1 Thermal History

2 First-order phase transitions

3 From micro to macro

4 Gravitational waves

5 Recent topics

高エネルギー初期宇宙における一次相転移と重力波生成

Ryusuke Jinno (RESCEU, UTokyo)

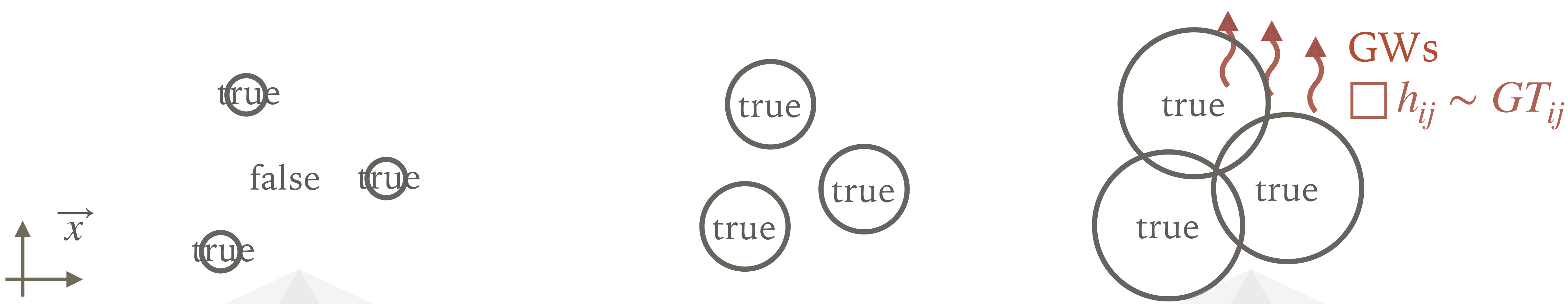
熱場の量子論とその応用 @KEK, 2023/8/28-30

概要

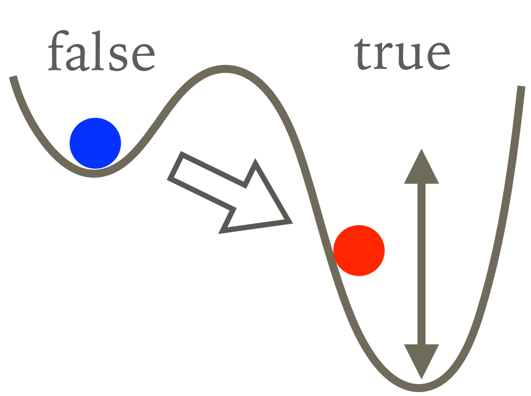
microphysics *macrophysics*

時間 / スケール →

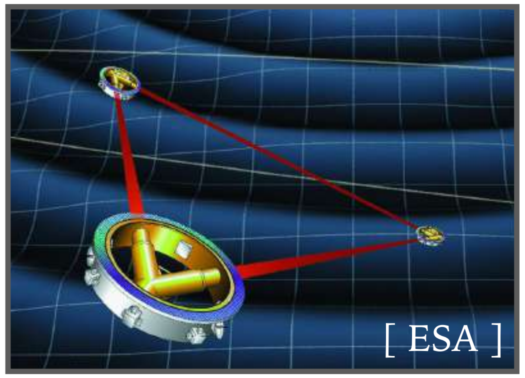
(1) 核生成 (nucleation) (2) 拡大 (expansion) (3) 衝突 (collision)



一次相転移の物理



重力波観測



概要

microphysics

泡のダイナミクス

macrophysics

時間 / スケール →

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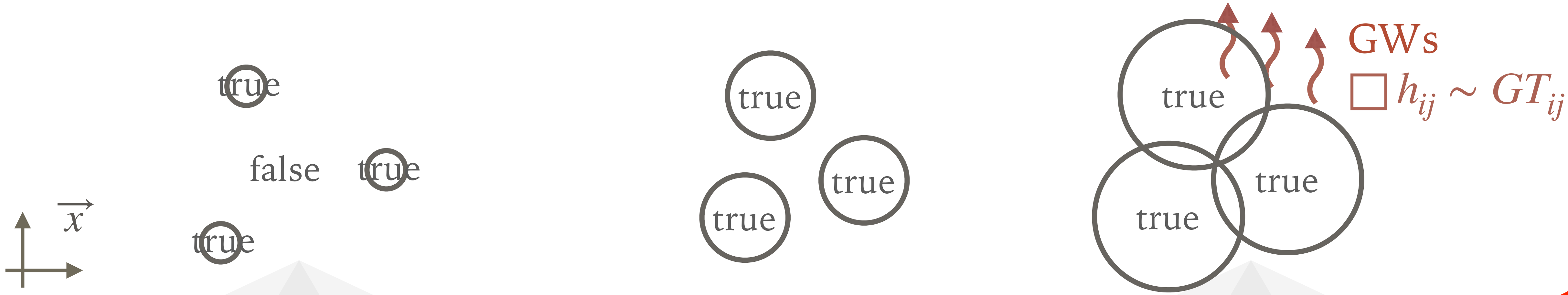

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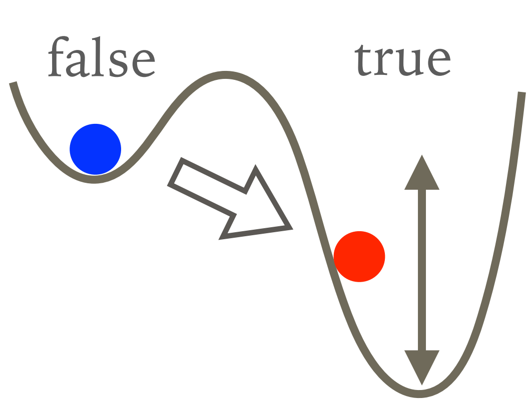
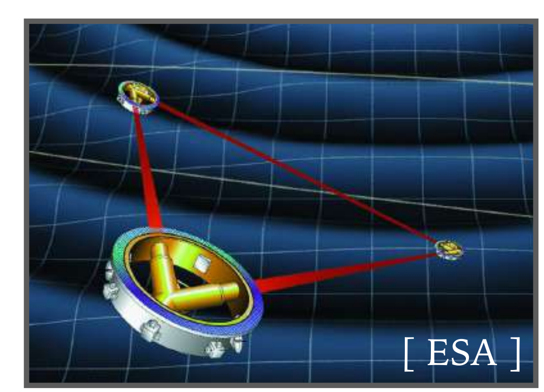


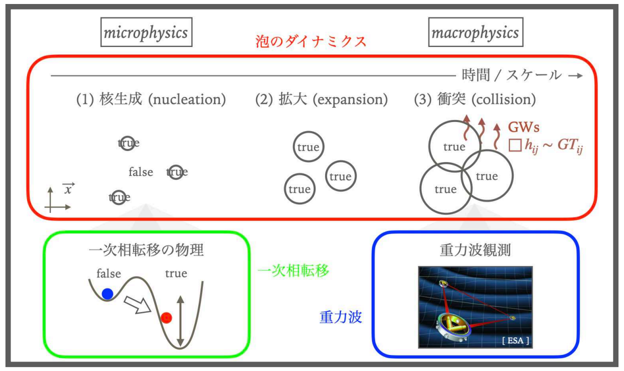
一次相転移

重力波

重力波観測



トーク構成



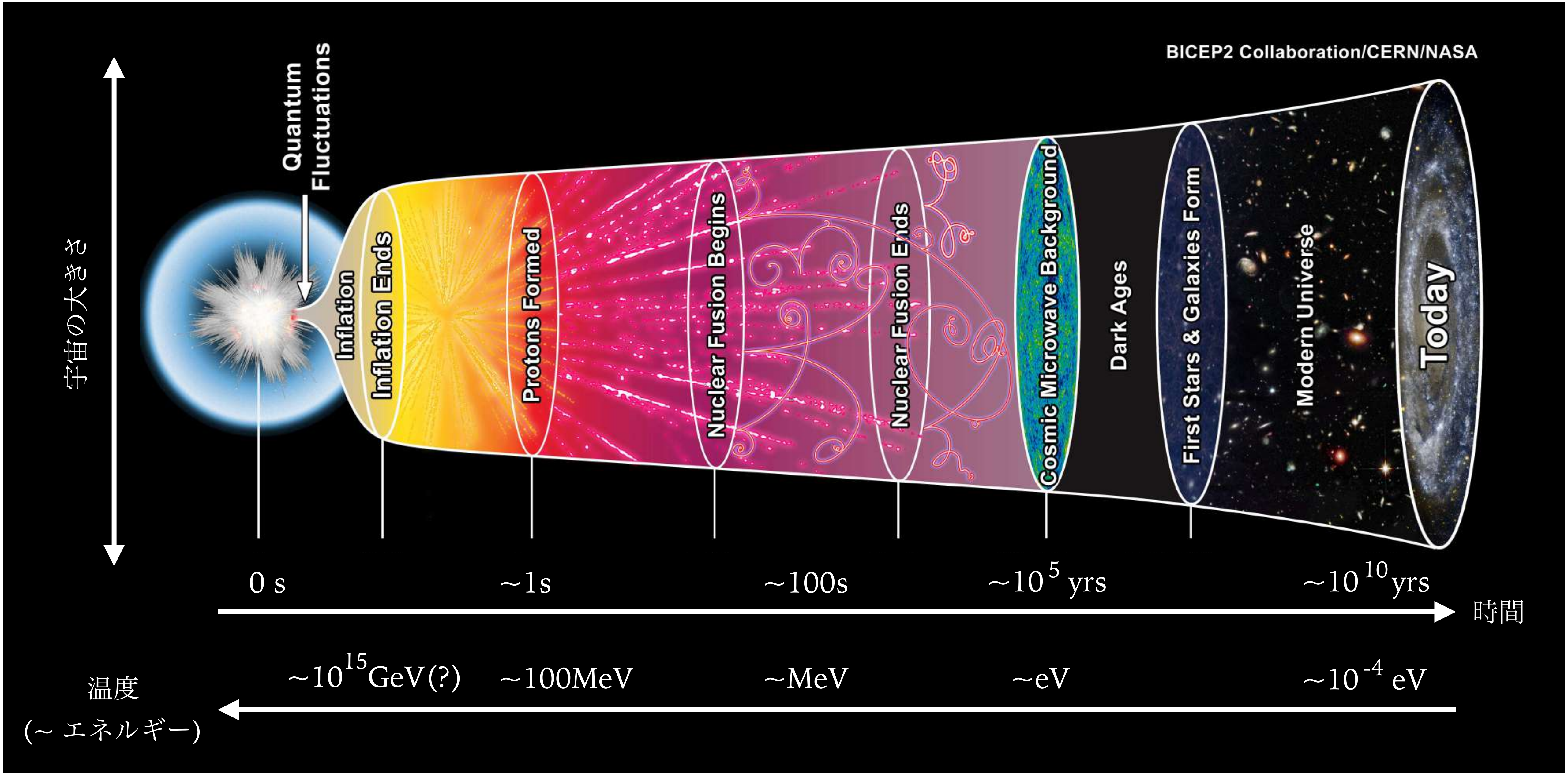
~10min　1. 宇宙の熱史 (Thermal history of the Universe)

~35min
- 2. 一次相転移 (First-order phase transitions)
- 3. ミクロの物理からマクロの物理へ (From microphysics to macrophysics)
- 4. 重力波生成 & 観測 (GW production & observations)

~10min　5. 最近の(個人的な)ホットトピック

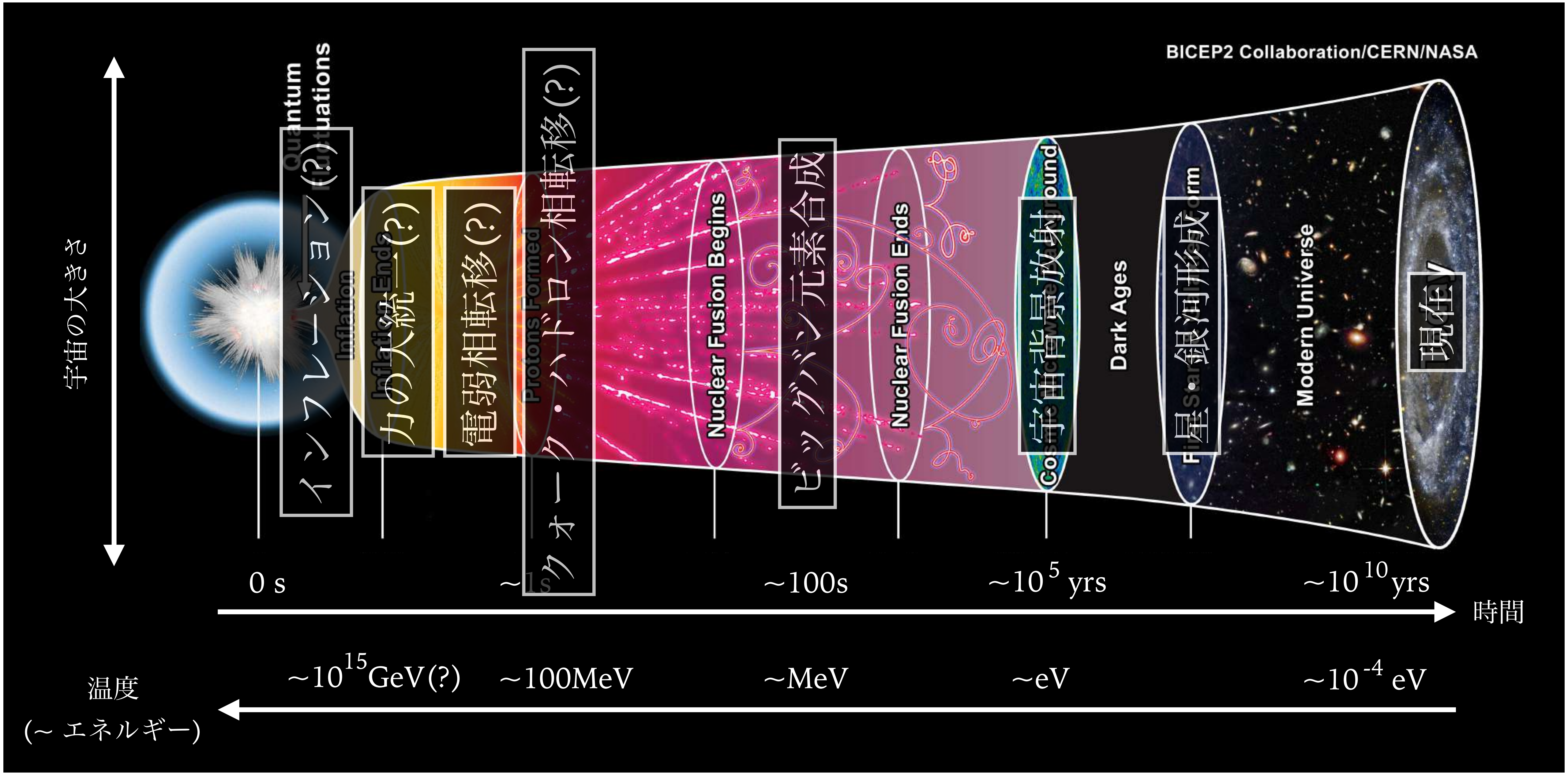
1. Thermal History
2. First-order phase transitions
3. From micro to macro
4. Gravitational waves
5. Recent topics

宇宙の熱史



宇宙の歴史 = 冷却の歴史

宇宙の熱史



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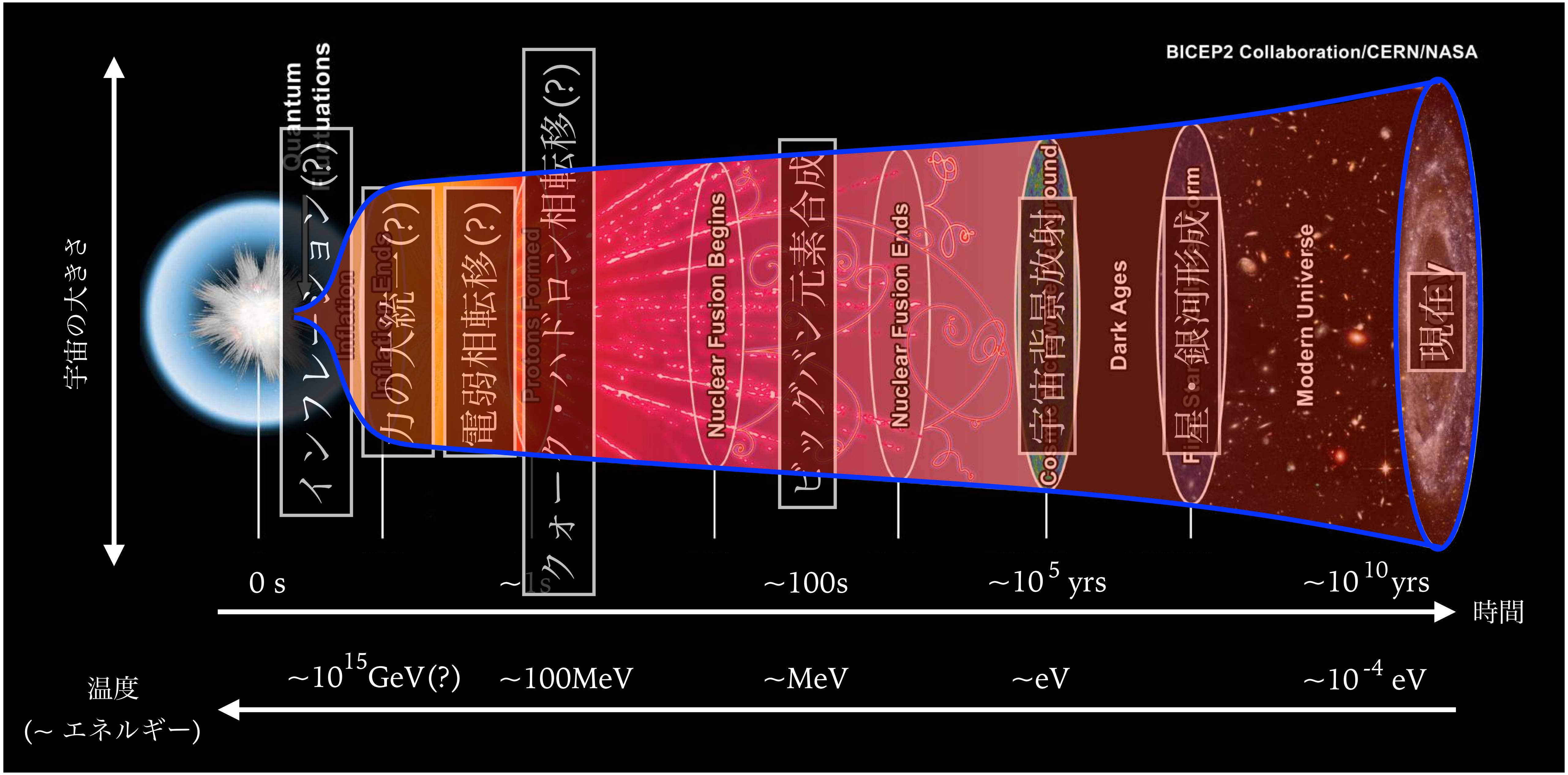
宇宙の熱史



宇宙の歴史 = 冷却の歴史

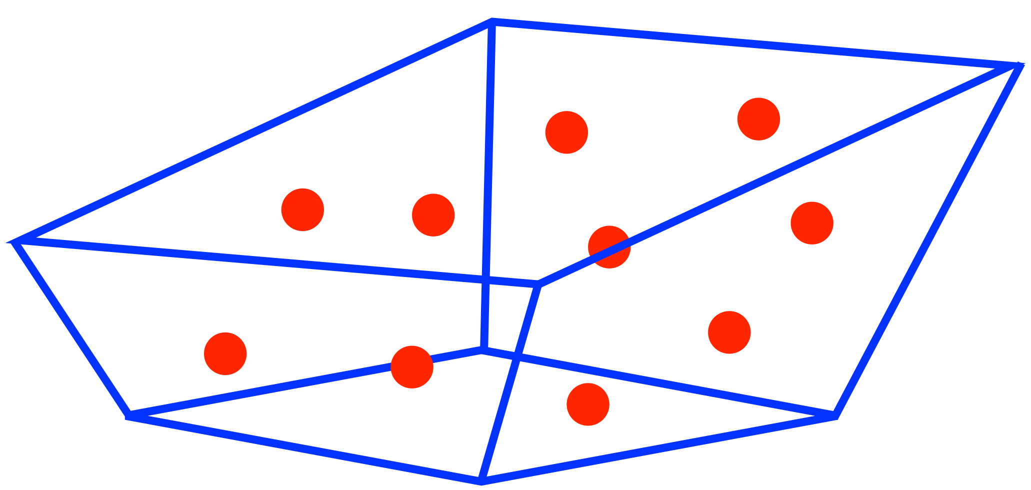
アインシュタイン方程式

➤ 宇宙の歴史を記述する方程式 = アインシュタイン方程式

$$\text{カタチ} \quad G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad \text{モノ}$$

"*Space(-time) tells matter how to move. Matter tells space(-time) how to curve.*"

John Wheeler



アインシュタイン方程式

➤ 宇宙の歴史を記述する方程式 = アインシ

Einstein tensor

determined from the metric

$G \simeq 6.67 \times 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}}$

: Newton constant

Energy-momentum tensor of matter

$$\text{カタチ} \quad G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad \text{モノ}$$

$c \simeq 3 \times 10^8\,\mathrm{m/s}$: speed of light

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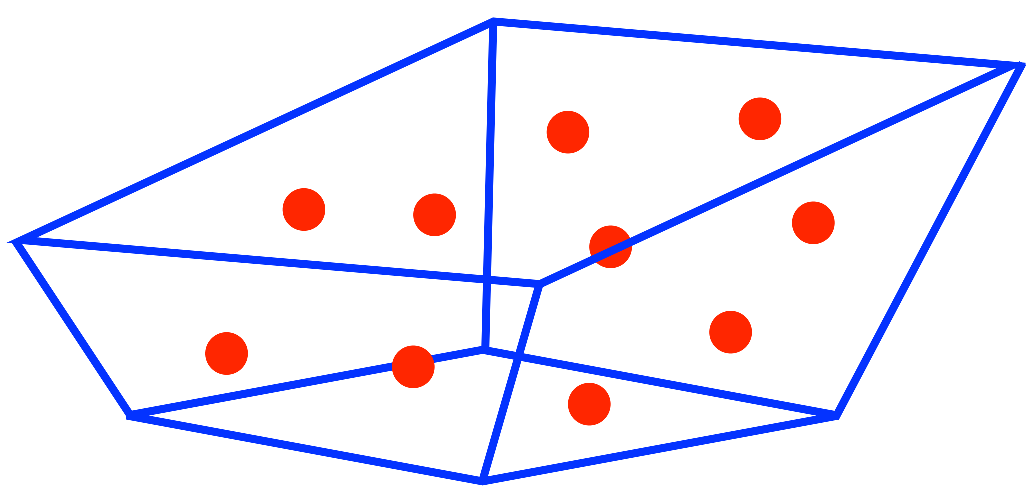
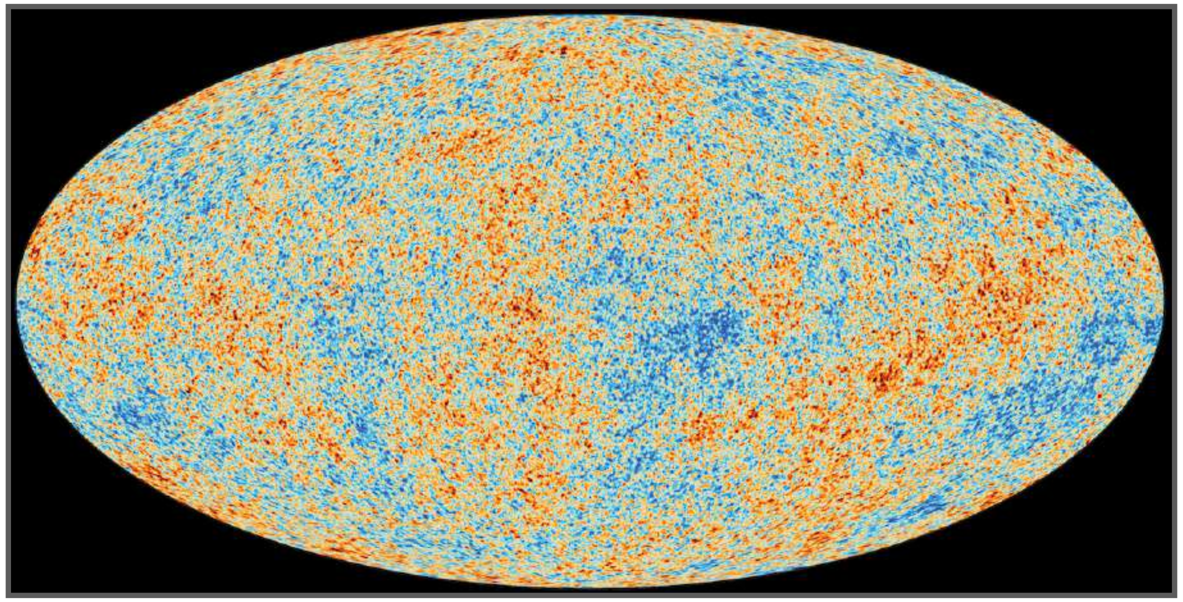


一様等方時空

➤ 宇宙は大きなスケールでは一様かつ等方

宇宙背景放射

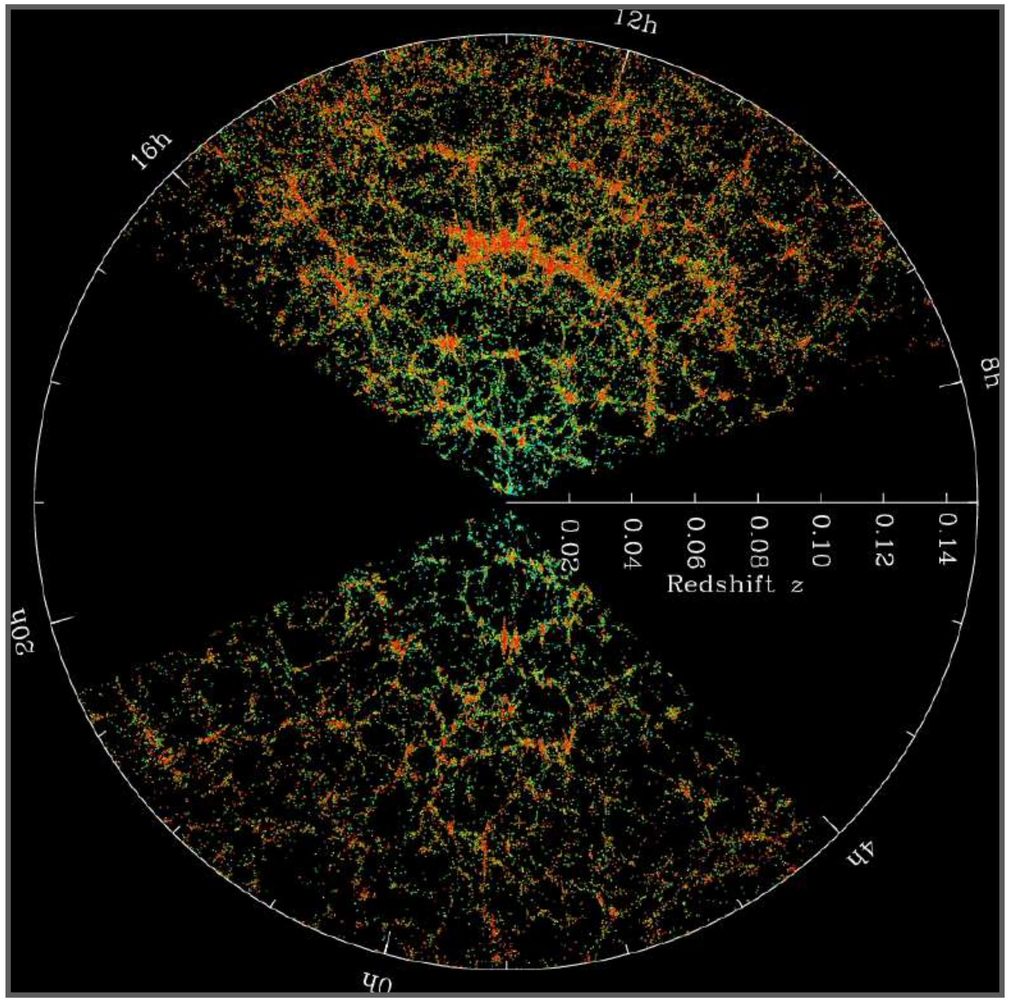
(CMB, Cosmic Microwave Background)



$$\frac{\delta T}{T} \sim 10^{-4}$$

宇宙の大規模構造

(LSS, Large Scale Structure)



一様等方時空

- 宇宙は大きなスケールでは一様かつ等方

- そのような宇宙を記述する計量は

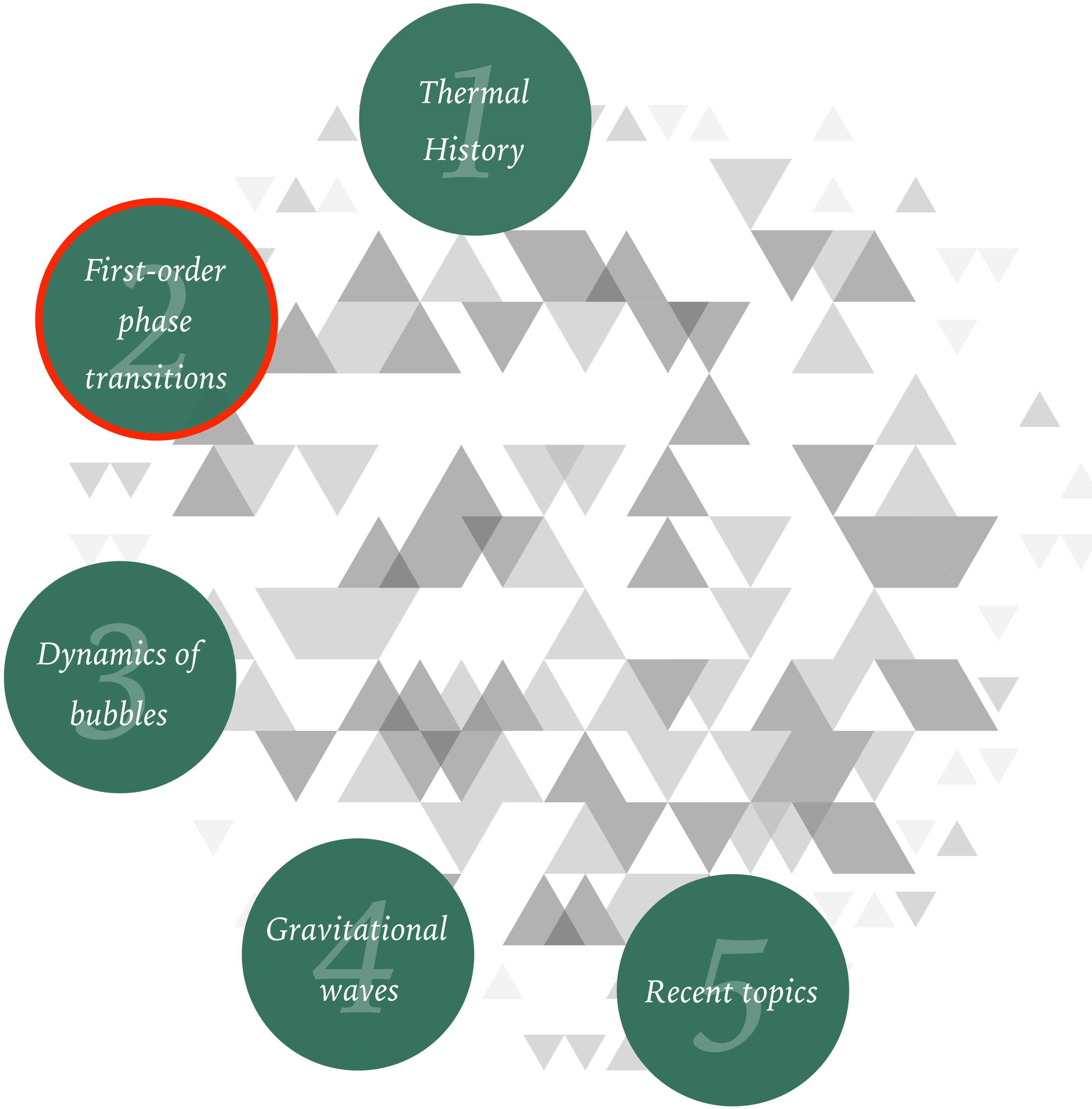
  FLRW (Friedmann-Lemaitre-Robertson-Walker) 計量と呼ばれる

$$\boxed{ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right]}$$

$a(t)$: スケールファクター,　　$H(t) \equiv \dfrac{\dot{a}(t)}{a(t)}$: ハッブルパラメータ

- 宇宙の温度は大まかにスケールファクターの逆冪で落ちる

$$\boxed{T(t) \sim a(t)^{-1}}$$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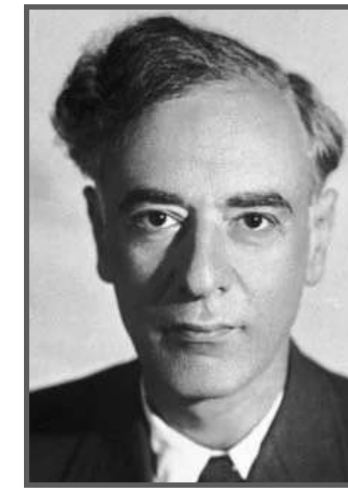
一次相転移

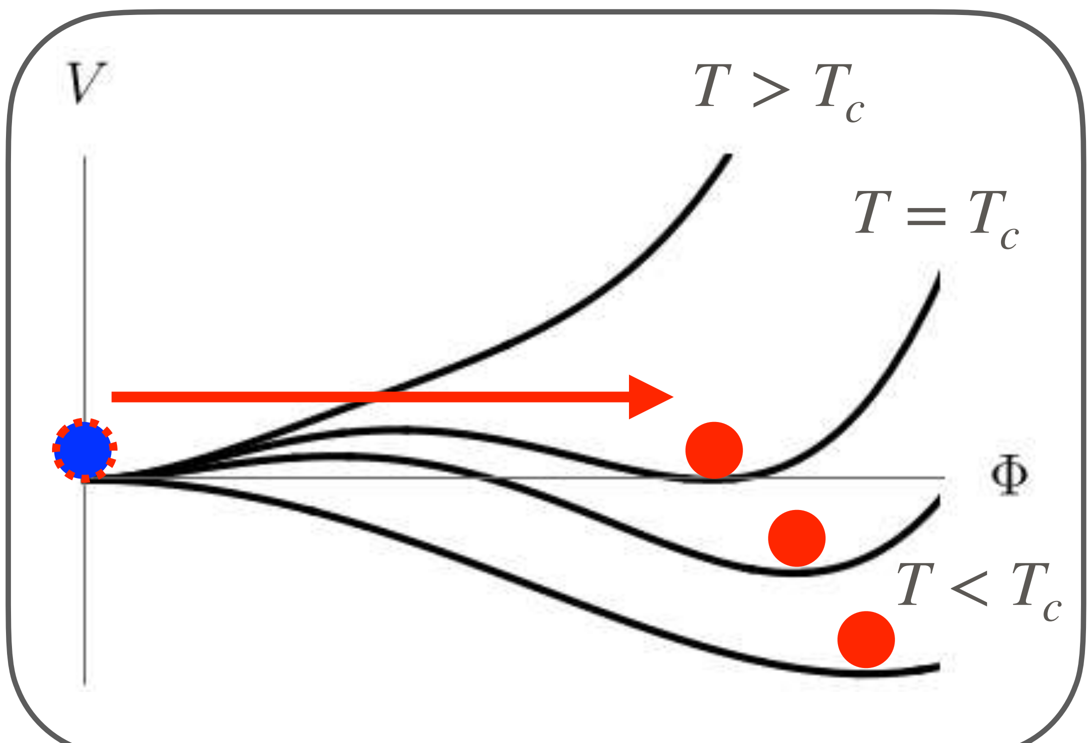
- 相転移の分類 (Landau)

