Higgs-confinement continuity in light of particle-vortex statistics

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熱場の量子論とその応用@KEK

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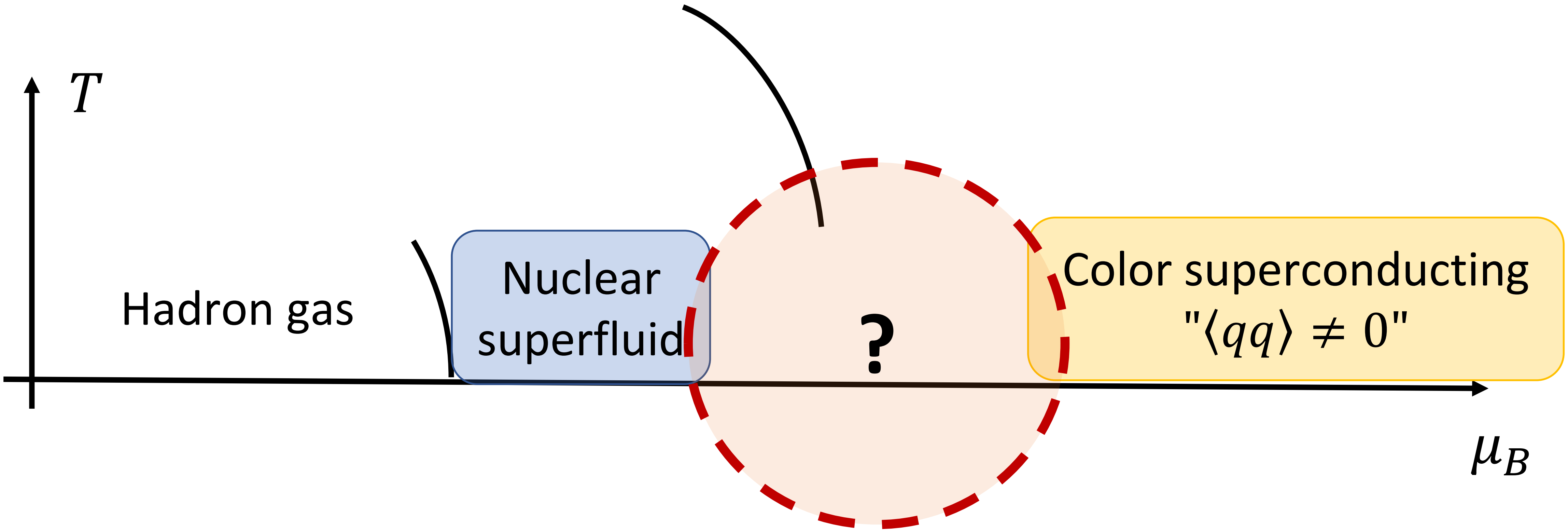
Dense QCD matter



same symmetry breaking pattern → No transition?

("quark-hadron continuity") [Schäfer-Wilczek '99]

Higgs-confinement continuity

An idea behind quark-hadron continuity:

