スピン軌道相互作用を有する冷却原子系における CP^2スキルミオン結晶とその派生相

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Based on *PRB* **106**, L100406 (2022) and arXiv:23XX.XXXX

Collaborators：Yutaka Akagi, Sven Gudnason, Muneto Nitta, Yakov Shnir

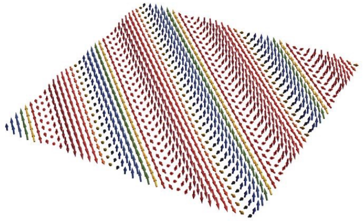
(U. Tokyo) (Henan) (Keio) (JINR)

熱場の量子論とその応用 @ KEK, 2023年8月28日

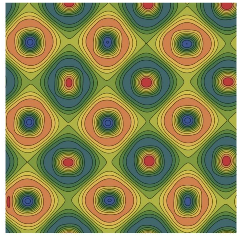
Summary of my talk

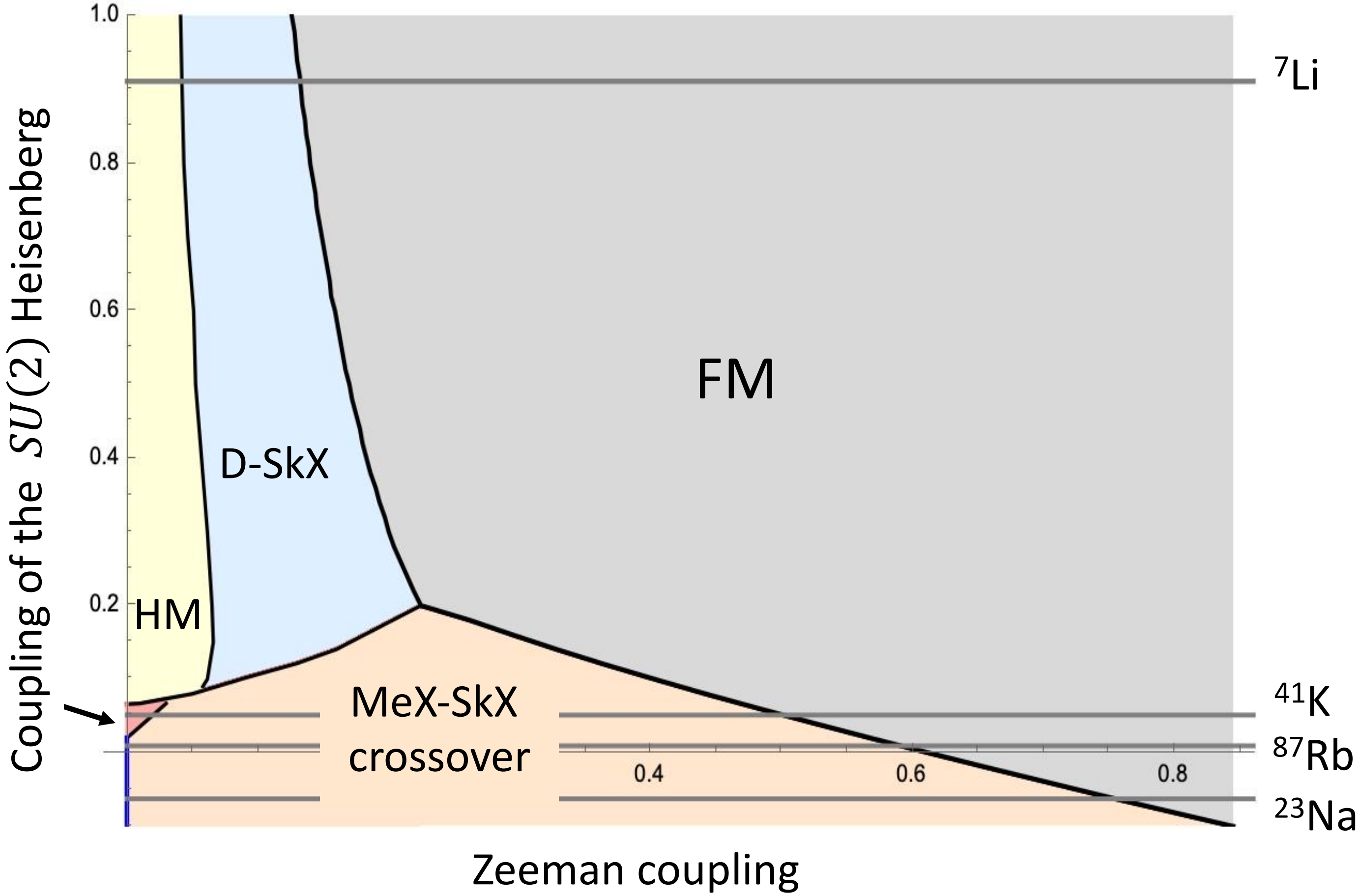
Hamiltonian = $SU(3)$ Heisenberg + $SU(2)$ Heisenberg + Zeeman + Generalized DM

Helimagnetic (HM)

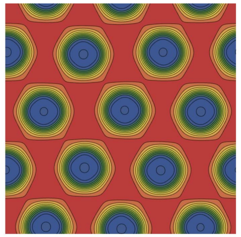


Skyrmionium crystal





Double-Skyrmion crystal (D-SkX)



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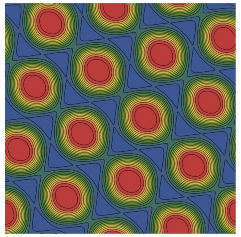
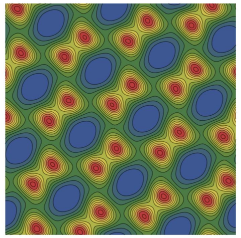
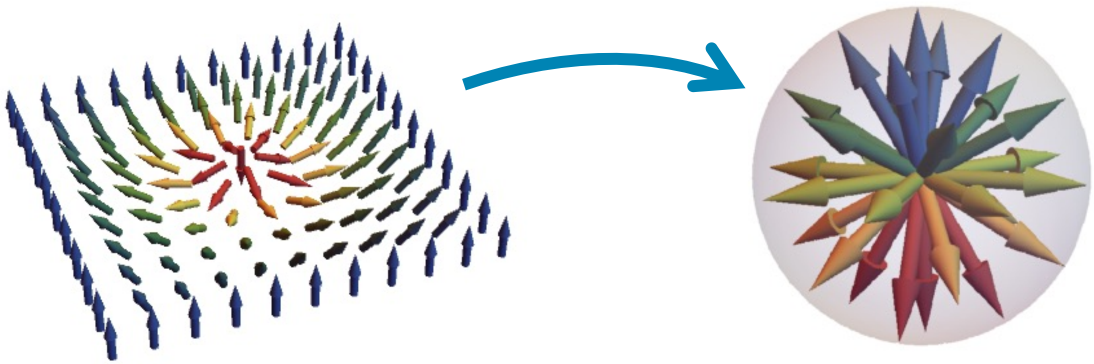
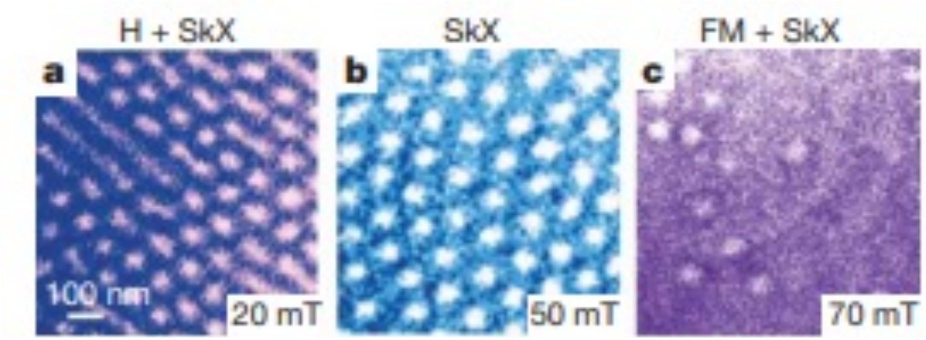


Meron crystal (MX)



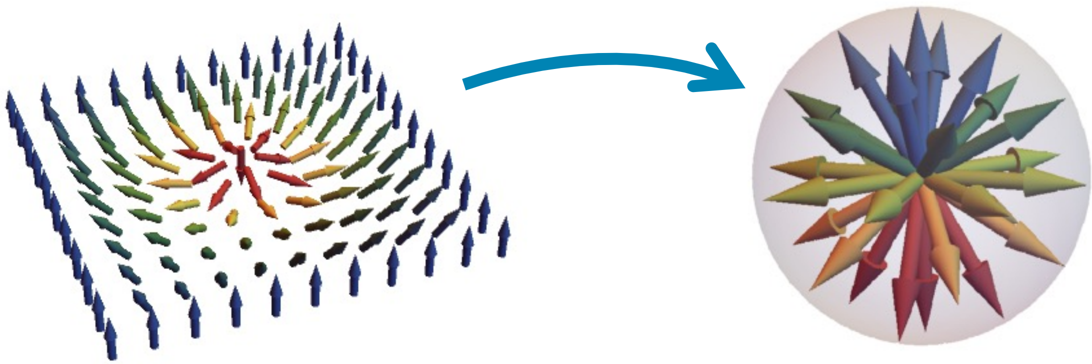
Magnetic Skyrmions

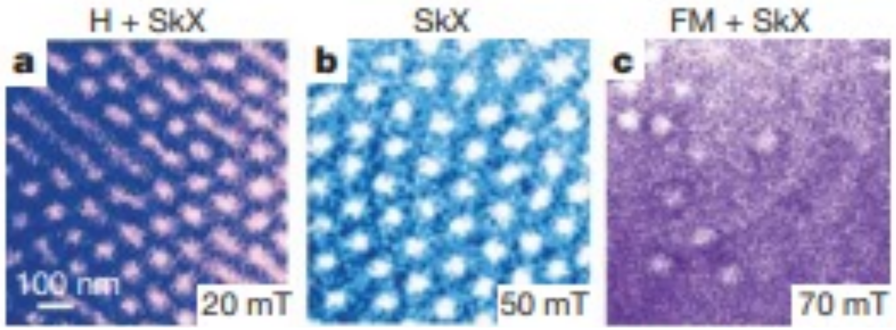




Experiments on a thin film of $Fe_{0.5}Co_{0.5}Si$
[X. Z. Yu et.al., Nature **465**, 901(2010)]

Magnetic Skyrmions





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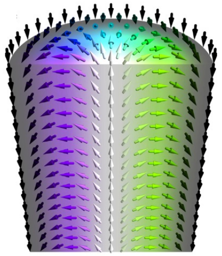
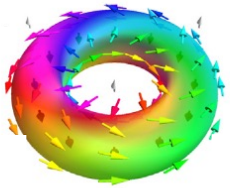
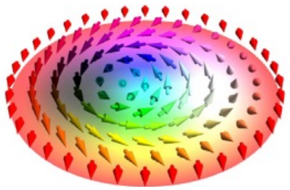
Recent trends





