QCD物質の音速のピーク構造と高密度極限
quark meson モデルによる解析

Sound velocity peak structure and conformality in QCD matter
quark meson model analysis

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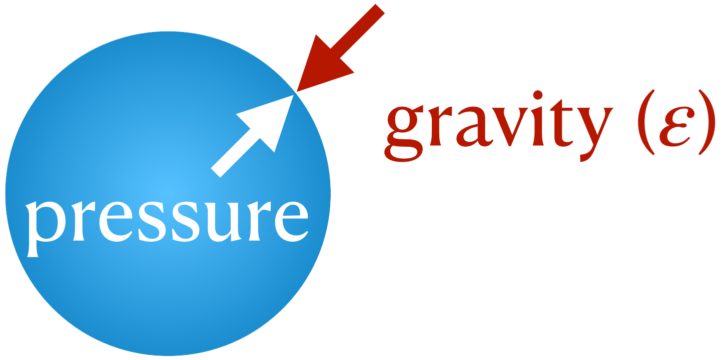
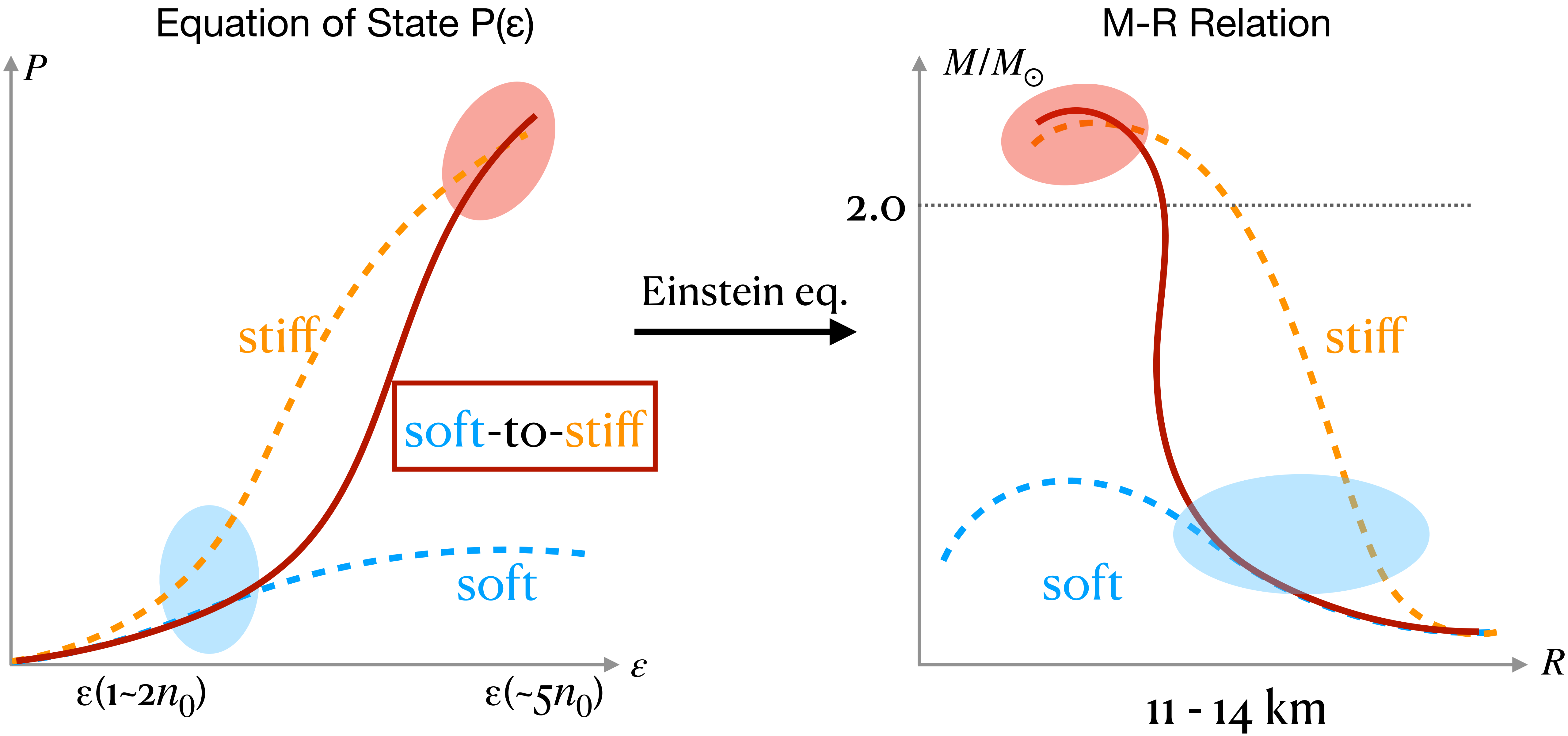
Ref.) Sound velocity peak and conformality in isospin QCD
Ryuji Chiba, Toru Kojo arXiv:2304.13920 [hep-ph]

Background: T~0 QCD Equation of State

EoS: input of Neutron Star M-R Relation

Einstein eq. $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ energy-momentum tensor




Equation of State P(ε)

