カイラル対称性の超回復

Super restoration of chiral symmetry

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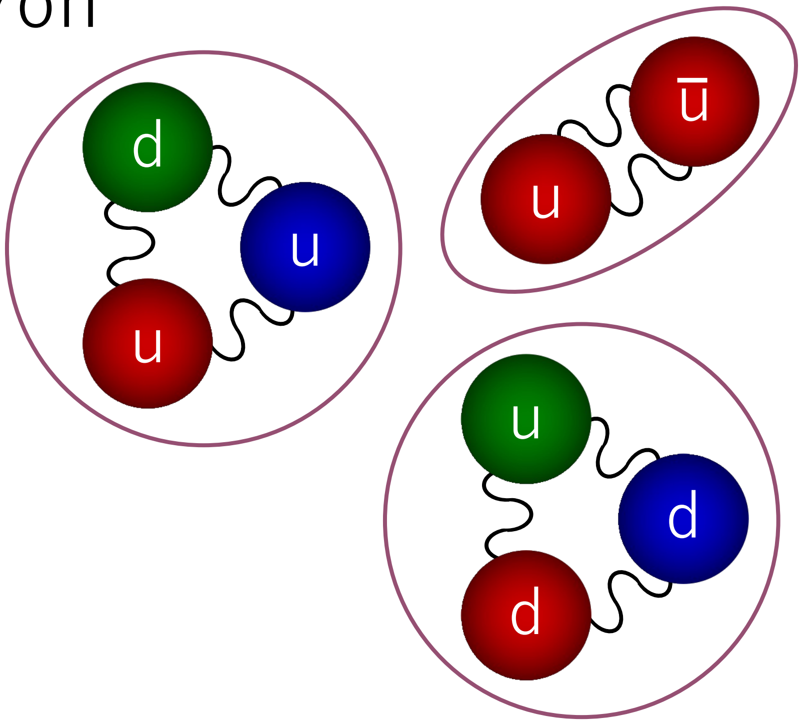
T.I., D. Kimura and H. Shimoji, arXiv:2306.00470

Phase Structure of QCD

Are symmetries restored or lost in extreme conditions?