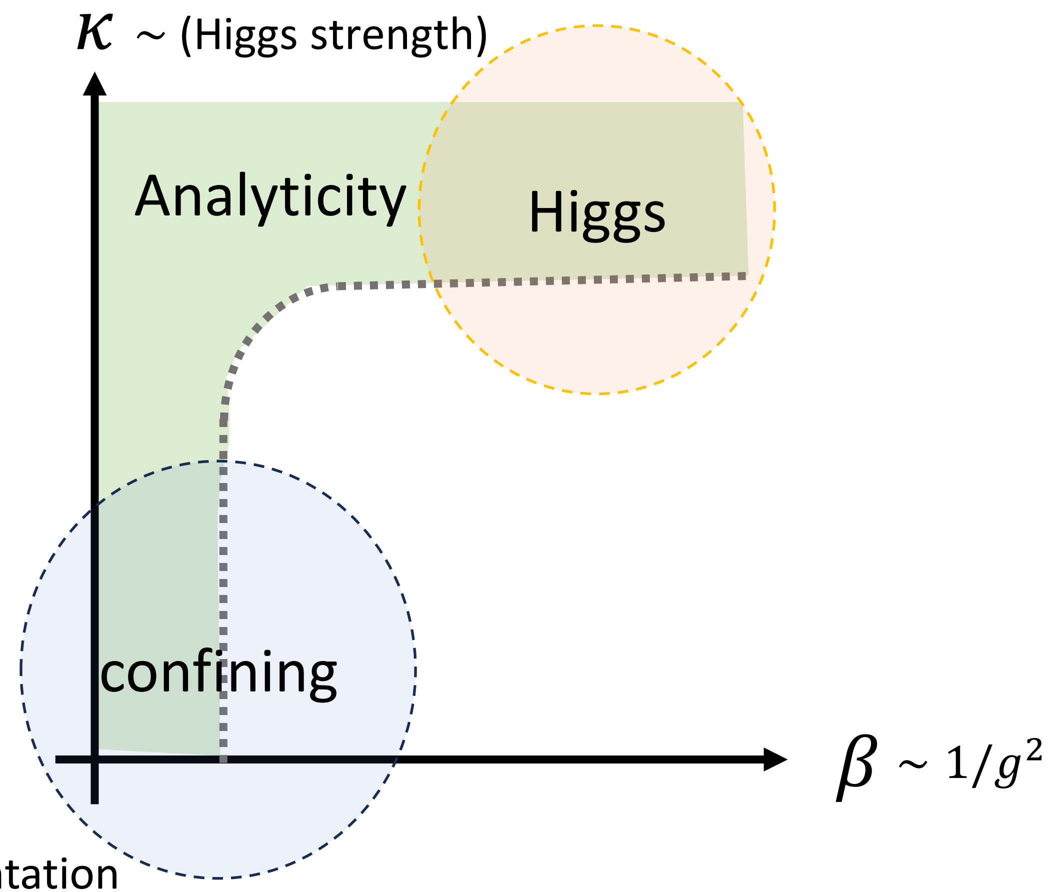
Higgs-confinement continuity

[Osterwalder-Seiler '78][Fradkin-Shenker '79][Banks-Rabinovici '79]

For fundamental gauge-Higgs systems, the confining regime (small β, κ) and Higgs regime (large β, κ) are connected by an analyticity region (without any transition).

⇨

Condensation of fundamental matter
"$\langle\phi\rangle \neq 0$" does not distinguish phases