M-R Relation

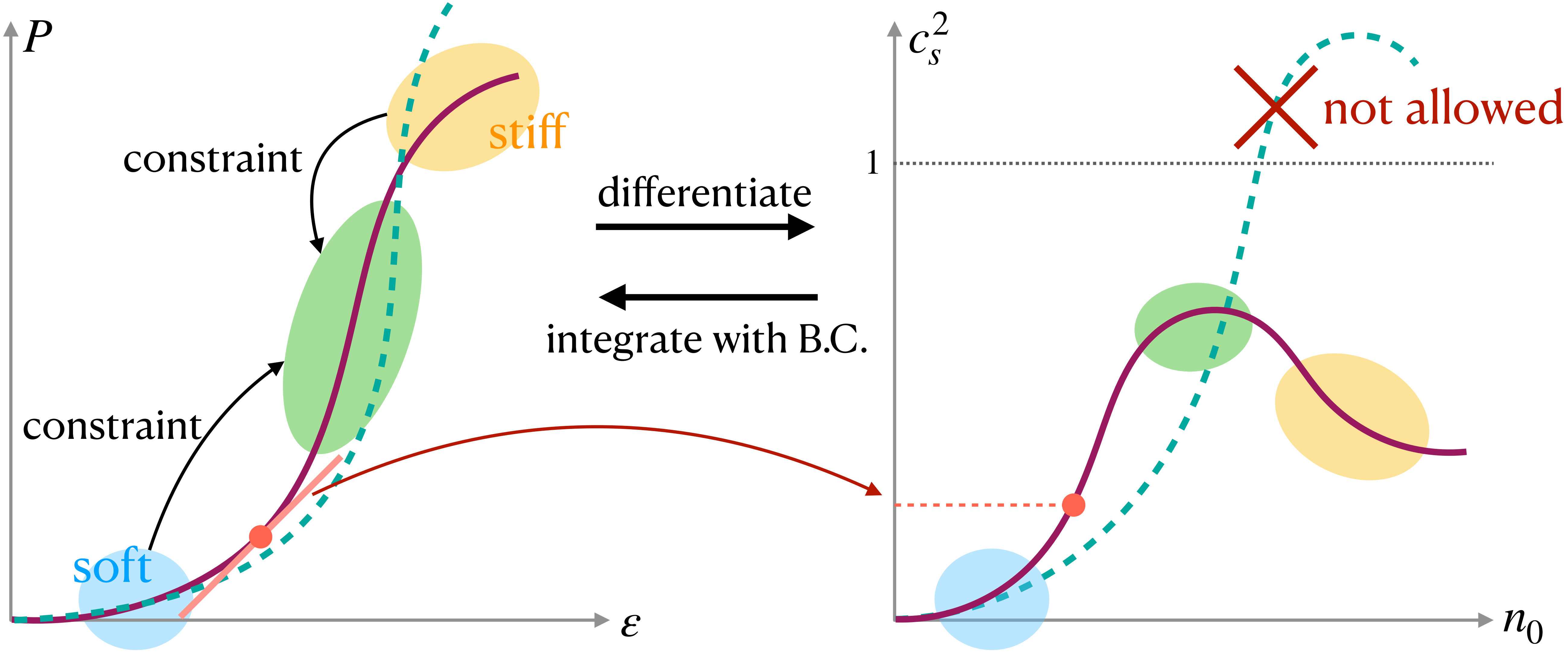
n_0 ~ 0.16/fm³

Background: T~0 QCD Equation of State

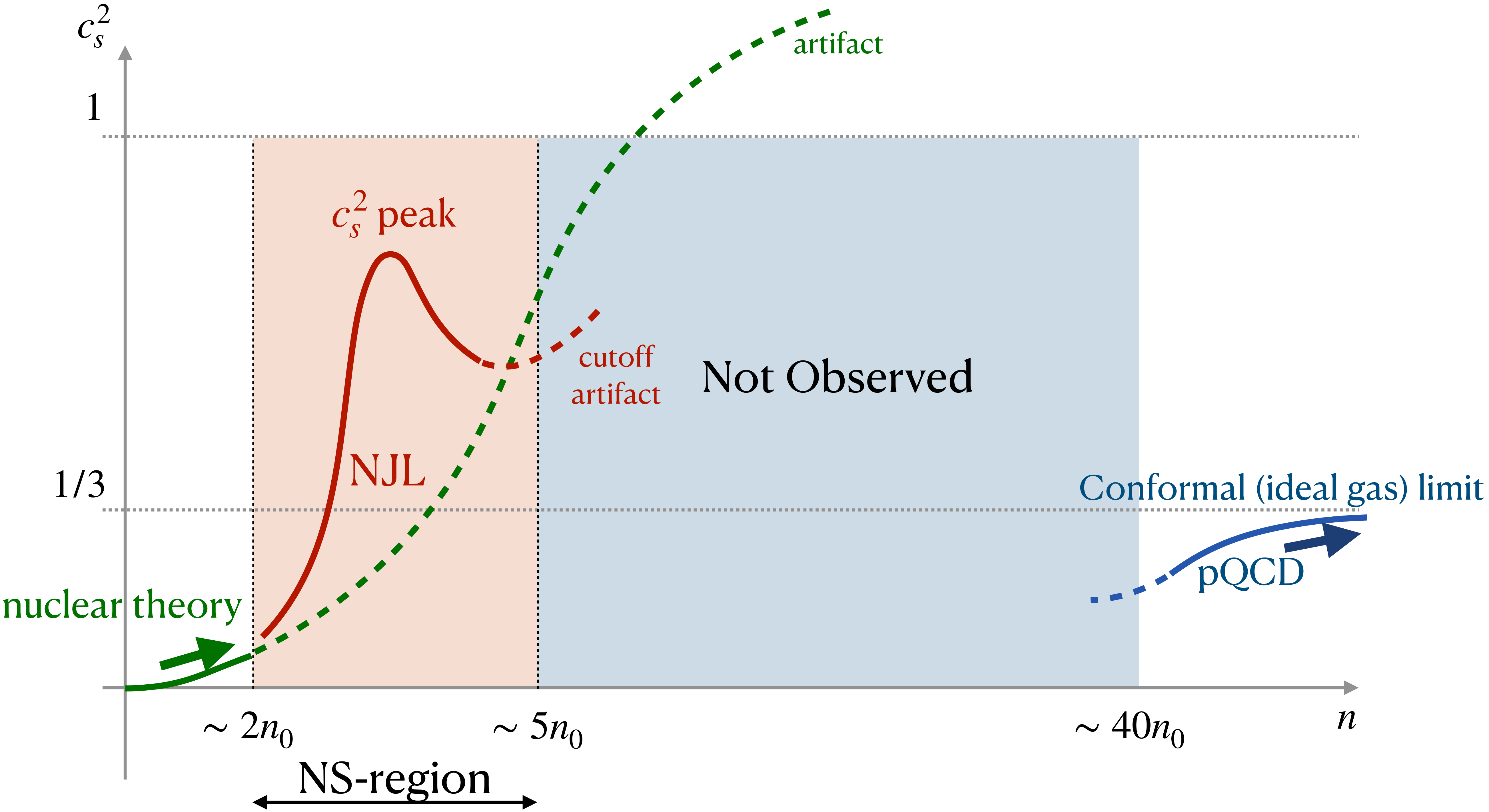
Sound velocity c_s^2

$$c_s^2 = \frac{\partial P}{\partial \varepsilon} < 1 \ (c = 1)$$ → indicator of **stiffening of EoS**

