


Phenomenologies of QCD equation of state in high-energy nuclear collisions

Akihiko Monnai (Osaka Institute of Technology)

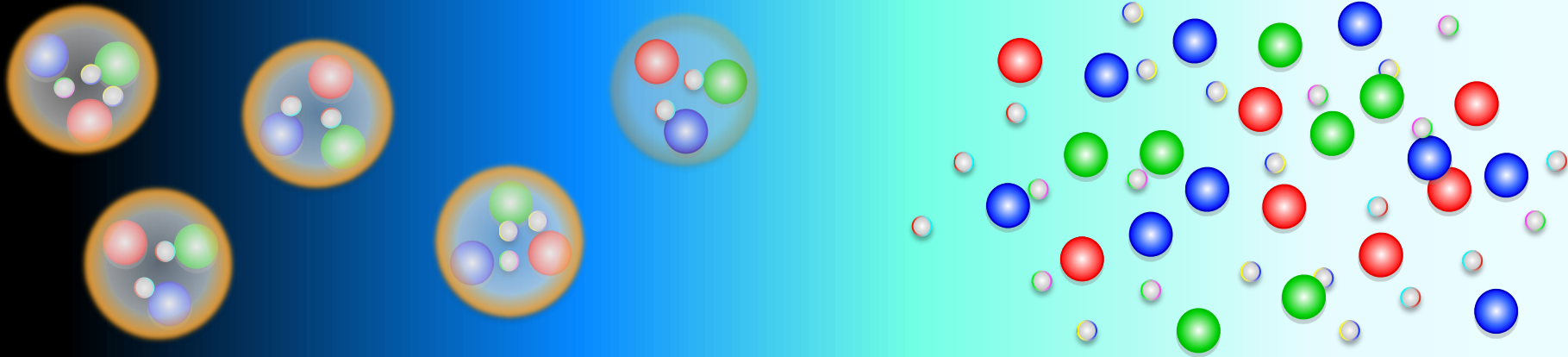
[AM, J. Phys. G 50, 095103 \(2023\)](#) 

Thermal Quantum Field Theory and Their Applications

29th August 2023, KEK, Japan

Introduction

- Phase transition of quarks

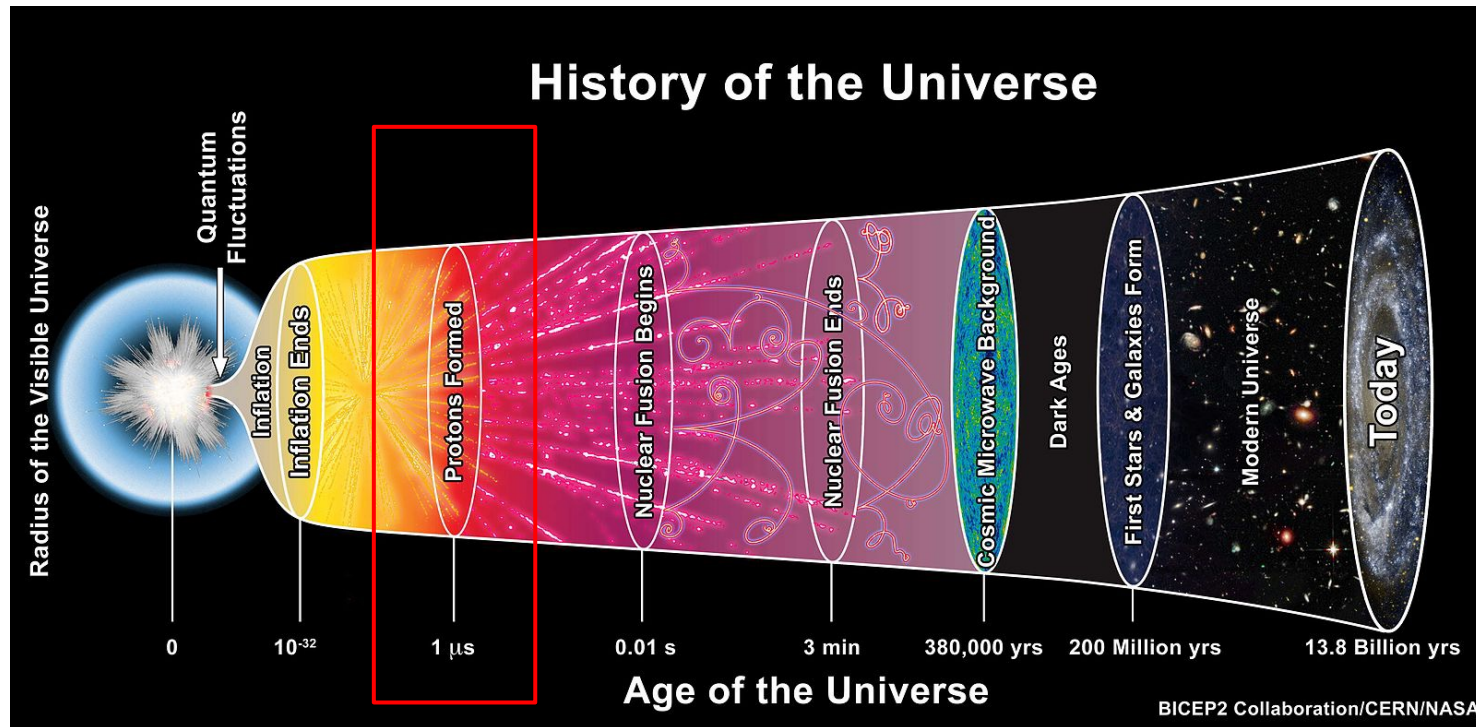


Protons and neutrons “melt” into **quark-gluon plasma (QGP)** above around 2 trillion degrees Kelvin

