冷却原子気体における交流スピン伝導率

Yuta Sekino, Hiroyuki Tajima, & Shun Uchino, arXiv:2103.02418 (accepted by PRResearch)
Hiroyuki Tajima, Yuta Sekino, & Shun Uchino, PRB 105, 064508 (2022)

関野 裕太　(理研CPR, 理研iTHEMS)

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熱場の量子論2022 @京大基研　20th Sep, 2022

Outline of this talk

1. Introduction
2. Optical spin conductivity
    - Proposal of measurement
3. Theoretical studies
    - Formalism
    - Fermi superfluids
    - Tomonaga-Luttinger liquid
4. Summary

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   - Formalism
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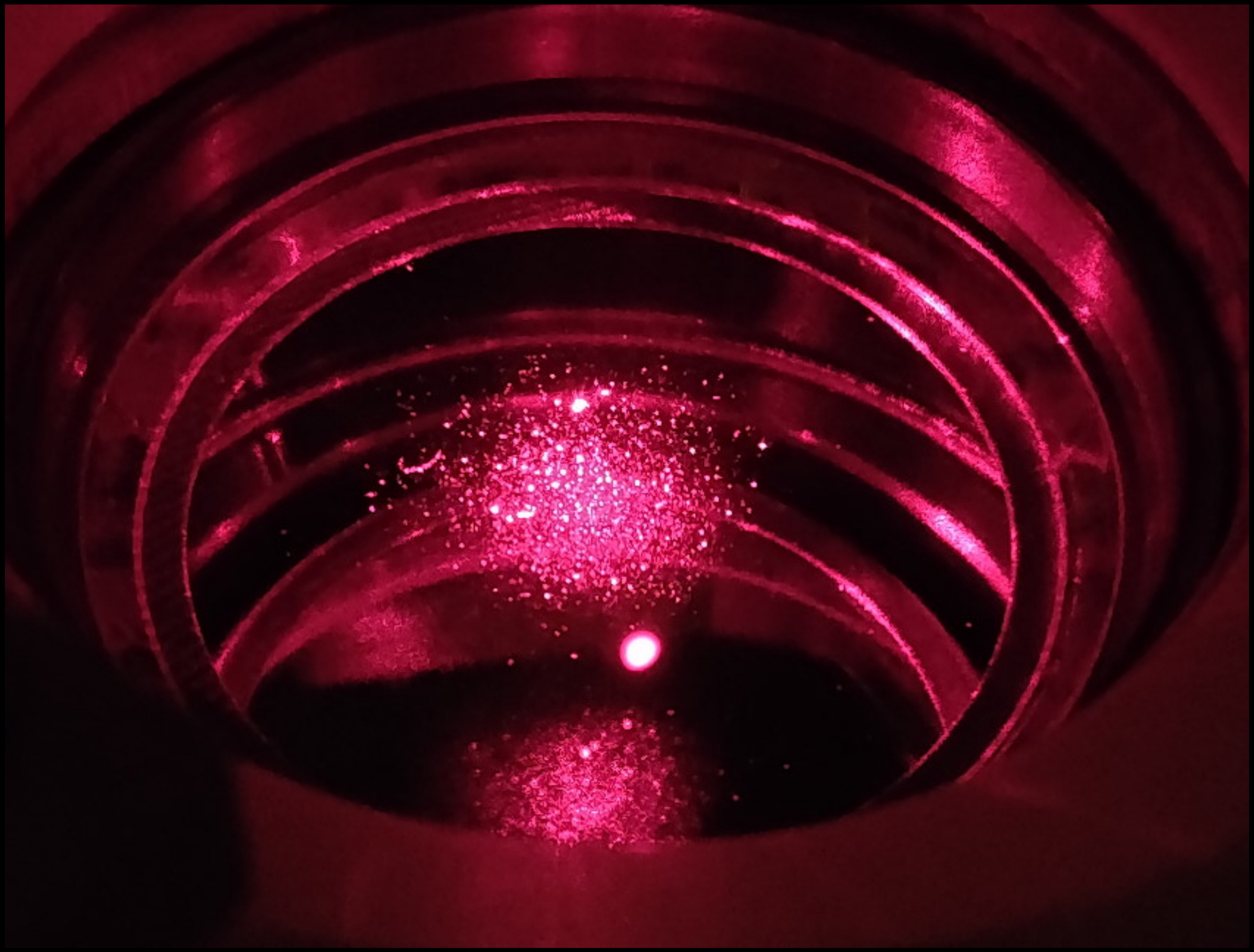
   - Tomonaga-Luttinger liquid
4. Summary