- High Temperature
- High Density
- …

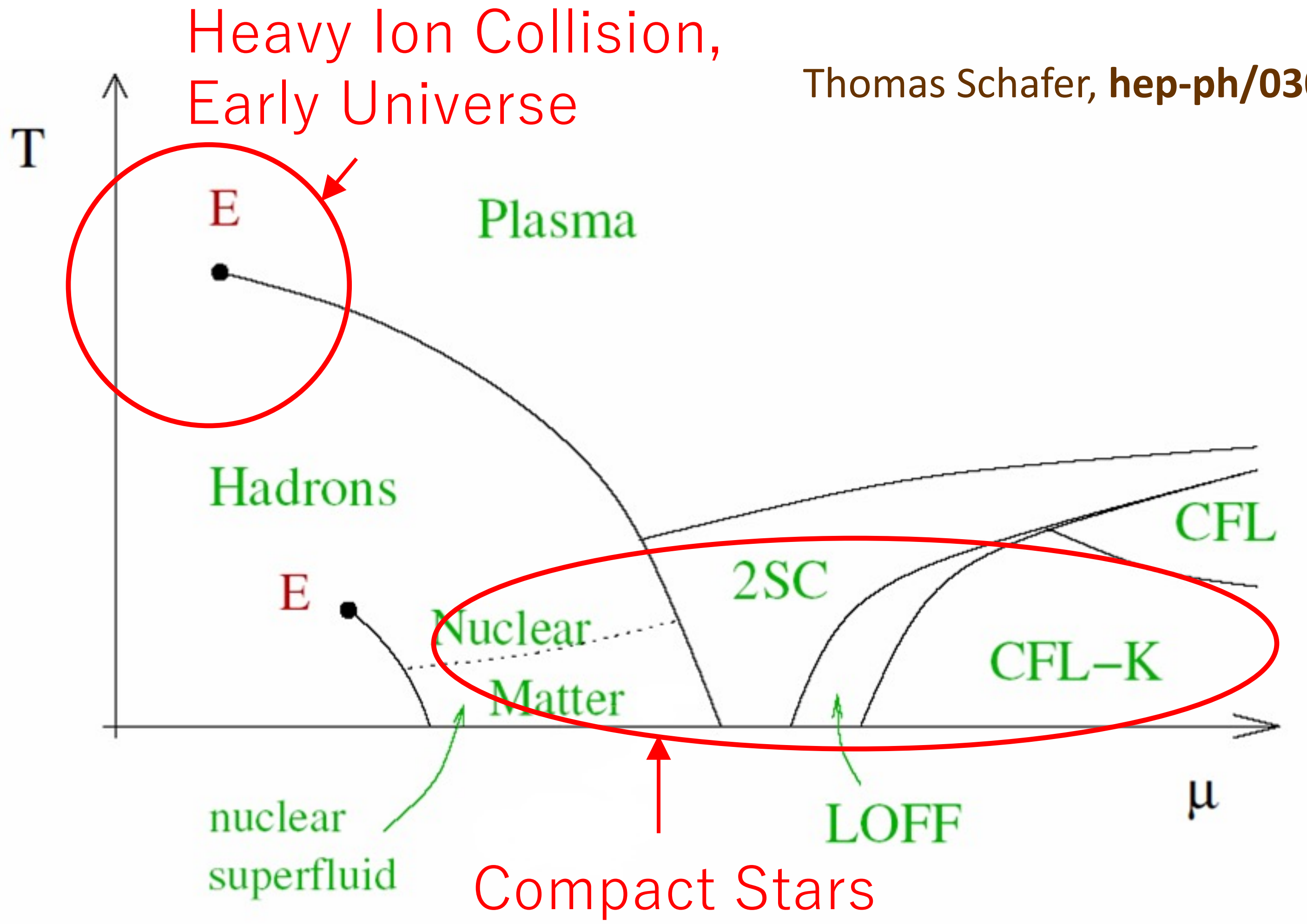
- Hadron



- Confinement

Phase Structure of QCD

Thomas Schafer, **hep-ph/0304281**



Conjectured phase diagram of three flavor QCD with realistic quark masses

Phenomenological Approach

- Because of the strong coupling, it is difficult to evaluate the ground state from the first principle, i. e. QCD.
- Phenomenological approach
  - Effective models
  - SD equation

Effective Model of QCD

- NJL model (2 flavors)

$$S=\int d^Dx\left[\bar{\psi}(x)(i\gamma^\mu\partial_\mu-m_0)\psi+\frac{G}{2N}\left((\bar{\psi}\psi)^2+(\bar{\psi}i\gamma_5\tau^a\psi)^2\right)\right]$$

Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1960) 345; 124 (1961) 246.

- GN model

$$S=\int d^Dx\left[\bar{\psi}(x)\left(i\gamma^\mu\partial_\mu-m_0\right)\psi(x)+\frac{\lambda_0}{2N}\left(\bar{\psi}(x)\psi(x)\right)^2\right]$$

D. J. Gross and A. Neveu, Phys. Rev. D10, 3235 (1974).

Chiral symmetry breaking

- Chiral symmetry

$\psi \to \exp(i\gamma^5 \theta^a \tau^a)\psi$: NJL model

$\psi \to \gamma^5 \psi$: GN model

- Auxiliary field method (GN model)

$$S = \int d^D x \left[\bar{\psi}(x) \left(i\gamma^\mu \partial_\mu - m_0 - \sigma(x) \right) \psi(x) - \frac{N}{2\lambda_0} \sigma(x)^2 \right]$$

Explicit breaking (m_0)   Spontaneous breaking ($\sigma(x)$)