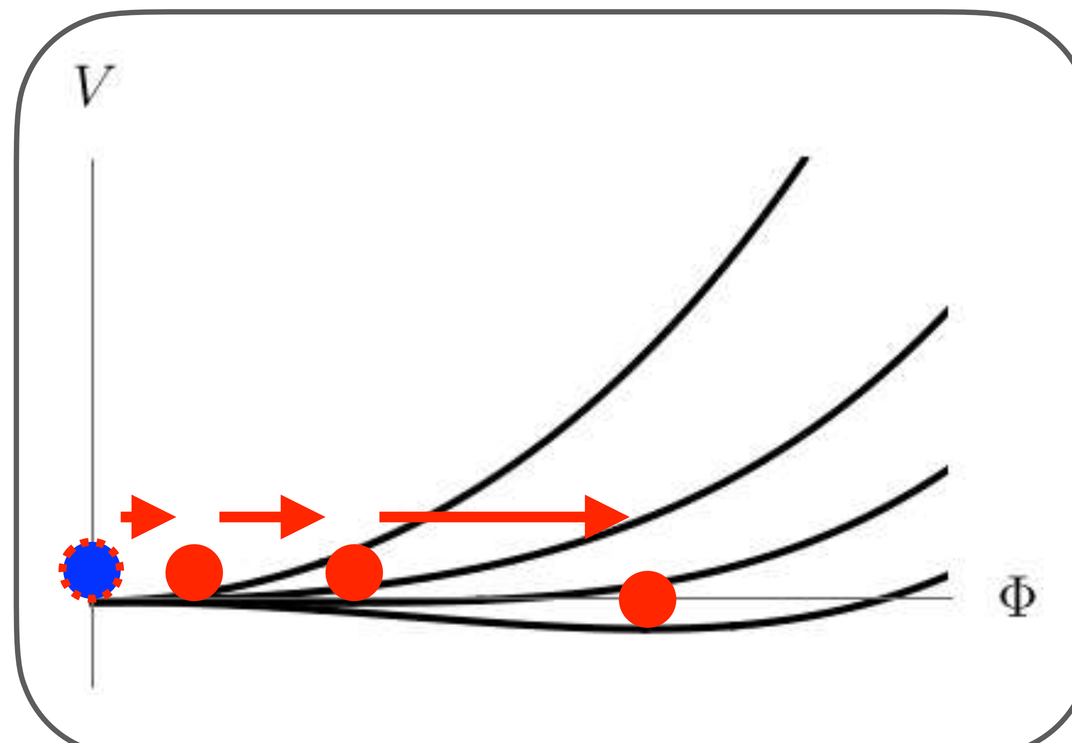


秩序パラメータ

不連続 ↙　　↘ 連続

一次相転移　　二次相転移

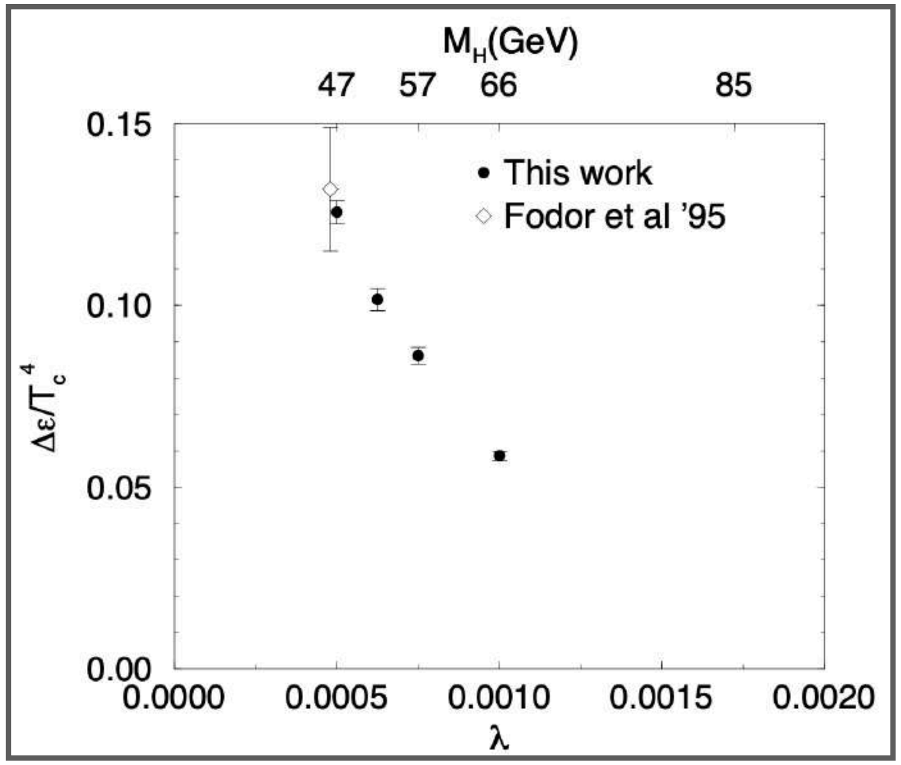


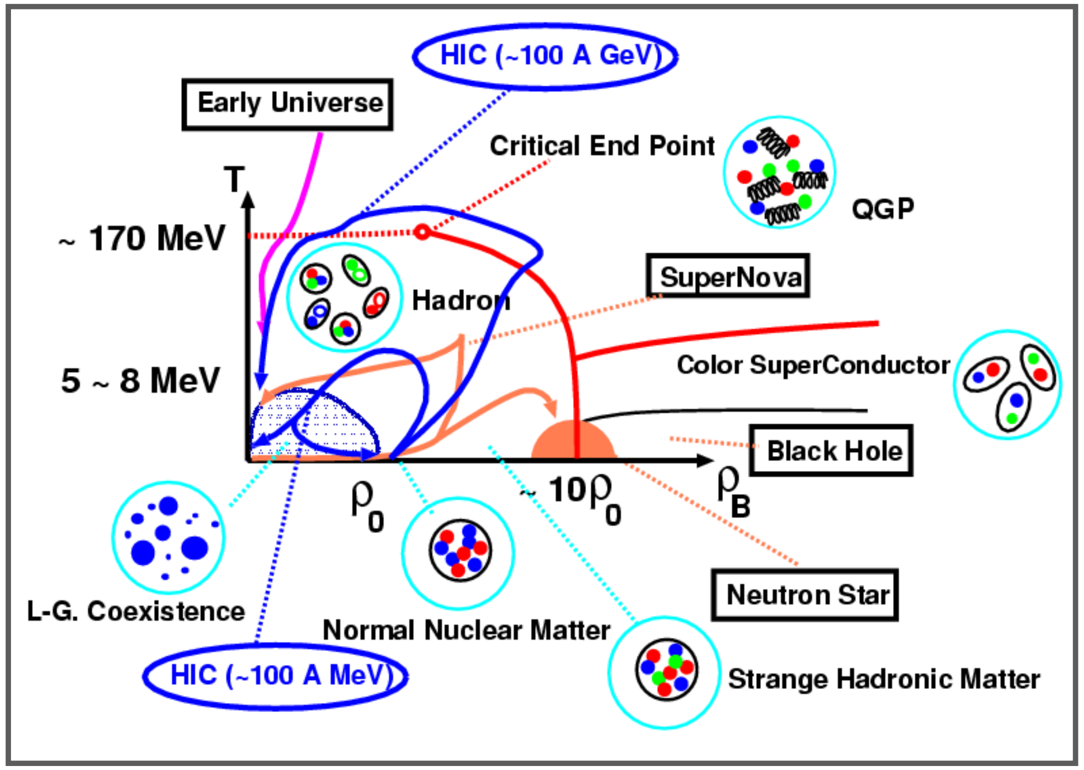


素粒子標準理論での一次相転移？

➤ 素粒子標準理論で一次相転移の可能性があると思われていた

電弱「相転移」 & クォーク・ハドロン「相転移」





[Aoki '97]
see also
[Kajantie, Laine, Rummukainen, Shaposhnikov '96]
[Karsch, Neuhaus, Patkós, Rank '97]

[Onishi, Sapporo Winter School '09]

→ 初期宇宙の発展においては残念ながらどちらもクロスオーバー

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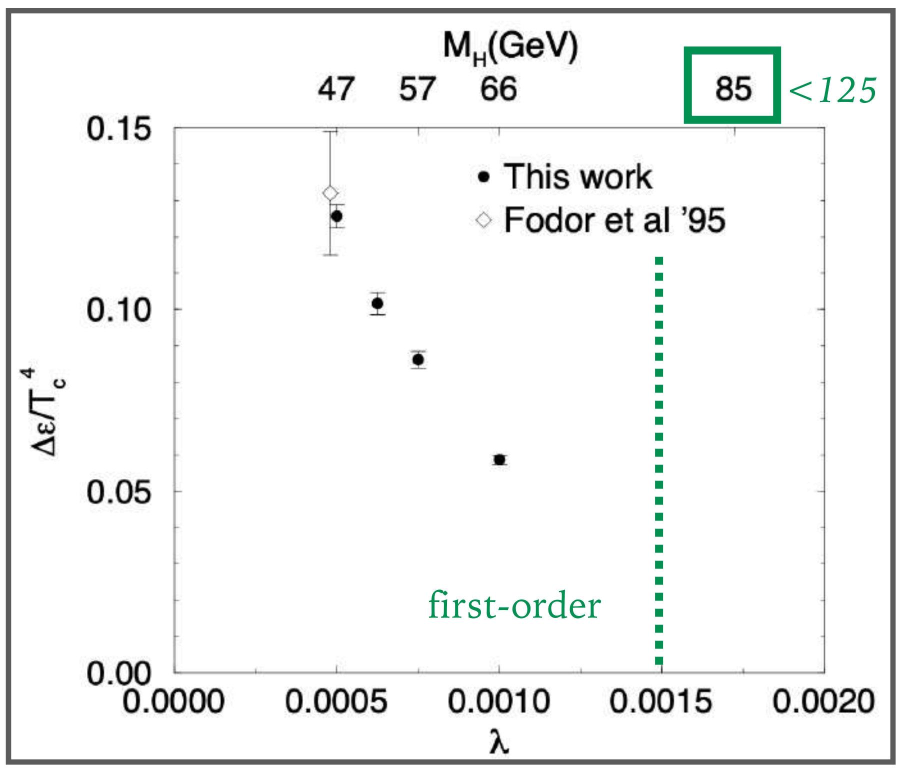

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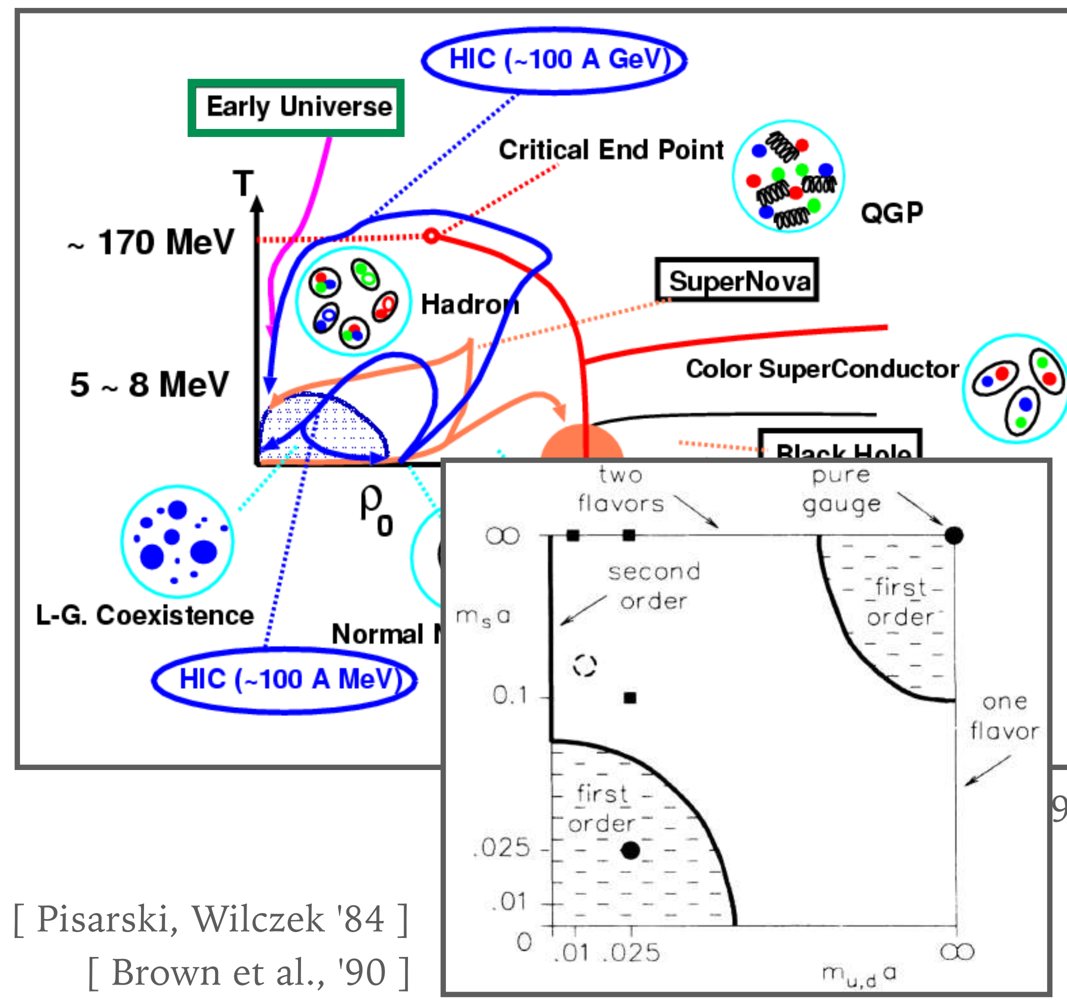


[Pisarski, Wilczek '84]
[Brown et al., '90]

9]

→ 初期宇宙の発展においては残念ながらどちらもクロスオーバー

一次相転移を考える動機

- インフレーション ($\lesssim 10^{15}$GeV) から現在 ($\sim 10^{-4}$eV) に至るまで
  宇宙が経験した広大なエネルギースケール

- その中で起こり得た自発的対称性の破れ

  - 大統一群の破れ (in relation to GUT)

  - Peccei-Quinn対称性 $U(1)_{\rm PQ}$ の破れ (in relation to strong CP)

  - B-L対称性 $U(1)_{\rm B-L}$ の破れ (in relation to neutrino mass)

  - dark群の破れ (in relation to dark matter?)

  → どこかで一次相転移が起きていた可能性を考えてみても良いのでは？

一次相転移を考える（伝統的な）動機

➤ 素粒子と宇宙の最大の謎の1つ

バリオン非対称性 (BAU) = なぜバリオンは反バリオンより多いのか

銀河

反銀河

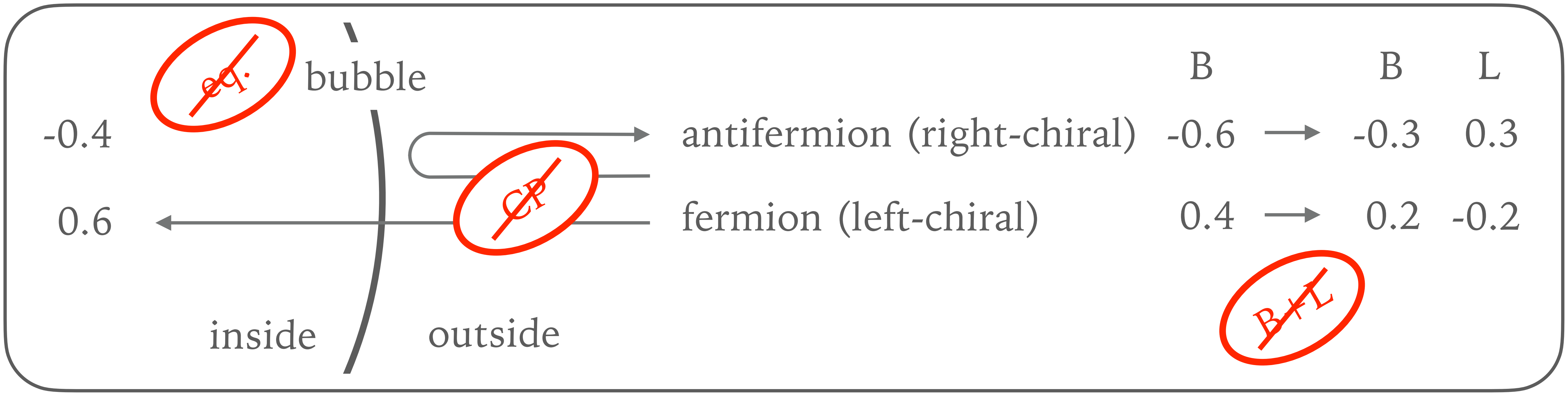
➤ バリオン非対称性を作り出すのに必要な3条件 (サハロフの3条件) [Sakharov '67]

1) Bの破れ　　2) CとCPの破れ　　3) 熱平衡からの乖離

一次相転移を考える（伝統的な）動機

- もし一次相転移が起きていると、サハロフの3条件の一部が満たされる

  (電弱バリオジェネシスと呼ばれる [Kuzmin, Rubakov, Shaposhnikov '85])



- しかし、electric dipole momentからの制限が厳しい

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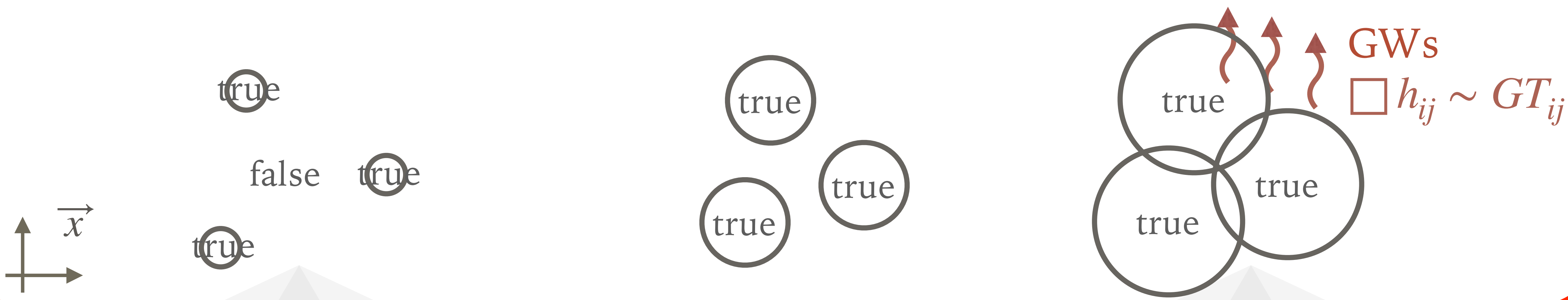

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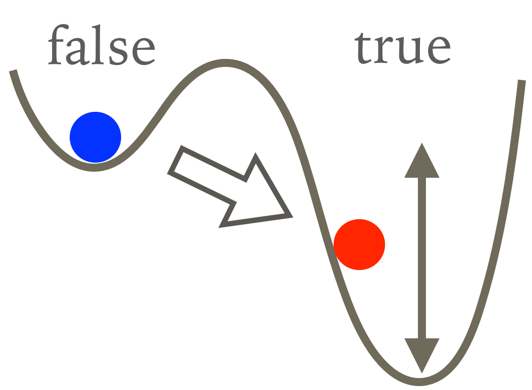
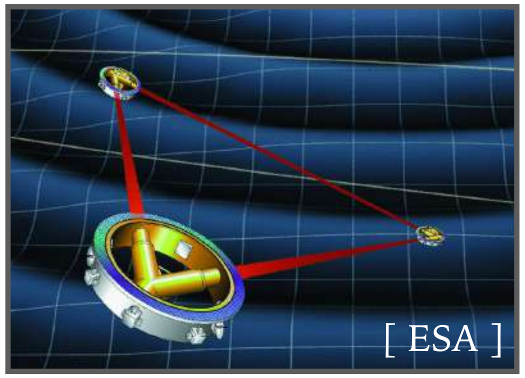


一次相転移

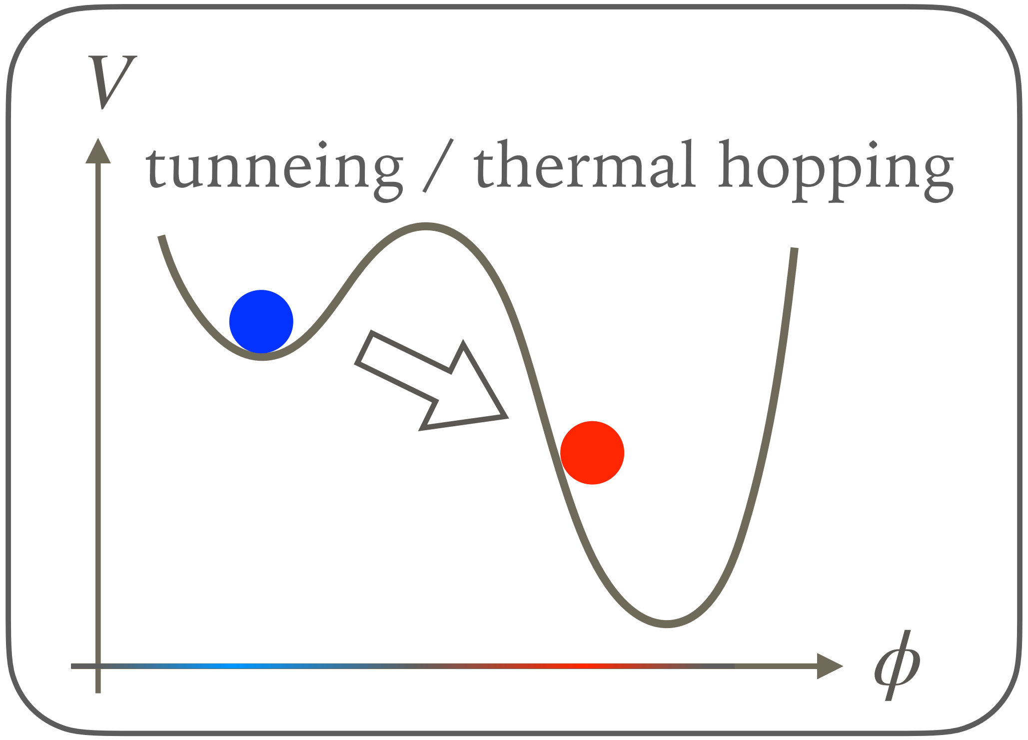
重力波

重力波観測

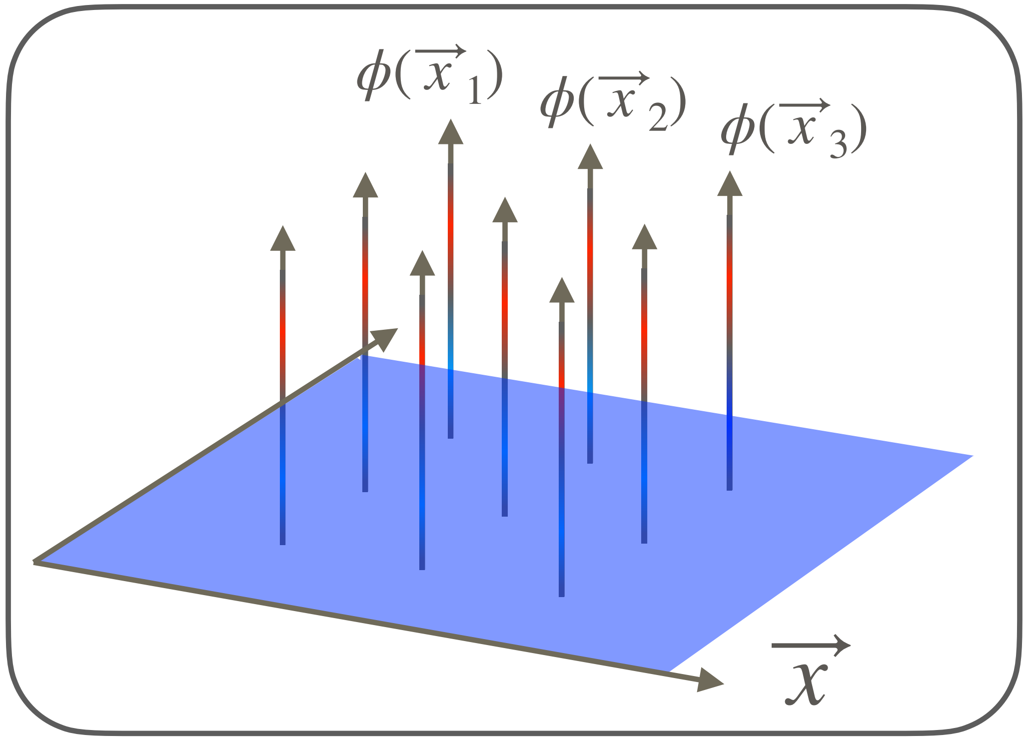


FIELD SPACEとPOSITION SPACEでの核生成

Field space



Position space



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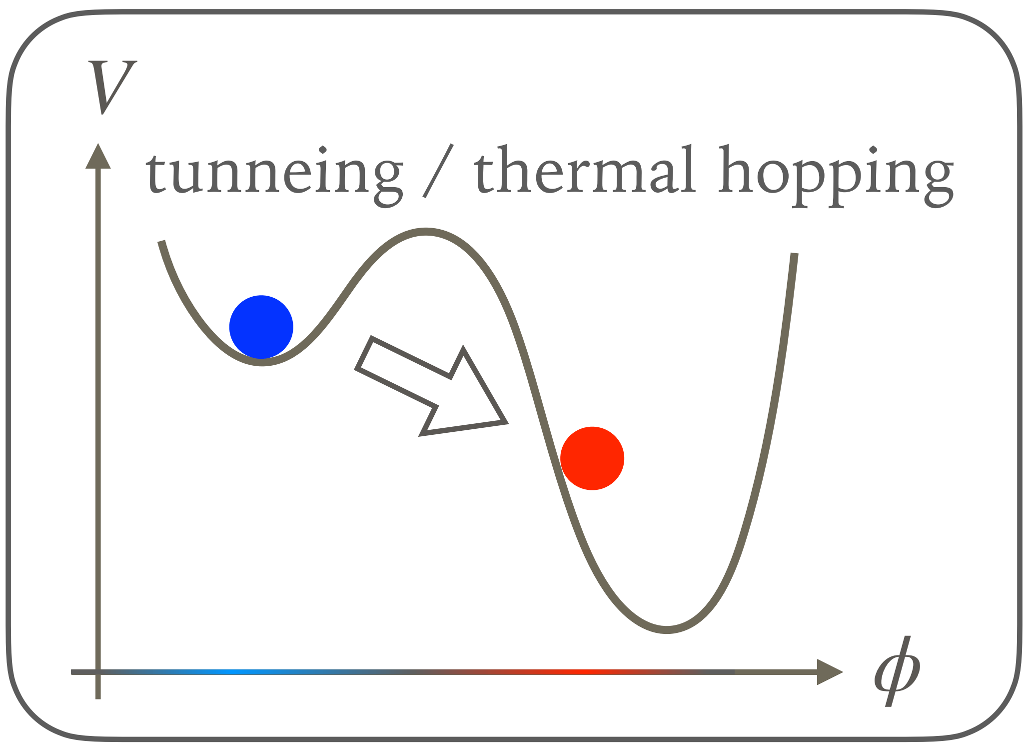
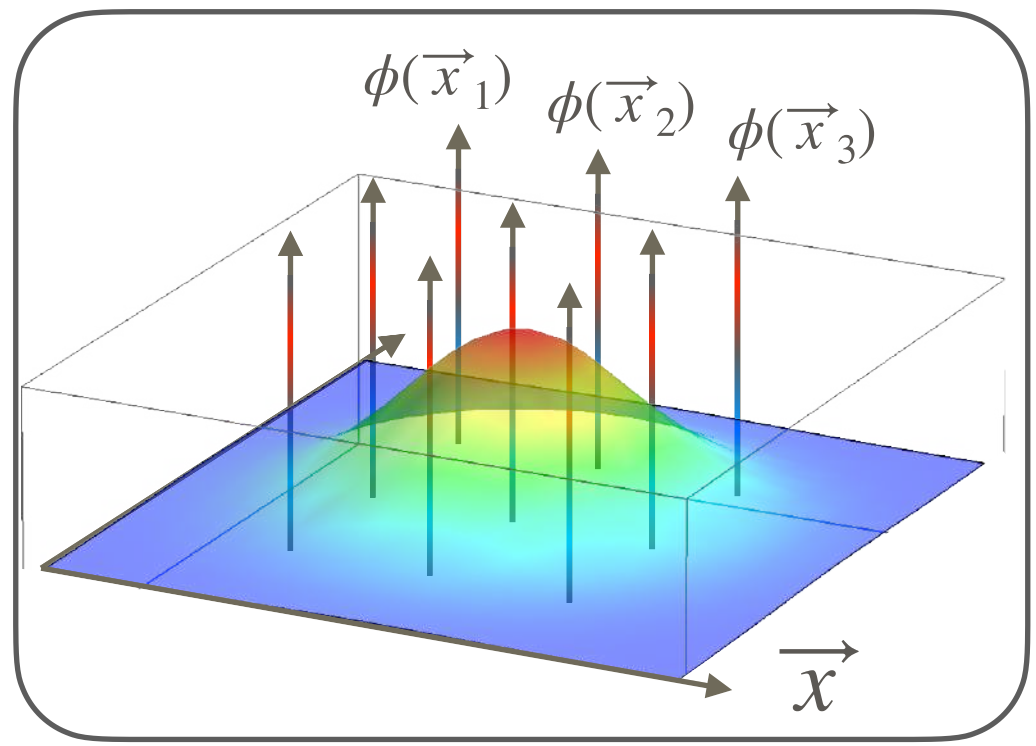
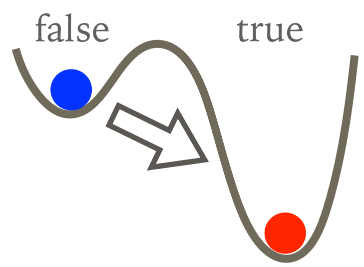


Position space



核生成 (nucleation)

バブルの拡大



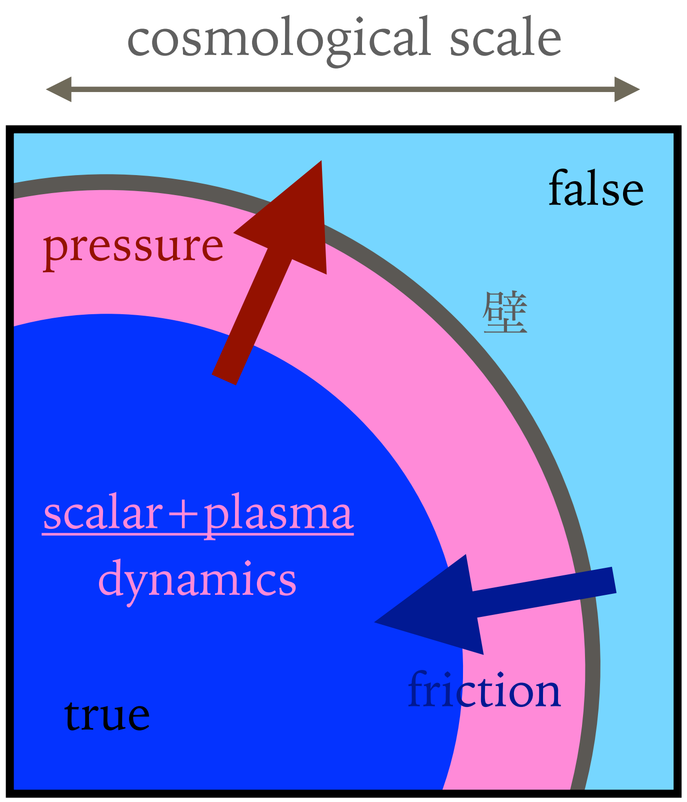
- "Pressure vs. Friction" がバブルの振る舞いを決める

    (1) Pressure: 壁が潜熱解放により外側に押される

    $\alpha \equiv \rho_{\rm vac}/\rho_{\rm plasma}$ でパラメトライズされる

    e.g. [Espinosa et al. '10, Hindmarsh et al. '15, Giese et al. '20]

    (2) Friction: 壁がプラズマとの相互作用で内側に押される



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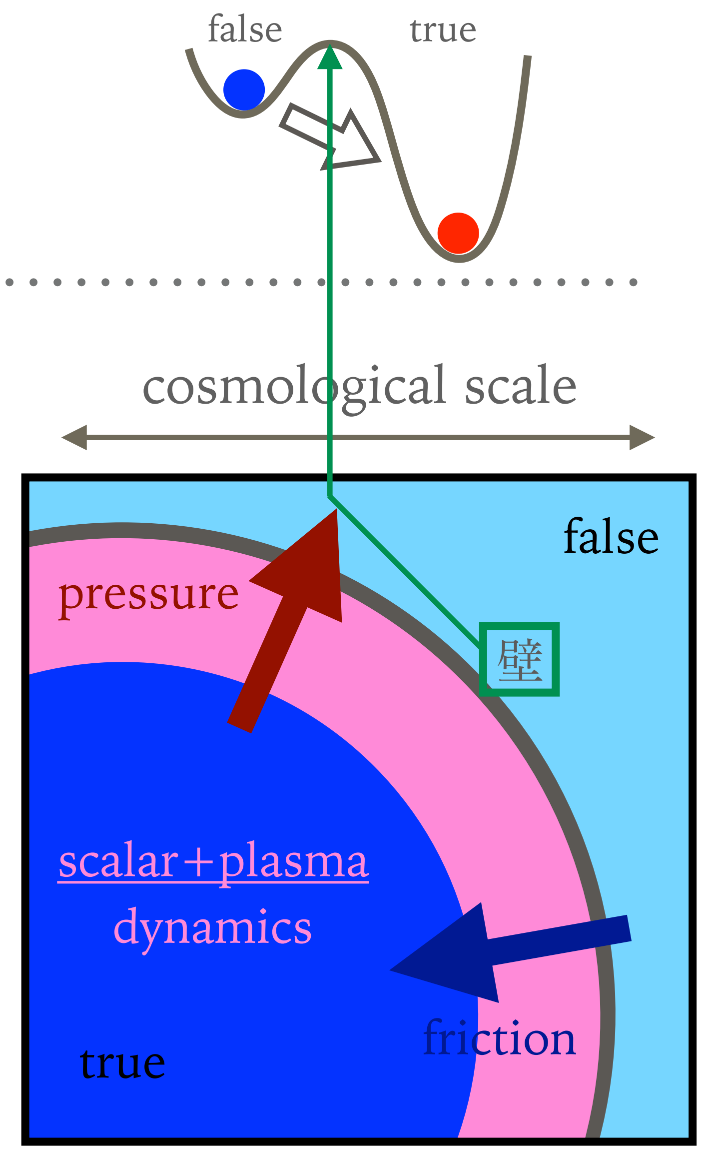

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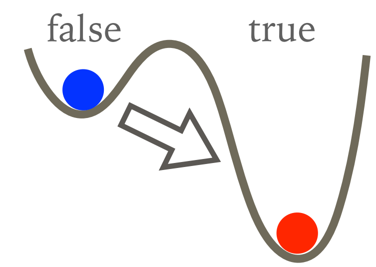

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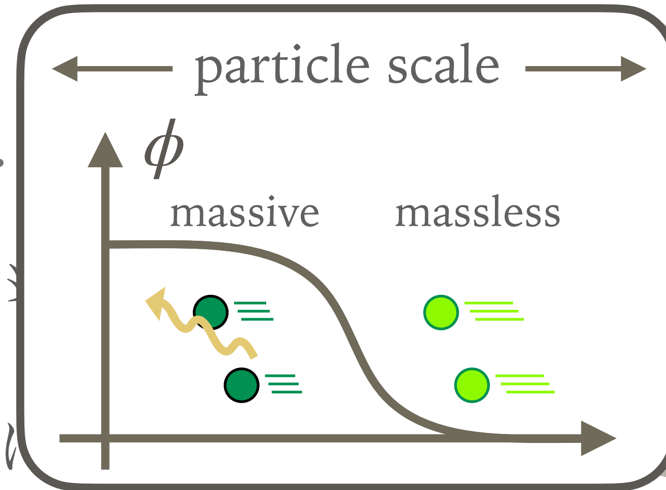

