1+1次元有限密度QCDのハミルトン形式による数値解析

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熱場の量子論 2024/9/11

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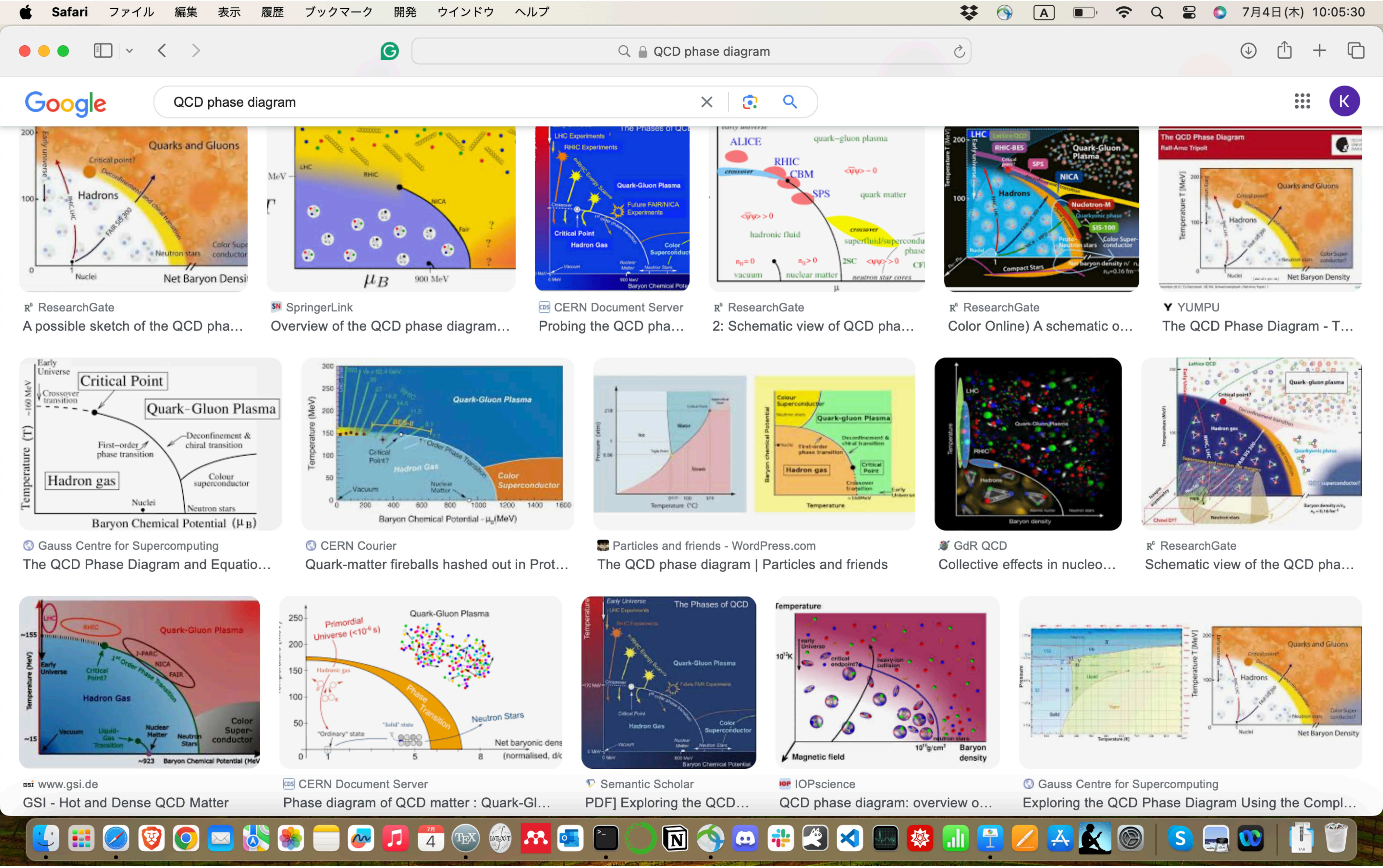

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Sign problem

- Monte Carlo simulation can not be applied to QCD at finite density.
  - Three-color QCD with isospin chemical potential:
    Kogut and Sinclair (2002)……
  - QCD at finite baryon density with $N_c=2$ and N_f=(even number):
    Hands, Kim and Skullerud (2006), Iida and Itou (2022), ……
- Finite density QCD
  - Interior of the Neutron stars See talk by T. Kojo
  - Compressed matter by the stopping in low-energy heavy ion collision experiment
- It is important to theoretically investigate QCD at finite density!

Hamiltonian formalism

- Lattice Hamiltonian on a spatial lattice, and time is continuous.

  Kogut and Susskind (1975)

- Minimization of the free energy without Monte Carlo → No sign problem
  - We use good variational ansatz and efficient algorithm for searching the ground state.

- There are many applications of Hamiltonian formalism.
  - SU(2) or SU(3) lattice gauge theory in (1+1) dim: Kuhn et.al., (2015); Silvi et.al., (2017); Banuls et.al., (2017); Sala et.al., (2018)
  - Schwinger model: Silvi et.al., (2017); Rigobello et.al., (2023)

- Numerical calculation of thermodynamic quantities 1+1 dim QCD in finite baryon chemical potential.
  - To our knowledge, comprehensive study of dense QCD_2 has not been done yet.

Matrix Product State and Density Matrix Renormalization Group

- **Matrix Product State (MPS):** Perez-Garcia, et.al., (2007) ; McCulloch, et.al., (2007)
  - This is frequently used for quantum spin systems in 1+1 dimension

- **Density Matrix Renormalization Group (DMRG)** White (1992)
  - Efficient algorithm for searching the ground state represented by MPS

- iTensor = A wonderful package that performs MPS and DMRG!

  https://itensor.github.io/ITensors.jl/stable/ITensorType.html

  Also see reference by Fishman, White and Stoudenmire (2022)

Hamiltonian Formalism for Lattice gauge theory in (1+1)D

- **Kogut-Susskind Hamiltonian:** $H = H_{\rm E} + H_{\rm hop} + H_{\rm m}$ Kogut and Susskind (1975)

$$H_{\rm E} = \frac{a g_0^2}{2} \sum_{n=1}^{N-1} E_i^2(n),$$

$$H_{\rm hop} = \frac{1}{2a} \sum_{n=1}^{N-1} \left(\chi^\dagger(n+1) U(n) \chi(n) + \chi^\dagger(n) U^\dagger(n) \chi(n+1) \right),$$

$$H_{\rm m} = m_0 \sum_{n=1}^{N} (-1)^n \chi^\dagger(n) \chi(n)$$

- **Quark (Baryon) number:** $N_{\rm q} = \sum_{n=1}^{N} \chi^\dagger(n) \chi(n)$

- **Full Hamiltonian:** $H_{\rm tot} \equiv H_{\rm E} + H_{\rm hop} + H_{\rm m} - \mu_{0\rm B} N_{\rm B}$

Dimensionless Hamiltonian

- **Full Hamiltonian w/ n_B normalized by g_0:** $\tilde{H}_{\rm tot} \equiv H_{\rm tot}/g_0$

$$H_{\rm E}/g_0 = J \sum_{n=1}^{N-1} E_i^2(n) \equiv \tilde{H}_{\rm E},$$

$$H_{\rm hop}/g_0 = \epsilon \sum_{n=1}^{N-1} \left(\chi^\dagger(n+1)U(n)\chi(n) + \chi^\dagger(n)U^\dagger(n)\chi(n+1)\right) \equiv \tilde{H}_{\rm hop},$$

$$H_{\rm m}/g_0 = m \sum_{n=1}^{N} (-1)^n \chi^\dagger(n)\chi(n) \equiv \tilde{H}_{\rm m},$$

