

# 音響ブラックホールにおける Hawking輻射と重力波発生過程

高橋淳一, 田島裕之<sup>A</sup>, 仲野英司, 飯田圭

高知大理工, 東大理<sup>A</sup>

基研研究会 「熱場の量子論とその応用」

2022年9月20日

# Bose-Einstein凝縮系と宇宙の類似性

凝縮体の線形揺らぎが従う方程式は曲がった時空の場の方程式と対応する

M. Novello, M. Visser, and G. Volovik,  
*Artificial black holes* (World Scientific, 2002)

Time dependent Gross-Pitaevskii equation

$$i\hbar \frac{\partial}{\partial t} \psi(t, x) = \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) + g|\psi(t, x)|^2 \right) \psi(t, x)$$

$$\Downarrow \psi = \sqrt{\rho} e^{i\theta}$$

Hydrodynamic equation (Quantum Hamilton-Jacobi equation)

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \rho}{\sqrt{\rho}} + \partial_t \theta + \frac{(\partial_x \theta)^2}{2m} + V + g\rho = 0$$

$\simeq 0$

$$\Downarrow \begin{aligned} \rho &= \rho_0 + \delta\rho \\ \theta &= \theta_0 + \delta\theta \end{aligned}$$

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left( \sqrt{-g} g^{\mu\nu} \frac{\partial \delta\theta}{\partial x^\nu} \right) = 0$$

where

$$x^\mu = (t, x)$$

$$g_{\mu\nu} = c_s \begin{pmatrix} c_s^2 - v_0^2 & v_0 \\ v_0 & -1 \end{pmatrix}$$

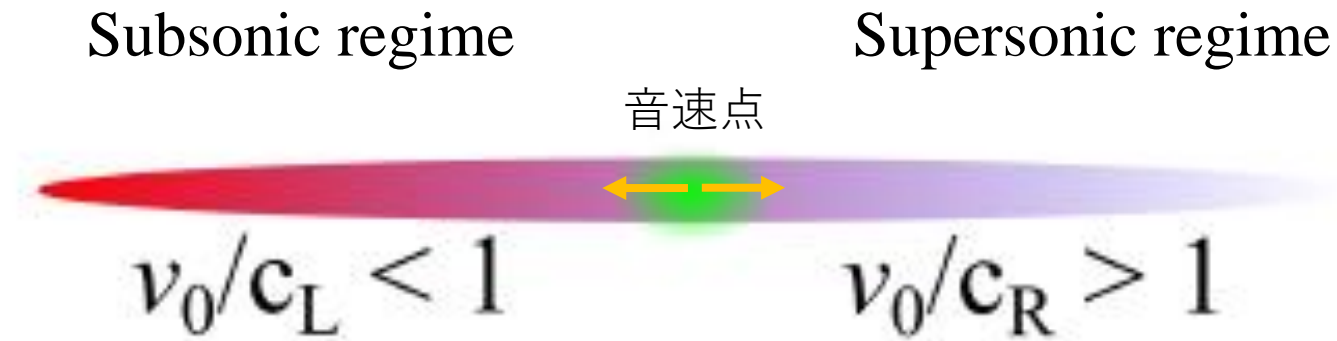
$$v_0 = \frac{\hbar}{m} \frac{\partial \theta_0}{\partial x}, \quad c_s = \sqrt{\frac{g\rho_0}{m}}$$

# 流体を用いた音響ブラックホール

流体を用いてブラックホール類似物を作ることができる

M. Novello, M. Visser, and G. Volovik,  
*Artificial black holes* (World Scientific, 2002)

遷音速流



$c_{L/R}$  : speed of sound  
 $v_0$  : stationary flow

# Bose-Einstein凝縮系による音響Hawking輻射

A. Recati, N. Pavloff, and I. Carusotto, Phys. Rev. A **80**, 043603 (2009)

TDGP eq. を解く

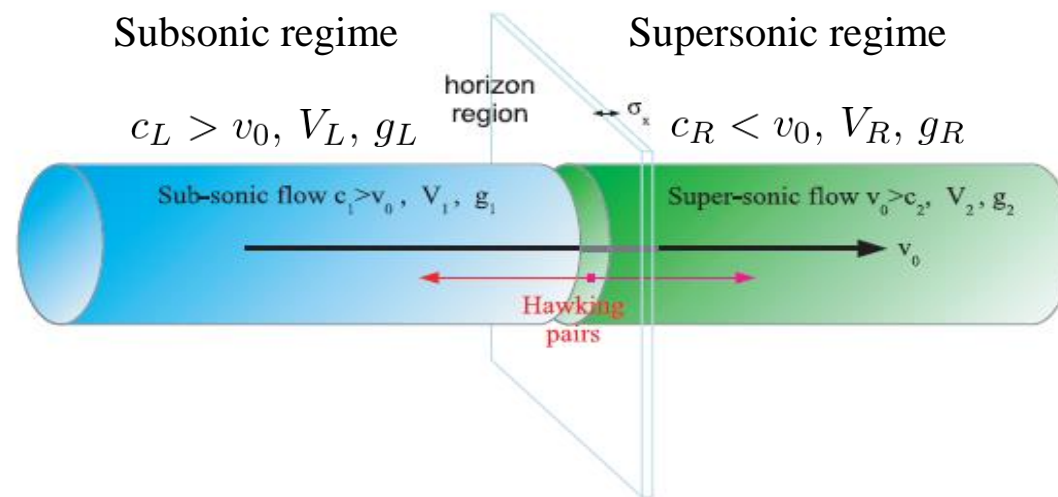
$$i\hbar \frac{\partial}{\partial t} \psi(t, x) = \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) + g(x)|\psi(t, x)|^2 \right) \psi(t, x)$$