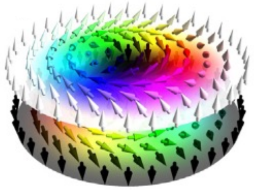
- **3D topological soliton**
  - Skyrmion string
  - Hopfion

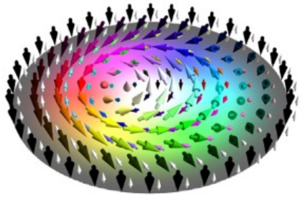
    ⋮



- **Composite & Constituent**
  - multi-Skyrmion
  - Skyrmionium
  - fractional Skyrmion (meron)

    ⋮
- **Different surroundings**
  - anti-ferromagnets
  - ferrimagnets
  - $SU(N)$ **magnets**





These figures are taken from
[B. Göbel, I. Mertig, & O. Tretiakov, Phys. Rep. **895**, 1 (2021)]

$SU(N)$ magnets

- **$SU(2I+1)$ magnets have been realized using cold atoms with nuclear spin I.**
  - ^{173}Yb $\rightarrow SU(6)$ [T. Fukuhara et.al, PRL **98**, 030401 (2007)]
  - ^{87}Sr $\rightarrow SU(10)$ [B. J. DeSalvo et.al, PRL **105**, 030402 (2010)]
  - ^{87}Rb $\rightarrow SU(3)$ [S. Will el.al., Nature **465**, 197–201 (2010)]

- **Spin-1 systems can be viewed as $SU(3)$ magnets**

Bilinear Biquadratic (BBQ) model

$$H_{\mathrm{BBQ}} = \sum_{\langle i,j\rangle} \left[J_1 \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_j + J_2 \left(\hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_j \right)^2 \right]$$

$$= \sum_{\langle i,j\rangle} \left[\frac{J_2}{2} \hat{T}_i^{\alpha} \hat{T}_j^{\alpha} + (J_1 - J_2) \hat{S}_i^{a} \hat{S}_j^{a} \right]$$

$\hat{S}_i^{a}$: Spin-1 operator, $\hat{T}_i^{\alpha}$: $SU(3)$ spin operator

$$\hat{\boldsymbol{S}}_i = \left(\frac{\hat{T}_i^1 + \hat{T}_i^6}{\sqrt{2}}, \frac{\hat{T}_i^2 + \hat{T}_i^7}{\sqrt{2}}, \frac{\hat{T}_i^3 + \sqrt{3}\hat{T}_i^8}{2} \right)$$

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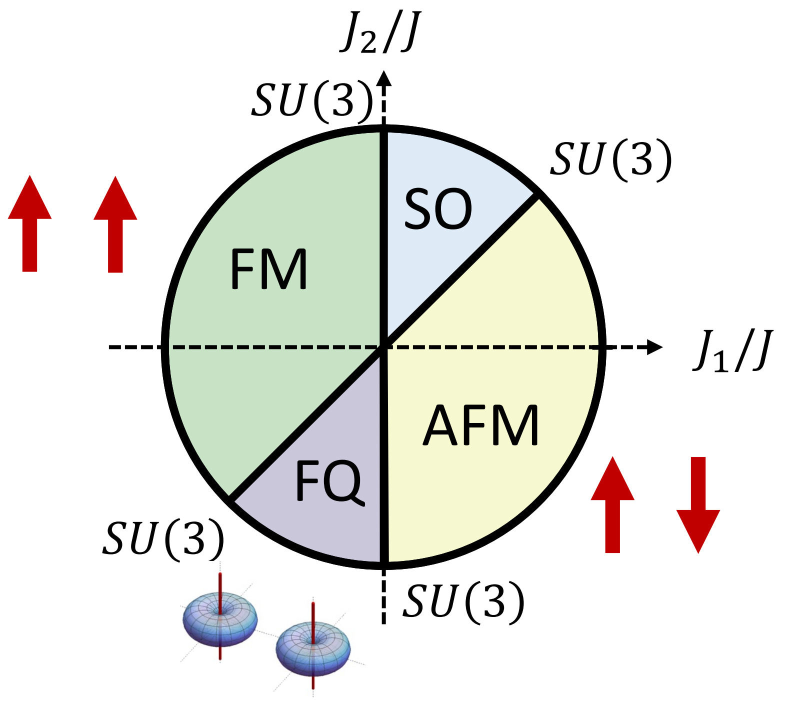
Mean-field phase diagram (square lattice)

[N. Papanicolaou, Nucl. Phys. B **305**, 367]

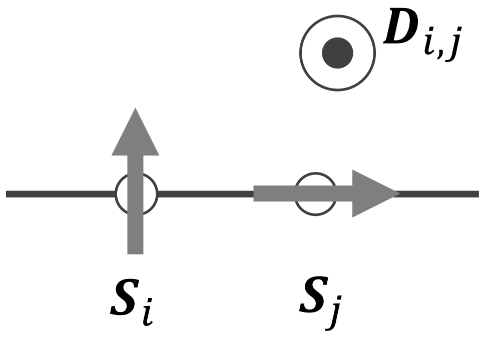


Quadrupolar tensor : $\hat{Q}_j = \hat{\boldsymbol{S}}_j \otimes \hat{\boldsymbol{S}}_j^{\mathrm{T}} - \frac{2}{3}\mathbf{1}$

How to introduce a stabilizing term?

Dzyaloshinskii-Moriya (DM) interaction

$$H_{\mathrm{DM}} = \sum_{\langle i,j \rangle} \boldsymbol{D}_{i,j} \cdot \left(\hat{\boldsymbol{S}}_i \times \hat{\boldsymbol{S}}_j \right)$$



- It favors to twist the spins.
- An effect of the *spin-orbit coupling*.

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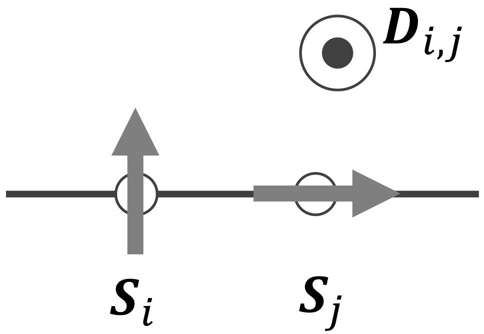

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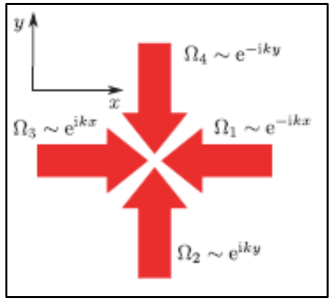
Spin-1 Bose-Hubbard model with spin-orbit coupling

$$H_{\rm BH} = -t \sum_{\langle ij \rangle} \left[\hat{b}^{\dagger}_{i,\sigma} \left(e^{iA_{i,j}} \right)_{\sigma\rho} \hat{b}_{j,\rho} + {\rm H.c.} \right] + \frac{1}{2} \sum_i \left[U_0\, \hat{n}_i (\hat{n}_i - 1) + U_2 \left(\hat{\boldsymbol{S}}_i^2 + 2\hat{n}_i \right) \right] - h \sum_i \hat{S}_i^z$$



$$A_{i,i\pm \boldsymbol{e}_x} = \pm \frac{\kappa}{\sqrt{2}} \tau^x, \quad A_{i,i\pm \boldsymbol{e}_y} = \pm \frac{\kappa}{\sqrt{2}} \tau^y$$

τ^a: Spin-1 matrix

[Juzeliūnas, Ruseckas, Dalibard, PRA **81**, 053403 (2010)]

Our model

$$H = \frac{1}{2}\sum_{\langle i,j\rangle}\left[\underbrace{J\,\hat{T}_i^{\alpha}\hat{T}_j^{\alpha}}_{SU(3)} + \underbrace{K\,\hat{S}_i^{a}\hat{S}_j^{a}}_{SU(2)} + \underbrace{2f_{\alpha\beta\gamma}A_{i,j}^{\alpha}\hat{T}_i^{\beta}\hat{T}_j^{\gamma}}_{\text{Generalized DM}}\right] - h\sum_i \hat{S}_i^z \qquad (\,J<0\,)$$

$$H_{\mathrm{DM}} = \sum_{\langle i,j\rangle}\varepsilon_{abc}D_{i,j}^{a}\hat{S}_i^{b}\hat{S}_j^{c}$$

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SU(3) operators $\hat{T}_j^\alpha = (\lambda^\alpha)_{\sigma\rho}|\sigma\rangle_j\langle\rho|_j$ where $\hat{S}_j^z|\sigma\rangle_j \equiv \sigma|\sigma\rangle_j$

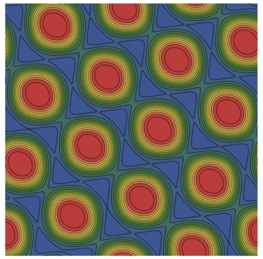
SU(3) spin coherent state $|Z\rangle = \otimes_j |Z_j\rangle$ with $|Z_j\rangle = Z_j^\sigma|\sigma\rangle_j$

- At the single site level, $|Z_j\rangle$ can describe any spin-1 state at site j.
- $\boldsymbol{Z}_j = (Z_j^{+1}, Z_j^0, Z_j^{-1})^{\mathrm{T}}$ takes its value on $S^5/S^1 = SU(3)/U(2) = CP^2$.

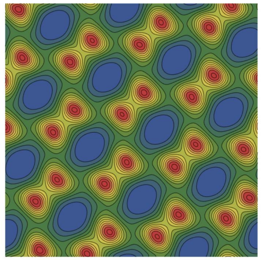
Topological charge $N = -\frac{1}{64\pi}\sum_{\langle ijk\rangle} f_{\alpha\beta\gamma}\langle\hat{T}_i^\alpha\rangle\langle\hat{T}_j^\beta\rangle\langle\hat{T}_k^\gamma\rangle \in \pi_2(CP^2) = \mathbb{Z}$

Ground state phase diagram

CP^2 *Skyrmion crystal* (SkX)

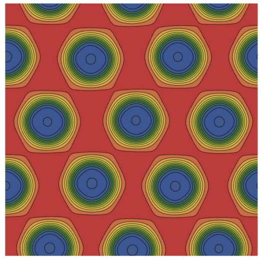


CP^2 *Meron crystal* (MeX)

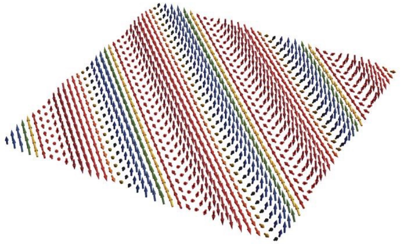


[**YA**, Y. Akagi, et al.,
PRB **106**, L100406 (2022)]

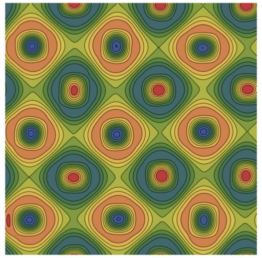
CP^2 *Double-Skyrmion crystal* (D-SkX)