Introduction

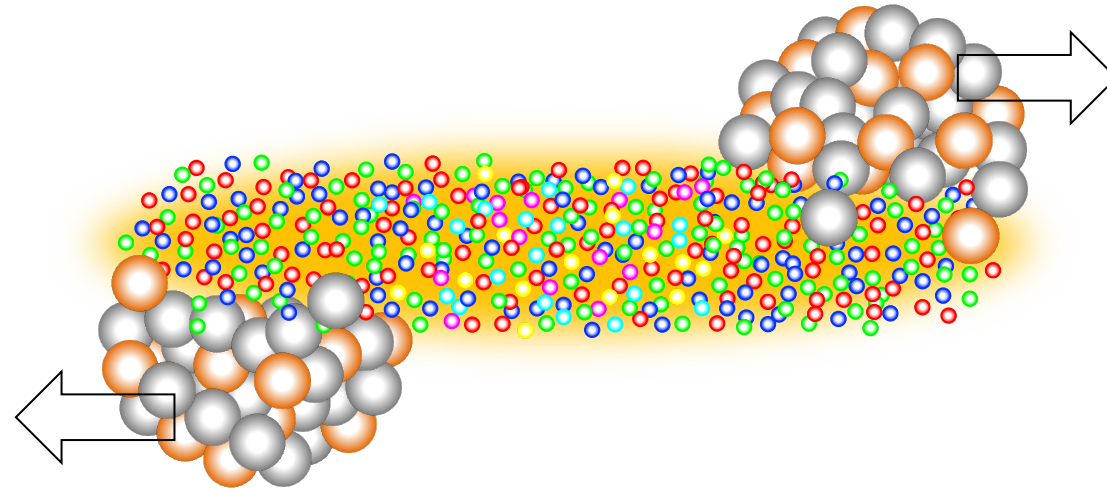
- Where is the quark-gluon plasma?

The QGP had filled the Universe around 10^{-5} seconds after the Big Bang



Introduction

- How to make the quark-gluon plasma **on earth**



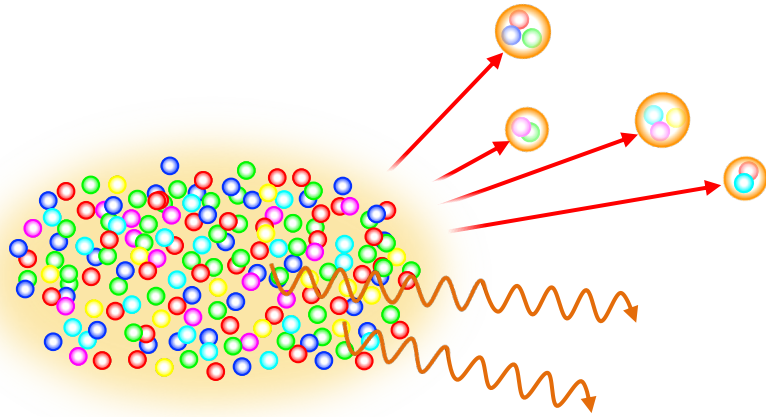
Smash two big nuclei (such as gold or lead) almost at the speed of light

BNL Relativistic Heavy Ion Collider (2000-)
CERN Large Hadron Collider (2010-)



Introduction

- What can be observed in nuclear collisions?



$T \sim 200\text{-}600 \text{ MeV}$
(primarily u, d, s, g)

Hadrons

- bulk observables
- informative of the medium “surface”
(QGP is color opaque)

Direct photons

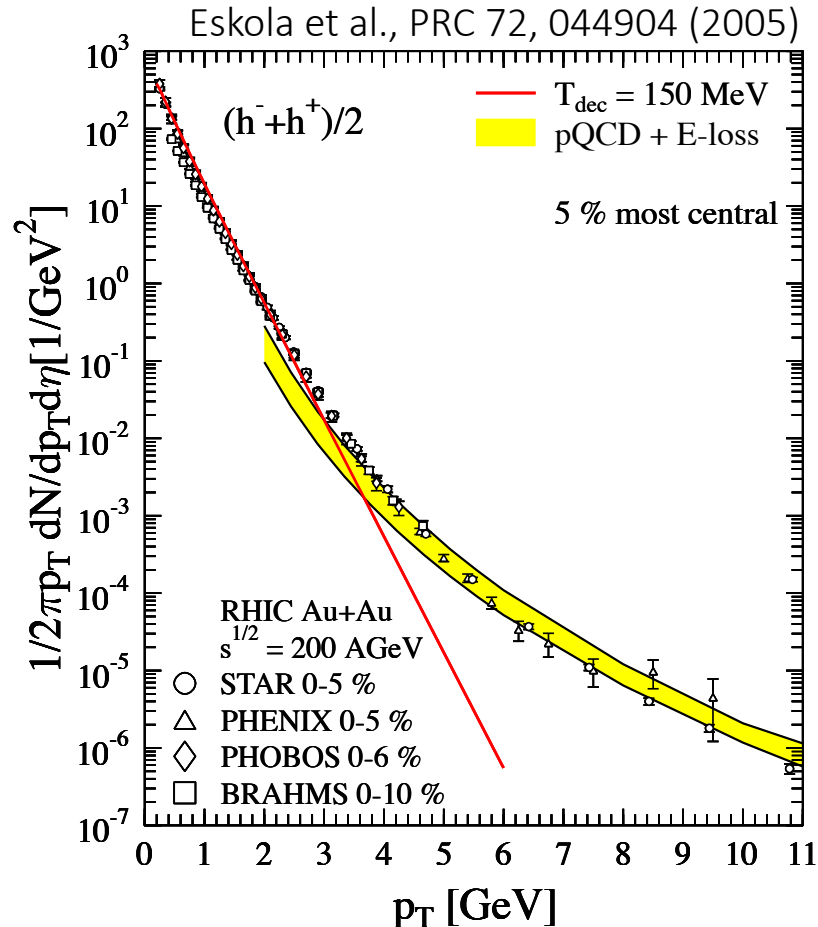
- photons w/o decay photons
- informative of the medium “interior”
(QGP is electroweak transparent)

