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New gauge-independent transition separating confinement-Higgs phase in the lattice gauge-fundamental scalar model

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arXiv:2308.13430 **[hep-lat]**

1. Introduction

■ We consider SU(2) LGST (with fundamental scalar)

Action of SU(2) LGST (with fund. scalar):

$$S[U,\hat{\Theta}] = \underbrace{\frac{\beta}{2}\sum_{x,\mu>\nu} \mathrm{Re\,tr}\left(\mathbf{1} - U_{x,\mu}U_{x+\mu,\nu}U^{\dagger}_{x+\nu,\mu}U^{\dagger}_{x,\nu}\right)}_{S_G[U]\ :\ \text{gauge part}} + \underbrace{\frac{\gamma}{2}\sum_{x,\mu} \mathrm{Re\,tr}(\mathbf{1} - \hat{\Theta}^{\dagger}_{x}U_{x,\mu}\hat{\Theta}_{x+\mu})}_{S_H[U,\hat{\Theta}]\ :\ \text{scalar part}}$$

- $U_{x,\mu} \in \mathrm{SU}(2)$: link variables, $\hat{\Theta}_x \in \mathrm{SU}(2)$: (normalized) scalar fields
- β : gauge coupling, γ : scalar coupling

$\hat{\Theta}_x$ transforms as the fundamental representation of the gauge group SU(2).

This model has the $\mathrm{SU}(2)_{\mathrm{local}} \times \mathrm{SU}(2)'_{\mathrm{global}}$ symmetry:

$$U_{x,\mu} \mapsto U'_{x,\mu} = \Omega_x U_{x,\mu}\Omega^{\dagger}_{x+\mu}\,, \quad \hat{\Theta}_x \mapsto \hat{\Theta}'_x = \Omega_x\hat{\Theta}_x\Omega'$$

where $\Omega_x \in \mathrm{SU}(2)_{\mathrm{local}}\,,\ \Omega' \in \mathrm{SU}(2)'_{\mathrm{global}}\,.$

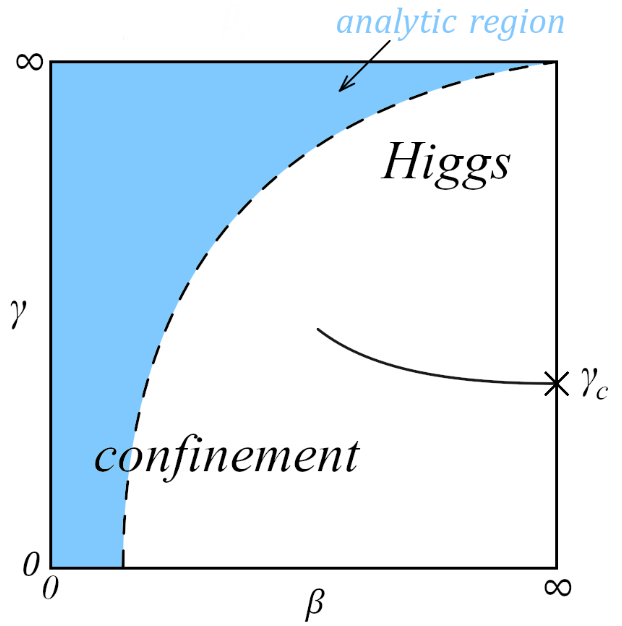
■ Motivation

□ In case of fundamental scalar field → This talk

Confinement ($\beta \geqslant 0, \gamma \ll 1$) and Higgs ($\beta \gg 1, \gamma_c \leqslant \gamma < \infty$) regions are subregions of an analytically continued single phase. The transition line starts from $(\beta, \gamma) = (\infty, \gamma_c)$ does not reach "analytic region" which connect these subregions.

[1] K. Osterwalder and E. Seiler, Annals Phys. **110**, 440 (1978)

[2] E. Fradkin and S. H. Shenker, Phys. Rev. **D19**, 3682 (1979)

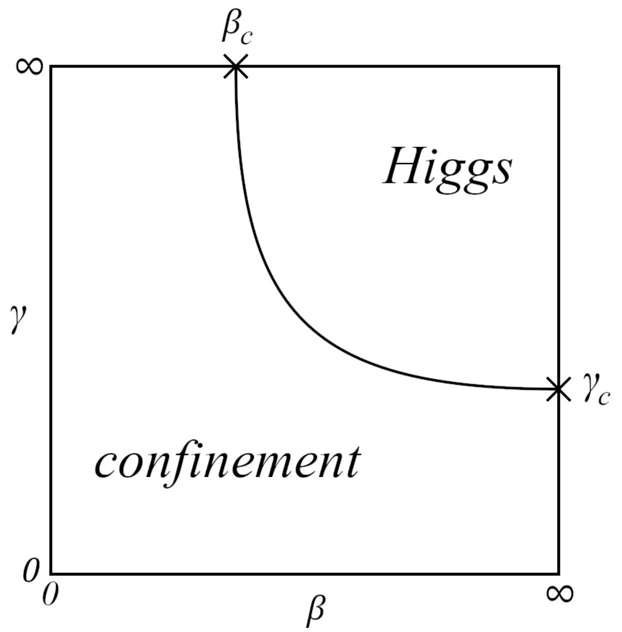


□ In case of adjoint scalar field → [3]

Confinement and Higgs regions are completely separated into the two different phases by the continuous transition line. The transition line has two endpoints, $(\beta, \gamma) = (\infty, \gamma_c)$ and $(\beta, \gamma) = (\beta_c, \infty)$.

[3] A. Shibata and K.-I. Kondo (2023), arXiv: 2307.15953 [hep-lat]

[4] R. C. Brower et al., Phys. Rev. **D25**, 3319 (1982)



■ Motivation and Results

Re-examine the 4D SU(2) lattice gauge scalar theory (LGST) with fund. scalar field,

- We found the gauge-invariant composite operator of the original scalar field and the new "color-direction field", which enables to separate the confinement phase and the Higgs phase completely and gauge-independently.
  (Greensite-Matsuyama discriminated 2 phases by detecting the SSB of $\mathrm{SU}(2)'_{\mathrm{global}}$)
  [5] J. Greensite and K. Matsuyama, Phys. Rev. D**98**, 074504 (2018); Phys. Rev. D**101**, 054508 (2020)

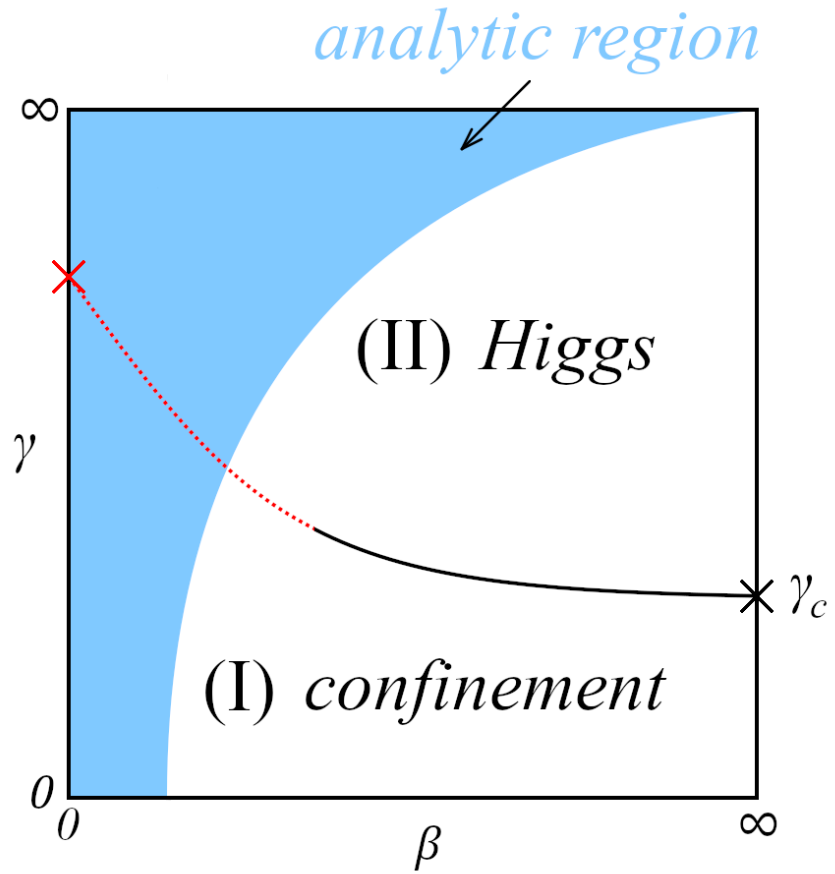
- We perform the numerical simulations for this model without any gauge fixing, and found a new transition line:
  - in the weak gauge coupling, it agrees with the conventional thermodynamical transition line.
  - in the strong gauge coupling, it divides the single confinement-Higgs phase into two separate phases, confinement and the Higgs.