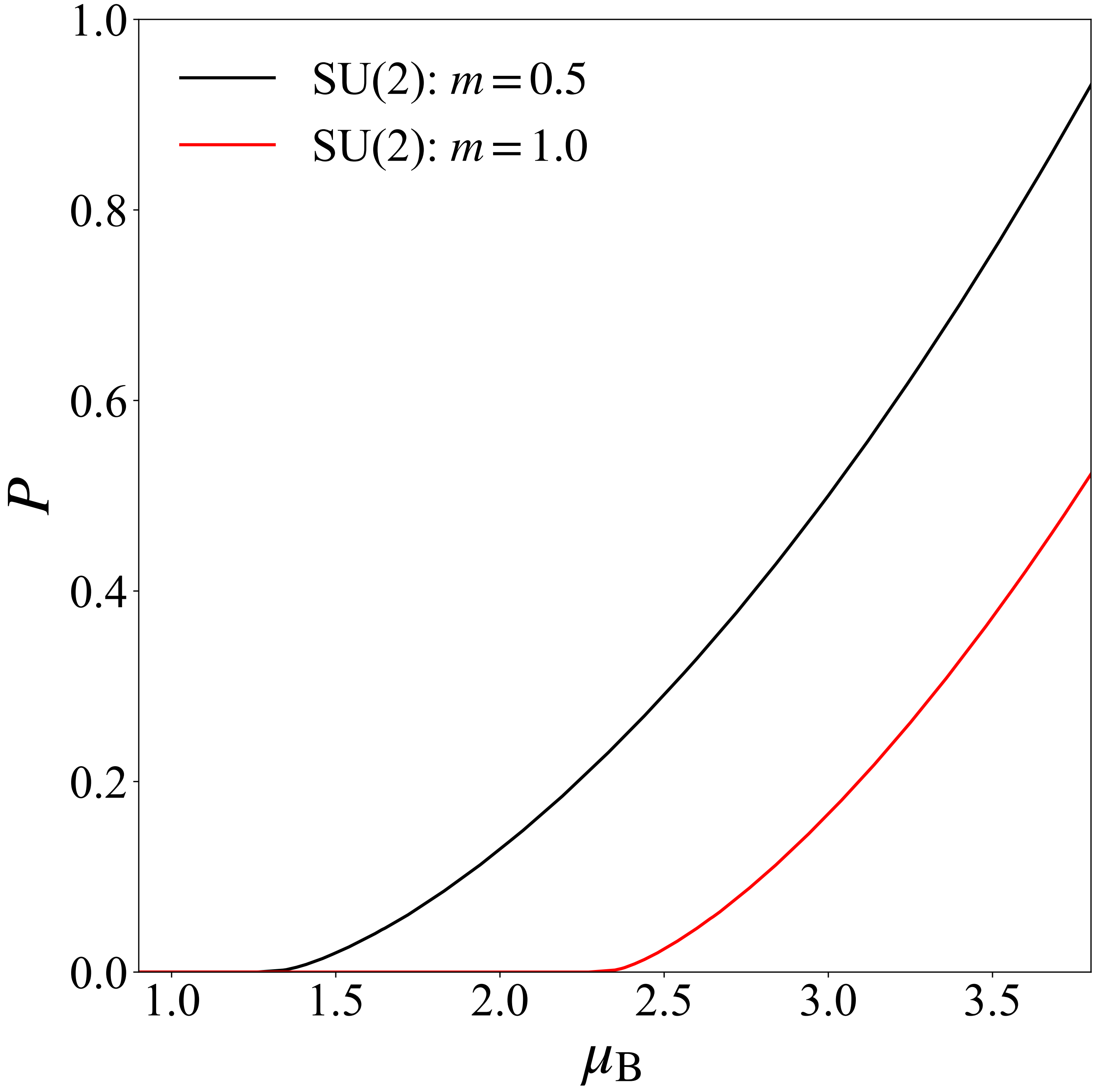
$$\mu_{0\rm B} N_{\rm B}/g_0 = \mu_{\rm B} \frac{1}{N_{\rm c}} \sum_{n=1}^{N} \chi^\dagger(n)\chi(n) \equiv \mu_{\rm B} N_{\rm B}.$$

- **Parameters:** $J = ag_0/2$, $\epsilon = 1/(2ag_0)$, $m = m_0/g_0$, $\mu = \mu_0/g_0$

- **Since I can solve Gauss law constraint in open boundary condition, U(n) can be eliminated and represented by Staggered fermions.**
- **After Wigner-Jordan transformation, our Hamiltonian is represented by spins, or Pauli matrices.**

SU(2) results

Pressure



Hayata, KN and Hidaka, (2024)

- **N=160, N_c=2, and w=2.**
- **Grand potential and Pressure:**

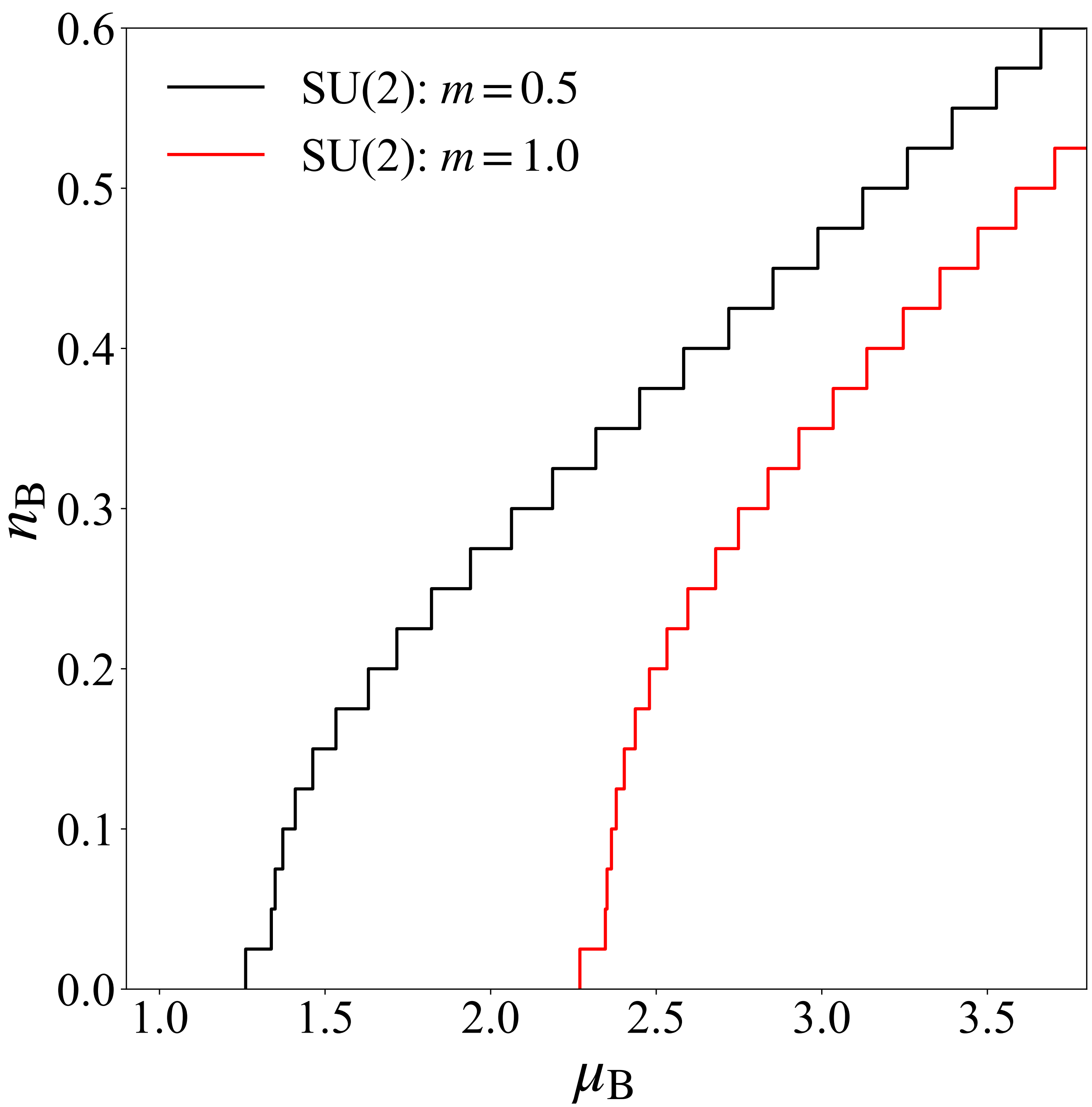
$$J = \langle \mathrm{GS}|(H - \mu_\mathrm{B} N_\mathrm{B})|\mathrm{GS}\rangle$$

$$P = -J/V$$

- **First derivative = baryon number:**

$$n_\mathrm{B} = \frac{\mathrm{d}P(\mu_\mathrm{B})}{\mathrm{d}\mu_\mathrm{B}} = \frac{1}{V}\langle N_\mathrm{B}\rangle$$

