揺らぐ流体力学・QCD 臨界点の理論研究

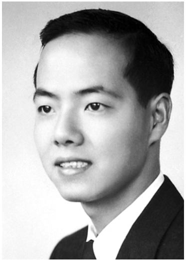
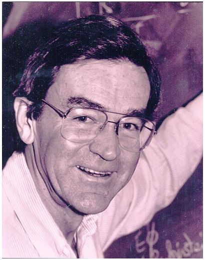
赤松　幸尚　（大阪大学）

「熱場の量子論とその応用」

京都大学基礎物理学研究所
2024 年 9 月 9 日-11 日

Why do we study QGP?

Last month, two giants, T.D.Lee and J.Bjorken, passed away

Wikipedia

The goal of heavy-ion collisions should be remembered

- Little Bang on the Earth to simulate Big Bang of the Universe
- I initiated my research to understand the hierarchy of matter
- Now I have more detailed questions, of course

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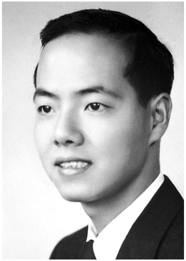

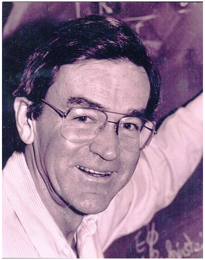

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Quarkonium: can we witness color fields in the QGP?

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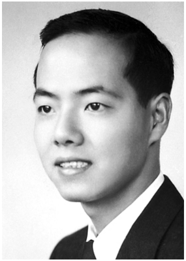

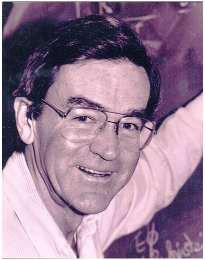

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Critical point: what signals can we expect?

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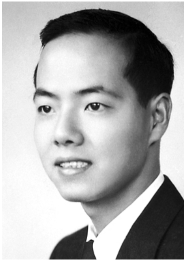
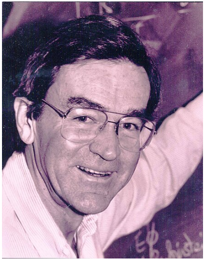

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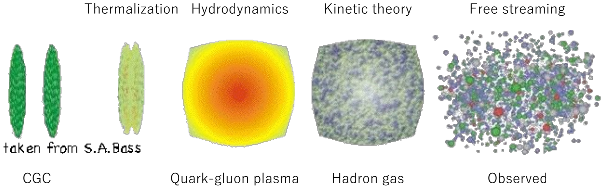
Ask a big question, and try to answer smaller questions
眼高手低（益川語録） never 眼低手低

Heavy-ions

Key achievements

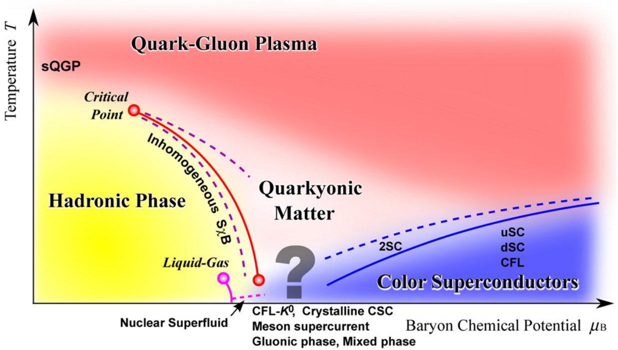
- Formation of Quark-Gluon Plasma (QGP)
- Strongly coupled nature of QGP $\leftarrow$ Nearly perfect fluid $\eta/s \sim 1/4\pi$



Challenges

- Hydrodynamization in a rapid expansion
- Hydrodynamic collectivity for small systems
- Dynamical properties of strongly coupled QGP $\eta, \zeta, D, \cdots$
- How to study critical point and dense matter at lower energies?
- How to deal with abundant charms in higher energies?

A speculated QCD phase diagram



[Fukushima-Hatsuda (11)]

Can we discover the critical point or 1st order transition in heavy-ion collisions or by lattice simulations?

Critical point search on the lattice and in the heavy-ions

1. QCD critical point in the phase diagram
   - Lattice + Pade + LYZ
2. Critical point search in heavy-ions: static properties
   - Ising universality class
   - Higher cumulants
   - Experimental data
3. Critical point search in heavy-ions: dynamic properties
   - Model H
   - Kibble-Zurek scaling

Contents

Lattice QCD has the sign problem at finite μ

Path integral for QCD partition function

$$
\begin{aligned}
Z(\beta, \mu) &= e^{P(T,\mu)V/T} = \mathrm{Tr} e^{-\beta(H-\mu N)} \\
&= \int \mathcal{D}U \underbrace{\det[\not{D}(U) + m_q - \mu\gamma_4]}_{\text{complex "probability" at } \mu \neq 0} e^{S_E[U]}
\end{aligned}
$$