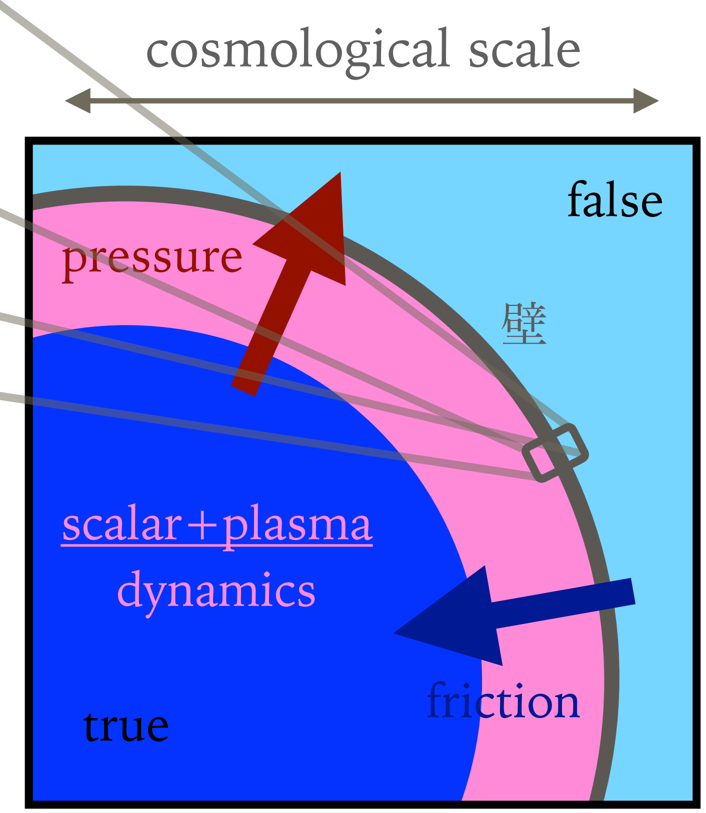

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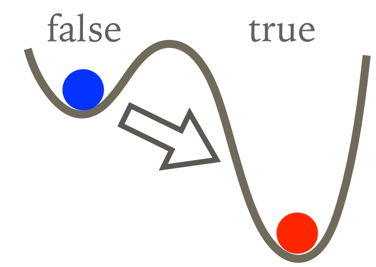

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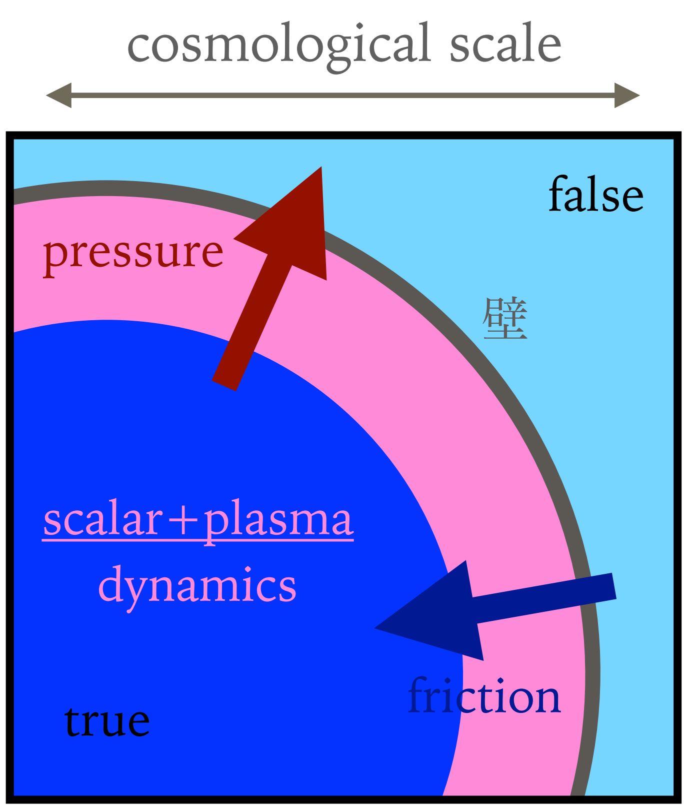
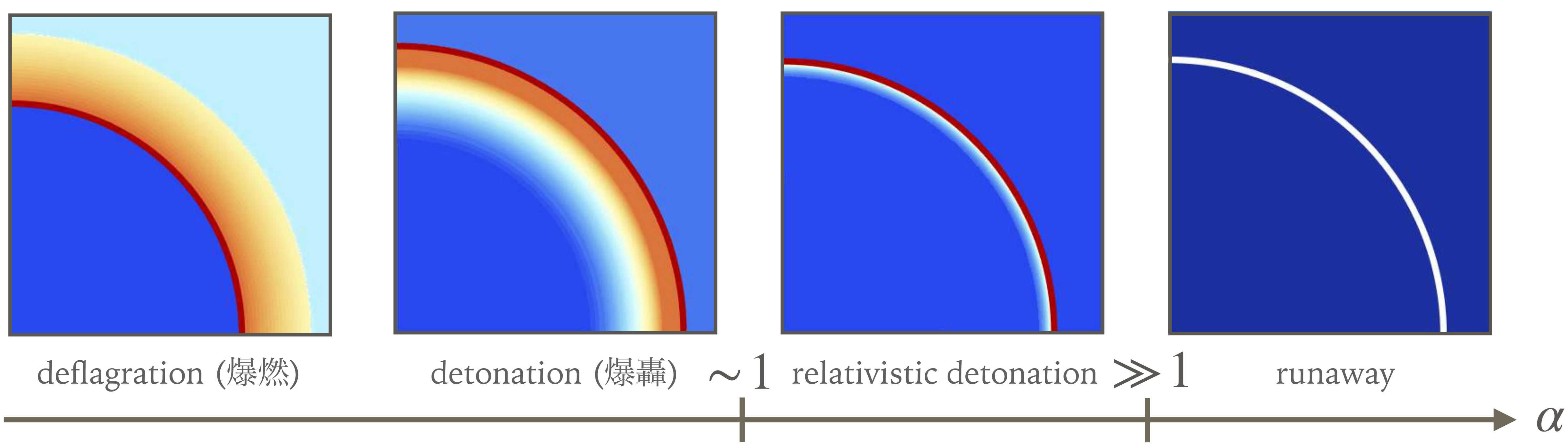
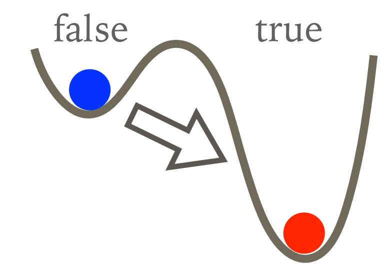


- バブル拡大のいくつかのパターン



バブルの拡大

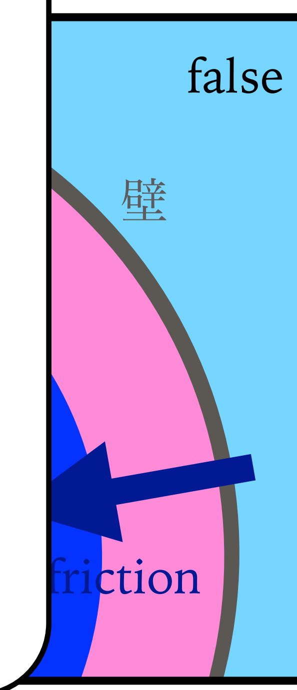


➤ "Pr

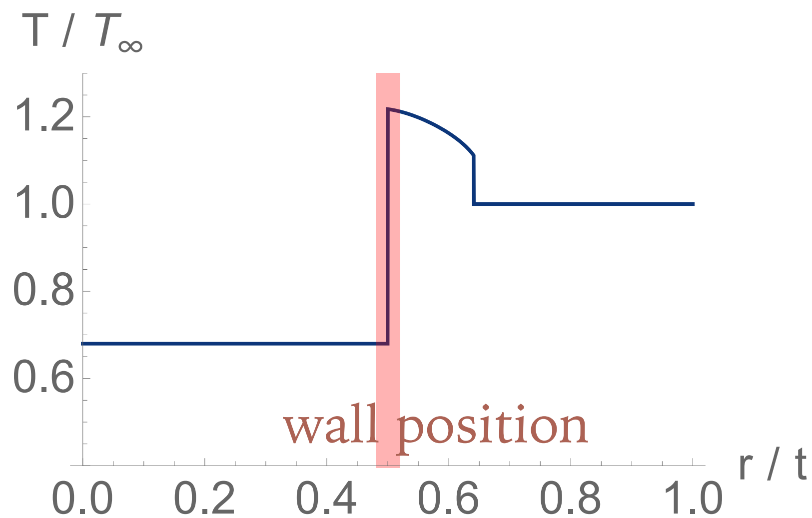
(1)

(2)

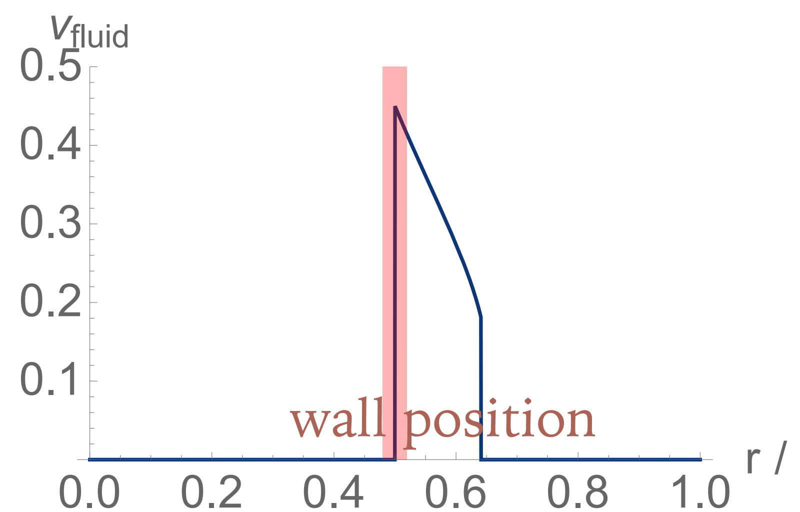
➤ バ

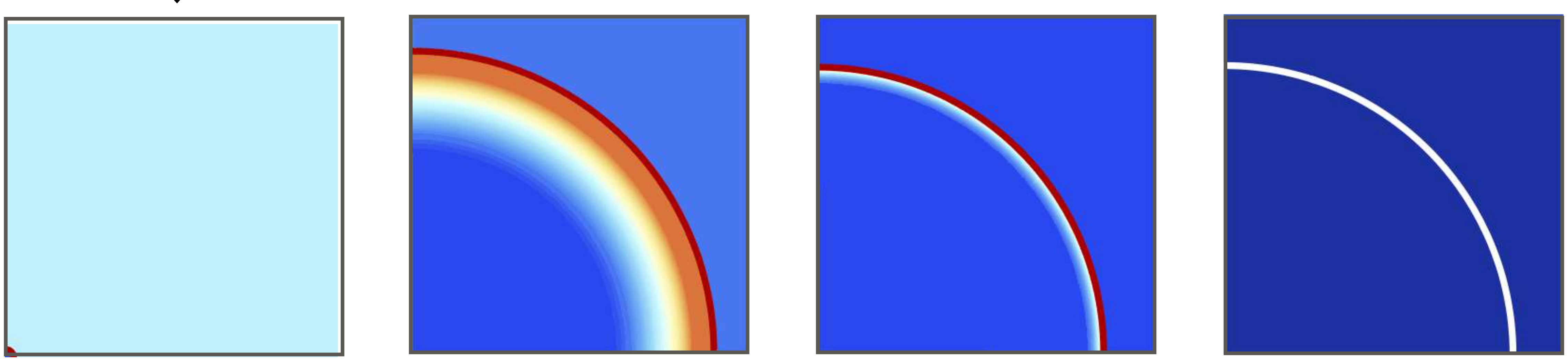


温度



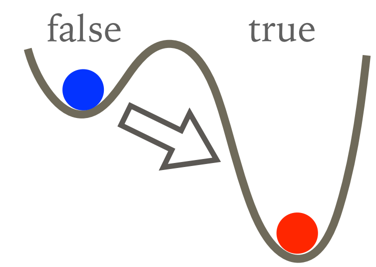
流体の外向き速度





deflagration (爆燃) detonation (爆轟) ~ 1 relativistic detonation $\gg 1$ runaway

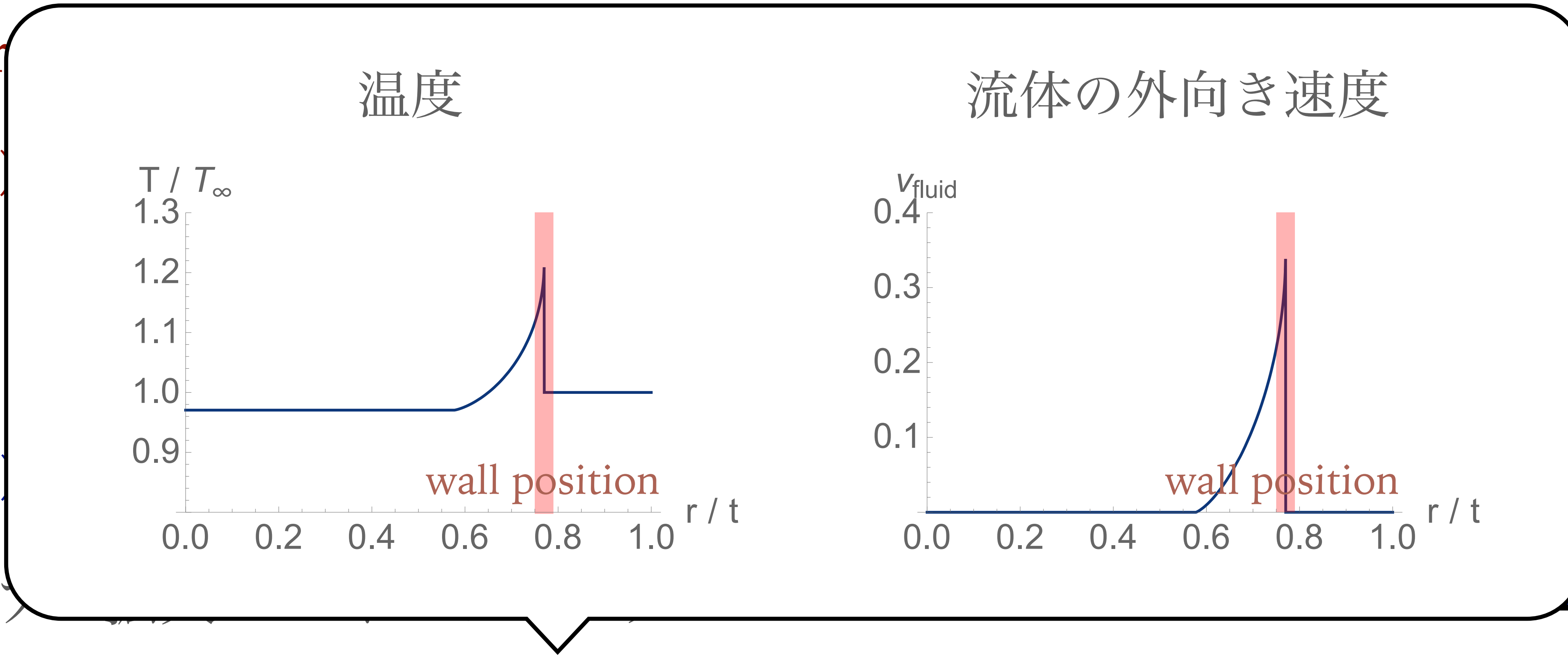
バブルの拡大

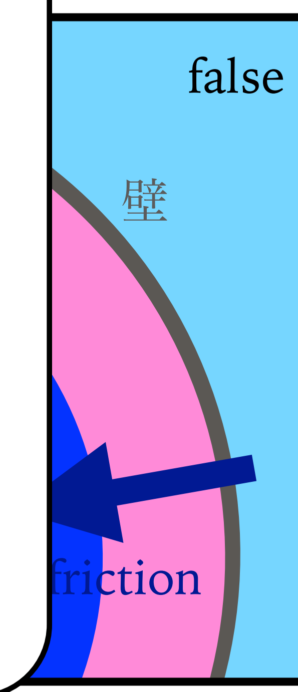


➤ "Pr … l scale

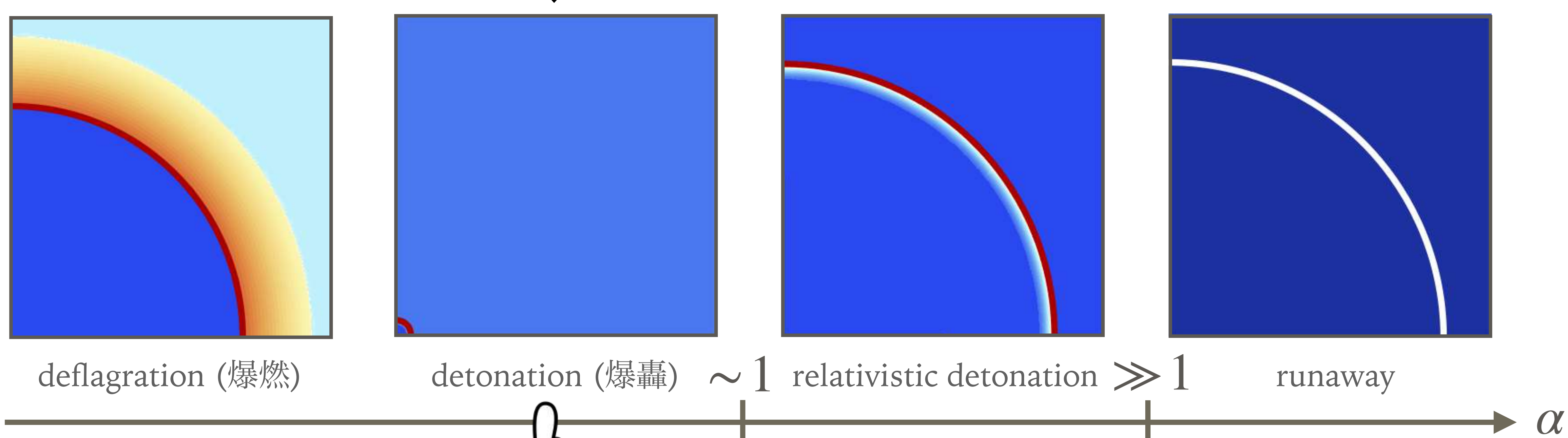
(1)

(2)

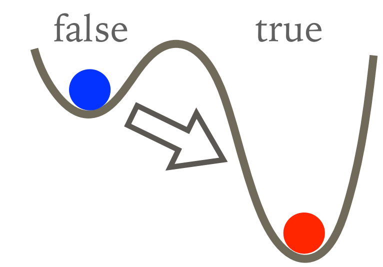




➤ バ



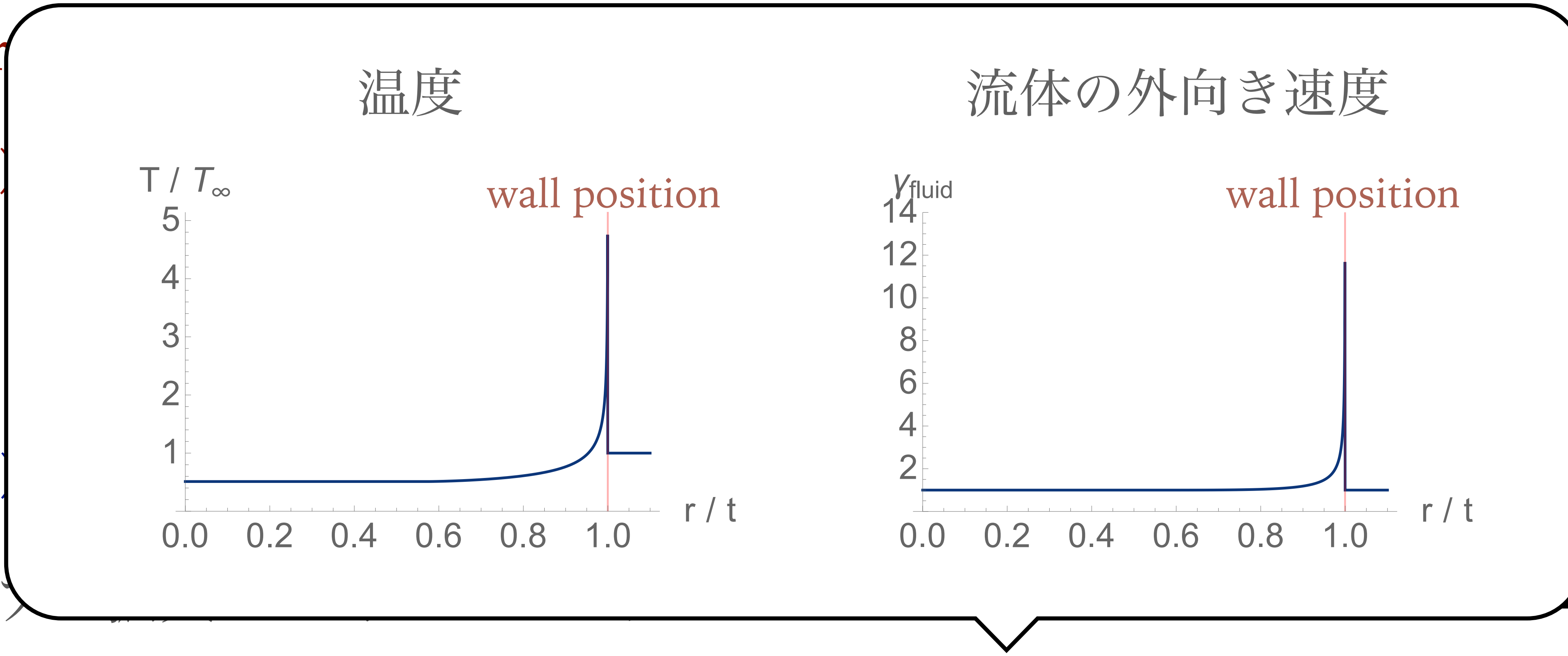
バブルの拡大

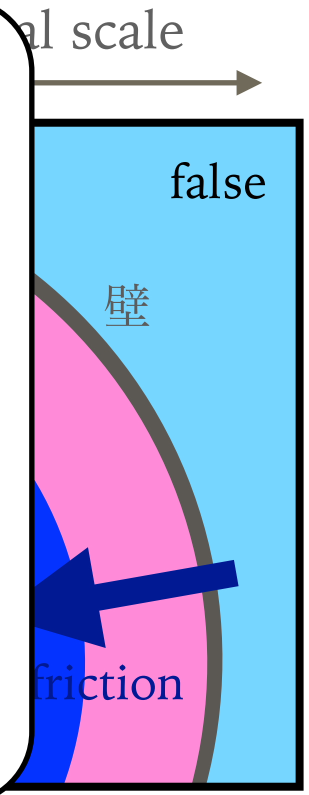


➤ "Pr ... l scale

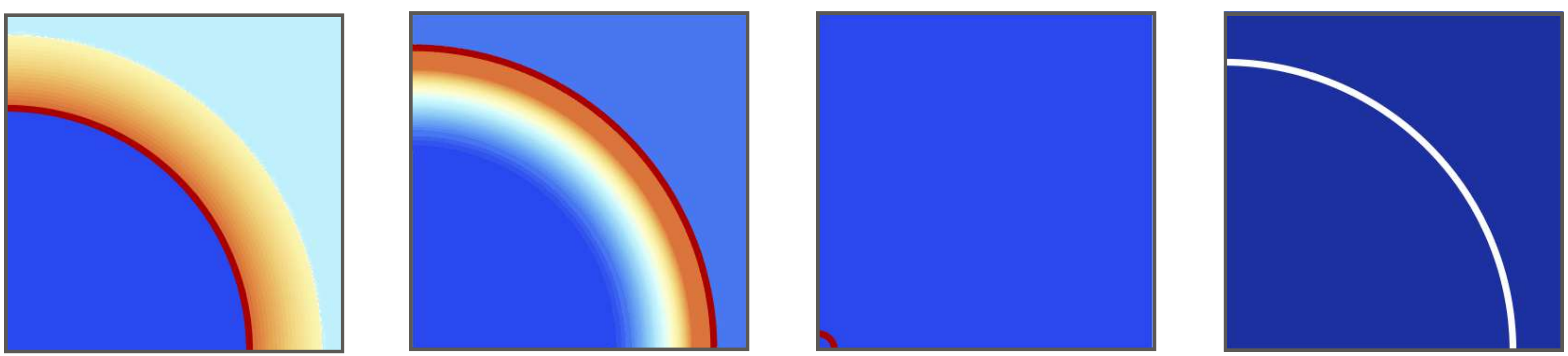
(1)

(2)





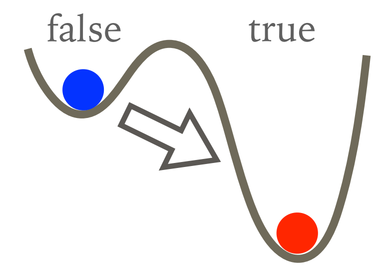
➤ バ



deflagration (爆燃) detonation (爆轟) ~ 1 relativistic detonation $\gg 1$ runaway

α

バブルの拡大



➤ "Pr

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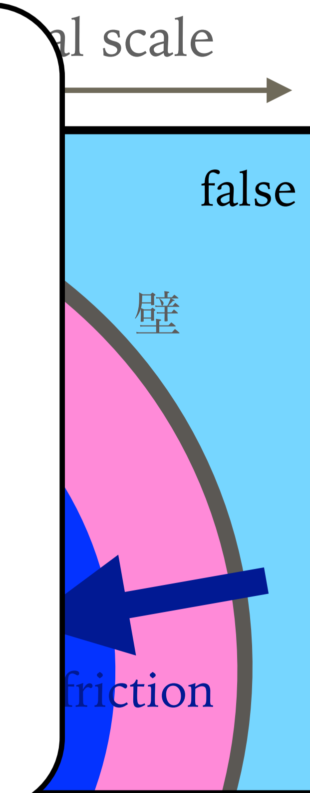
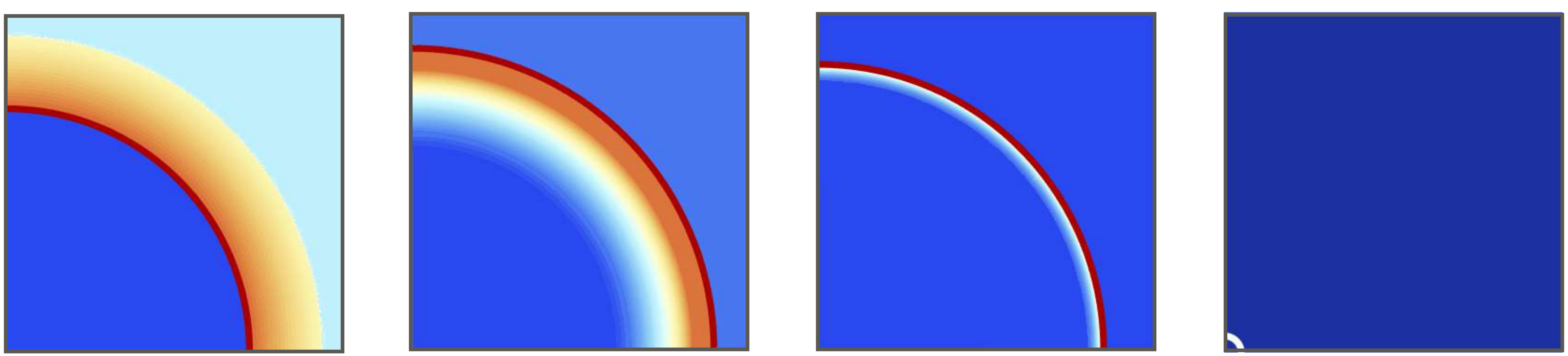
➤ バ



プラズマ粒子が壁の加速を止められない

→ 壁は他のバブルと当たるまで加速し続ける

[Bodeker & Moore '09, '17]

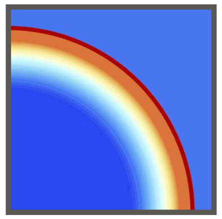


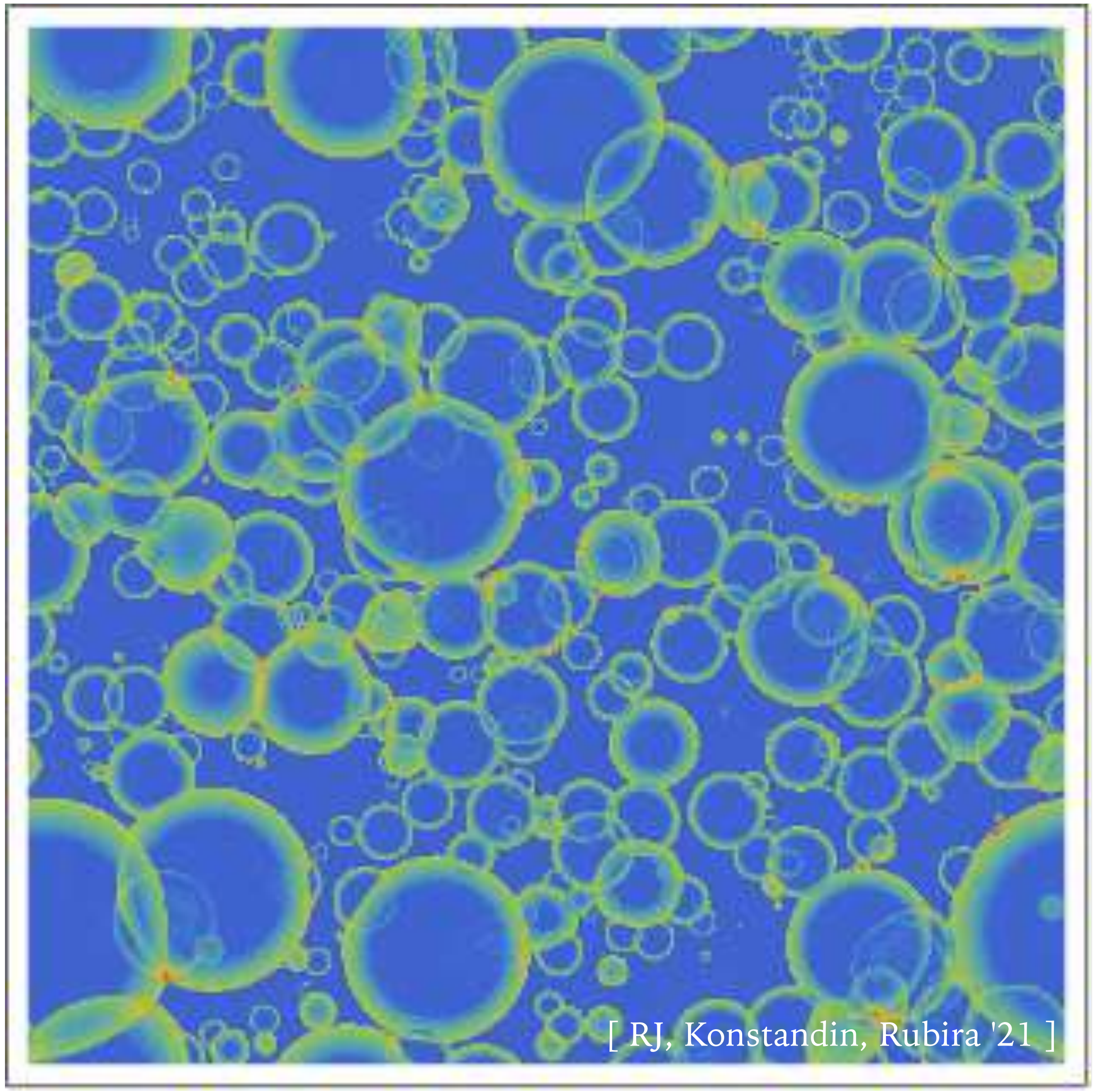
deflagration (爆燃)　detonation (爆轟) ~ 1 relativistic detonation $\gg 1$ runaway

α


Youtube "Explosions: 100 ton test detonation"

バブルの衝突と流体のダイナミクス

➤ バブルが衝突し、流体のダイナミクスが始まる ( の例)



転移パラメータ（熱力学的パラメータ）

- 熱力学の精神：

  系の巨視的な性質を記述するには、いくつかのパラメータで十分

- 今考えている系でそのようなパラメータは何か？

Particle physics $\longrightarrow$ Transition parameters $\longrightarrow$ Prediction on GWs

Lagrangian $\mathscr{L}$

α : transition strength

β : nucleation increase rate

v_w : wall velocity

T_* : transition temperature

GW spectrum Ω_{GW}

転移パラメータ（熱力学的パラメータ）

see e.g. [Caprini et al. '16]
[Caprini et al. '20]

➤ 転移の強さ $\alpha \equiv \rho_{\rm vac}/\rho_{\rm plasma}$

- 周囲のプラズマのエネルギーに比べ、どのくらいのエネルギーが解放されたか

- 分子の $\rho_{\rm vac} = \rho_{\rm vac,false} - \rho_{\rm vac,true}$ は $U = F + TS = F - T\left(\frac{\partial F}{\partial T}\right)_V$ を用いて

$$\rho_{\rm vac,true} = V_{\rm eff}(\phi_{\rm true}, T) - T\left(\frac{\partial V_{\rm eff}(\phi_{\rm true}, T)}{\partial T}\right)$$

$$\rho_{\rm vac,false} = V_{\rm eff}(\phi_{\rm false}, T) - T\left(\frac{\partial V_{\rm eff}(\phi_{\rm false}, T)}{\partial T}\right)$$

のように（ヘルムホルツの）自由エネルギーから求められる

転移パラメータ（熱力学的パラメータ）

see e.g. [Caprini et al. '16]
[Caprini et al. '20]

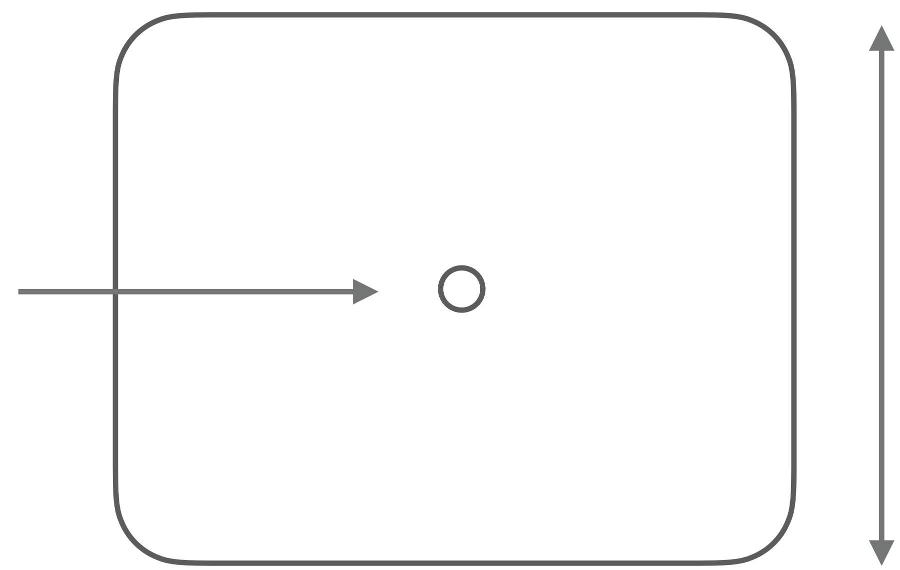
➤ 核生成率の増加率 β ： $\Gamma(t) \propto e^{\beta(t-t_*)+\cdots}$

- 有限温度の場の理論で核生成率 $\Gamma(T)$ を温度の関数として計算

- 温度の関数 $\Gamma(T)$ を (宇宙論的温度) $\Leftrightarrow$ (宇宙論的時間) の関係を用いて $\Gamma(t)$ に変換

- 指数部分を典型的な転移時刻 $t = t_*$ 周りでテーラー展開

ハッブル半径内で
最初にできたバブル



ハッブル半径

$$H^{-1} = \left(\frac{\dot{a}}{a}\right)^{-1}$$

転移パラメータ（熱力学的パラメータ）

see e.g. [Caprini et al. '16]
[Caprini et al. '20]