Charged leptons, weak bosons, etc.

Introduction

$$1 \text{ GeV} \approx 1.6 \times 10^{-10} \text{ J}$$
$$1 \text{ GeV}/c \approx 5.3 \times 10^{-19} \text{ kg} \cdot \text{m/s}$$

■ Momentum distribution of charged hadrons



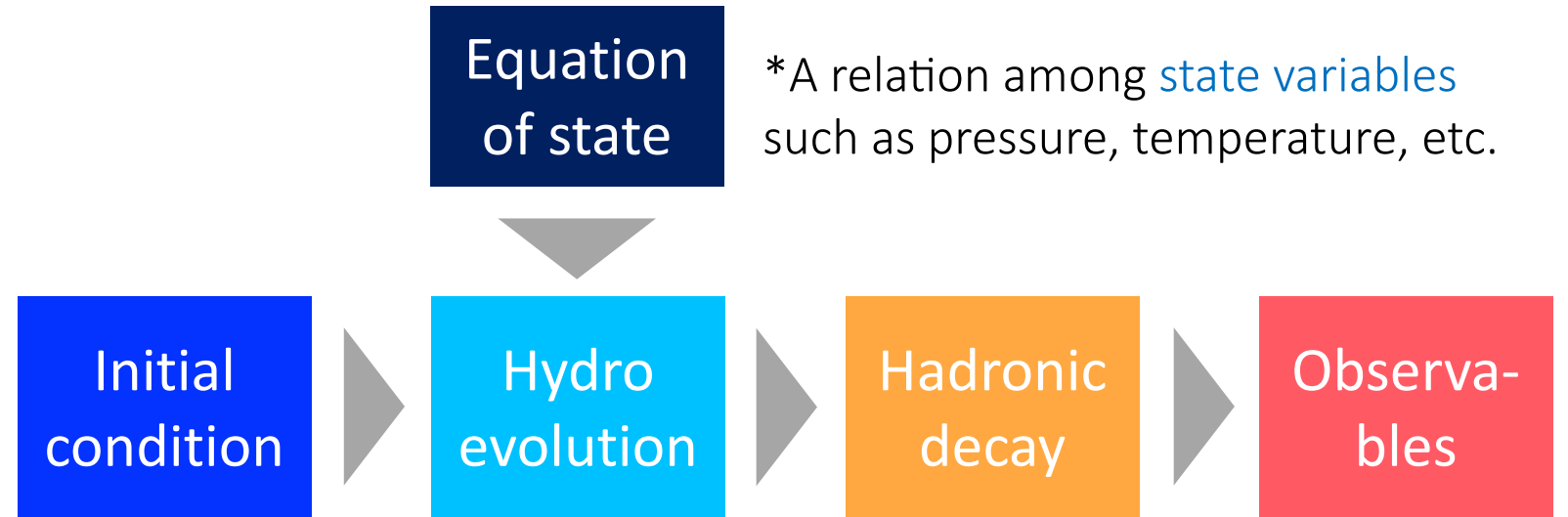
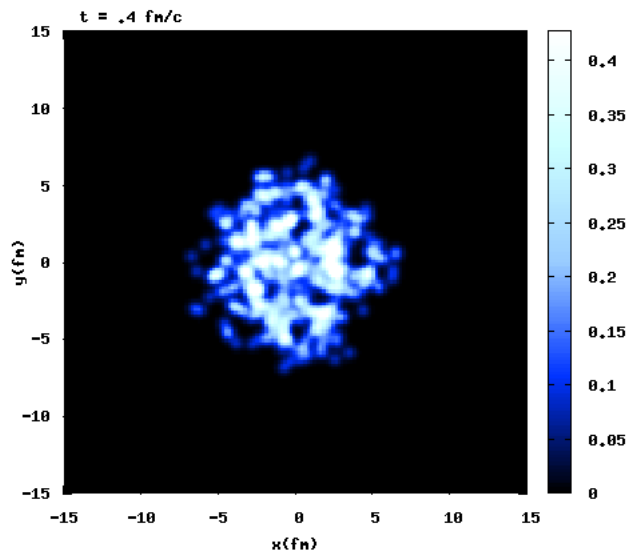
Low momentum region ($< 2\text{-}4 \text{ GeV}$)
Hydrodynamic model (strongly-coupled)