2. New transition line and Color-direction field

■ Newly found transition line

- As the result of the numerical simulations for the 4D SU(2) LGST with fund. scalar, we found a new transition line which separates confinement and Higgs regions completely, without any gauge fixing.

- This transition line is obtained in the gauge-independent way by introducing the new composite operator of the original scalar field and the new "color-direction field", based on the gauge-covariant decomposition of the gauge field due to [6-9].



[6] Y.M. Cho, Phys. Rev. D**21**, 1080 (1980); Phys. Rev. D**23**, 2415 (1981)

[7] Y.S. Duan and M.L. Ge, Sinica Sci. **11**, 1072 (1979)

[8] S.V. Shabanov, Phys. Lett. B**463**, 263 (1999)

[9] L.D. Faddeev and A.J. Niemi, Phys. Rev. Lett. **82**, 1624 (1999); Nucl. Phys. B**776**, 38 (2007)

■ gauge-covariant decomposition of the gauge field

$U_{x,\mu} \in \mathrm{SU}(2)$ is gauge-covariantly decomposable: $U_{x,\mu} = X_{x,\mu} V_{x,\mu} \quad (X_{x,\mu}, V_{x,\mu} \in \mathrm{SU}(2))$

(We required the transformations, $X_{x,\mu} \mapsto X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega_x^\dagger\,,\quad V_{x,\mu} \mapsto V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega_{x+\mu}^\dagger$)

- This decomposition is given uniquely by solving the defining equations for the "color-direction field" $\boldsymbol{n}_x \in su(2) - u(1)$ with a unit :

$$D_\mu[V]\boldsymbol{n}_x := V_{x,\mu}\boldsymbol{n}_{x+\mu} - \boldsymbol{n}_x V_{x,\mu} = 0\,,\quad \mathrm{tr}\,(\boldsymbol{n}_x X_{x,\mu}) = 0$$

  (We required the transformation, $\boldsymbol{n}_x \mapsto \boldsymbol{n}'_x = \Omega_x \boldsymbol{n}_x \Omega_x^\dagger$)

- For a given set of gauge fields $\{U_{x,\mu}\}$, a set of color-direction fields $\{\boldsymbol{n}_x\}$ is determined as the configuration minimizing the reduction functional:

$$F_{\mathrm{red}}[\boldsymbol{n};U] = \sum_{x,\mu} \frac{1}{2}\,\mathrm{tr}\left\{ (D_\mu[U]\boldsymbol{n}_x)^\dagger\,(D_\mu[U]\boldsymbol{n}_x) \right\} = \sum_{x,\mu} \mathrm{tr}\,(\mathbf{1} - \boldsymbol{n}_x U_{x,\mu} \boldsymbol{n}_{x+\mu} U_{x,\mu}^\dagger)$$

- $F_{\mathrm{red}}[\boldsymbol{n};U]$ has the same form as the Higgs action of SU(2) LGST (with adj. scalar)

  (minimization of $F_{\mathrm{red}}[\boldsymbol{n};U]$ extracts the DOF of $\{\boldsymbol{n}_x\}$ from the gauge fields $\{U_{x,\mu}\}$)

  [10] Kondo et al., Phys. Rep. **579**, 1-226 (2015)

■ construction of the scalar-color density $\bar{\boldsymbol{R}}$

- We required that $\hat{\Theta}_x \in \mathrm{SU}(2)$ and $\boldsymbol{n}_x \in su(2) - u(1)$ transform as

$$\hat{\Theta}_x \mapsto \hat{\Theta}'_x = \Omega_x \hat{\Theta}_x \Omega' , \quad \boldsymbol{n}_x \mapsto \boldsymbol{n}'_x = \Omega_x \boldsymbol{n}_x \Omega_x^\dagger$$

where $\Omega_x \in \mathrm{SU}(2)_{\mathrm{local}}$, $\Omega' \in \mathrm{SU}(2)'_{\mathrm{global}}$.

- We can define a global covariant local field $\boldsymbol{R}_x$ and its spacetime average $\bar{\boldsymbol{R}}$:

$$\boldsymbol{R}_x := \hat{\Theta}_x^\dagger \boldsymbol{n}_x \hat{\Theta}_x , \quad \boldsymbol{R}_x \mapsto \boldsymbol{R}'_x = \Omega'^\dagger \boldsymbol{R}_x \Omega'$$

$$\bar{\boldsymbol{R}} := \frac{1}{V} \sum_x \boldsymbol{R}_x , \quad \bar{\boldsymbol{R}} \mapsto \bar{\boldsymbol{R}}' = \Omega'^\dagger \bar{\boldsymbol{R}} \Omega' \qquad \left(\bar{R}^A = \frac{1}{2} \mathrm{tr}(\sigma^A \bar{\boldsymbol{R}}) \right)$$

- Then there exists a non-trivial gauge invariant defined as

$$\|\bar{\boldsymbol{R}}\| = \sqrt{\frac{1}{2} \mathrm{tr}(\bar{\boldsymbol{R}}^\dagger \bar{\boldsymbol{R}})} , \quad \|\bar{\boldsymbol{R}}\| \mapsto \|\bar{\boldsymbol{R}}'\| = \|\bar{\boldsymbol{R}}\|$$

$\|\bar{\boldsymbol{R}}\|$ also can be defined as the absolute value of the two eigenvalues of $\bar{\boldsymbol{R}}$.

3. Numerical simulation

■ Setting for lattice simulation

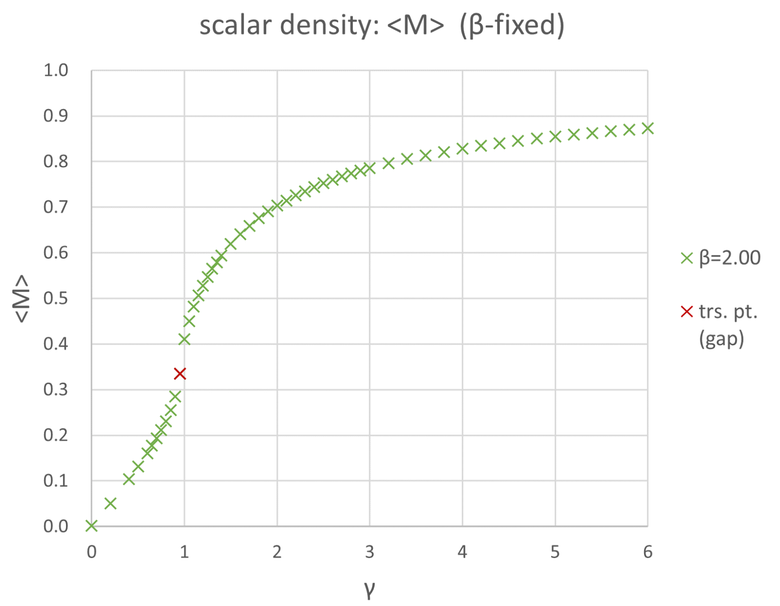
- 8^4 lattice, pseudo heat bath method
- cold start ($U_{x,\mu} = \mathbf{1}$, $\hat{\Theta}_x = \mathbf{1}$)
- 5000 sweeps for thermalization before sampling
- a gauge/scalar config. sampling per 100 sweeps (100 samples for average)