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有限温度の場の理論 　　　　 宇宙論

$$\boxed{\Gamma(T) \sim T^4 e^{-S_3/T}} \quad = \quad \boxed{H^4 \sim (T^2/M_P)^4}$$

転移パラメータ（熱力学的パラメータ）

see e.g. [Caprini et al. '16]
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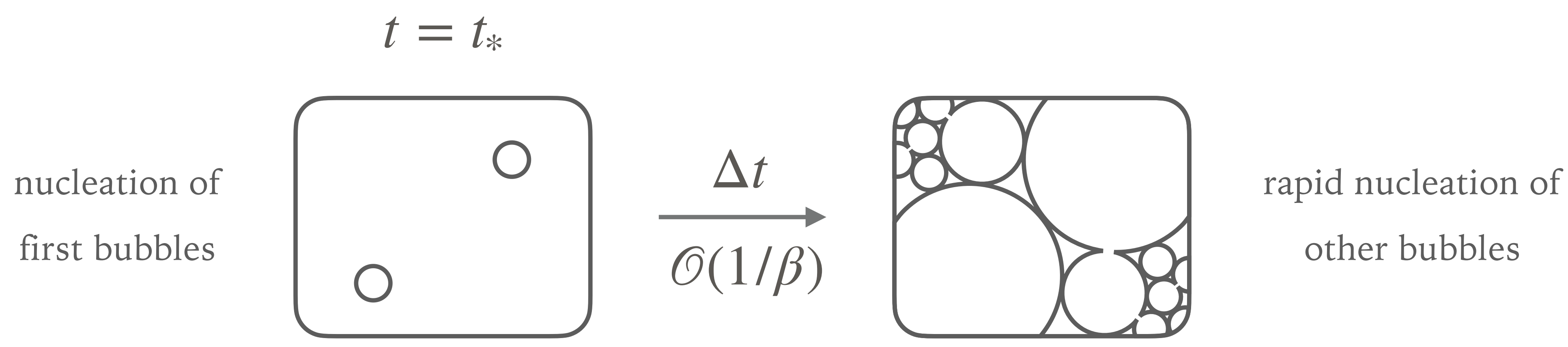
- 指数部分を典型的な転移時刻 $t = t_*$ 周りでテーラー展開

- 面白い性質 : v_w/β が衝突時の典型的なバブルサイズを与える

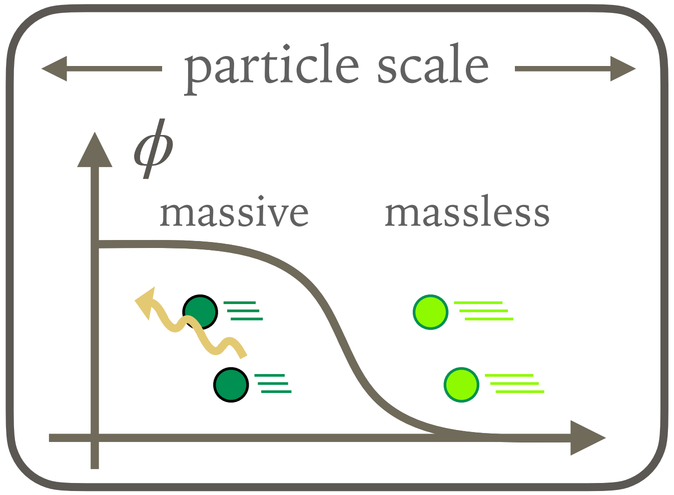


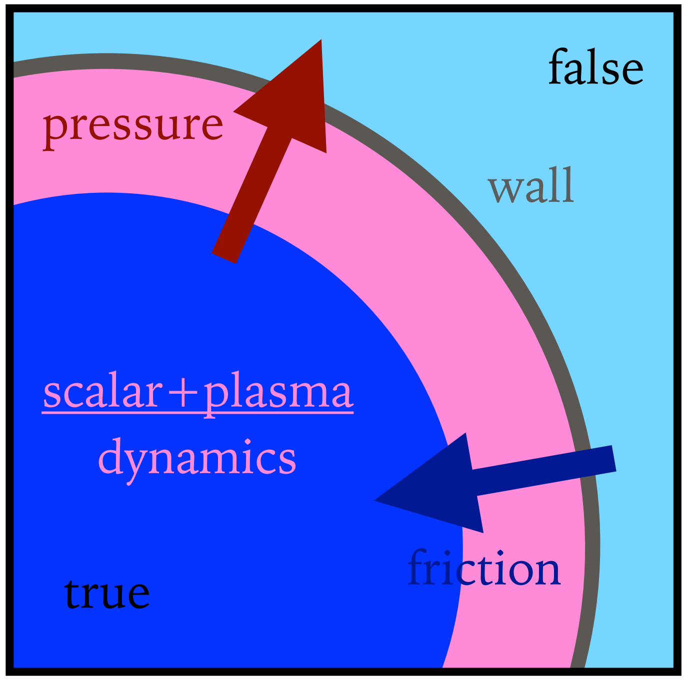
転移パラメータ（熱力学的パラメータ）

➤ 壁の速度 v_w

- pressure と friction のバランスから決まる

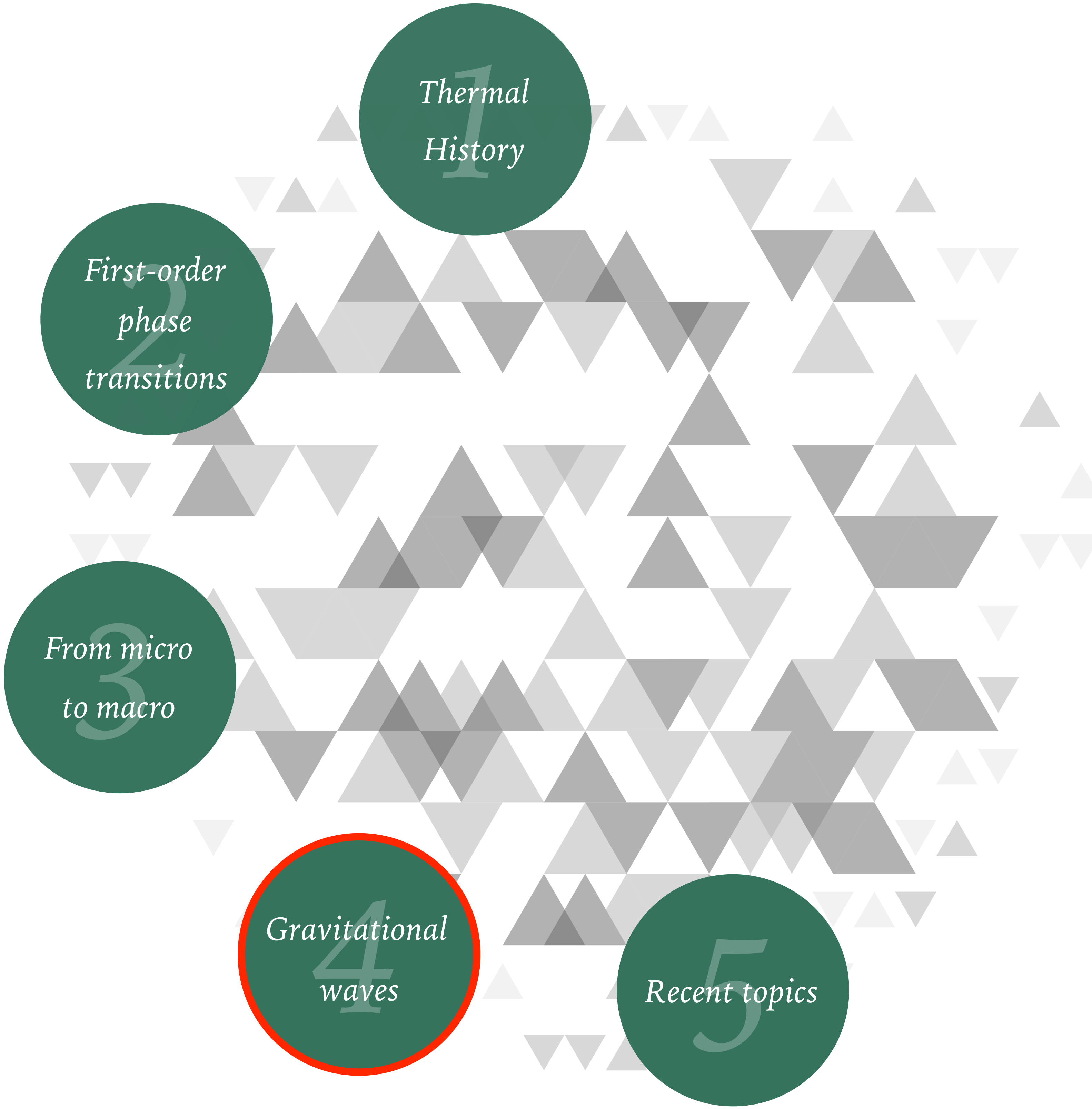
- 本来は壁周りのボルツマン方程式を解いて決めるが、
  手で置いてしまうことも多い
  (その場合、スカラー場とプラズマの結合定数の取り方の
  自由度を壁の速度の取り方に変換した、とも見れる)





➤ 転移温度 T_*

- ミクロな理論の典型的なエネルギースケールで決まる


1
Thermal History
2
First-order phase transitions
3
From micro to macro
4
Gravitational waves
5
Recent topics

重力波：宇宙への新たなプローブ

- アインシュタイン方程式

$$\boxed{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}}$$

"Space(-time) tells matter how to move. Matter tells space(-time) how to curve."

- 重力波： 計量のtransverse-traceless部分

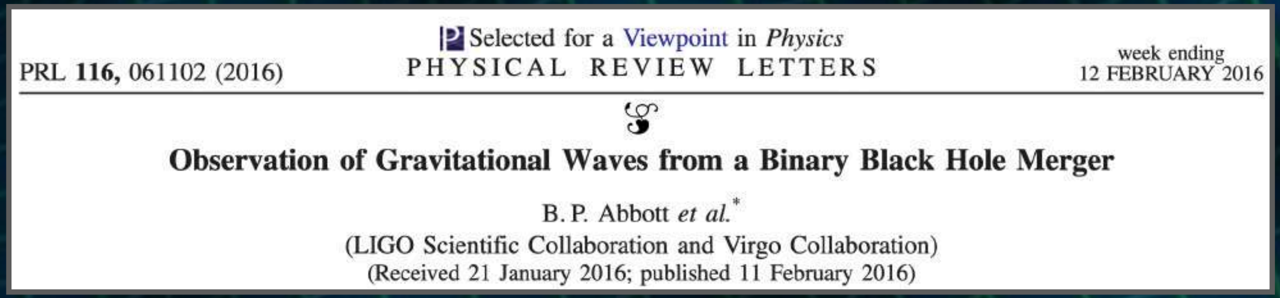
$$ds^2 = -dt^2 + a^2(\delta_{ij} + h_{ij})dx^i dx^j \qquad \partial_i h_{ij} = h_{ii} = 0$$

- アインシュタイン方程式を展開すると、線形近似のオーダーで

  系のエネルギー運動量テンソルによってソースされる波動方程式に従うことがわかる

$$\Box h_{ij} = 16\pi G \Lambda_{ij,kl} T_{kl}$$

- LIGO/Virgoが2015年にブラックホール連星系からの重力波を初検出



Selected for a Viewpoint in *Physics*

PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.**

(LIGO Scientific Collaboration and Virgo Collaboration)

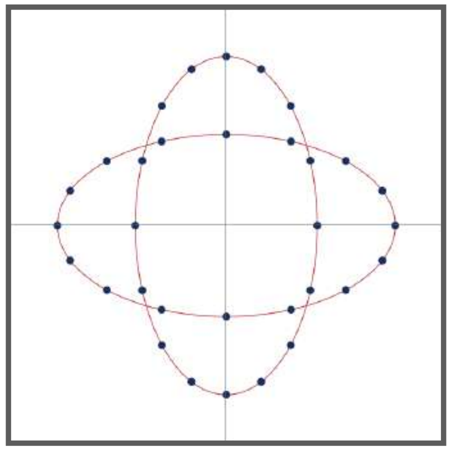
(Received 21 January 2016; published 11 February 2016)

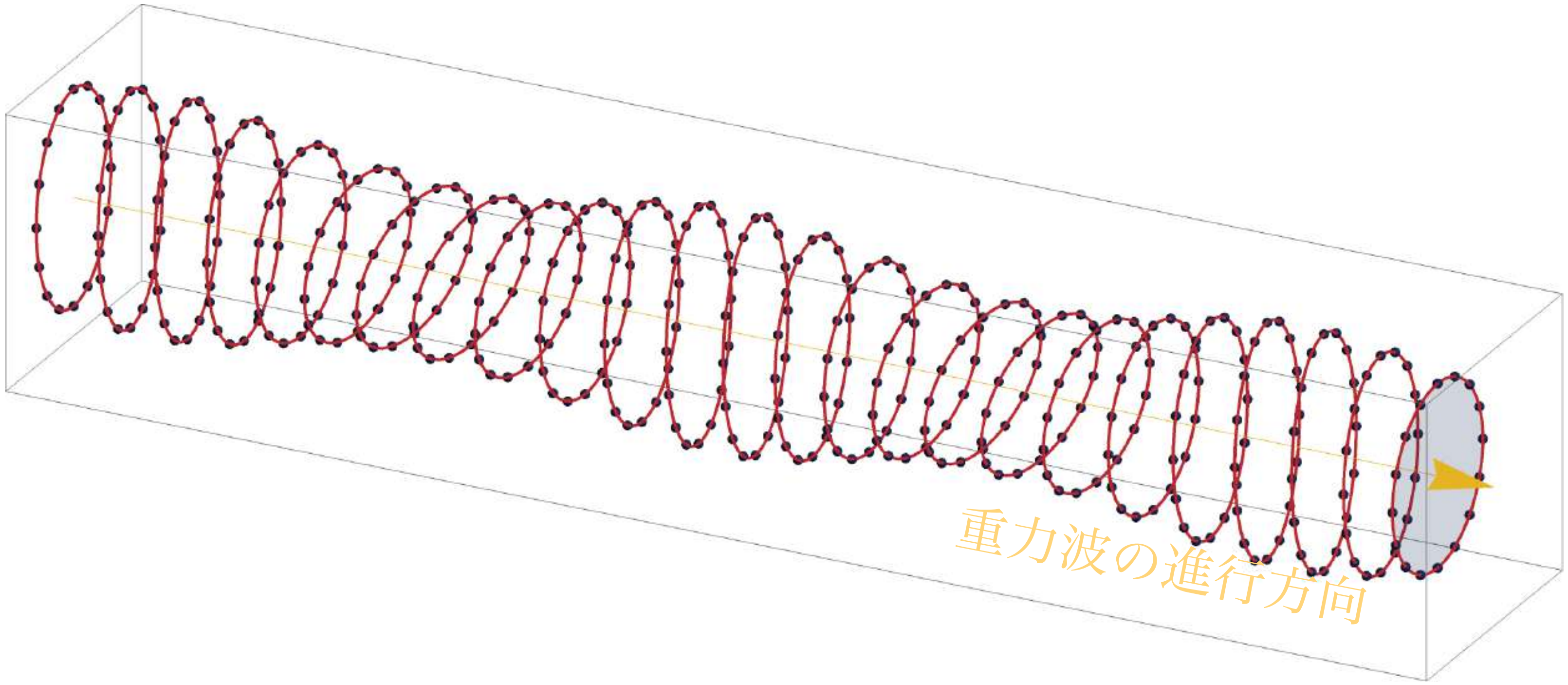
$$36 M_\odot + 29 M_\odot \rightarrow 62 M_\odot + 3 M_\odot \text{ (GWs)}$$

[R. Hurt/Caltech-JPL]

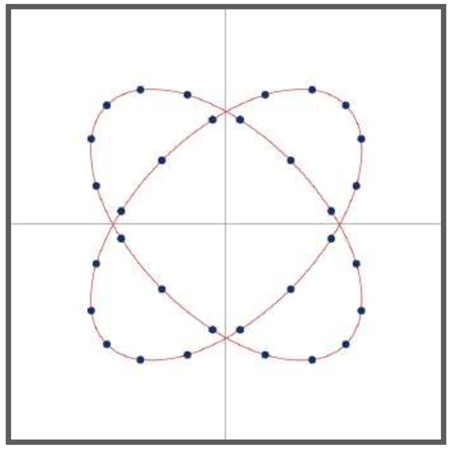
重力波の伝播

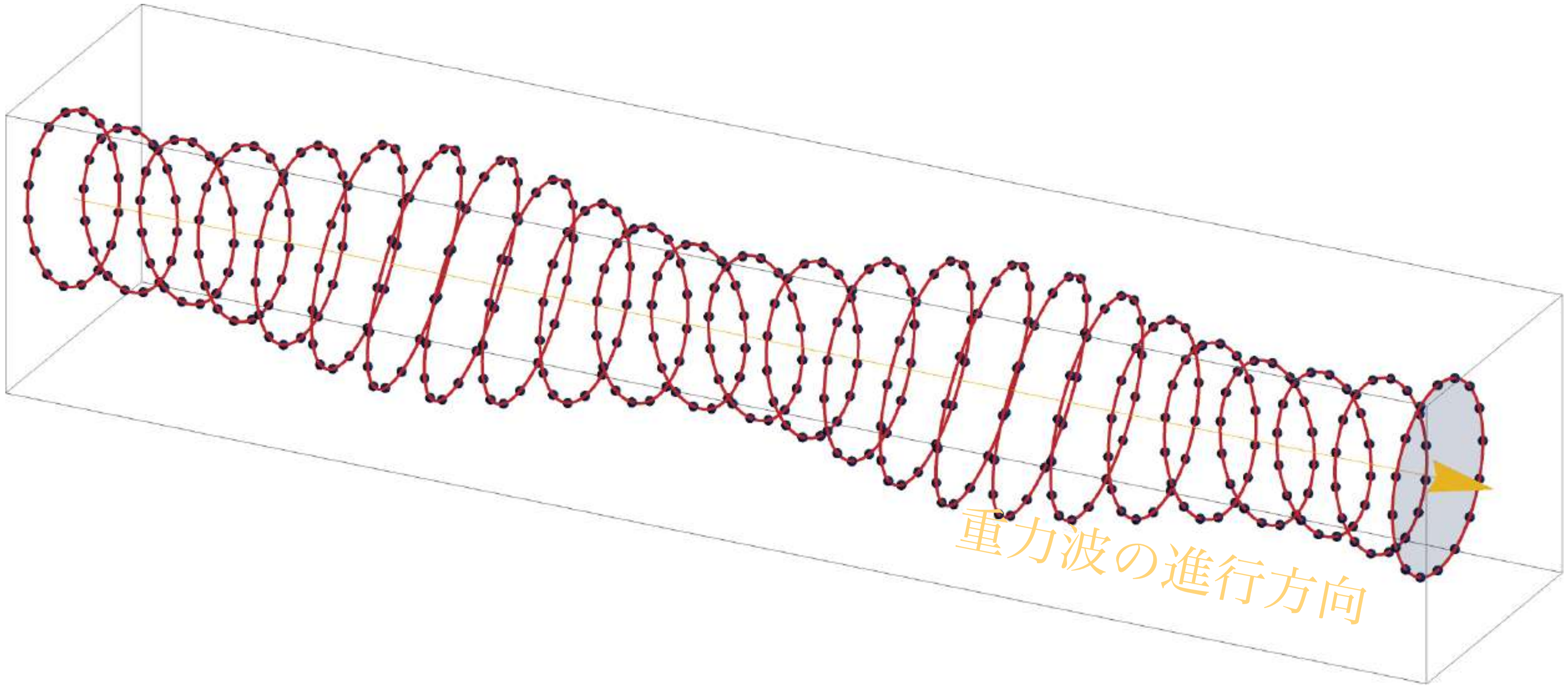
＋モード



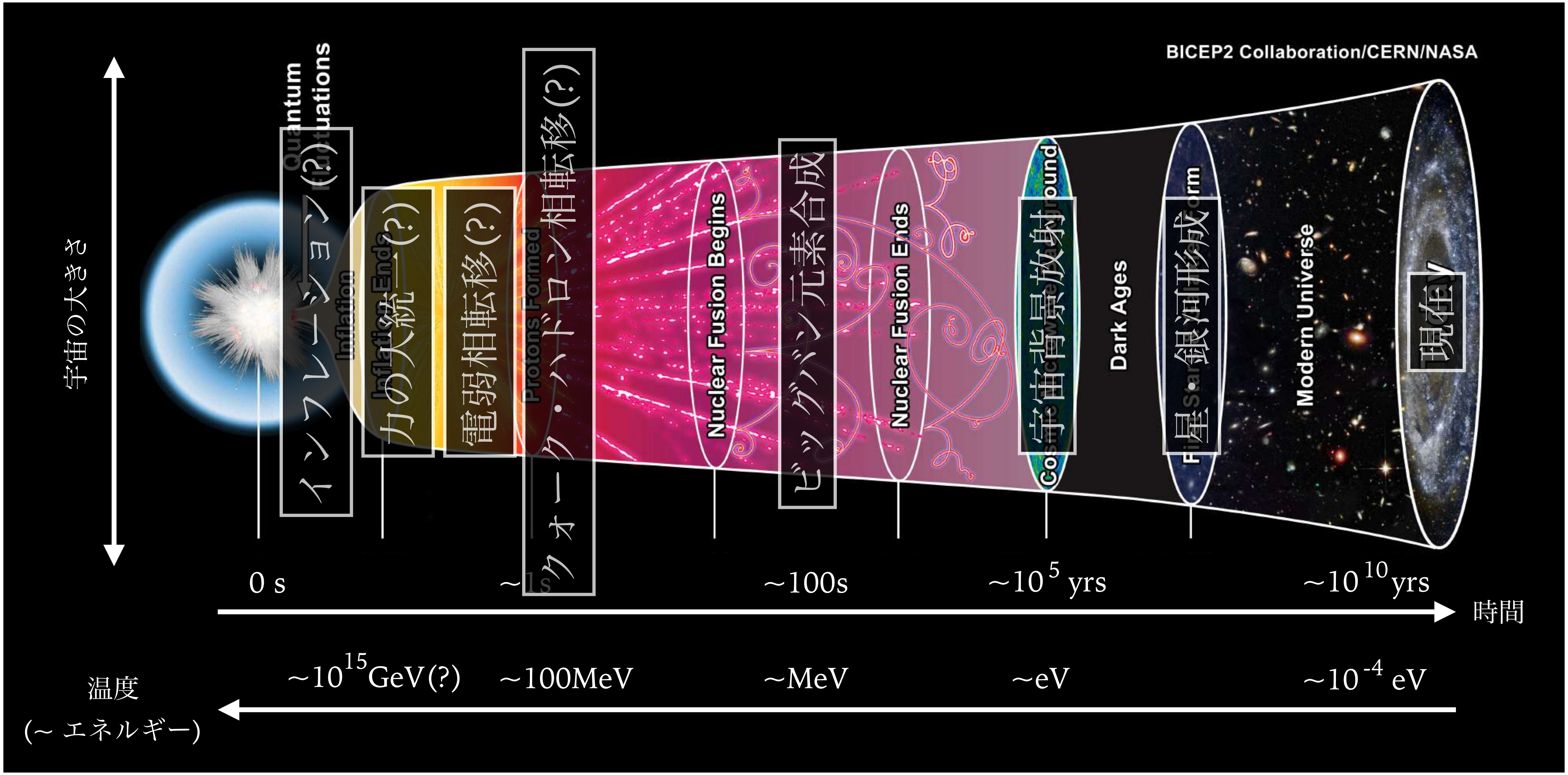


×モード





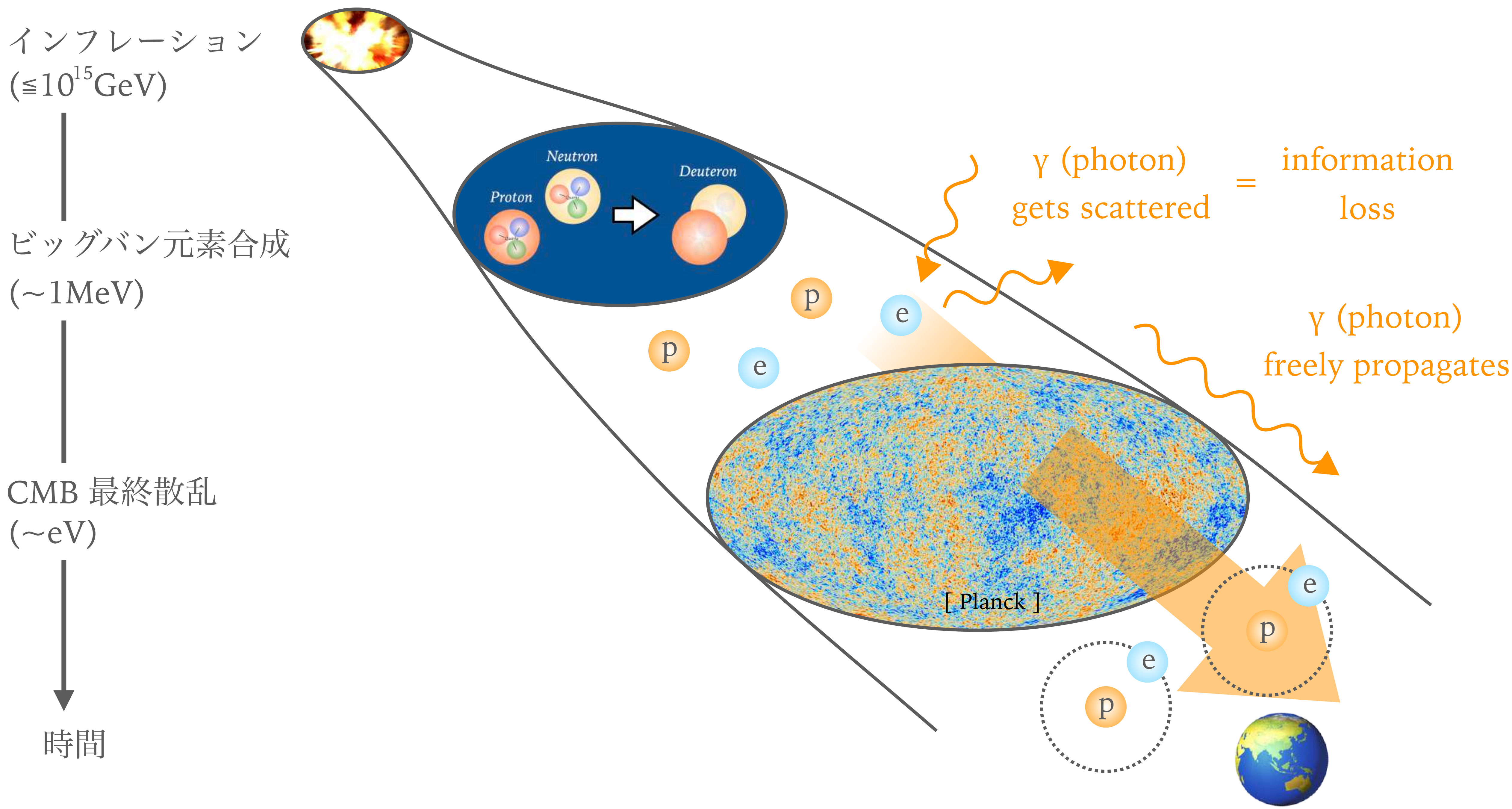
宇宙の熱史



宇宙の歴史 = 冷却の歴史

初期宇宙のプローブ

➤ 宇宙背景放射 (CMB = Cosmic Microwave Background)



初期宇宙のプローブ

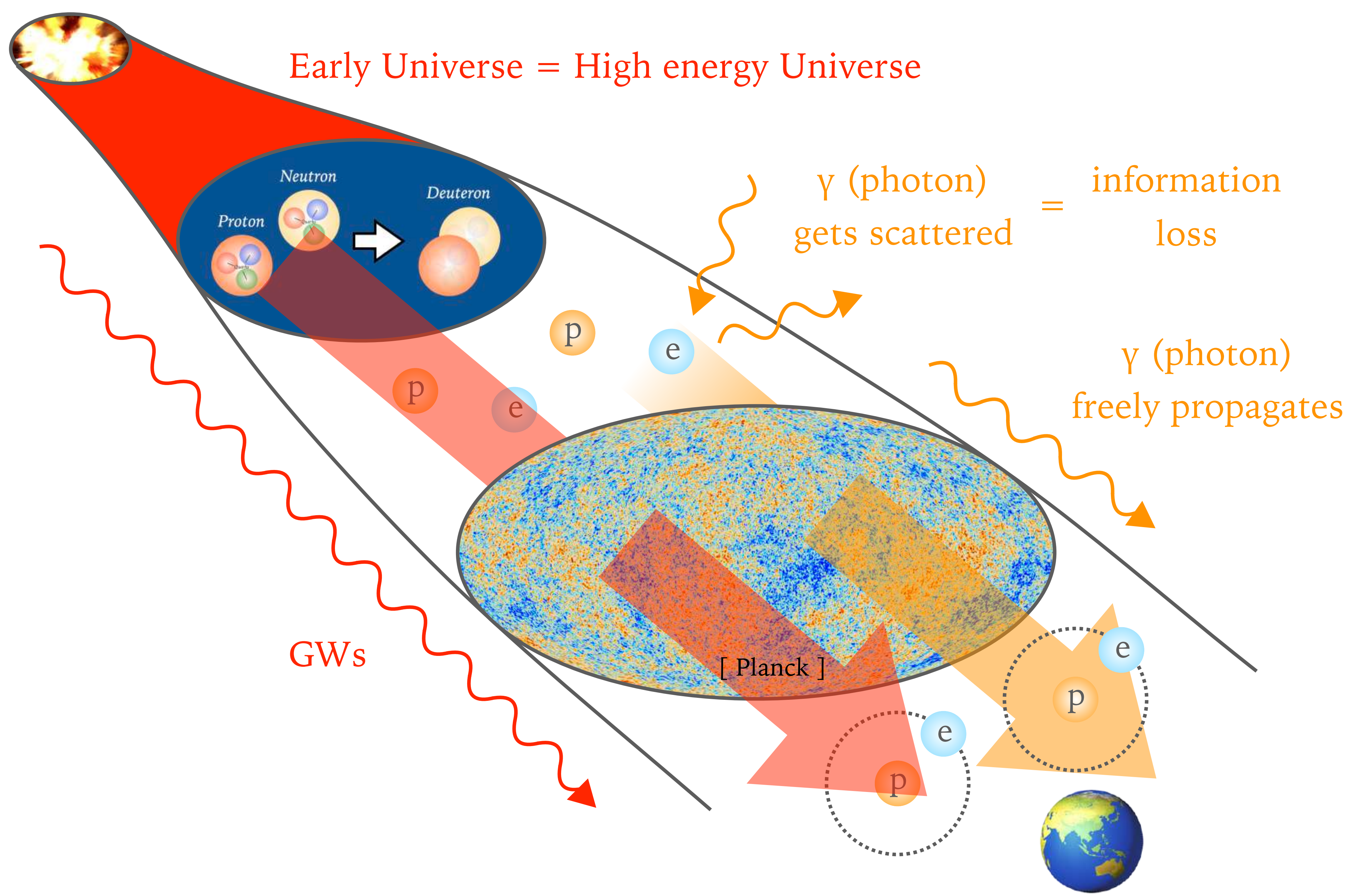
➤ 重力波

インフレーション
(≲10^{15}GeV)

ビッグバン元素合成
(~1MeV)

CMB 最終散乱
(~eV)

時間



初期宇宙のプローブ

- 宇宙背景放射 (CMB = Cosmic Microwave Background)

  - 最終散乱面から放出された光子

  - 探れる温度は典型的に ~eV まで (それ以上の温度では散乱されてしまうため)

  - y-distortion や μ-distortion を用いると ~keV まで頑張れる

  - よくインフレーションを探れると言われるが、それは少し例外

    (curvature perturbationの保存というちょっとトリッキーな事実を用いているため)

- 重力波 (Gravitational Waves)

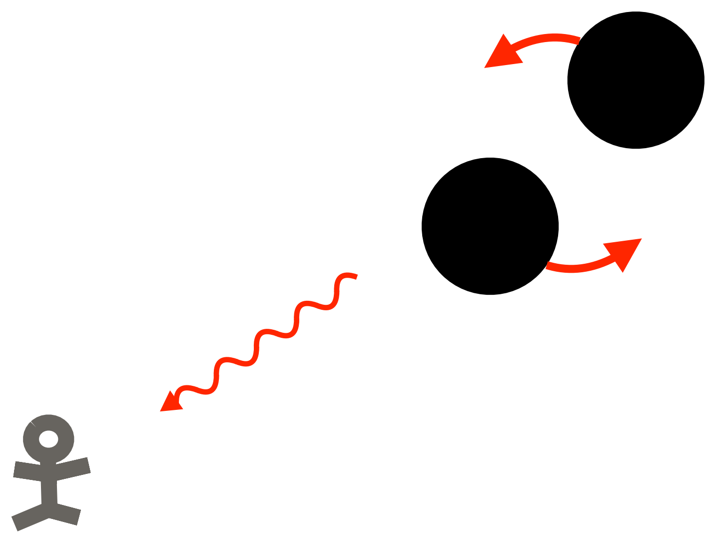
  - 物質と非常に弱い相互作用しかしない

  - そのため、一旦生成されると生成当時の情報を保ったまま現在まで伝搬する

天文学的重力波 VS. 宇宙論的重力波

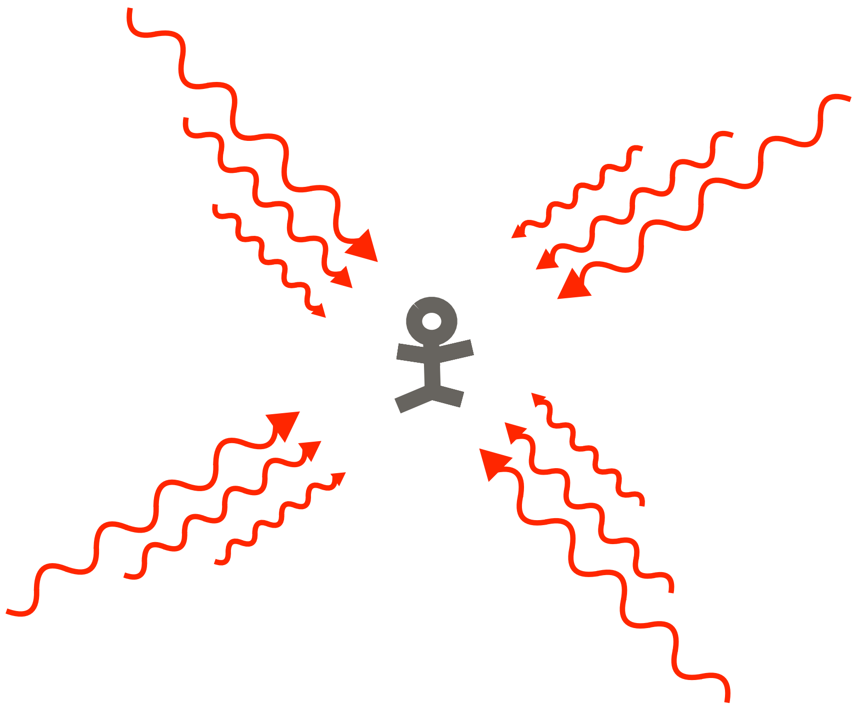
➤ 天文学的重力波 = 点源からの重力波

- 決まった方向
- 狭い周波数帯
- 検出器がそれぞれのソースを
  特定できないときに限り、stochasitcに見える see e.g. [Romano, Cornish '17] for further discussion