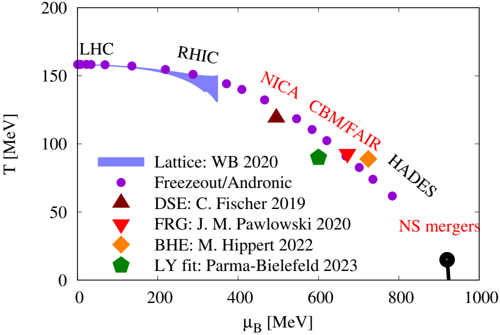
Monte Carlo method fails → several attempts

- Taylor expansion
- Canonical approach
- Reweighting method
- Imaginary chemical potential
- Lefschetz thimble
- Path optimization
- Complex Langevin method

⋮

Several approaches presented at CPOD2024

- Lattice QCD with Taylor expansions
- Lee-Yang edge singularities
- Dyson-Schwinger equations
- Functional renormalization group
- Black hole engineering



[Borsanyi @CPOD2024]

Lattice QCD with Taylor expansions

Basic idea: Expand $P(T,\mu)$ in terms of μ

$$\frac{P(T,\mu)}{T^4} = \frac{P(T,0)}{T^4} + \frac{1}{2!}\frac{\mu^2}{T^2}\chi_2(T) + \frac{1}{4!}\frac{\mu^4}{T^4}\chi_4(T) + \cdots$$

$$e^{P(T,\mu)V/T} = \int \mathcal{D}U \det[\not{D}(U) + m_q - \mu\gamma_4] e^{S_E[U]}$$

Lattice calculation of χ_{2n} involves expansion of $\det(\cdots)$

$$\chi^{uds}_{200} = +\langle B_u\rangle + \langle A_u^2\rangle - \langle A_u\rangle^2 \tag{B.5}$$

$$\chi^{uds}_{110} = +\langle A_u^2\rangle - \langle A_u\rangle^2 \tag{B.6}$$

$$\chi^{uds}_{101} = +\langle A_u A_s\rangle - \langle A_s\rangle\langle A_u\rangle \tag{B.7}$$

$$\chi^{uds}_{300} = +\langle C_u\rangle + 3\langle A_u B_u\rangle + \langle A_u^3\rangle - 3\langle B_u\rangle\langle A_u\rangle - 3\langle A_u\rangle\langle A_u^2\rangle + 2\langle A_u\rangle^3 \tag{B.8}$$

$$\chi^{uds}_{210} = +\langle A_u B_u\rangle + \langle A_u^3\rangle - \langle B_u\rangle\langle A_u\rangle - 3\langle A_u\rangle\langle A_u^3\rangle + 2\langle A_u\rangle^3 \tag{B.9}$$

$$\chi^{uds}_{120} = +\langle A_u B_u\rangle + \langle A_u^3\rangle - \langle B_u\rangle\langle A_u\rangle - 3\langle A_u\rangle\langle A_u^2\rangle + 2\langle A_u\rangle^3 \tag{B.10}$$

$$\chi^{uds}_{111} = +\langle A_u A_u A_s\rangle - \langle A_s\rangle\langle A_u^2\rangle - 2\langle A_u\rangle\langle A_u A_s\rangle + 2\langle A_s\rangle\langle A_u\rangle^2 \tag{B.11}$$

$$\begin{aligned}\chi^{uds}_{400} = &+\langle D_u\rangle + 3\langle B_u B_u\rangle + 4\langle A_u C_u\rangle + 6\langle A_u^2 B_u\rangle + \langle A_u^4\rangle \\ &- 4\langle C_u\rangle\langle A_u\rangle - 3\langle B_u\rangle^2 - 6\langle B_u\rangle\langle A_u^2\rangle - 12\langle A_u\rangle\langle A_u B_u\rangle \\ &- 4\langle A_u\rangle\langle A_u^3\rangle - 3\langle A_u A_u\rangle\langle A_u^2\rangle + 12\langle B_u\rangle\langle A_u\rangle^2 \\ &+ 12\langle A_u\rangle^2\rangle\langle A_u^2\rangle - 6\langle A_u\rangle^4\end{aligned} \tag{B.12}$$

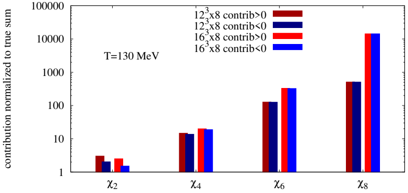
$$\begin{aligned}\chi^{uds}_{310} = &+\langle A_u C_u\rangle + 3\langle A_u^2 B_u\rangle + \langle A_u^4\rangle - \langle C_u\rangle\langle A_u\rangle - 3\langle B_u\rangle\langle A_u^2\rangle \\ &- 6\langle A_u\rangle\langle A_u B_u\rangle - 4\langle A_u\rangle\langle A_u^3\rangle - 3\langle A_u^2\rangle\langle A_u^2\rangle \\ &+ 6\langle B_u\rangle\langle A_u\rangle^2 + 12\langle A_u\rangle\langle A_u\rangle\langle A_u^2\rangle - 6\langle A_u\rangle^4\end{aligned} \tag{B.13}$$

