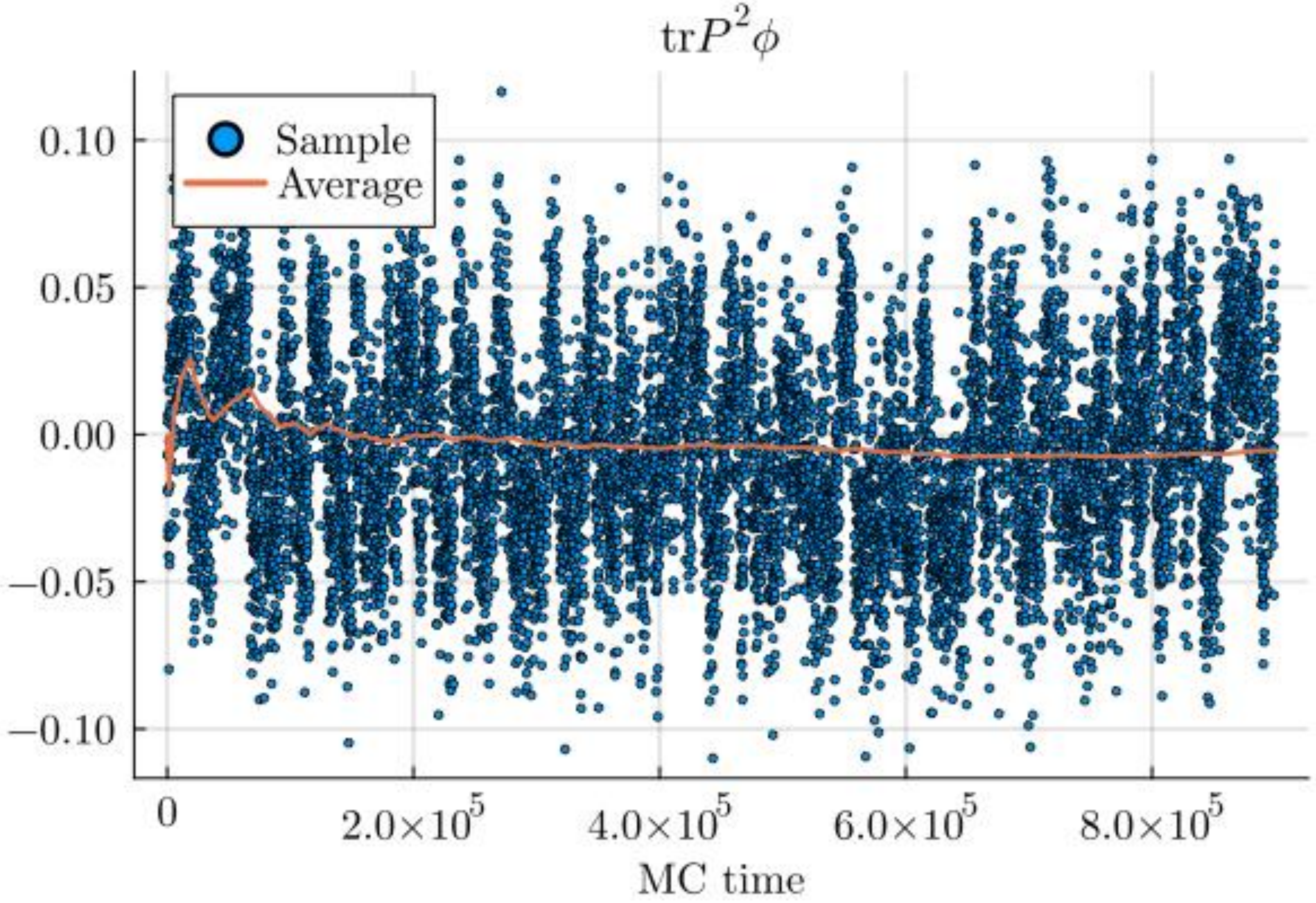


Analysis (3)