➤ 宇宙論的重力波 = stochastic

- ランダムな方向
- 幅広い周波数帯
- パワースペクトルにより特徴付けられる

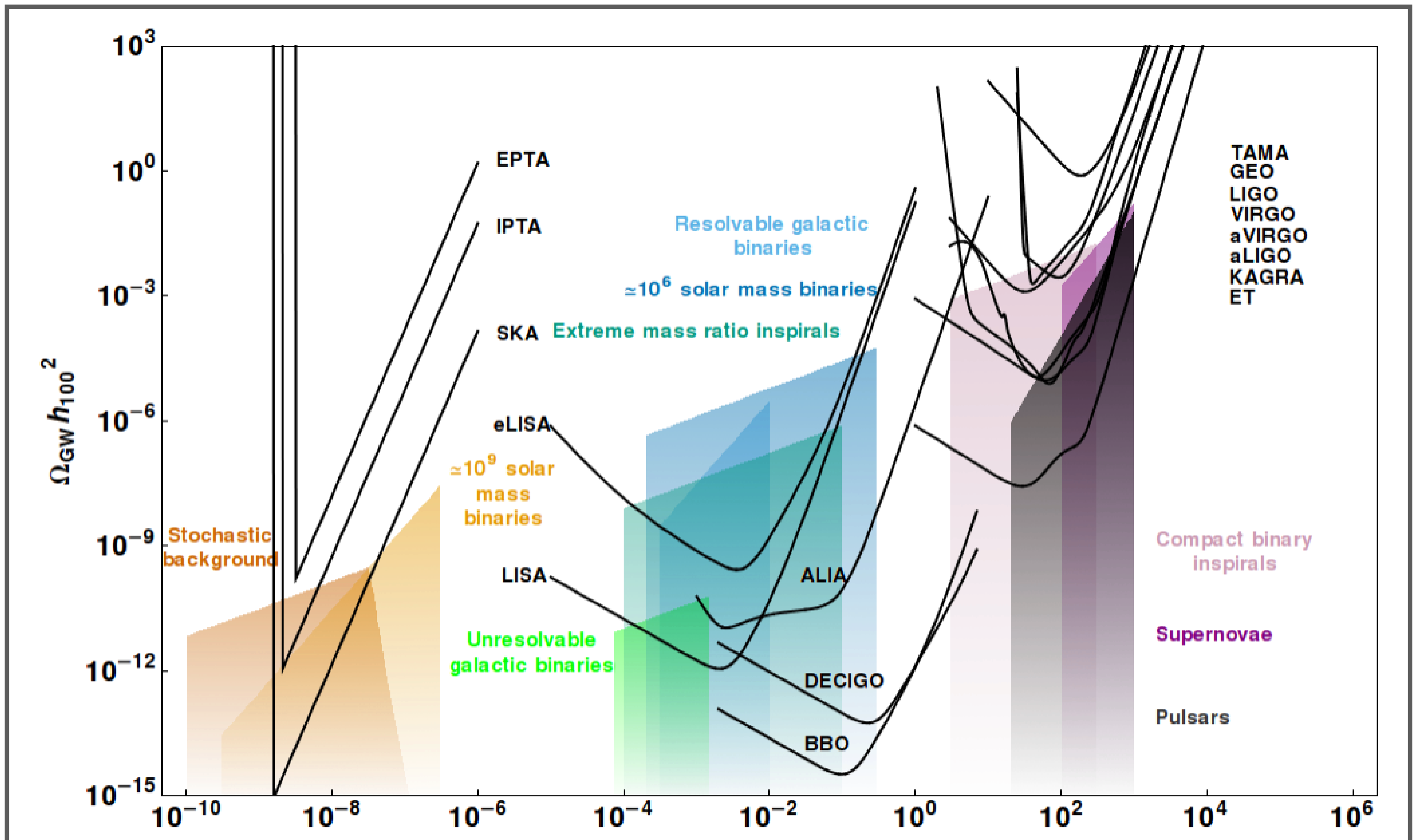


重力波の現在 & 将来観測

Pulsar timing arrays	Space-borne interferometers	Ground-based interferometers
$\sim 10^{-8}$ Hz	$\sim$ mHz – Hz	$\sim$ 100 Hz

log周波数あたりの重力波のエネルギー密度

現在の宇宙のエネルギー密度



[Moore, Cole, Berry '14]

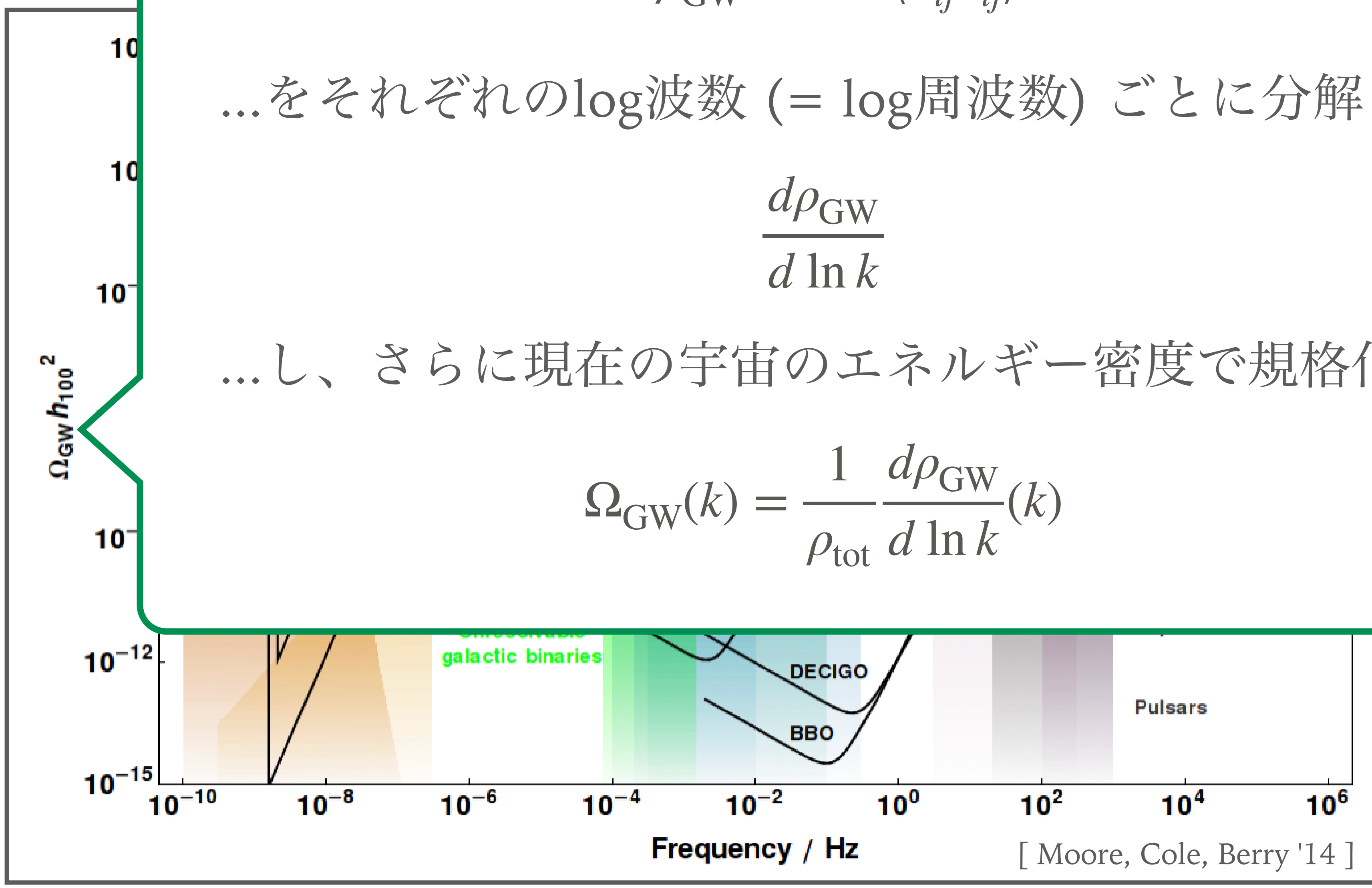
重力波の現在 & 将来観測

Puls

$\sim$

log周波数あたりの重力波のエネルギー密度

現在の宇宙のエネルギー密度



全周波数合わせた重力波のエネルギー密度

$$\rho_{\rm GW} \sim G^{-1}\langle \dot{h}_{ij}\dot{h}_{ij}\rangle$$

...をそれぞれのlog波数 (= log周波数) ごとに分解

$$\frac{d\rho_{\rm GW}}{d\ln k}$$

...し、さらに現在の宇宙のエネルギー密度で規格化

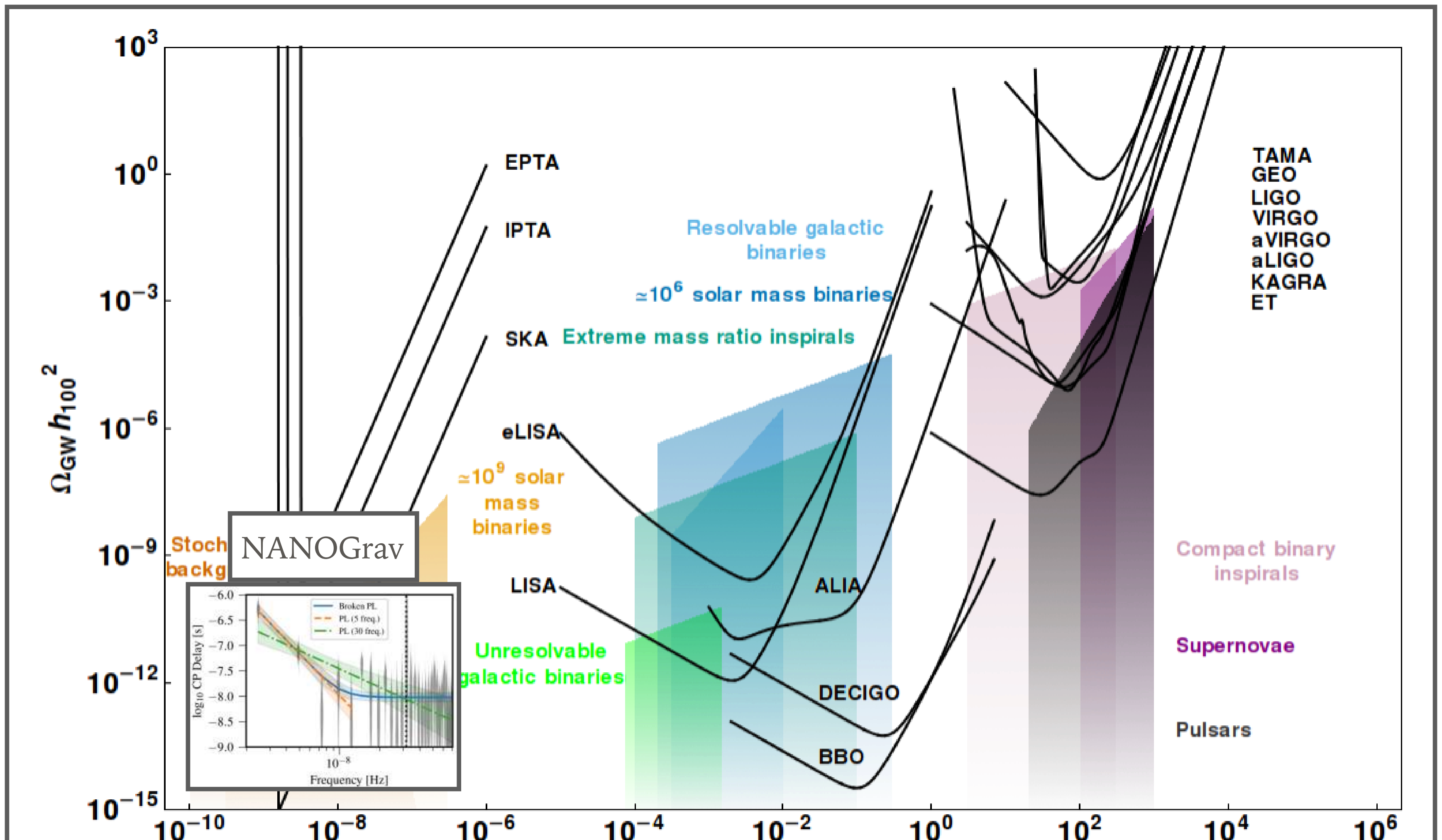
$$\Omega_{\rm GW}(k) = \frac{1}{\rho_{\rm tot}}\frac{d\rho_{\rm GW}}{d\ln k}(k)$$

[Moore, Cole, Berry '14]

重力波の現在 & 将来観測

Pulsar timing arrays	Space-borne interferometers	Ground-based interferometers
$\sim 10^{-8}$ Hz	$\sim$ mHz – Hz	~ 100 Hz

log周波数あたりの重力波のエネルギー密度 / 現在の宇宙のエネルギー密度



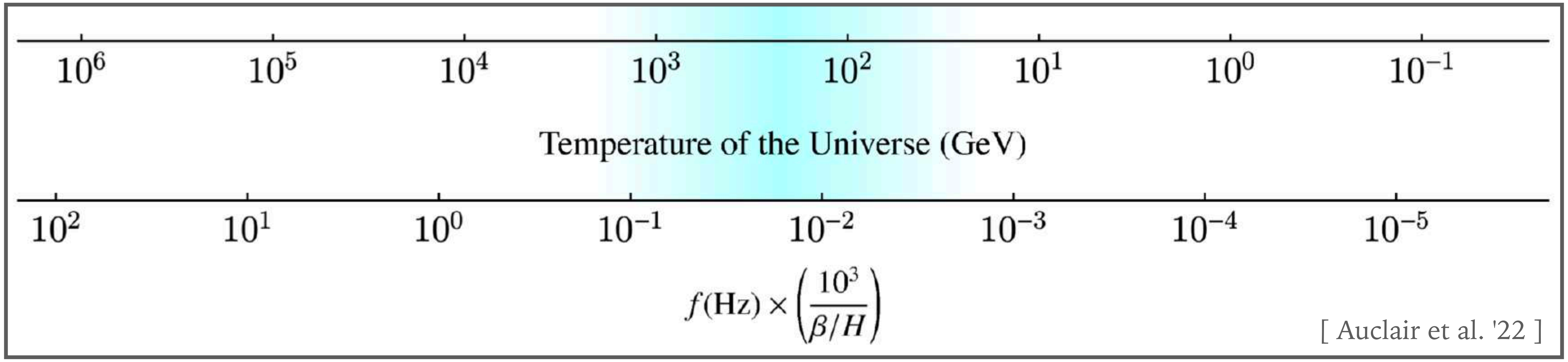
[NANOGrav Collab. '20]

[Moore, Cole, Berry '14]

初期宇宙で巨視的に (~ハッブル半径サイズで) 生成した重力波の

現在の周波数は、大まかに生成時のエネルギースケールと比例

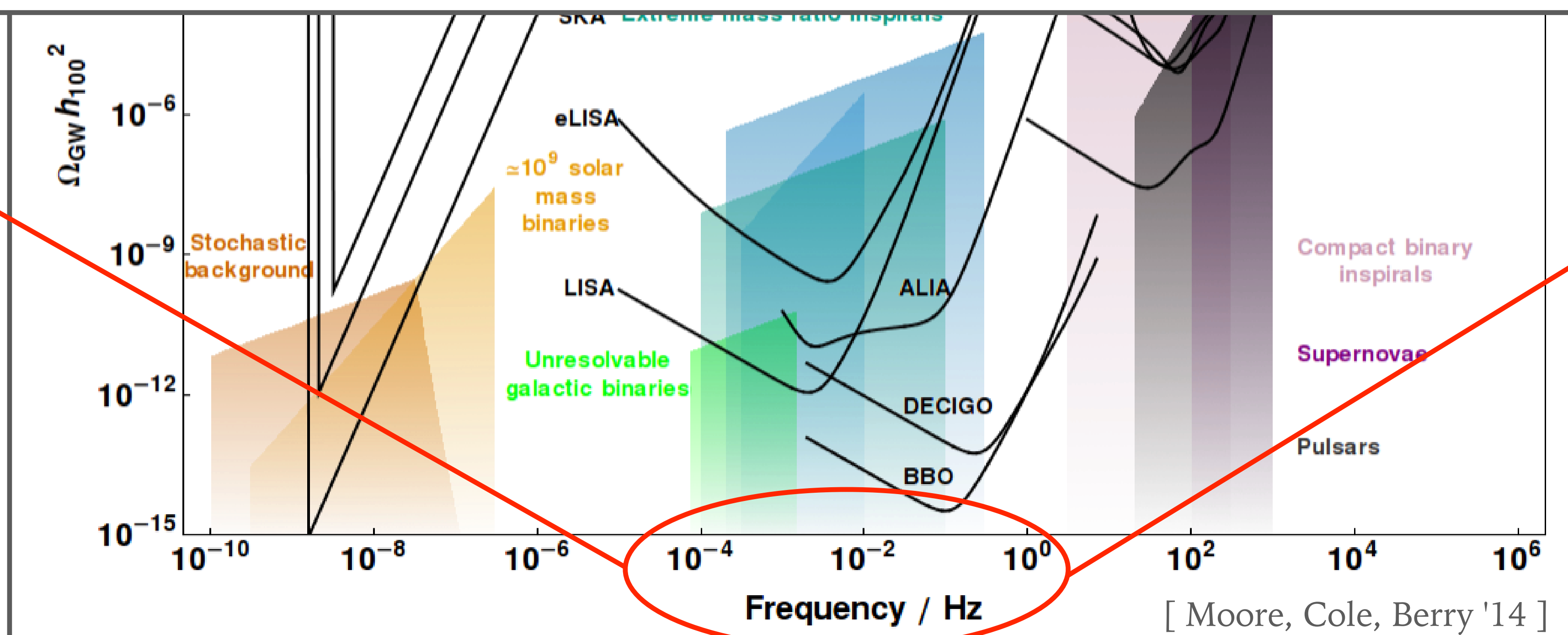
mHz-Hz ⇔ TeVスケールの物理



[Auclair et al. '22]

log周波数あたりの重力波

現在の宇宙のエ

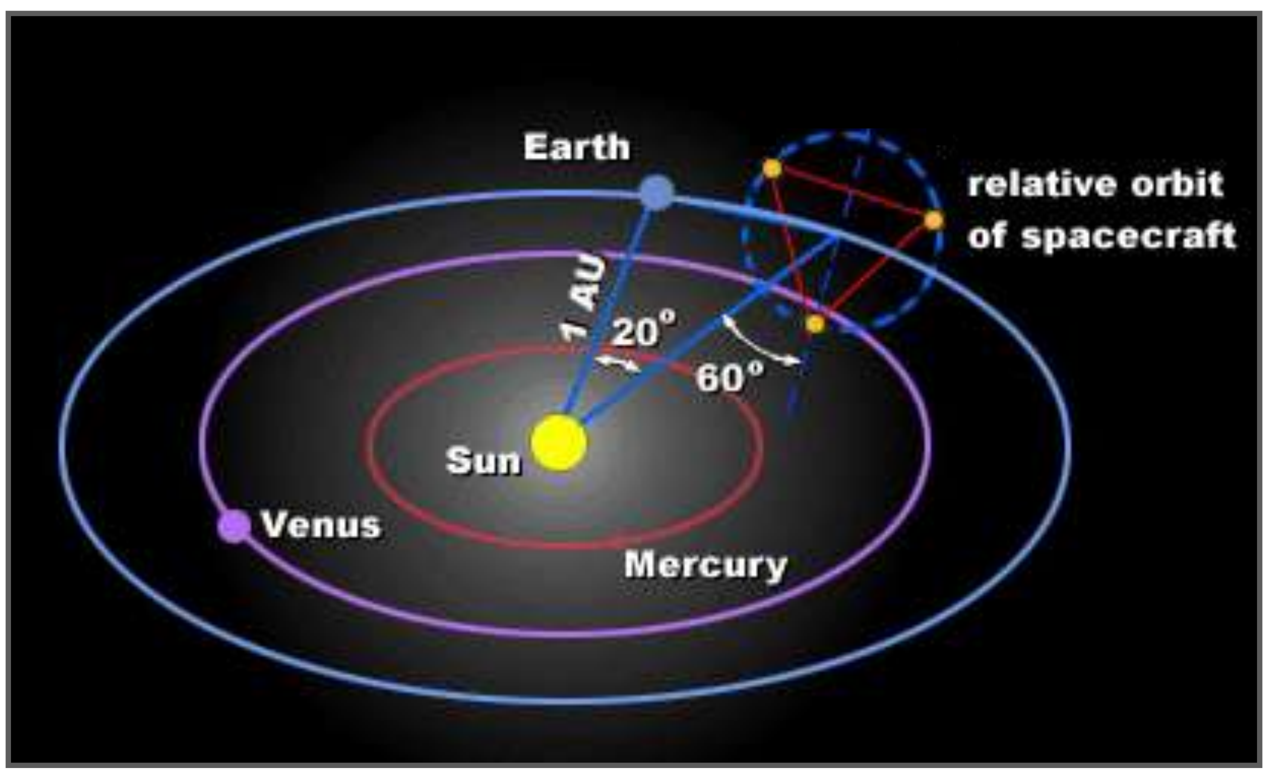


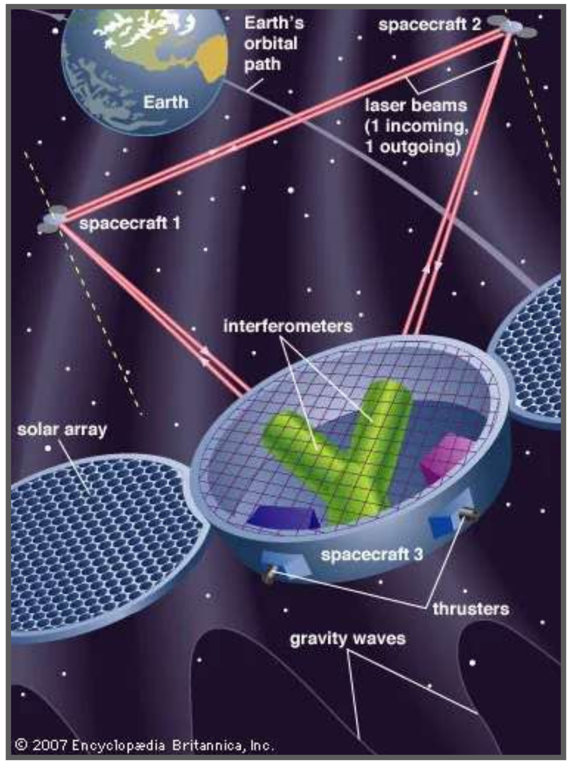
[Moore, Cole, Berry '14]

LISA (LASER INTERFEROMETER SPACE ANTENNA)

[LISA Mission L3 Proposal, https://www.elisascience.org/files/publications/LISA_L3_20170120.pdf] [Auclair et al. '22]

- ESA (European Space Agency) が NASAと共同で進めるプロジェクト
- 2017年にL3ミッションに選定され、打ち上げが計画されているのは2034年
- 3つの衛星が正三角形型に配置され、地球の軌道上を周回
- 衛星間の距離 ＝ 2.5×10^6 km
- ミッションは6年間で、duty cycleは75%





[https://sci.esa.int/web/lisa/-/31704-schematic-of-lisa-orbit] [https://www.britannica.com/science/physics-science/The-study-of-gravitation]

LISA (LASER INTERFEROMETER SPACE ANTENNA)

_20170120.pdf] [Auclair et al. '22]

- ESA … ジェクト
- 201 … るのは~~2034~~年 ***2037***
- 3つ
- 衛星
- ミッ

THE EUROPEAN SPACE AGENCY

esa

LISA factsheet

ESA / Science & Exploration / Space Science

Overview of the LISA mission.

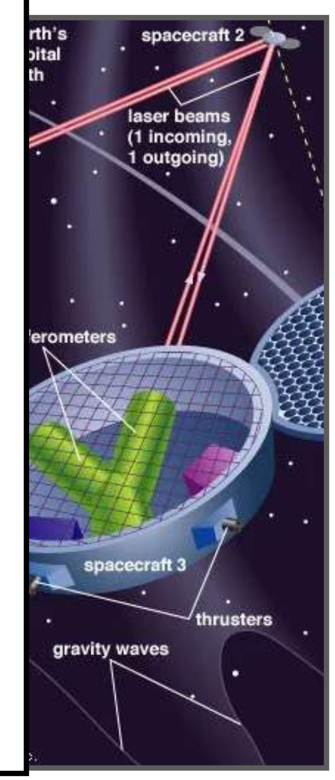
Name: The Laser Interferometer Space Antenna (LISA)

Planned launch: 2037

Mission theme: The gravitational Universe

Status: On 20 June 2017, LISA was selected as the third large-class mission, L3, under ESA's Cosmic Vision 2015-2025. LISA is currently in a more detailed phase of study and will be proposed for 'adoption' around 2024, after which construction can begin

[https://www.esa.int/Science_Exploration/Space_Science/LISA_factsheet]



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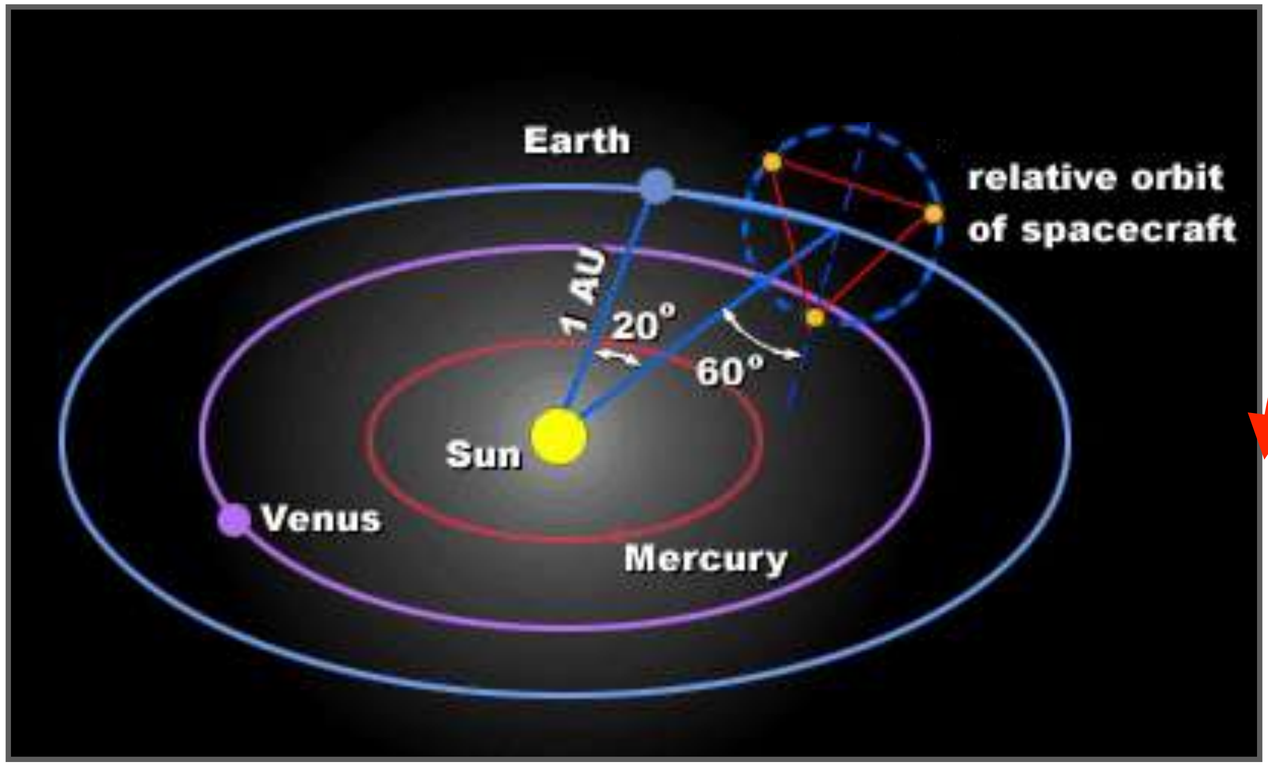
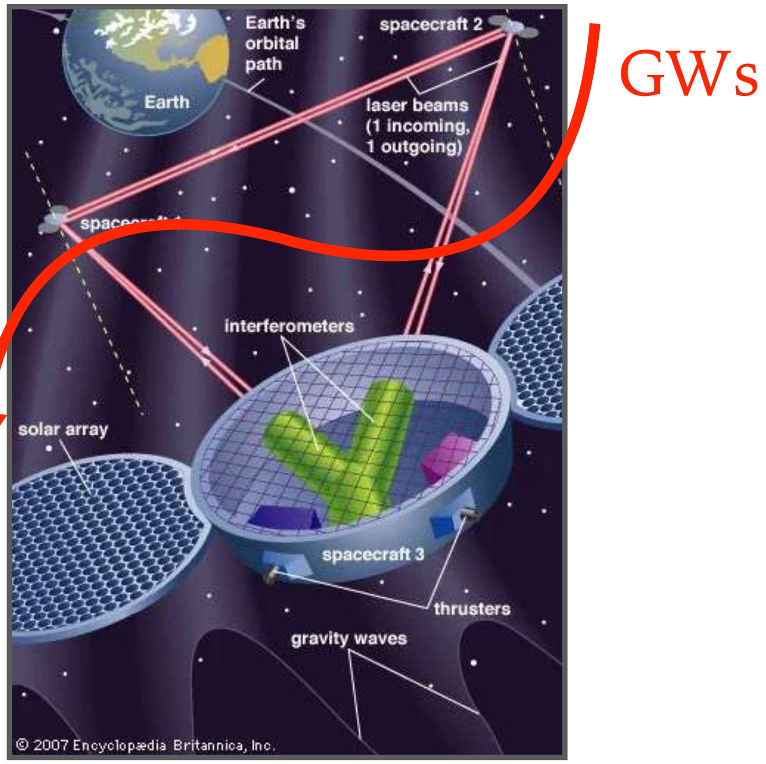
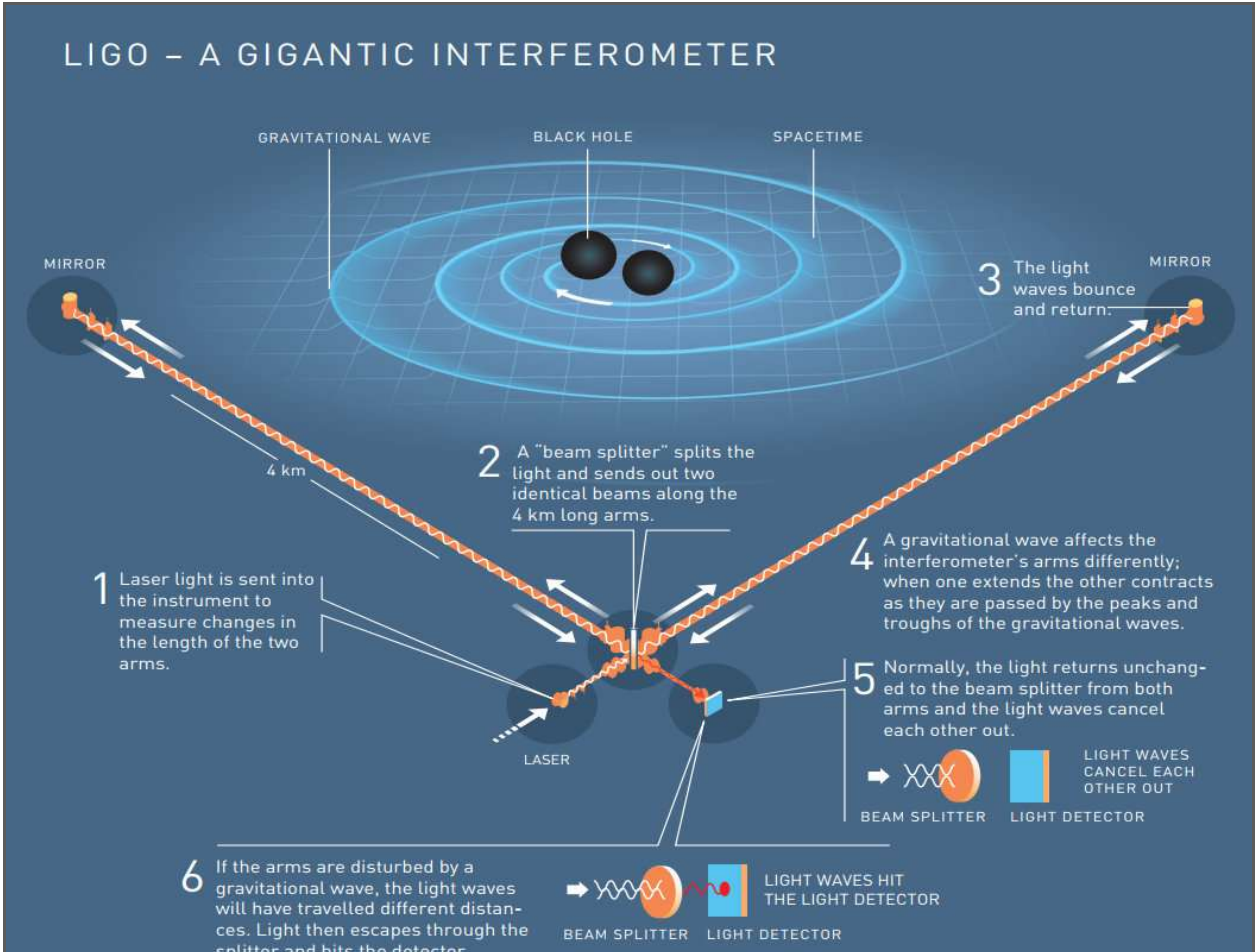




[https://sci.esa.int/web/lisa/-/31704-schematic-of-lisa-orbit] [https://www.britannica.com/science/physics-science/The-study-of-gravitation]


LIGO – A GIGANTIC INTERFEROMETER
GRAVITATIONAL WAVE
BLACK HOLE
SPACETIME
MIRROR
MIRROR
3 The light waves bounce and return.
4 km
2 A "beam splitter" splits the light and sends out two identical beams along the 4 km long arms.
1 Laser light is sent into the instrument to measure changes in the length of the two arms.
4 A gravitational wave affects the interferometer's arms differently; when one extends the other contracts as they are passed by the peaks and troughs of the gravitational waves.
5 Normally, the light returns unchanged to the beam splitter from both arms and the light waves cancel each other out.
LASER
LIGHT WAVES CANCEL EACH OTHER OUT
BEAM SPLITTER
LIGHT DETECTOR
6 If the arms are disturbed by a gravitational wave, the light waves will have travelled different distances. Light then escapes through the splitter and hits the detector.
LIGHT WAVES HIT THE LIGHT DETECTOR
BEAM SPLITTER
LIGHT DETECTOR

[LIGO]

一次相転移における重力波源

[Kosowsky, Turner, Watkins '92]
[Kosowsky, Turner '92]
[Kamionkowski, Kosowsky, Turner '93]
and e.g. [Caprini et al. '16] [Caprini et al. '20]

➤ バブル衝突 (bubble collision)

- スカラー場 (秩序パラメータ場) の
  kinetic & gradient エネルギー

- 転移が極端に強く、
  壁がrunawayする場合に主要となる

➤ 音波 (sound waves)

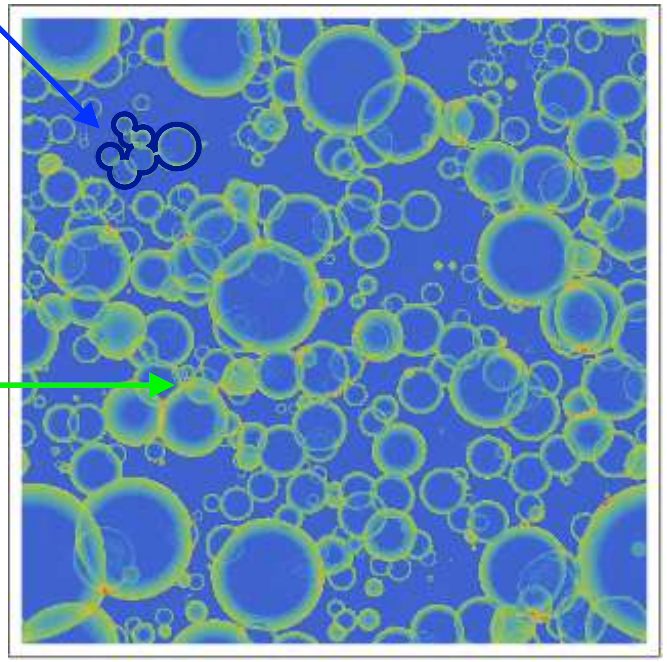
- 流体の圧縮波

- 転移が極端に強くない限りはこれが主要

➤ 乱流 (turbulence)