$$\begin{aligned}\chi^{uds}_{220} = &+\langle B_u^2\rangle + 2\langle A_u^2 B_u\rangle + \langle A_u^4\rangle - \langle B_u\rangle^2 - 2\langle B_u\rangle\langle A_u^2\rangle \\ &- 4\langle A_u\rangle\langle A_u B_u\rangle - 4\langle A_u\rangle\langle A_u^3\rangle - 3\langle A_u^2\rangle\langle A_u^2\rangle \\ &+ 4\langle B_u\rangle\langle A_u\rangle\langle A_u\rangle + 12\langle A_u\rangle\langle A_u\rangle\langle A_u^2\rangle - 6\langle A_u\rangle^4\end{aligned} \tag{B.14}$$

$$\begin{aligned}\chi^{uds}_{211} = &+\langle A_u B_u A_s\rangle + \langle A_u^3 A_s\rangle - \langle A_s\rangle\langle A_u B_u\rangle - \langle A_s\rangle\langle A_u^3\rangle - \langle B_u\rangle\langle A_u A_s\rangle - \langle B_u A_s\rangle\langle A_u\rangle \\ &- 3\langle A_u\rangle\langle A_u^2 A_s\rangle - 3\langle A_u A_s\rangle\langle A_u^2\rangle + 2\langle A_s\rangle\langle B_u\rangle\langle A_u\rangle + 6\langle A_s\rangle\langle A_u\rangle\langle A_u^2\rangle \\ &+ 6\langle A_u\rangle^2\langle A_u A_s\rangle - 6\langle A_s\rangle\langle A_u\rangle^3\end{aligned} \tag{B.15}$$

[Borsanyi et al, JHEP10(2018)205]

Large cancellation in χ_{2n}



[Borsanyi @CPOD2024]

Taylor expansion up to μ^8

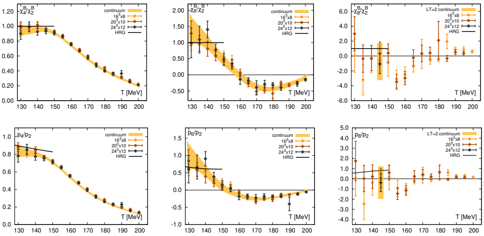
Hot QCD

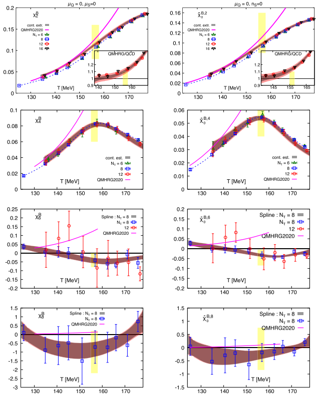
[Bollweg et al, PRD105(2022)074511]

Budapest-Wuppertal

[Borsanyi et al, 2312.07528]

upper (lower) panels: $\mu_S = 0$ $(n_S = 0)$





$$\frac{P(\mu) - P(0)}{T^4} = P_2\hat{\mu}^2 + P_4\hat{\mu}^4 + P_6\hat{\mu}^6 + P_8\hat{\mu}^8, \quad \hat{\mu} = \frac{\mu}{T}$$

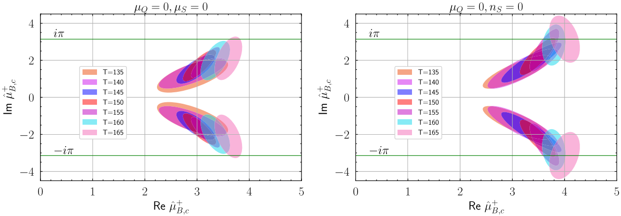
$$\frac{P_4 \Delta P(\mu)}{P_2^2 T^4} = \bar{x}^2 + \bar{x}^4 + c_6\bar{x}^6 + c_8\bar{x}^8, \quad \bar{x} = \sqrt{\frac{P_4}{P_2}}\hat{\mu}$$

What can we learn from χ_{2-8}? – Padé approximation

1. Taylor series up to μ^8 + [4,4] Padé approximation

$$\frac{P_4 \Delta P(\mu)}{P_2^2 T^4} \simeq \underbrace{\frac{(1-c_6)\bar{x}^2 + (1-2c_6+c_8)\bar{x}^4}{(1-c_6)+(c_8-c_6)\bar{x}^2+(c_6^2-c_8)\bar{x}^4}}_{\text{4 poles in complex } \bar{x} \text{ plane}} = P[4,4]$$

2. Poles in complex $\hat{\mu}$ plane (nearest to the origin)



[Bollweg et al, PRD105(2022)074511]

3. Critical point is unlikely to exist in $135 \leq T \leq 165$ MeV
   - Because poles are away from real $\hat{\mu}$
   - It may exist below $T = 135$ MeV