Ultracold atoms

- Very pure & highly controllable atomic gases
- Ideal research platform for quantum many-body phenomena



From Schauss Group, University of Virginia (https://ultracold.phys.virginia.edu/public_html/)

High controllability

1. Quantum statistics & spin degrees of freedom

Bose atom: ^{7}Li, ^{23}Na, ^{39}K,⋯

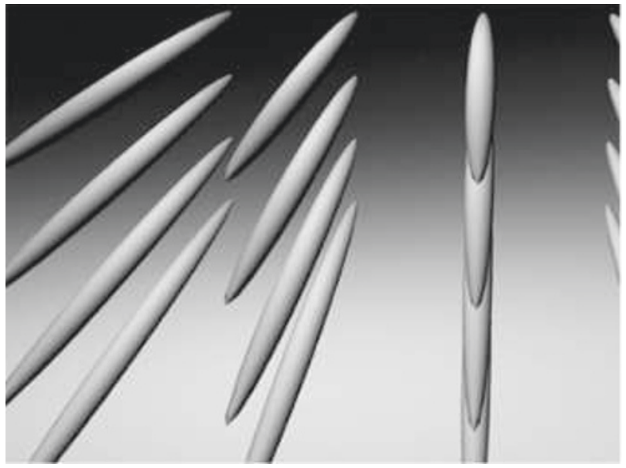
Fermi atom: ^{6}Li, ^{40}K,⋯

Hyperfine states ⟷ Spin

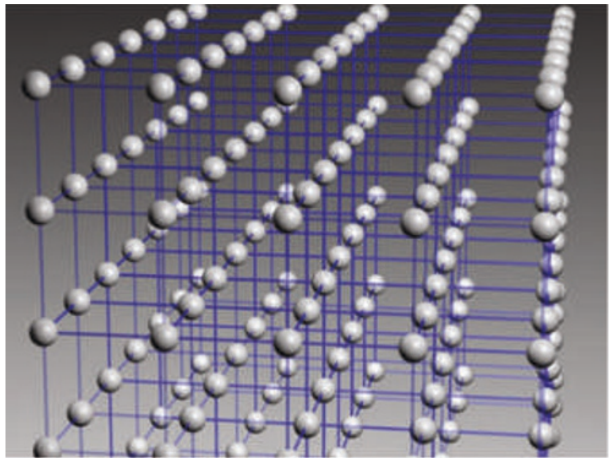
2. Spatial geometry of gas

3D, 2D, 1D, Lattice, ⋯

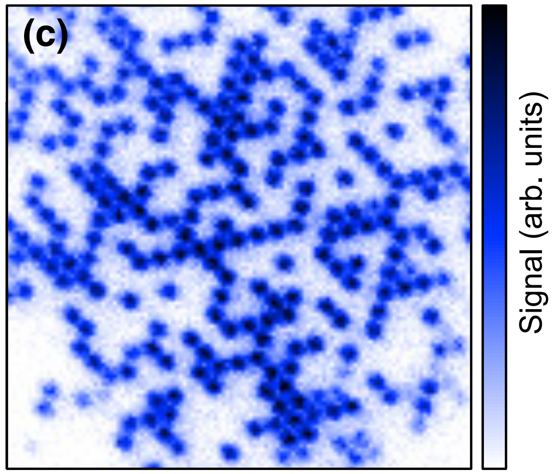
1D



Cubic lattice



Triangular lattice



Bloch, Nat. Phys. (2005)

Yang et al.(Virginia), PRX Quantum (2021)

