Estimating a scalar conductivity for quark-gluon plasma in a relativistic magneto-hydrodynamic model

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Collaborators:

T. Miyoshi, C. Nonaka, and H. R. Takahashi

Phenomenological model used in this work:

[1] K. Nakamura, T. Miyoshi, C. Nonaka, and H. R. Takahashi, Eur. Phys. J. C 83, 229 (2023)
[2] K. Nakamura, T. Miyoshi, C. Nonaka, and H. R. Takahashi, Phys. Rev. C 107, 014901 (2023)
[3] K. Nakamura, T. Miyoshi, C. Nonaka, and H. R. Takahashi, Phys. Rev. C 107, 034912 (2023)

What is our work?

New phenomenological model using relativistic resistive magneto-hydrodynamics (RRMHD) for quark-gluon plasma (QGP)

$$\nabla_\mu N^\mu = 0 \qquad \nabla_\mu F^{\mu\nu} = -J^\nu$$
$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu {}^*F^{\mu\nu} = 0$$
$$\nabla_\mu S^\mu \geq 0 \qquad J^\mu = q u^\mu + \sigma e^\mu$$

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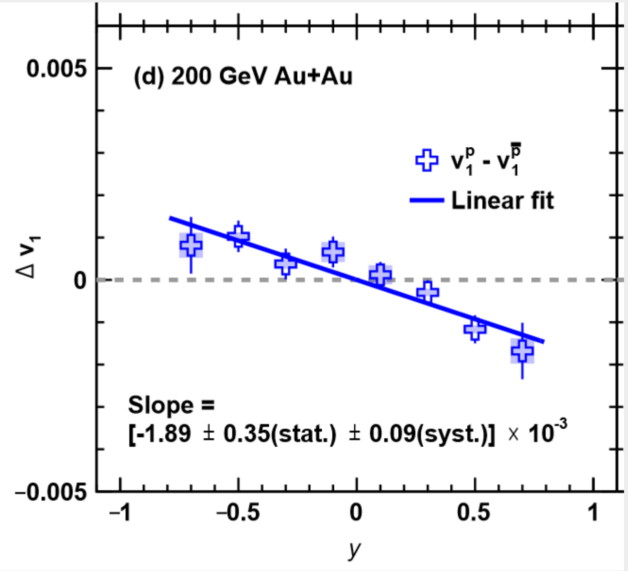
New 2024 results from STAR
Phys. Rev. X 14, 011028 (2024)
arXiv:2304.03430 [hep-ex]



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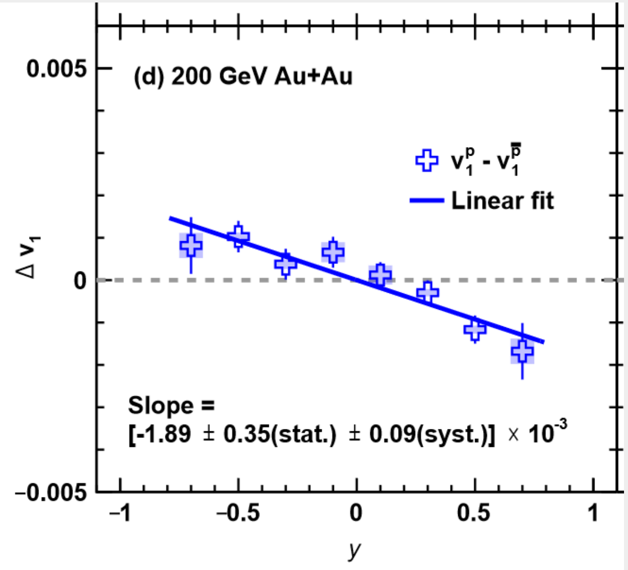


Estimation the electric conductivity of QGP independent of lattice and pQCD calculations.

What is QGP conductivity?

Conductivity is a QGP transport parameter

- They characterize the space-time evolution of QGP

Shear viscosity（せん断粘度）	η	Energy-momentum
Bulk viscosity（体積粘性）	ζ	
Charm-diffusion coefficient（ charm の拡散係数）	D	Heavy-flavor quantum numbers
Thermal conductivity（熱伝導率）	κ	Heat via baryon current
Electric conductivity（電気伝導率）	σ	Electrical charges via the electric current

Why study QGPs electric conductivity?

3 motivations

1) Important for new phenomena

- Chiral Magnetic Effect (CME)
- Magnetic reconnection
- EM probe diffusion

Interesting physics!

2) Advances in models + new experimental data

- STAR 2024 data
- Our RRMHD model

Good timing!

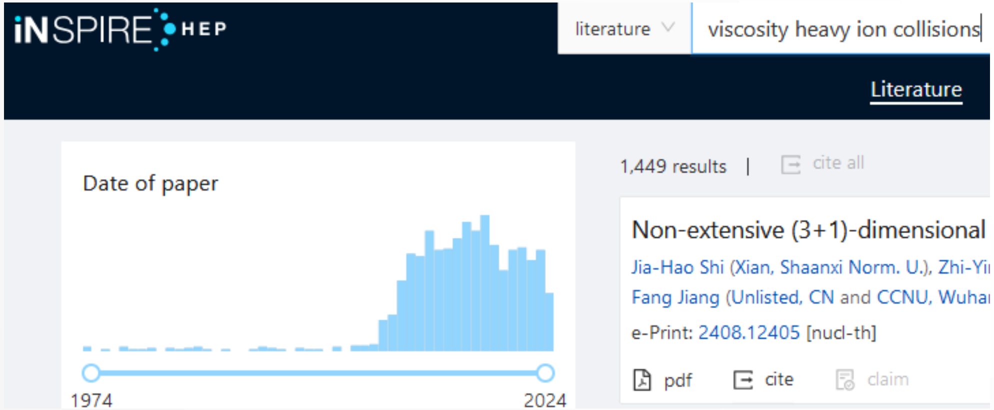
Colossal Magnetic Field Detected in Nuclear Matter

February 23, 2024 • *Physics* 17, 31

Collisions of heavy ions briefly produced a magnetic field 10^{18} times stronger than Earth's, and it left observable effects.

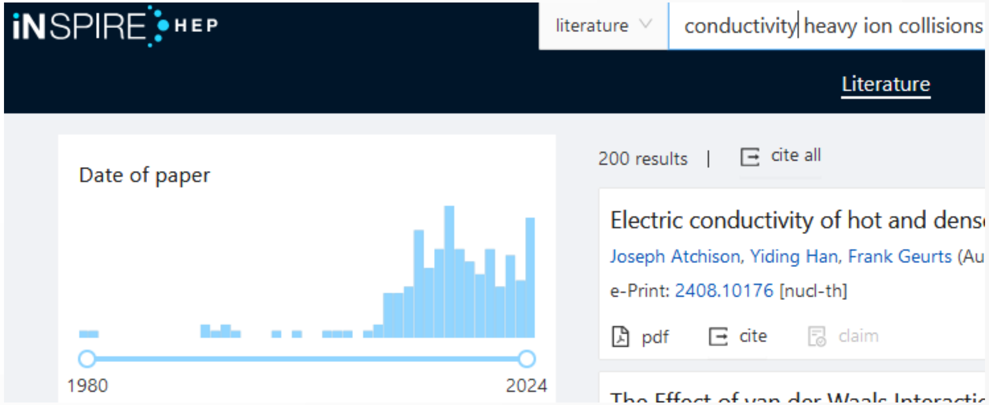
Why study QGPs electric conductivity?

3) Viscosity has a lot of studies in high-energy heavy-ion collisions (~1450)



Captured on 2024/08/27

3) Conductivity has comparability been poorly studied in high-energy heavy-ion collisions (~200)



Bump is because of interest in the chiral magnetic effect (CME)

Quark-gluon plasma (QGP) electric conductivity

What is QGP electric conductivity (σ)?