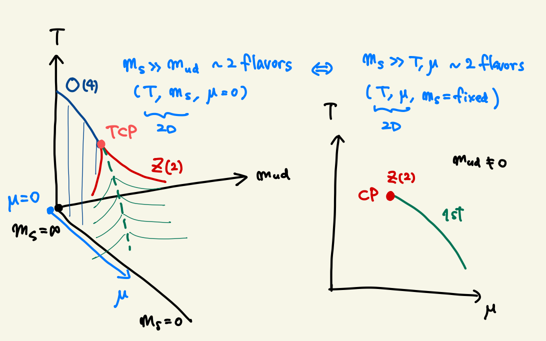
Contents

Ising model ∼ QCD critical point

1. Massless 2-flavor QCD has $SU(2)_L \otimes SU(2)_R \simeq O(4)$ symmetry

$$\mathcal{L}|_{\text{near CP}} \simeq \frac{1}{2}(\nabla\vec{\psi})^2 + \frac{a}{2}\vec{\psi}^2 + \frac{b}{4}(\vec{\psi}^2)^2 + \frac{c}{6}(\vec{\psi}^2)^3 - h\psi_0$$

2. Speculated mapping from (a, b, h) to $(T, \mu(m_s), m_{ud})$ of QCD



We can use the critical exponents of Ising model for *static* observables

Critical fluctuations of QCD critical point

1. Ising model in (r, h) phase diagram

$$\underbrace{\frac{\partial}{\partial h} \leftrightarrow \phi}_{\text{critical}}, \quad \underbrace{\frac{\partial}{\partial r} \leftrightarrow \phi^2}_{\text{less singular}}$$

2. Chiral condensate fluctuation at QCD critical point $\varphi \equiv \langle \bar{q}q \rangle - \langle \bar{q}q \rangle_c$

$$\frac{\partial}{\partial \hat{h}} \leftrightarrow \varphi, \quad \frac{\partial}{\partial \hat{r}} \leftrightarrow \varphi^2$$

3. Baryon and energy densities mix with φ

$$\frac{\partial}{\partial \hat{h}} = c_1 \frac{\partial}{\partial \beta} + c_2 \frac{\partial}{\partial (\beta\mu)} \leftrightarrow c_1 \underbrace{(e - e_c)}_{\Delta e} + c_2 \underbrace{(n - n_c)}_{\Delta n} \leftrightarrow \varphi,$$

$$\frac{\partial}{\partial \hat{r}} = c_3 \frac{\partial}{\partial \beta} + c_4 \frac{\partial}{\partial (\beta\mu)} \leftrightarrow c_3 (e - e_c) + c_4 (n - n_c) \leftrightarrow \varphi^2$$

Almost any linear combinations of Δe and Δn are critical

Scaling with correlation length

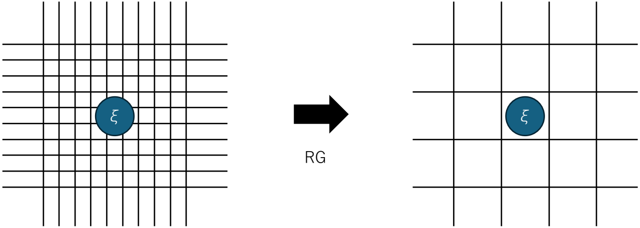
1. Mean field approximation around ground state $\phi - \langle\phi\rangle = \delta\phi \to \phi$

$$f(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{3}b_3\phi^3 + \frac{1}{4}b_4\phi^4$$

2. Roughly, kinetic term drops at $\Delta x \sim \xi$

$$F[\phi] \simeq \xi^3 \sum_i \left[\frac{1}{2}m^2\phi_i^2 + \frac{1}{3}b_3\phi_i^3 + \frac{1}{4}b_4\phi_i^4\right] \sim T,$$

$$\phi_i \sim \sqrt{T/\xi}, \quad b_3 \sim \bar{b}_3/T^{1/2}\xi^{3/2}, \quad b_4 \sim \bar{b}_4/T\xi$$



Non-Gaussian fluctuations

1. Cumulants near the critical point

$$V\kappa_2 = \frac{1}{V}\int_{x,y}\langle\phi(x)\phi(y)\rangle_c \sim \frac{\xi^6}{V}\sum_i\langle\phi_i^2\rangle_c \sim \frac{\xi^6}{V}\frac{V}{\xi^3}\frac{T}{\xi} \sim T\xi^2,$$

$$V^{n-1}\kappa_n = \frac{1}{V}\int_{x_1,\cdots,x_n}\langle\phi(x_1)\cdots\phi(x_n)\rangle_c \sim \frac{\xi^{3n}}{V}\sum_i\langle\phi_i^n\rangle_c$$

$$\sim \frac{\xi^{3n}}{V}\frac{V}{\xi^3}\left(\frac{T}{\xi}\right)^{n/2} \sim T^{n/2}\xi^{5n/2-3}$$

2. Formula [Stephanov (09)]

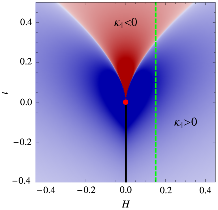
$$\therefore \kappa_n \sim \frac{T^{n/2}\xi^{n(5-\eta)/2-3}}{V^{n-1}}, \quad \eta \approx 0.04$$