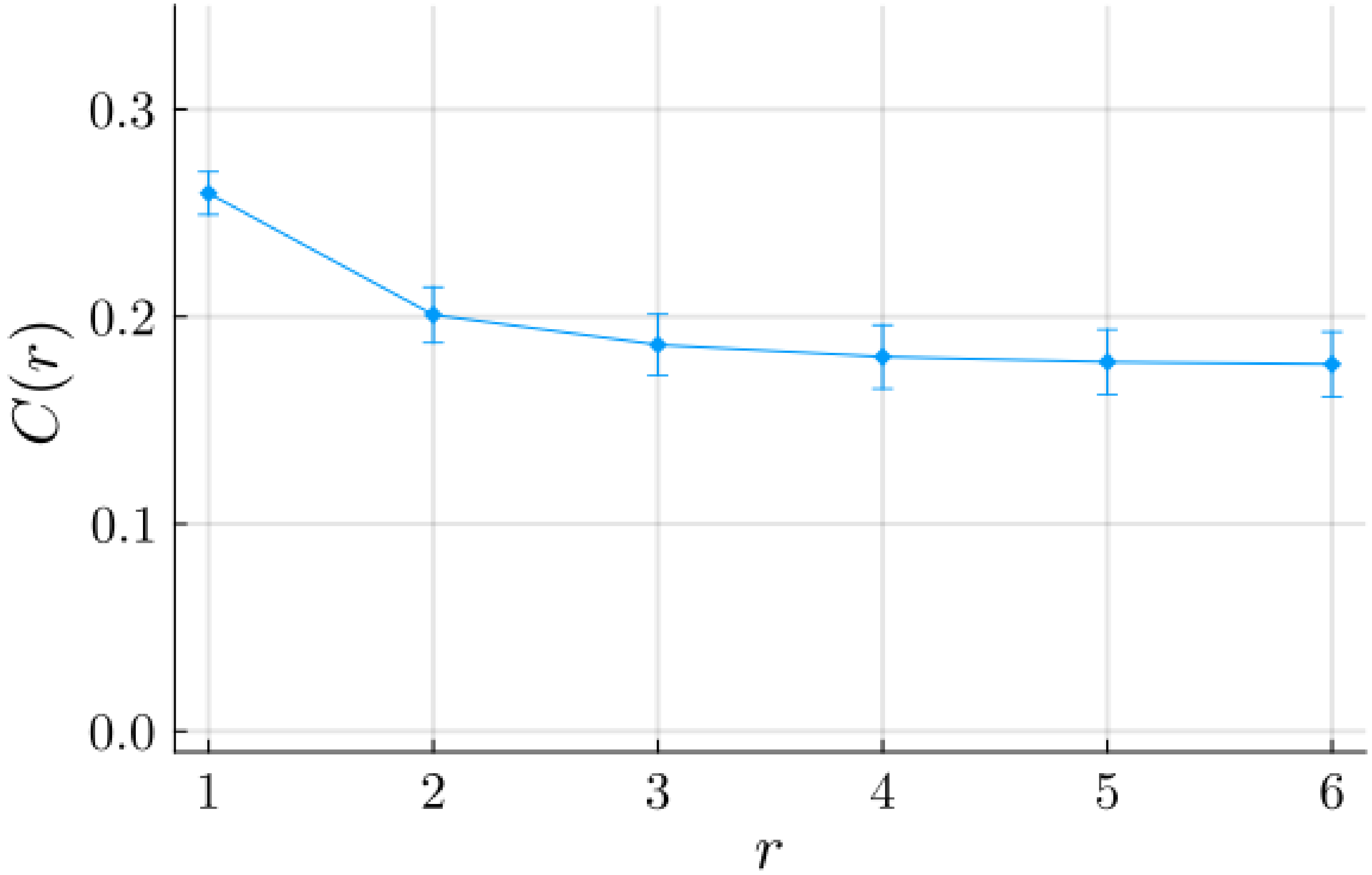
$$N_t \times N_s^3 = 6 \times 12^3$$

$$\beta = 2.0 \quad \beta_{\mathrm{H}} = 2.8$$



$\mathrm{tr}P^2\phi$
Sample
Average
0.10
0.05
0.00
−0.05
−0.10
0
2.0×10^5
4.0×10^5
6.0×10^5
8.0×10^5
MC time

$$C(r) \coloneqq \langle \mathrm{tr}(P)(\vec{x})\mathrm{tr}(P)(\vec{y}) \rangle$$