High controllability

1. Quantum statistics & spin degrees of freedom

   Bose atom: ^{7}Li, ^{23}Na, ^{39}K,⋯

   Fermi atom: ^{6}Li, ^{40}K,⋯

   Hyperfine states ⟷ Spin

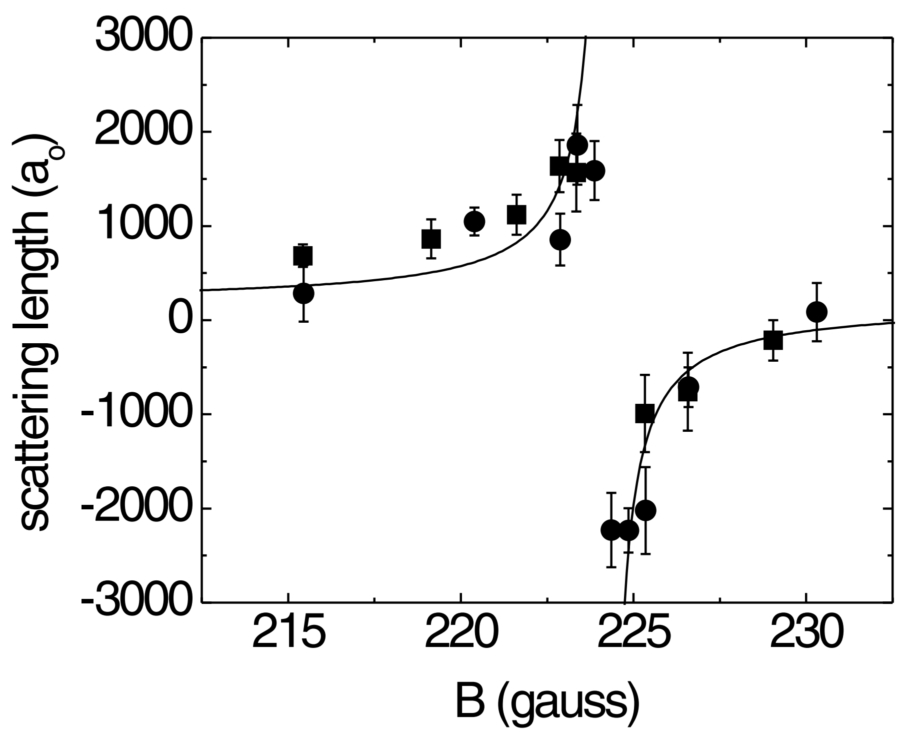
2. Spatial geometry of gas

   3D, 2D, 1D, Lattice, ⋯

3. Interaction between atoms

   Feshbach resonance, Lattice depth, ⋯

^{40}K



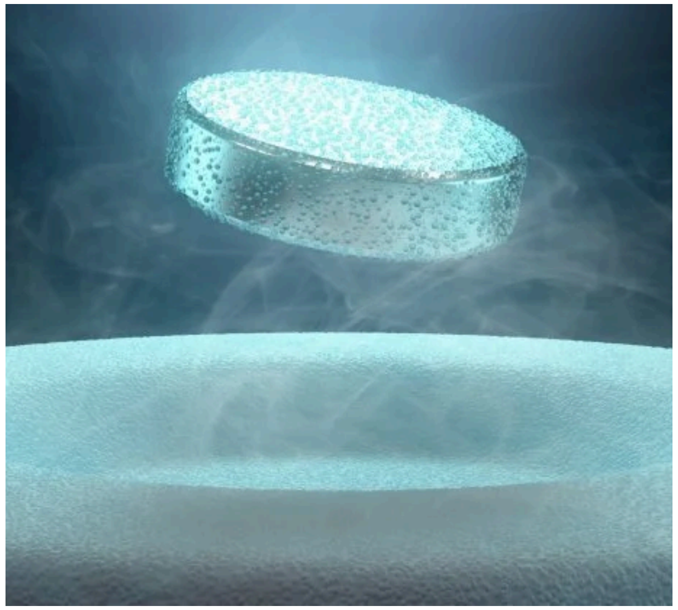
Regal & Jin (JILA), PRL(2003)