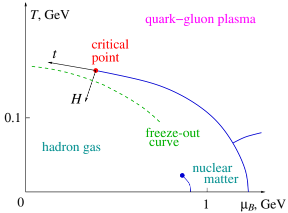
Higher-order cumulants are sensitive to ξ but suppressed by $1/V$

$\phi =$ almost any linear combination of Δn and Δe

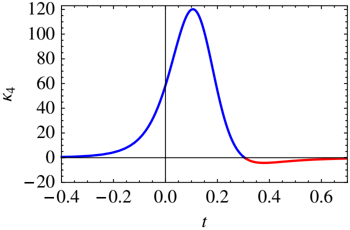
Theoretical expectation of kurtosis [Stephanov (11)]

1. Signs of kurtosis κ_4 in Ising model and freezeout surface





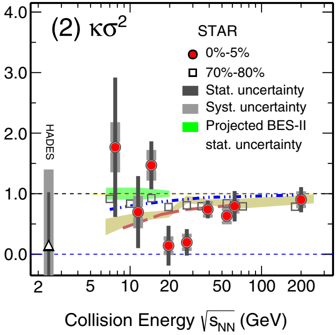
2. Sign change expected in Beam Energy Scan experiment



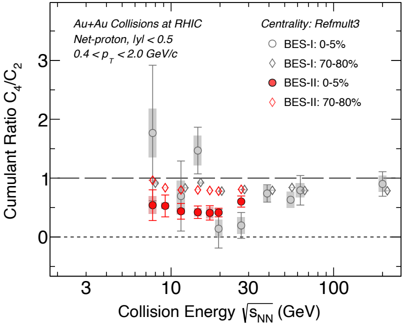
lower energy ← → higher energy

Experimental data of kurtosis

STAR data updated



[STAR (21)]



[Esumi-san's slide at HIC tutorial 2024]

Peak gone ..., but dip still there ? (I cannot see)
Deviation from normal phase base line should be discussed?

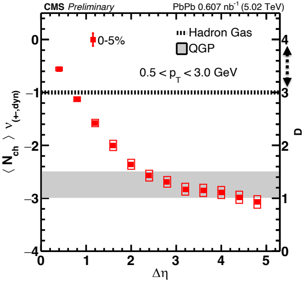
Some thoughts about rapidity window dependence?

Diffusion of conserved densities in expanding system

$$\kappa \frac{\partial^2}{\partial z^2} = \frac{\kappa}{\tau^2} \frac{\partial^2}{\partial \eta^2} = \frac{1}{\tau} \quad \rightarrow \quad \text{diffusion frozen at } \tau_D \sim \kappa/(\Delta\eta)^2$$

CMS data of D measure ($\simeq 3$ for hadron gas / $\simeq 1$ for QGP)

$$D = 4\langle \Delta Q^2 \rangle / \langle N_{\rm ch} \rangle, \quad Q = N_+ - N_-, \quad N_{\rm ch} = N_+ + N_-$$



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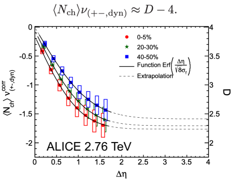


Phys. Rev. Lett. 110, 152301 (2013)

[Shengquan Tuo's slide at QM2023]

Can we make a 2D scan at low baryon density regions?

[Sakaida-Asakawa-Kitazawa (14), Sakaida-Asakawa-Fujii-Kitazawa (17)]

Contents

Hydrodynamic description

Long-time and long wavelength phenomena

- Conserved densities $\rightarrow$ change only through surface
- Nambu-Goldstone modes $\rightarrow$ massless boson
- Critical amplitudes near the critical point $\rightarrow$ large correlation length
- Gauge fields $\rightarrow$ unscreened magnetic field

Modes with $\lim_{k\to 0}\omega(\boldsymbol{k}) = 0$ are relevant

Soft modes near the QCD critical point

1. Recall that almost any linear combinations of Δe and Δn are critical

$$\frac{\partial}{\partial \hat{h}} = c_1 \frac{\partial}{\partial \beta} + c_2 \frac{\partial}{\partial(\beta\mu)} \leftrightarrow c_1 \underbrace{(e - e_c)}_{\Delta e} + c_2 \underbrace{(n - n_c)}_{\Delta n} \leftrightarrow \varphi$$

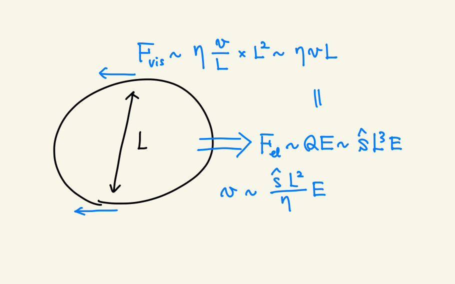
2. QCD critical point is $Z(2)$ symmetry breaking $\rightarrow$ no NG modes
3. Color magnetic screening at long distance
4. Mixed with momentum density $\boldsymbol{g}$ $\rightarrow$ Hydrodynamics $\rightarrow$ Model H