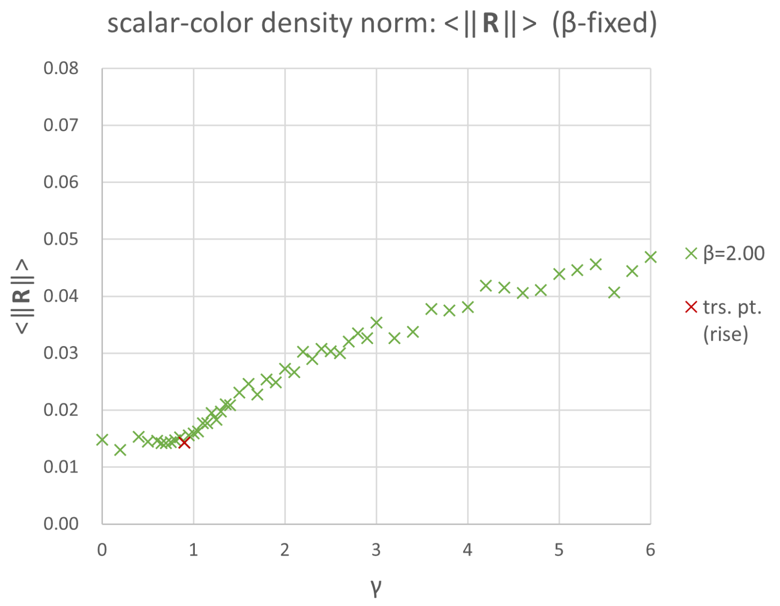
To determine a set of $\{\boldsymbol{n}_x\}$ from a given set of $\{U_{x,\mu}\}$,

- reduction functional $F_{\text{red}}[\boldsymbol{n}; U]$ is minimized by the iterative method with over-relaxation.
- 10 trials are taken for searching global optimal config..

Performed for 17 × 52=884 sets of parameter points (β, γ).

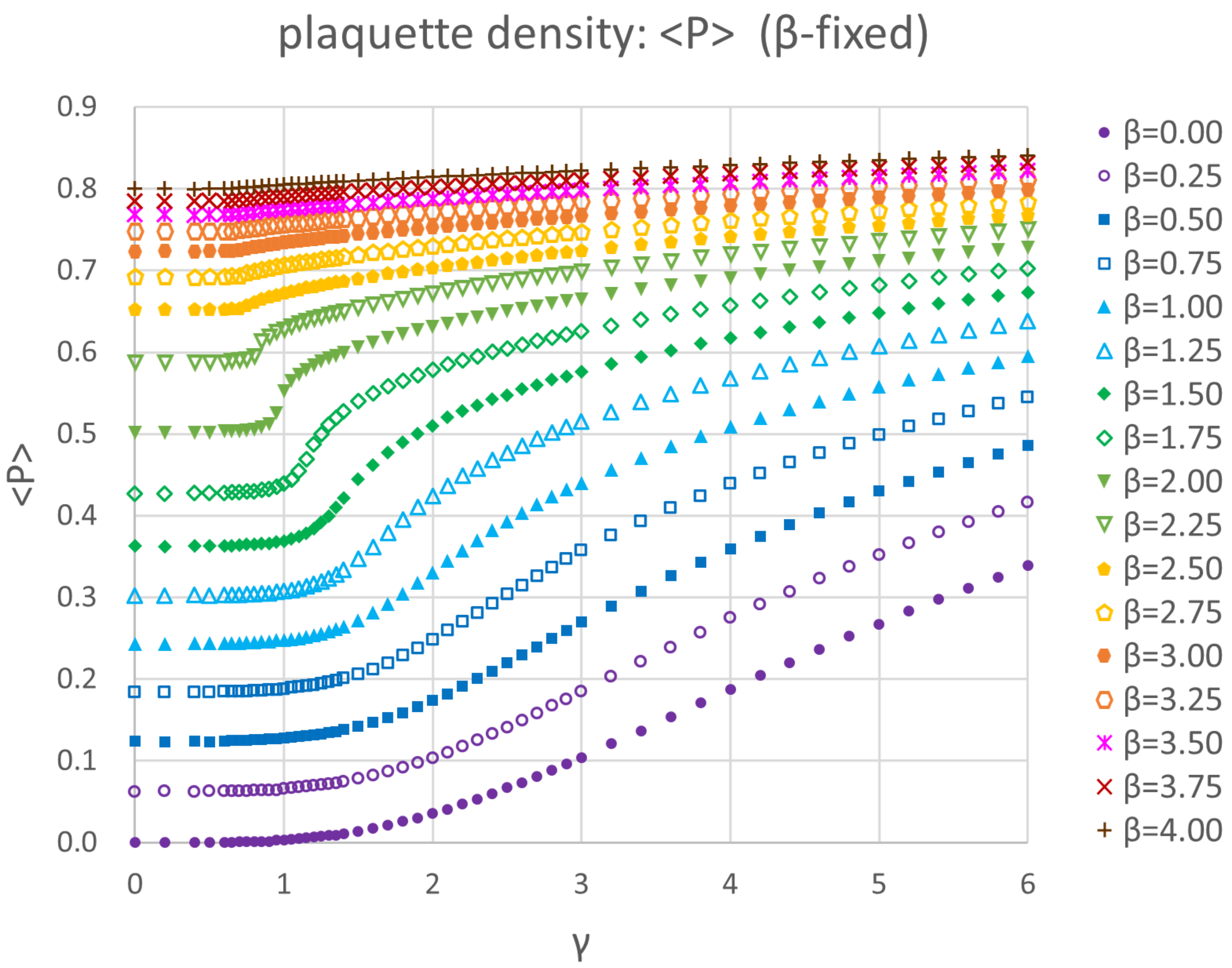
Transition lines were identified from gaps/rises of the plots.