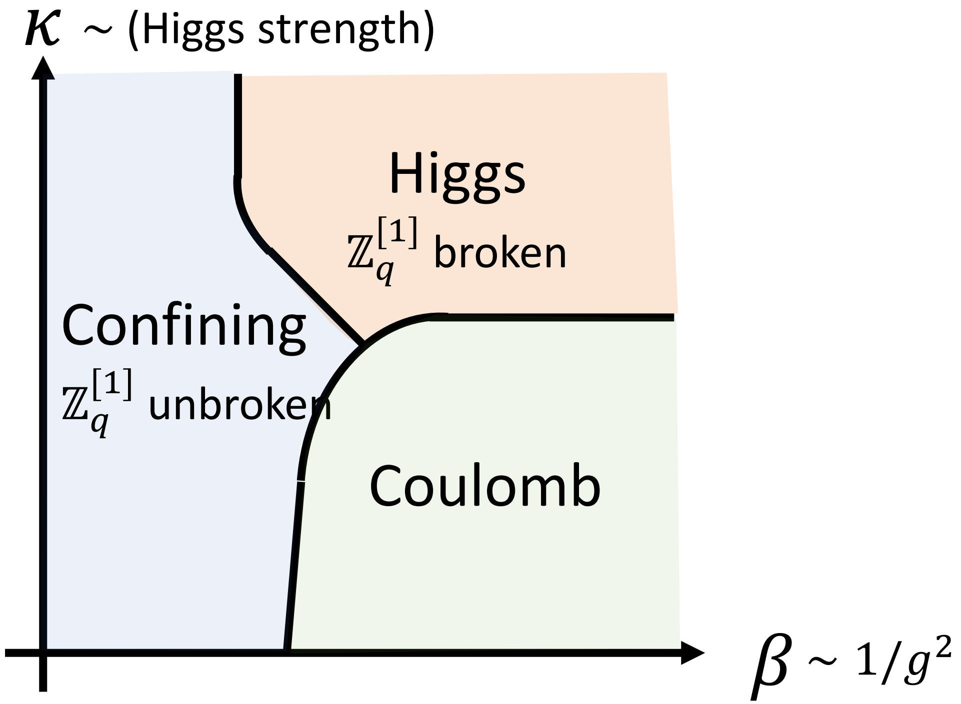


The CFL diquark condensation is in (anti-)fundamental representation

When distinguishable? to introduce the recent discussion……

e.g.) **charge-q Abelian Higgs Model** $(q > 1)$

The Higgs/confining phases are distinguished by perimeter-law/area-law of the Wilson loop
(equivalently $\mathbb{Z}_q^{[1]}$ broken/unbroken)

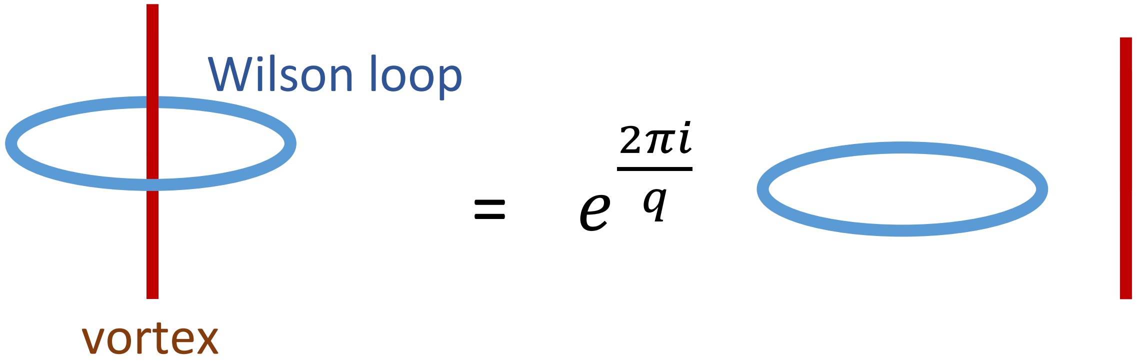


Another characterization: **topological ordering**

In the (low-energy effective theory of) Higgs phase, we have two topological operators, showing nontrivial mutual statistics:

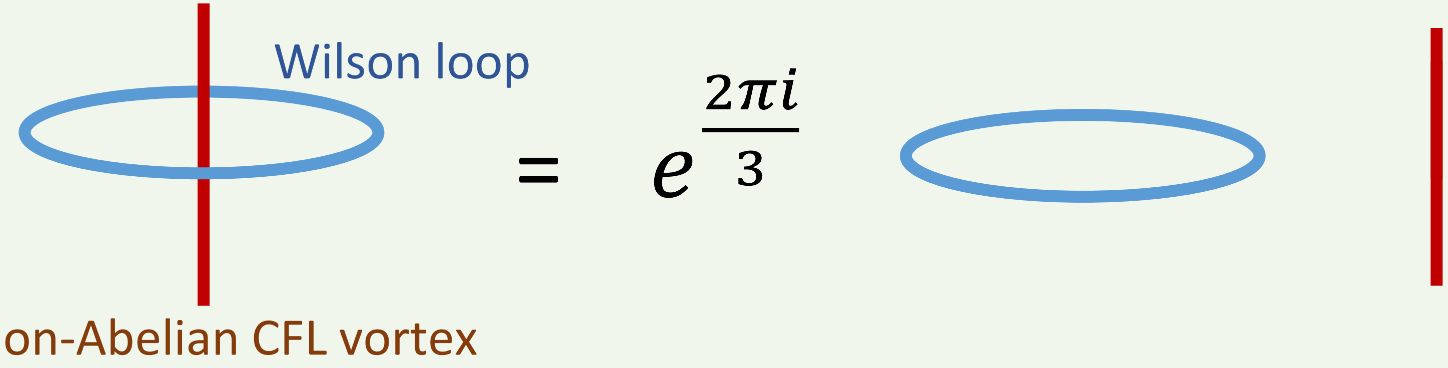
- Wilson loop: $W(C)$
- Vortex worldsheet: $V(S)$

$$\langle W(C)V(S)\rangle = e^{\frac{2\pi i}{q}\mathrm{Link}(C,S)}\langle W(C)\rangle\langle V(S)\rangle$$