+

Mid-high momentum region ($> 4\text{-}5 \text{ GeV}$)
perturbative QCD (weakly-coupled)

Motivation

■ Relativistic hydrodynamic model



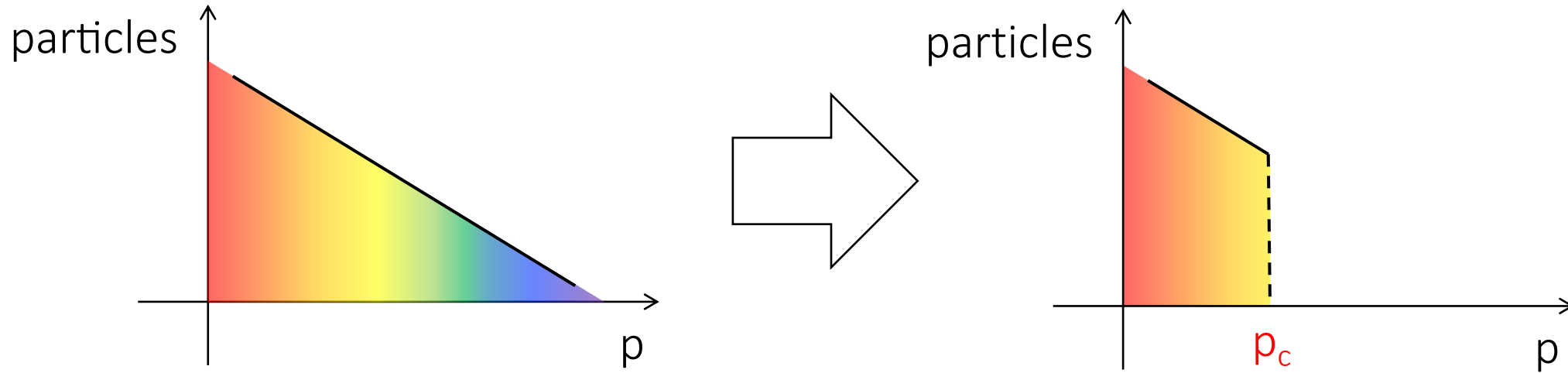
A caveat of the conventional model:
Equation of state & particle production include the contribution of *high-momentum particles*



This work

■ “Red” hydrodynamic model

*low momentum = long wavelength = “red”

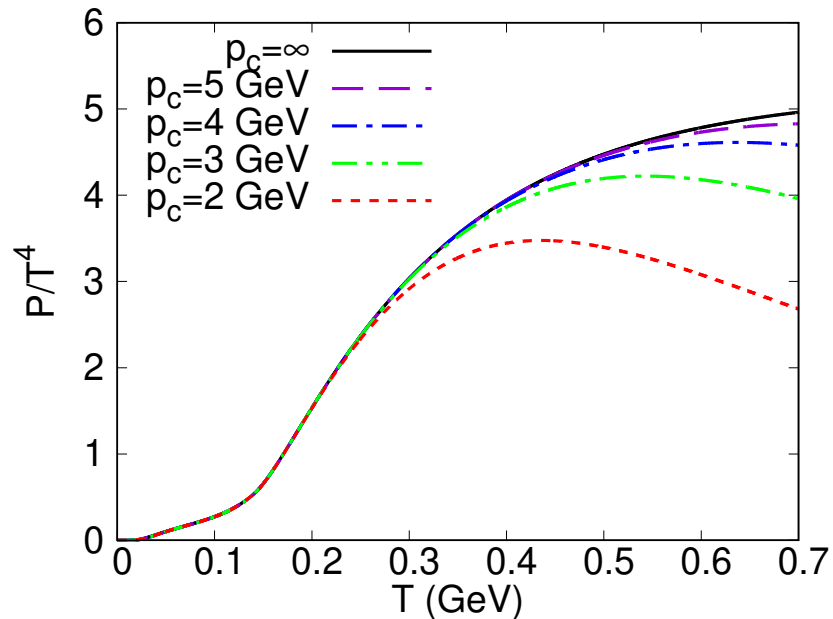


We introduce an **upper limit (p_c)** for the momenta of strongly-coupled components

Equation of state



$0.1 \text{ GeV} \simeq 1.16 \times 10^{12} \text{ K}$

■ Results



$$\text{Pressure: } P = \pm T \sum_i \int_0^{p_c} \frac{g_i d^3 p}{(2\pi)^3} \ln \left[1 \pm \exp \left(-\frac{E_i}{T} \right) \right]$$

i : index for particle species

Hadron resonance gas  } Smoothly connect the
(2+1)-flavor parton gas  } pressures of the 2 phases

Effects of momentum cutoff is **larger at higher temperatures**;
mostly negligible in the hadronic phase

Hydrodynamic evolution

■ (2+1)-dimensional inviscid model

AM, PRC 90, 021901(R) (2014)

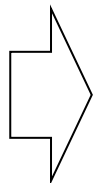
Initial condition: Monte-Carlo Glauber model (event-averaged)
2.76 TeV Pb+Pb collisions at $b = 4.6$ fm (~ 0 -20%)

Initial time: $\tau_{\text{ini}} = 0.4$ fm/c

Freezeout temperature: $T_f = 140$ MeV

Thermal photon estimated down to: $T_{\text{ph}} = 110$ MeV

Hadronic decay: Sollfrank, Koch, and Heinz, Phys. Lett. B 252, 256 (1990)