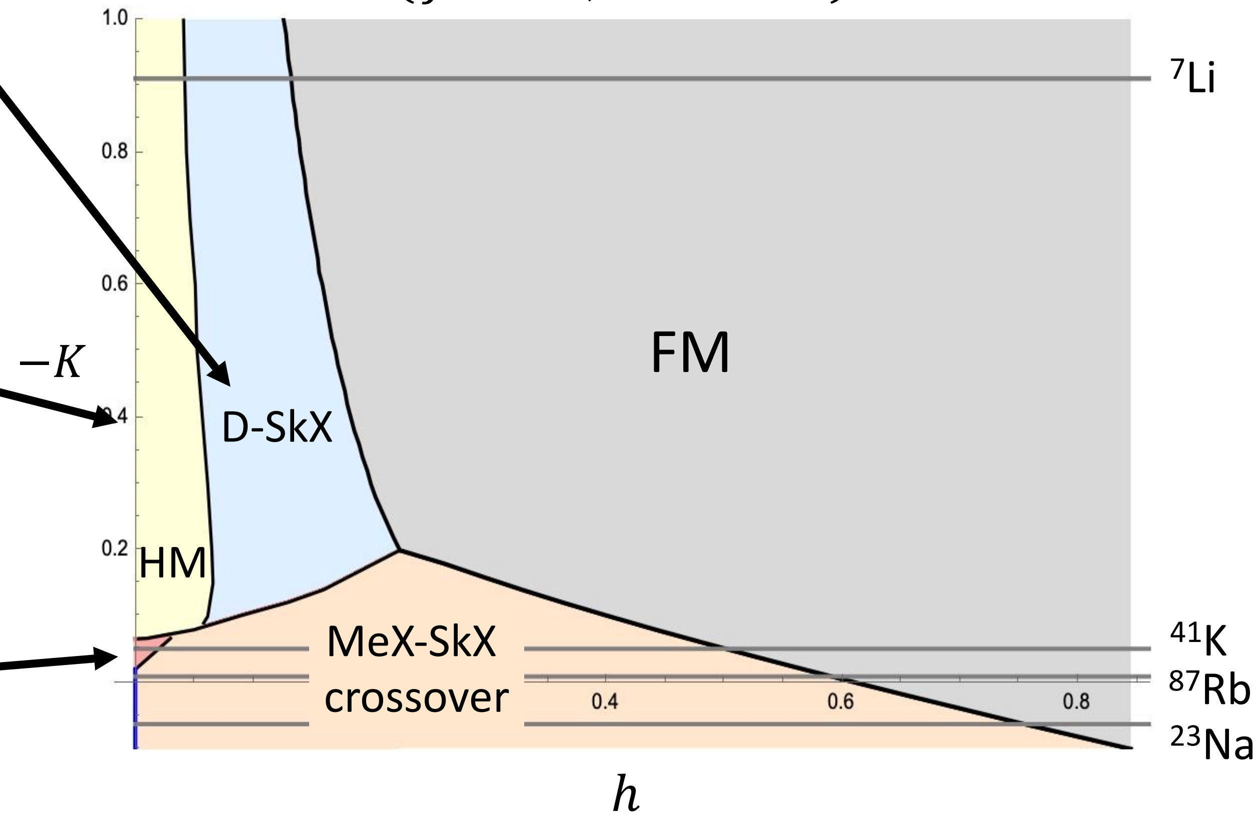
CP^2 *Helimagnetic* (HM)



CP^2 *Skyrmionium crystal*



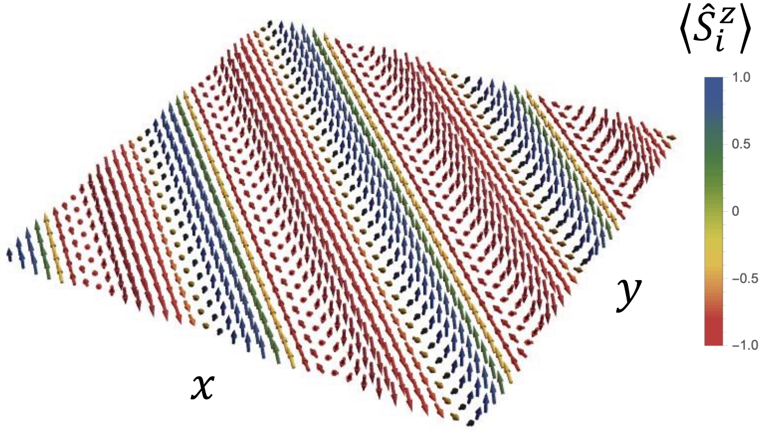
$(J = -1, \kappa = -0.4)$

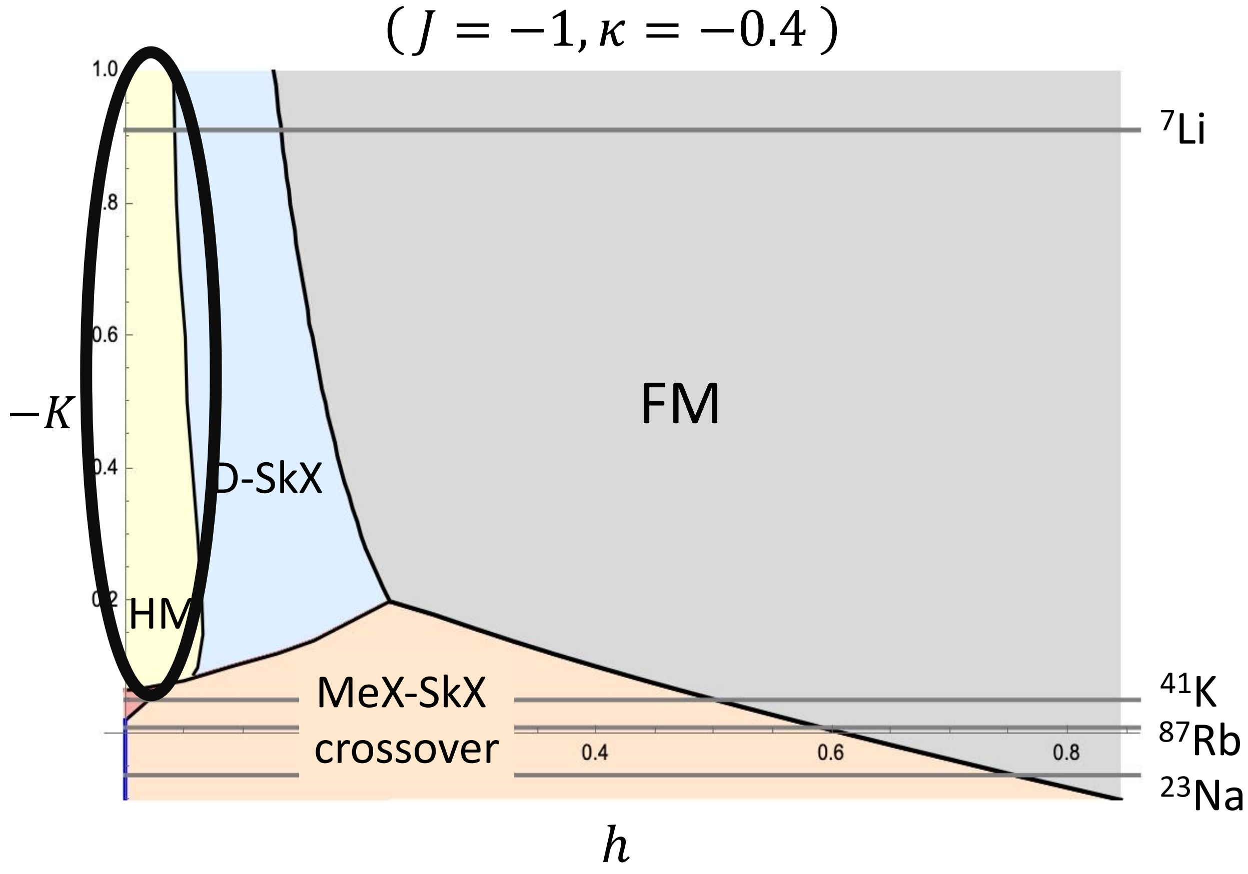


CP^2 Helical structure

- Helical structure with small modulation along the stripes
- Single q-state both in $\mathcal{S}(\boldsymbol{q})$ and $\mathcal{Q}(\boldsymbol{q})$

Magnetization vectors $\langle\hat{\boldsymbol{S}}_i\rangle$





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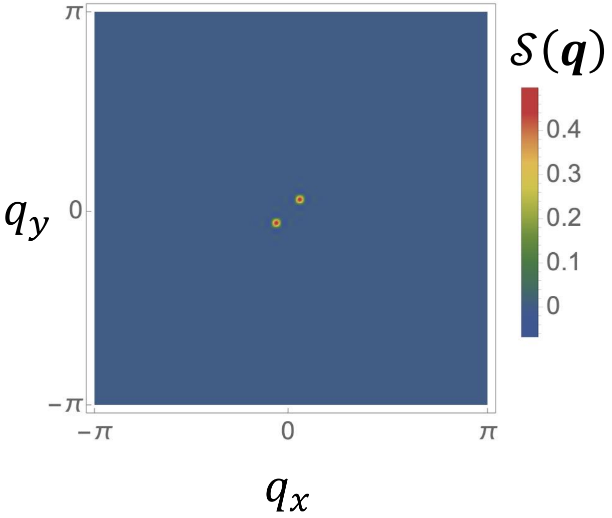
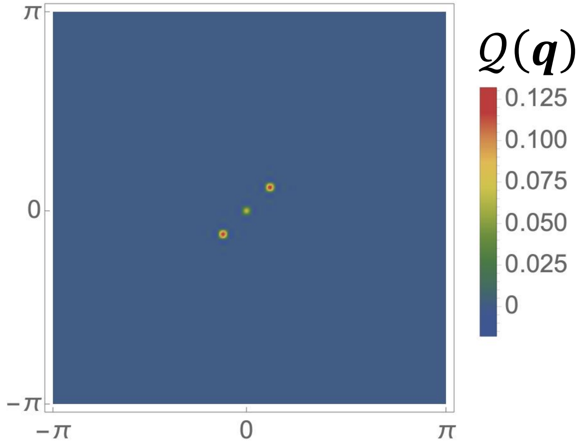
Spin Structure Factor



Quadrupole Structure Factor



$$\mathcal{S}(\boldsymbol{q}) = \mathcal{N}^{-2} \sum_{j,k} \langle \hat{\boldsymbol{S}}_j \rangle \cdot \langle \hat{\boldsymbol{S}}_k \rangle \, e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_j - \boldsymbol{r}_k)}$$

$$\mathcal{Q}(\boldsymbol{q}) = \mathcal{N}^{-2} \sum_{j,k} \frac{1}{2} \mathrm{Tr}\left(\langle \hat{Q}_j \rangle \langle \hat{Q}_k \rangle\right) e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_j - \boldsymbol{r}_k)}$$