Recent discussion

- Particle-vortex statistics in the CFL phase [Cherman-Sen-Yaffe '18]



$$= e^{\frac{2\pi i}{3}}$$

Does this nontrivial AB phase signal a quark-hadron transition?

how connected, concretely?

- This AB phase does *not* mean topological order [Hirono-Tanizaki '18 '19]

(∵ The CFL vortex is (partially) a global vortex, which does not become topological)

At least, the previous logic does not apply here → continuity?

More constructively?

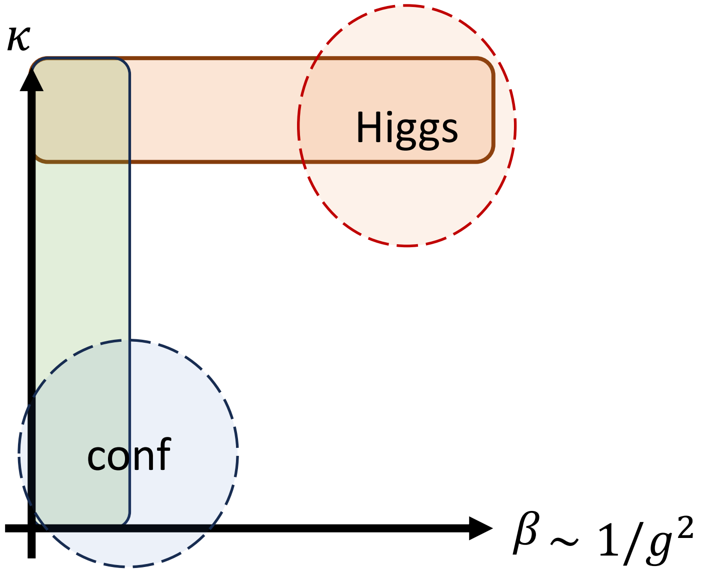
- Still, it was conjectured that **the AB phase can be an order parameter** for a Higgs-confinement transition, by studying an Abelian toy model (detailed later) [Cherman-Jacobson-Sen-Yaffe '20]. → transition?

Main claim by a Fradkin-Shenker-like analysis

For superfluid fundamental gauge-Higgs systems, the **Aharonov-Bohm phase around the vortex** is **continuous** (or constant, if protected by symmetry) in the strong-coupling and deep-Higgs regions, connecting confining and Higgs regimes

Below, we illustrate this claim in the following two lattice models analogous to:

1) The Abelian toy model
2) Ginzburg-Landau model for CFL diquark

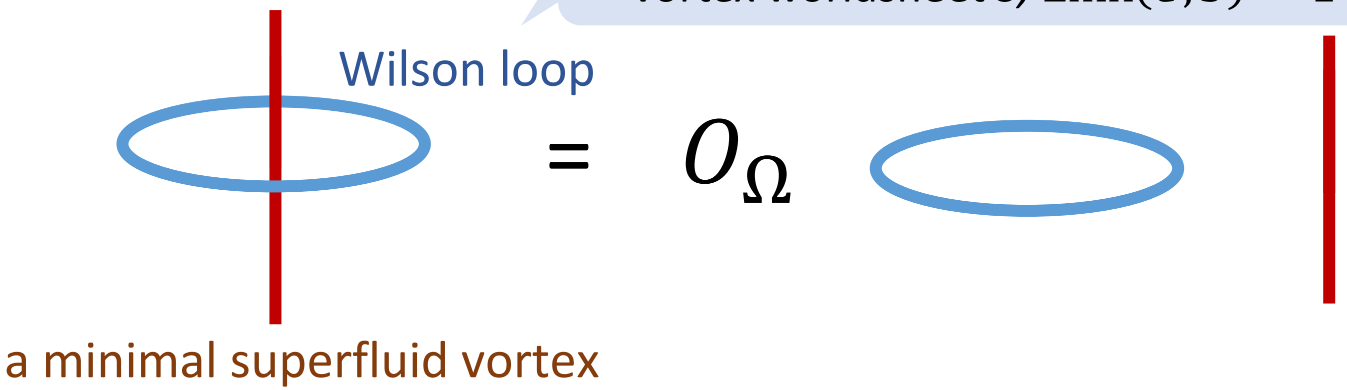


An order parameter?: Aharonov-Bohm phase

In what follows, we evaluate the AB phase O_Ω around a minimal superfluid vortex, schematically defined by,

$$O_\Omega := \lim_{|C|\to\infty} \frac{\langle W(C) V(S) \rangle}{\langle W(C) \rangle \langle V(S) \rangle}$$

asymptotically large loop linking with the vortex worldsheet S, $\mathrm{Link}(C, S) = 1$



Example 1: the Abelian toy model

used to argue a Higgs-confinement transition [Cherman-Jacobson-Sen-Yaffe '20].

Field contents:

3d compact U(1) gauge a + charge-(± 1) matters (ϕ_+, ϕ_-) + neutral scalar ϕ_0

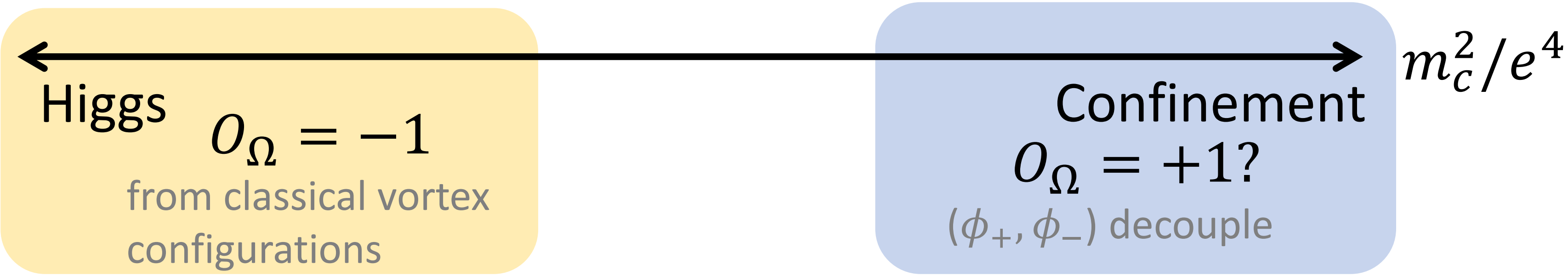
Action:

$$S = \int \frac{1}{2e^2}|da|^2 + |D\phi_+|^2 + |D\phi_-|^2 + |d\phi_0|^2 + V_c(\phi_+) + V_c(\phi_-) + V_0(\phi_0) + \epsilon\phi_+\phi_-\phi_0 + c.c.$$

$V_0(\phi_0)$: wine bottle potential → ϕ_0 condensation (superfluidity)

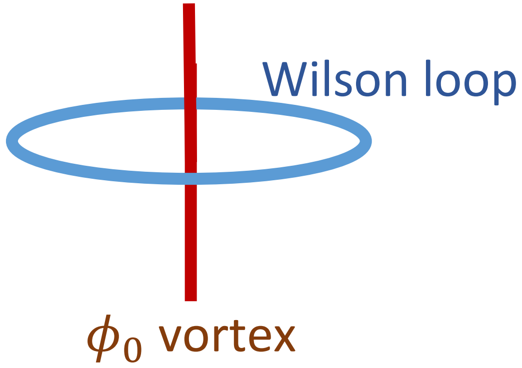
$V_c(\phi_\pm) = m_c^2|\phi_\pm|^2 + \lambda_c|\phi_\pm|^4$: identical potential for charged matters (ϕ_+, ϕ_-)

AB phase as an order parameter?:



The AB phase must be ± 1 due to $\mathbb{Z}_2$ symmetry $[\phi_\pm \to \phi_\mp, a \to -a]$ → **transition somewhere?**