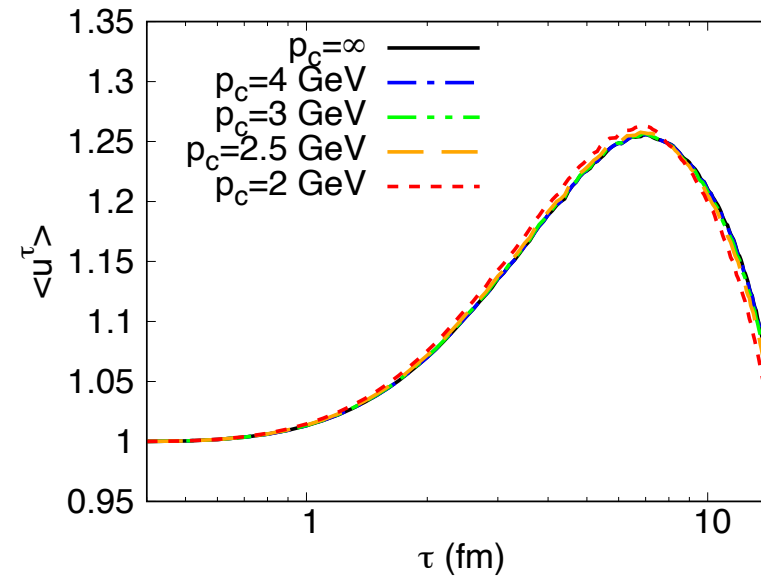
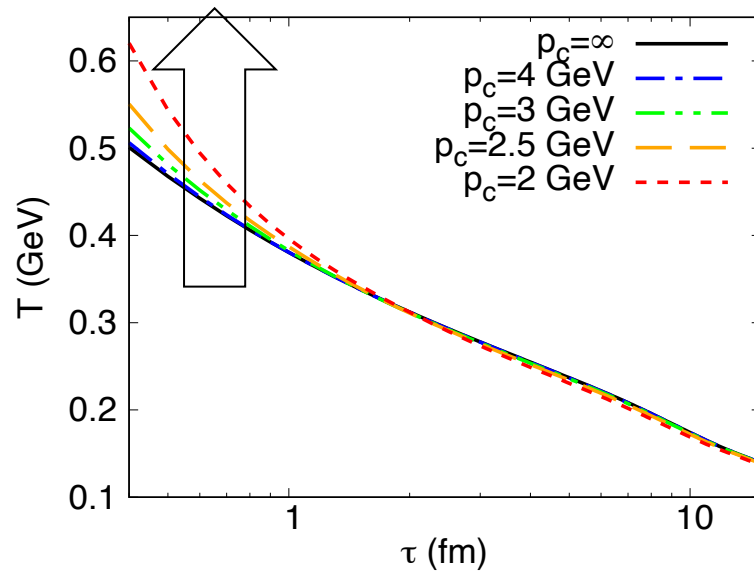
Simulations of nuclear collisions at the Large Hadron Collider



Hydrodynamic evolution


$$1 \text{ fm}/c \simeq 3.3 \times 10^{-24} \text{ s}$$

■ Numerical results

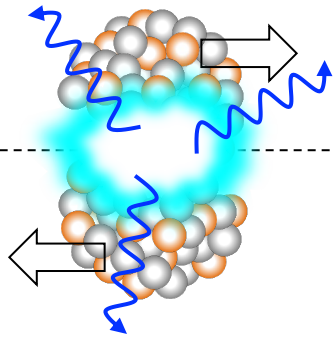


Initial temperature is **higher with lower cutoff momentum**; the flow development is mostly unaffected

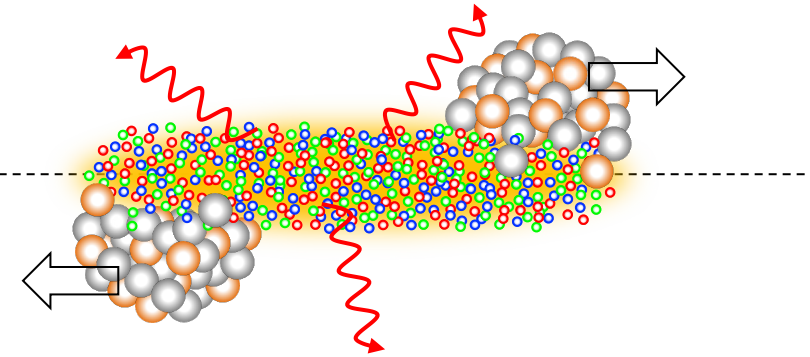
Photons

See e.g. AM, Int. J. Mod. Phys. A 37(11n12), 2230006 (2022)
for a review 

■ Sources of direct photons (conventional)