Baryon number



Hayata, KN and Hidaka, (2024)

- **Threshold value:**

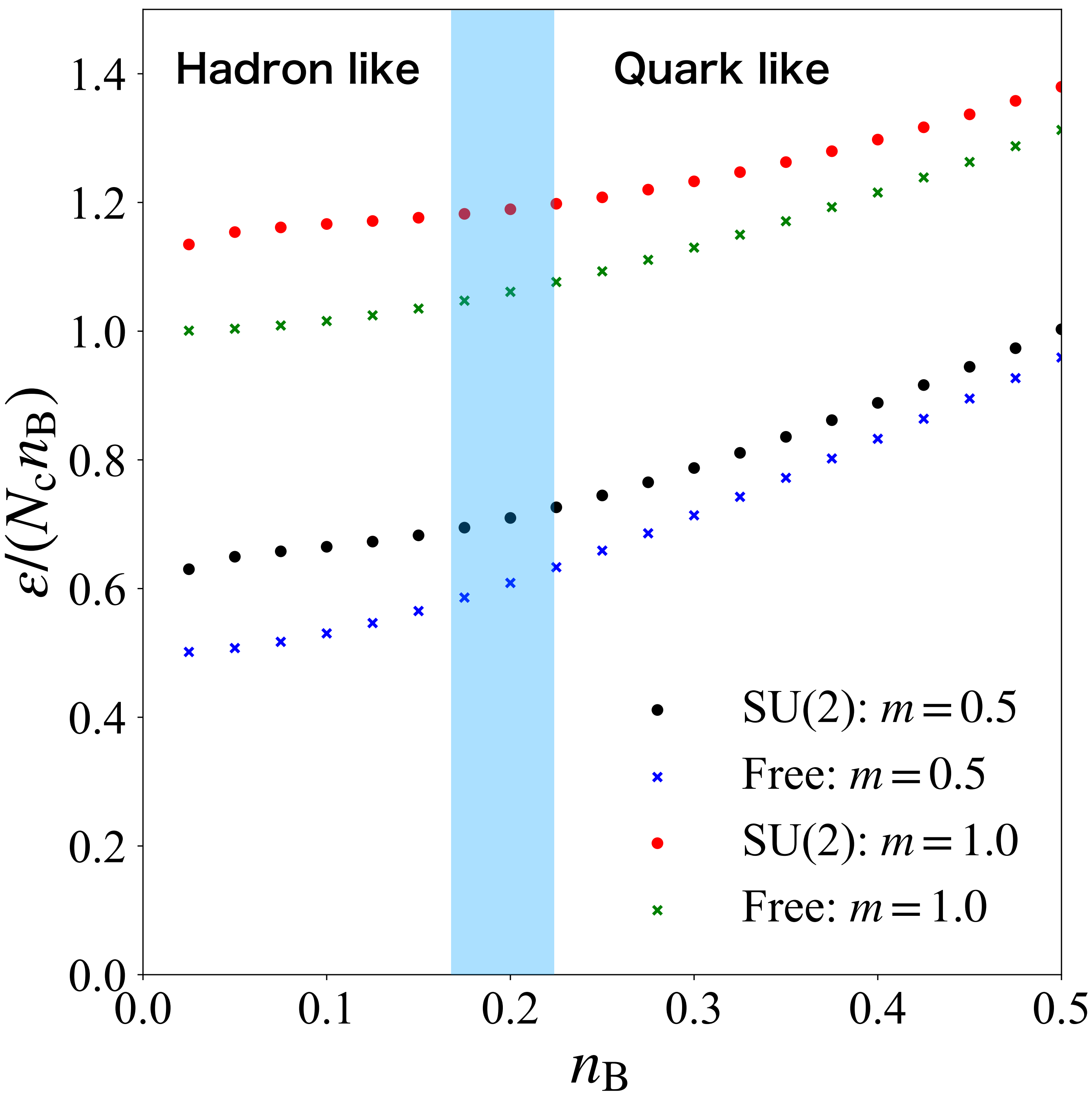
$$\mu_c = \begin{cases} 1.26 > 2 \times 0.5 & (m = 0.5) \\ 2.27 > 2 \times 1.0 & (m = 1.0) \end{cases}$$

- **At high density $g_0/\mu \ll 1$ and $m/\mu \ll 1$, the contribution from g_0 and m can be negligible→Free-quark!**

$$n_B = N_c \mu/\pi = \mu_B/\pi$$

- **How does it behave at low density?**

From hadron to quark



Hayata, KN and Hidaka, (2024)

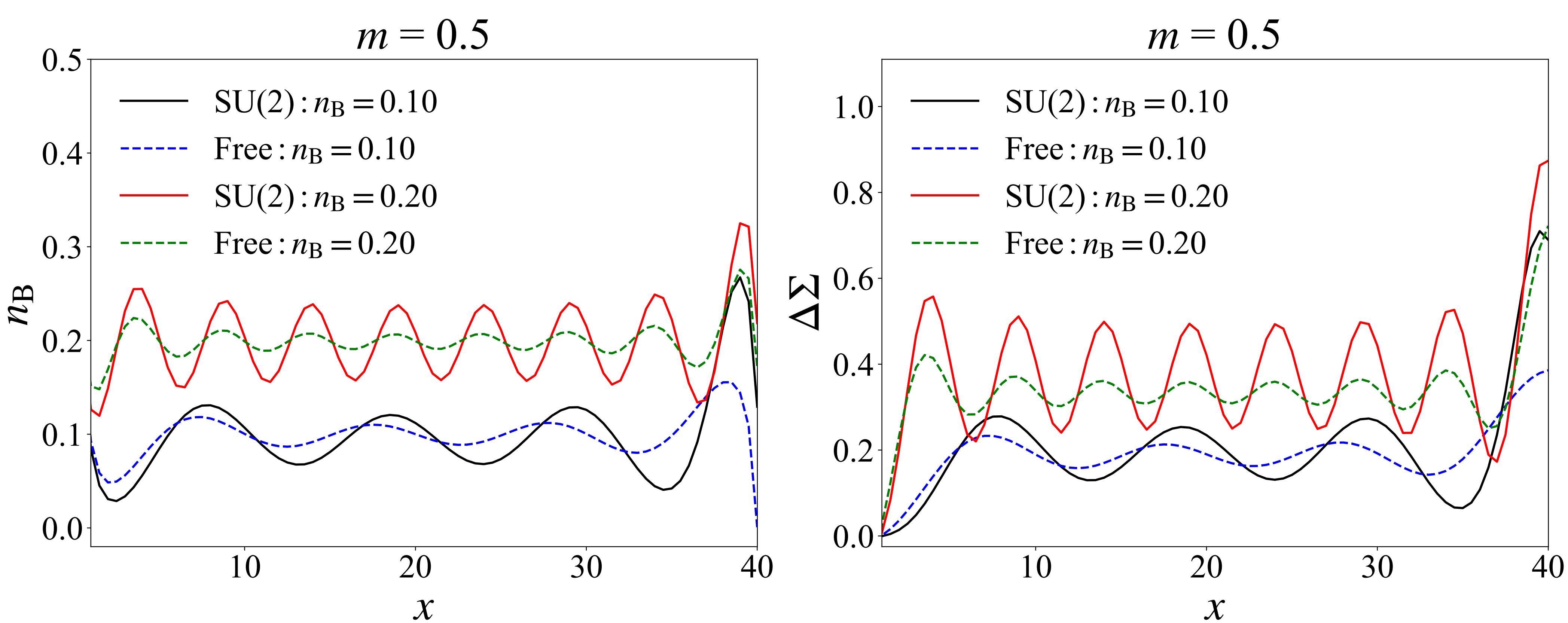
- Energy per one quark
- At high density, ε/n_q behaves like the free theory.
- At low density, ε/n_q is larger due to the confining energy.
- The behavior changes to free theory around n_B~0.2.
- If interaction can be negligible, the baryons will degenerate to the lowest energy state.

$$\epsilon/(N_c n_B) = \mathrm{const}$$

Inhomogeneous phase (m=0.5)

- **Can the inhomogeneous state realize beyond mean field approximation?**
  - Gross-Neveu, chiral Gross-Neveu, and QCD in (1+1) dim w/ Mean field approximation:
    Schon and Thies, (2000); Thies and Urlichs, (2003); Kojo, (2012)