Example 1: the Abelian toy model



Result: *the AB phase is -1 (constant!) in both regions* (skip lattice details)

- Deep Higgs region

In the deep Higgs limit, the gauge field a is frozen to be $\frac{d\varphi_+ - d\varphi_-}{2}$ (with $\phi_\pm = v\, e^{i\,\varphi_\pm}$).
The minimal ϕ_0 vortex rotates $(\phi_+\phi_-)$ by 2π asymptotically.

$$\langle W(C)V(S)\rangle \sim \left\langle e^{i\,\int_C \frac{d\phi_+ - d\phi_-}{2}}\right\rangle_{\text{vortex}} \sim -1$$

in accordance with [Cherman-Jacobson-Sen-Yaffe ‘20].

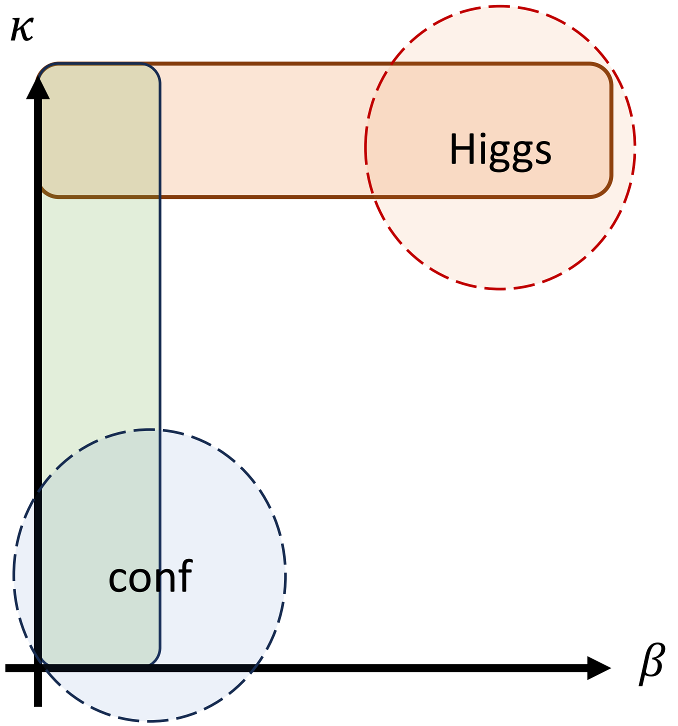
- Strong coupling region

Even if charged matters are heavy, an asymptotically large Wilson loop is dominated by screened perimeter-law part, which can be affected by ϕ_0 vortex.

For example, in the “deep confining regime” ($\beta \to +0$, small κ),

$$\langle W(C)V(S)\rangle = \left\langle \prod_{\ell\in C} U_\ell \right\rangle_{\text{vortex}} \sim \left\langle \prod_{\ell\in C} [(\phi_+\ \text{hopping})^* + (\phi_-\ \text{hopping})] \right\rangle_{\text{vortex}}$$

$$e^{i\,\theta_1} + e^{i\,\theta_2} = (e^{i\,\theta_1}\sqrt{e^{i\,(-\theta_1+\theta_2)}})|e^{i\,\theta_1} + e^{i\,\theta_2}| \to \quad \sim \left\langle e^{i\,\int_C \frac{d\varphi_+ - d\varphi_-}{2}}\right\rangle_{\text{vortex}} \sim -1 \ \text{(matched!)}$$



(Actually, $O_\Omega = -1$ without κ expansion)

Example 2: Ginzburg-Landau model for diquark

(anti-)fundamental
CFL diquark $\Phi^{ai} \sim \epsilon^{abc}\,\epsilon^{ijk}\,q^t_{bj} C\gamma^5 q_{ck}$

We add superfluid "dibaryon" for nuclear superfluid phase

<u>Field contents</u>:

SU(N) gauge a + $(N \times N)$-matrix-valued fundamental matter Φ + neutral scalar ϕ_0