with

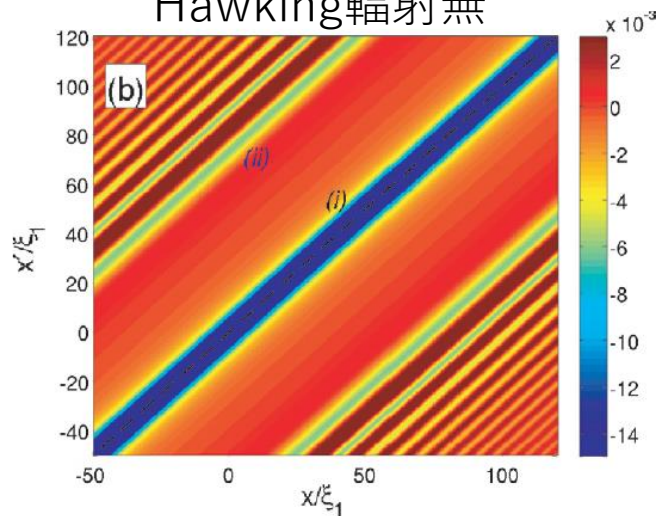
$$V(x) = V_L \theta(-x) + V_R \theta(x)$$

$$g(x) = g_L \theta(-x) + g_R \theta(x)$$

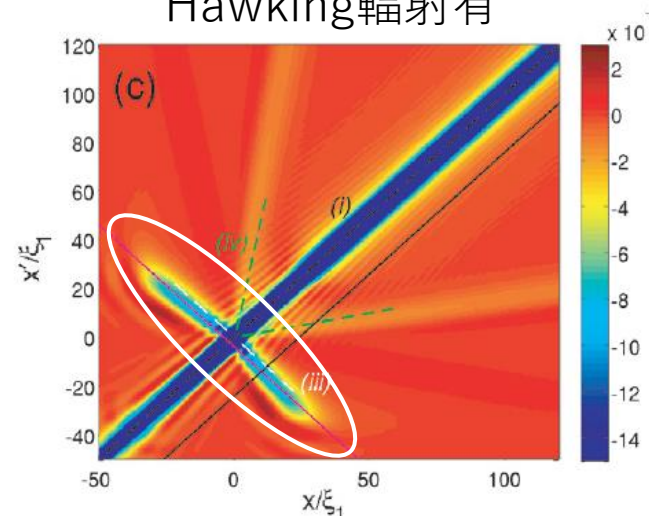
density-density Correlation function



Hawking輻射無



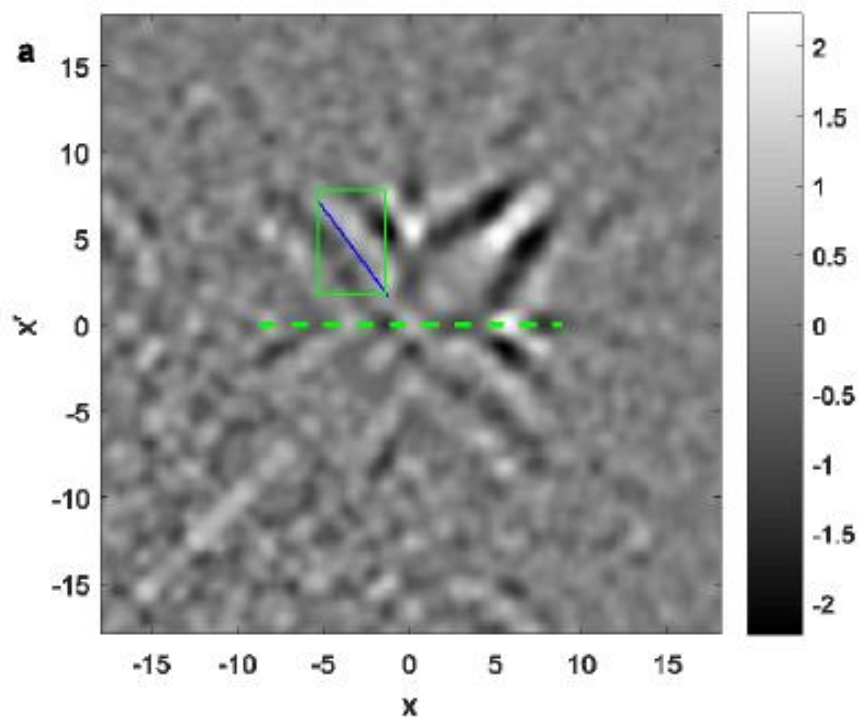
Hawking輻射有



# 冷却原子系によるHawking輻射の観測

実験

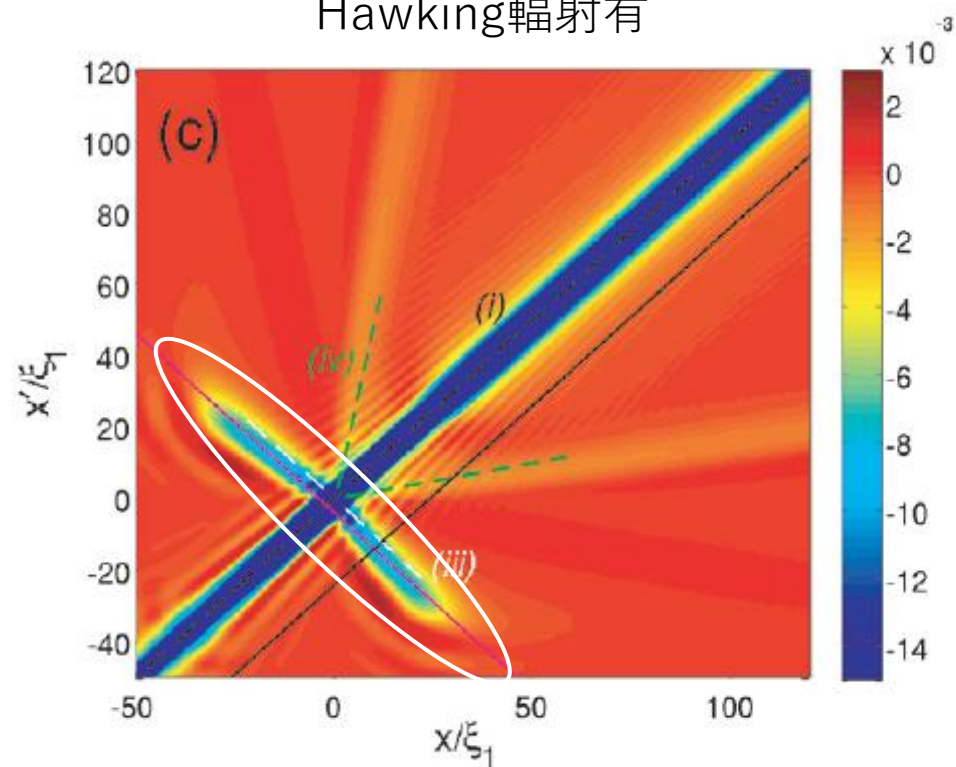
Hawking輻射有



J. Steinhauer, *Nature Physics*, **12**, 959 (2016)

理論

Hawking輻射有

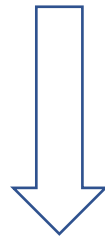


I. Carusotto, *et al.*, *New Journal of Physics* **10**, 103001 (2008)

# Bogoliubov理論による音響Hawking輻射の理解

TDGP equation

$$i\hbar \frac{\partial}{\partial t} \psi(t, x) = \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) + g(x)|\psi(t, x)|^2 \right) \psi(t, x)$$