causality constraint



Closer look of c_s^2



Closer look of c_s^2

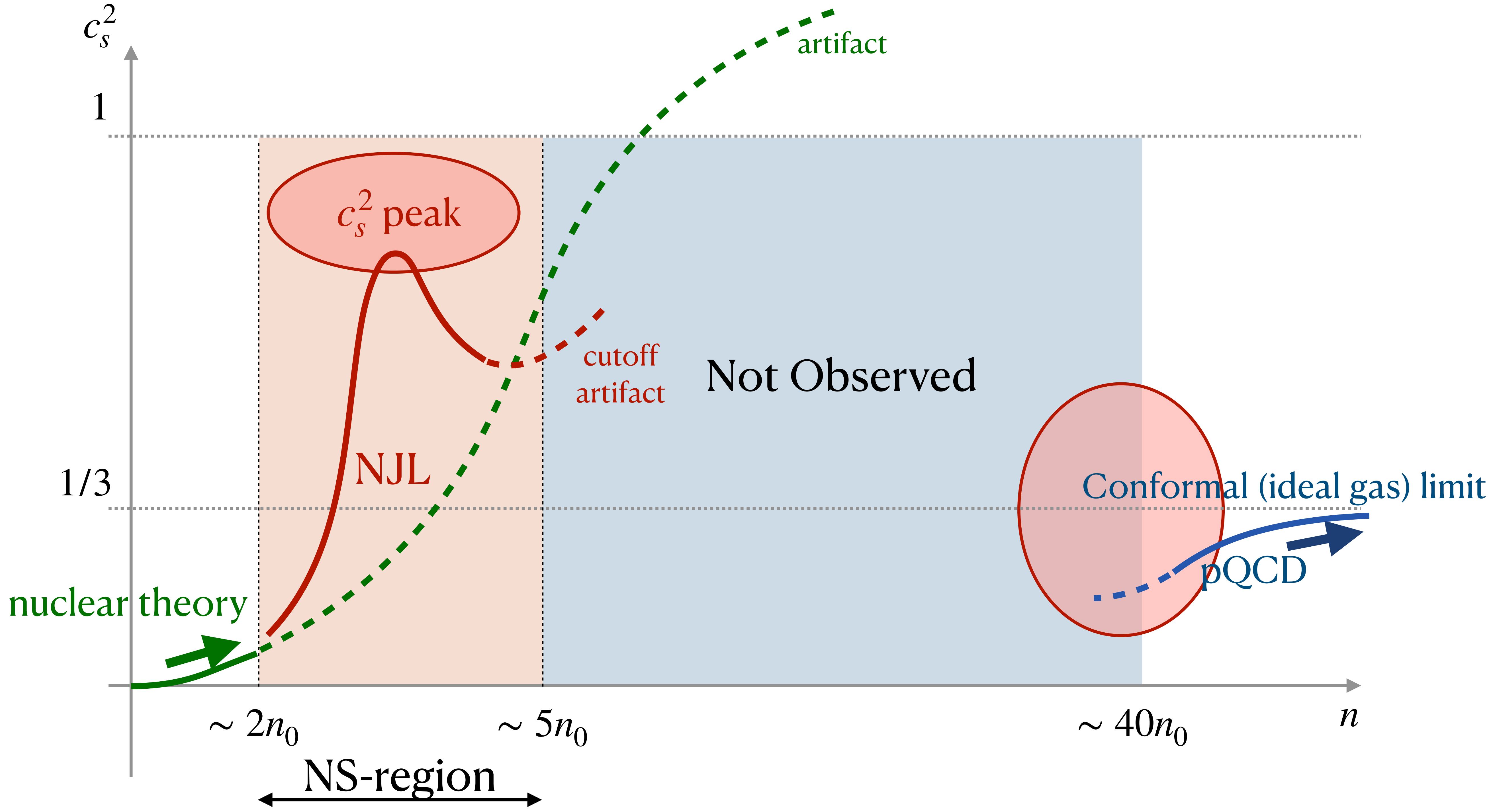


Table of Contents

Isospin QCD: $\mu_I = \mu_u - \mu_d \neq 0$ and $\mu_B = \mu_u + \mu_d = 0$

Isospin QCD

- LQCD: w/o sign problem

↕ comparison ✅

- Model analysis:
  - Confinement
  - **BCS-like pairing** etc.

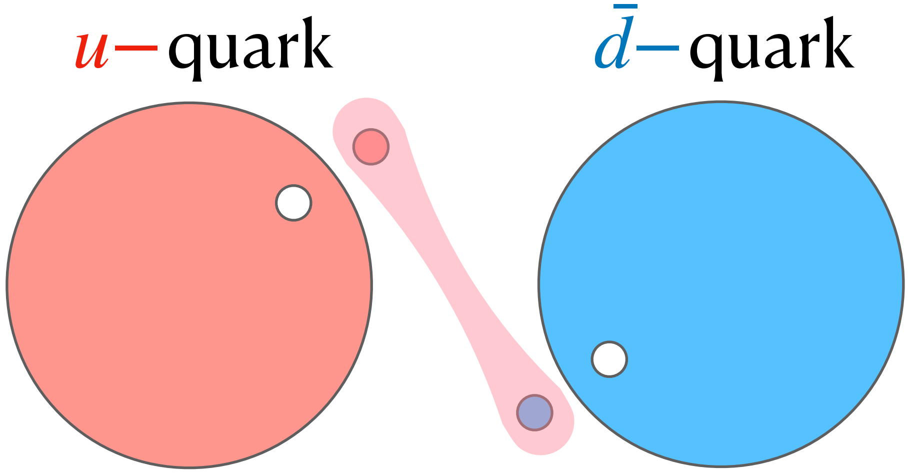
⇨

Real QCD

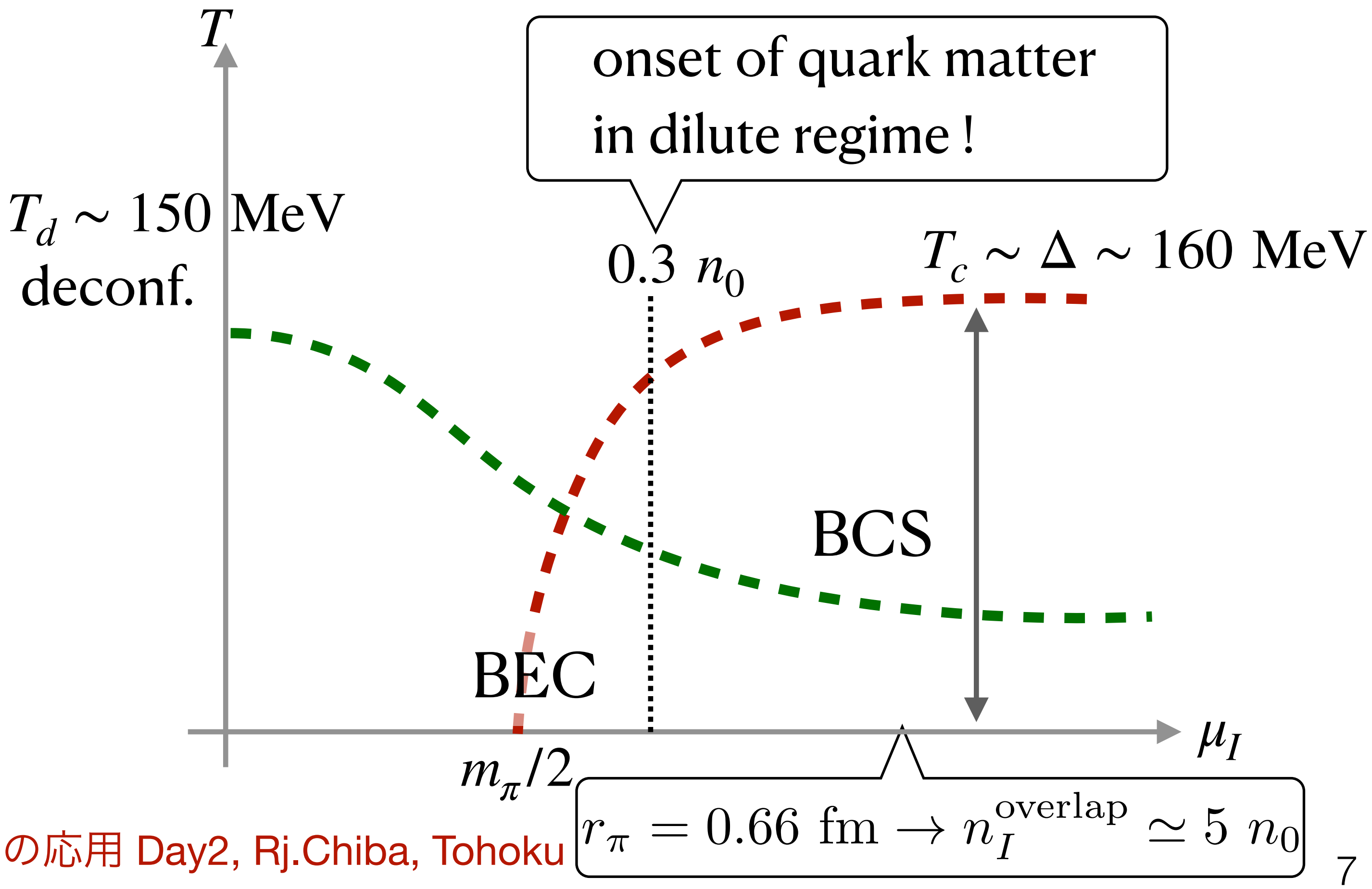
- LQCD: w/ Sign problem

↕ comparison ❌

- Model analysis:
  → Prediction



BCS gap of Pion Condensate Δ



Quark Meson Model (at T = 0)

meson int.