- 流体の非線形性により現れる

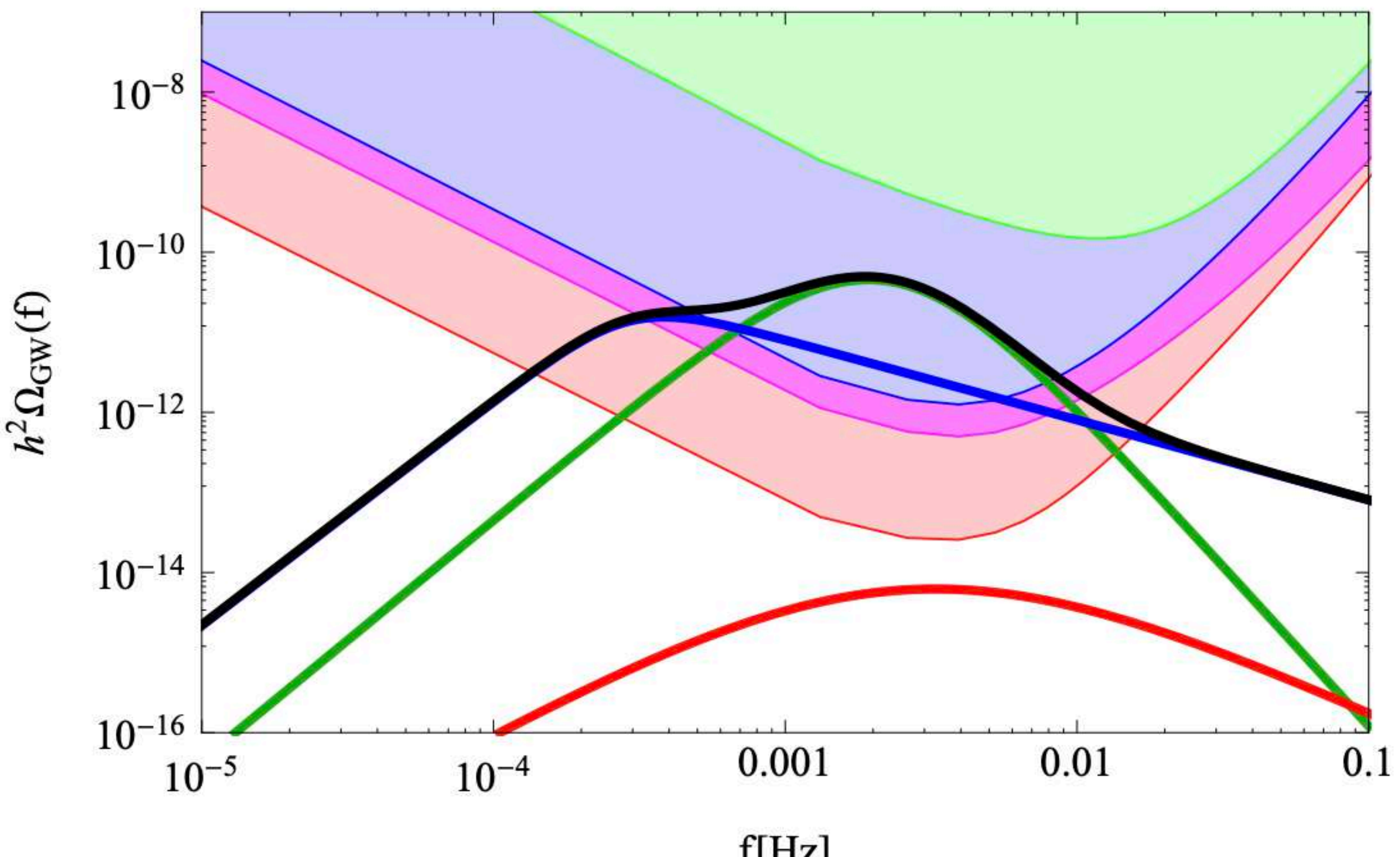
- 音波よりも後のフェーズで出現すると考えられている



important at later stage

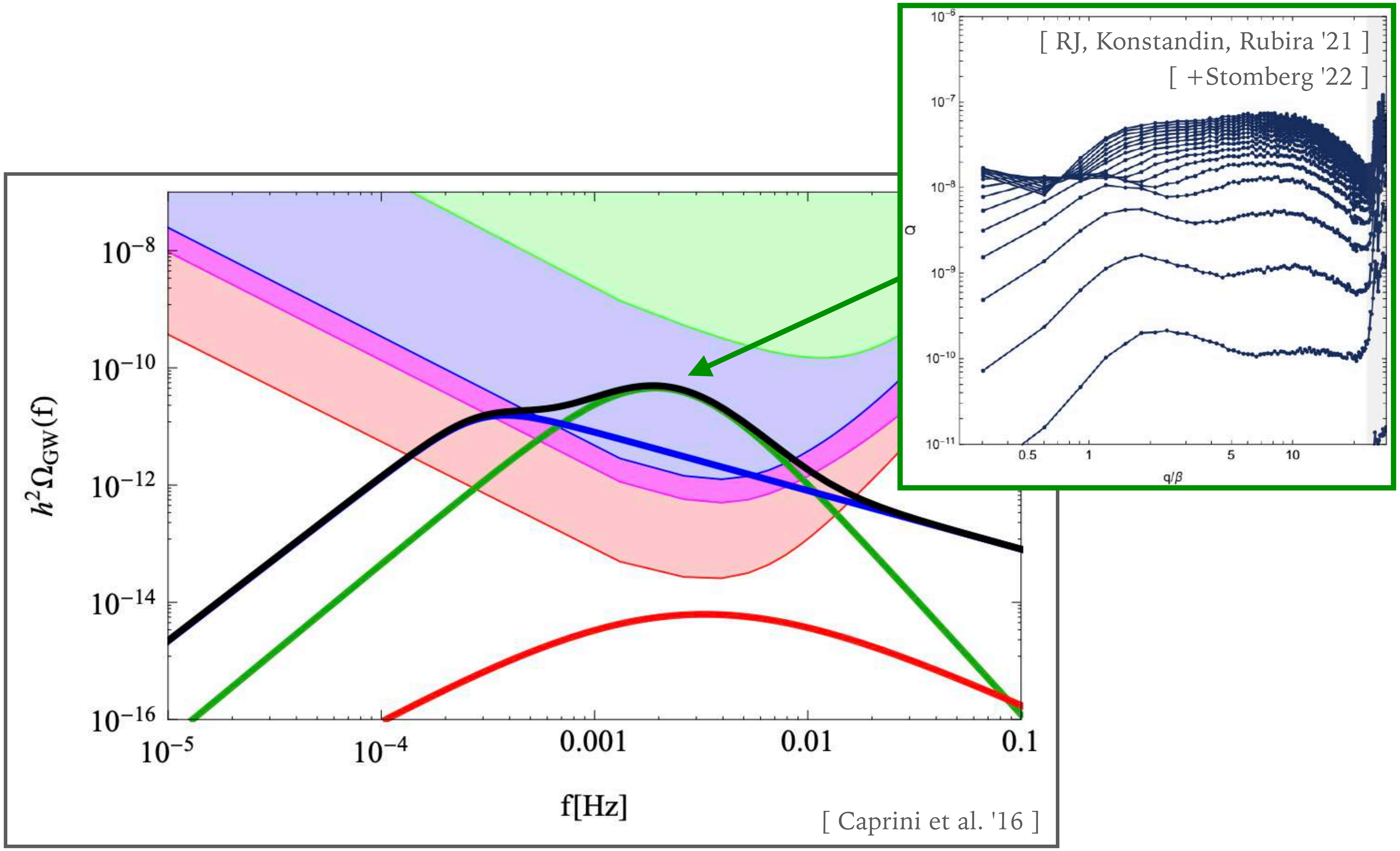
$$\Box h_{ij} = 16\pi G \Lambda_{ij,kl} T_{kl}$$

重力波スペクトル



[Caprini et al. '16]

重力波スペクトル



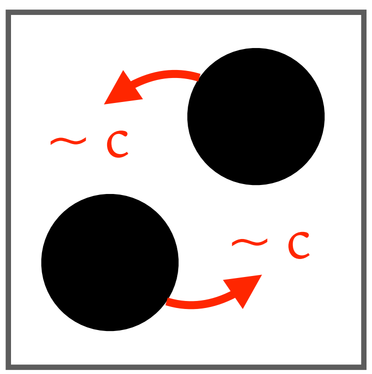
スペクトルの詳細が、一次相転移を起こす高エネルギーの物理の情報を持っている

重力波はどのくらい生成する？ - 直観的な議論

➤ 「最大で」どのくらい生成するか考えてみよう

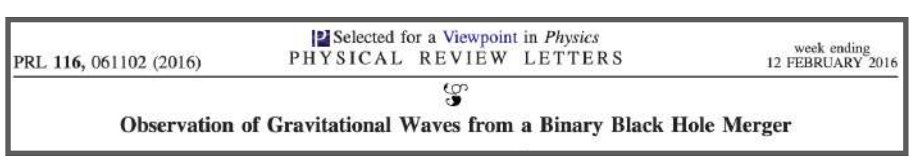
最大？ → horizonサイズの物体がほぼ光速度 ~c で動いている状況

ブラックホール連星



合体する直前

系のサイズ ~ Schwartzschild 半径

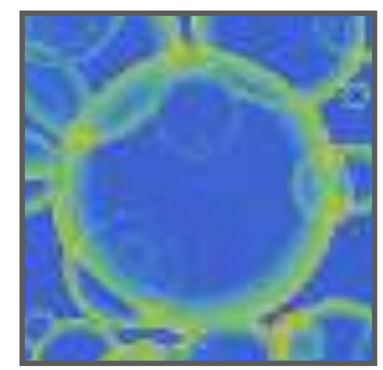

Selected for a Viewpoint in *Physics*
PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

$$36M_\odot + 29M_\odot \rightarrow 62M_\odot + 3M_\odot \,(\mathrm{GWs})$$

(観測から知ってる)

理論的に最大強度の一次相転移



壁の速度 ~ c

流体の速度 ~ c

系のサイズ ~ バブルのサイズ ~ ハッブル半径

解放される潜熱 ~ 系の全エネルギーのO(10%)

相転移時には、

系のエネルギーの O(1%) が重力波に渡っているはず

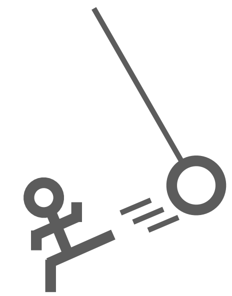
重力波はどのくらい生成する？ - 直観的な議論

- BIG & RELATIVISTIC なバブルほど重力波を生成しやすい

  - 重力波の運動方程式をソースのcoherence time Δt (~ソースがactiveな時間) で積分

$$\Box h_{ij} \sim G T_{ij} \xrightarrow[\text{coherence time } \Delta t]{\text{integration over}} \dot{h}_{ij} \sim G T_{ij} \Delta t$$

oscillator



kicked oscillator

  - 重力波のエネルギー密度 $\rho_{\mathrm{GW}} \sim G^{-1} \dot{h}_{ij}^2 \propto T_{ij}^2 \Delta t^2$

Note but:
GWs from sound waves behave differently

1. relativisticなバブルほど大きな $T_{ij} \propto \alpha$ を持つ

2. bigなバブルほど、当たり始めてから当たり切るまで長い $\Delta t \propto \beta^{-1}$

概要

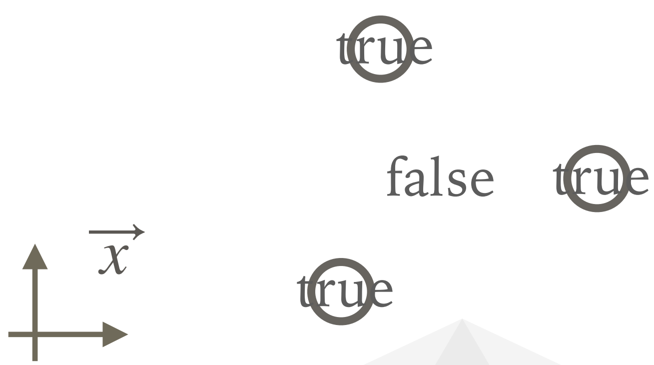
microphysics

泡のダイナミクス

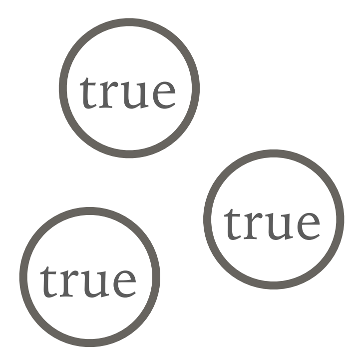
macrophysics

時間 / スケール →

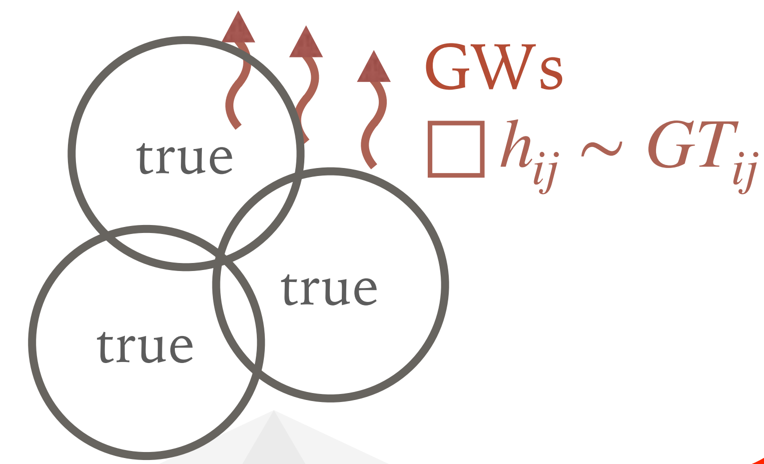
(1) 核生成 (nucleation)



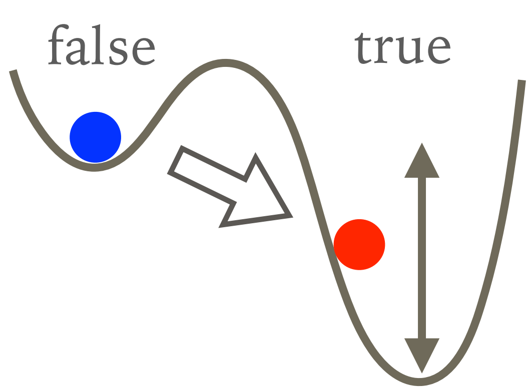
(2) 拡大 (expansion)



(3) 衝突 (collision)



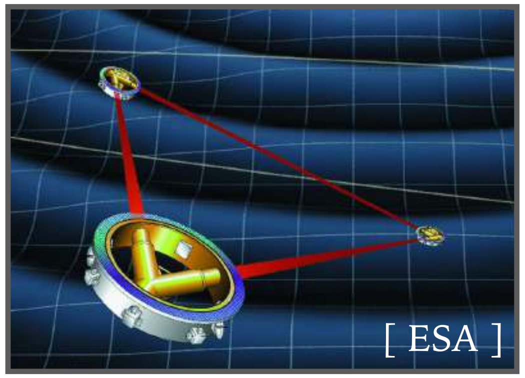
一次相転移の物理



一次相転移

重力波

重力波観測



まとめ

- 一次相転移が宇宙の熱史のどこかで起きていた場合、
  バブルの「核生成 → 拡大 → 衝突」として実現する

- これは一般に、スカラー場と流体の複雑なダイナミクスを引き起こす

- このプロセスにより生成された重力波が、将来観測で見えるかもしれない

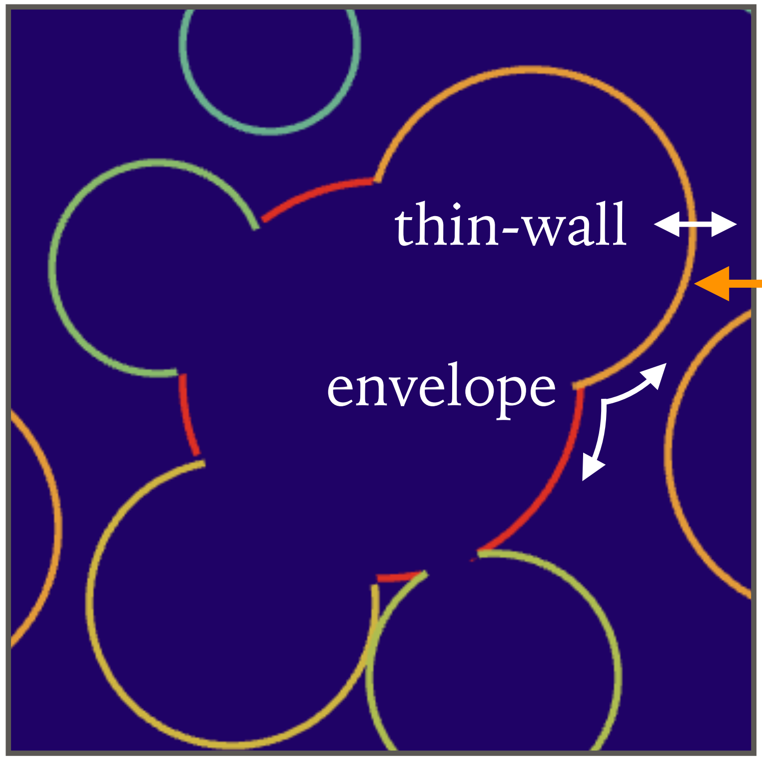

Thermal History
First-order phase transitions
From micro to macro
Gravitational waves
Recent topics

①

A SOLVABLE SYSTEM

➤ A bit of history: GW production from first-order phase transitions has been modeled by so-called thin-wall and envelope approximations

[Kosowsky, Turner, Watkins '92]



T_{ij} grows as $T_{ij} \propto \underbrace{\text{(radius)}}_{} \times \hat{n}_i \hat{n}_j$

why? because

$$\frac{\text{(released energy)}}{\text{(surface area)}} \propto \text{(radius)}$$

Thin-wall : the energy-momentum tensor is localized along the thin surface

Envelope : the surface damps as soon as it collides

THIN-WALL & ENVELOPE MODEL

- Thin-wall & envelope model is "Ising model for statistical physics":

  The simplest modeling, but still captures many physical features

- Numerical simulations have been performed to obtain the GW spectrum

  [e.g. Huber, Konstandin, '08]

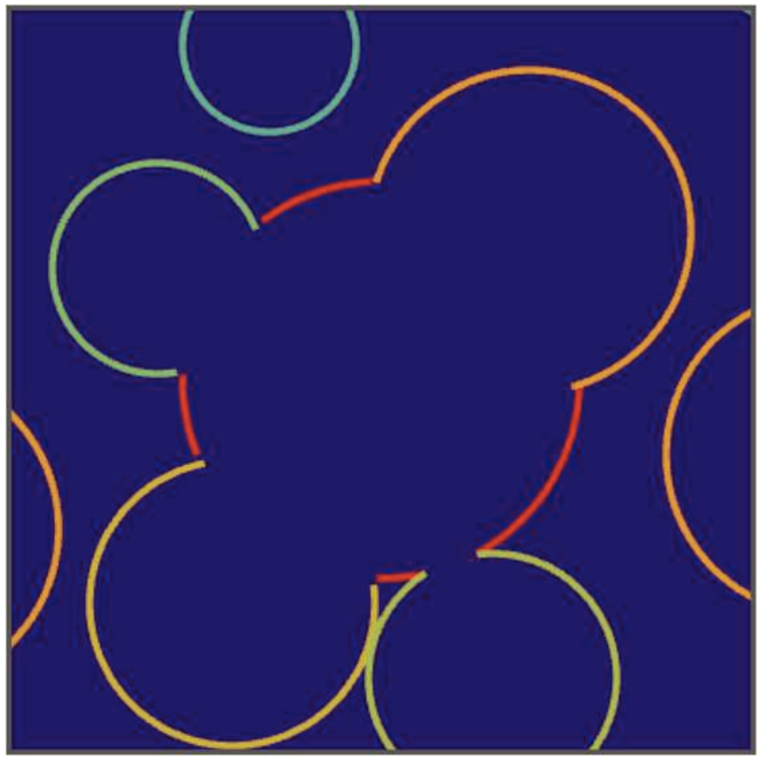
- We showed that the GW spectrum is calculable analytically [RJ, Takimoto, '16]

$$\boxed{\Omega_{\rm GW}(k) = \Omega^{(s)}_{\rm GW}(k) + \Omega^{(d)}_{\rm GW}(k)}$$

$$\Omega^{(s)}_{\rm GW} \propto k^3 \int_{-\infty}^{\infty} dt \int_{|t|}^{\infty} dr \; \frac{e^{-\beta r/2}}{e^{\beta t/2} + e^{-\beta t/2} + \frac{\beta^2 t^2 - (\beta^2 r^2 + 4\beta r)}{4\beta r} e^{-\beta r/2}} \times \left[j_0(kr) S_0(t,r) + \frac{j_1(kr)}{kr} S_1(t,r) + \frac{j_2(kr)}{k^2 r^2} S_2(t,r) \right] \cos(kt)$$

$$\Omega^{(d)}_{\rm GW} \propto k^3 \int_{-\infty}^{\infty} dt \int_{|t|}^{\infty} dr \; \frac{e^{-\beta r/2}}{\left[e^{\beta t/2} + e^{-\beta t/2} + \frac{\beta^2 t^2 - (\beta^2 r^2 + 4\beta r)}{4\beta r} e^{-\beta r/2} \right]^2} \times \left[\frac{j_2(kr)}{k^2 r^2} D(t,r) D(-t,r) \right] \cos(kt)$$

HOW TO SOLVE IT: ALL YOU NEED IS CAUSALITY [RJ, Takimoto, '16]



➤ Setup

- Linearized gravity $\Box h_{ij} = 16\pi G \Lambda_{ij,kl} T_{kl}$

- Bubbles nucleate with the rate $\Gamma(t) \propto e^{\beta(t-t_*)}$
  per unit time & vol., with β = const.

- Cosmic expansion neglected, walls moving with c

- Energy-momentum tensor grows as $T_{ij} \propto (\text{radius}) \times \hat{n}_i \hat{n}_j$,
  and localizes at the bubble surface (thin-wall)

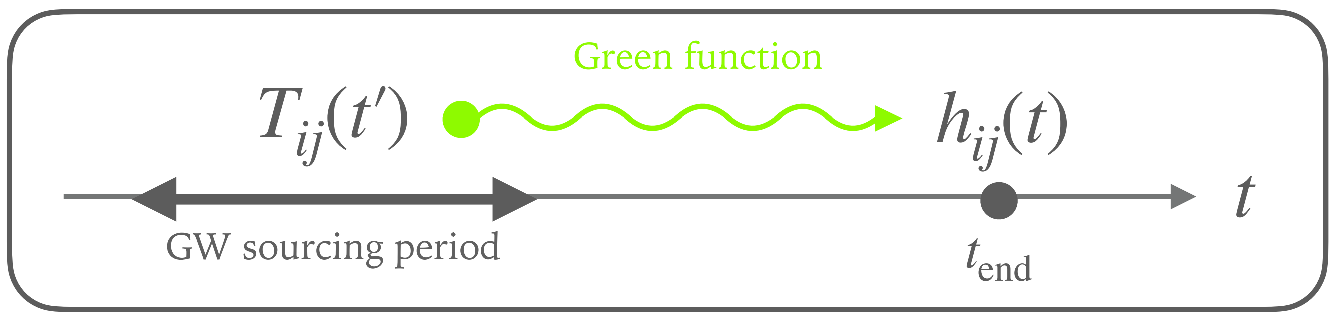
- Energy-momentum tensor damps upon collision (envelope)

HOW TO SOLVE IT: ALL YOU NEED IS CAUSALITY [RJ, Takimoto, '16]

➤ Rough sketch of the derivation

① GW equation of motion = wave equation $\Box h_{ij} \sim T_{ij}$

→ formally solved with the Green function $h_{ij}(t) \sim \int^t dt' \, \mathrm{Green}(t, t') \, T_{ij}(t')$



② GW spectrum = 2-point ensemble average of h_{ij} = 2-point ensemble average of T_{ij}

$$\langle h_{ij}(t_{\rm end}, \vec{k}) h^*_{kl}(t_{\rm end}, \vec{k}) \rangle_{\rm ens} \sim \int^{t_{\rm end}} dt_x \int^{t_{\rm end}} dt_y \ \langle T_{ij}(t_x, \vec{k}) T^*_{kl}(t_y, \vec{k}) \rangle_{\rm ens} \ \cos(k(t_x - t_y))$$

HOW TO SOLVE IT: ALL YOU NEED IS CAUSALITY [RJ, Takimoto, '16]

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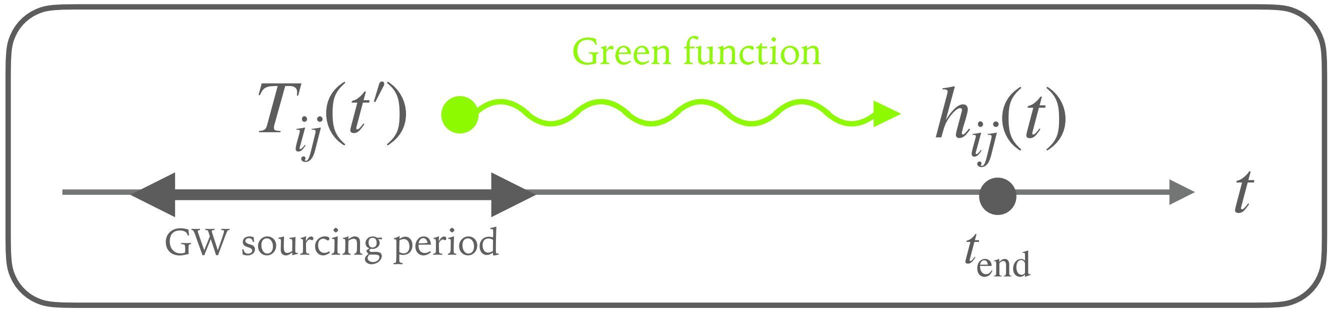

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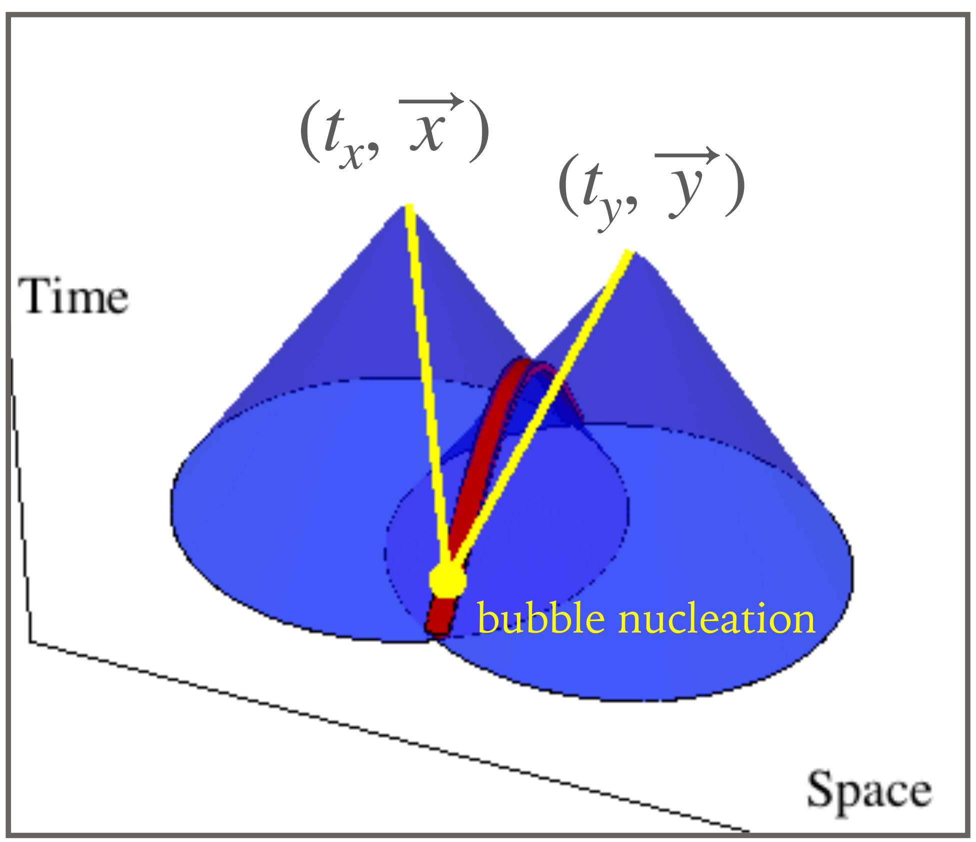
HOW TO SOLVE IT: ALL YOU NEED IS CAUSALITY [RJ, Takimoto, '16]

➤ Rough sketch of the derivation

③ $\langle T_{ij}(t_x, \vec{x}) T_{kl}(t_y, \vec{y}) \rangle_{\rm ens}$ (Fourier tramsform of $\langle T_{ij}(t_x, \vec{k}) T^*_{kl}(t_y, \vec{k}) \rangle_{\rm ens}$)

is calculable from the consideration on the causal cones:

2 possibilities for $\langle T_{ij}(t_x, \vec{x}) T_{kl}(t_y, \vec{y}) \rangle_{\rm ens}$ to have nonzero contributions



1. One bubble nucleates at the intersection of the past cones

2. Two bubbles nucleates on each past cone

HOW TO SOLVE IT: ALL YOU NEED IS CAUSALITY [RJ, Takimoto, '16]

➤ Rough sketch of the derivation

③ $\langle T_{ij}(t_x, \vec{x})T_{kl}(t_y, \vec{y})\rangle_{\rm ens}$ (Fourier tramsform of $\langle T_{ij}(t_x, \vec{k})T^*_{kl}(t_y, \vec{k})\rangle_{\rm ens}$)

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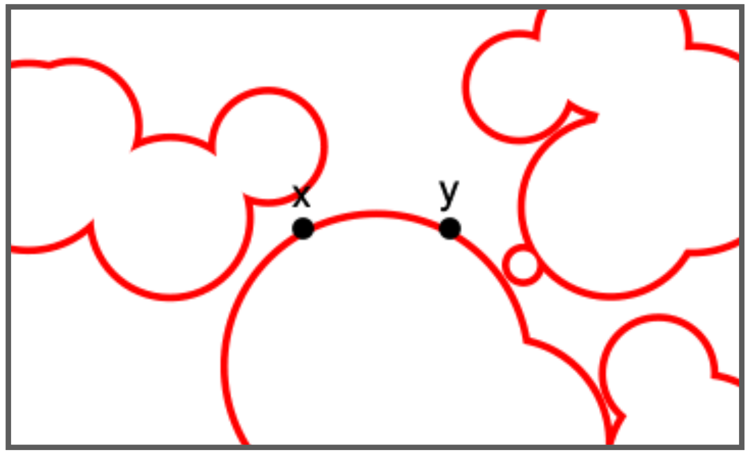

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HOW TO SOLVE IT: ALL YOU NEED IS CAUSALITY [RJ, Takimoto, '16]

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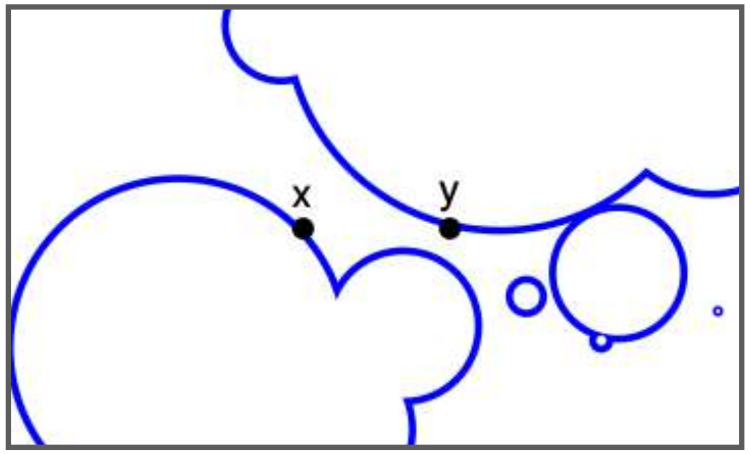


1. One bubble nucleates
   at the intersection of the past cones
2. Two bubbles nucleates
   on each past cone

HOW TO SOLVE IT: ALL YOU NEED IS CAUSALITY [RJ, Takimoto, '16]

- Result (just for completeness)

$$\Omega_{\rm GW}(k) = \Omega_{\rm GW}^{(s)}(k) + \Omega_{\rm GW}^{(d)}(k)$$

spherical Bessel of order 0 | of order 1 | of order 2

$$\Omega_{\rm GW}^{(s)} \propto k^3 \int_{-\infty}^{\infty} dt \int_{|t|}^{\infty} dr \; \frac{e^{-\beta r/2}}{e^{\beta t/2} + e^{-\beta t/2} + \frac{\beta^2 t^2 - (\beta^2 r^2 + 4\beta r)}{4\beta r} e^{-\beta r/2}} \times \left[j_0(kr) S_0(t,r) + \frac{j_1(kr)}{kr} S_1(t,r) + \frac{j_2(kr)}{k^2 r^2} S_2(t,r) \right] \cos(kt)$$

$$\Omega_{\rm GW}^{(d)} \propto k^3 \int_{-\infty}^{\infty} dt \int_{|t|}^{\infty} dr \; \frac{e^{-\beta r/2}}{\left[e^{\beta t/2} + e^{-\beta t/2} + \frac{\beta^2 t^2 - (\beta^2 r^2 + 4\beta r)}{4\beta r} e^{-\beta r/2} \right]^2} \times \left[\frac{j_2(kr)}{k^2 r^2} D(t,r) D(-t,r) \right] \cos(kt)$$

$$S_0(t,r) = \frac{2}{3} \frac{(\beta^2 t^2 - \beta^2 r^2)^2}{\beta^3 r^3} (\beta^2 r^2 + 6\beta r + 12) \qquad S_1(t,r) = \frac{2}{3} \frac{\beta^2 t^2 - \beta^2 r^2}{\beta^3 r^3} \left[-\beta^2 t^2 (\beta^3 r^3 + 12\beta^2 r^2 + 60\beta r + 120) + \beta^2 r^2 (\beta^3 r^3 + 4\beta^2 r^2 + 12\beta r + 24) \right]$$