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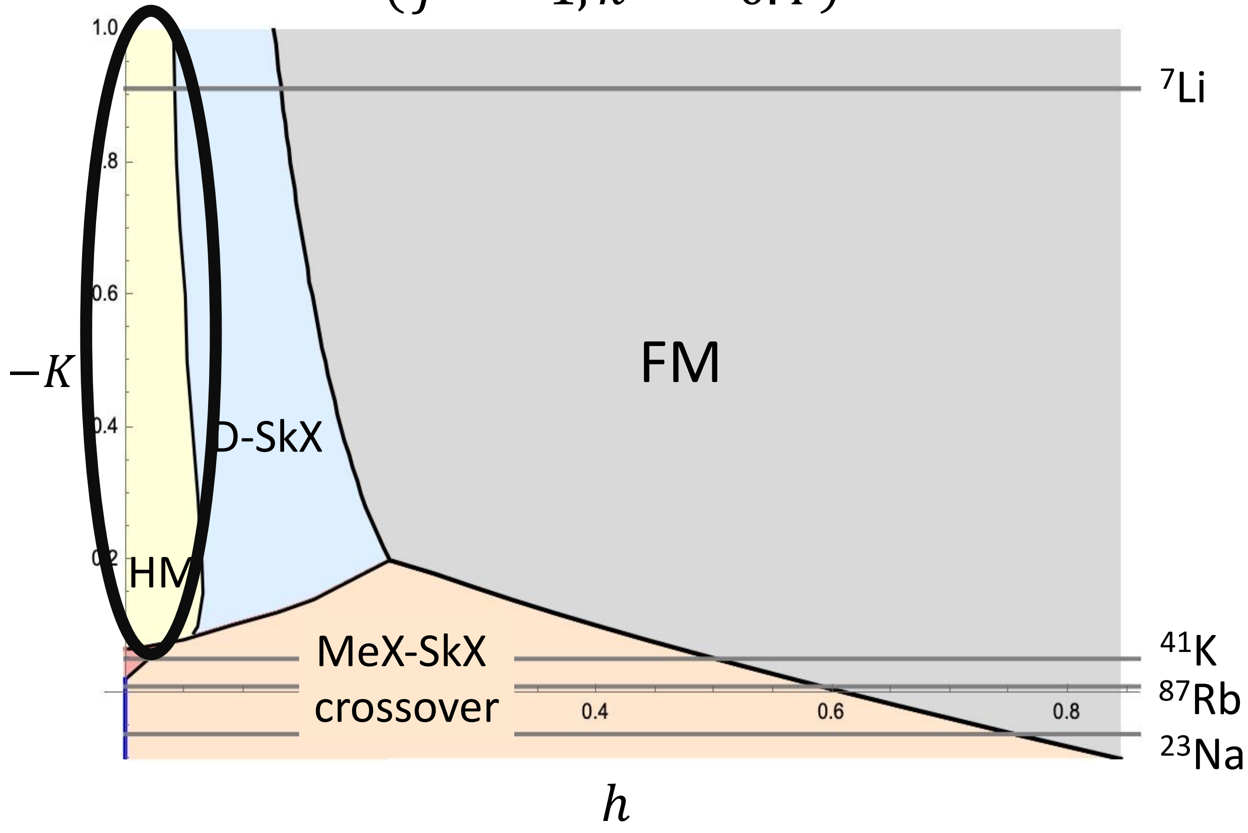
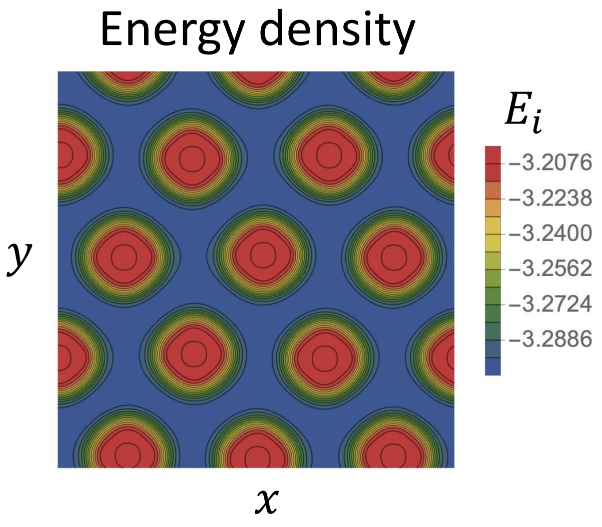
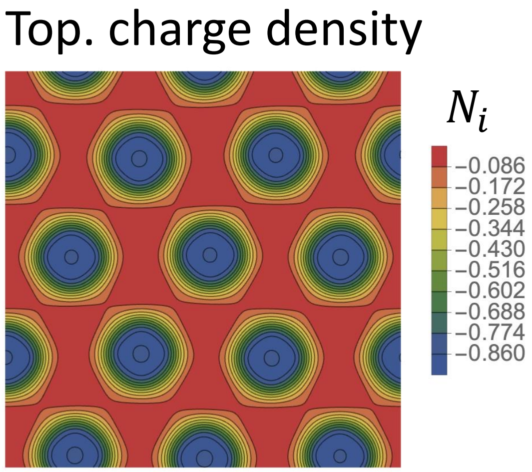
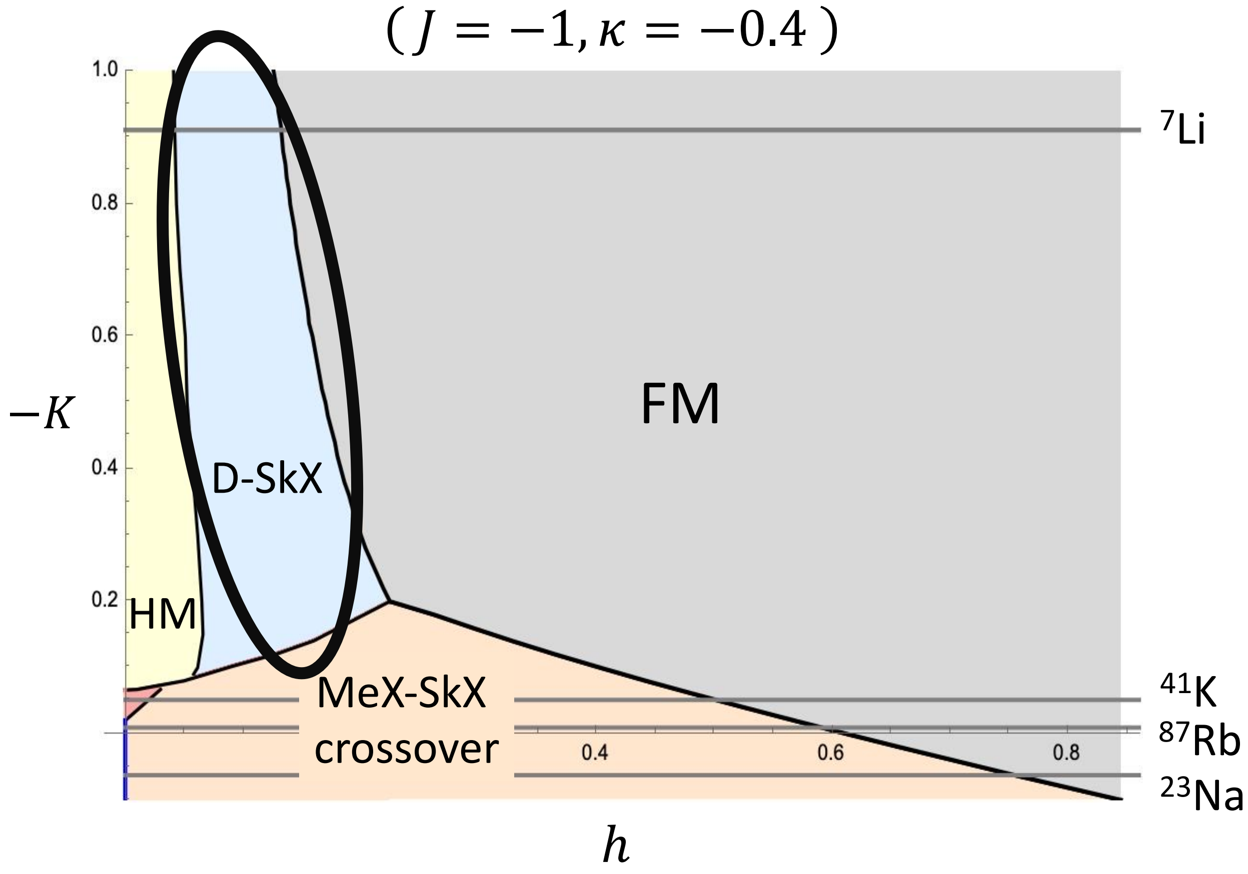


CP^2 Double-Skyrmion crystal

- Triangular lattice of $N = -2\ CP^2$ Skyrmions
- Magnetic Skyrmion-like magnetic structure
- But, non-trivial quandrupole structure







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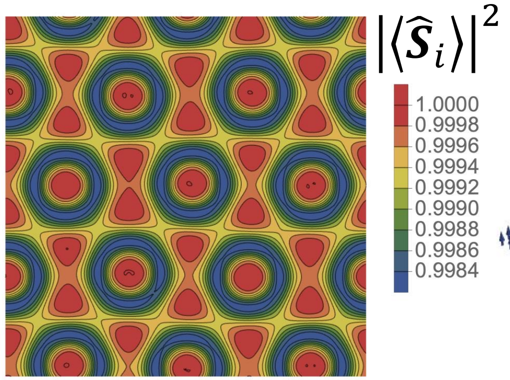
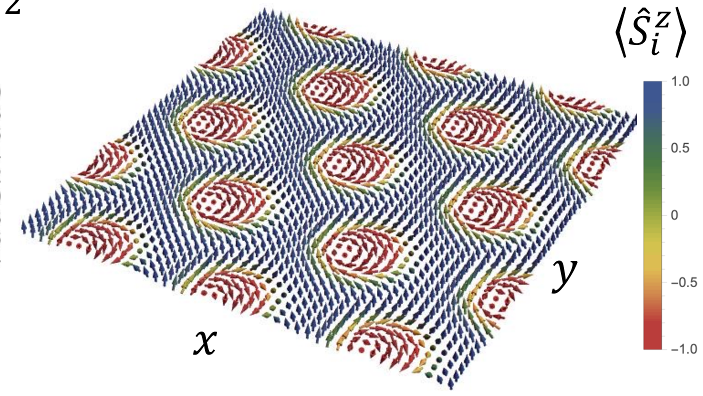
Norm of spins