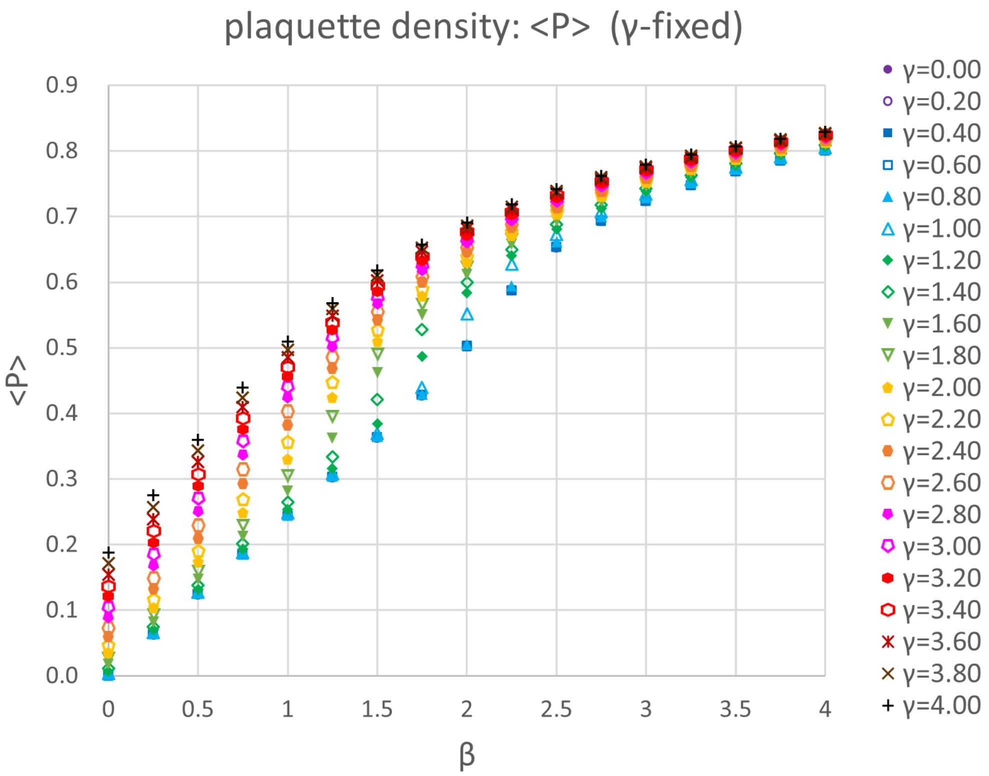




■ plaquette density $\langle P \rangle$

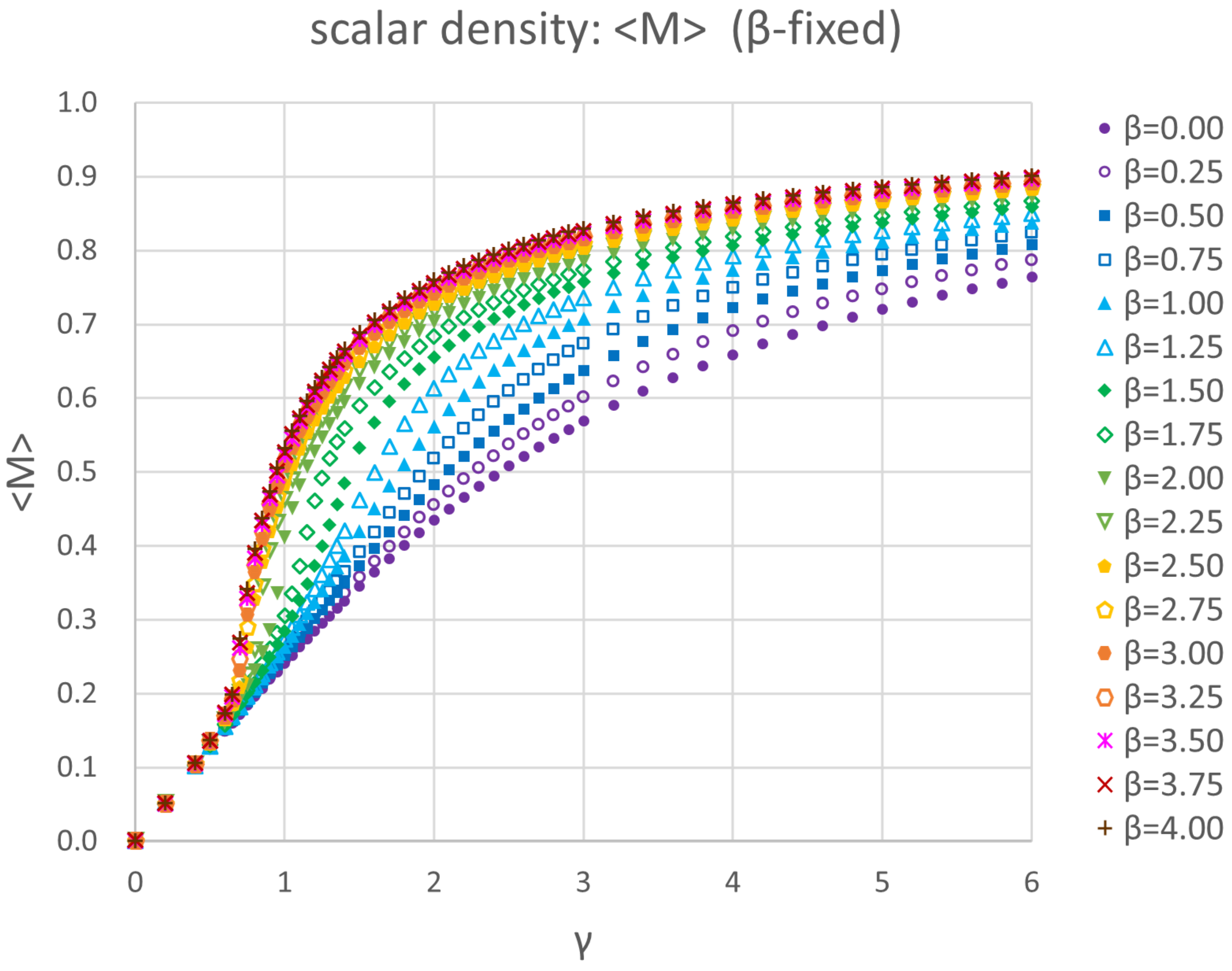
$$P = \frac{1}{6V} \sum_{x,\mu<\nu} \mathrm{tr}\left(U_{x,\mu} U_{x+\mu,\nu} U^{\dagger}_{x+\nu,\mu} U^{\dagger}_{x,\nu} \right)$$

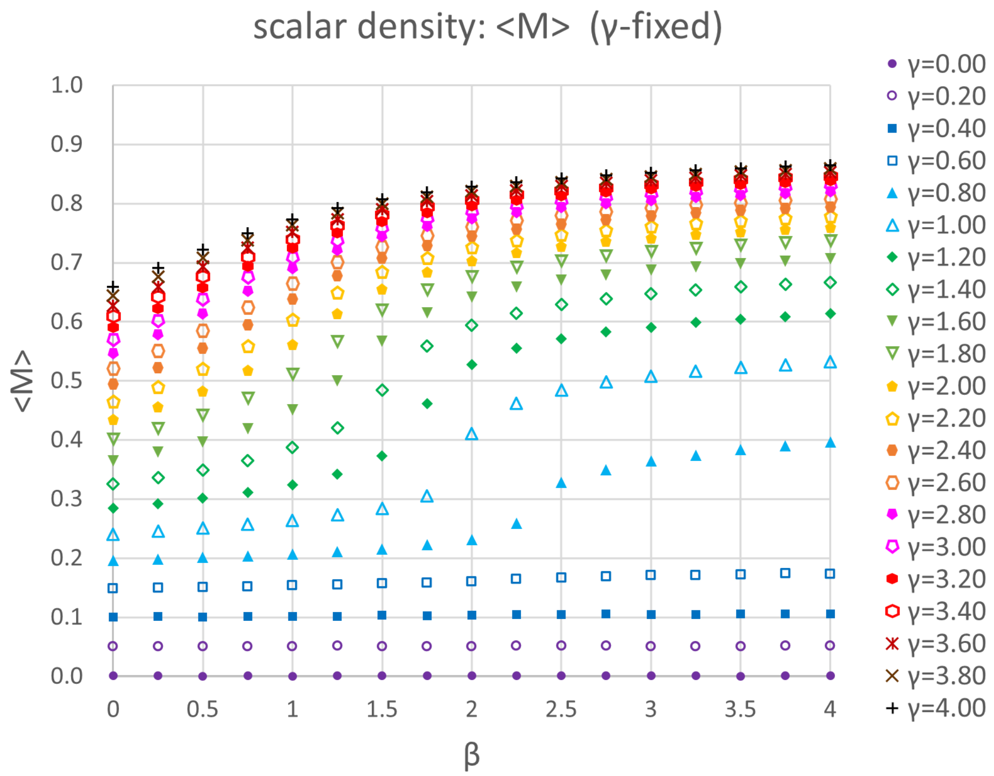




■ scalar density $\langle M \rangle$

$$M = \frac{1}{4V} \sum_{x,\mu} \mathrm{tr}\left(\hat{\Theta}_x^\dagger U_{x,\mu} \hat{\Theta}_{x+\mu}\right)$$