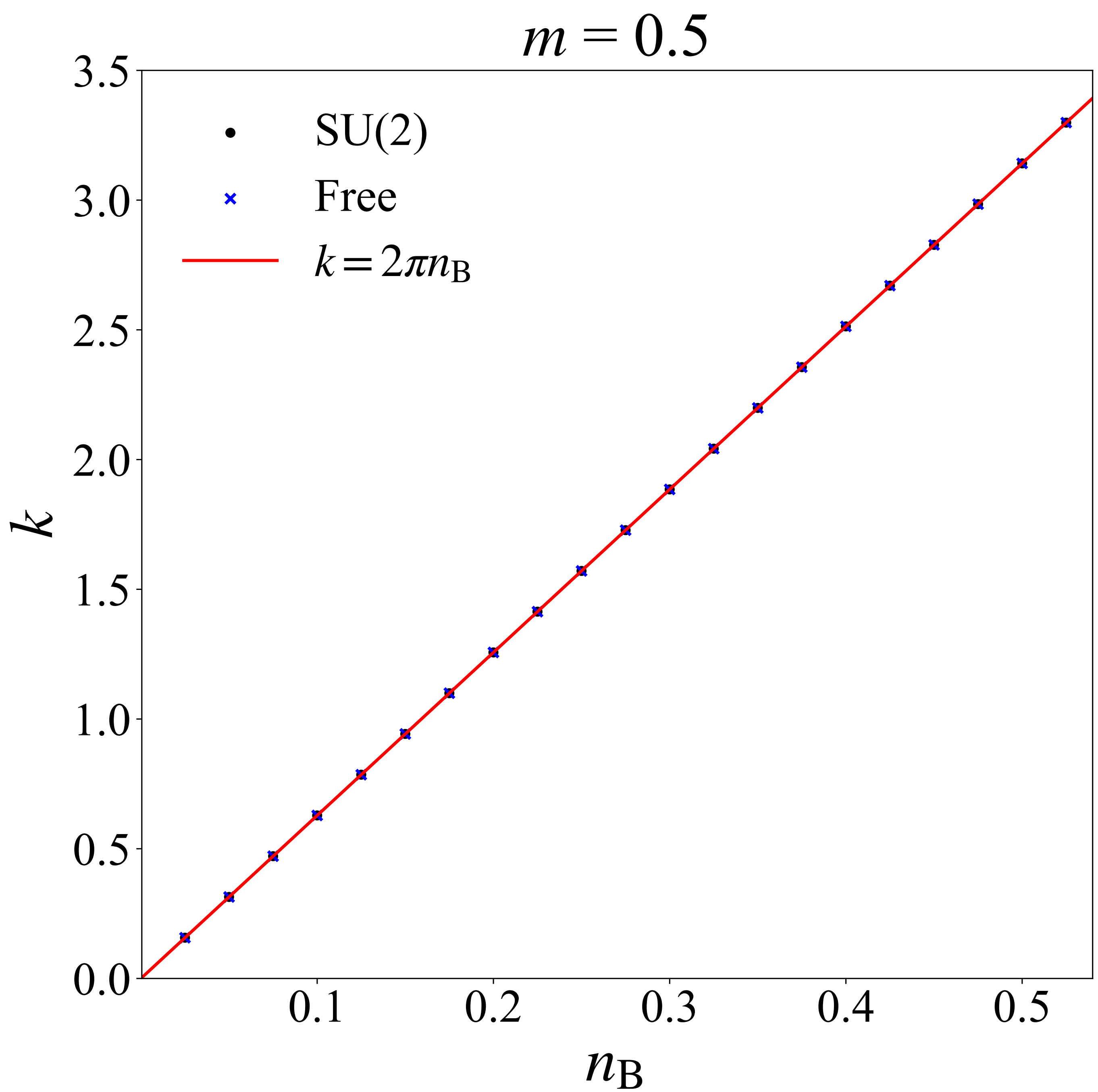
- **SSB of continuous symmetry is prohibited.**
  - The spatial modulation is induced by the open boundary condition.



- **In free theory, the modulations vanish proportionally to 1/L.**

Wave number (period)

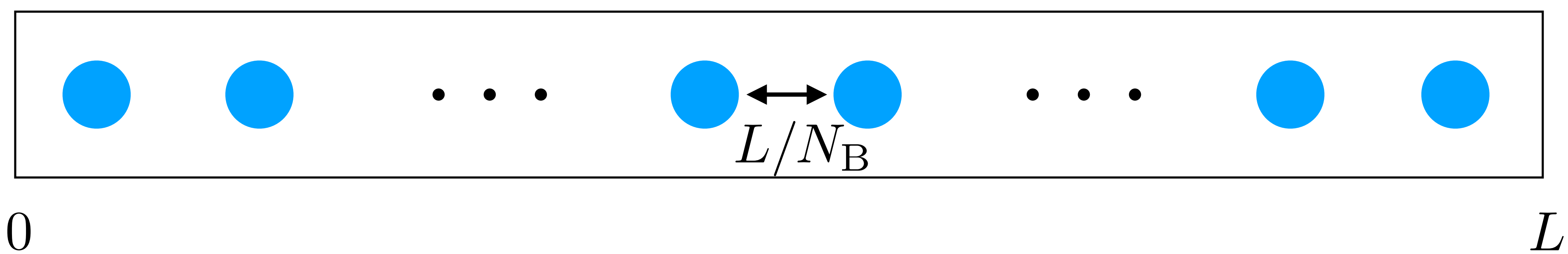
- Fourier transform on $n_B(x)$ and wave number of the largest modulation.



Baryon crystal and chiral density wave

- If I assume repulsive force, the baryons are periodically aligned.

  cf) Bosonization with large-N_c expansion Kojo, (2012)



- $k=2\pi/(\text{period})=2\pi/(L/N_B)=2\pi n_B$

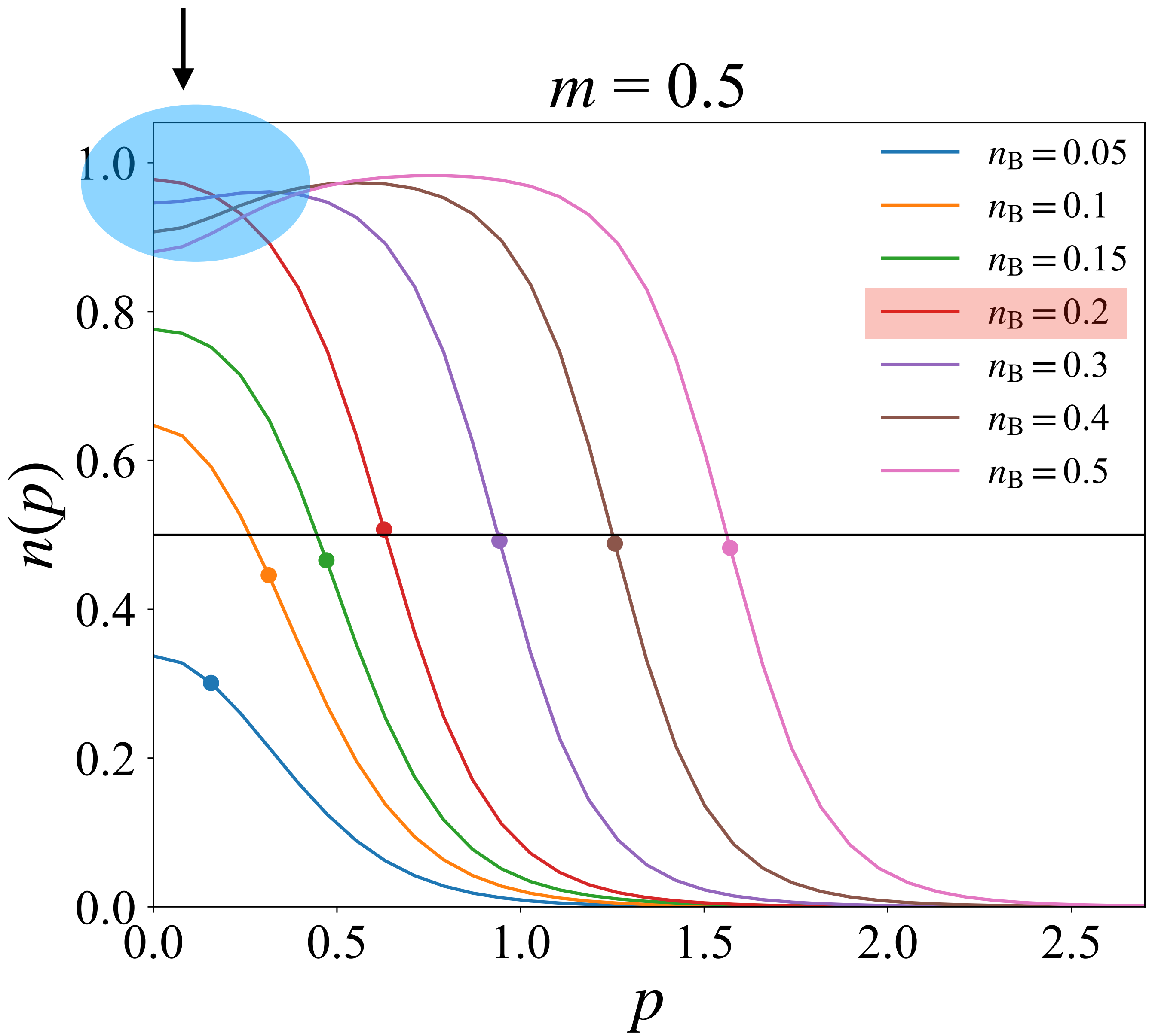
- Due to the Peierls instability, the particle and hole form the condensate.