Magnetization vectors



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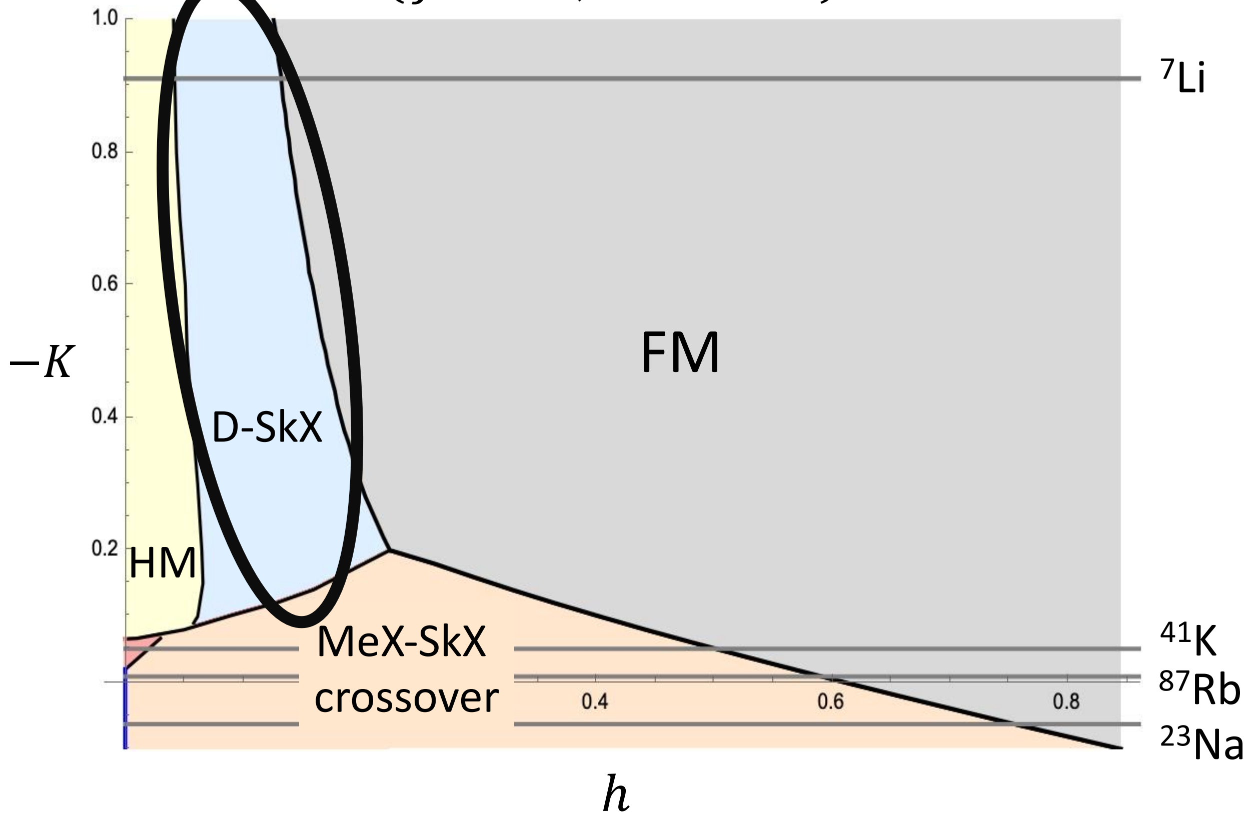

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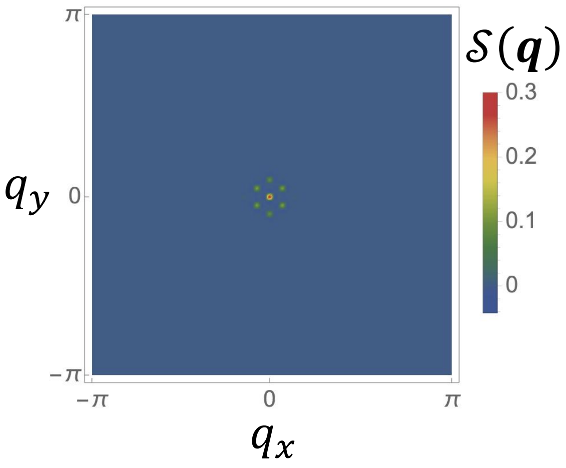
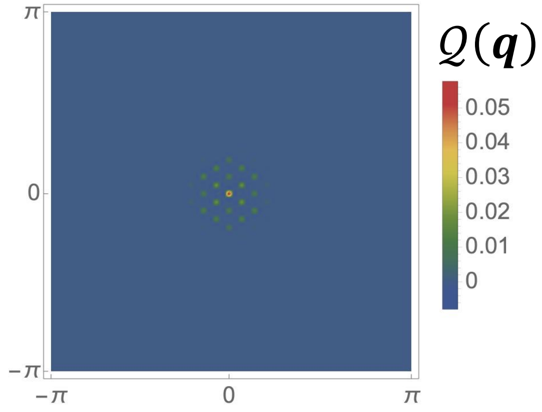
Spin structure factor



Quadrupole structure factor



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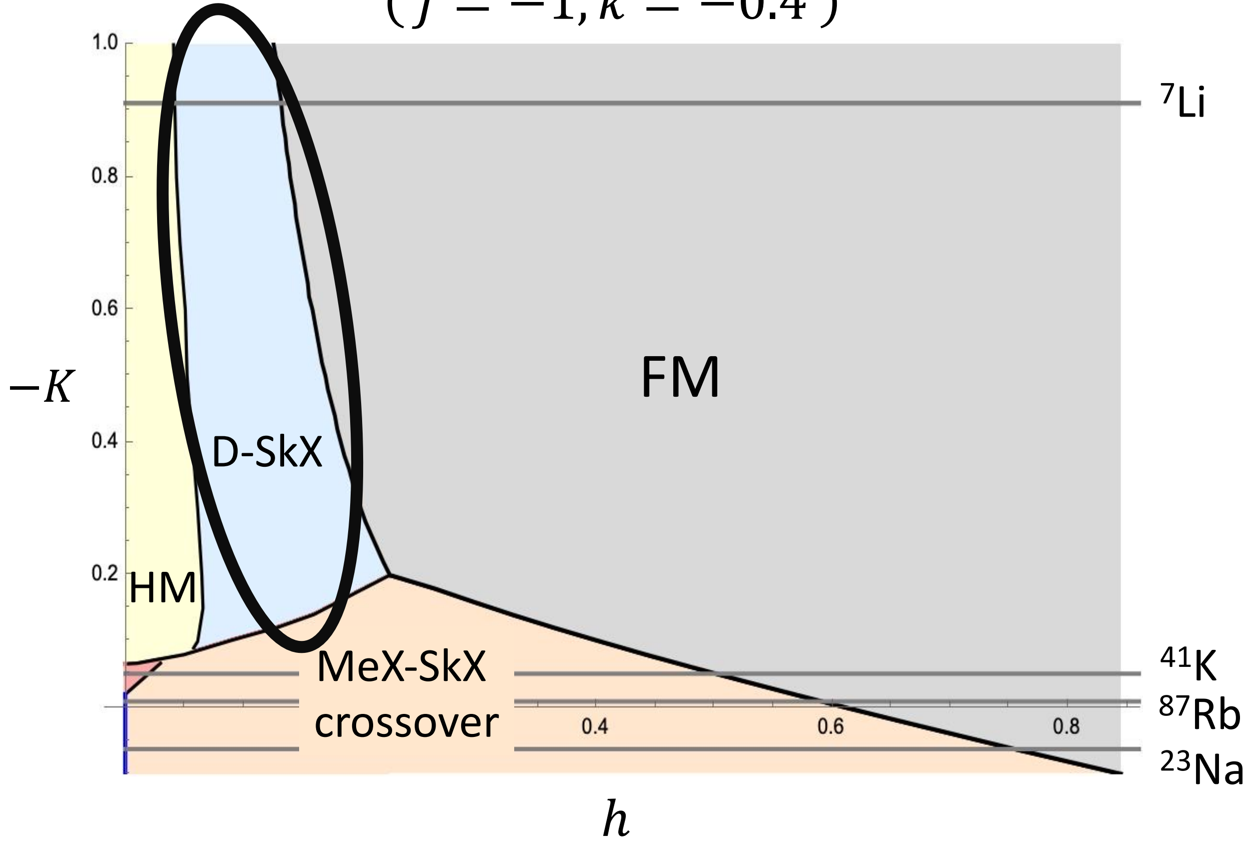
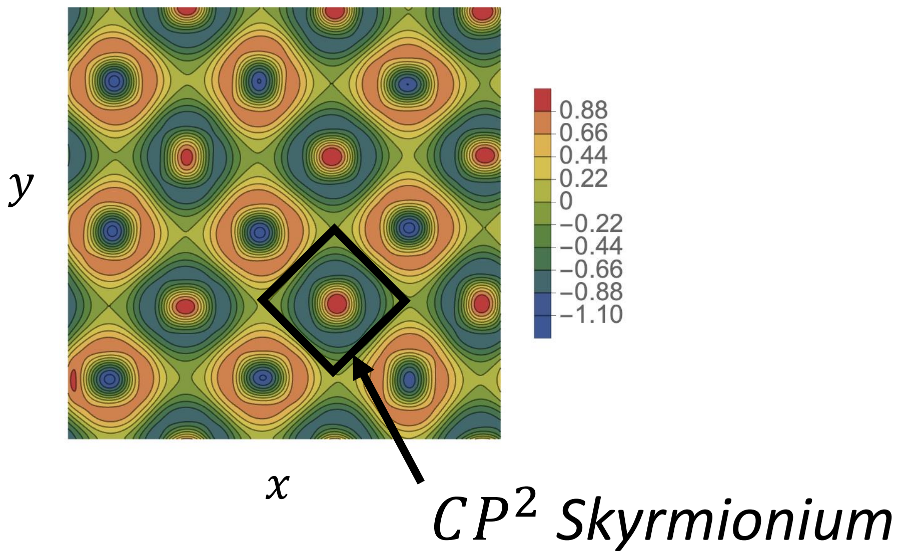


CP^2 Skyrmionium crystal

= Skyrmion surrounded by an anti-Skyrmion

- Square lattice of CP^2 Skyrmioniums
- Spin nematic realize outside of Skyrmioniums
- Double q-structure in $\mathcal{S}(\boldsymbol{q})$ and $\mathcal{Q}(\boldsymbol{q})$