$$\mathcal{L}_{\rm dense} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 - \frac{1}{2}m_0^2\sigma^2 - \frac{1}{2}(m_0^2 - 4\mu_I^2)\vec{\pi}^2 - \frac{\lambda}{24}(\vec{\phi}^2)^2 + h\sigma$$

Yukawa int.

$$+\bar{\psi}\left(i\partial\!\!\!/ + \mu_I \frac{\tau_3}{2}\gamma^0\right)\psi - g\bar{\psi}(\sigma + i\gamma^5\vec{\tau}\cdot\vec{\pi})\psi$$

meson: $\vec{\phi} = (\sigma, \vec{\pi})$

quark: $\psi = \begin{pmatrix} q_u \\ q_d \end{pmatrix}$

mean field approximation

$$\sigma = \tilde{\sigma} + \langle\sigma\rangle, \quad \pi_1 = \tilde{\pi}_1 + \langle\pi_1\rangle$$

$$g\langle\sigma\rangle = M_q, \quad g\langle\pi_1\rangle = \Delta$$

$$\to V_{\rm eff} = V_{\rm eff}(M_q, \Delta; \mu_I)$$

$$(\ V_{\rm eff} = V_0 + \mathcal{O}(\hbar)\ )$$

gap equation: $\left.\frac{\partial V_{\rm eff}}{\partial \Delta}\right|_{\Delta^*, M_q^*} = \left.\frac{\partial V_{\rm eff}}{\partial M_q}\right|_{\Delta^*, M_q^*} = 0$

rewrite potential: $V_{\rm EOS}(\mu_I) = V_{\rm eff}(M_q^*, \Delta^*; \mu_I)$

thermodynamic quantities: $P = -V_{\rm EOS}, \quad n_I = \frac{\partial P}{\partial \mu_I}$

$$\varepsilon = n_I\mu_I - P, \quad c_s^2 = \frac{\partial P}{\partial \varepsilon} = \frac{n_I}{\mu_I \chi_I}, \quad \chi_I = \frac{\partial^2 P}{\partial \mu_I^2}$$

Quark Meson Model (at T = 0)

Effective potential

Diverge

$$V_{\rm eff} = V_0 + \underbrace{V_q + V_{\rm c.t.}^{\overline{\rm MS}}}_{V^{\rm R}_{1-{\rm loop}}} + V_{\rm c.t.}^{\rm finite}$$

Determine to reproduce physical quantities

$$m_\pi = 140{\rm MeV}, \quad m_\sigma = 600{\rm MeV},$$
$$f_\pi = 90{\rm MeV}, \quad M_q = 300{\rm MeV}$$
$$\longrightarrow g \simeq 3.333, \quad \lambda \simeq 126.1$$

tree-level (meson contribution)

$$V_0 = \frac{m_0^2}{2g^2} M_q^2 + \frac{m_0^2 - 4\mu_I^2}{2g^2} \Delta^2 + \frac{\lambda}{24 g^4} \left(M_q^2 + \Delta^2\right)^2 - \frac{h}{g} M_q$$

1-loop (quark single-particle energy)

$$V_q = -N_c \int_{\mathbf{p}} \left(E_u + E_d + E_{\bar{u}} + E_{\bar{d}}\right)$$

quasiparticle energy

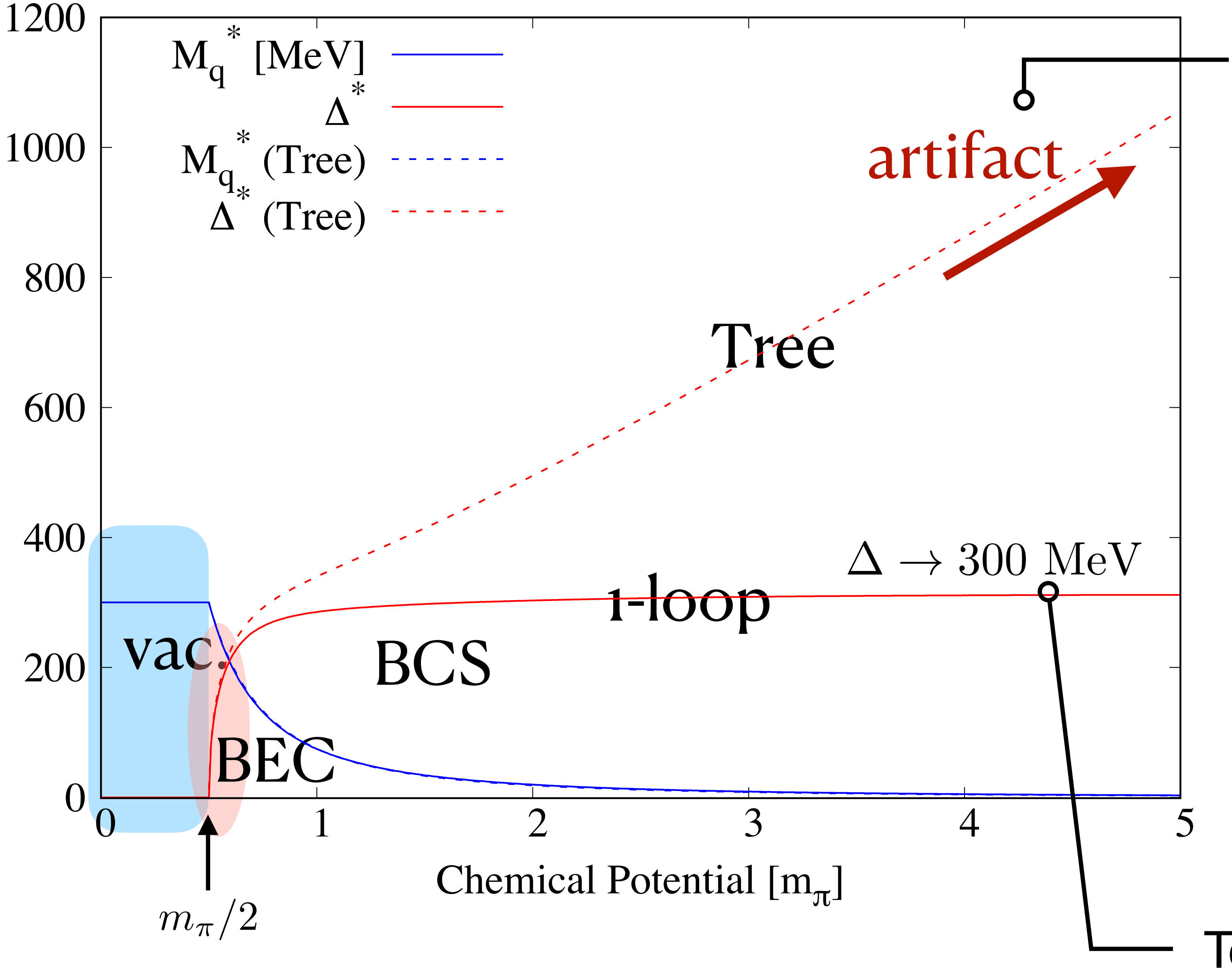
$$E_{u,\bar{d}} = E(\mu_I)\,, \quad E_{d,\bar{u}} = E(-\mu_I)$$
$$E(\mu_I) = \sqrt{\left(E_D - \mu_I\right)^2 + \Delta^2}, \quad E_D = \sqrt{\mathbf{p}^2 + M_q^2}$$