$$\cancel{\left(\mathbb{Z}_2^{[0]}\right)}_{\mathrm{Center}}$$

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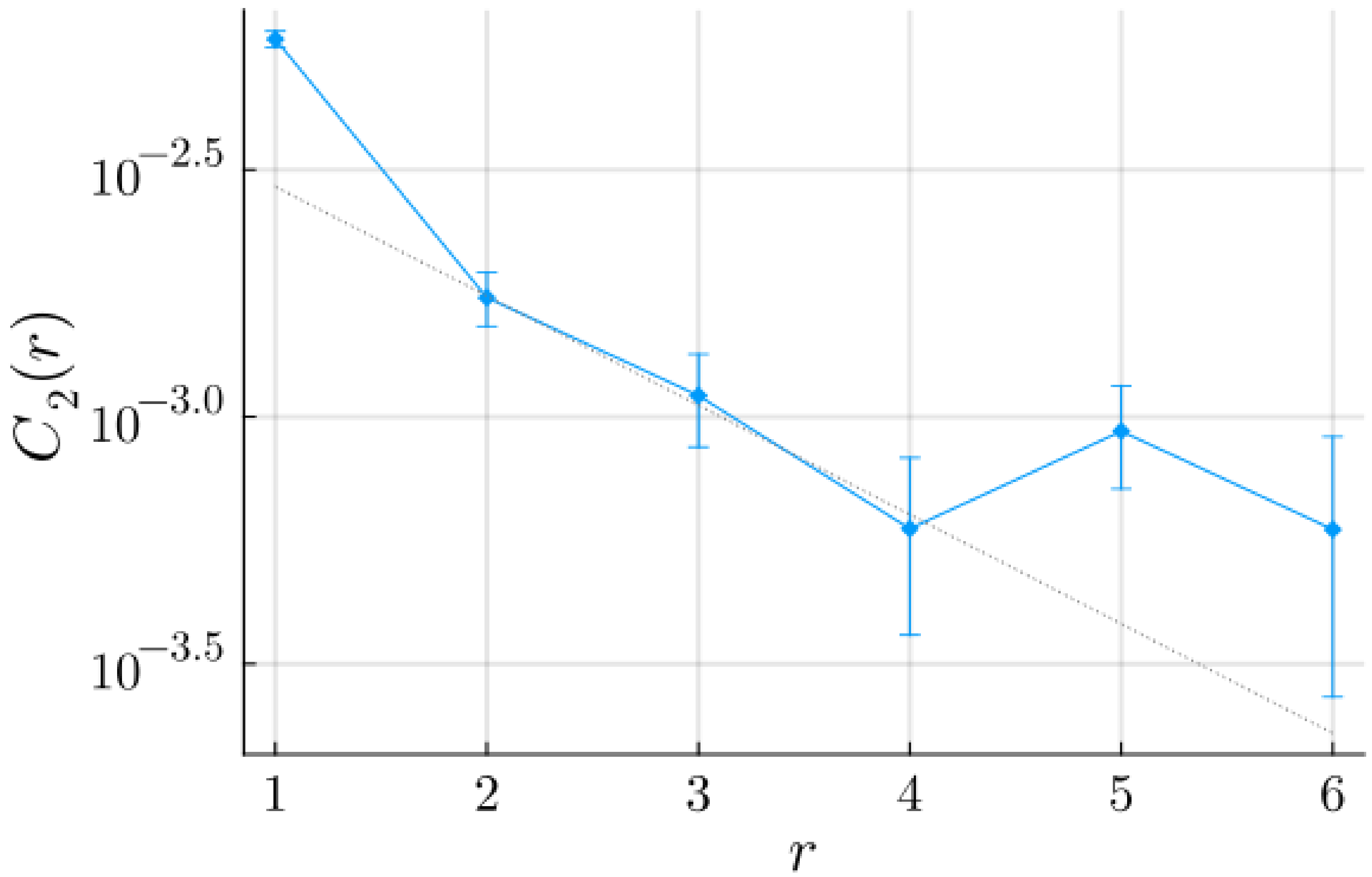
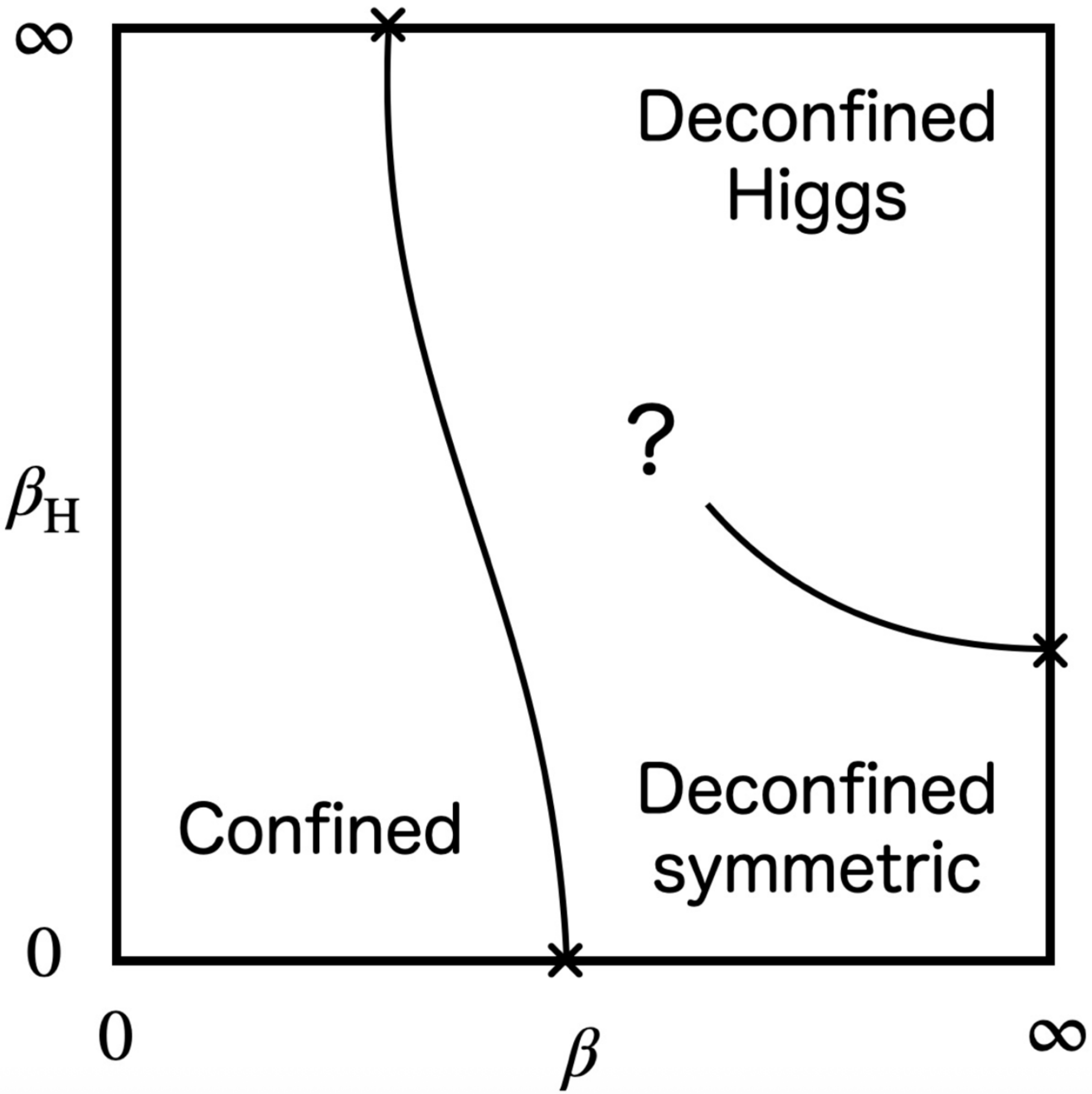