Candidate: Hydrodynamics with $\Delta e, \Delta n, \boldsymbol{g}$

Keep relevant modes: Model H with $\hat{s} = \Delta(s/n), \boldsymbol{g}_T$

Conductivity $\sigma \propto \xi$ in Model H, intuitively [Hohenberg-Halperin (77)]

1. A lump of $\hat{s}$ with linear dimension L in an electric field E



2. Electric current $j \sim \hat{s}v$ in the equilibrium

$$j \sim \hat{s}v = \underbrace{\frac{\hat{s}^2}{\eta}}_{\sim\sigma} L^2 E, \quad \hat{s}^2 \sim \underbrace{L^{-3}\frac{1}{L^{-2}+\xi^{-2}}}_{d^3k/(k^2+\xi^{-2})}, \quad \sigma \sim \frac{1}{\eta L}\frac{1}{L^{-2}+\xi^{-2}} \lesssim \frac{\xi}{\eta}$$

Conductivity scales with $\sigma \propto \xi$

Dynamical critical exponent $z \simeq 3$ in Model H

1. We can think of electric field generated by chemical potential slope

$$\partial_t \hat{s} = -\nabla \cdot j = -\sigma \nabla \cdot E = \sigma \nabla \cdot \underbrace{\nabla \mu}_{= -E} = \underbrace{\frac{\sigma}{\chi}}_{= D} \nabla^2 \hat{s}$$

2. Diffusion constant scales with

$$D = \frac{\sigma}{\chi} \sim \frac{\xi}{\xi^2} = \frac{1}{\xi}$$

3. Time scale of diffusion for wavelength ξ

$$\frac{1}{t} \sim D\nabla^2 \sim \frac{1}{\xi} \cdot \frac{1}{\xi^2} \sim \frac{1}{\xi^3}, \quad t \sim \xi^3 (=: \xi^z) \quad \therefore z \simeq 3$$

Relaxation time diverges $\propto \xi^z$ (critical slowing down)

Dynamical simulations of critical models

Langevin models for critical fluctuations

$$\dot{\phi} = \{\phi, H\} - \gamma\delta H/\delta\phi + \xi$$

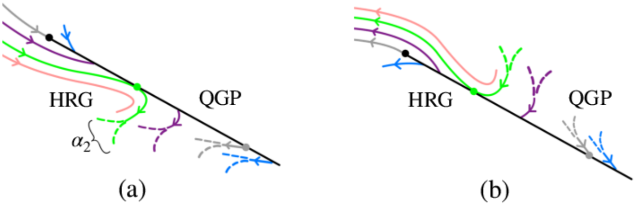
	Model	Method
[Berges-Schlichting-Sexty (10)]	C	CSS[1]
[Schlichting-Smith-vonSmekal (20)]	G	CSS
[Schweitzer-Schlichting-vonSmekal (20)]	C(A)	CSS (+damping)
[Schweitzer-Schlichting-vonSmekal (22)]	B/BC[2]	Langevin
[Nahrgang-Bluhm-Schäfer-Bass (19)]	B	Langevin
[Schäfer-Skokov (22)]	A	Langevin
[Chattopadhyay-Ott-Schäfer-Skokov (23)]	B	Langevin + KZ scaling
[Chattopadhyay-Ott-Schäfer-Skokov (24)]	H	Langevin
[Florio-Grossi-Soloviev-Teaney (22)]	G	Langevin
[Florio-Grossi-Soloviev-Teaney (24)]	G	Langevin

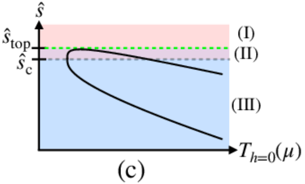
Basically, they confirm the known dynamical scaling exponents

[1] CSS = Classical Statistical Simulation, i.e. initial distribution + Hamilton dynamics
[2] BC = A certain limit conserving energy, with a propagating mode unlike in D

Isentropic trajectories on QCD phase diagram

Ideal hydrodynamics follows isentropic trajectories
Non-monotonicity of s/n on $T_{h=0}(\mu)$





	(a)	(b)
(I)	Crossover	
(II)	HRG → HRG	QGP → QGP
(III)	QGP → HRG	

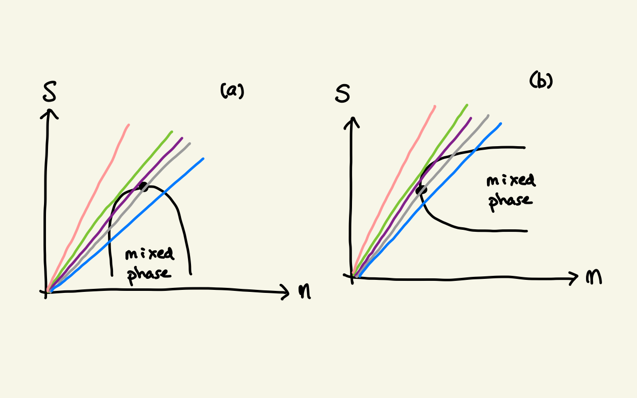
(d)

[Pradeep-Sogabe-Stephanov-Yee (24)]

Passing the QCD critical point → What can we expect as a signal?