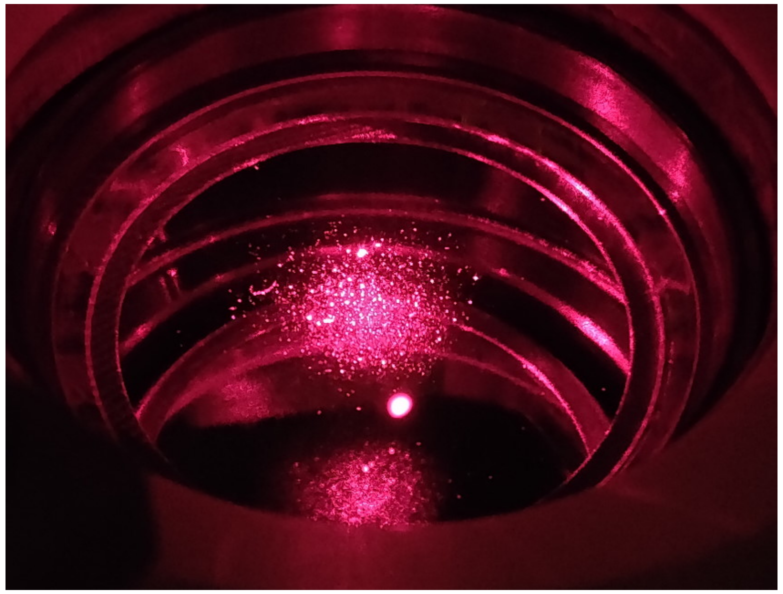
Major research directions

Ideal platform to study quantum many-body phenomena

1. Novel quantum phenomena
2. Quantum computation
3. Analog quantum simulation:
   cold-atomic systems equivalent/similar to other interesting systems

Superconductors ⟷ Fermi atoms in superfluid phase ⟷ Neutron superfluid in neutron stars

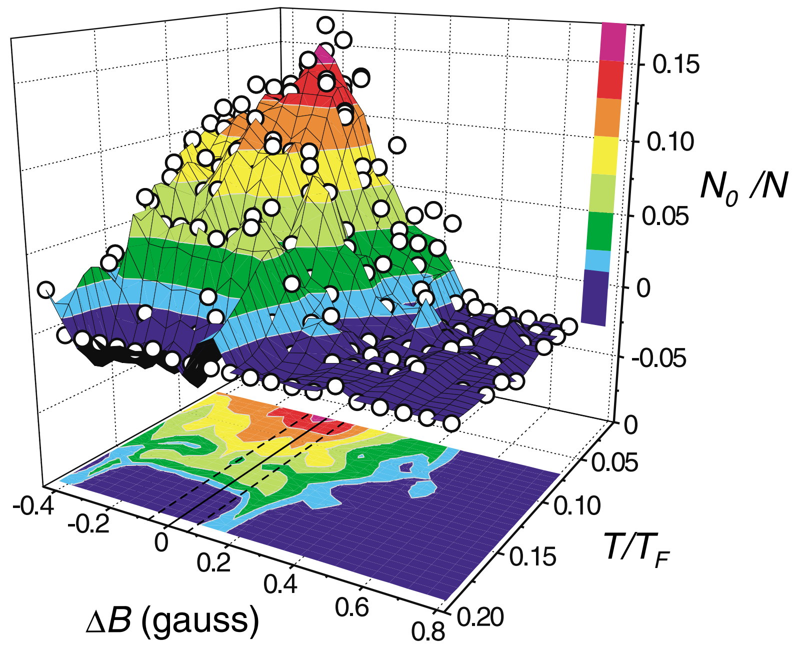




https://www.riken.jp/en/news_pubs/research_news/rr/20200207_2/index.html

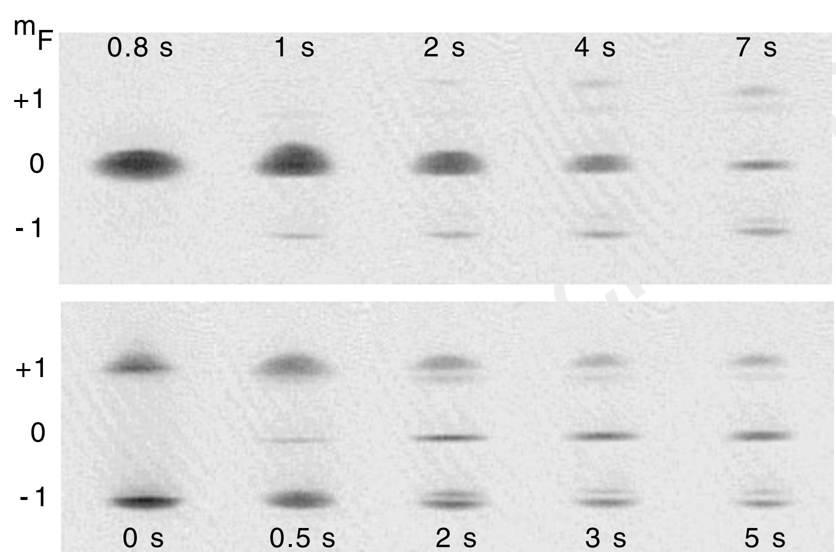
Various many-body states with spin

Fermi superfluid (BCS-BEC crossover)