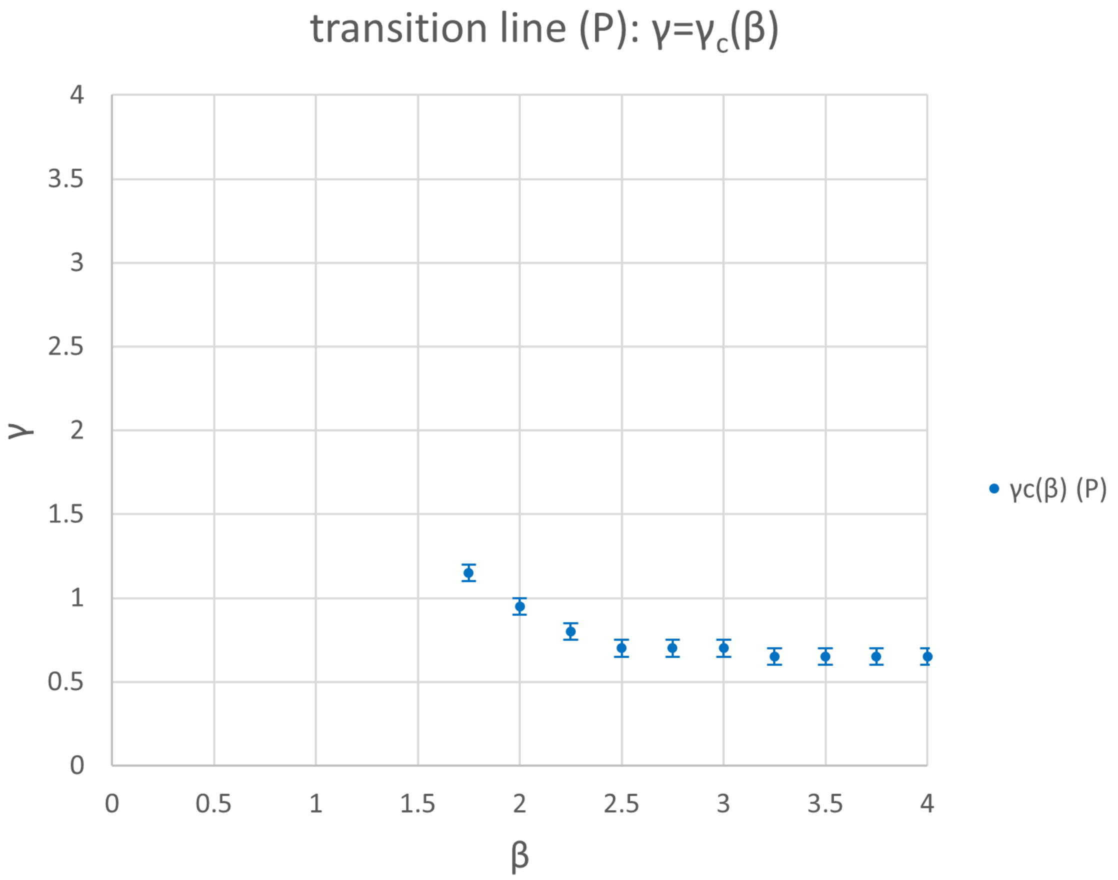


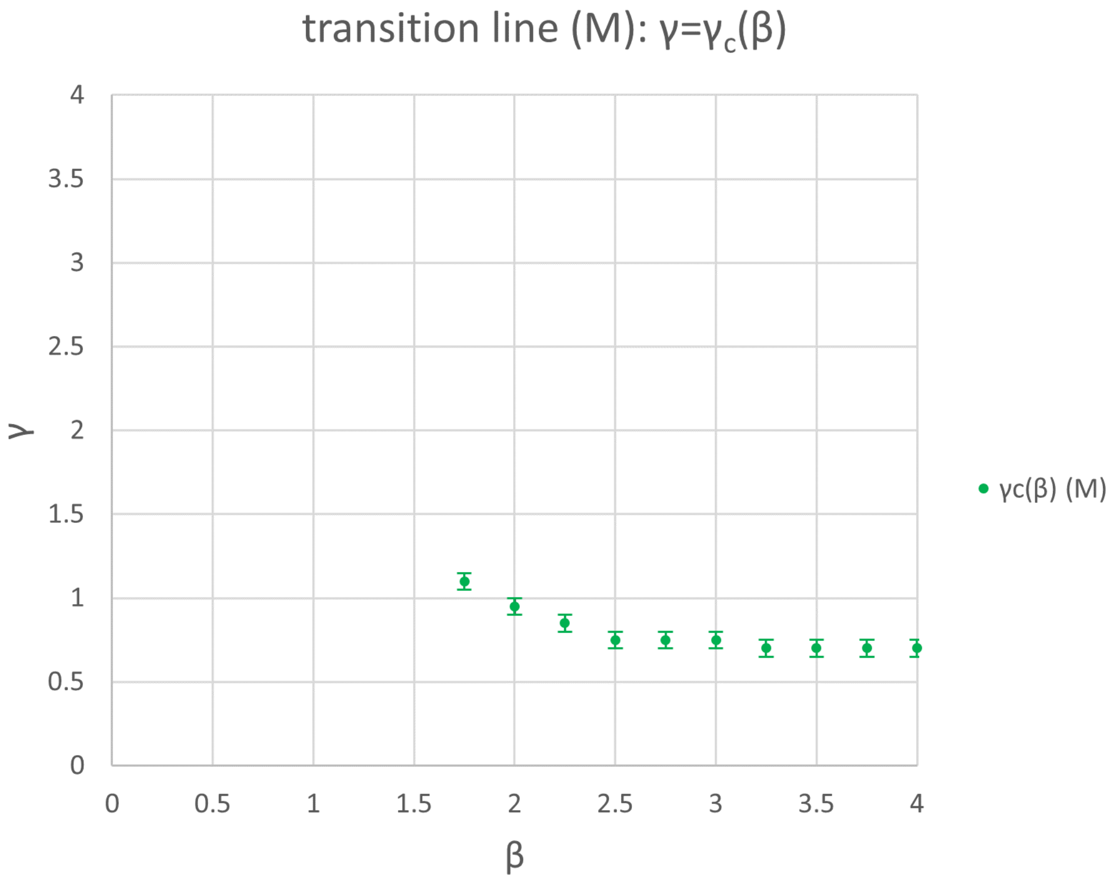


Transition lines determined by $\langle P \rangle$, $\langle M \rangle$

$$P = \frac{1}{6V} \sum_{x,\mu<\nu} \mathrm{tr}\left(U_{x,\mu} U_{x+\mu,\nu} U^{\dagger}_{x+\nu,\mu} U^{\dagger}_{x,\nu}\right)$$

$$M = \frac{1}{4V} \sum_{x,\mu} \mathrm{tr}\left(\hat{\Theta}^{\dagger}_{x} U_{x,\mu} \hat{\Theta}_{x+\mu}\right)$$