Top. charge density



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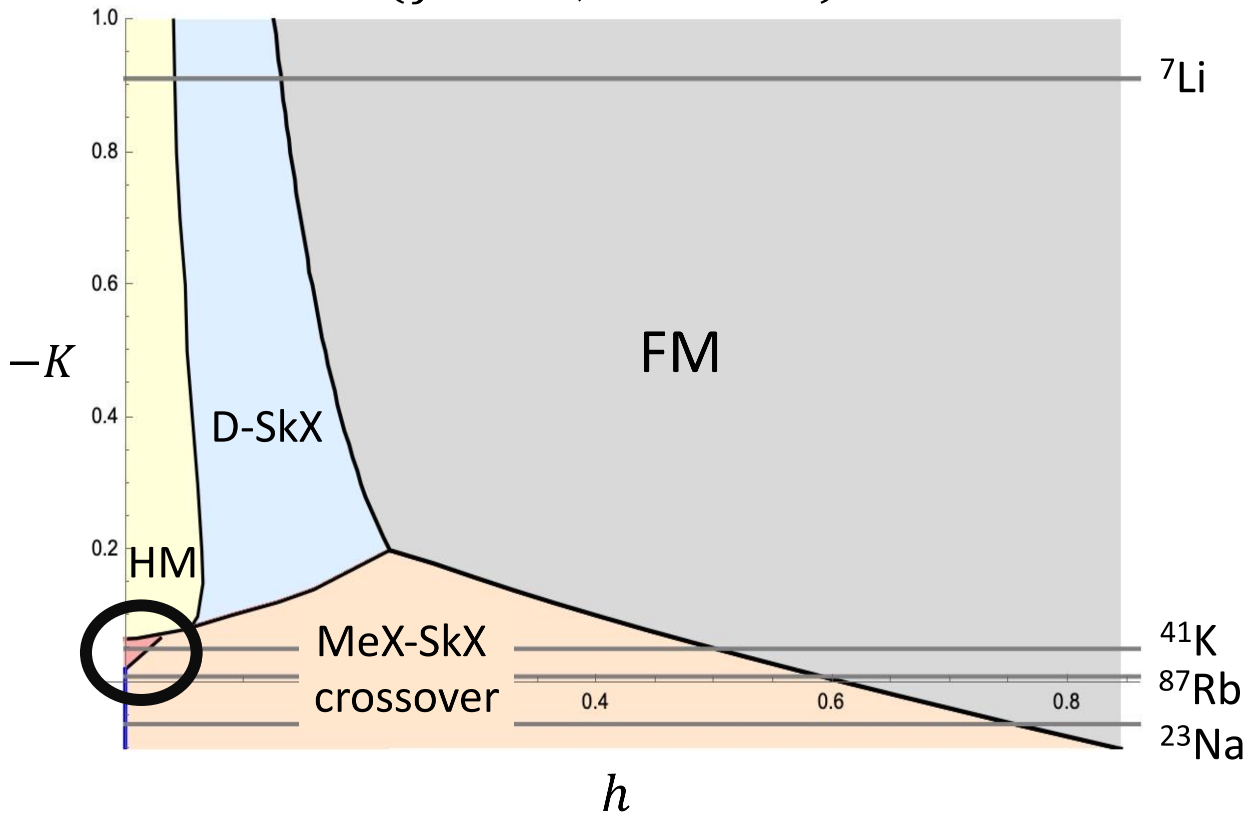

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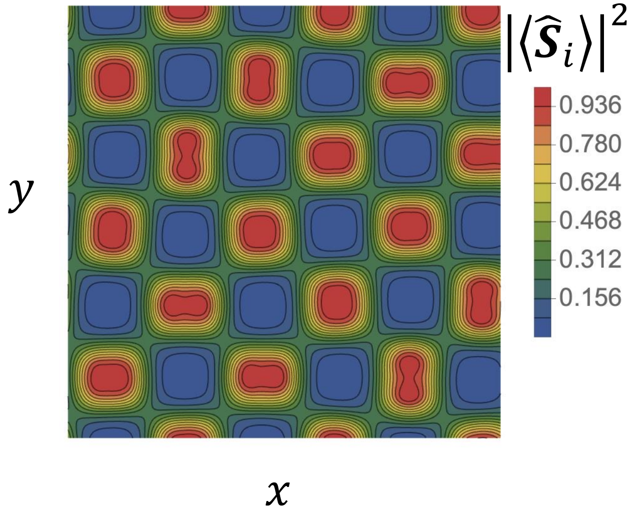

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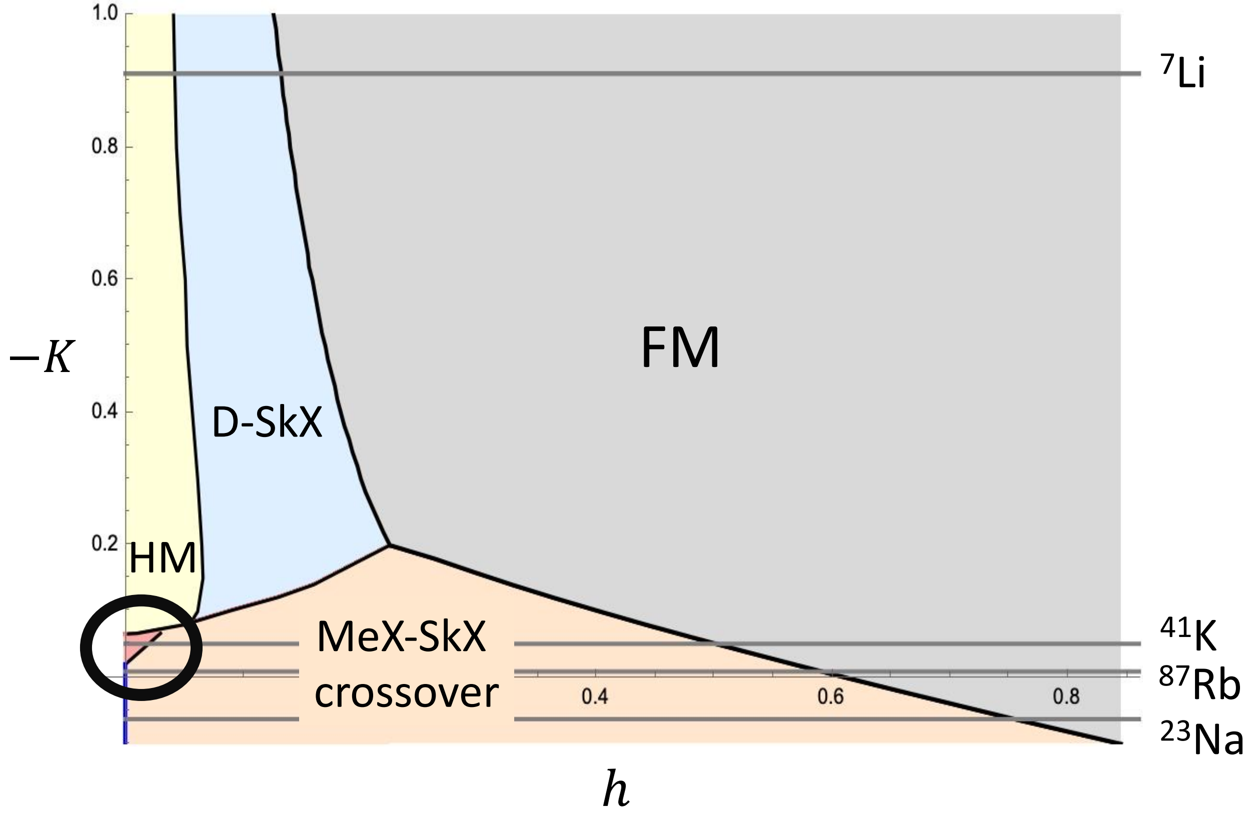

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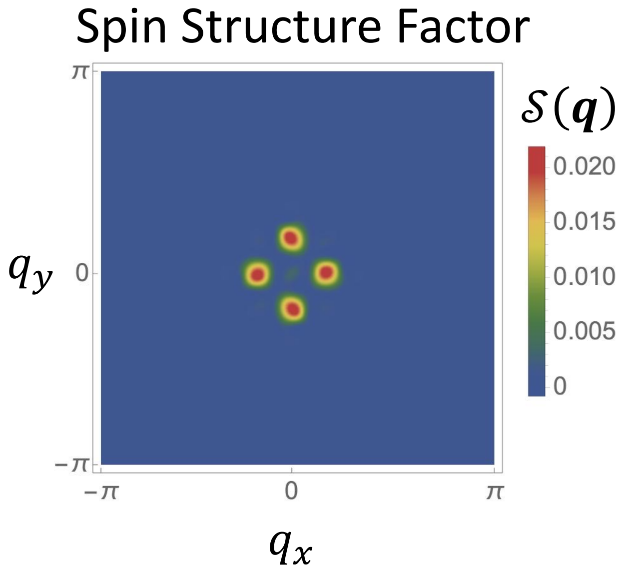
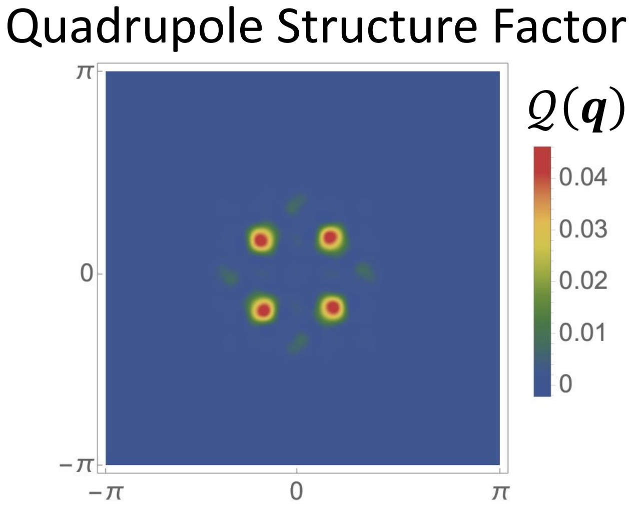
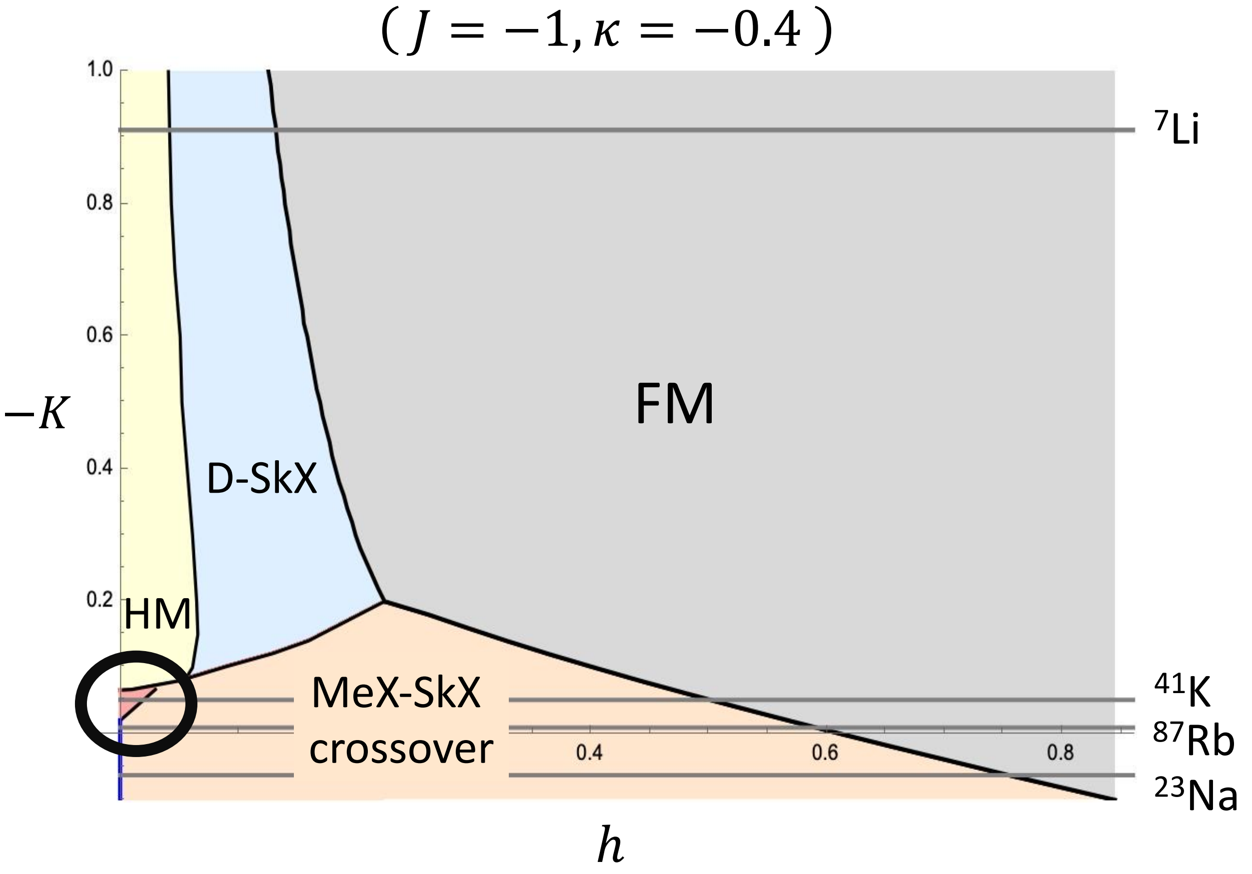