- Studied by lattice calculations (~10 papers), pQCD, and kinetic transport theories

Review: G. Aarts and A. Nikolaev, Eur. Phys. J. A 57, 118 (2021); 2008.12326 [hep-lat]

$$\sigma = \frac{1}{6}\frac{\partial}{\omega}\left(\int d^4x e^{i\omega t}\langle[j_\mu^{\mathrm{em}}(t,x), j_\mu^{\mathrm{em}}(0,0)]\rangle\right)|_{\omega=0}$$

using linear-response theory (Kubo formula)

Low energy limit of the electromagnetic spectral function

$$j_\mu^{\mathrm{em}}(x) = \sum_{f=1}^{N_f}(eq_f)\overline{\psi}^f(x)\gamma_\mu\psi^f(x)$$

where the EM current depends on the quark flavors

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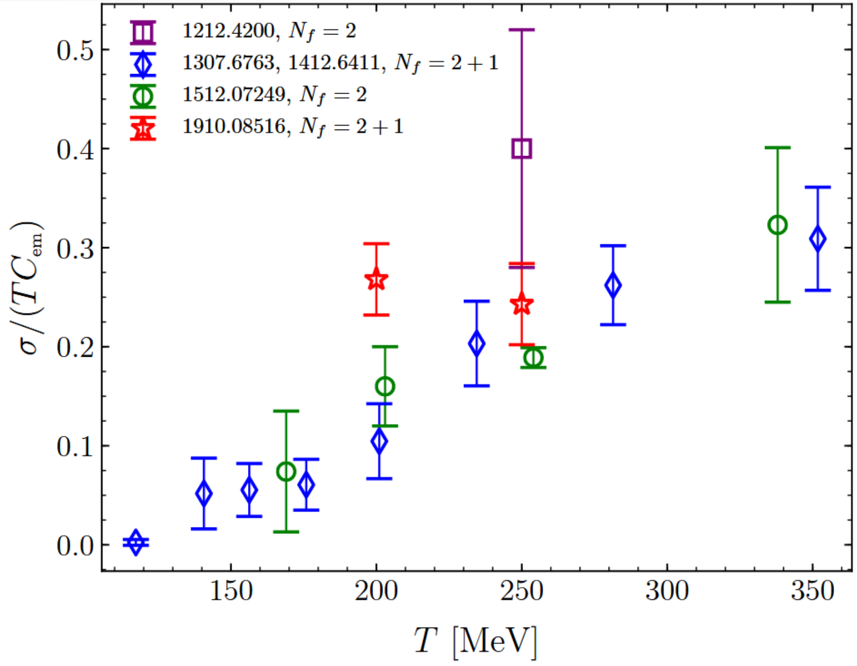

- Does not include external magnetic field effects
- Uses approximately realistic pion mass
- General agreement among results using a variety of methods and parameters (see backup or paper)

Review: G. Aarts and A. Nikolaev, Eur. Phys. J. A 57, 118 (2021); 2008.12326 [hep-lat]

Quark-gluon plasma (QGP) electric conductivity

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Unfortunately lattice calculations, pQCD, and kinetic transport calculations can disagree

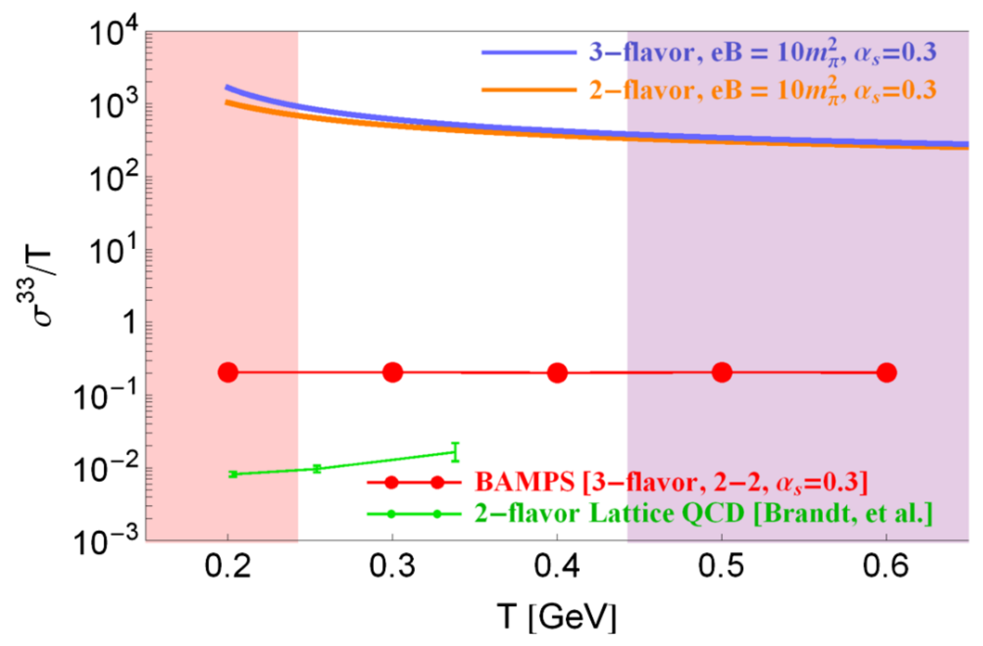

Fig. 8 from K. Hattori and D. Satow, Phys. Rev. D 94, 114032 (2016)

Quark-gluon plasma (QGP) electric conductivity

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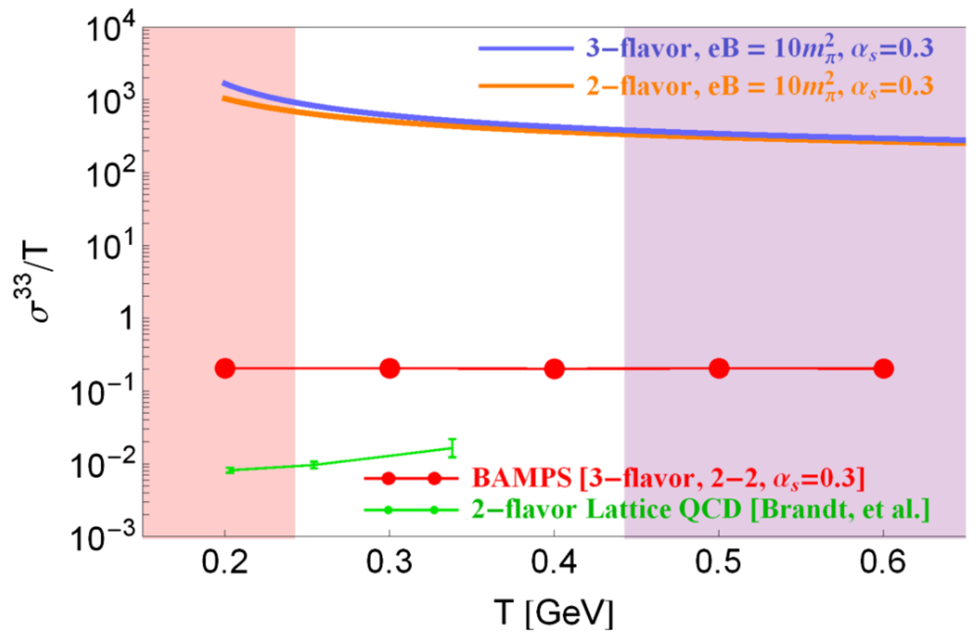
Unfortunately lattice calculations, pQCD, and kinetic transport calculations can disagree

Includes an external magnetic field

- Several orders of magnitude difference
- Blue+Orange is Strong Field calculation
- Red is Boltzman Transport
- Green is Lattice

Fig. 8 from K. Hattori and D. Satow, Phys. Rev. D 94, 114032 (2016)



Results also vary depending on the details of the calculation

What about heavy-ion collisions (HIC)?

Is it possible to measure electric conductivity in HICs?

- Yes! Collision environment has QGP + EM fields

What observable can we use?

- Dileptons → i.e., Y. Akamatsu, H. Hamagaki, T. Hatsuda, and T. Hirano, Phys. Rev. C 85, 054903 (2012).
- Photons → i.e., J.-A. Sun and L. Yan, Phys. Rev. C 109, 034917 (2024).

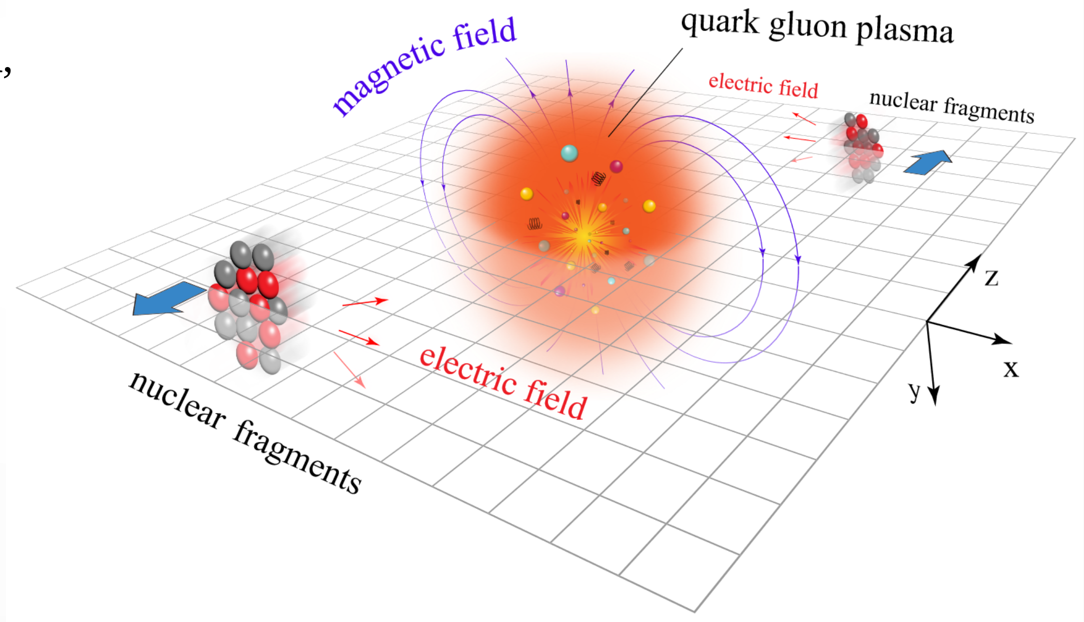


Image: STAR Collaboration (2024)

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- Photons → i.e., J.-A. Sun and L. Yan, Phys. Rev. C 109, 034917 (2024).