counter term

$$V_{\rm c.t.} = -\frac{1}{2}\delta m_{0\sigma}^2 \left(\frac{M_q}{g}\right)^2 - \frac{1}{2}\delta m_{0\pi}^2 \left(\frac{\Delta}{g}\right)^2 + \delta h \frac{M_q}{g}$$
$$-\frac{1}{24}\left[\delta\lambda_{4\sigma}\left(\frac{M_q}{g}\right)^4 + \delta\lambda_{4\pi}\left(\frac{\Delta}{g}\right)^4 + 2\delta\lambda_{2\sigma\pi}\left(\frac{M_q}{g}\right)^2\left(\frac{\Delta}{g}\right)^2\right]$$

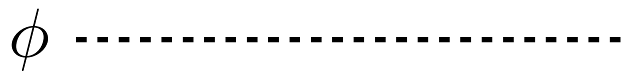
Importance of quark substructure

Gap equation $$\left.\frac{\partial V_{\text{eff}}}{\partial \Delta}\right|_{\Delta^*, M_q^*} = \left.\frac{\partial V_{\text{eff}}}{\partial M_q}\right|_{\Delta^*, M_q^*} = 0$$

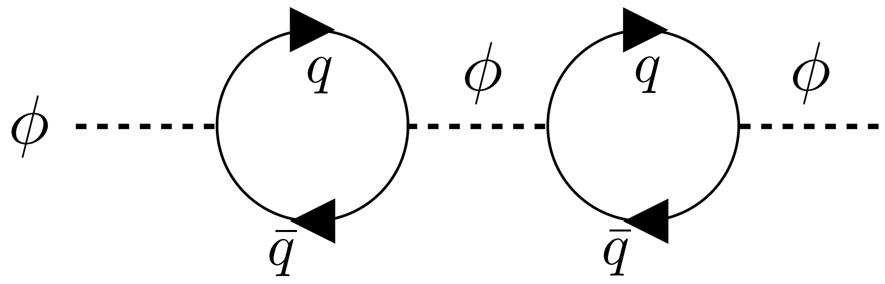


Difference on physical picture of meson

Tree: mesons as elementary particles



1-loop: mesons as composite particles



→ quark Pauli blocking

Tc ~ 0.57Δ → 170 MeV

agreement with lattice

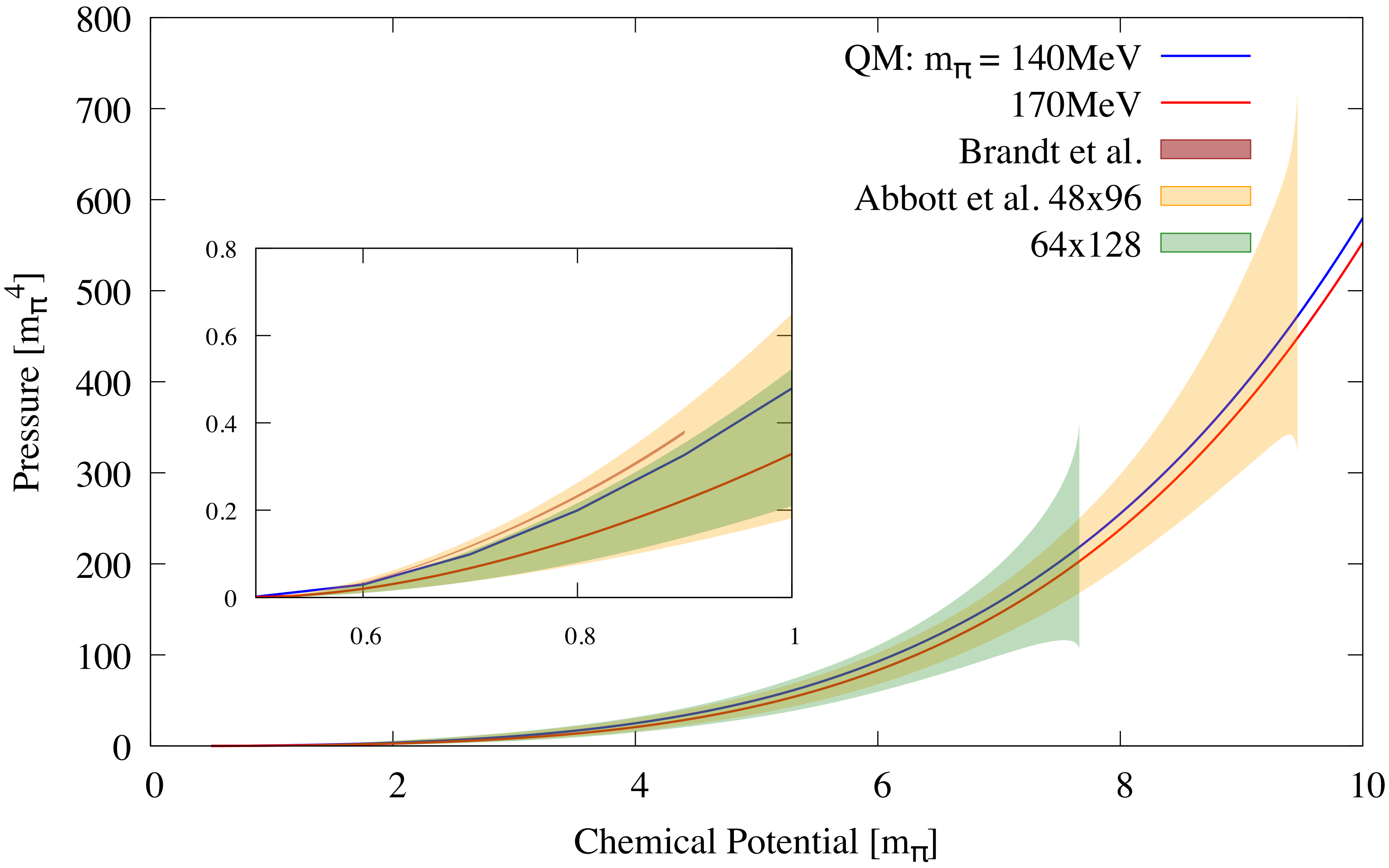
Equation of State

- ChPT should work in dilute regime

  QM model respects Chiral symmetry → reproduce ChPT at low energy

- Good agreements with the LQCD result even at higher density ✅

  (Also trace anomaly)