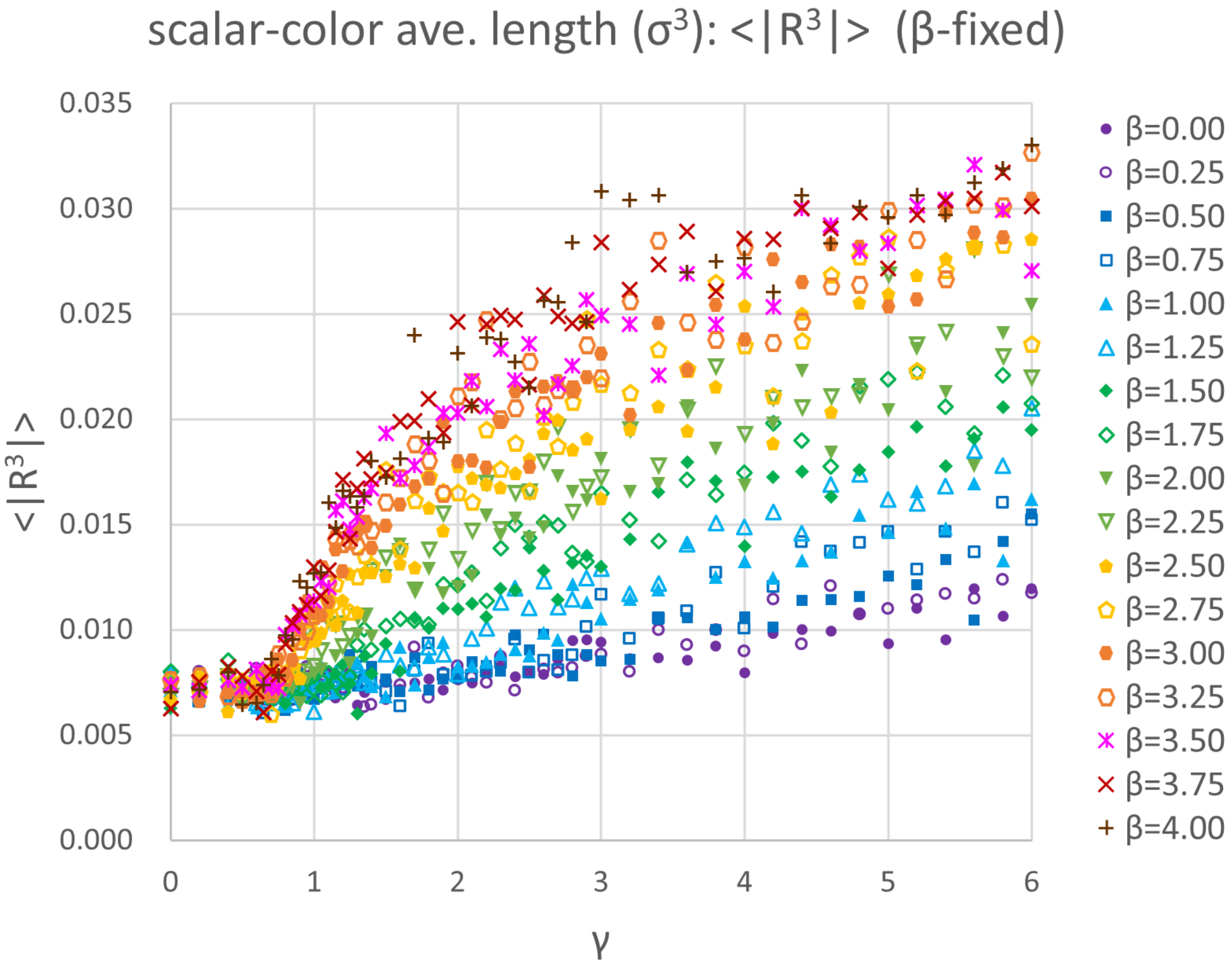


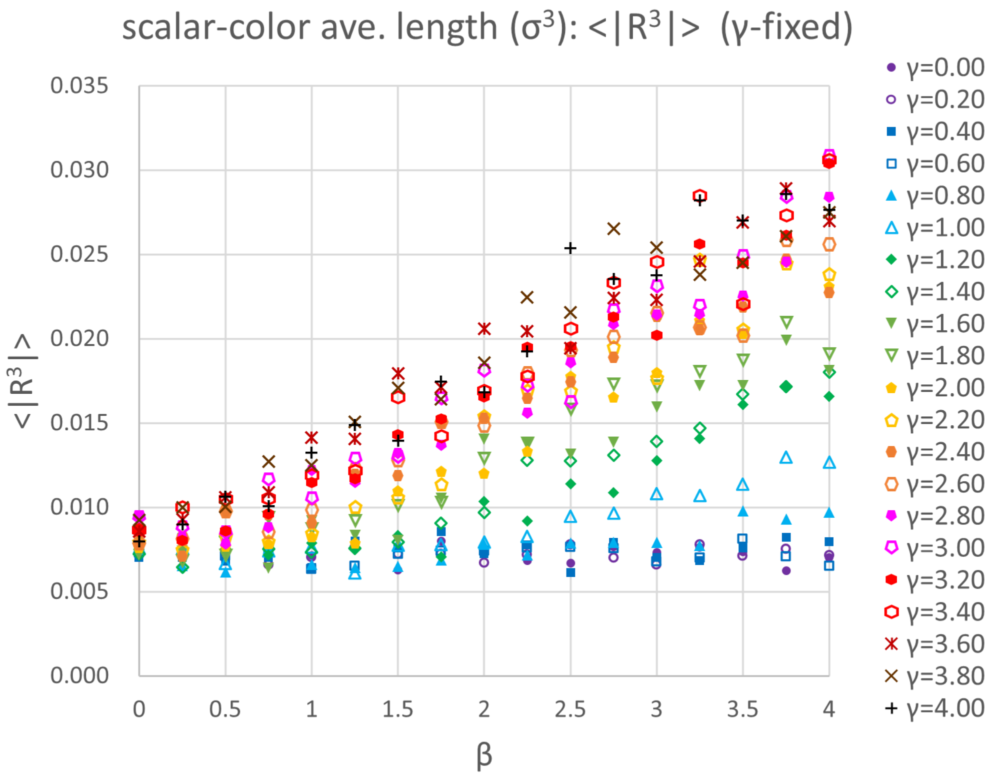


■ modulus of scalar-color density (3rd component) $\langle|\bar{R}^3|\rangle$ NEW

$$|\bar{R}^3| = \left|\frac{1}{2}\mathrm{tr}(\sigma^3\bar{\boldsymbol{R}})\right|$$

$$\bar{\boldsymbol{R}} = \frac{1}{V}\sum_x \hat{\Theta}_x^\dagger \boldsymbol{n}_x \hat{\Theta}_x \qquad (\bar{\boldsymbol{R}} \mapsto \Omega'^\dagger \bar{\boldsymbol{R}} \Omega')$$