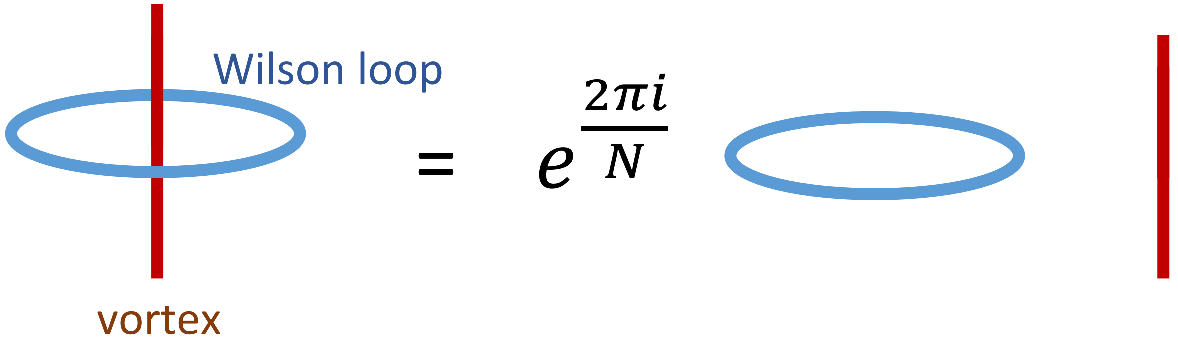
<u>Action</u>:

$$S = \int |f|^2 + |D\Phi|^2 + |D\phi_0|^2 + V\left(\mathrm{tr}\,\Phi^\dagger\Phi\right) + V_0(\phi_0) + \epsilon\,\phi_0^*(\det\Phi) + c.c.$$

$V_0(\phi_0)$: wine bottle potential → ϕ_0 condensation (superfluidity)

By tuning $V\left(\mathrm{tr}\,\Phi^\dagger\Phi\right)$, this model has superfluid confining regime [nuclear superfluidity] and Higgs regime [CFL].

<u>(apparent) mismatch of AB phase:</u>



$$, \quad O_\Omega = \begin{cases} +1\ ? & \text{(Confining limit)} \\ e^{\frac{2\pi i}{N}} & \text{(Higgs limit)} \end{cases}$$

Example 2: Ginzburg-Landau model for diquark

We can perform the similar analysis on an analogous lattice model

- **Deep Higgs region**

$$\langle W(C)V(S)\rangle \sim e^{\frac{2\pi i}{N}}$$

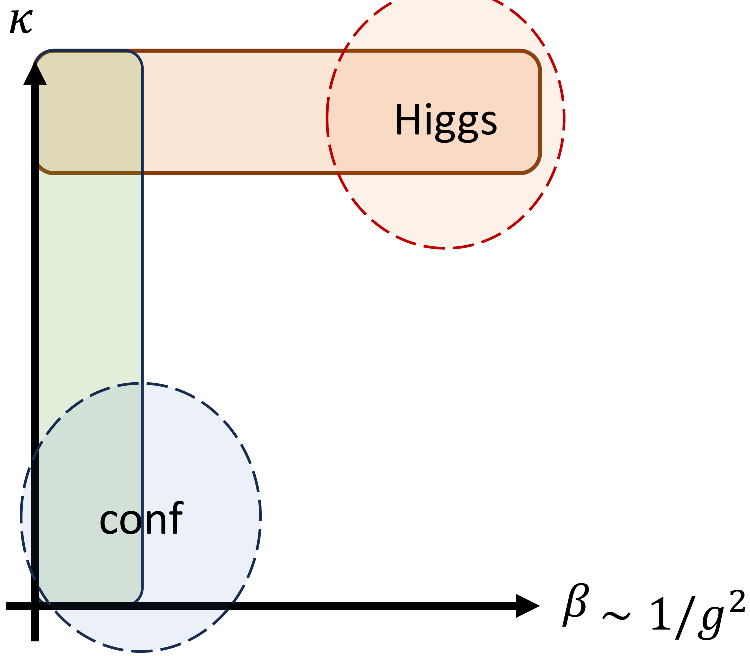
reproducing [Cherman-Sen-Yaffe '18]