Charge dependent directed flow?

- Asymmetic collisions → i.e., Y. Hirono, M. Hongo, and T. Hirano, Phys. Rev. C 90, 021903 (2014).
- Symmetric collisions?

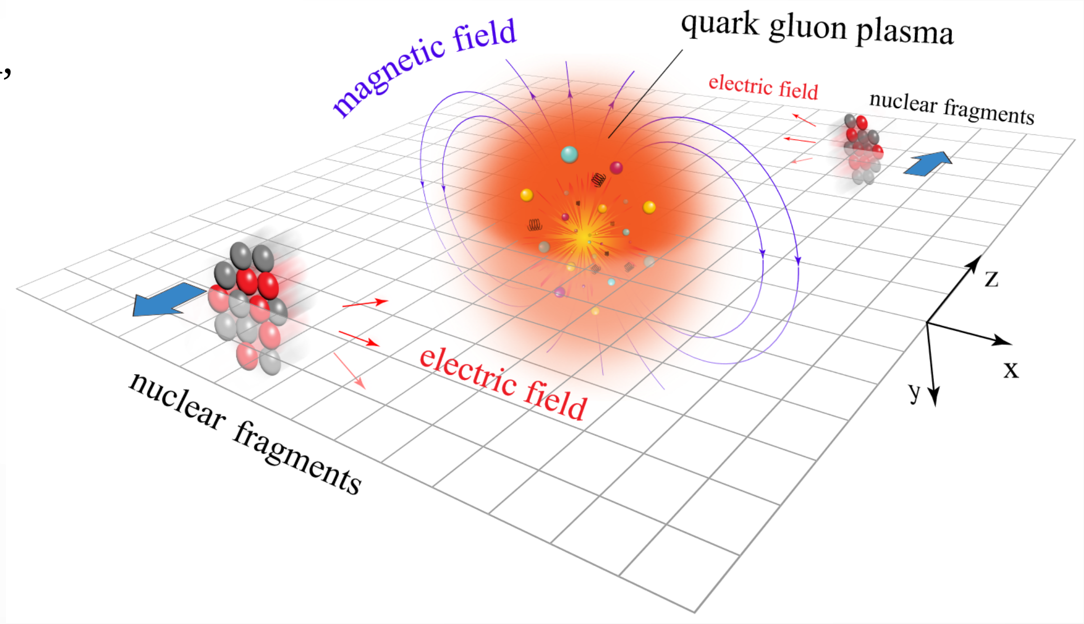


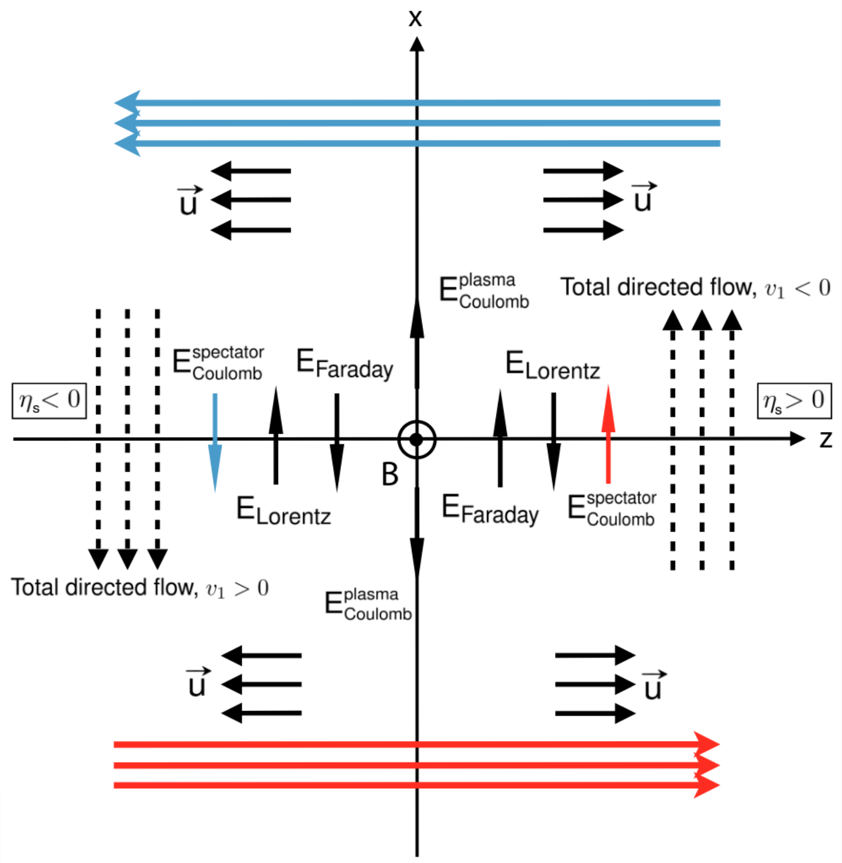
Image: STAR Collaboration (2024)

Charge dependent directed flow

Electromagnetic fields inside QGP

- Faraday: the decay of the fragments magnetic field
- Lorentz Force or Hall Effect: from the longitudinal expansion of the fluid
- Coulomb: produced by spectators and fluid

Size of the effects depend upon the collision



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Charge dependent directed flow

Electromagnetic fields inside QGP

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Size of the effects depend upon the collision

What model can be used to study all of those effects?

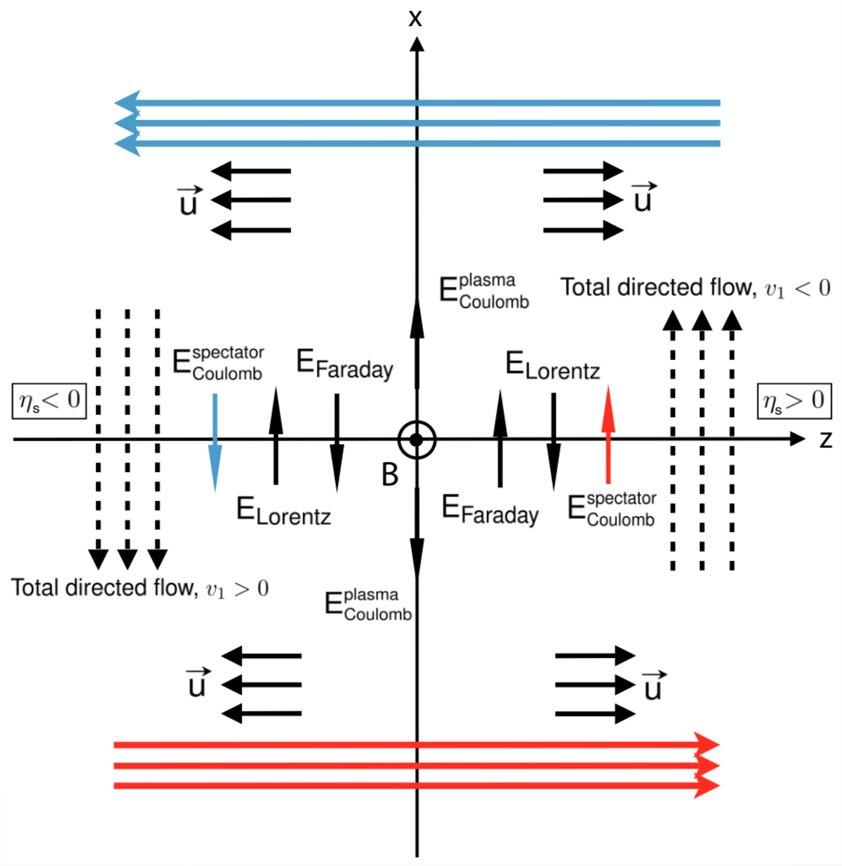


Image: STAR Collaboration (2024)

Introduction to our model

Model created by K. Nakamura, T. Miyoshi, C. Nonaka, and H. R. Takahashi

- Start with ideal relativistic hydrodynamics
- Assume the plasma has a main fluid-current

$$\nabla_\mu N^\mu = 0$$ Continuity equation (i.e. net-baryon current)

$$\nabla_\mu T^{\mu\nu} = 0$$ Total energy-momentum tensor

$$\nabla_\mu S^\mu \geq 0$$ 2nd law of thermodynamics

- Assume a secondary current in the plasma creates the EM fields
- Then, assume the EM fields are too short-lived for nonlinear effects
- Also the plasma is highly ionized, so magnetization/polarization are ignored

$$\alpha = \frac{n_i}{n_i + n_n} \approx 1$$

Ionization ratio captures how much of the plasma number density is neutral

$$\nabla_\mu F^{\mu\nu} = -J^\nu$$
$$\nabla_\mu {}^*F^{\mu\nu} = 0$$

Maxwell's equations
Faraday + Coulomb effects

Introduction to our model

Model created by K. Nakamura, T. Miyoshi, C. Nonaka, and H. R. Takahashi

- Ideal relativistic hydrodynamics + Maxwell's equations

$$\nabla_\mu T^{\mu\nu} = 0$$

Energy-momentum tensor

$$\nabla_\mu F^{\mu\nu} = -J^\nu$$
$$\nabla_\mu {}^*F^{\mu\nu} = 0$$

Maxwell's equations

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{m}} + T^{\mu\nu}_{\mathrm{f}}$$

$$T^{\mu\nu}_{\mathrm{m}} = (\epsilon + p_{\mathrm{gas}})\, u^\mu u^\nu + p_{\mathrm{gas}} g^{\mu\nu}$$

$$T^{\mu\nu}_{\mathrm{f}} = F^{\mu\lambda} F^\nu_\lambda - \frac{1}{4} g^{\mu\nu} F^{\lambda\delta} F_{\lambda\delta}$$