- Both of the particle and hole have p_F, then the total momentum is $2p_F$.

- $k=2p_F=2\pi n_B$ - In (1+1) dim, the fermi momentum is πn_B.

Distribution function

This depression is a lattice artifact.



- **N=240, w=2.0**
  - to reduce the finite volume effect

- **As n_B increases, maximum of n(p) increases and forms Fermi sea.**

cf) Quarkyonic matter in (3+1) dim
BCS-BEC crossover of ultracold atom gases
Kojo, (2012); Astrakharchik et.al., (2005); Regal et.al., (2005)

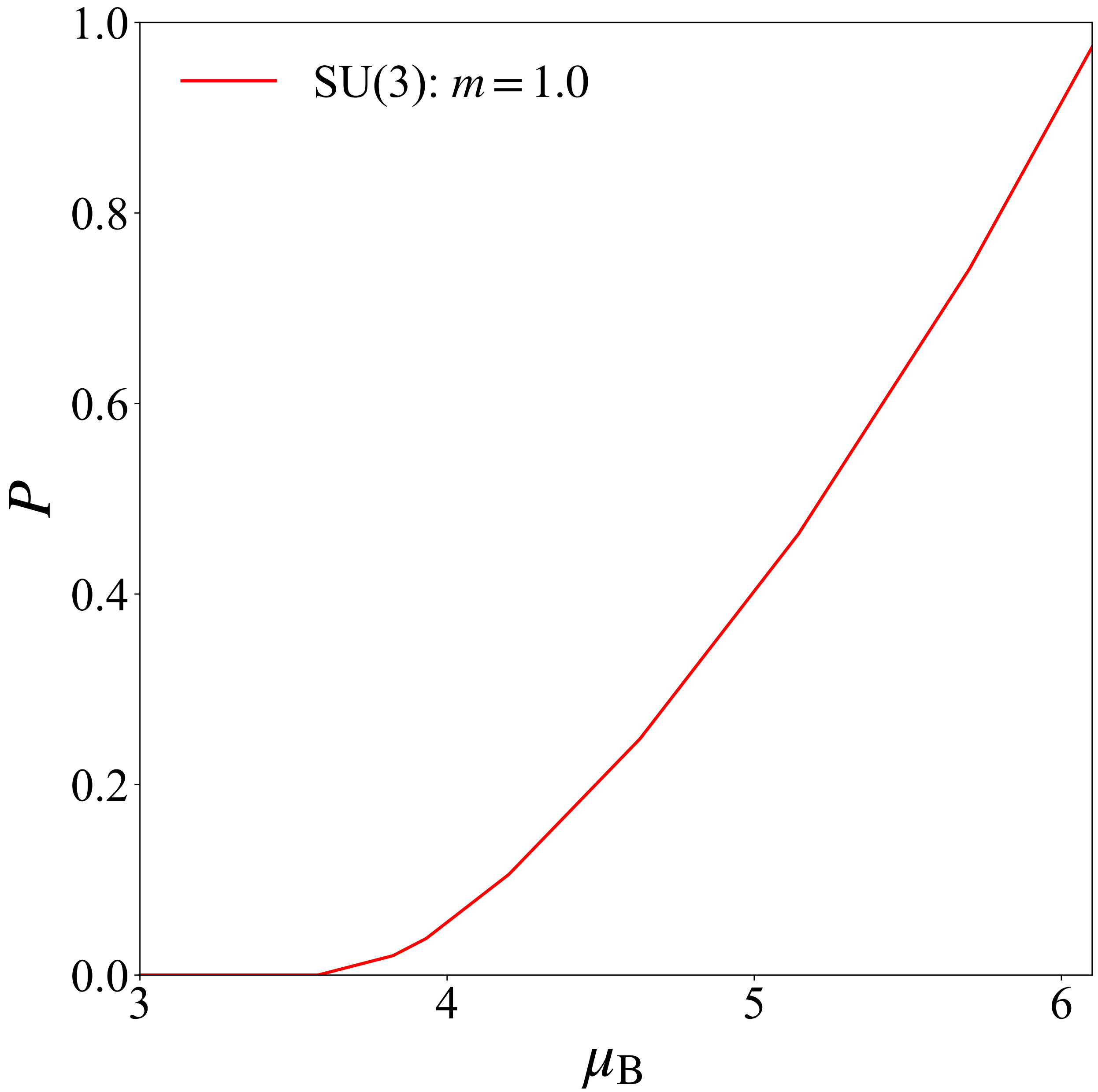
- **Fermi sea forms near n_B~0.2, which is consistent with behavior of ε / n_q**

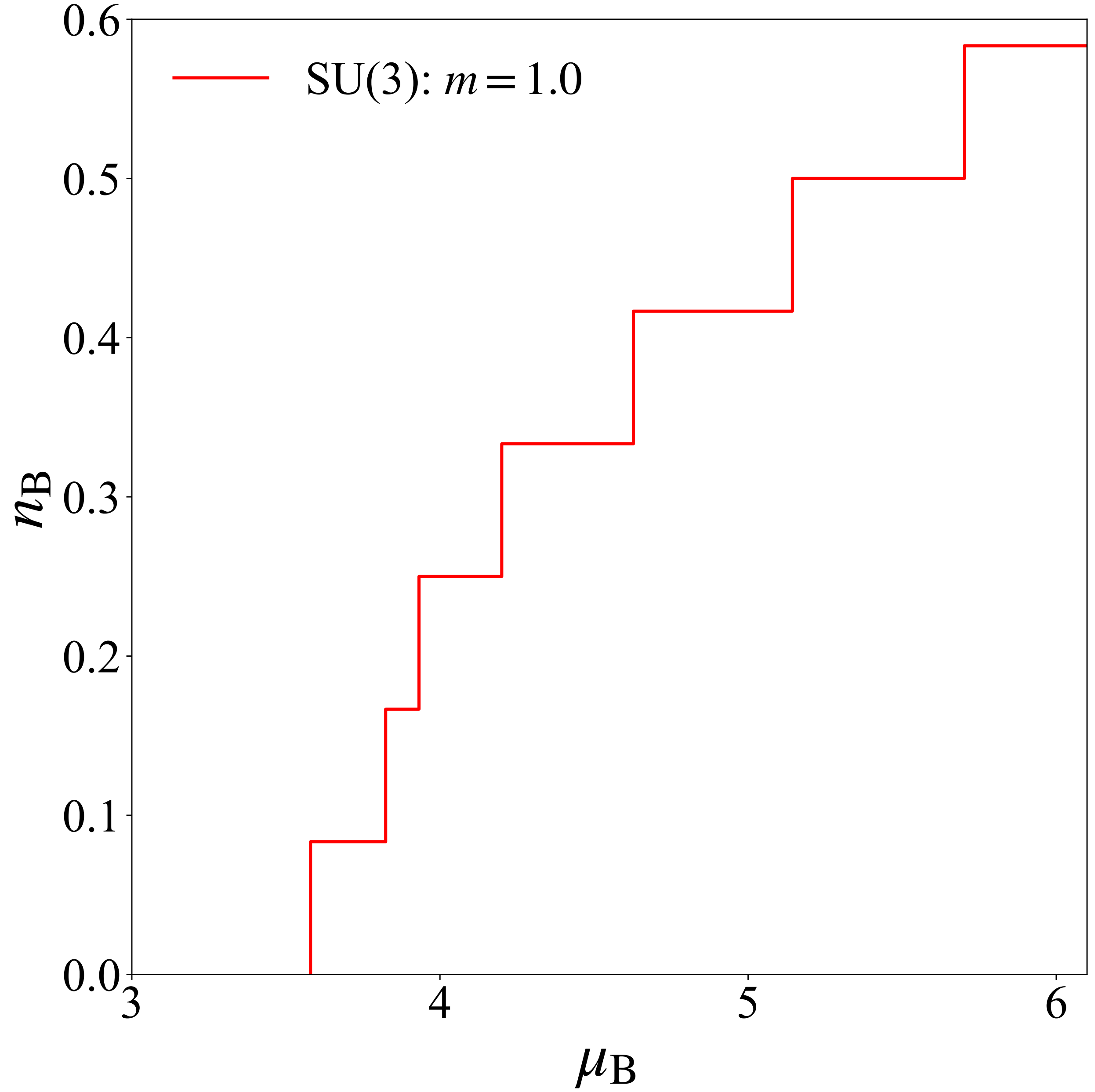
- **Fermi surface is smooth, not sharp.**
  →Instability of the Fermi surface