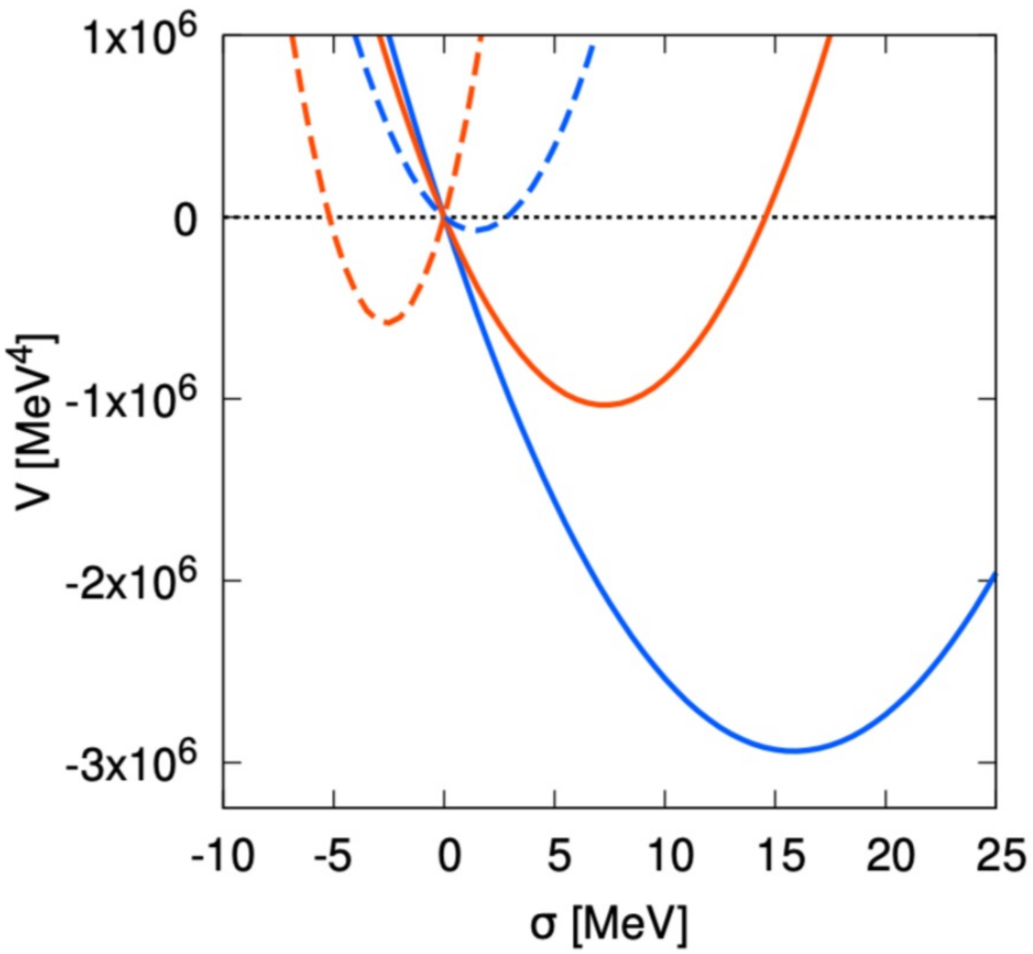
Effective potential analysis

We assume

- Homogeneous chiral condensation

and calculated the effective potential at the

- Leading order of 1/N expansion



Renormalization at T=μ=0

- Effective potential

$$V_D(\tilde{\sigma}) = \frac{\tilde{\sigma}^2 - 2m_0\tilde{\sigma}}{2\lambda_0} - \frac{C_D}{D}(\tilde{\sigma}^2)^{D/2}, \qquad \tilde{\sigma} = \sigma + m_0$$

- Renormalization condition

$$\left.\frac{\partial^2 V_D(\tilde{\sigma})}{\partial\tilde{\sigma}^2}\right|_{\tilde{\sigma}=\mu_r} = \frac{\mu_r^{D-2}}{\lambda_r}$$

$$\frac{1}{\mu_r^{D-1}} \left.\frac{\partial V_D(\tilde{\sigma})}{\partial\tilde{\sigma}}\right|_{\tilde{\sigma}=\mu_r} = \frac{1}{\lambda_r}\left(1 - \frac{m_r}{\mu_r}\right) + C_D(D-2),$$

Effective potential analysis

- Following the imaginary time formalism, we introduce the temperature and the chemical potential and obtain

$$V_D(\tilde{\sigma}; m_r, \mu_r; T, \mu) = \left(\frac{1}{\lambda_r} + (D-1)C_D\right)\frac{\tilde{\sigma}^2 \mu_r^{D-2}}{2} - \frac{m_r \tilde{\sigma} \mu_r^{D-2}}{\lambda_r} - \frac{C_D}{D}(\tilde{\sigma}^2)^{D/2}$$
$$- \tilde{C}_D T \int_0^\infty \mathrm{d}q\, q^{D-2}\left[\ln\left(1 + e^{-\left(\sqrt{q^2+\tilde{\sigma}^2}-\mu\right)/T}\right) + (-\mu \to \mu)\right]$$

Super restoration

- Order parameter of symmetry breaking (constituent mass)