- Brandt+ (2022) $m_\pi = 135\mathrm{MeV}$

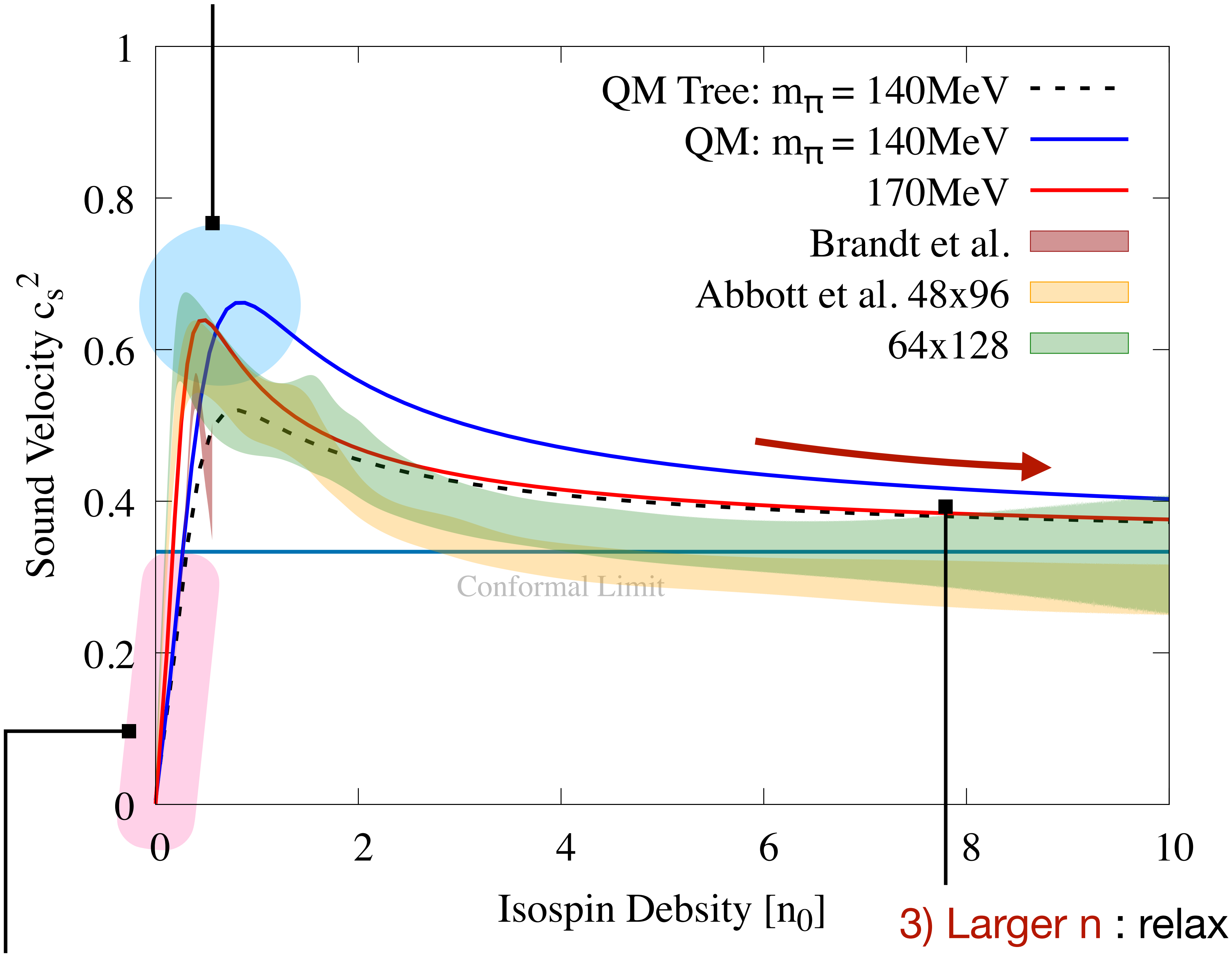
  arXiv 2212.14016

- Abbott+ (2023) $m_\pi = 170\mathrm{MeV}$

  arXiv 2307.15014

Sound velocity

2) $n \sim 0.8\ n_0$: peak structure



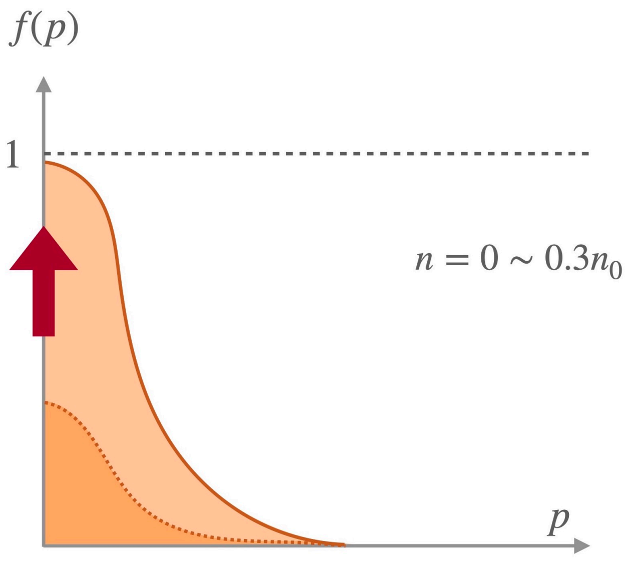
1) $n < 0.8\ n_0$: rapid increase

3) Larger n : relax to the conformal value **from above**

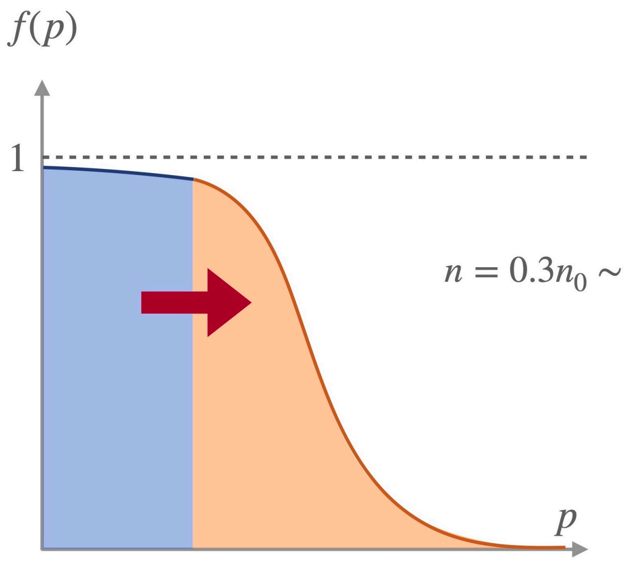
$\neq$ pQCD

Occupation Probability $f(p)$; onset of quark substructure

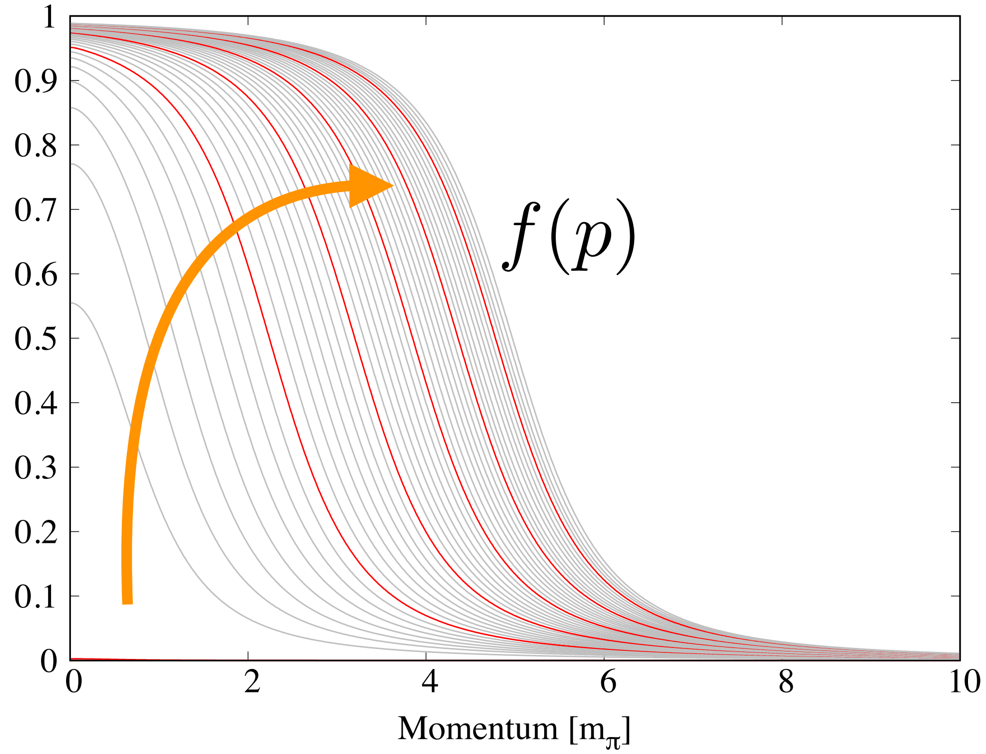