This is the Lorentz force acting on the plasma

The Faraday tensor is decomposed as

$$F^{\mu\nu} = u^\mu e^\nu - u^\nu e^\mu + \epsilon^{\mu\nu\lambda\delta} b_\lambda u_\delta$$
$${}^*F^{\mu\nu} = u^\mu b^\nu - u^\nu b^\mu + \epsilon^{\mu\nu\lambda\delta} e_\lambda u_\delta$$

where

$$e^\mu u_\mu = 0$$
$$b^\mu u_\mu = 0$$

Fields in the fluid's co-moving (rest) frame

Introduction to our model

Model created by K. Nakamura, T. Miyoshi, C. Nonaka, and H. R. Takahashi

- EM fields exist in a medium (the quark-gluon plasma)
- Conductivity models the coupling between the fluid and the EM fields
- Acts as energy dissipation away from the fluid

$$\nabla_\mu F^{\mu\nu} = -J^\nu$$
$$\nabla_\mu {}^*F^{\mu\nu} = 0$$

$$J^\mu = q u^\mu + \boxed{\sigma e^\mu}$$

We assume a scalar conductivity.

We do not include any other kinds of dissipation, i.e., viscosity

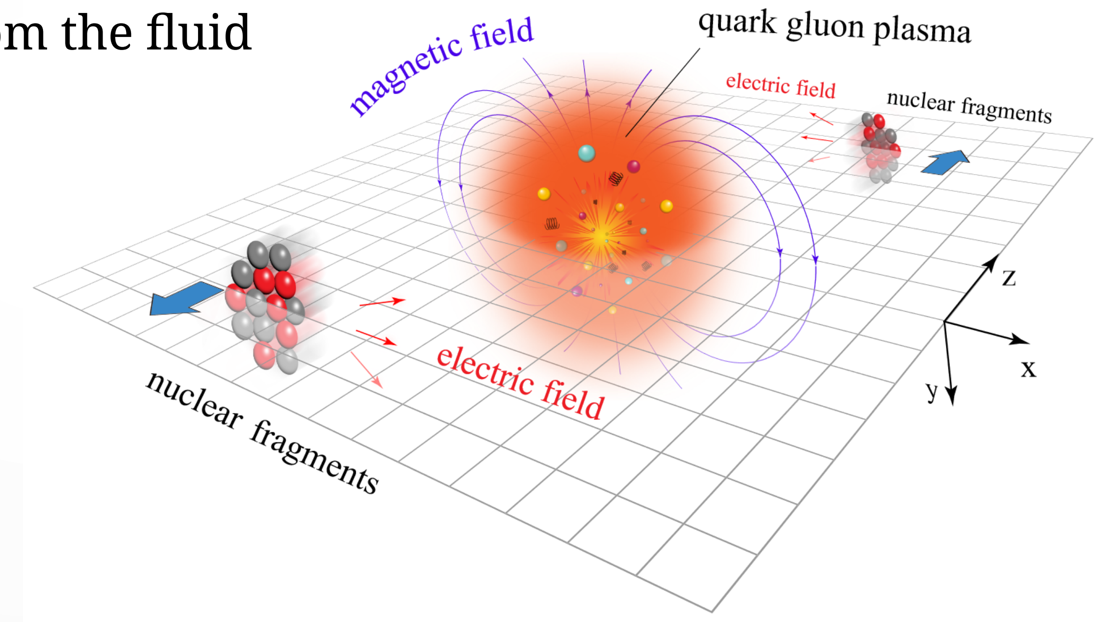


Image: STAR Collaboration (2024)

Introduction to our model

Model created by K. Nakamura, T. Miyoshi, C. Nonaka, and H. R. Takahashi

- To summarize, we use ideal relativistic hydrodynamics + Maxwell's equations

$$\nabla_\mu N^\mu = 0 \qquad \nabla_\mu F^{\mu\nu} = -J^\nu$$
$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu {}^*F^{\mu\nu} = 0$$
$$\nabla_\mu S^\mu \geq 0 \qquad J^\mu = qu^\mu + \sigma e^\mu$$

system of semi-hyperbolic differential equations

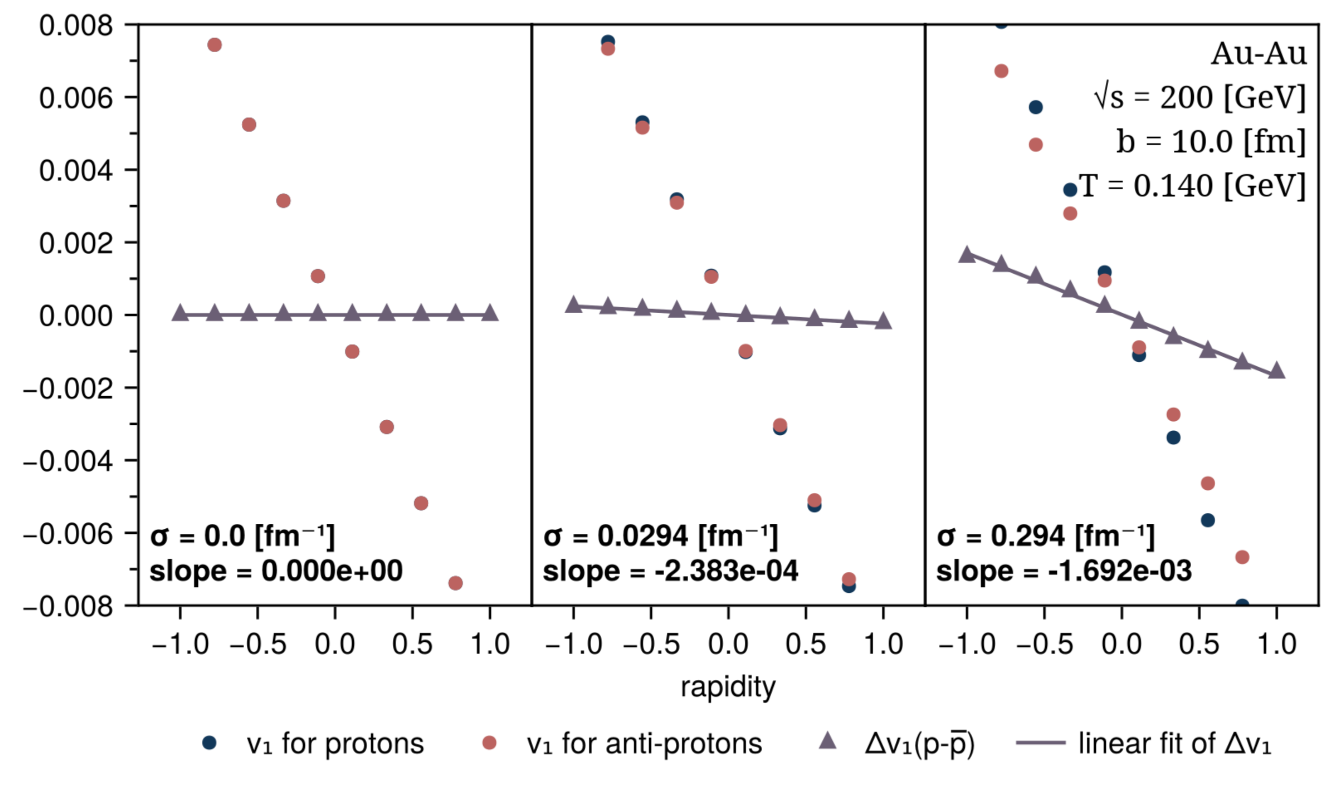
$$\longrightarrow \quad \partial_0[\sqrt{-g}\mathbf{U}] + \partial_i[\sqrt{-g}\mathbf{F}^i] = \sqrt{-g}(\mathbf{S}_e + \mathbf{S}_s)$$

conserved variables, fluxes, explicit source terms, stiff source terms

$$\mathrm{U} = \begin{pmatrix} D \\ \Pi_j \\ \epsilon \\ B^j \\ E^j \\ q \\ \vdots \end{pmatrix} \quad \mathrm{F}^i = \begin{pmatrix} Dv^i \\ T^i_j \\ \Pi^i \\ \epsilon^{jik}E_k + g^{ij}\psi \\ -\epsilon^{jik}B_k + g^{ij}\phi \\ J^i \\ \vdots \end{pmatrix} \quad \mathrm{S}_e = \begin{pmatrix} 0 \\ \frac{1}{2}T^{ik}\partial_j g_{ik} \\ -\frac{1}{2}T^{ik}\partial_0 g_{ik} \\ 0 \\ -qv^i \\ 0 \\ \vdots \end{pmatrix} \quad \mathrm{S}_s = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -J^i_c \\ 0 \\ \vdots \end{pmatrix}$$