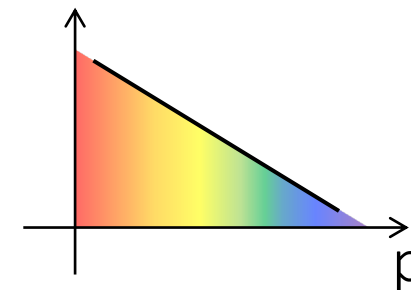


Prompt photons
produced at the collision




Thermal photons
produced from the medium

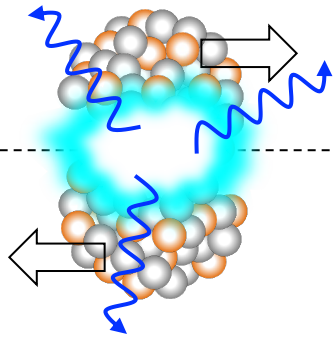
Turbide, Rapp and Gale, PRC 69, 014903
Heffernan, Hohler, and Rapp, PRC 91, 027902
Holt, Hohler, and Rapp, NPA 945, 1
Berges et. al., PRC 95, 054904 (2017)
Tanji and Venugopalan PRD 95, 094009 (2017)



Photons

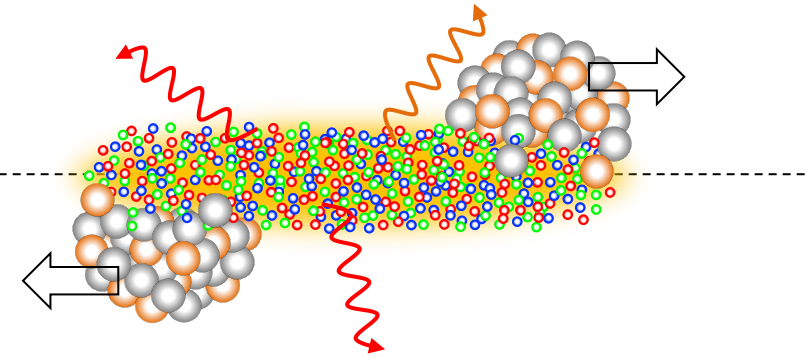
See e.g. AM, Int. J. Mod. Phys. A 37(11n12), 2230006 (2022)
for a review 

■ Sources of direct photons (**this work**)

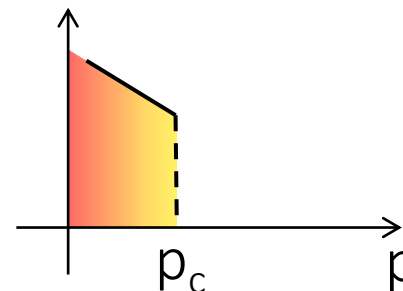


Prompt photons
produced at the collision

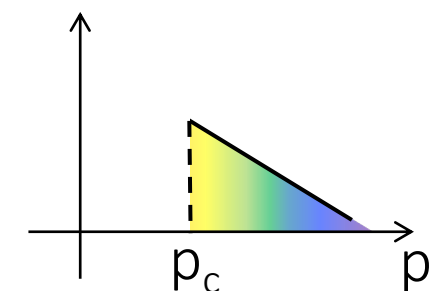
Turbide, Rapp and Gale, PRC 69, 014903
Heffernan, Hohler, and Rapp, PRC 91, 027902
Holt, Hohler, and Rapp, NPA 945, 1
Berges et. al., PRC 95, 054904 (2017)
Tanji and Venugopalan PRD 95, 094009 (2017)



Thermal photons
produced from the medium
(low momentum only)