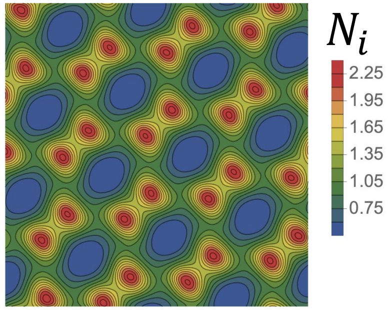




Meron crystal – Skyrmion crystal crossover

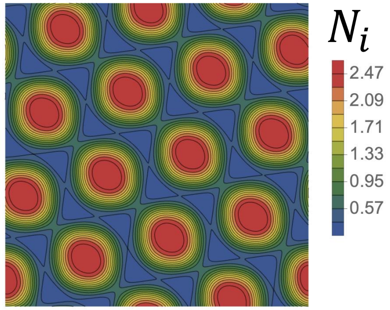
- Meron crystal = honeycomb lattice of merons
  Skyrmion crystal = triangular lattice of Skyrmions
- Spin nematic state is realized at the core of Skyrmions and outside of merons.
- Triple q-structure in $\mathcal{S}(\boldsymbol{q})$ and $\mathcal{Q}(\boldsymbol{q})$
- These state are smoothly connected.

Meron crystal



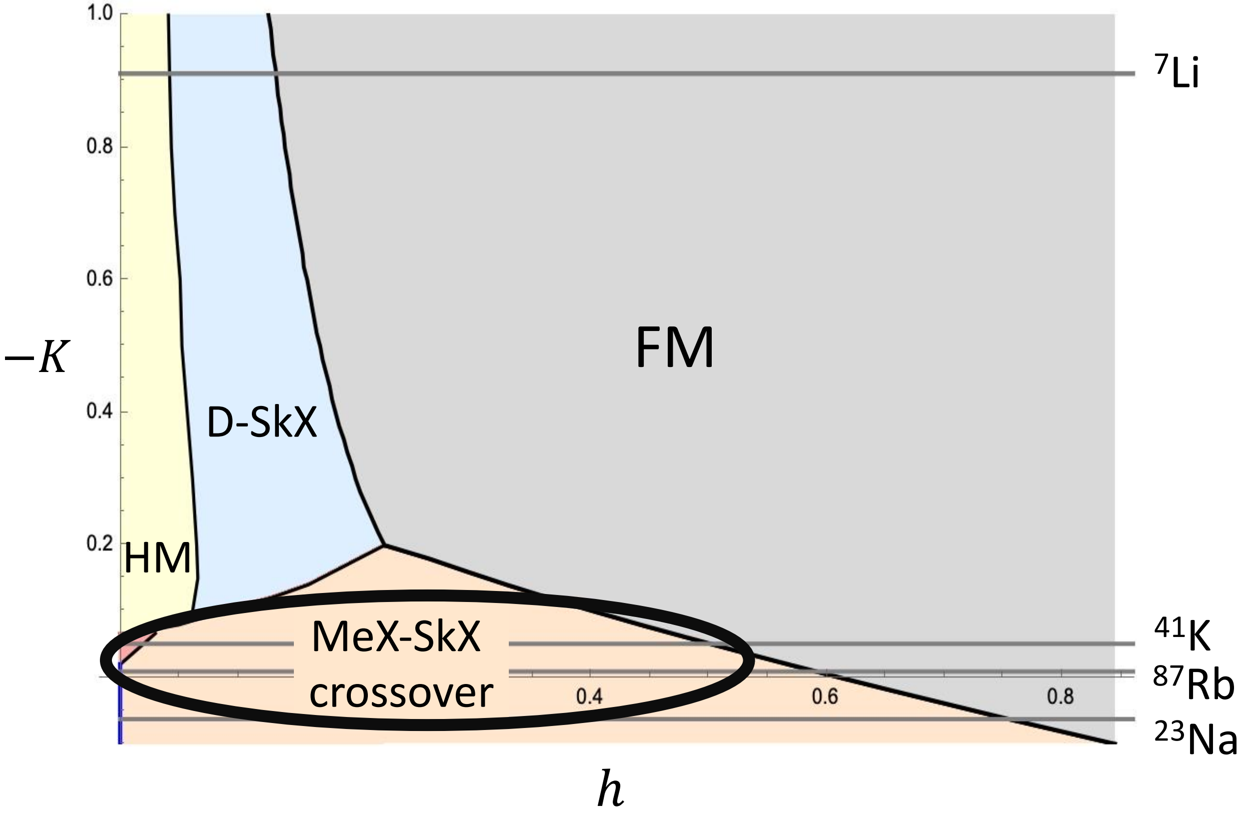
$h = 0.05$

Skyrmion crystal



$h = 0.45$

$(J = -1, \kappa = -0.4)$



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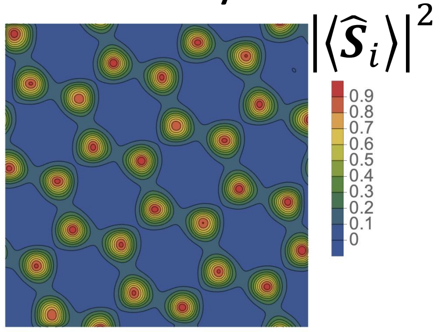
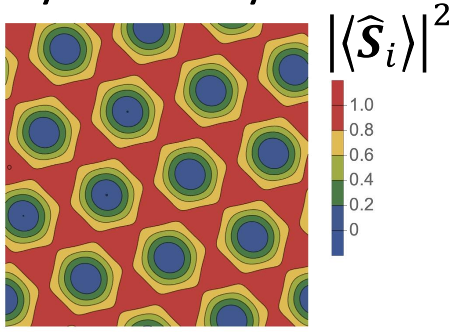


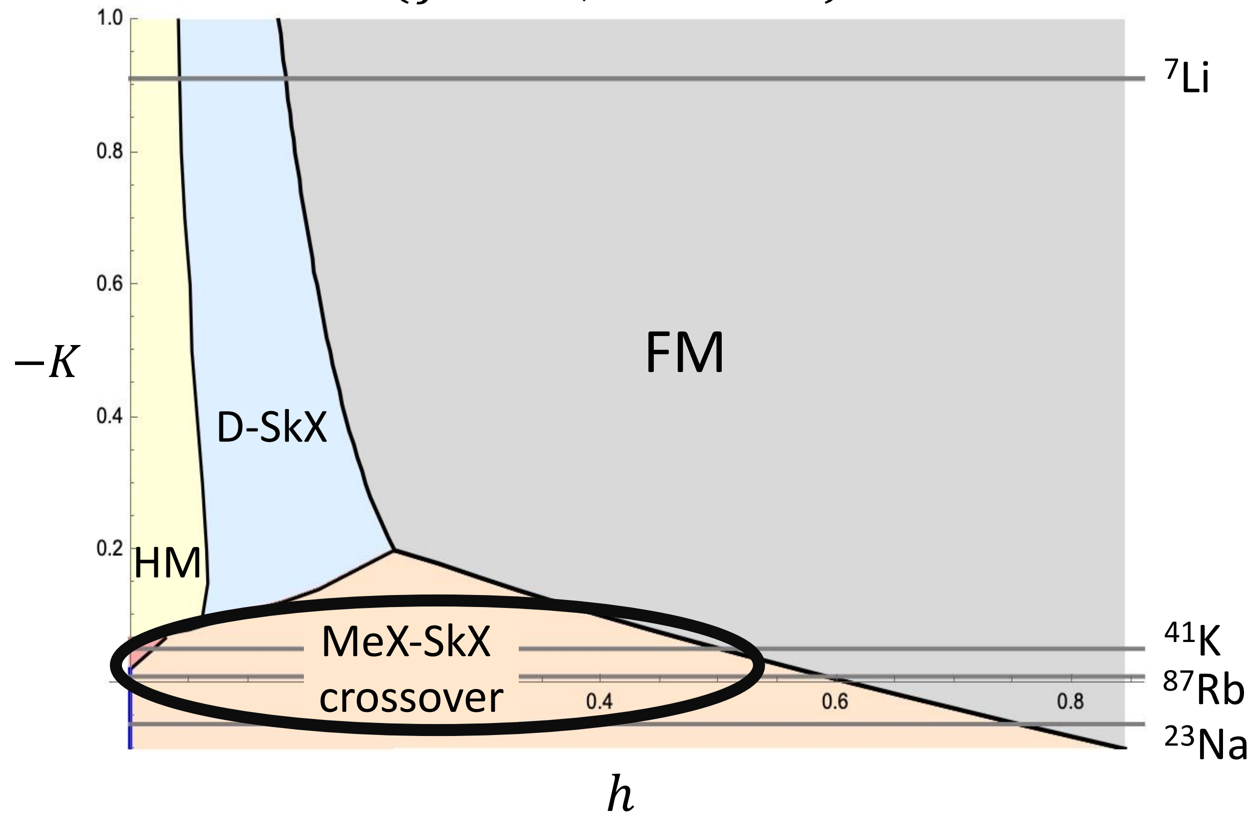
$h = 0.05$

Skyrmion crystal



$h = 0.45$

$(J = -1, \kappa = -0.4)$



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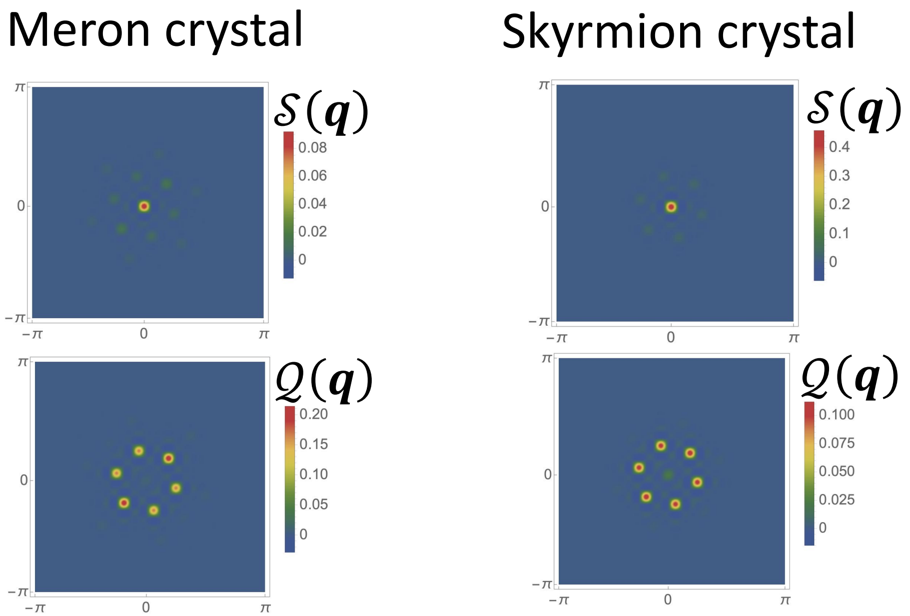