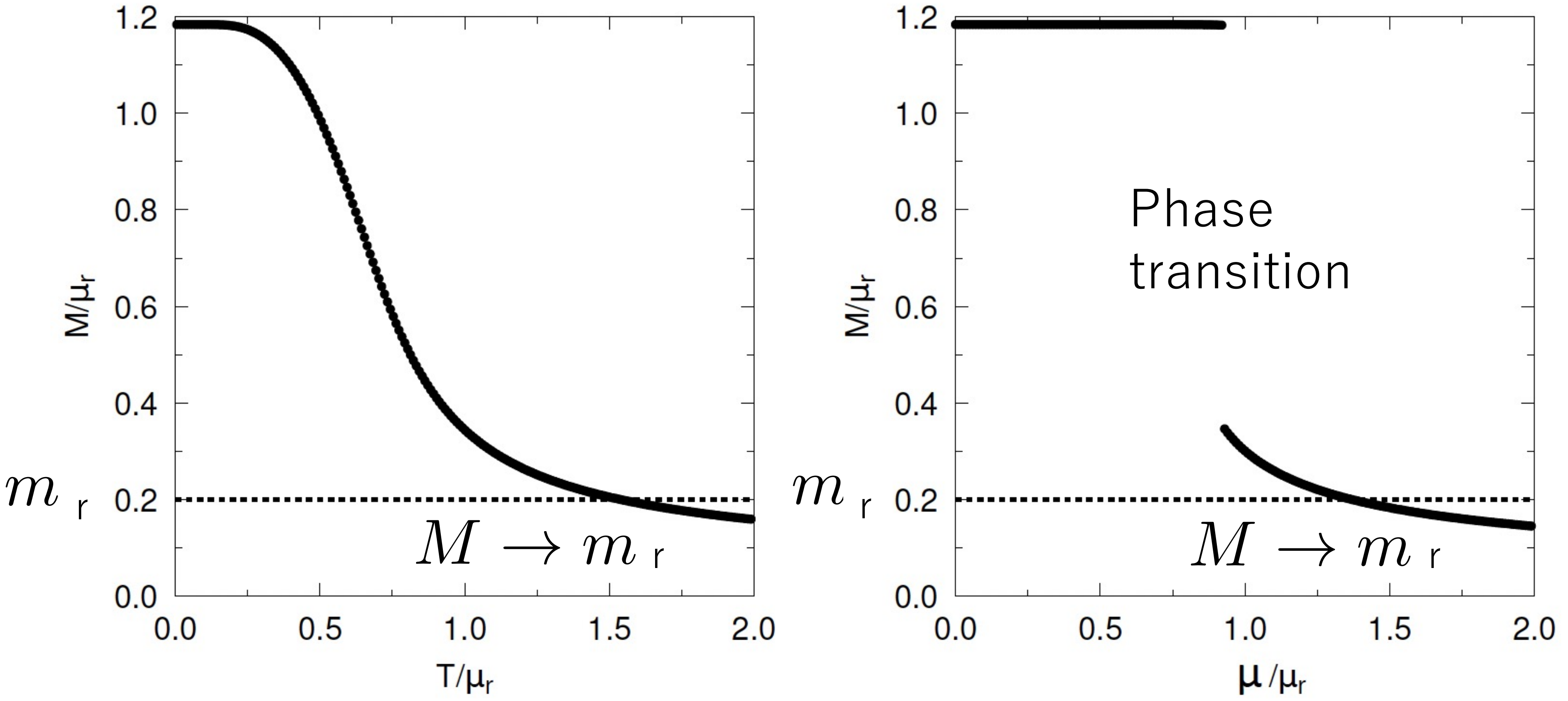
$$M = \langle \tilde{\sigma} \rangle = m_{\mathrm{r}} + \langle \sigma \rangle$$

- It has been expected that the spontaneously broken chiral symmetry is restored at high T and μ.