SU(3) results

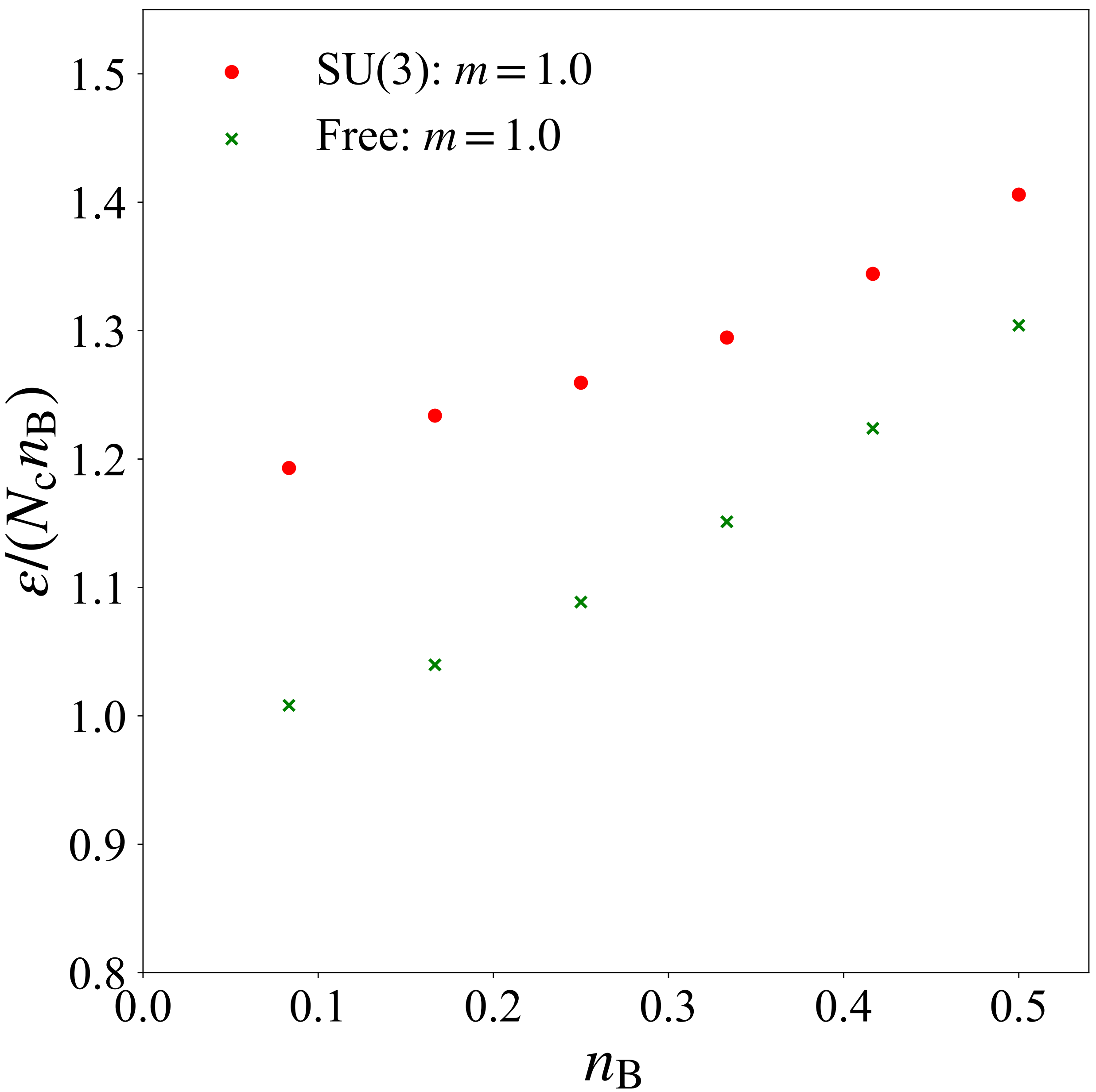
Pressure and Baryon number

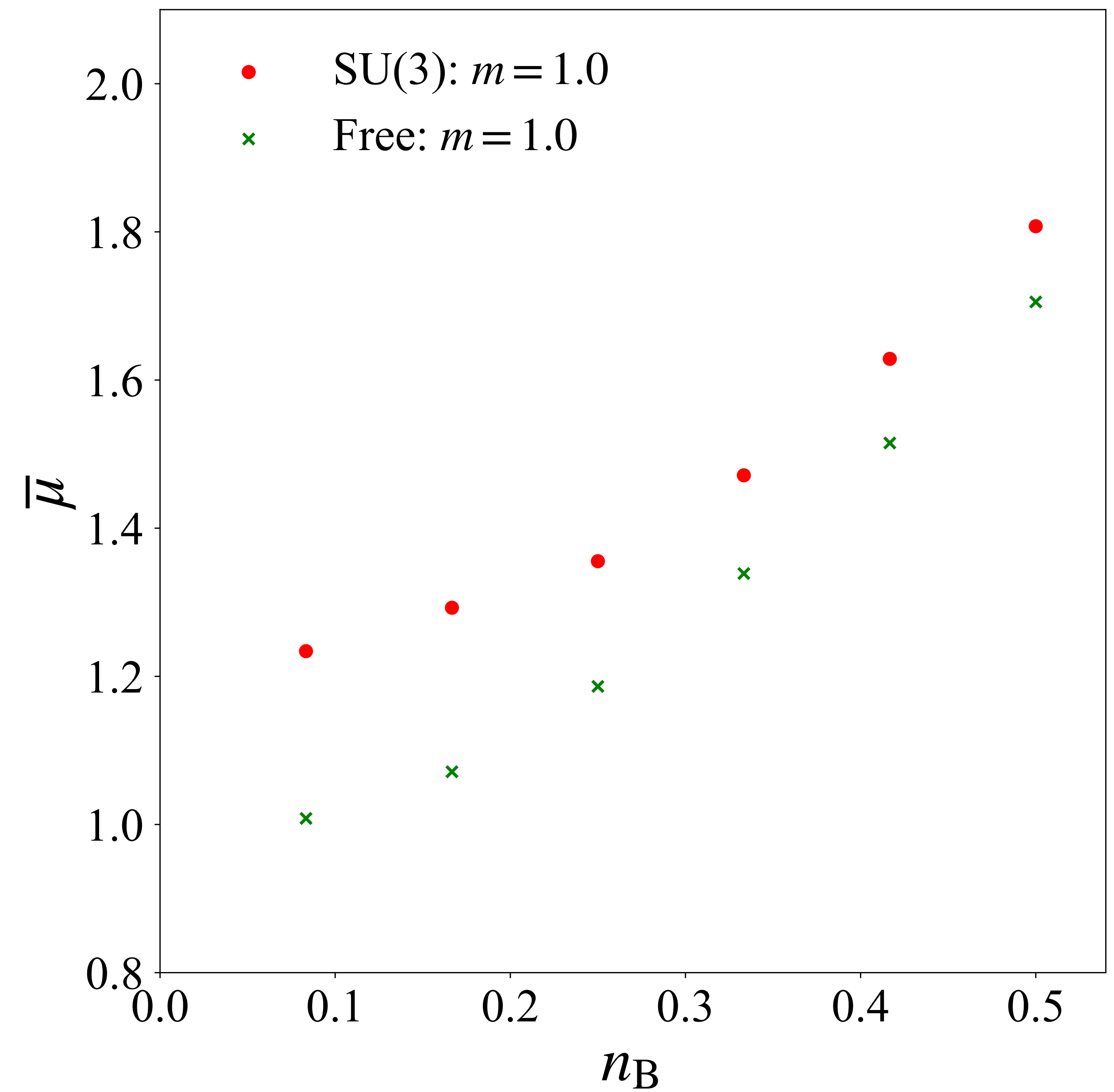
- N=48, w=2. Threshold value is 3.58.