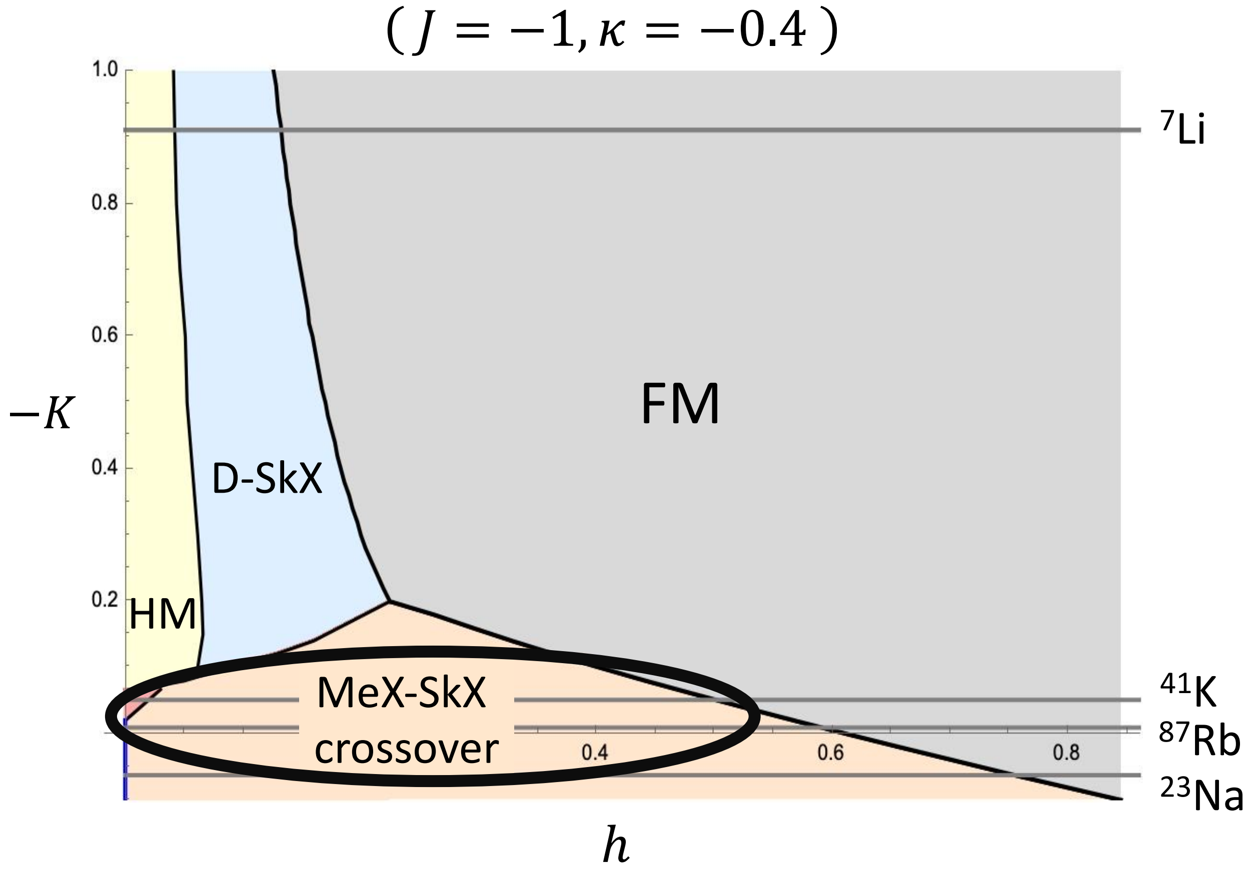

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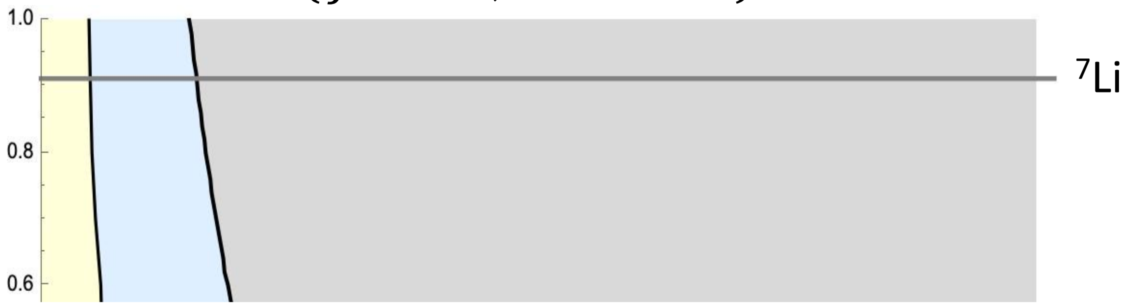
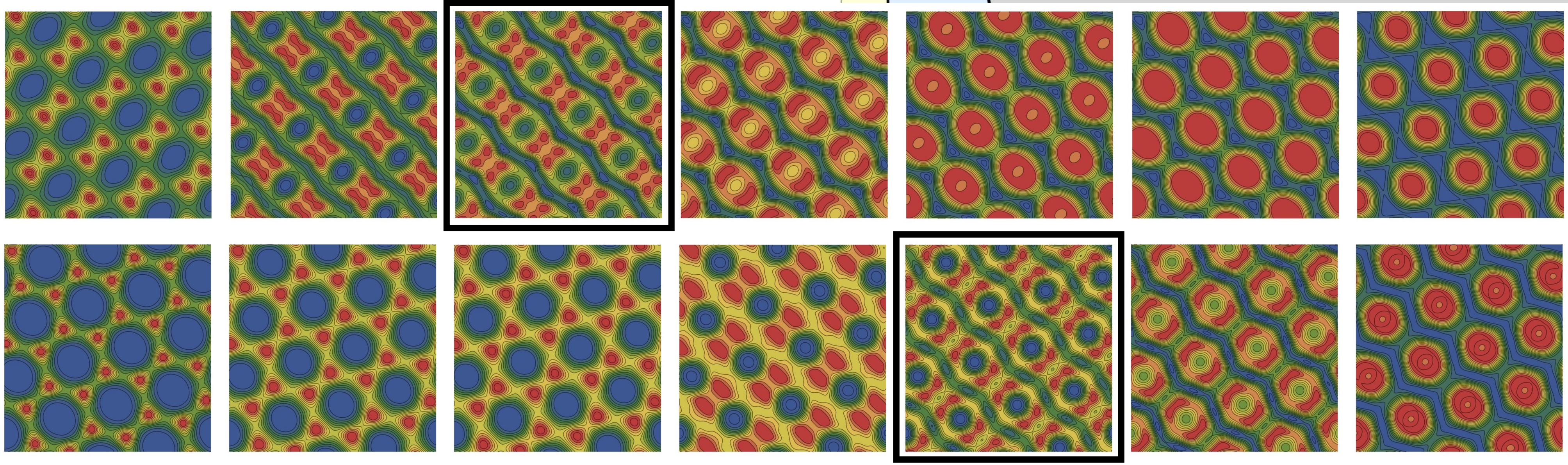


Top. charge density

Energy density

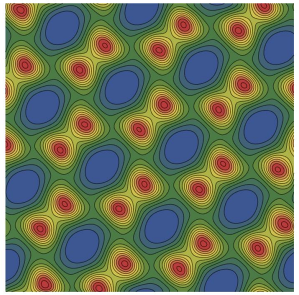


$h = 0.05$ $h = 0.15$ $h = 0.18$ $h = 0.25$ $h = 0.31$ $h = 0.33$ $h = 0.45$

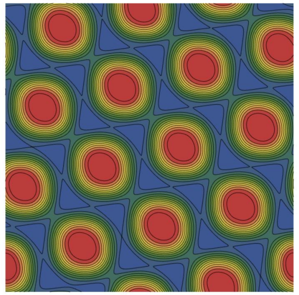
Summary

- ✓ We have studied the ground states in an $SU(3)$ spin system obtained as an effective theory of the spin-orbit coupled spin-1 Bose-Hubbard model.
- ✓ **The $SU(3)$ spin systems host various exotic phases**:

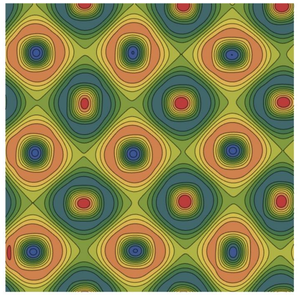
CP^2 *Meron crystal*



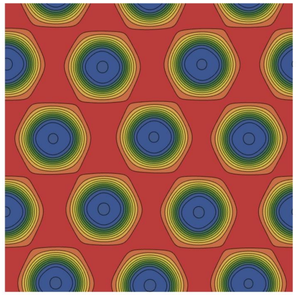
CP^2 *Skyrmion crystal*



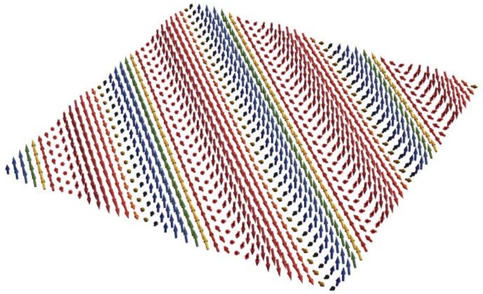
CP^2 *Skyrmionium crystal*



CP^2 *Double-Skyrmion crystal*



CP^2 *Helix*



- ✓ They possess *non-trivial dipole and quadrupole moment* structures, unlike the standard magnetic Skyrmions.

Thank you for your attention!