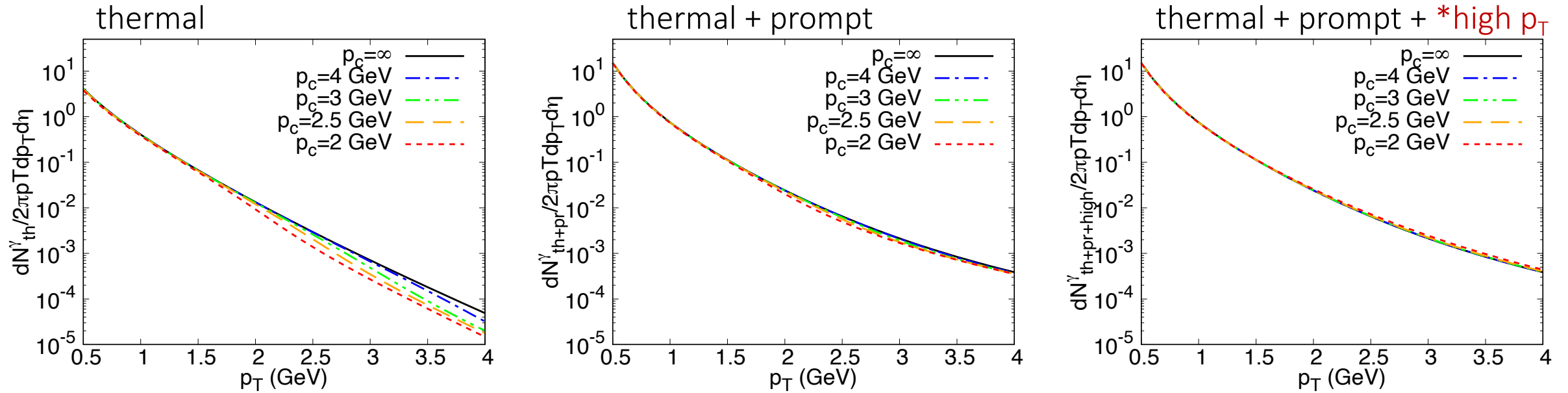


High p_T photons
produced from the non-thermal components



Particle spectra

■ Numerical results



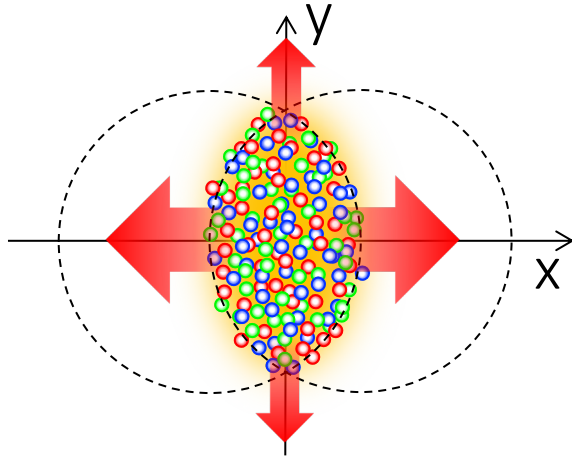
Thermal photon distribution is reduced above p_c
Total distribution is **mostly unaffected** by the choice of p_c

Elliptic flow

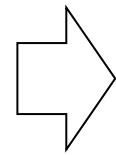
Ollitrault, PRD 46, 229 (1992)

Poskanzer and Voloshin, PRC 58, 1671 (1998)

- From spatial anisotropy to momentum anisotropy



Pressure gradients drive particles **faster** in the direction of the minor axis



The momentum distribution of hadrons has anisotropy (**elliptic flow v_2**)

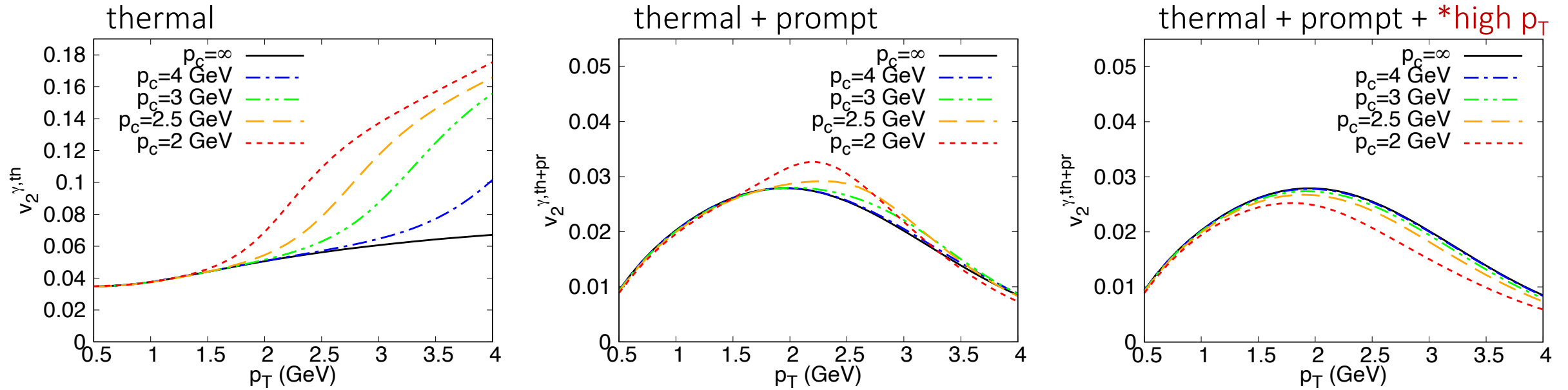
Direct photons are also **anisotropic**

- Prompt photons: isotropic
- Thermal photons: **anisotropic**
- High p_T photons: isotropic

**High p_T photons are conjectured as thermal photons with no anisotropy in this study*

Elliptic flow

■ Numerical results

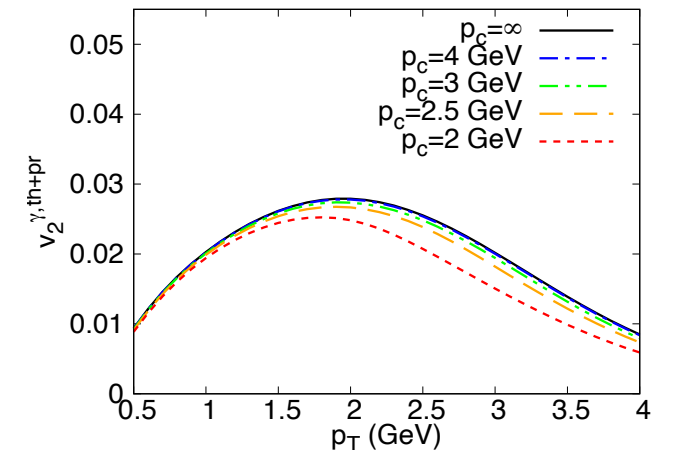
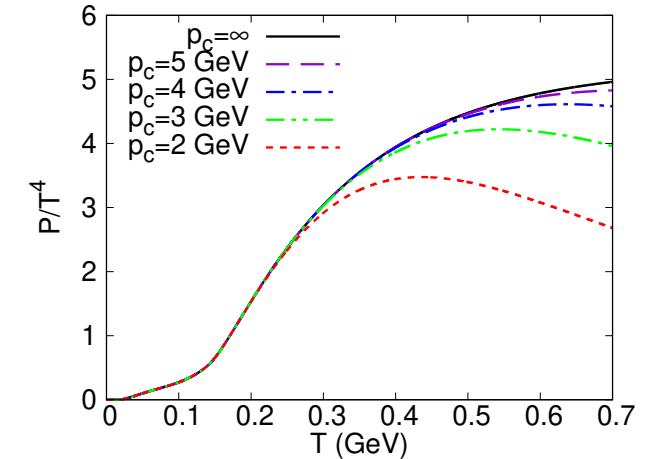