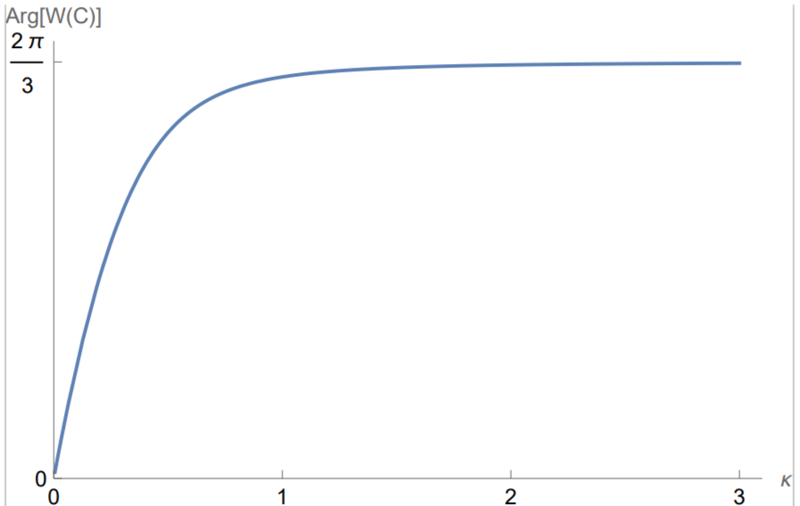
- **Strong coupling region**

The AB phase is not a constant [at $N = 3$] and trivial in the deep confining limit ($\beta \to 0$, small κ)

$$\langle W(C)V(S)\rangle \sim 1$$

Still, the AB phase is **continuous** and smoothly interpolates between 1 ($\kappa \to +0$) and $e^{\frac{2\pi i}{N}}$($\kappa \to +\infty$) in the strong coupling limit ($\beta \to 0$):



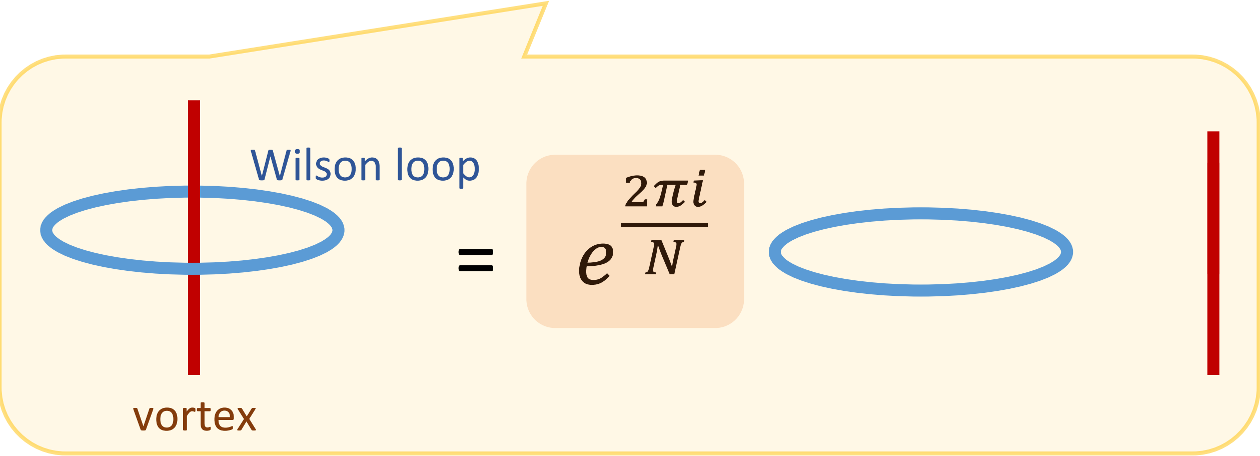


Summary

e.g.) diquark condensation in dense QCD

In some superfluid gauge-Higgs systems,

Higgs ⟷ Confinement



<u>Recently-debated issue</u>:
Does the nontrivial AB phase necessitate a transition?

<u>Claim:</u> For fundamental superfluid gauge-Higgs systems,
the AB phase respects the Higgs-confinement continuity.
⇒ the quark-hadron continuity is still a possible scenario.