$\left(\mathbb{Z}_2^{[0]}\right)_{\mathrm{Higgs}}$???

Natural scenario

$\left(\mathbb{Z}_2^{[0]}\right)_{\mathrm{Center}}$

$\left(\mathbb{Z}_2^{[0]}\right)_{\mathrm{Higgs}}$



$\cancel{\left(\mathbb{Z}_2^{[0]}\right)}_{\mathrm{Center}}$

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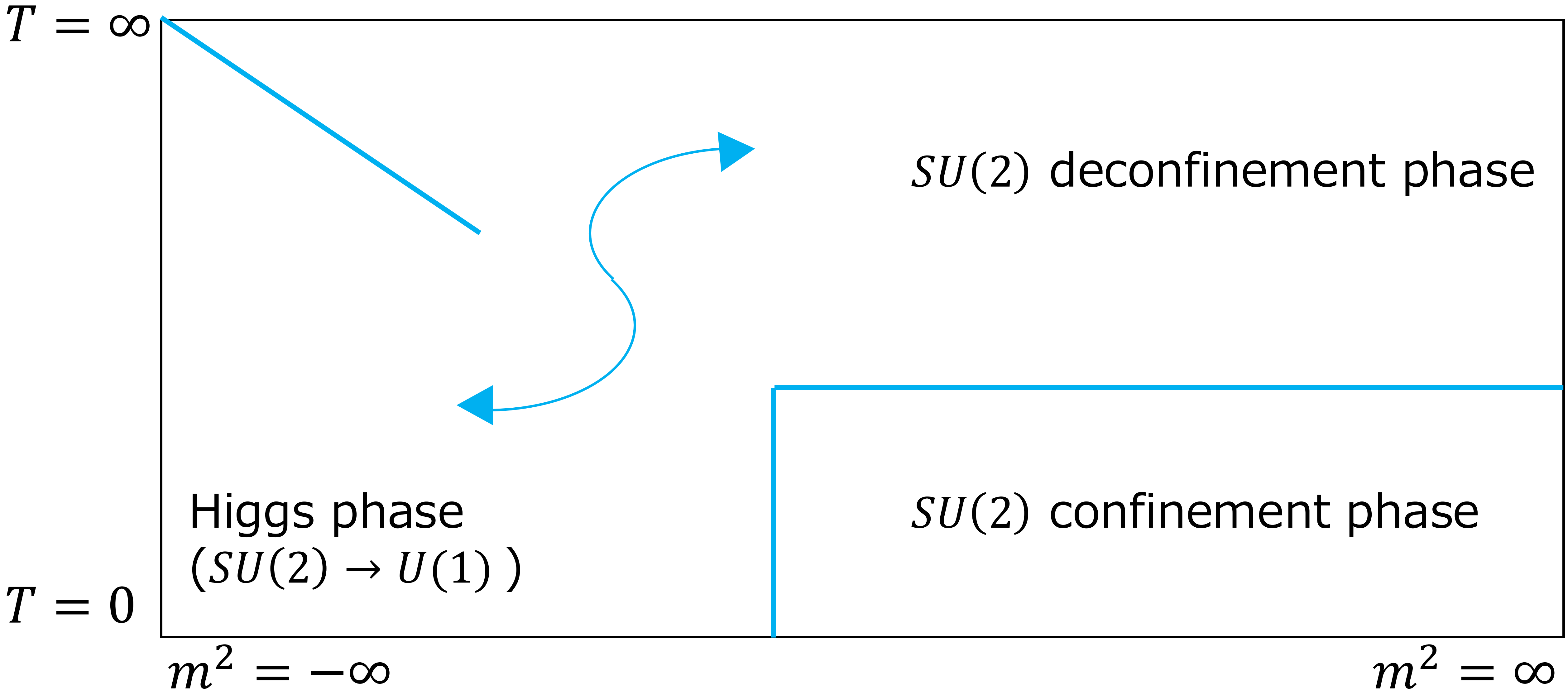

Discussion

- More analysis of $\left(\mathbb{Z}_2^{[0]}\right)_{\mathrm{Higgs}}$.
- Generalization from $SU(2)$ to $SU(N)$.

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$N = 2$ ▶ $SU(2) \to U(1)$

$N = 5$ ▶ $SU(5) \to \dfrac{U(1) \times SU(2) \times SU(3)}{\mathbb{Z}_6}$

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$N = 5$ ▶ $SU(5) \to \dfrac{U(1) \times SU(2) \times SU(3)}{\mathbb{Z}_6}$

Toy model of GUT