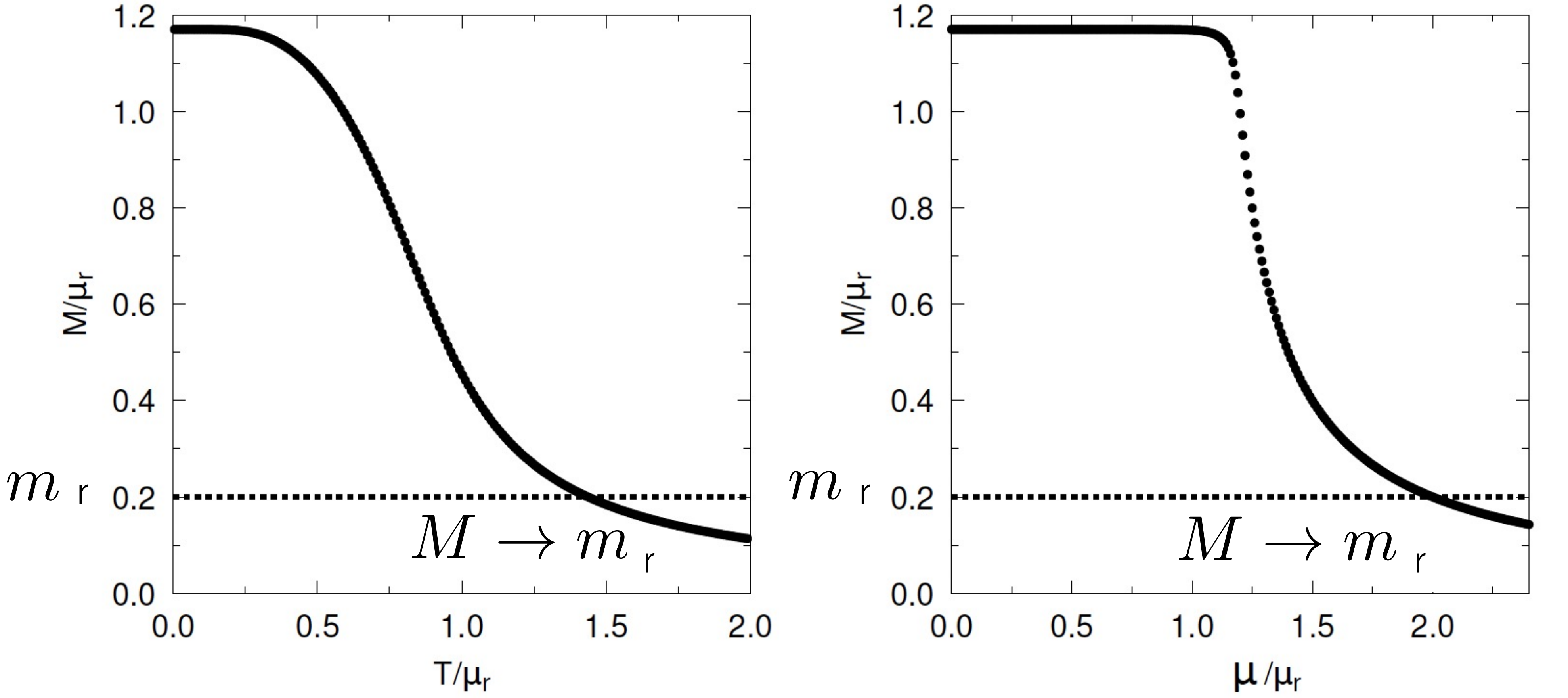
$$M \to m_{\mathrm{r}}$$

- Super restoration: If σ develops a negative expectation value, the explicitly broken chiral symmetry can be also restored.

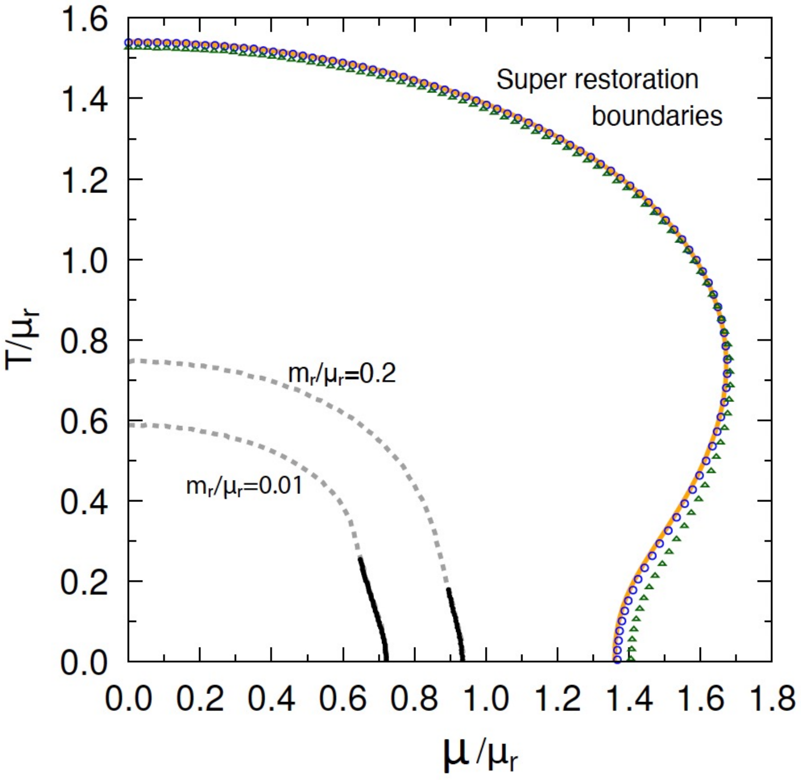
GN model (D=2)