fluctuation

$$\psi(x, t) = f(x, t) e^{i \frac{mv_0}{\hbar} x} := f(x, t) e^{ik_0 x}$$

$$f(x, t) = \sqrt{n_0} + \sum_k \left[ \alpha_k(t) u_k(x) e^{i(kx - \omega_k t)} + \alpha_k^*(t) v_k^*(x) e^{-i(kx - \omega_k^* t)} \right]$$

Bogoliubov equations

$$\begin{pmatrix} \mathcal{L}_k & \mathcal{M}_k \\ -\mathcal{M}_k & -\mathcal{L}_{-k} \end{pmatrix} \begin{pmatrix} u_k \\ v_k \end{pmatrix} = \hbar \omega_k \begin{pmatrix} u_k \\ v_k \end{pmatrix}$$

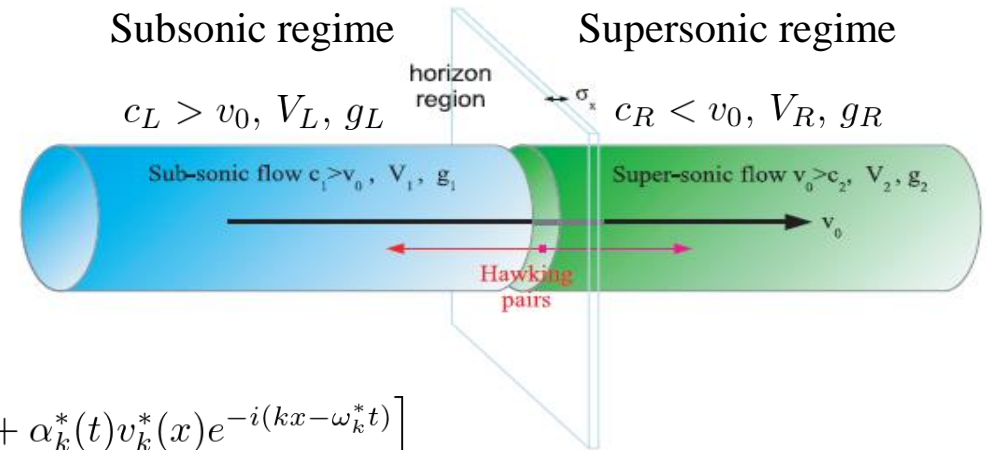
where

$$\mathcal{L}_k = \frac{\hbar^2}{2m} (k + k_0)^2 + V_i(x) + 2g_i(x)n_0$$

$$\mathcal{M}_k = g_i(x)n_0$$

Bogoliubov spectrum

$$\hbar \omega^\pm(k) = v_0 k \pm c_{L/R} k \sqrt{g_{L/R} n_0 + \frac{1}{4} (k \xi_{L/R})^2}$$