Isentropic trajectories on QCD phase diagram

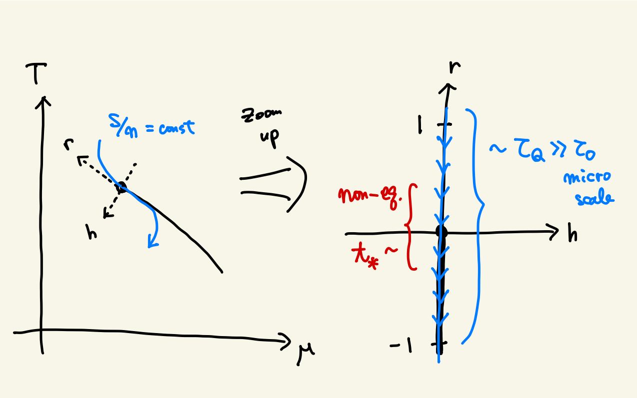
Ideal hydrodynamics follows isentropic trajectories
Non-monotonicity of s/n on $T_{h=0}(\mu)$



Generalization of Fig.1 of [Akamatsu-Teaney-Yan-Yin (19)]

Passing the QCD critical point $\rightarrow$ What can we expect as a signal?

QCD critical point in an expanding system



Trajectory on the Ising phase diagram

$$r(t) = t/\tau_Q, \quad h(t) = 0, \quad \xi(t) \sim \ell_o |r(t)|^{-\nu} \quad (\nu \approx 0.5)$$

Slow passing vs critical slowing down [Chandran-Erez-Gubser-Sondhi (12)]

1. Relaxation time for modes $k \sim 1/\xi(t)$

$$\tau_R(t) = \tau_o \left(\frac{\xi(t)}{\ell_o}\right)^z \quad \text{critical slowing down } \xi \to \infty$$

2. Effective time scales near the critical point

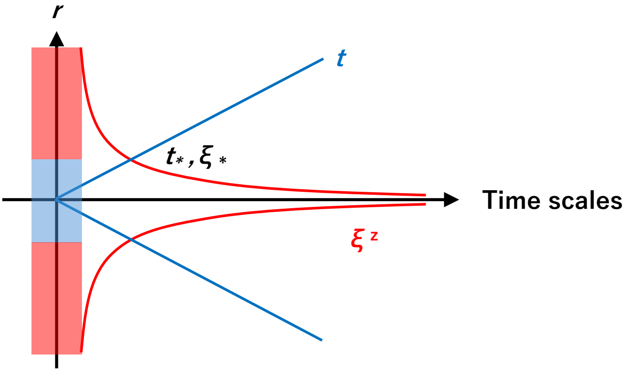
$$\text{Power-laws of } r(t) \quad \to \quad \frac{\dot{r}(t)}{r(t)} = \frac{1}{t}$$

3. Scales when the critical mode ξ starts to get out of equilibrium

$$\underbrace{t_* = \tau_o \left(\frac{\xi(t_*)}{\ell_o}\right)^z \sim \tau_o \left(\frac{t_*}{\tau_Q}\right)^{-\nu z}}_{\text{passing vs relaxation}} \to t_* = \tau_o \left(\frac{\tau_Q}{\tau_o}\right)^{\frac{\nu z}{1+\nu z}}, \quad \ell_* = \xi(t_*)$$

$$\tau_o \ll \underbrace{t_* \ll \tau_Q}_{r(t_*) \ll 1}, \quad \ell_o \ll \ell_* = \ell_o \left(\frac{\tau_Q}{\tau_o}\right)^{\frac{\nu}{1+\nu z}} \ll \underbrace{\ell_o \left(\frac{\tau_Q}{\tau_o}\right)^{\frac{1}{2}}}_{\text{kinetic regime}}$$

Time scales



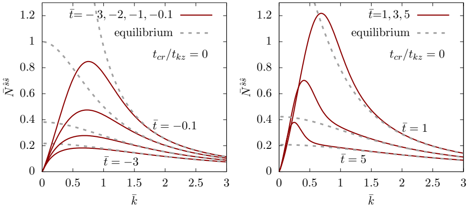
Kibble-Zurek scaling [Chandran-Erez-Gubser-Sondhi (12)]

1. In the slow passing limit $\tau_Q/t_o \gg 1$, new scaling with t_* and ℓ_*

$$\langle \phi(t_1, x_1)\phi(t_2, x_2)\rangle = \left(\frac{1}{\ell_*}\right)^{2\Delta} \mathcal{G}\left(\frac{t_1}{t_*}, \frac{t_2}{t_*}, \frac{x_1 - x_2}{\ell_*}\right) \quad : \quad \text{KZ scaling}$$