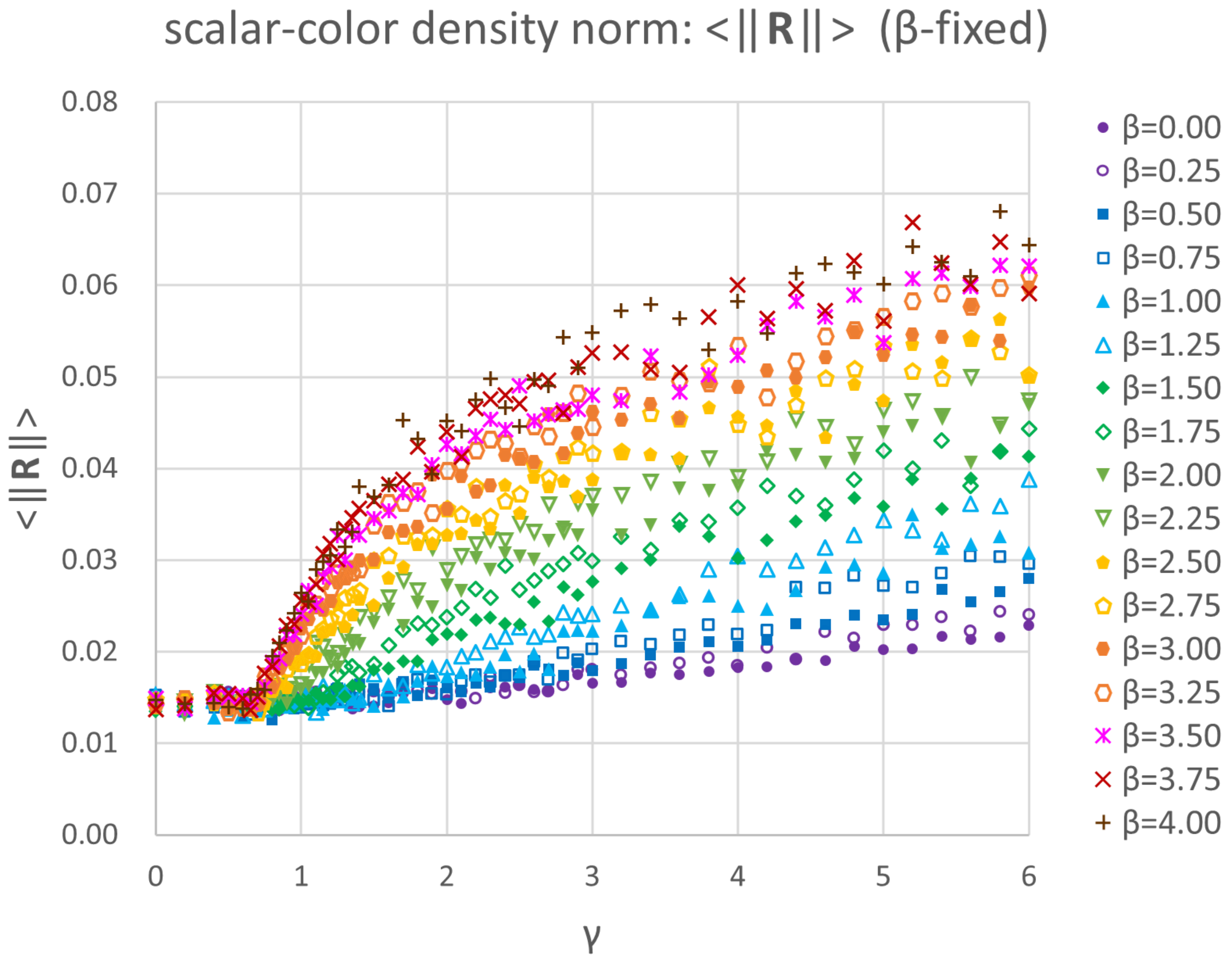


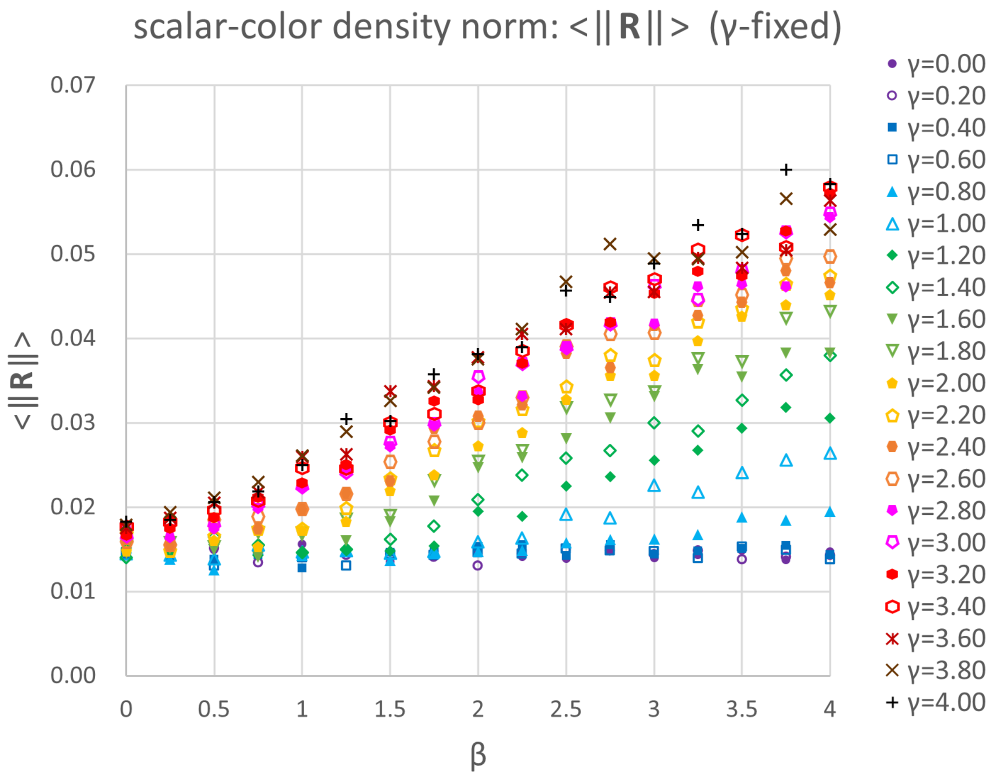


■ scalar-color density norm $\langle\|\bar{\boldsymbol{R}}\|\rangle$ NEW

$$\|\bar{\boldsymbol{R}}\| := \sqrt{\frac{1}{2}\,\mathrm{tr}(\bar{\boldsymbol{R}}^\dagger \bar{\boldsymbol{R}})} \qquad (\|\bar{\boldsymbol{R}}\| \mapsto \|\bar{\boldsymbol{R}}\|)$$

$$\bar{\boldsymbol{R}} := \frac{1}{V}\sum_x \hat{\Theta}_x^\dagger \boldsymbol{n}_x \hat{\Theta}_x \qquad (\bar{\boldsymbol{R}} \mapsto \Omega'^\dagger \bar{\boldsymbol{R}} \Omega')$$