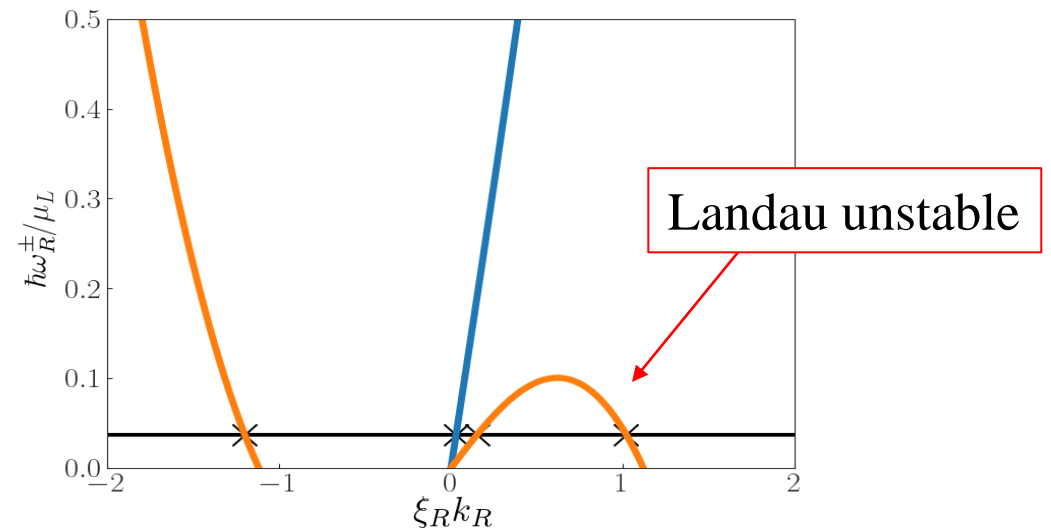
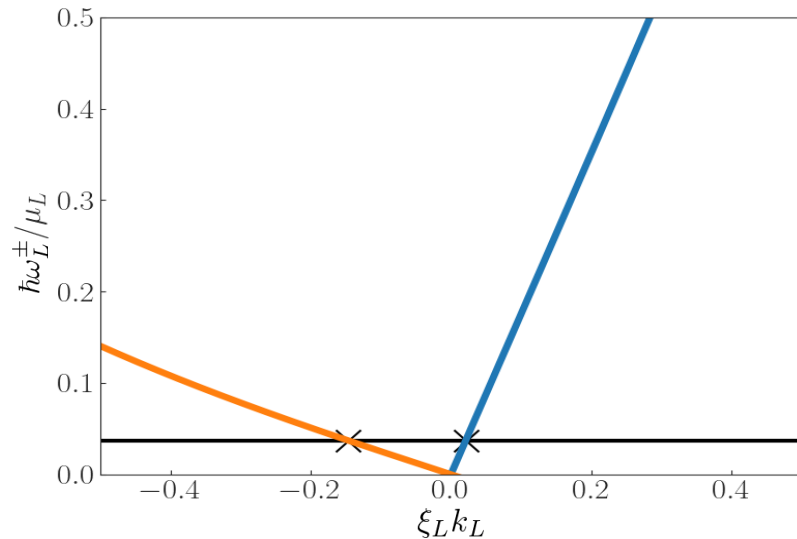
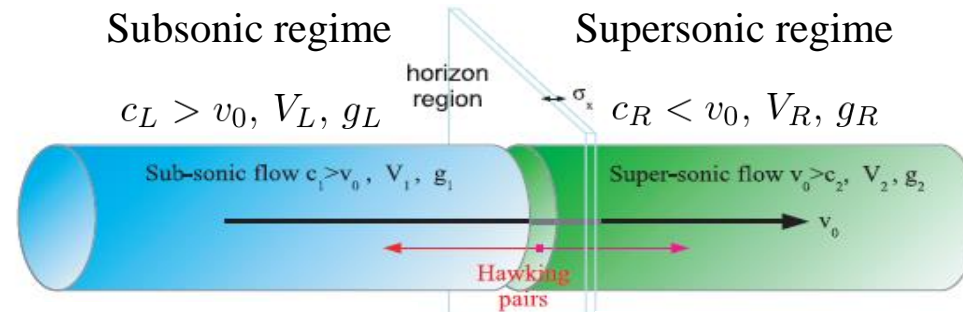


# Bogoliubov理論による音響Hawking輻射の理解

A. Recati, N. Pavloff, and I. Carusotto, Phys. Rev. A **80**, 043603 (2009)

Bogoliubov spectrum

$$\hbar\omega^\pm(k) = v_0 k \pm c_{L/R} k \sqrt{g_{L/R} n_0 + \frac{1}{4} (k \xi_{L/R})^2}$$

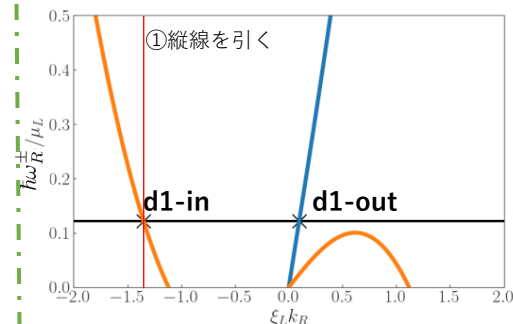
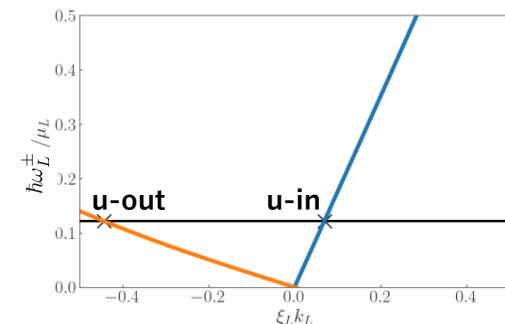
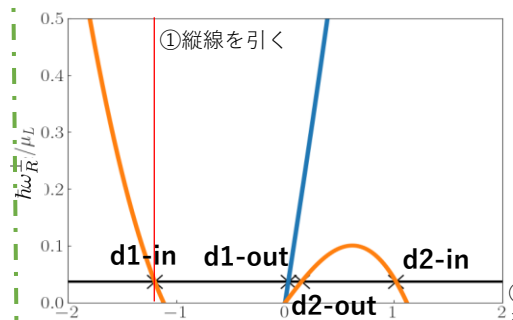
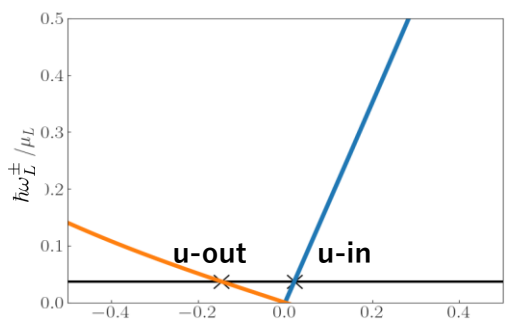
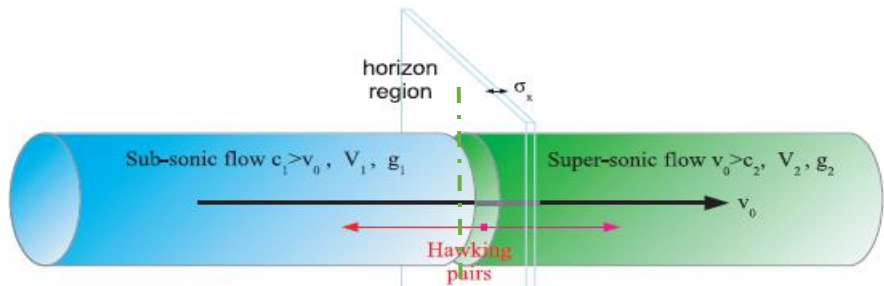