Photon v_2 is **sensitive** to the choice of p_c ; whether it is enhanced or not depends on the **high p_T contributions**

Summary

- We developed a “red” hydrodynamic model of the QGP

- ▶ Equation of state is constructed only with **low momentum components**
- ▶ High p_T photon emission is assumed to be non-thermal
- ▶ **Initial temperature** can be higher
- ▶ **Direct photon v_2** can be sensitive to the cutoff momentum p_c



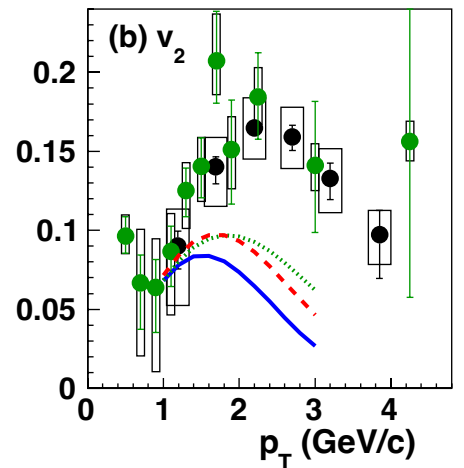
Outlook

■ Future directions

- ▶ Introduction of **viscous corrections** (including estimation of transport coefficients)
- ▶ Event-by-event estimation for **quantitative analyses of elliptic flow**
- ▶ Comparison with the **experimental data** for understanding the *photon puzzle**

*The discrepancy between the theoretical estimations and experimental data of direct photon v_2

PHENIX, Phys. Rev. C 94, 064901 (2016)



Backup slides

Equation of state

- Hadron resonance gas (HRG) + parton gas

Particle Data Group: PRD 98, 030001 (2018)

P_{had} : hadron resonances < 2 GeV with u,d,s components

P_{QGP} : u,d,s, and g

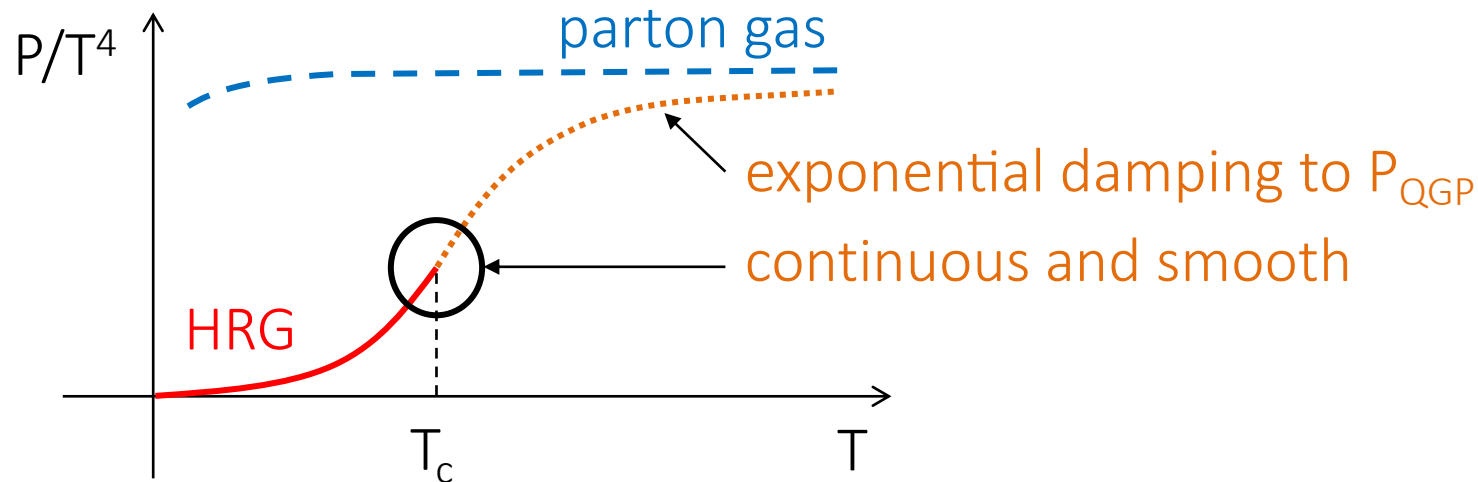
- ▶ Connection of the hadronic and QGP pressures

$$\begin{cases} P(T) = P_{\text{had}}(T) & (T < T_c) \\ P(T) = P_{\text{had}}(T_c) + [P_{\text{QGP}}(T) - P_{\text{had}}(T_c)] \times \{1 - \exp[-c(T - T_c)]\} & (T \geq T_c) \end{cases}$$

where $c = \frac{P'_{\text{had}}(T_c)}{P_{\text{QGP}}(T) - P_{\text{had}}(T_c)}$ so that $\frac{dP(T_c)}{dT} = \frac{dP_{\text{had}}(T_c)}{dT}$

Equation of state

■ Benefits of the connection method



1. It puts emphasis on **HRG**, which is more reliable than parton gas according to lattice QCD
2. Kinetic freezeout is safe below T_c
3. The difference from the lattice QCD pressure is within 10% for $0 < T < 1 \text{ TeV}$