Charged directed flow results

Our RRMHD model vs the STAR 2024 experimental results



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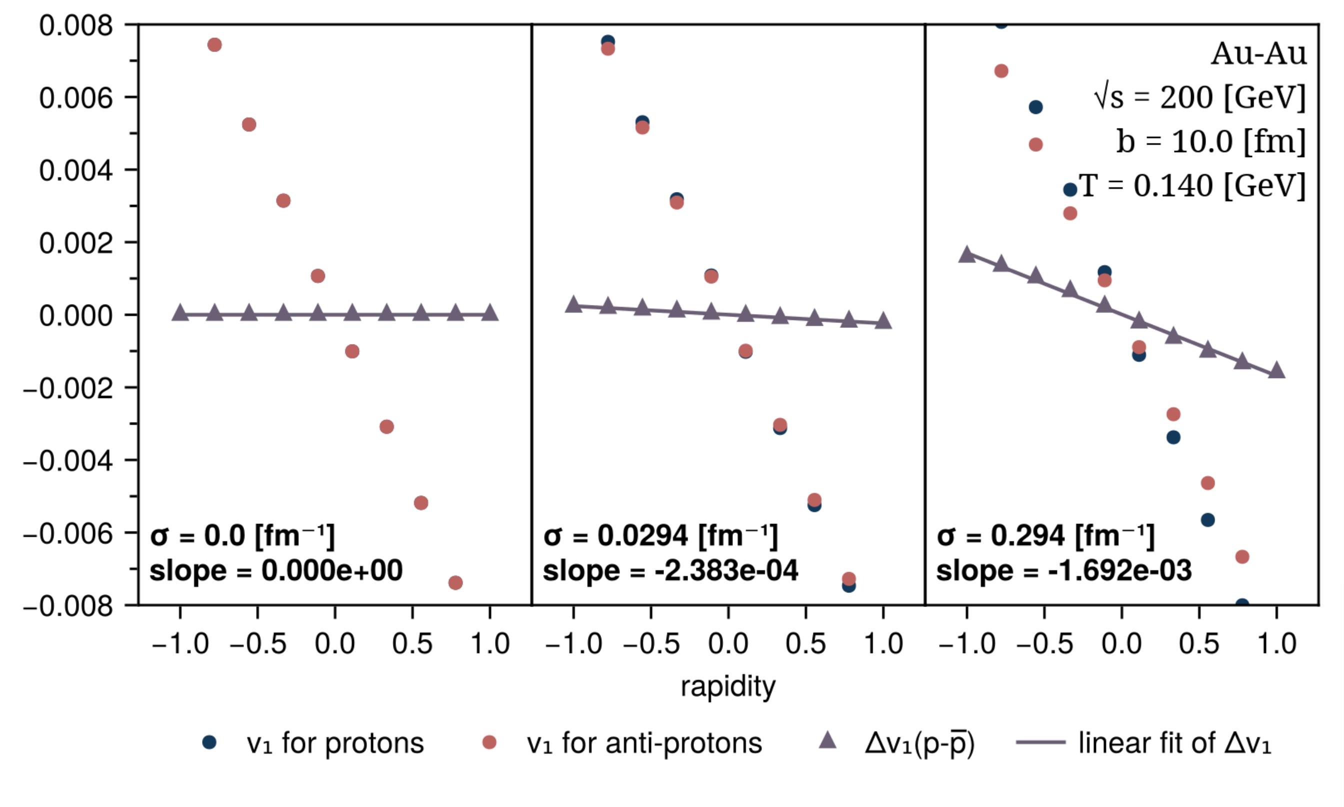

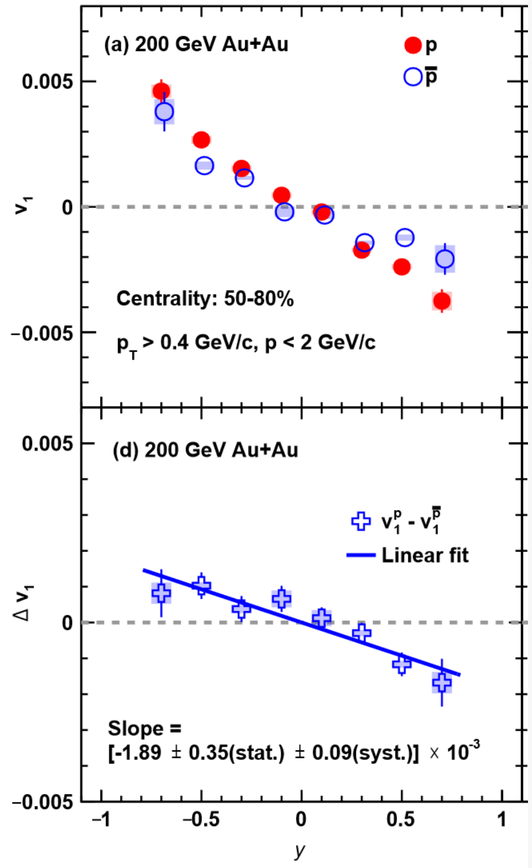

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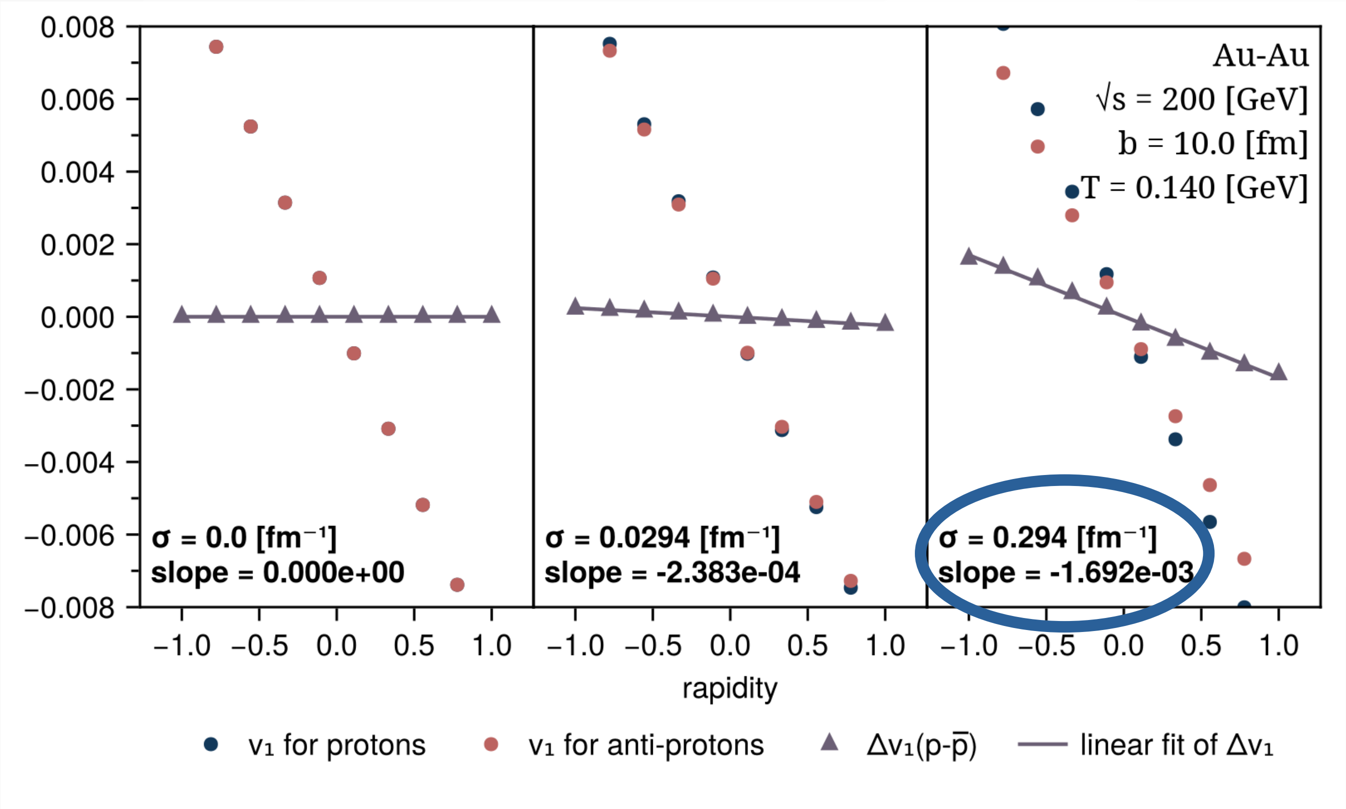
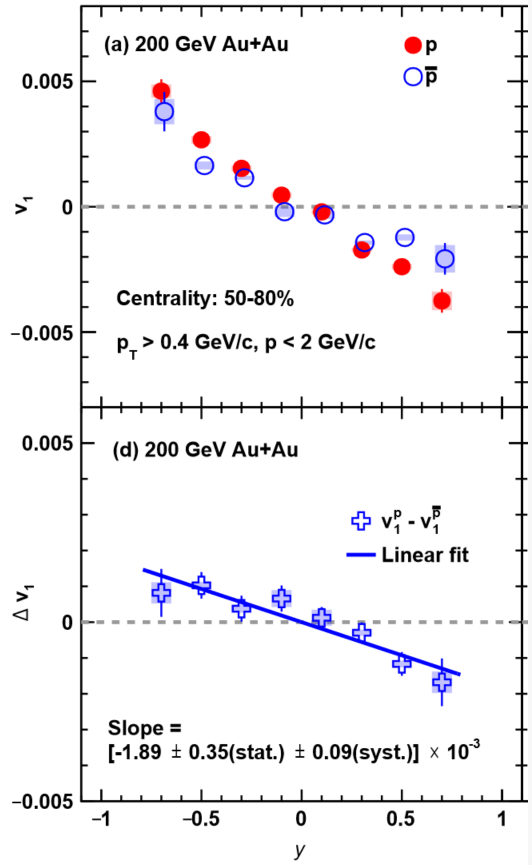