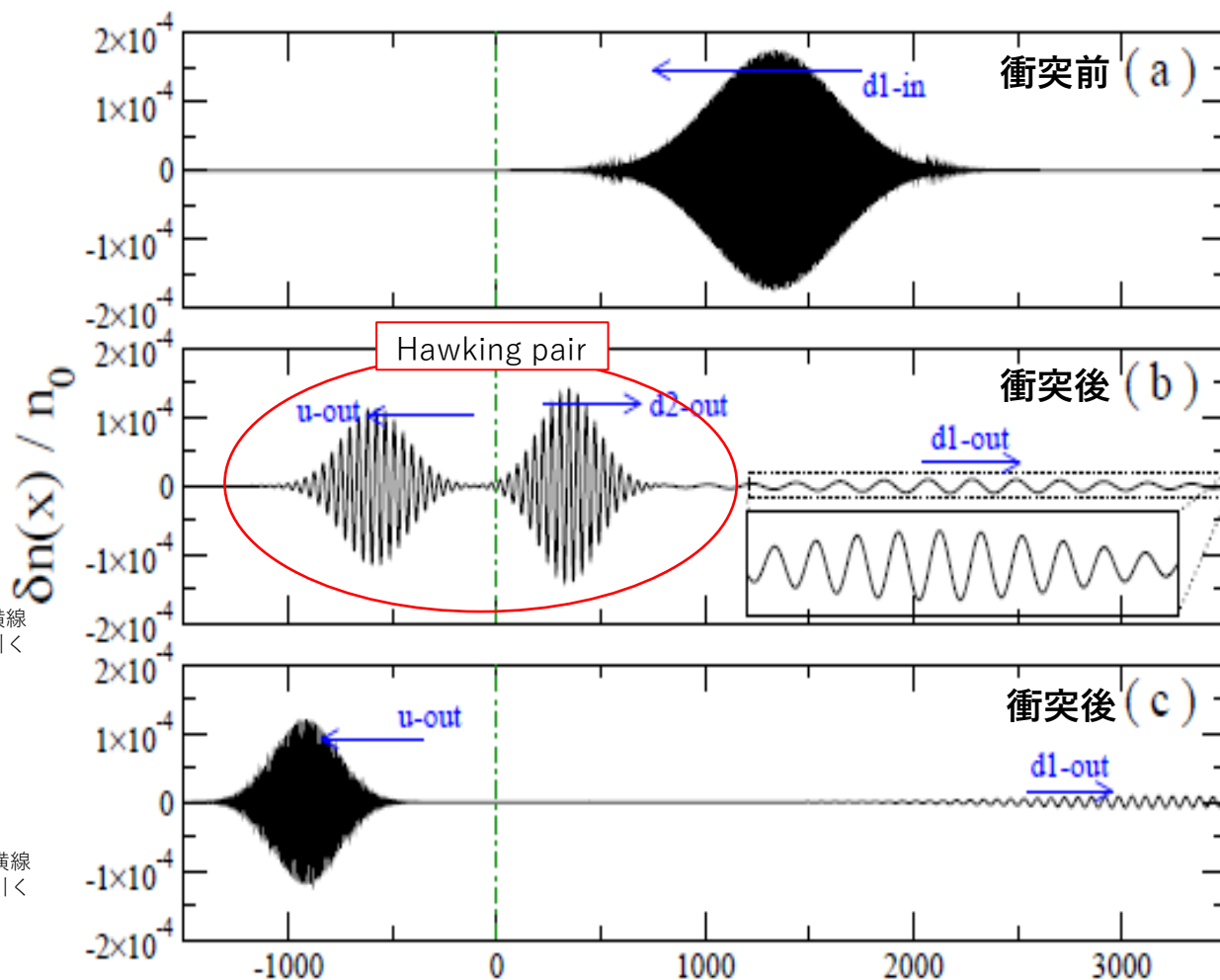
# Bogoliubov理論による音響Hawking輻射の理解

A. Recati, N. Pavloff, and I. Carusotto, Phys. Rev. A **80**, 043603 (2009)

condition



凝縮体の揺らぎダイナミクス





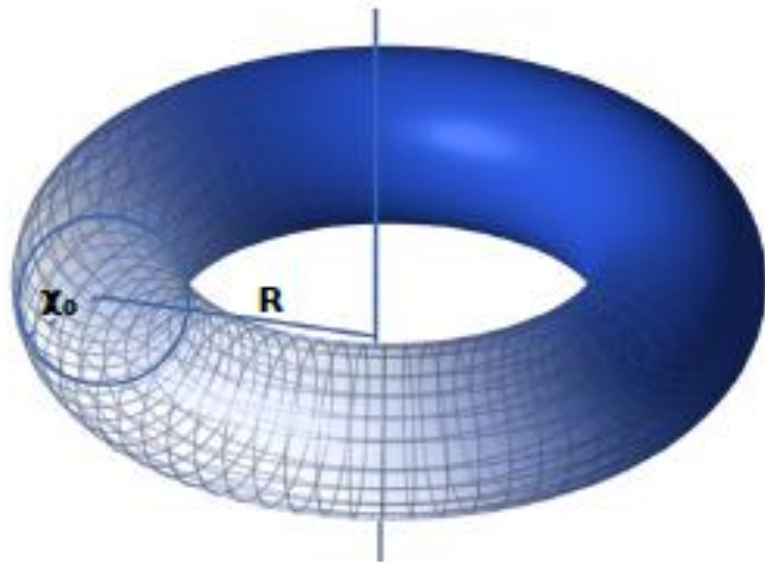
# [背景] 冷却原子系と重力波

## 重力波に対する音響類似物の提案

[S. Datta, Phys. Rev. D 98, 064049 (2018)]