2. Mean field approx. of model B & H [Akamatsu-Teaney-Yan-Yin (19)]

$$\bar{N}_{\hat{s}\hat{s}}(t,k) = N_{\hat{s}\hat{s}}(t,k)/C_p(t_*), \quad \text{Note: } \hat{s} = n\Delta(s/n) \text{ in this paper}$$



Correlation enhances to $C_p(t_*)$ for $k_* \sim 1/\ell_*$

Fluctuation $\propto C_p(t_* \neq 0)$ is finite even in the luckiest case

How to relate with observables? [Akamatsu-Teaney-Yan-Yin (19)]

1. Baryon number fluctuation

$$\hat{n} \equiv s\delta\left(\frac{n}{s}\right) = -\frac{n}{s}\hat{s}$$

$$N_{\hat{n}\hat{n}}(t_*, k_*) \sim \left(\frac{n}{s}\right)^2 C_p(t_*) \propto \ell_*^{2-\eta} \propto \left(\frac{\tau_Q}{\tau_o}\right)^{\frac{\nu(2-\eta)}{1+\nu z}}$$

2. Numerical estimate

$$\epsilon \equiv \frac{\tau_o}{\tau_Q} \sim \frac{2\ \text{fm}}{10\ \text{fm}} \sim 0.2, \quad \frac{N_{\hat{n}\hat{n}}(t_*, k_*)}{\text{baseline}} \sim \epsilon^{-0.43} \sim 2,$$

$$\ell_* = \ell_o \epsilon^{-0.22} \sim 1\ \text{fm} \times 1.4 = 1.4\ \text{fm}$$

3. Two particle correlation enhanced at relatively short distance

$$\left.\frac{\left\langle \frac{dN}{d\eta_1}\frac{dN}{d\eta_2}\right\rangle}{\left\langle \frac{dN}{d\eta_1}\right\rangle\left\langle \frac{dN}{d\eta_2}\right\rangle}\right|_{\Delta p \sim 140\ \text{MeV}} \sim 200\%$$

Numbers are not quantitative!!

Contents

Critical point search: current status and future

1. Location and existence of QCD critical point is still unclear
2. Experimental data on kurtosis updated and peak is gone
3. Investigation of nonequilibrium effect by Kibble-Zurek scaling
4. Within a few years, Kibble-Zurek scaling will be simulated
   - Semi-analytical studies? Mode coupling (or FRG) for model H? [Kitao-Akamatsu+, in progress?]
5. Electromagnetic probes?
   - Dileptons from NJL calculation [Nishimura-Kitazawa-Kunihiro (23)]
   - Photons from model H + phonon calculation [Akamatsu-Asakawa-Hongo-Stephanov-Yee, in prep.]
6. Heavy quark probes?
   - Unlikely for model H / likely for model B [Akamatsu-Asakawa (24)]

Backup slides

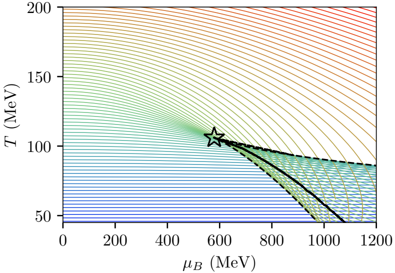
What can we learn from χ_{2-8}? – Black hole engineering (1/2)

1. Black hole solutions of Einstein-Maxwell-Dilaton equations

$$S = \frac{1}{2\kappa_5}\int_{\mathcal{M}_5} d^5x\sqrt{-g}\Big[R - \frac{(\partial\phi)^2}{2} - \underbrace{V(\phi)}_{s^{\rm lat}} - \frac{F_{\mu\nu}^2}{4}\underbrace{f(\phi)}_{\chi_2^{\rm lat}}\Big]$$

- Determine $V(\phi)$ and $f(\phi)$ by fitting $s^{\rm lat}(T,\mu=0)$ and $\chi_2^{\rm lat}(T,\mu=0)$
- Introduce dilaton ϕ for non-conformal systems

2. Calculate the phase diagram for a particular $V(\phi)$ and $f(\phi)$

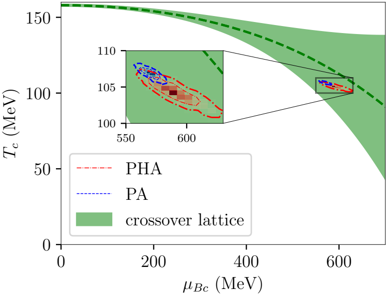


[Hippert et al, 2309.00579]

What can we learn from χ_{2-8}? – Black hole engineering (2/2)

3. Systematics with Bayesian inference

$$\underbrace{P(V,f|s,\chi_2)}_{\text{posteriori}} P(s,\chi_2) = \underbrace{P(s,\chi_2|V,f)}_{\text{likelihood}} \underbrace{P(V,f)}_{\text{prior}}$$



[Hippert et al, 2309.00579]

- PHA and PA: parametrizations of $V(\phi)$ and $f(\phi)$ with ~ 10 parameters
- No critical point in 20% of prior samples
- Predicts a critical point at $(T_c, \mu_c) \sim (105\ \mathrm{MeV}, 580\ \mathrm{MeV})$