Summary

Used a hydrodynamic model to study Quark-Gluon plasma (QGP) in relativistic heavy-ion collisions

Worked with a new relativistic resistive magneto-hydrodynamics model

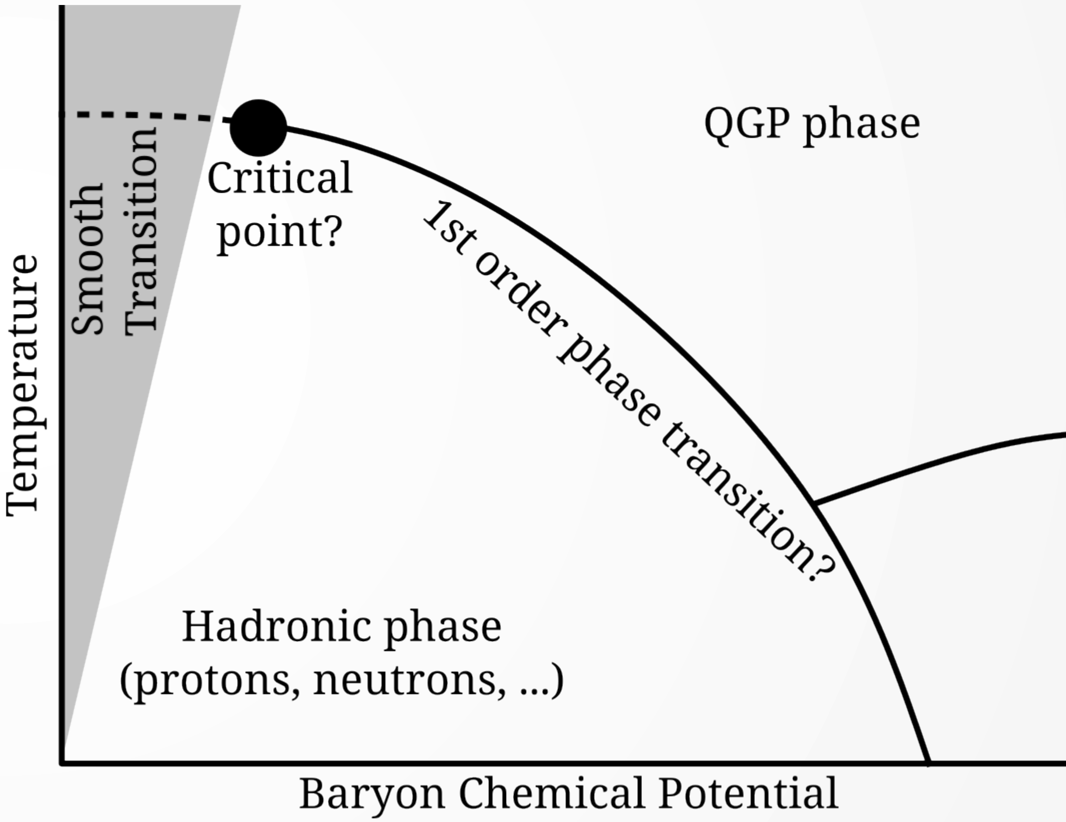
Discussed how we can use that model + the STAR data to estimate QGPs electric conductivity

Backups

Why study quark-gluon plasma (QGP)?

Quantum Chromodynamics (QCD)

- At high temperatures hadrons are melted into a strongly interacting quark-gluon plasma (QGP)
- In QGP quarks and gluons are a many-body system that acts with cohernet motion
- The motion is described using various transport parameters



Quark-gluon plasma (QGP) electric conductivity

What is QGP electric conductivity (σ)?

- Studied by lattice calculations (~10 papers), pQCD, and kinetic transport theories

Table 1 Details of the lattice QCD ensembles to compute the electrical conductivity. Fermion properties refer to sea quarks. Here a_τ and a_s denote the temporal and spatial lattice spacing respectively

Ref.	arXiv number	N_f (sea)	Fermion type	m_π [MeV]	a_τ [fm]	a_s/a_τ	Discretisation
[15]	hep-lat/0301006	0	Quenched	–	$a_\tau \to 0$	1	Continuum limit
[16]	hep-lat/0703008	0	Quenched	–	0.0488, 0.0203	1	Fixed cutoff
[17]	1012.4963	0	Quenched	–	$a_\tau \to 0$	1	Continuum limit
[18]	1112.4802	0	Quenched	–	0.015	1	Fixed scale
[19]	1212.4200	2	Wilson-clover	270	0.0486(4)(5)	1	Fixed cutoff
[20]	1307.6763	2 + 1	Wilson-clover	384(4)	0.0350(2)	3.5	Fixed scale
[21]	1412.6411	2 + 1	Wilson-clover	384(4)	0.0350(2)	3.5	Fixed scale
[22]	1512.07249	2	Wilson-clover	270	0.0486(4)(5)	1	Fixed scale
[23]	1604.06712	0	Quenched	–	$a_\tau \to 0$	1	Continuum limit
[24]	1910.08516	2 + 1	Staggered	134.2(6)	0.0618, 0.0493	1	Fixed cutoff

Review: G. Aarts and A. Nikolaev, Eur. Phys. J. A 57, 118 (2021); 2008.12326 [hep-lat]

Motivation of my research

Traditionally, the study of powerful magnetic fields has been limited to astrophysics

~ 10 T	Weak	MRI
~ 45 T		Strongest continuous magnetic fields in labs (China & USA)
10^5 ~ 10^8 T	Strong	Non-magnetar neutron stars
~10^9 T		EM fields become nonlinear (Schwinger limit)
~10^{13} T		Strongest magnetic pulsars
~10^{14} T		Possible field strength at 200 GeV RHIC collisions

Studying powerful magnetic fields in labs on earth allows for more control over testing and better accessibility

Magnetic fields in heavy-ion collisions

Collision system with the same two nuclei

- Electromagnetic fields are a superposition (summation) from all the protons in the nuclear fragments

$$E_{x,y} = \int d^3xq \frac{\vec{x}\sigma}{4(t-z/v)^2} e^{\frac{-\vec{x}^2\sigma}{4(t-z/v)}},$$

$$E_z = -\int d^3xq \frac{(t-z/v)-\vec{x}^2\sigma/4}{\gamma^2(t-z/v)^3} e^{\frac{-\vec{x}^2\sigma}{4(t-z/v)}}$$