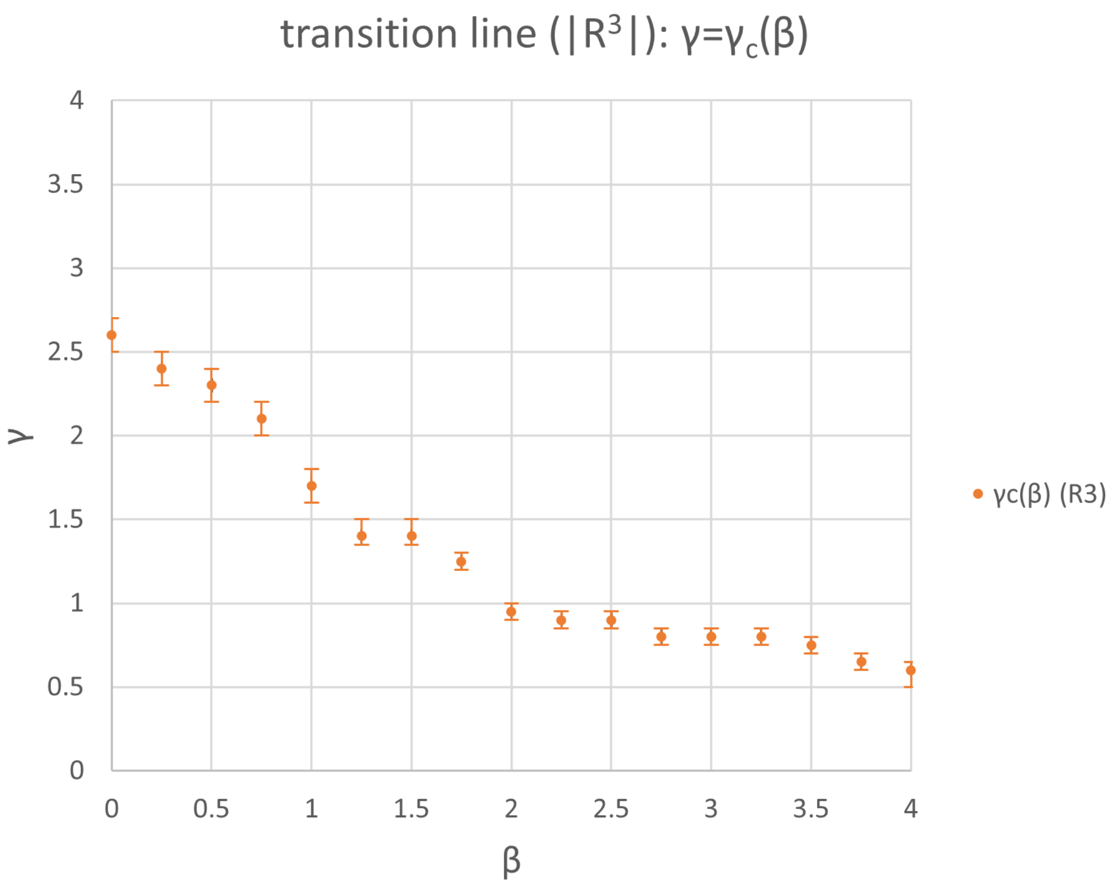


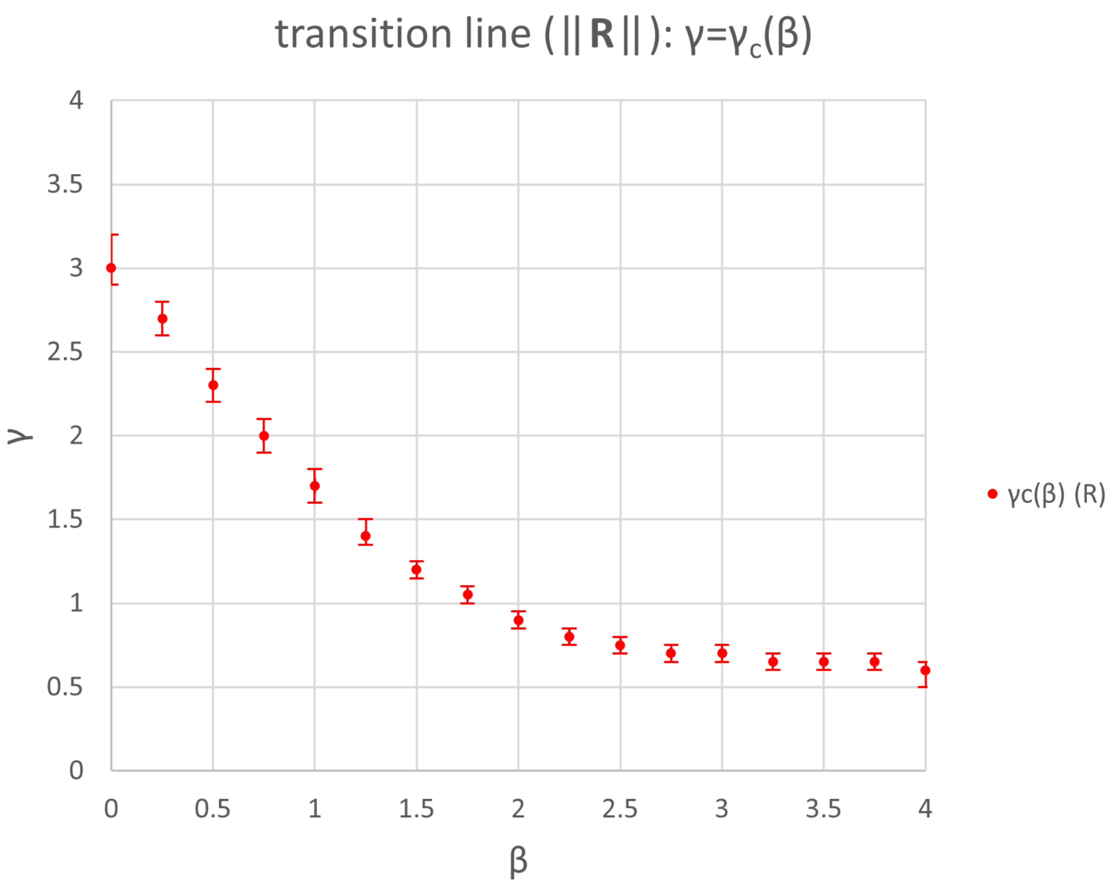


■ Transition lines determined by $\langle|\bar{R}^3|\rangle$, $\langle\|\bar{\boldsymbol{R}}\|\rangle$ NEW

$$|\bar{R}^3| = \left|\frac{1}{2}\,\mathrm{tr}(\sigma^3 \bar{\boldsymbol{R}})\right|$$

$$\|\bar{\boldsymbol{R}}\| := \sqrt{\frac{1}{2}\,\mathrm{tr}(\bar{\boldsymbol{R}}^\dagger \bar{\boldsymbol{R}})}$$





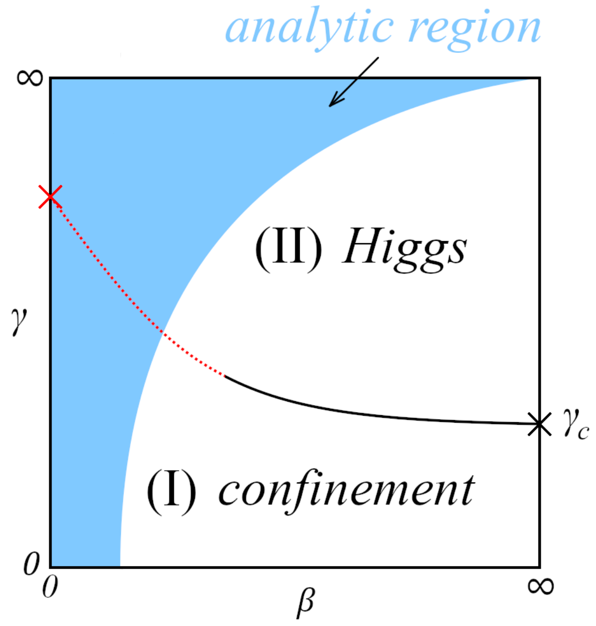
■ Phase structure and physical interpretation

(Ⅰ) confinement phase ($\langle |R^A| \rangle = 0$) $\cdots$ $\mathrm{SU}(2)'_{\mathrm{global}}$ unbroken phase

- confinement similar to the pure SU(2) gauge theory
- Isotropic color-direction field config. $\{\boldsymbol{n}_x\}$ in color space
- very small correlation between the color-direction field $\boldsymbol{n}_x$ and the scalar field $\hat{\Theta}_x$
- Massive gauge fields due to self-interactions among themselves

(Ⅱ) Higgs phase ($\langle |R^A| \rangle > 0$) $\cdots$ $\mathrm{SU}(2)'_{\mathrm{global}}$ broken phase

- conventional BEH mechanism with complete SSB SU(2)→**1**
- Anisotropic color-direction field config. $\{\boldsymbol{n}_x\}$ in color space
- stronger correlation between the color-direction field $\boldsymbol{n}_x$ and the scalar field $\hat{\Theta}_x$ (which tends to align to a particular direction)
- Massive gauge fields due to the BEH mechanism



4. Summary

We studied the new type of operator in the SU(2) LGST with fundamental scalar field.

- Combining the scalar field $\hat{\Theta}_x$ and newly introduced "color-direction field" $\boldsymbol{n}_x$ (representing the gauge DOF) as the composite operator, we found the complete and gauge-independent separation between the confinement phase and the Higgs phase.
- We performed the gauge-fixing-free numerical simulations and checked that there is a new transition line which overlaps with the known thermodynamical transition line in the weak gauge coupling, and divides a single confinement-Higgs phase into the two phases in the strong gauge coupling.
- New transition lines can be interpretated as the consequence of the SSB of $\mathrm{SU}(2)'_{\mathrm{global}}$.

Outlook:

- More physical meaning and implications of the new transition
  (e.g. quark-hadron continuity, Schäfer and Wilczek, 1999, $\cdots$)
- Improvement of the accuracy for the numerical simulation
- Extension to the case of LGST with SU(N) gauge group ($N \geq 3$)

5. Discussion

■ Consistency with the OSFS theorem

- In Osterwalder-Seiler [1], the lattice action is written in terms of the scalar field in the SU(2) doublet, therefore the custodial symmetry does not assumed.
- In Fradkin-Shenker [2], the lattice action agrees with ours, however in estimating the analytic region, the unitary gauge $\Theta(x) \to \mathbf{1}$ breaks the custodial symmetry.

→ New transition line does not contradict with the OSFS theorem.

■ Absence of the massless Nambu-Goldstone particles

- In the Higgs phase, the continuous global symmetry is spontaneously broken.
- The color-direction field $\boldsymbol{n}_x$ obtained by minimizing the global reduction functional is intrinsically non-local, which violates one of the assumptions of the NG theorem.

→ There are no massless particles (gapless excitations) in the Higgs phase, although the continuous global symmetry is spontaneously broken.