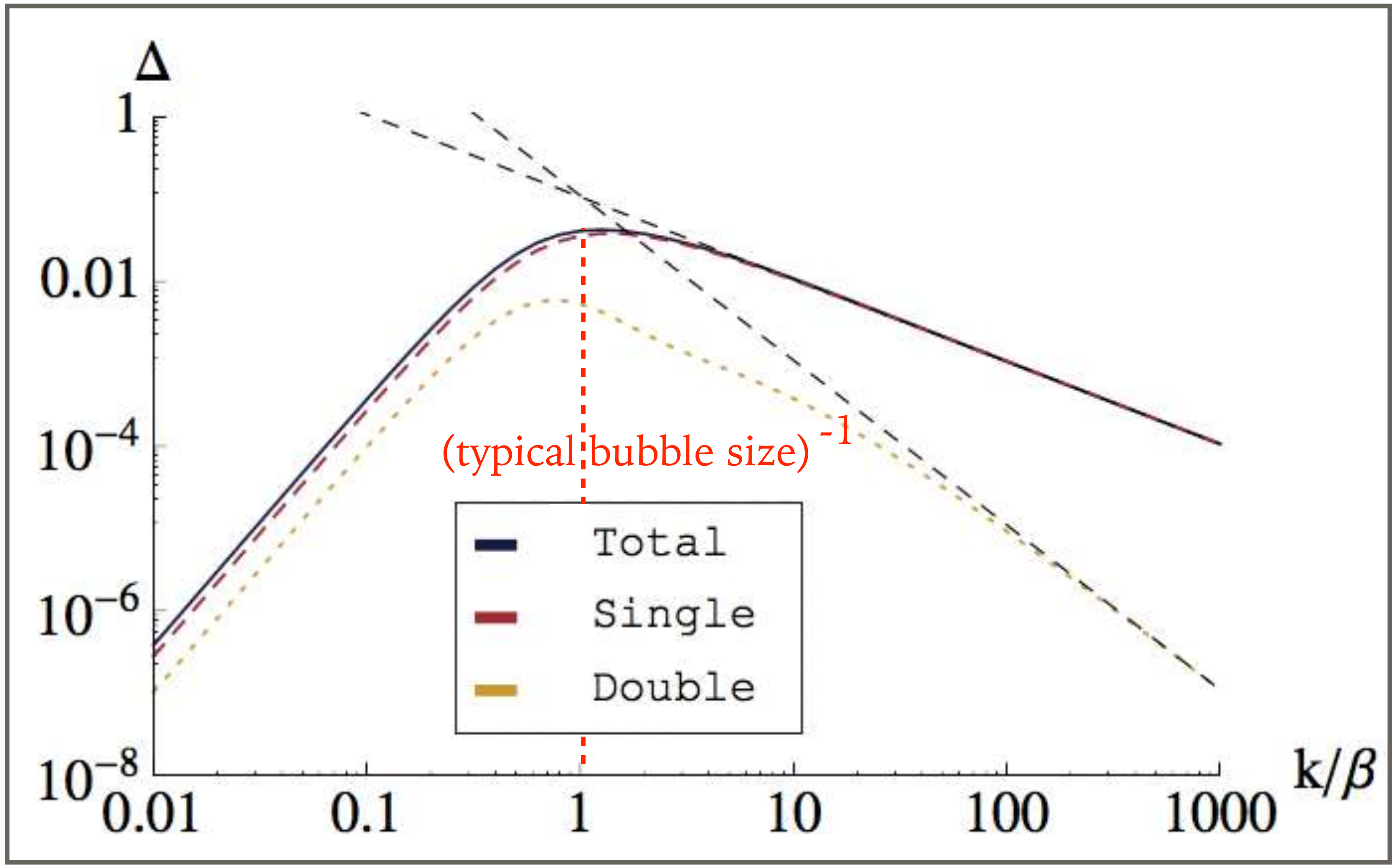
$$S_2(t,r) = \frac{1}{6} \frac{1}{\beta^3 r^3} \left[\beta^4 t^4 (\beta^4 r^4 + 20\beta^3 r^3 + 180\beta^2 r^2 + 840\beta r + 1680) - 2\beta^4 t^2 r^2 (\beta^4 r^4 + 12\beta^3 r^3 + 84\beta^2 r^2 + 360\beta r + 720) \right.$$
$$\left. + \beta^4 r^4 (\beta^4 r^4 + 4\beta^3 r^3 + 20\beta^2 r^2 + 12\beta r + 24) \right]$$

$$D(t,r) = \sqrt{\frac{\pi}{3}} \frac{\beta^2 t^2 - \beta^2 r^2}{\beta^2 r^2} \left[\beta t (\beta^2 r^2 + 6\beta r + 12) + (\beta^3 r^3 + 2\beta^2 r^2) \right]$$

HOW TO SOLVE IT: ALL YOU NEED IS CAUSALITY [RJ, Takimoto, '16]

➤ Result (just for completeness)

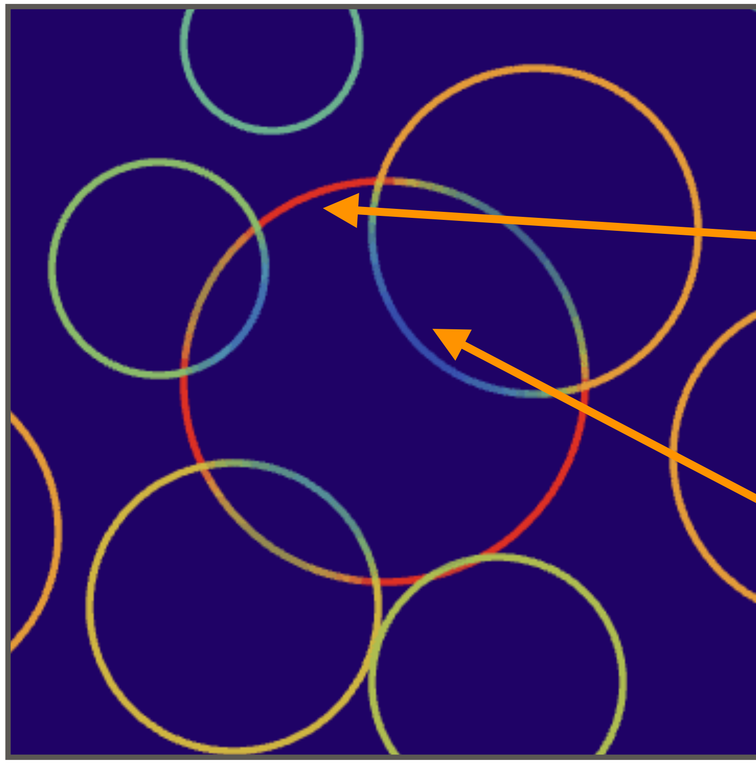
$$\Omega_{\rm GW}(k) = \Omega_{\rm GW}^{(s)}(k) + \Omega_{\rm GW}^{(d)}(k)$$



BEYOND THE TRADITIONAL MODELING

➤ We proposed a modeling that takes the collided shells into account

[Jinno, Takimoto '19]



Before collision

$$T_{ij} \text{ grows as } \propto \frac{\text{(released energy)}}{\text{(surface area)}} \propto \text{(radius)}$$

After collision

$$T_{ij} \text{ decreases as } \propto \frac{1}{\text{(surface area)}} \propto \text{(radius)}^{-2}$$

➤ ...and solved

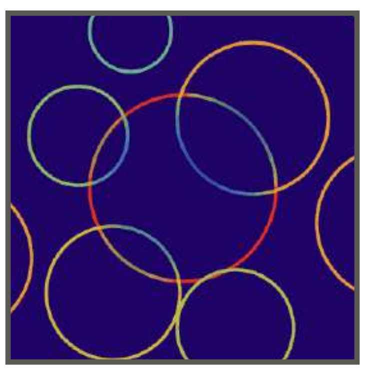
$$\boxed{\Omega_{\rm GW}(k) = \Omega_{\rm GW}^{(s)}(k) + \Omega_{\rm GW}^{(d)}(k)}$$

$$\begin{aligned}
\Delta^{(s)} = &\int_{-\infty}^{\infty} dt_x \int_{-\infty}^{\infty} dt_y \int_{v|t_{x,y}|}^{\infty} dr \int_{-\infty}^{t_{\rm max}} dt_n \int_{t_n}^{t_x} dt_{xi} \int_{t_n}^{t_y} dt_{yi} \\
&\frac{k^3}{3} \left[\begin{aligned} &e^{-I(x_i,y_i)}\,\Gamma(t_n)\,\frac{r}{r_{xn}^{(s)} r_{yn}^{(s)}} \\ &\times \left[j_0(kr)\mathcal{K}_0(n_{xn\times}, n_{yn\times}) + \frac{j_1(kr)}{kr}\mathcal{K}_1(n_{xn\times}, n_{yn\times}) + \frac{j_2(kr)}{(kr)^2}\mathcal{K}_2(n_{xn\times}, n_{yn\times}) \right] \\ &\times \partial_{txi}\left[r_B(t_{xi}, t_n)^3 D(t_x, t_{xi}) \right] \partial_{tyi}\left[r_B(t_{yi}, t_n)^3 D(t_y, t_{yi}) \right] \cos(kt_{x,y}) \end{aligned} \right]
\end{aligned}$$

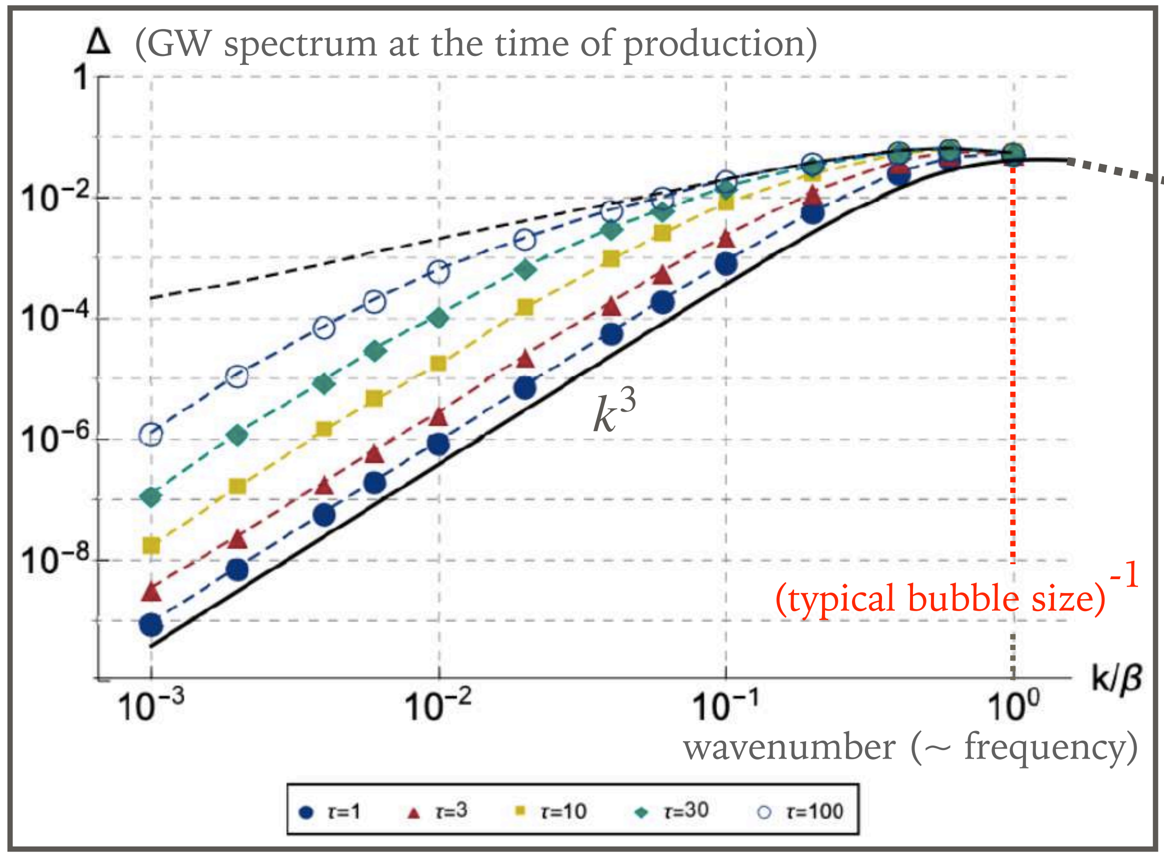
$$\begin{aligned}
\Delta^{(d)} = &\int_{-\infty}^{\infty} dt_x \int_{-\infty}^{\infty} dt_y \\
&\int_0^{\infty} dr \int_{-\infty}^{t_x} dt_{xn} \int_{-\infty}^{t_y} dt_{yn} \int_{t_{xn}}^{t_x} dt_{xi} \int_{t_{yn}}^{t_y} dt_{yi} \int_{-1}^{1} dc_{xn} \int_{-1}^{1} dc_{yn} \int_0^{2\pi} d\phi_{xn,yn} \\
&\frac{k^3}{3} \left[\begin{aligned} &\Theta_{\rm sp}(x_i, y_n)\Theta_{\rm sp}(x_n, y_i) e^{-I(x_i,y_i)}\Gamma(t_{xn})\Gamma(t_{yn}) \\ &\times r^2 \left[j_0(kr)\mathcal{K}_0(n_{xn}, n_{yn}) + \frac{j_1(kr)}{kr}\mathcal{K}_1(n_{xn}, n_{yn}) + \frac{j_2(kr)}{(kr)^2}\mathcal{K}_2(n_{xn}, n_{yn}) \right] \\ &\times \partial_{txi}\left[r_B(t_{xi}, t_{xn})^3 D(t_x, t_{xi}) \right] \partial_{tyi}\left[r_B(t_{yi}, t_{yn})^3 D(t_y, t_{yi}) \right] \cos(kt_{x,y}) \end{aligned} \right]
\end{aligned}$$

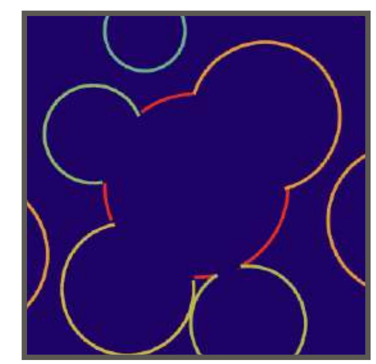
BEYOND THE TRADITIONAL MODELING

- One interesting feature found: IR enhancement of the spectrum



Beyond Envelope

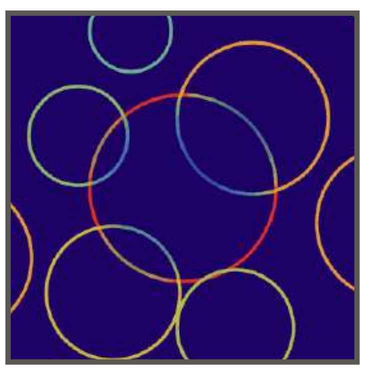




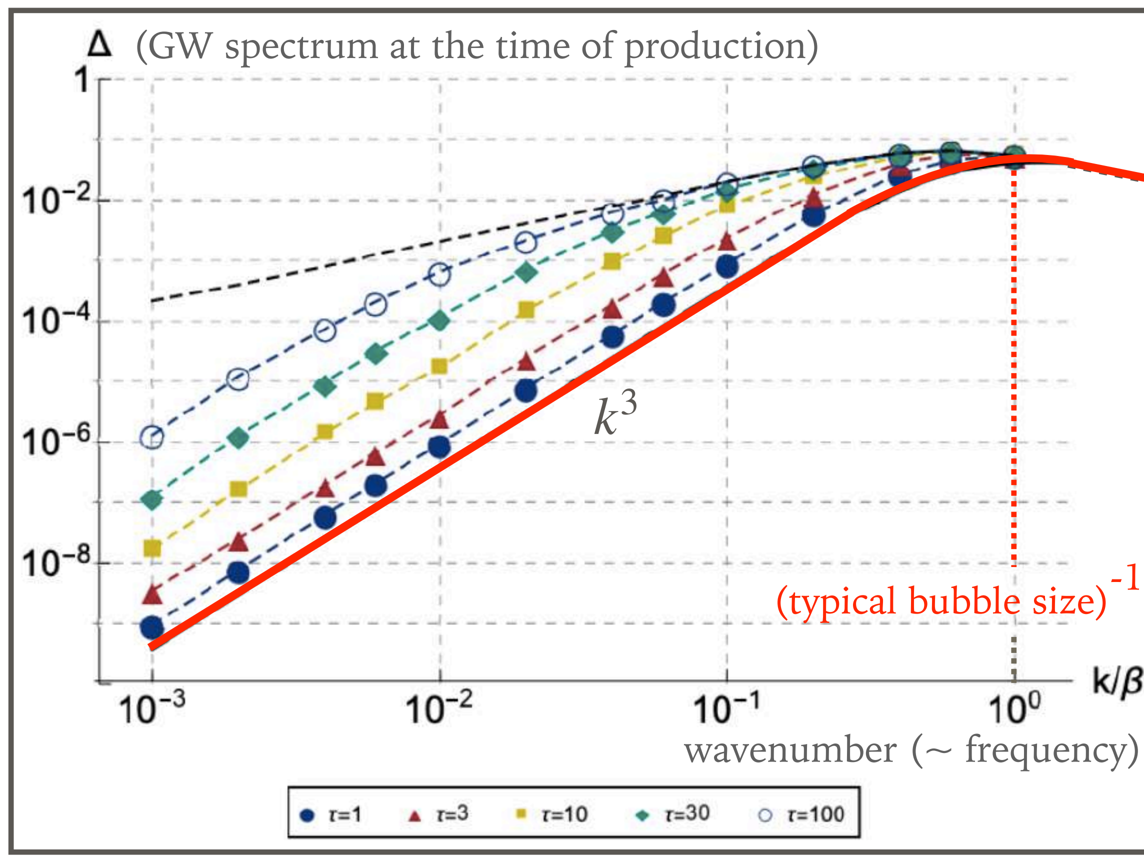
Envelope

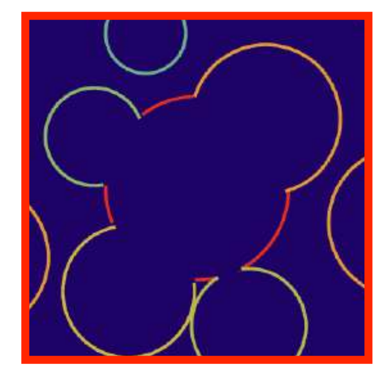
BEYOND THE TRADITIONAL MODELING

➤ One interesting feature found: IR enhancement of the spectrum



Beyond Envelope





Envelope

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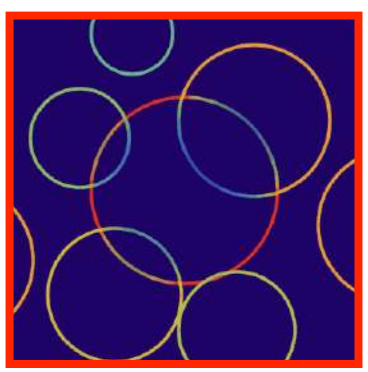

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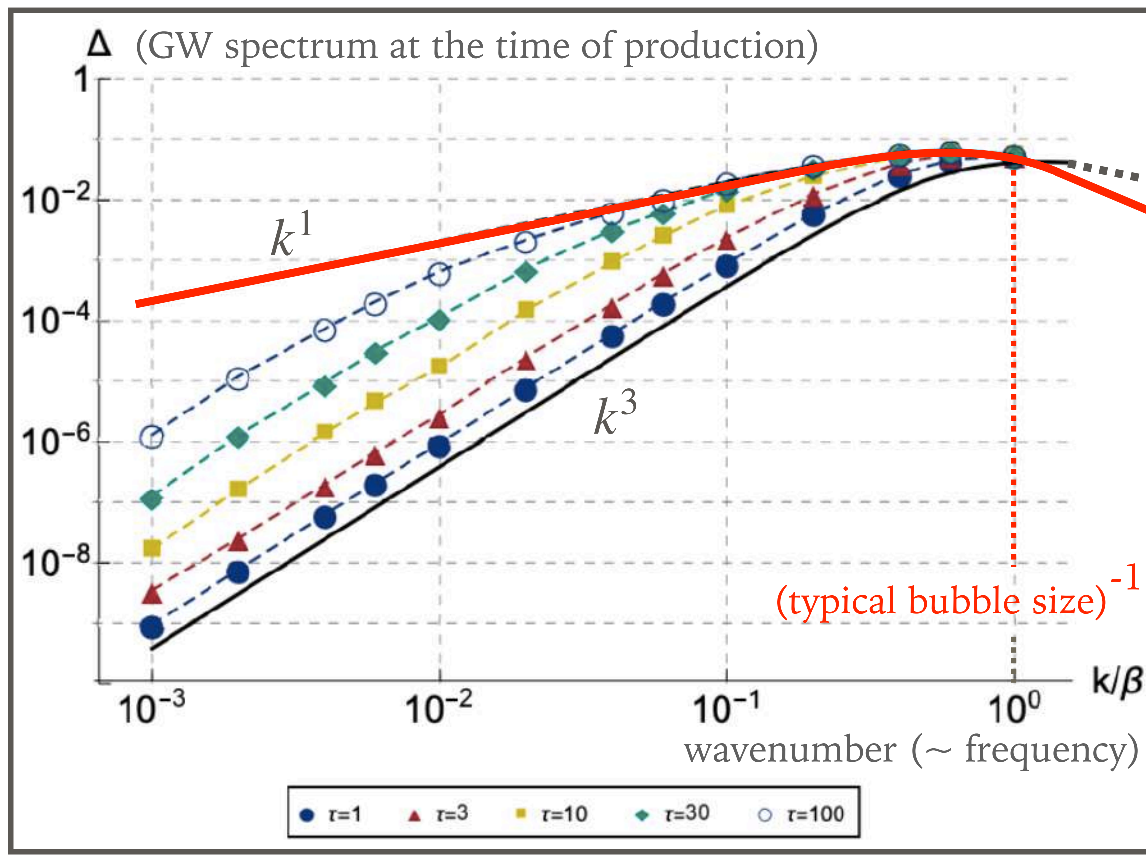

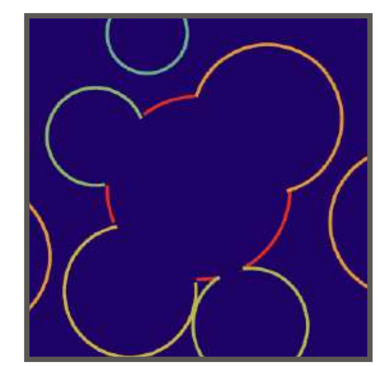

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BEYOND THE TRADITIONAL MODELING

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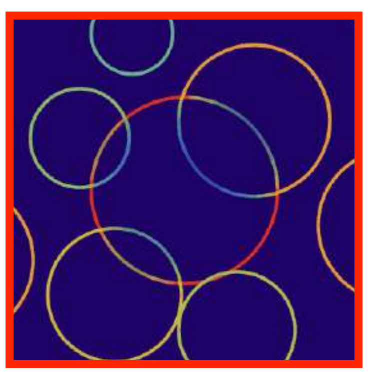

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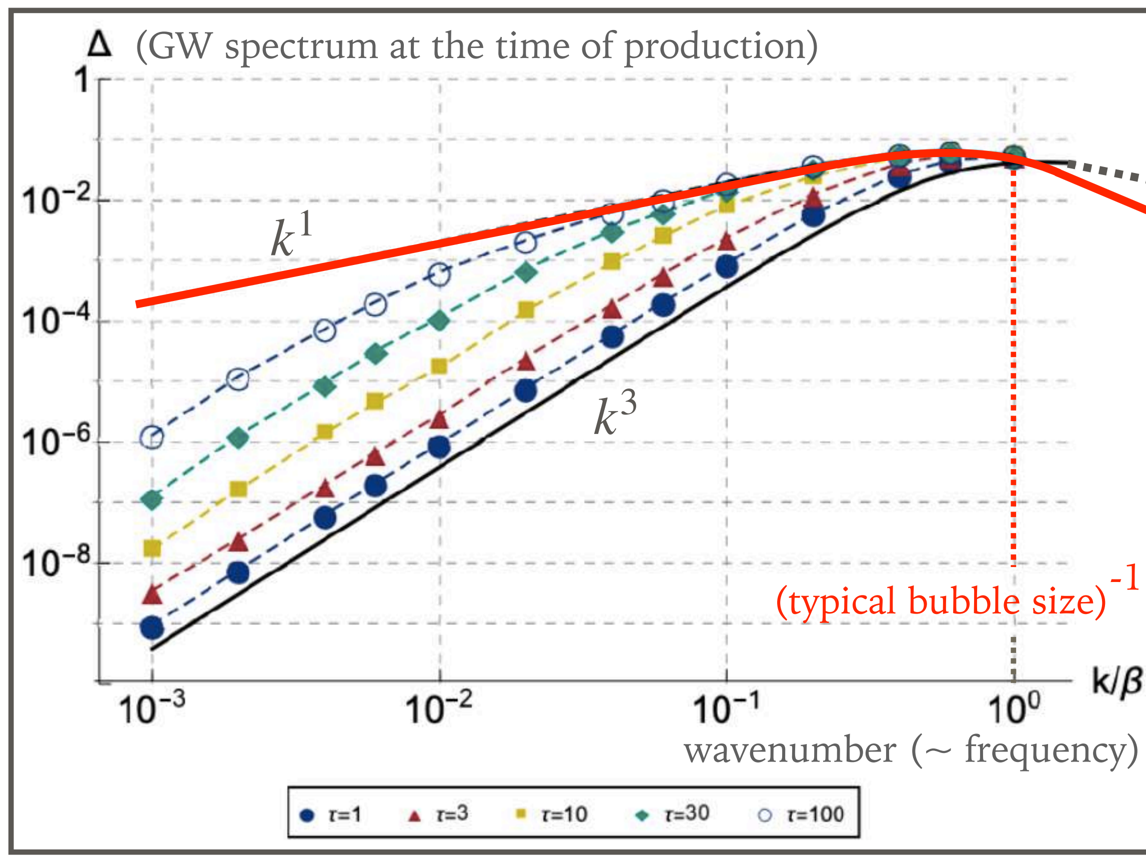
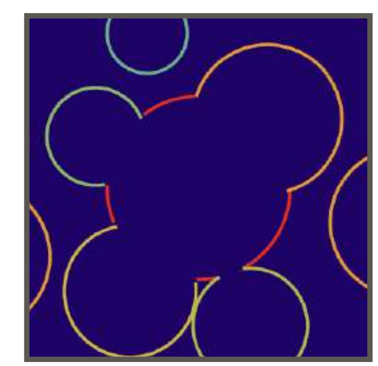




Envelope

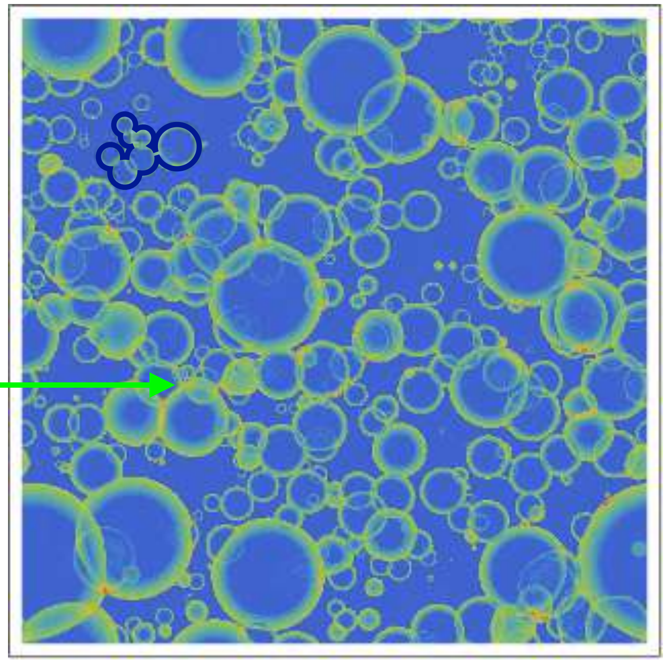
- Nowadays the model is called "the bulk flow model"

②

GRAVITATIONAL WAVE SOURCES

[Kosowsky, Turner, Watkins '92]
[Kosowsky, Turner '92]
[Kamionkowski, Kosowsky, Turner '93]
and e.g. [Caprini et al. '16] [Caprini et al. '20]

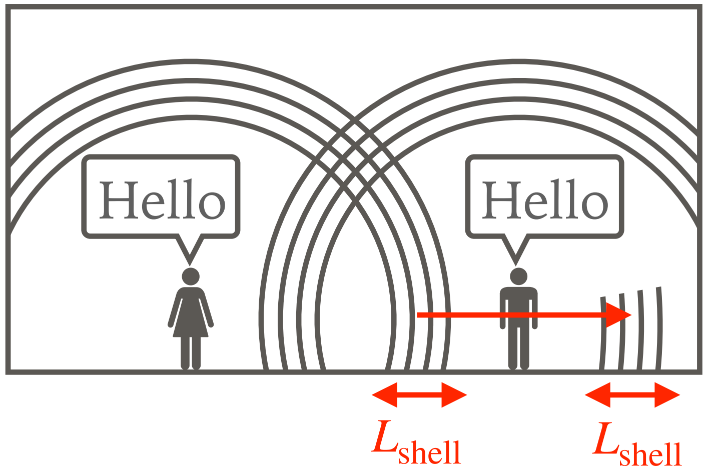
- Bubble collision
  - Kinetic & gradient energy of the scalar field (= order parameter field)
  - Dominant when the transition is extremely strong and the walls runaway
- Sound waves
  - Compression mode of the fluid motion
  - Dominant unless the transition is extremely strong
- Turbulence
  - Turbulent motion caused by fluid nonlinearity
  - Expected to develop at a later stage



important at later stage

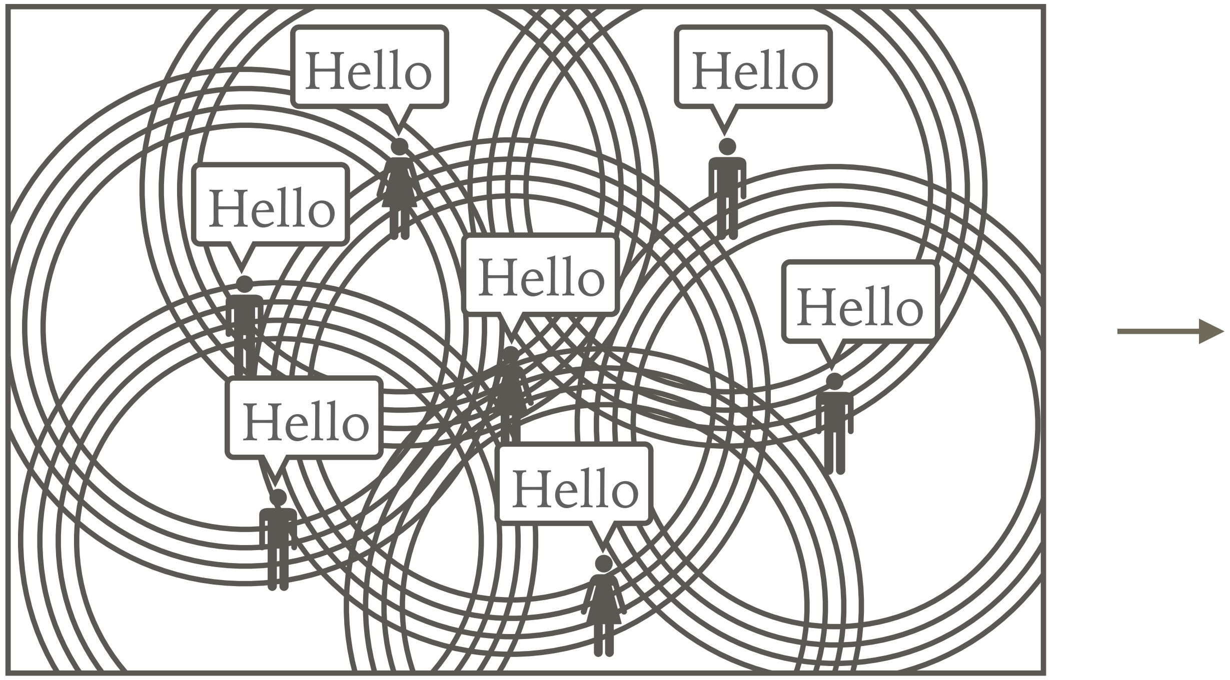
GRAVITATIONAL WAVES FROM SOUND WAVES

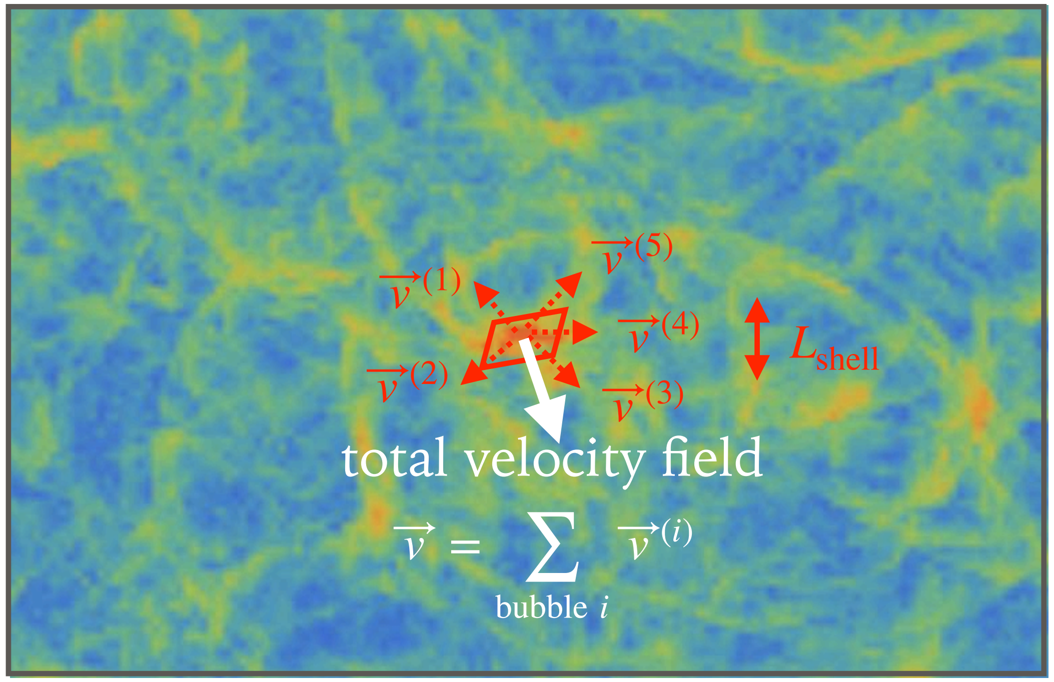
- ➤ Sound shells continue to propagate inside other bubbles



- ➤ Shell overlap creates random velocity fields everywhere, sourcing GWs

[Hindmarsh, Huber, Rummukainen, Weir '14, '15, '17] [Hindmarsh '15, +Hijazi '19]





SOUND WAVE SIMULATIONS

[RJ, Konstandin, Rubira '21] [RJ, Konstandin, Rubira, Stomberg '22]

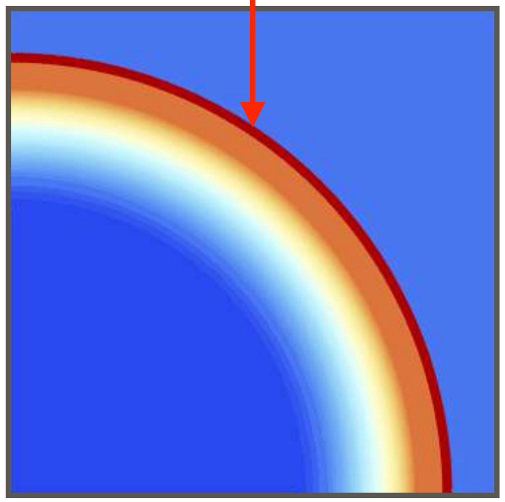
- Fluid 3d simulation may not so easy (though much easier than lattice QCD...)
  - Shock waves must be cared
  - Numerical viscosity must be cared
  - You need cluster computers

- We are proposing "Higgsless scheme"
  - We do ***not*** solve ***both*** the scalar field and fluid
  - But rather "integrate out" the scalar field

    (= treating the scalar field as non-dynamical)

energy-injecting boundary for fluid

(non-dynamical)



HOW TO INTEGRATE THE HIGGS OUT

- The fluid evolution is determined in principle from

  ① Energy-momentum conservation of the fluid $\partial_\mu T^{\mu\nu} = 0$

  ② Change in the equation of state across symmetric/broken phases

- But how can we implement these in simulations?