Momentum distribution of quark wavefunction
… two different mode



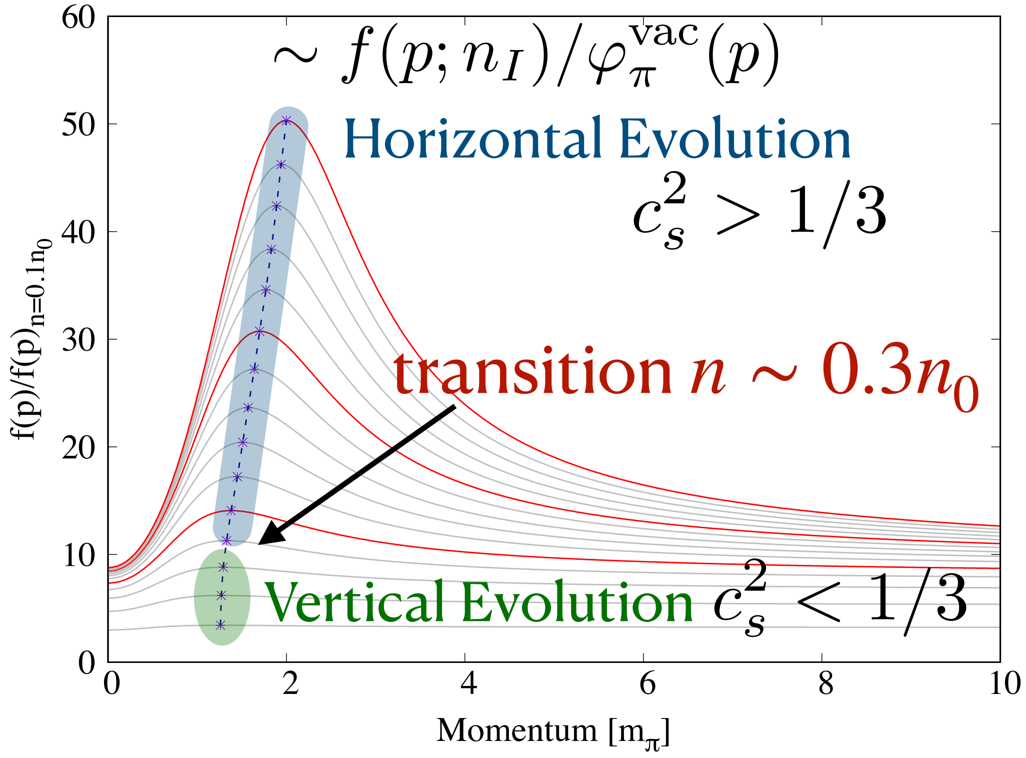
Vertical Evolution



Horizontal Evolution



$$P = n_I^2 \frac{\partial \varepsilon / n_I}{\partial n_I}$$



$$r_\pi = 0.66\mathrm{fm} \to \mathrm{n_I^{overlap}} \sim 5.2\mathrm{n_0}$$

Vertical Evolution: increasing uniformly

$f(p) \sim n_I \varphi_\pi^{\rm vac}(p)$ (low density limit)

→ $\varepsilon / n_I \sim \mathrm{const.}$ small P, soft EoS

Horizontal Evolution: increasing large p mode

(∵ Interactions and Pauli blocking)