1.2
1.0
0.8
0.6
0.4
0.2
0.0
M/μr
m_r
$M \to m_r$
0.0
0.5
1.0
1.5
2.0
T/μr
Phase
transition
μ/μr

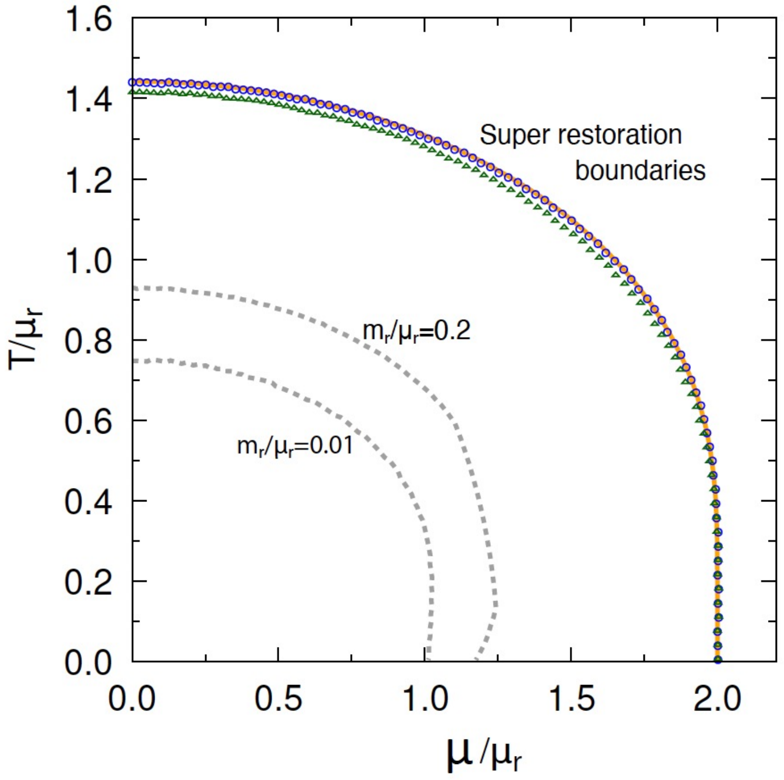
GN model (D=3)



Phase structure (GN model)



(a) $D = 2$



(b) $D = 3$

Gap equation with $M=m_0$

- At the limit $m_0=0$ for the gap equation with $M=m_0$

$$\frac{D-1}{2\pi^{1/2}}\Gamma\left(1-\frac{D}{2}\right)\Gamma\left(\frac{D-1}{2}\right)$$
$$=\Gamma(D-2)\left(\frac{T}{\mu_r}\right)^{D-2}\left[\mathrm{Li}_{D-2}\left(-e^{-\mu/T}\right)+\mathrm{Li}_{D-2}\left(-e^{\mu/T}\right)\right]$$

Specific points

- μ=0

$$T/\mu_r = e^{1+\gamma}/\pi$$ D=2

$$T/\mu_r = 1/\ln 2$$ D=3

- T=0

$$\mu/\mu_r = e/2$$ D=2

$$\mu/\mu_r = 2$$ D=3

Phase boundary at m_0=0

$$T/\mu_r = e^{\gamma}/\pi$$

$$T/\mu_r = 1/\ln 4$$

$$\mu/\mu_r = 1/\sqrt{2}$$

$$\mu/\mu_r = 1$$

2-flavor NJL model (D=4)

- The results depend on the regularization. Here we employ a simple cut off.
- 3DRT

$$\langle \sigma \rangle = 2G \left[i\mathrm{tr} S^0(M, \Lambda) + i\mathrm{tr} S^T(M, \Lambda) \right]$$