  ① Assume relativistic perfect fluid & bag EOS for simplicity

  ② Define $K^\mu \equiv T^{\mu 0}$, then $\partial_\mu T^{\mu\nu} = 0$ reduces to $\begin{cases} \partial_0 K^0 + \partial_i K^i = 0 \\ \partial_0 K^i + \partial_j T^{ij}(K^0, K^i) = 0 \end{cases}$

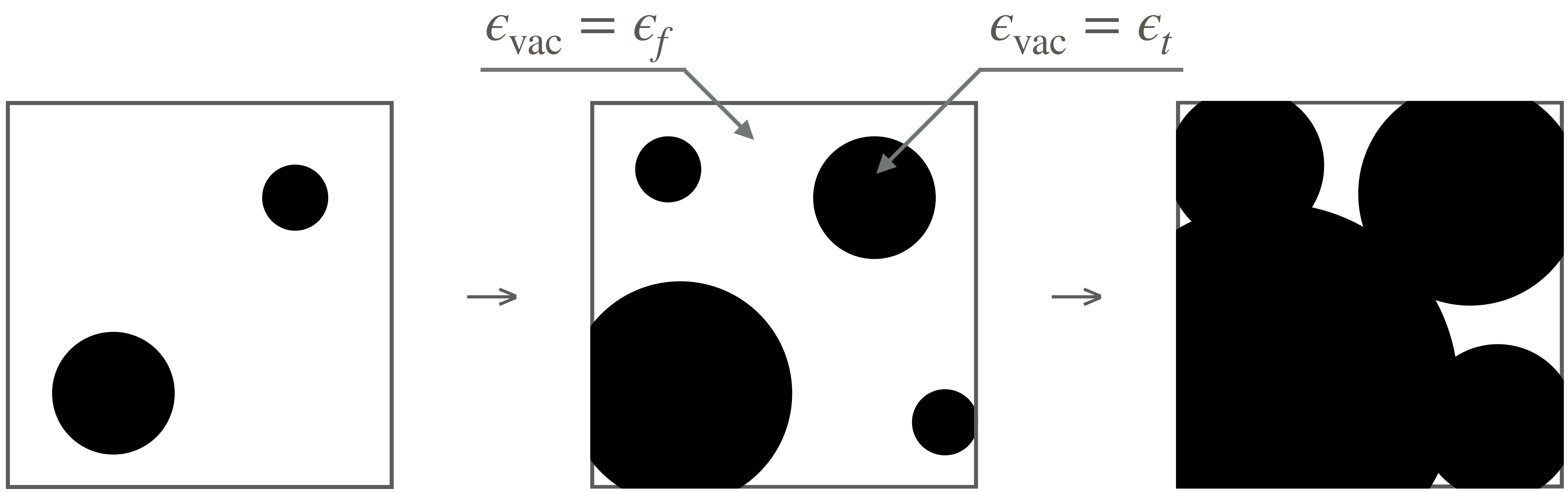
  ③ Where does the energy injection enter? The answer is in $T^{ij}(K^0, K^i)$:

$$T^{ij}(K^0, K^i) = \frac{3}{2}\frac{K^i K^j}{(K^0 - \epsilon_{\rm vac}) + \sqrt{(K^0 - \epsilon_{\rm vac})^2 - \frac{3}{4}K^i K^i}} \qquad \epsilon_{\rm vac} = \begin{cases} \epsilon_f & \text{(false vac.)} \\ \epsilon_t & \text{(true vac.)} \end{cases}$$

RECIPE FOR THE HIGGSLESS SIMULATION

- We numerically generate nucleation points,

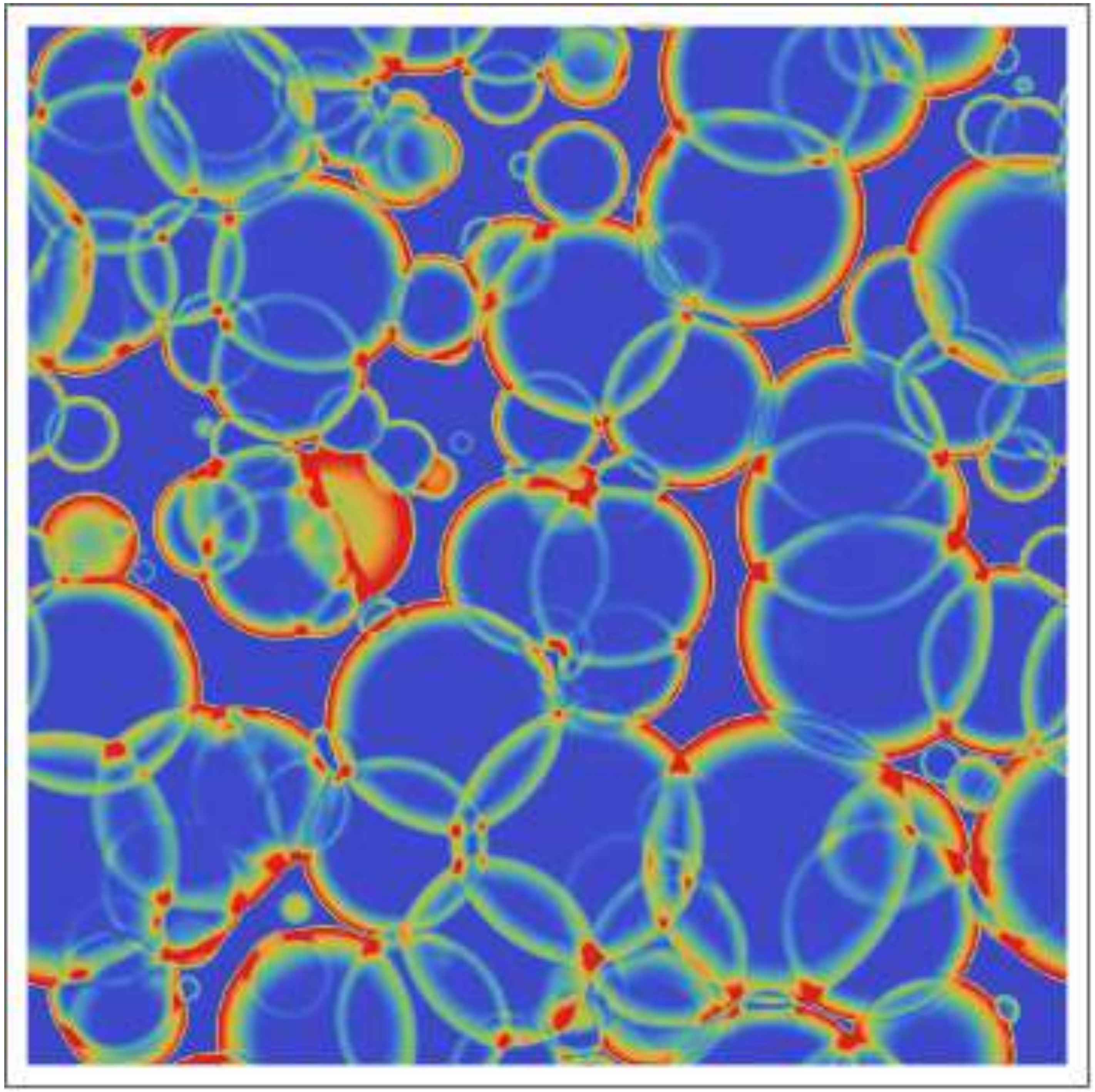
  and pre-determine the true-false evloution without fluid



- We then evolve the fluid in this box according to $\begin{cases} \partial_0 K^0 + \partial_i K^i = 0 \\ \partial_0 K^i + \partial_j T^{ij}(K^0, K^i) = 0 \end{cases}$

  $\rightarrow$ Fluid automatically develops profiles

RECIPE FOR THE HIGGSLESS SIMULATION



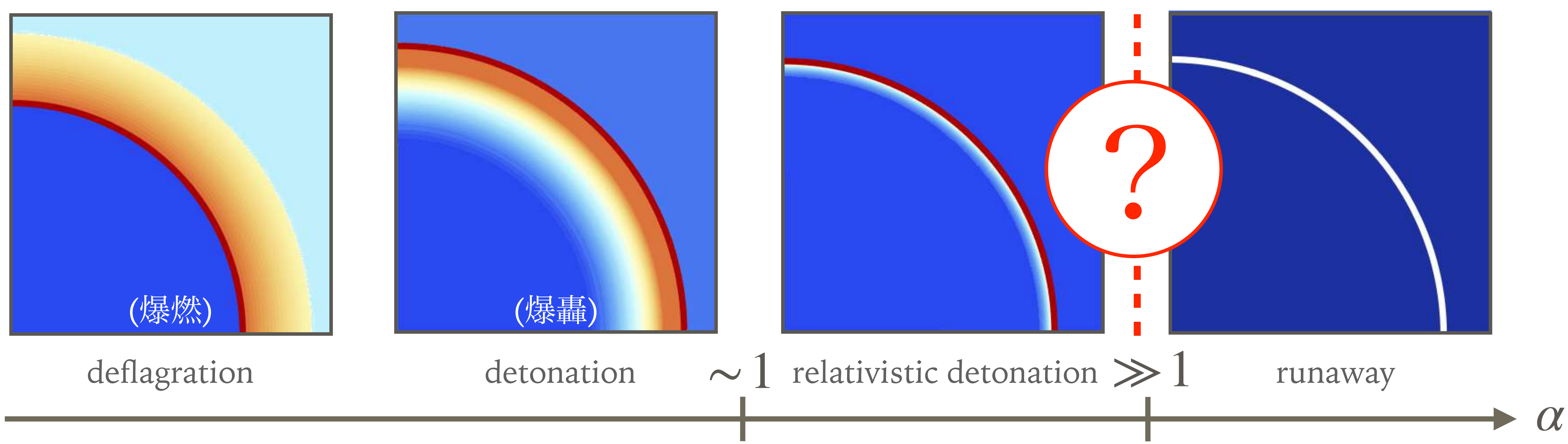
③

FRICTION TO RELATIVISTIC WALLS

➤ Transition point from relativistic detonation to runaway is not yet clear

- Understanding before 2017: $\alpha \sim \mathcal{O}(1)$ [Bodeker & Moore '09]

- Understanding after 2017: $\alpha \ggg \mathcal{O}(1)$ [Bodeker & Moore '17]



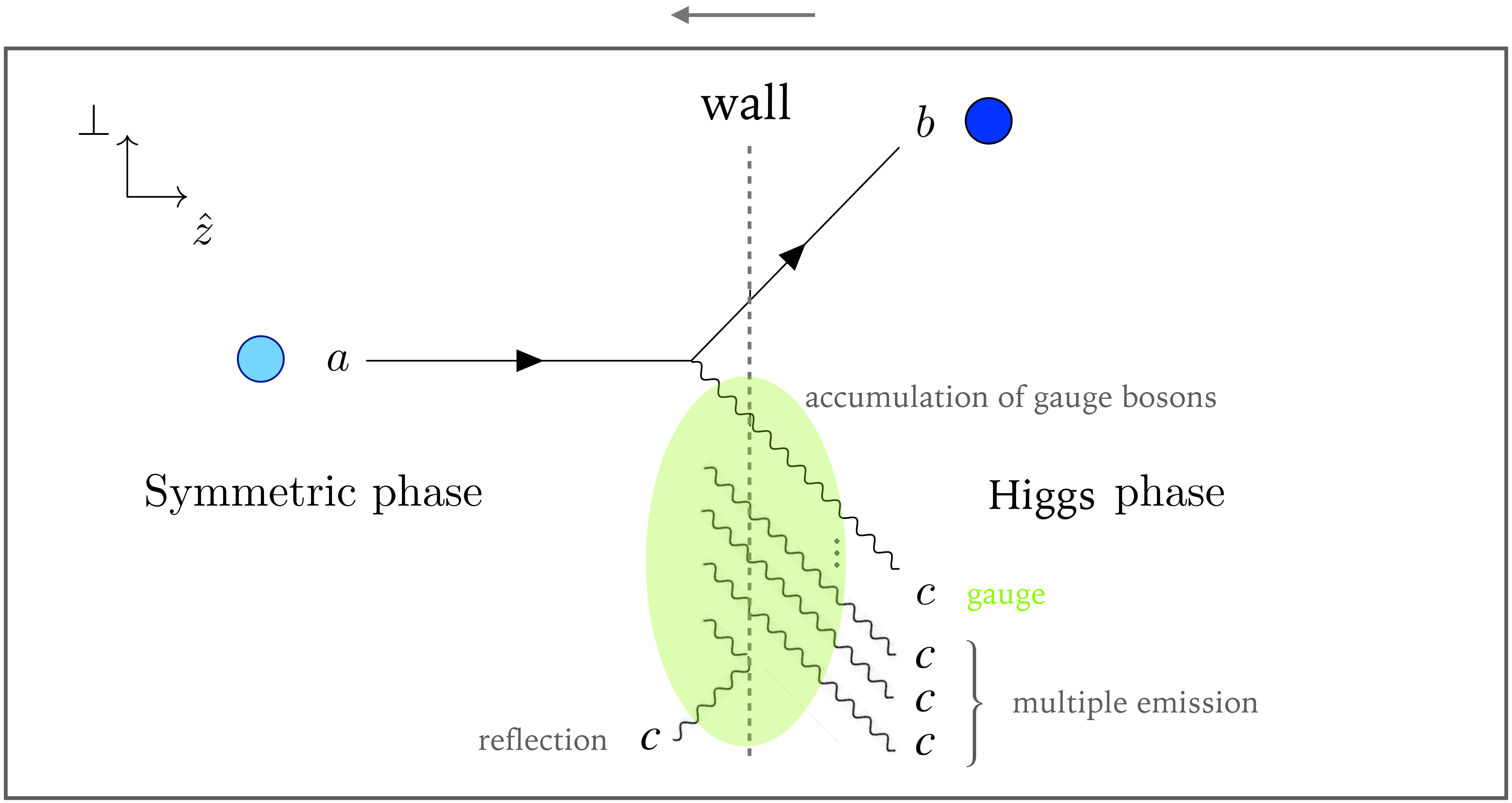
➤ To answer this, we need to understand friction to relativistic walls

FRICTION TO RELATIVISTIC WALLS

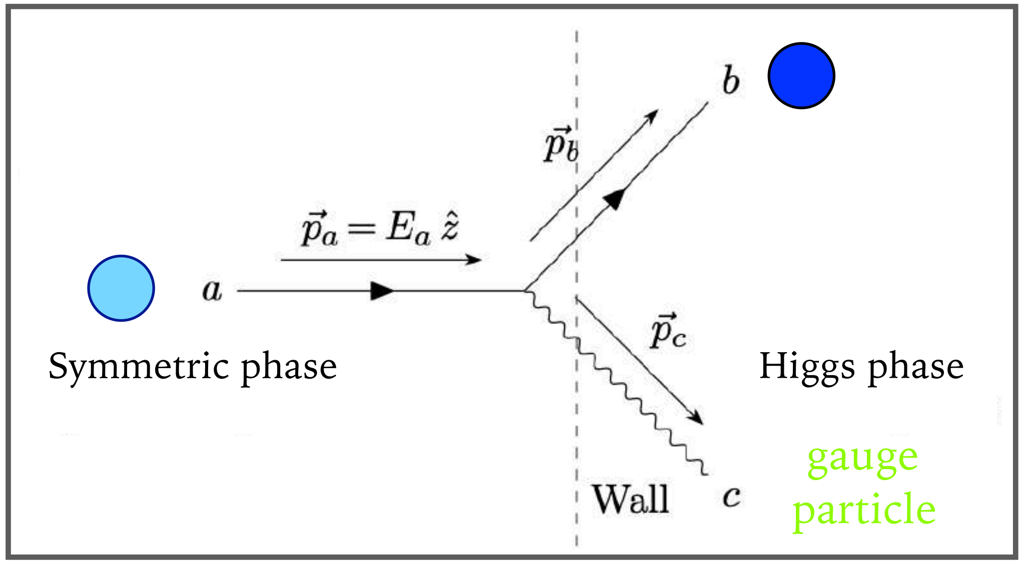
➤ Splitting radiation is found to be dominant source of friction [Bodeker & Moore '17]

see e.g. [Jackson "Classical Electrodynamics]

$$\gamma = 1/\sqrt{1-v^2} \gg 1$$



FRICTION TO RELATIVISTIC WALLS



- Results reported from several groups

(1) [Bodeker & Moore '17]

$a \to bc$ process gives friction propto the wall γ factor: $\mathscr{P} \sim \gamma m_c T^3$

(2) [Hoeche, Kozaczuk, Long, Turner, Wang '20]

multiple splitting $a \to bccc\cdots$ gives even stronger friction: $\mathscr{P} \sim \gamma^2 T^4$

(3) [Azatov & Vanvlasselaer '20]

points out (2) does not vanish in the transition-less limit: masses $(\propto \phi) \to 0$

(4) [Gouttenoire, Jinno, Sala '21]

multiple splitting $a \to bccc\cdots$ rather gives $\mathscr{P} \sim \gamma m_c T^3$

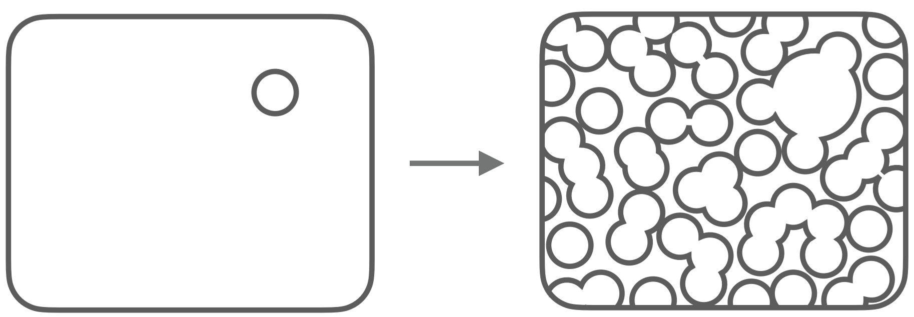
Backup

SCALE INVARIANT MODELS

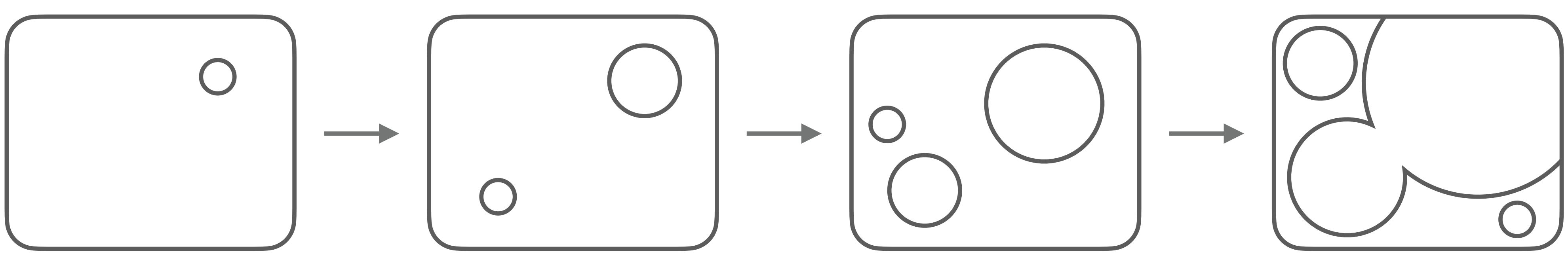
[Randall, Servant '07]
[Espinosa, Konstandin, No, Quiros '08]

- If the microphysics model has nearly scale invariance, the resulting GW production can be huge

Typical models



Nearly scale-invariant models



the system looks almost the same at different temperatures → gradual nucleation of bubbles

time ↗
temperature ↘

$T = T_{\rm initial}$ e.g. $T = 0.1T_{\rm initial}$

SCALE INVARIANT MODELS

[Iso, Okada, Orikasa '09]
[RJ, Takimoto '16]

➤ One example: Classically conformal B-L model

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$
q_L^i	**3**	**2**	+1/6	+1/3
u_R^i	**3**	**1**	+2/3	+1/3
d_R^i	**3**	**1**	-1/3	+1/3
l_L^i	**1**	**2**	+1/6	-1
e_R^i	**1**	**1**	-1	-1
ν_R^i	**1**	**1**	0	-1
H	**1**	**2**	-1/2	0
Φ	**1**	**1**	0	+2

TABLE I: Matter contents of the classically conformal $B-L$ model. In addition to the standard model matters, three generations of right-handed neutrinos ν_R^i and a $B-L$ charged complex scalar field Φ are introduced.

SCALE INVARIANT MODELS

[Iso, Okada, Orikasa '09]
[RJ, Takimoto '16]

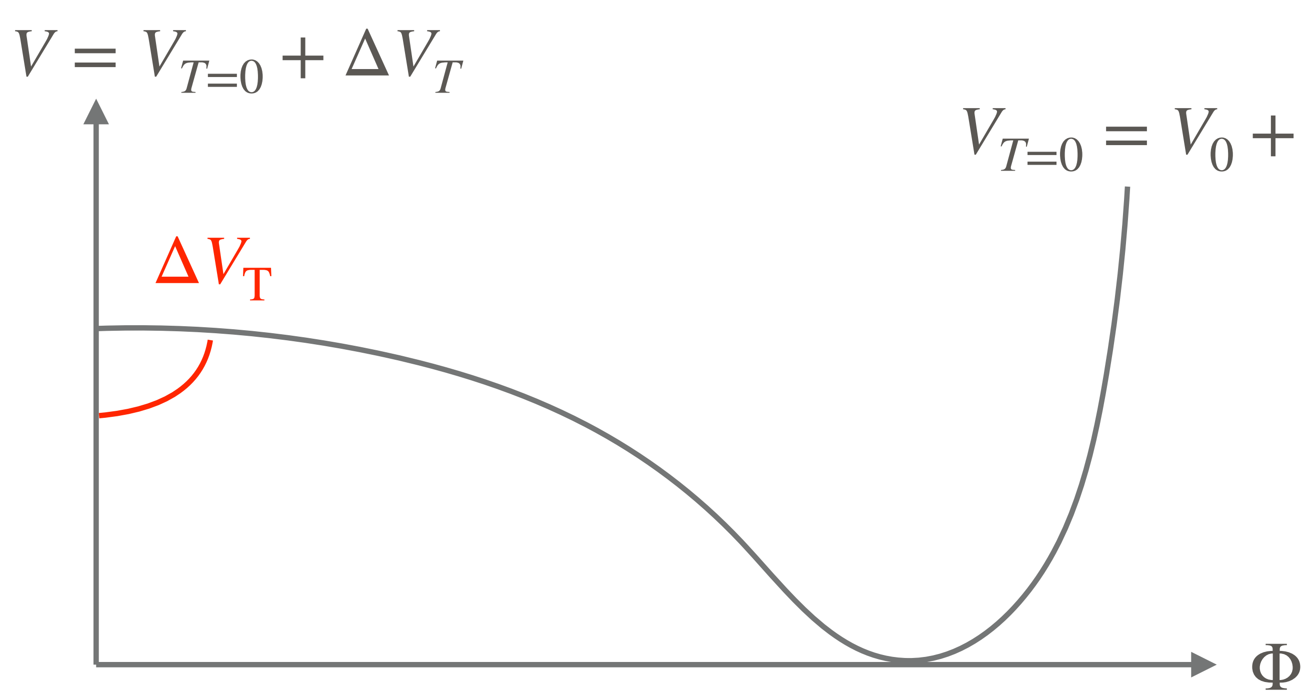
- Assumption: absence of mass scales

$$V_0 = \lambda_H |H|^4 + \lambda |\Phi|^4 - \lambda' |\Phi|^2 |H|^2$$

- Only quartic terms in the potential, no parameters with mass dim.

- Scale dependence enters only through running of couplings

- Transition in Φ direction can be strong



SCALE INVARIANT MODELS

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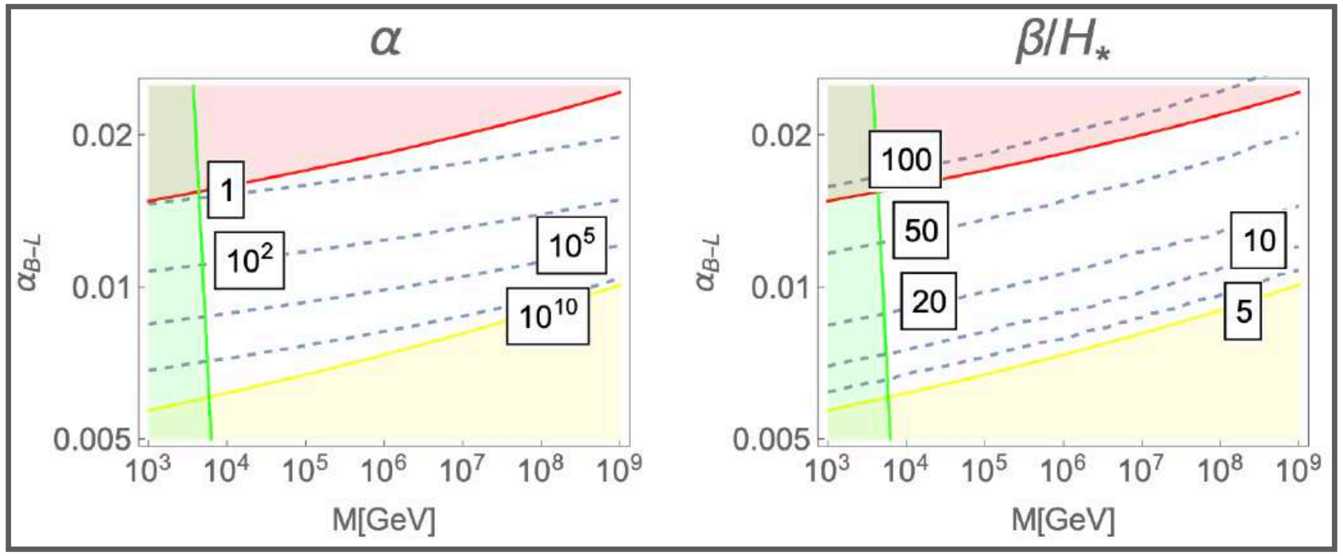
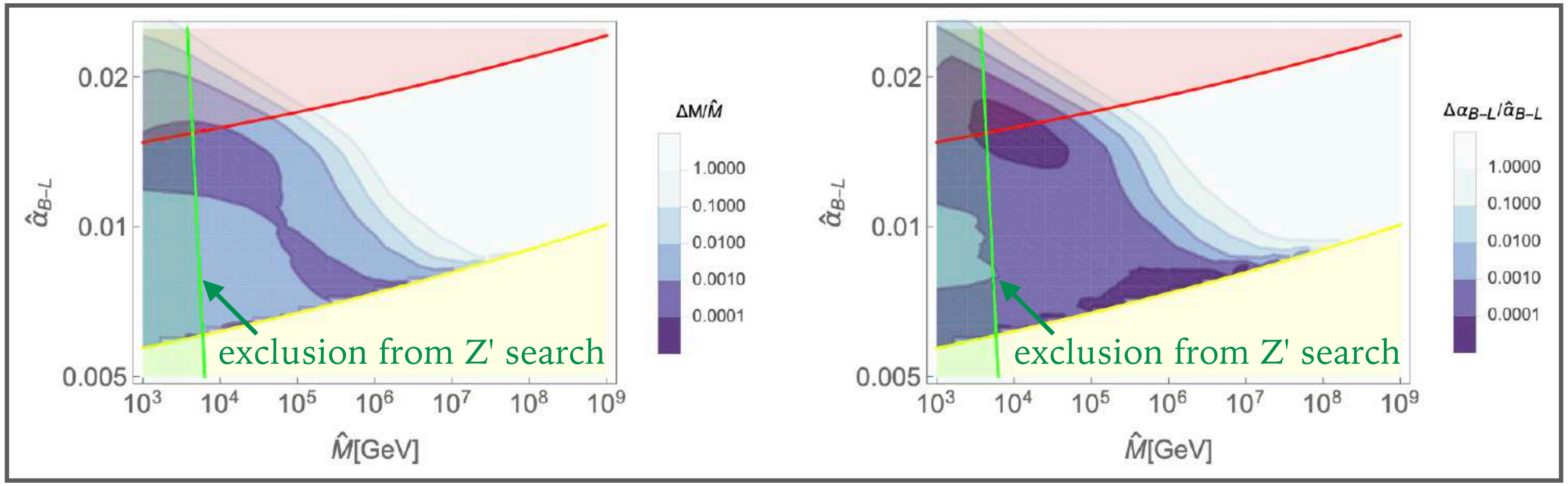
[RJ, Takimoto '16]

➤ Transition parameters



➤ Synergy between collider and GW experiments



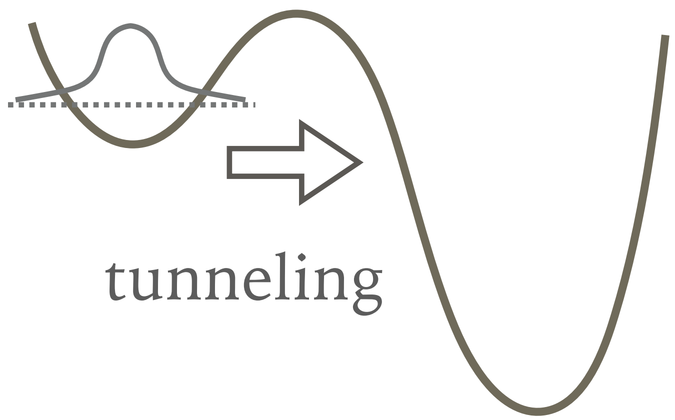
HOW TO CALCULATE THE TUNNELING RATE

[Coleman '77] [Callan, Coleman '77]
see also [Andreassen, Farhi, Frost, Schwartz '16]

➤ First let's consider tunneling in vacuum (= no plasma) for simplicity

- Tunneling rate appears as the imaginary part of the ground-state energy E_0

$$\sim e^{-iE_0 t} \sim e^{-iRe[E_0]t} e^{Im[E_0]t}$$



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- To extract it, consider $e^{-E_0 T} \sim \langle \phi_{\rm false} | e^{-HT} | \phi_{\rm false} \rangle = \int_{\phi(t_i)=\phi_{\rm false}}^{\phi(t_f)=\phi_{\rm false}} \mathscr{D}\phi \ e^{-S_E[\phi]}$

- The action is Euclidean $S_E = \int d^4x \left[\frac{1}{2} (\partial_E \phi)^2 + V(\phi) \right] \underset{\text{assumption}}{\overset{\text{O(4) symmetric}}{=}} 2\pi^2 \int dr \ r^3 \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial r} \right)^2 + V(\phi) \right]$

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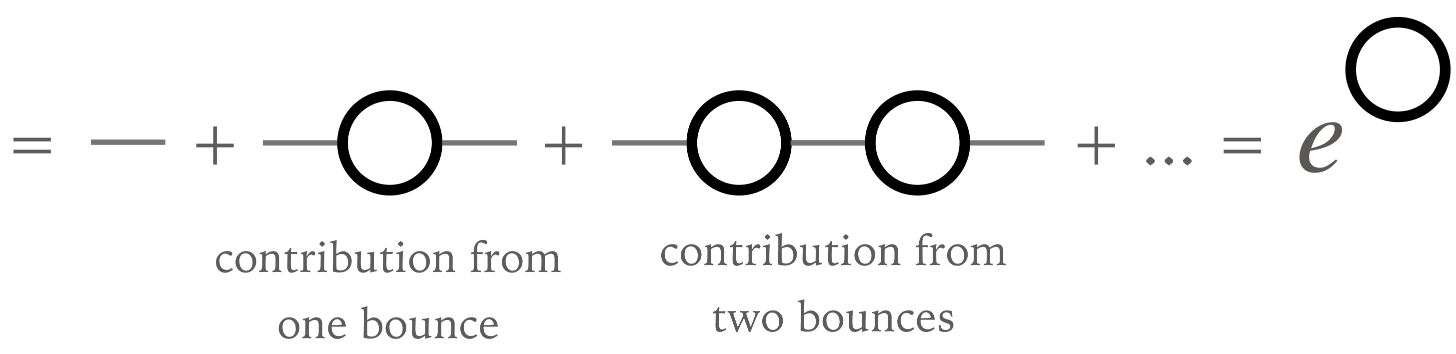
  - The imaginary part arises from a saddle-point configuration called "bounce"

$$\int_{\phi(t_i)=\phi_{\rm false}}^{\phi(t_f)=\phi_{\rm false}} \mathscr{D}\phi \; e^{-S_E[\phi]} =$$



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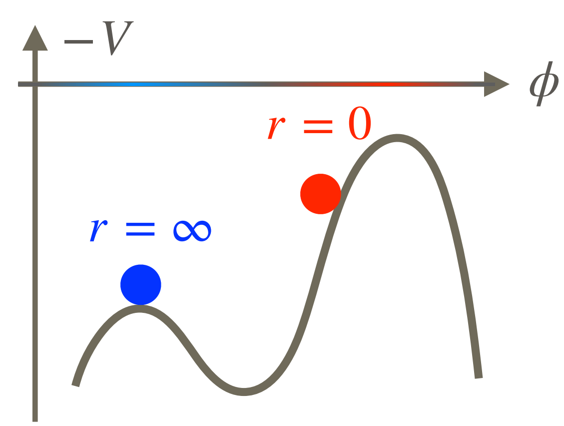
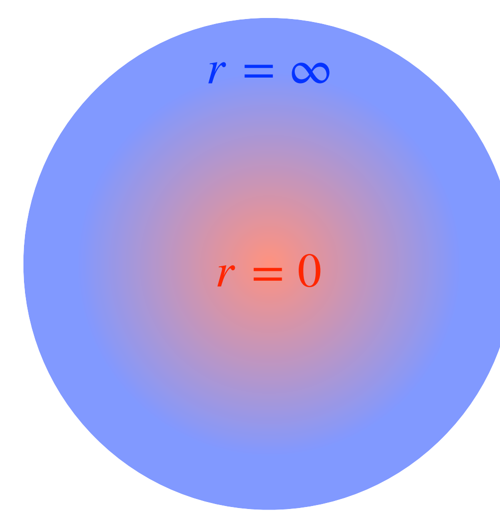
bounce $\bar{\phi}$ = solution of $\partial_r^2 \phi + \frac{3}{r} \partial_r \phi - V'(\phi) = 0$ with



=

x^4



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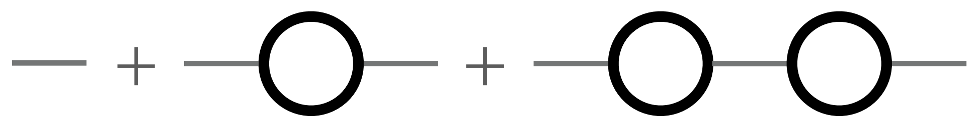


$$+ \ldots = e^{i\Lambda^4 \cdot VT \cdot e^{-S_E[\bar{\phi}]}}$$

contribution from one bounce

contribution from two bounces

$$\to \quad \Gamma = -\frac{2 \mathrm{Im} E_0}{V} \sim \Lambda^4 e^{-S_E[\bar{\phi}]}$$

HOW TO CALCULATE THE TUNNELING RATE

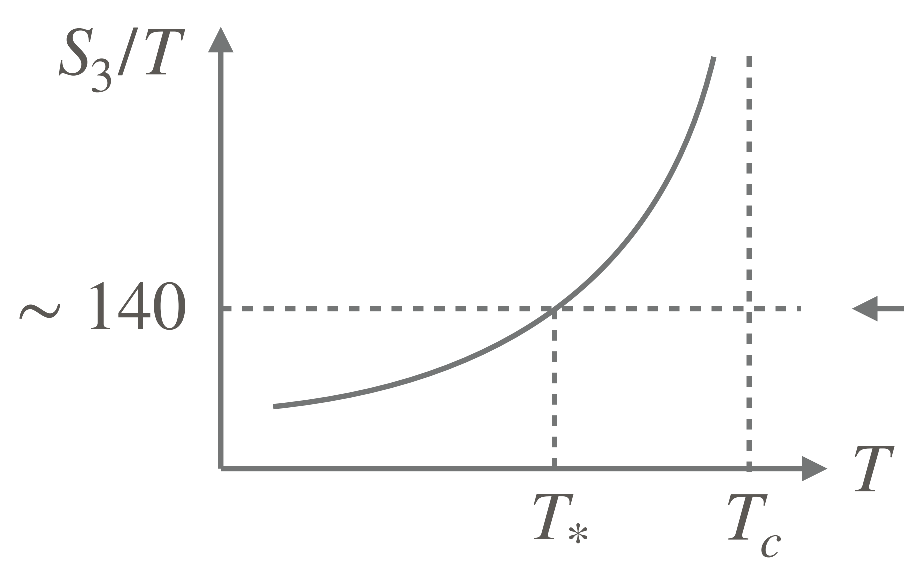
[Linde '83]

- In finite-temperature environment, the expression is similar

$$\Gamma(T) \sim \Lambda^4 e^{-\frac{S_3[\bar{\phi},T]}{T}}$$

$$S_3[\phi,T] = \int d^3x \left[\frac{1}{2}(\partial\phi)^2 + V(\phi,T)\right] \underset{\text{assumption}}{\overset{\text{O(3) symmetric}}{=}} 4\pi \int dr\ r^2 \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial r}\right)^2 + V(\phi,T)\right]$$

- Typical behavior



$$\longleftarrow \quad \Gamma \sim \Lambda^4 e^{-S_3/T} \sim H^4 \sim \frac{\Lambda^8}{M_P^4}$$

- To translate it to the transition parameter β , we use cosmology

$$\beta = \frac{d}{dt}\left(\frac{S_3}{T}\right)\bigg|_{t=t_*} = \frac{dT}{dt}\frac{d}{dT}\left(\frac{S_3}{T}\right)\bigg|_{t=t_*} \simeq H(t_*)\left[T\frac{d}{dT}\left(\frac{S_3}{T}\right)\right]_{T=T_*}$$