$$B_{\text{fragment}} = v \times E$$

$$\mathcal{B}_{\text{total}} = B_{\text{fragment 1}} + B_{\text{fragment 2}}$$

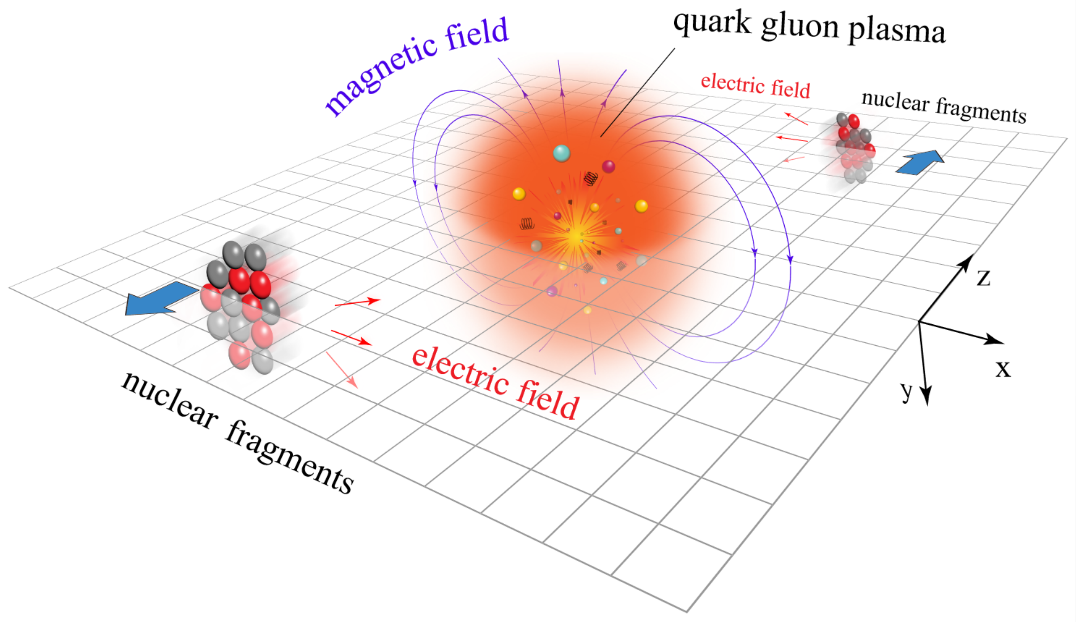
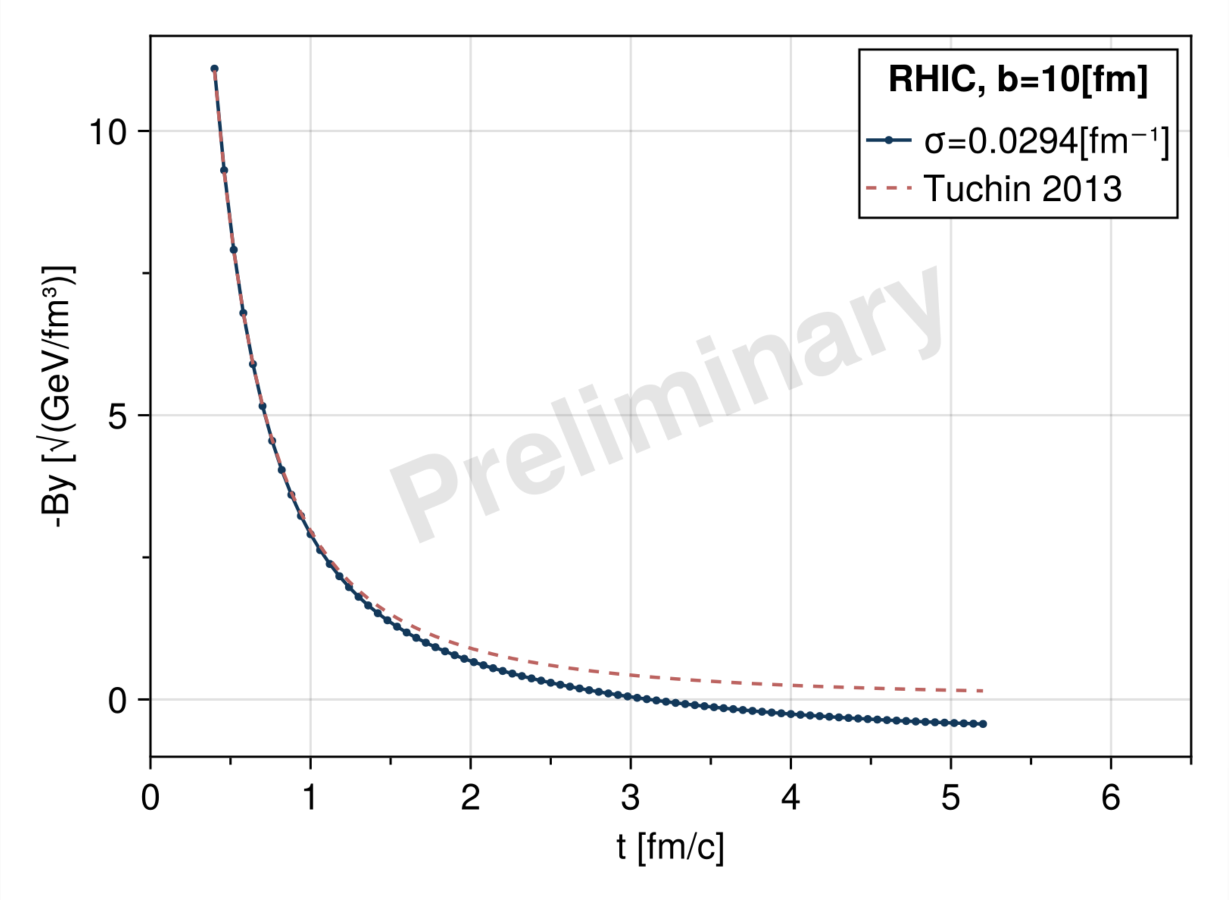


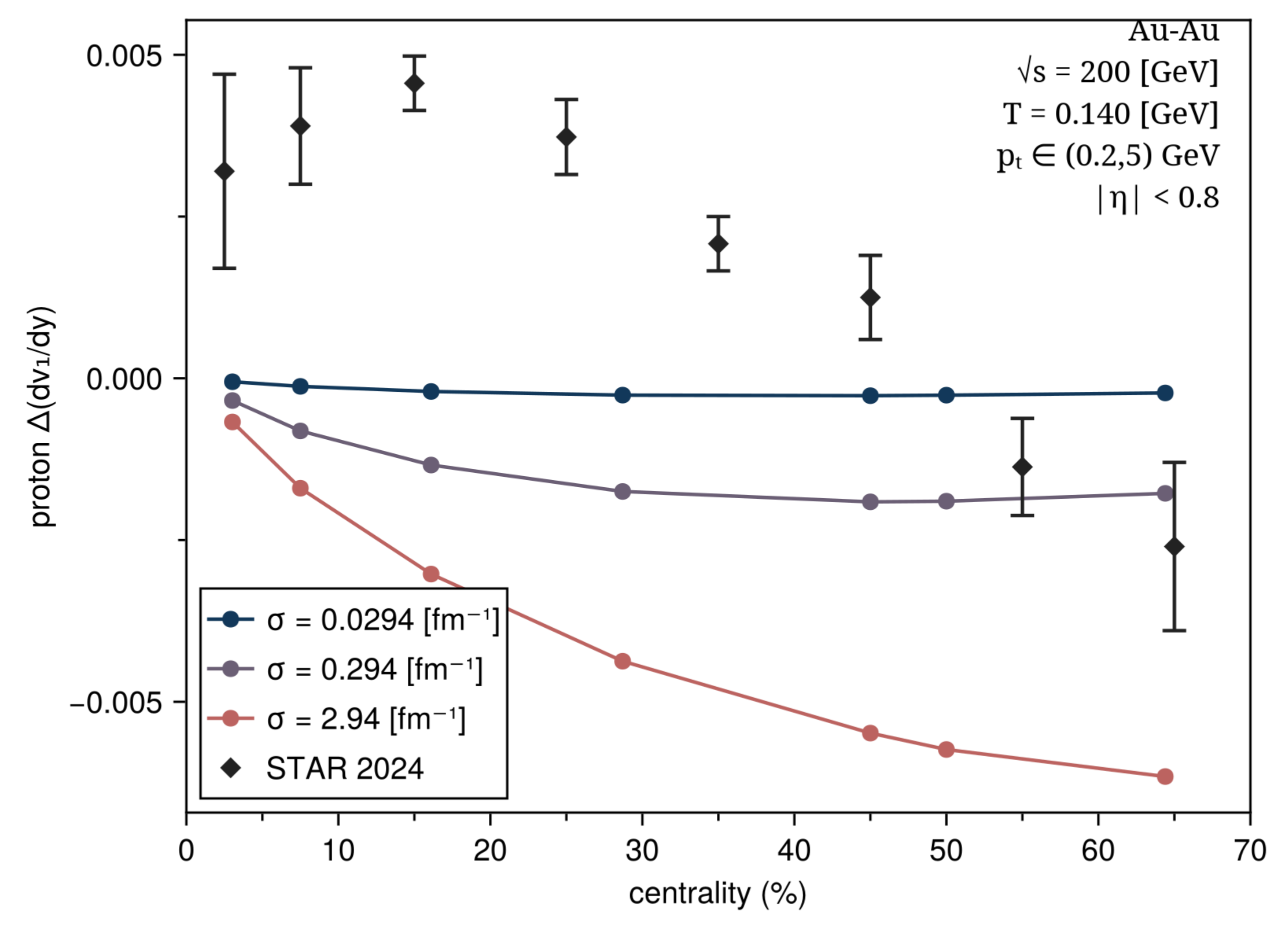
Image: STAR Collaboration (2024)

Time evolution of the magnetic field RRMHD



- Time evolution of the magnetic field at the center of the grid (x=0, y=0, \eta = 0) comparing our RRMHD model to an analytic calculation of the field generated by the collision fragments.
- The first point is equal, because the initial condition of the RRMHD model is the analytic solution.
- We attribute the differences between the models at later times to be due to the EM sources. The analytic solution only includes the ions as a source, whereas the RRMHD model only includes QGP charges as a source.