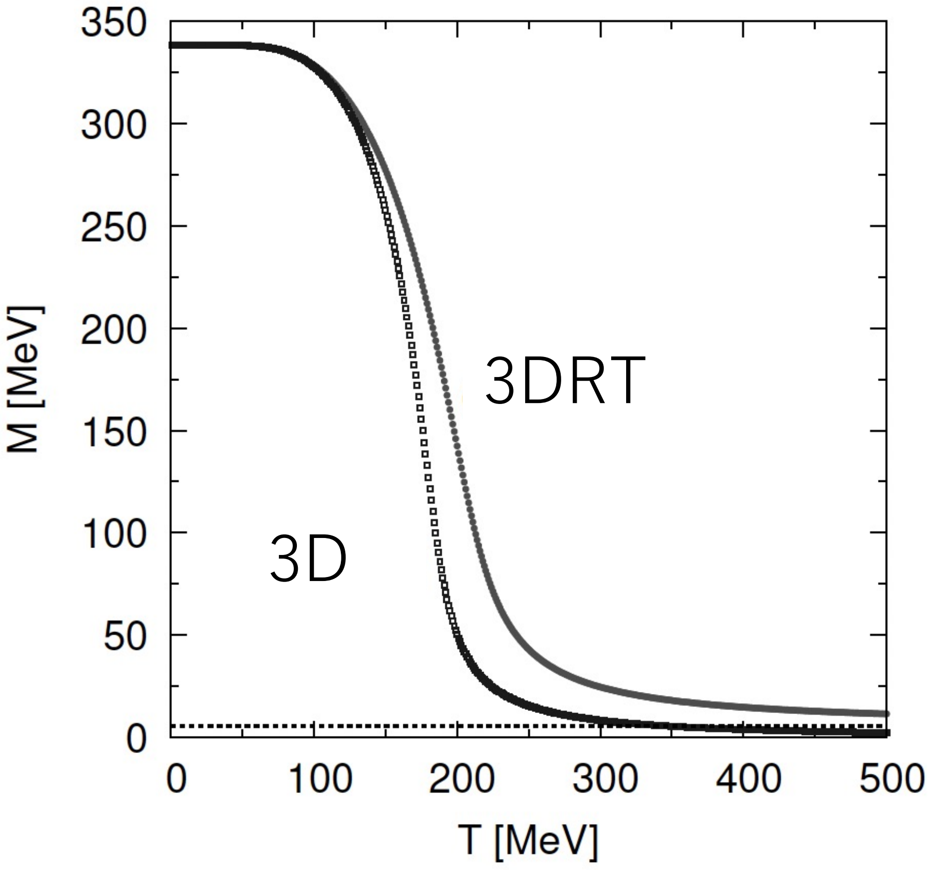
- 3D

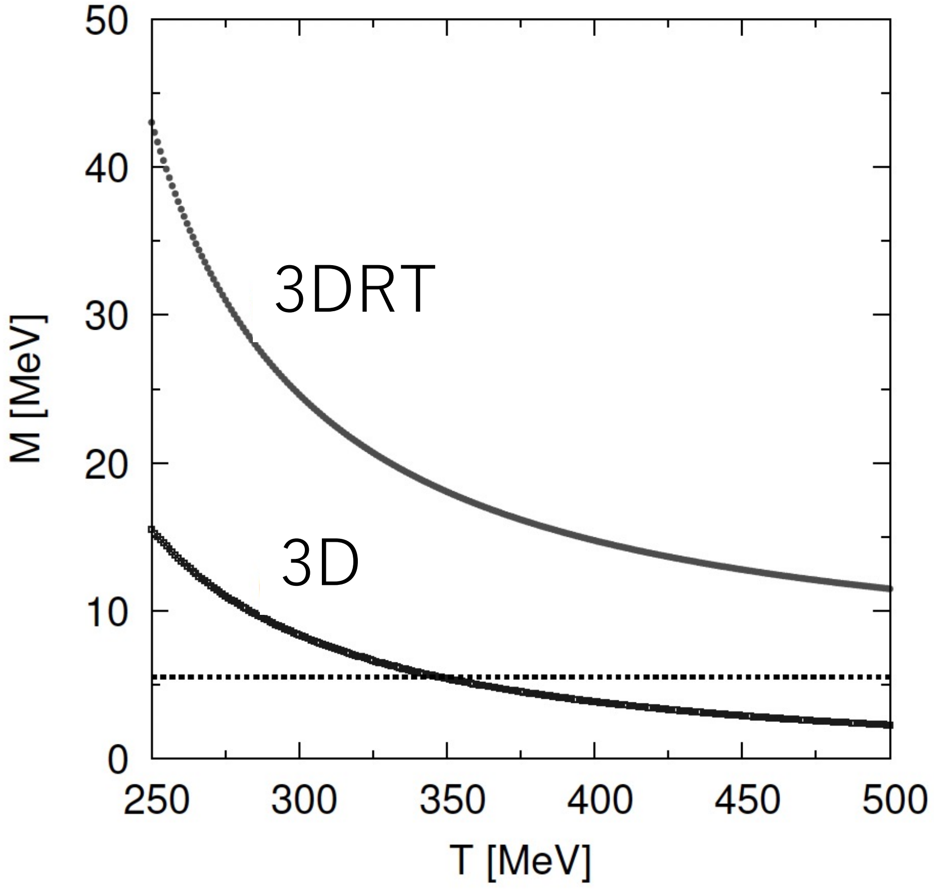
$$\langle \sigma \rangle = 2G \left[i\mathrm{tr} S^0(M, \Lambda) + i\mathrm{tr} S^T(M, \infty) \right]$$