- 2次摂動まで考慮
- 非線形計量はMinkowski時空での重力波を記述する計量と同型

例：タイトなリング型トラップ中の超流体



- 擬一次元GP方程式

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2mR^2} \frac{\partial^2 \Psi}{\partial \varphi^2} + U|\Psi|^2\Psi$$

- 摂動  $\Psi = \Psi_0 + \Psi_1(t, \varphi)$

- 計量

$$g_{\mu\nu}(t, R\varphi) = \eta_{\mu\nu} + h_{\mu\nu}(t, R\varphi) := \begin{pmatrix} -c_s^2 \left(1 + \frac{3}{2} \frac{\rho_1}{\rho_0}\right) & -(v_\varphi)_1 \\ -(v_\varphi)_1 & \left(1 + \frac{1}{2} \frac{\rho_1}{\rho_0}\right) \end{pmatrix}$$

- 非線形計量の満たす波動方程式

$$\left(-\frac{1}{c_s^2} \partial_t^2 + \partial_{R\varphi}^2\right) h_{\mu\nu}(t, R\varphi) = 0$$

# [本研究] 今回の課題

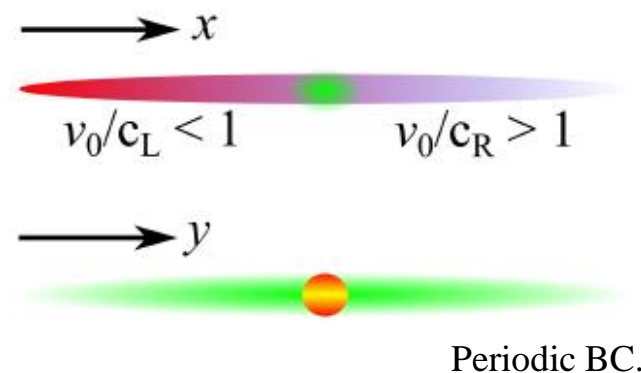
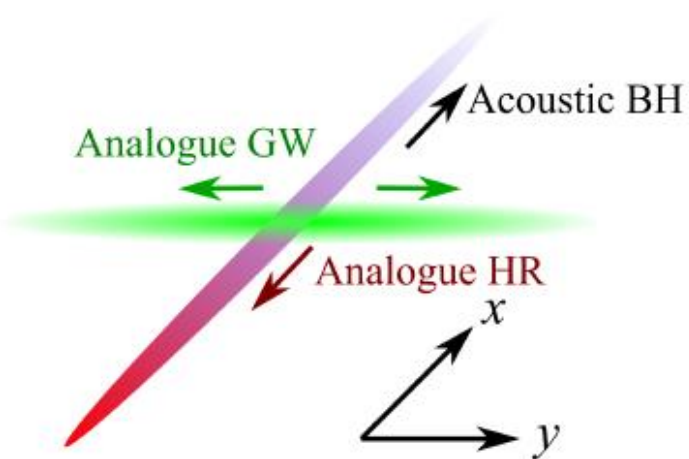
重い物質とブラックホール衝突時にはHawking輻射と重力波の両方が発生する

⇒ 冷却原子で理解を深めることはできるか？

簡単のため 1D × 1D を想定

$$i\hbar \frac{\partial}{\partial t} \psi_1(t, x) = \left( -\frac{\hbar^2}{2m_1} \frac{\partial^2}{\partial x^2} + V_1(x) + g_{11}(x)|\psi_1(t, x)|^2 + g_{12}|\psi_2(t, 0)|^2 \right) \psi_1(t, x)$$

$$i\hbar \frac{\partial}{\partial t} \psi_2(t, y) = \left( -\frac{\hbar^2}{2m_2} \frac{\partial^2}{\partial y^2} + V_2(y) + g_{22}(y)|\psi_2(t, y)|^2 + g_{12}|\psi_1(t, 0)|^2 \right) \psi_2(t, y)$$