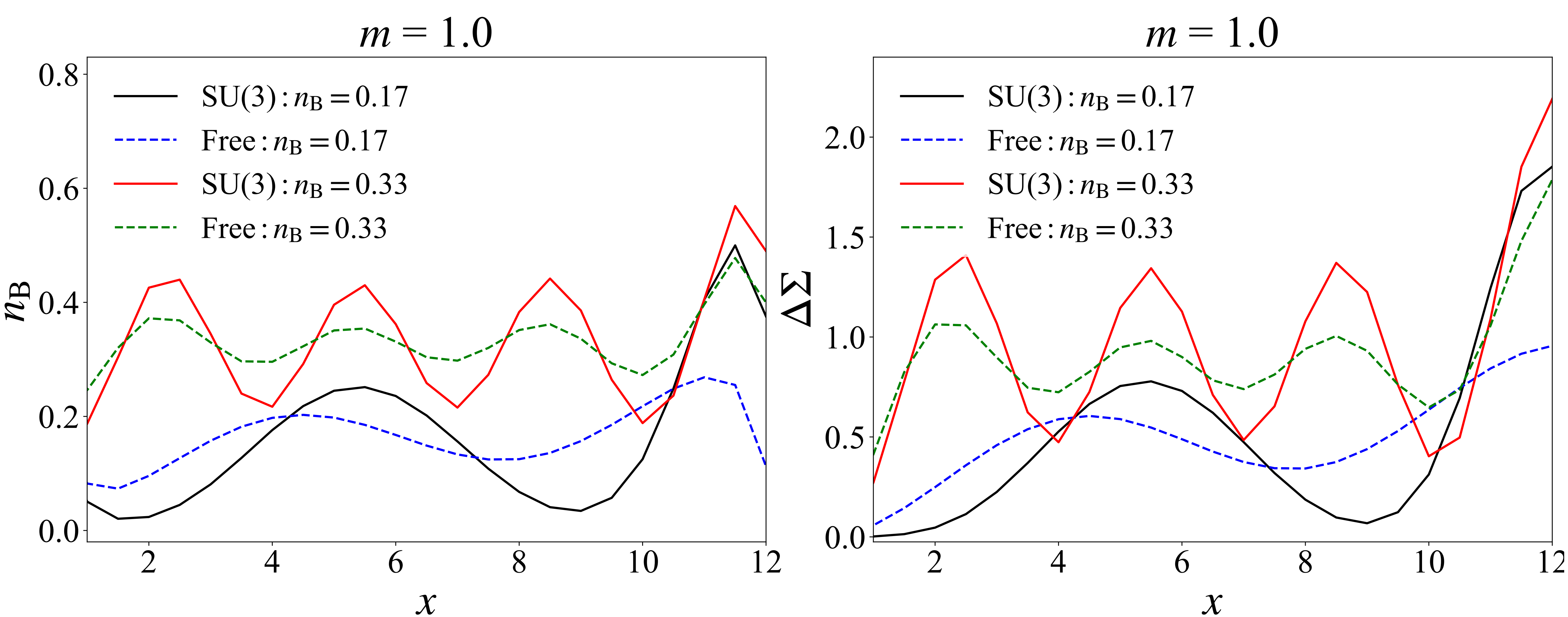


Average of μ_B and ratio

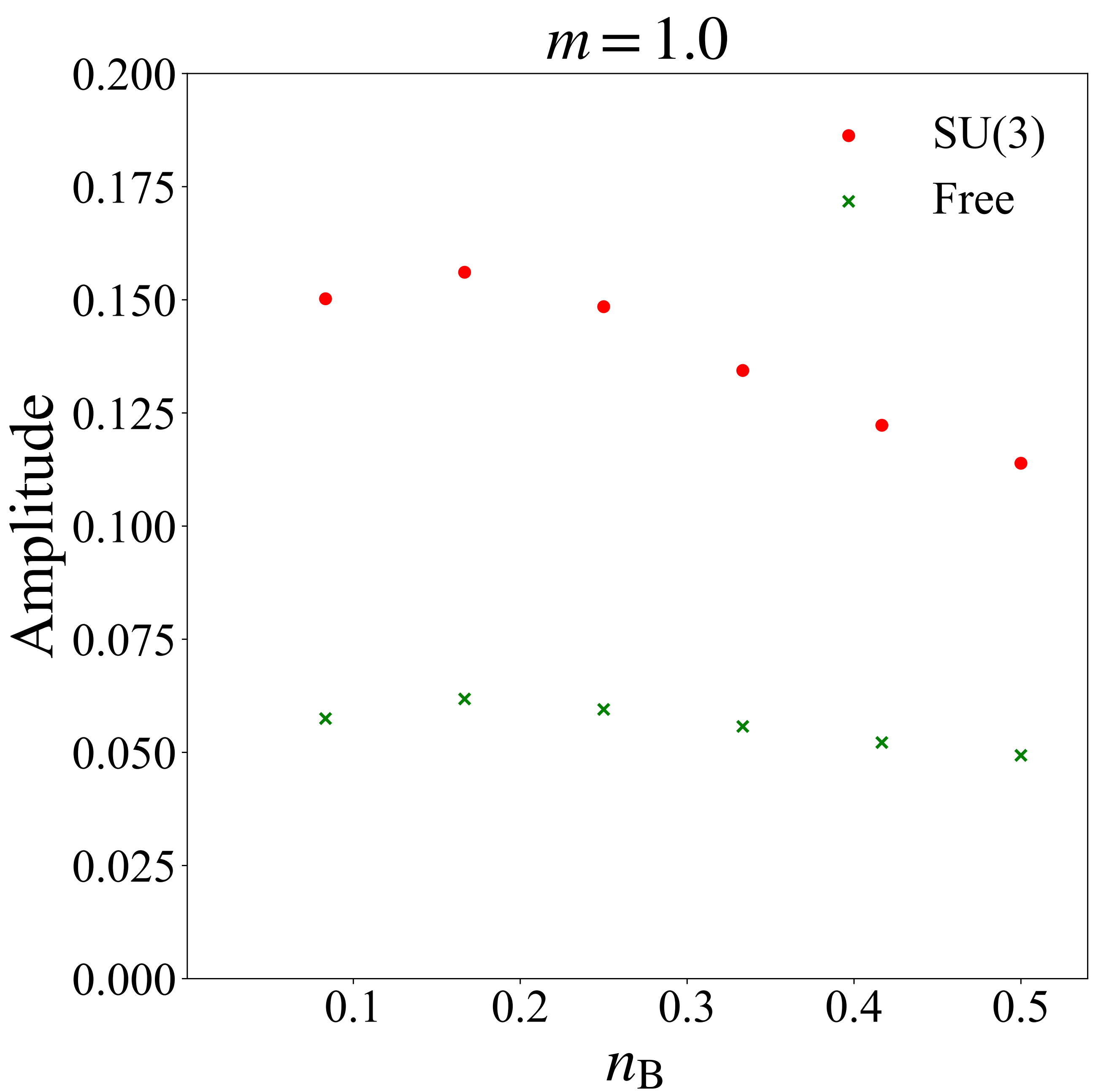


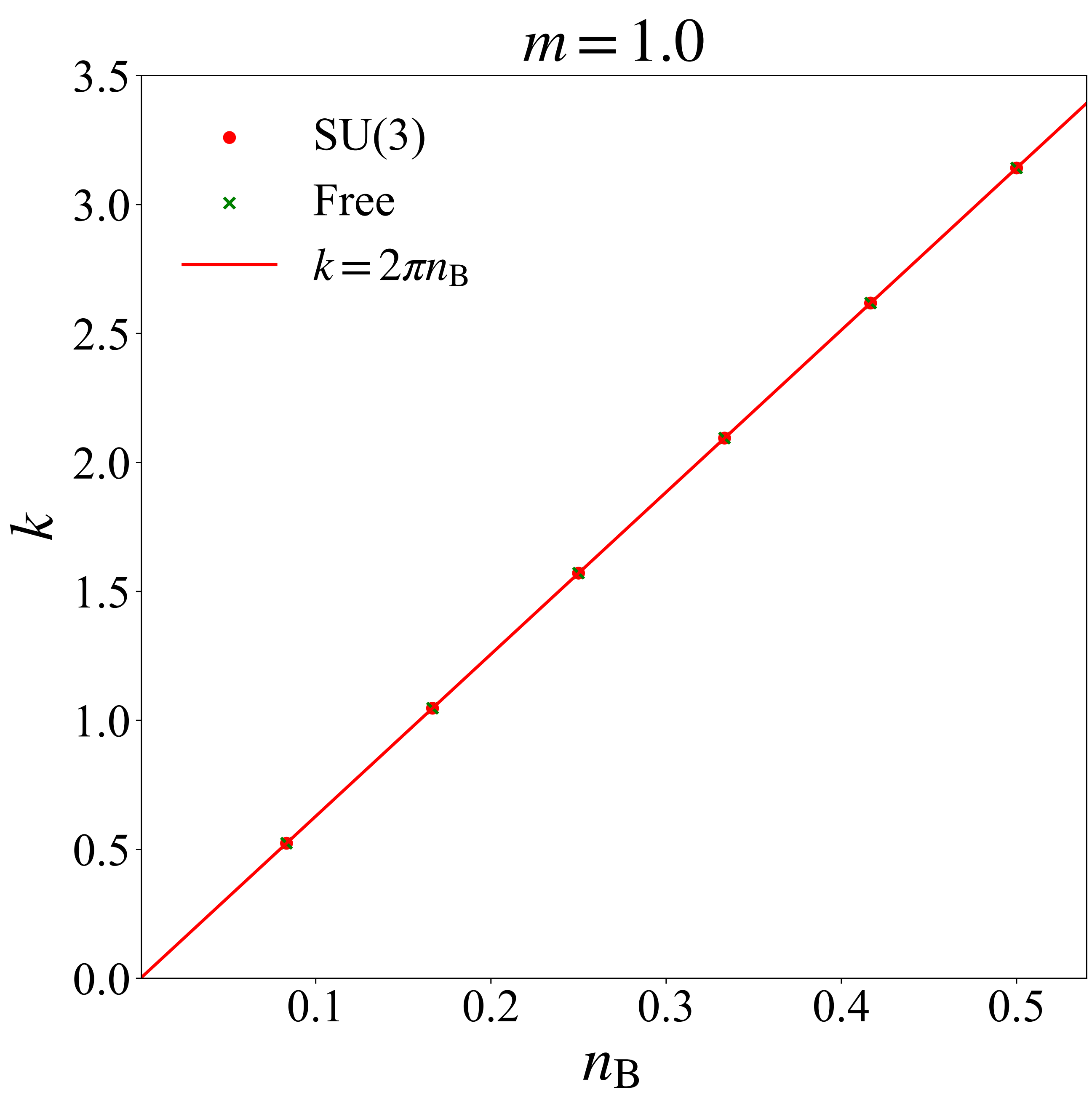


Spatial modulation

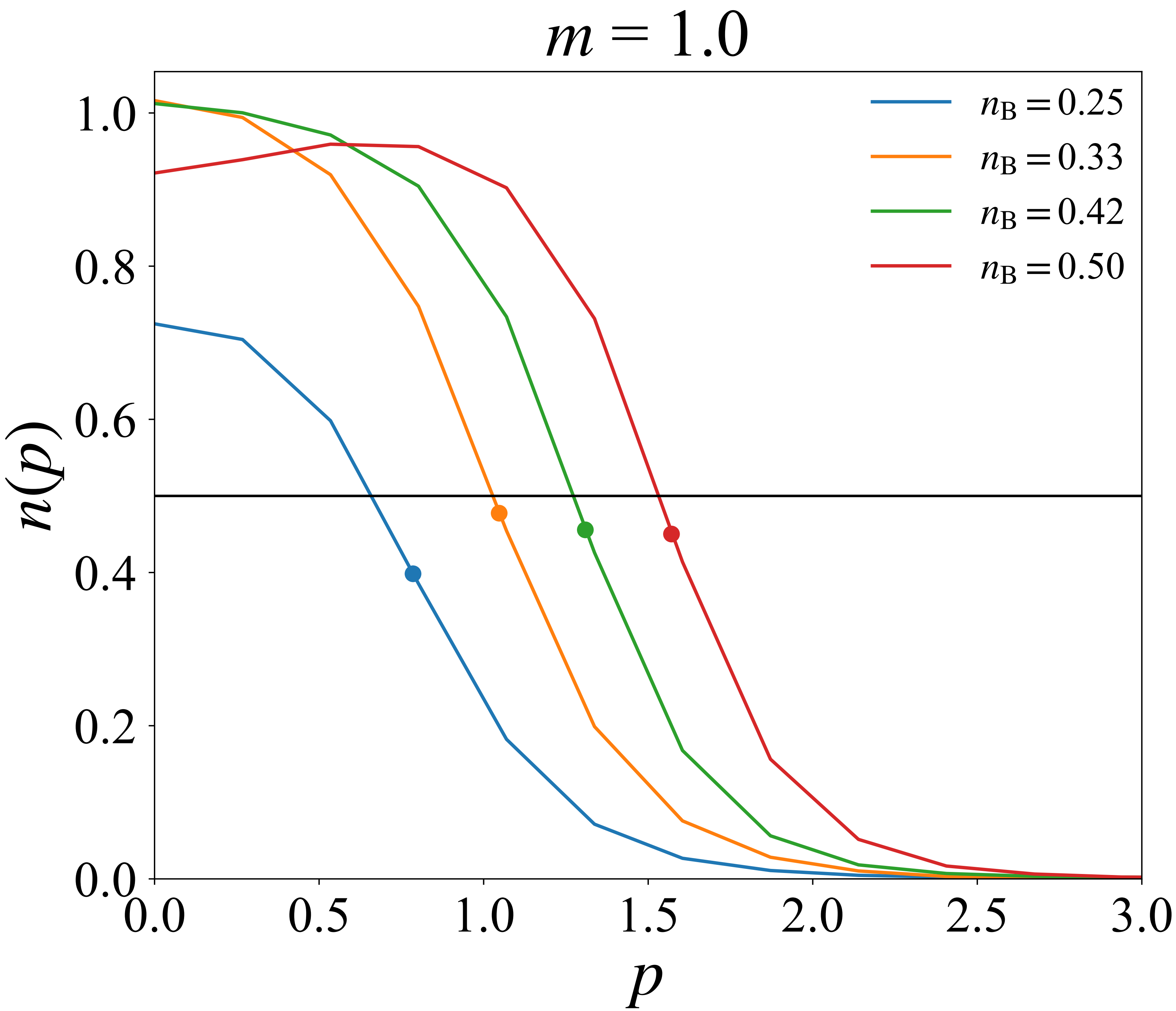


Wavenumber and amplitude





Quark distribution function



Summary

- Variational numerical calculation based on Hamiltonian formalism.
  - The transition from baryonic matter to quark matter occurs near $n_B \sim 0.2$.
  - Inhomogeneous state in QCD_2 occurs and is more enhanced than in free theory.
  - Fermi surface forms about $n_B \sim 0.2$.
- Baryon occupation probability? See talk by T. Kojo
- Real-time dynamics? Florio, et.al., (2023)