H. Kohyama, D. Kimura and T. I. , Nucl. Phys. **B896** (2015) 682.

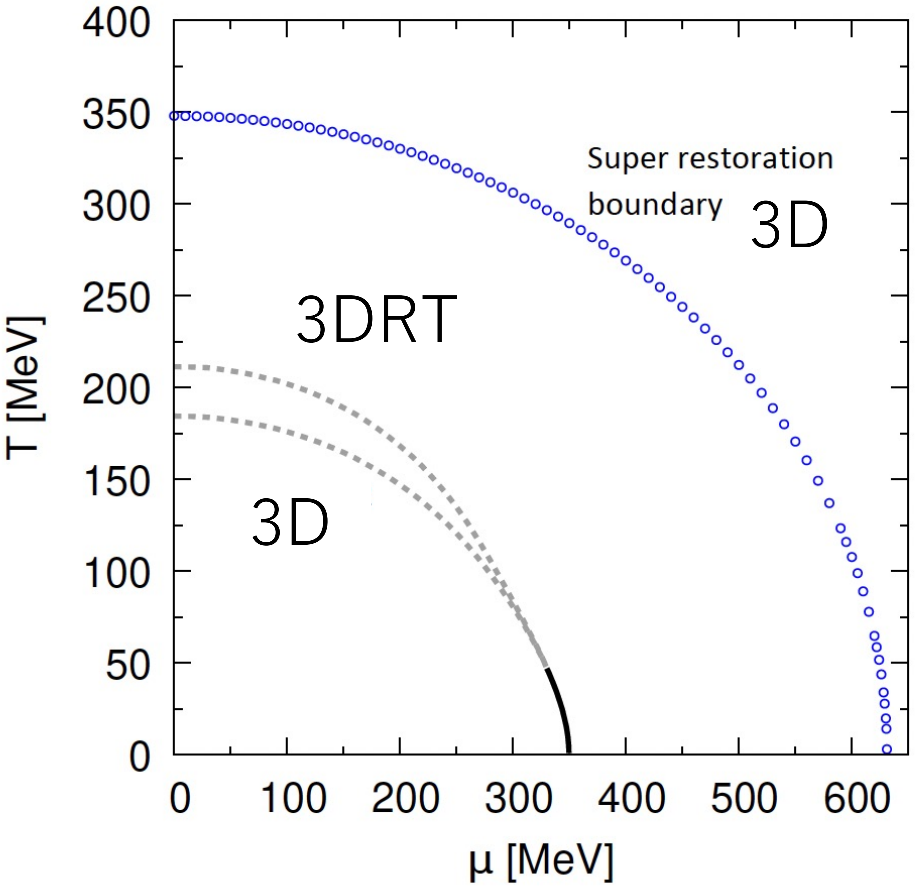
NJL model (D=4)

T.I., D. Kimura and H. Shimoji, arXiv:2306.00470,
E. B. Pasqualotto, R. L. S. Farias, W. R. Tavares, S. S. Avancini, and G. Krein, Phys. Rev. D 107, 096017 (2023)





NJL model (D=4)



$$\langle\sigma\rangle_{3D} = 2G\left[i\mathrm{tr}S^0(M,\Lambda) + i\mathrm{tr}S^T(M,\infty)\right]$$

Summary and concluding remarks

Super restoration in four-fermion interaction models:

- The spontaneously broken chiral symmetry can be restored at extremely high temperature or chemical potential.
- We can not find interesting phenomena around the boundary of the super restoration except for the chiral susceptibility.

What happens in a gauge theory?

- Preliminary result for SD equation in strong coupling QED: Super restoration has not been observed!?