$$V_1(x) = V_L \theta(-x) + V_R \theta(x)$$

$$g_{11}(x) = g_L \theta(-x) + g_R \theta(x)$$

$$V_2(y) = 0$$

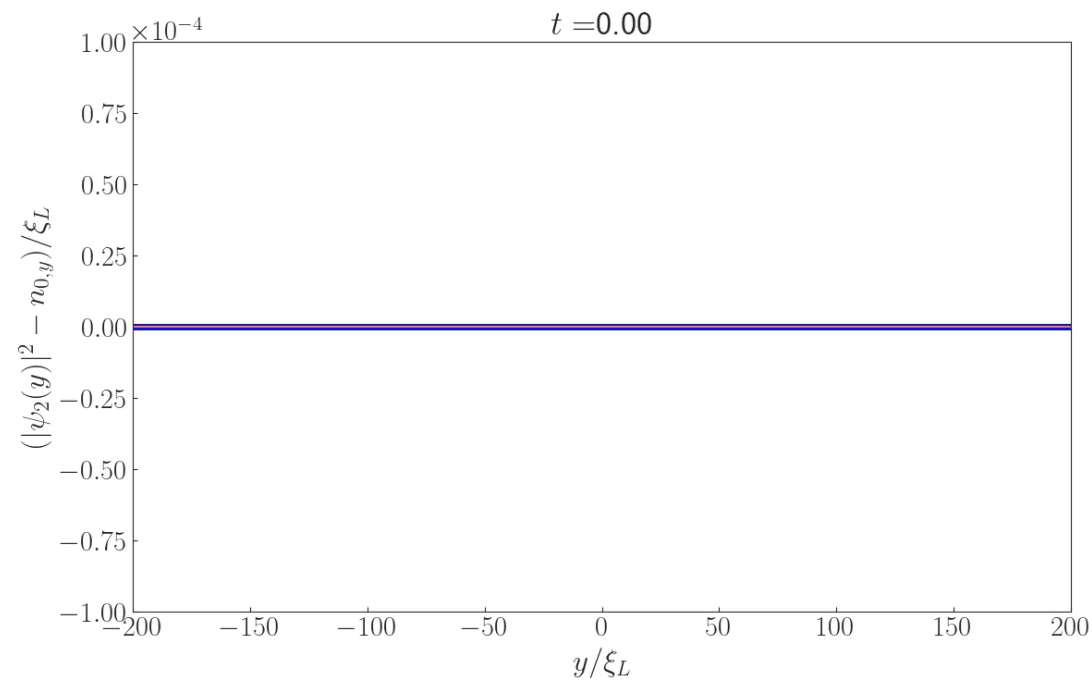
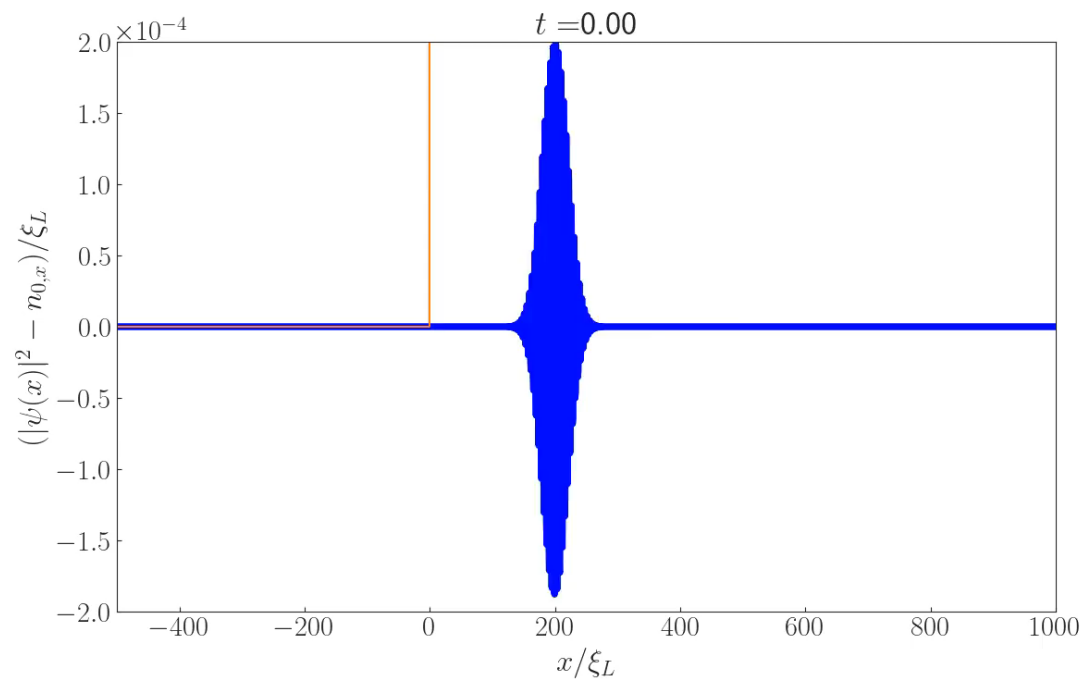
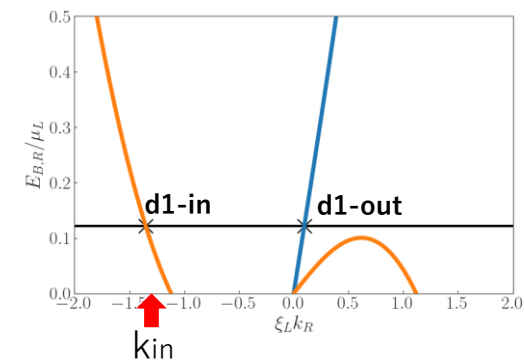
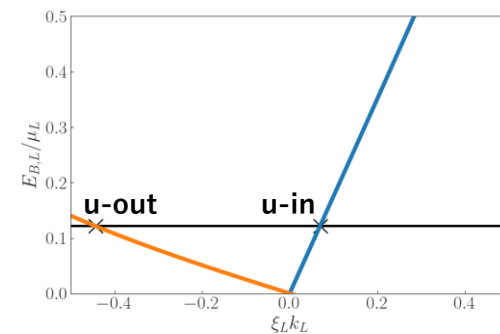
$$g_{22}(y) = g_{22}$$