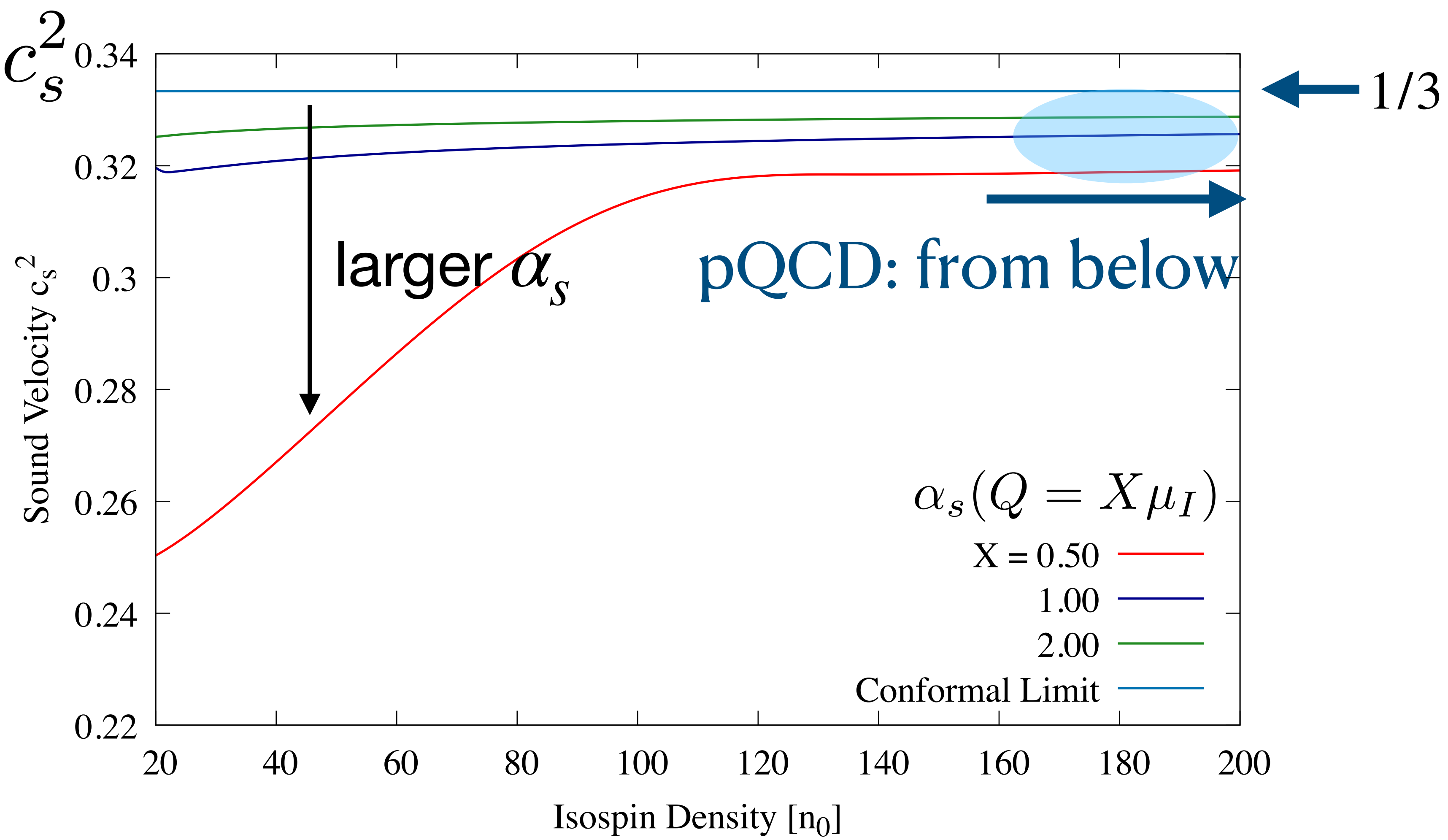
→ stiffen EoS

QM model vs pQCD; non-perturbative effect

c_s^2 asymptotic behavior

QM model:
approaches from above

pQCD:
approaches from below



$$a_2 \sim \Lambda_{\text{QCD}}^2$$

$$\Lambda_{\text{QCD}} = Q \exp\left(-\frac{2\pi}{\beta_0 \alpha_s(Q^2)}\right)$$

Adding the power correction (non-perturbative)

$$P_{\text{withpowers}} = a_0 \mu_I^4 + a_2 \mu_I^2$$

$$c_s^2 = \frac{2a_0\mu_I^2 + a_2}{6a_0\mu_I^2 + a_2} = \frac{1}{3}\left[1 + \frac{2a_2}{6a_0\mu_I^2 + a_2}\right]$$

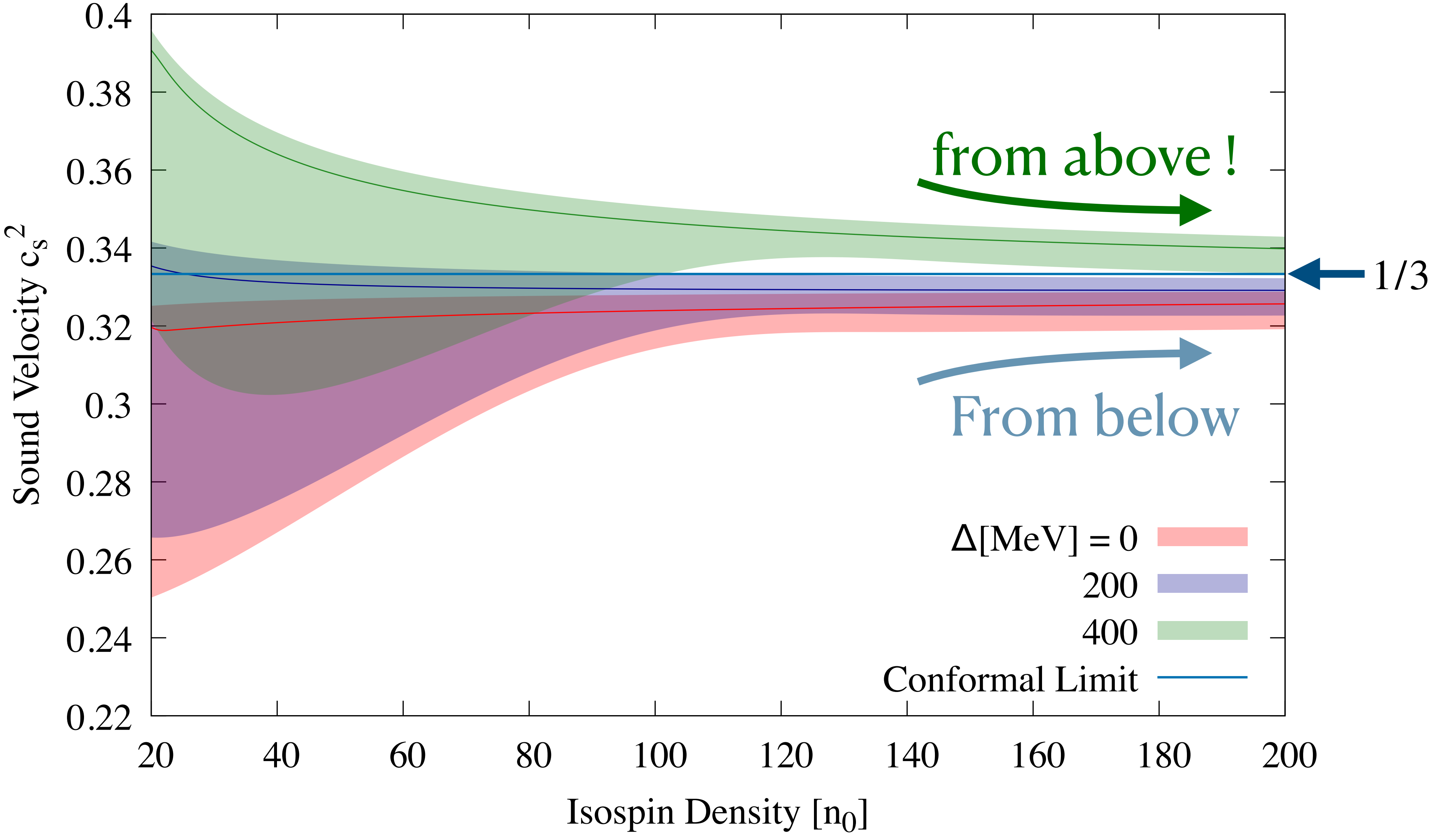
Conformality from above is possible
for large power correction.

pQCD + Power Corrections

$$P = P_{\mathrm{pQCD}} + \frac{\Delta^2}{\pi^2}\mu_I^2$$

$X = 0.5 \sim 2$

solid line is for $X = 1$



power correction in pressure … 5-10 % ↔ change the qualitative behavior of c_s^2

= origin of difference between QM and pQCD

Summary

- ◉ **Soft-to-stiff EOS and quark dof**
  - quark dof may appear before the hadron overlap
  - Importance of quark substructure

- ◉ **Quark-meson model and low- and high density physics**
  - low density: respects ch. symmetry → reproduce Ch.PT and LQCD
  - high density: includes quark dof, matched to LQCD ✅

- ◉ **Non-perturbative effects (= BSC-like pairing)**
  - low density: attractive power correction → stiffens EOS
  - high density: change the asymptotic behavior from pQCD