Centrality dependence of charged directed flow



Solving Relativistic Resistive Magneto-Hydrodynamics

These are difficult differential equations

- Require non-trivial initial conditions to solve (non-equilibrium)
- General analytic solutions are not known (semi-hyperbolic equations)
- We will use a curvilinear coordinate system (Milne coordinates)

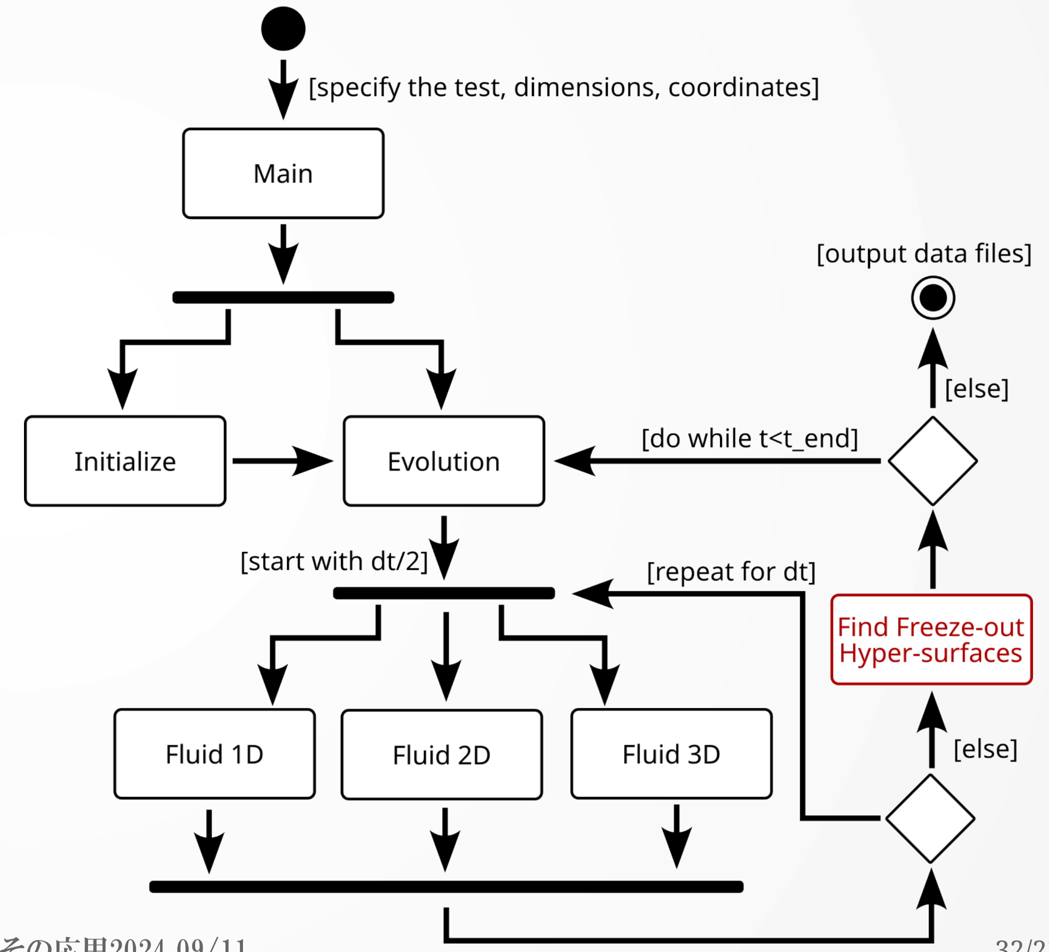
Solution

Use numerical methods from computational physics

Solving Relativistic Resistive Magneto-Hydrodynamics

Overview of the numerical model

- Create initial conditions
- Solve differential equations
  - Strang Splitting method
  - Runge-Kutta 2
  - primitive variable second order interpolation
- At freezeout, stop time evolution and output data
- Process data using Cooper-Frye method



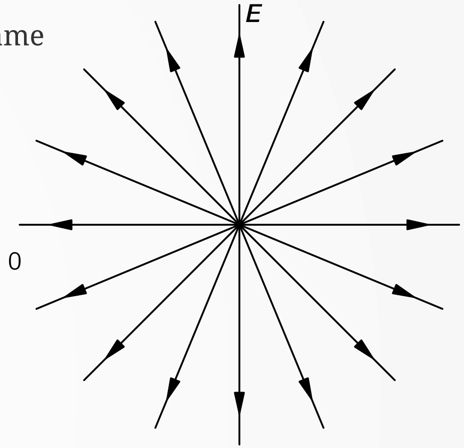
Magnetic fields of relativistic charges

Resources: Jackson (1975), Feynman (2010)

Charges moving at relativistic velocities

- Consider a point charge moving at a constant velocity in the z-direction
- In the lab frame, electric fields appear squished and stretched

Rest Frame



(a) $v = 0$

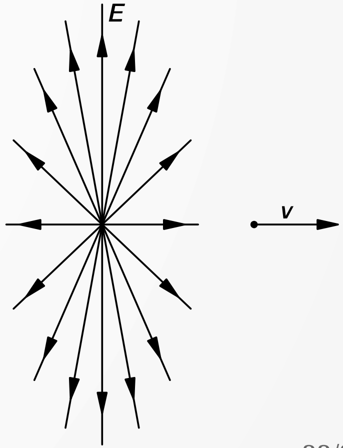
$$E_{x,y} = q\gamma \frac{\vec{x}}{[\vec{x}^2 + \gamma^2 (z - vt)^2]^{3/2}}, \qquad E_z = q\gamma \frac{z - vt}{[\vec{x}^2 + \gamma^2 (z - vt)^2]^{3/2}}$$

aka: Lienard-Wiechert Fields for a constant velocity

- Magnetic fields are formed around the moving charge (assume vacuum for all of these results)

$$\vec{B} = \vec{v} \times \vec{E}$$

Lab Frame



(b) $v = 0.9c$

Magnetic fields of relativistic charges

Relativistic charges in heavy-ion collisions

- Heavy nuclei are accelerated to relativistic velocities to study QCD
  - High-energy could be 200 GeV, ~2.56 TeV, ~6.78 TeV
  - Heavy nuclei = e.g., Au, Pb, Cu, U, Ru, Zr, …

- Near the collision point we can consider the nuclei to be moving with a constant velocity

Nuclei are positivly charged, so can we apply Lienard-Wiechert Fields?

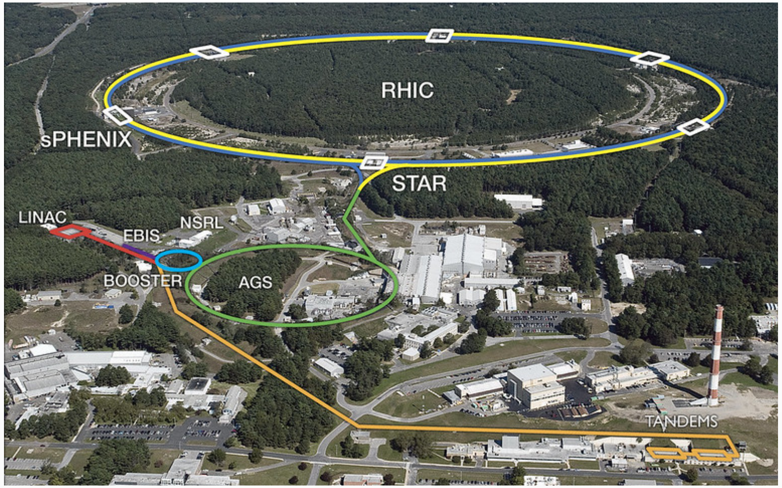


Image: Brookhaven National Lab (BNL)
https://www.bnl.gov/rhic/

Magnetic fields of relativistic charges

Unfortunately, nuclei are complicated

- Starting from a simple picture:
  - Treat protons inside the nuclei as point charges
  - Assume protons (charge) is equally distributed
  - Nuclei are Lorentz contracted, so all protons are perpendicular to the beam-line

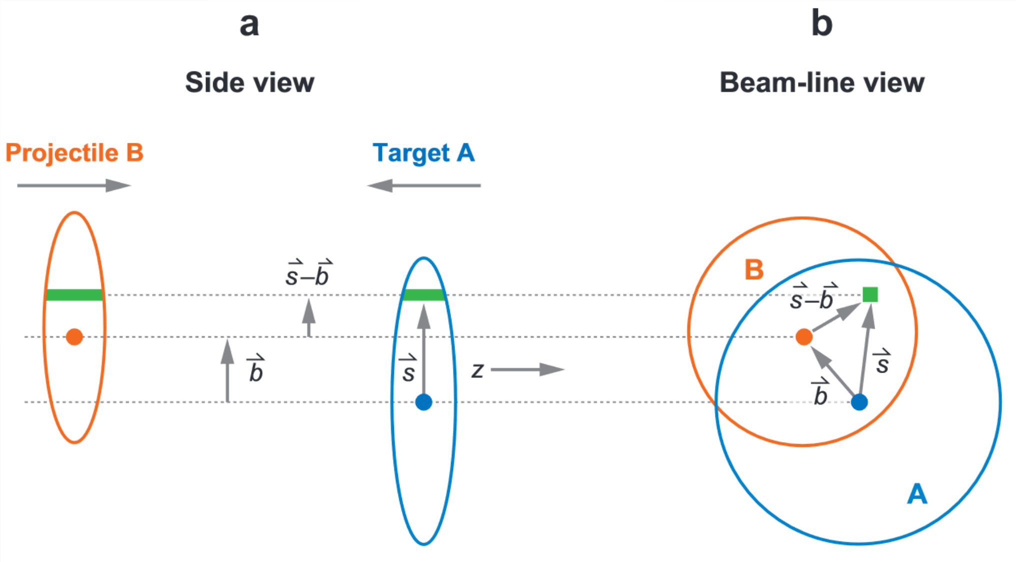


Image: M. L. Miller, K. Reygers, S. J. Sanders, P. Steinberg (2007)