$$m_2 = m_1$$

# [結果] Hawking 輻射なし

初期条件 :  $\psi(x) = \psi_0(x)(1 + \delta\psi(x))$

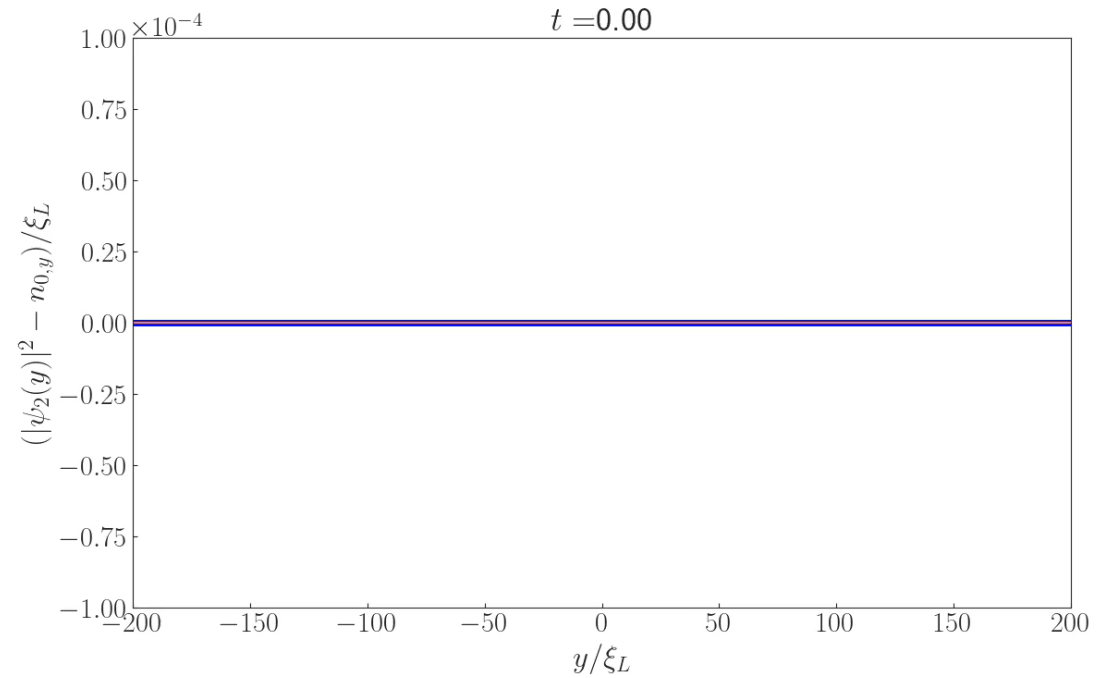
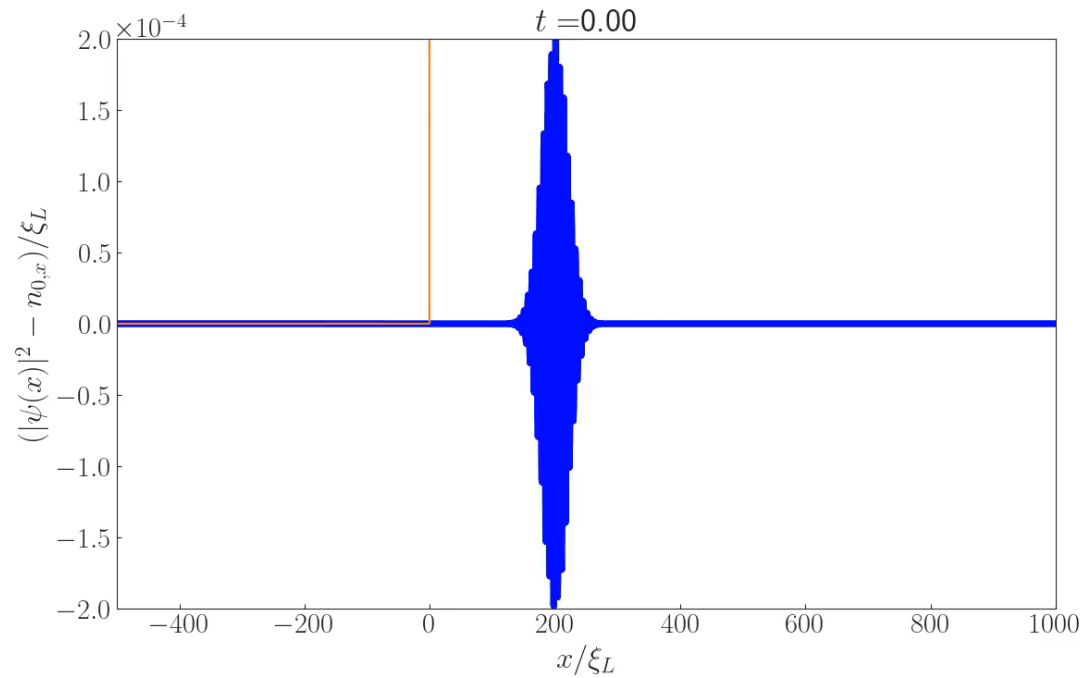
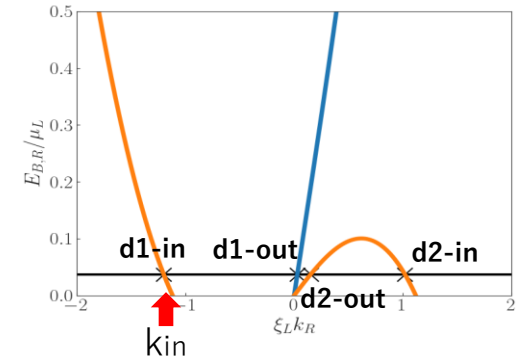
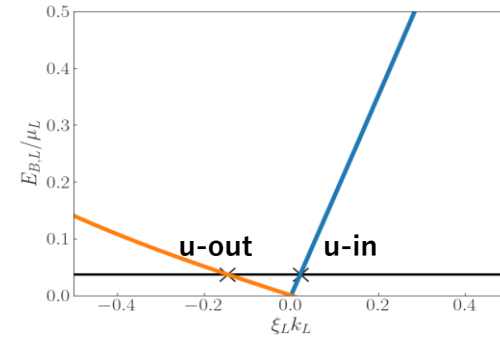
$$\delta\psi(x) = Ae^{-\frac{(x-x_0)^2}{2B}} + ik_{in}(x-x_0)$$



# [結果] Hawking 輻射あり

初期条件 :  $\psi(x) = \psi_0(x)(1 + \delta\psi(x))$

$$\delta\psi(x) = Ae^{-\frac{(x-x_0)^2}{2B}} + ik_{in}(x-x_0)$$



# まとめと今後の展望

## ●まとめ

- 冷却原子系による重い物質とブラックホール衝突のtoyモデルを提案
- 音響Hawking輻射と音響重力波の定性的な関係性を調べた
  - ✓ 音響BHのHawking輻射 = d2-outの波束
  - ✓ 音響Hawking輻射の存在により音響重力波の発生時間長い

## ●今後の展望

- Hawking温度の計算
- 重力波類似物の伝搬をになうBECの条件改善