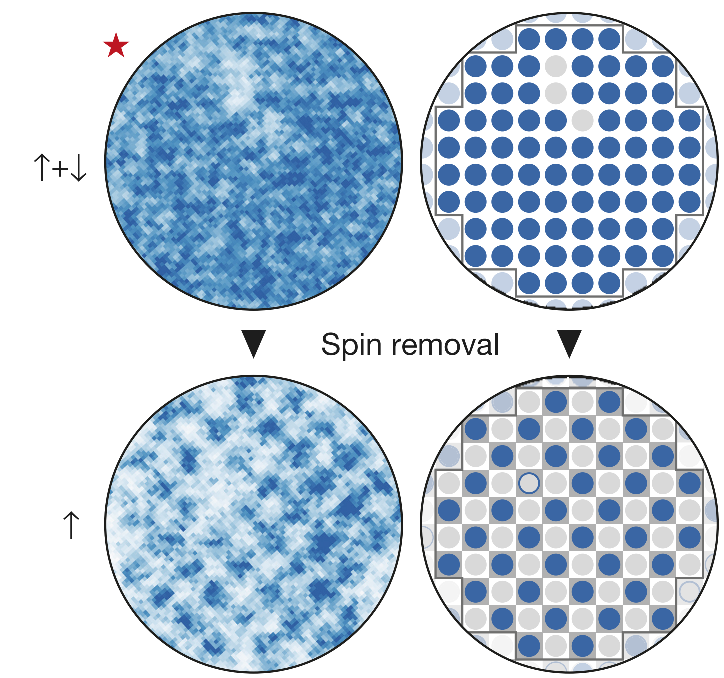
Regal et al., (JILA) PRL (2004)

Spinor Bose-Einstein Condensate (BEC)



Stenger et al., (MIT) Nature (1998)

Heisenberg antiferromagnet



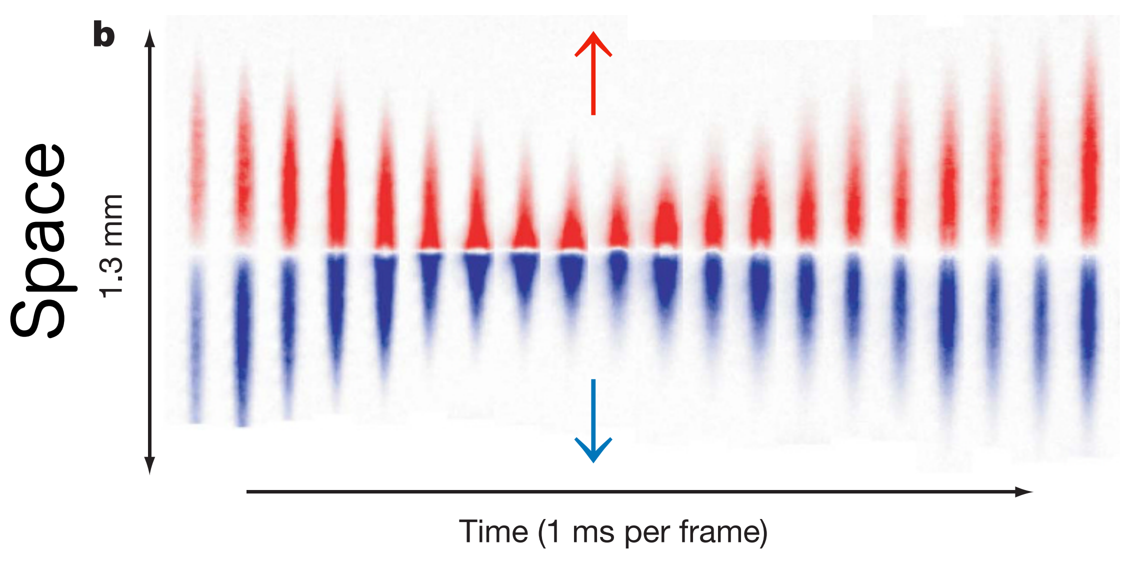
Mazurenko et al.,(Harvard) Nature (2017)

Spin dynamics with cold atoms

- Ideal experimental grounds to study spin dynamics

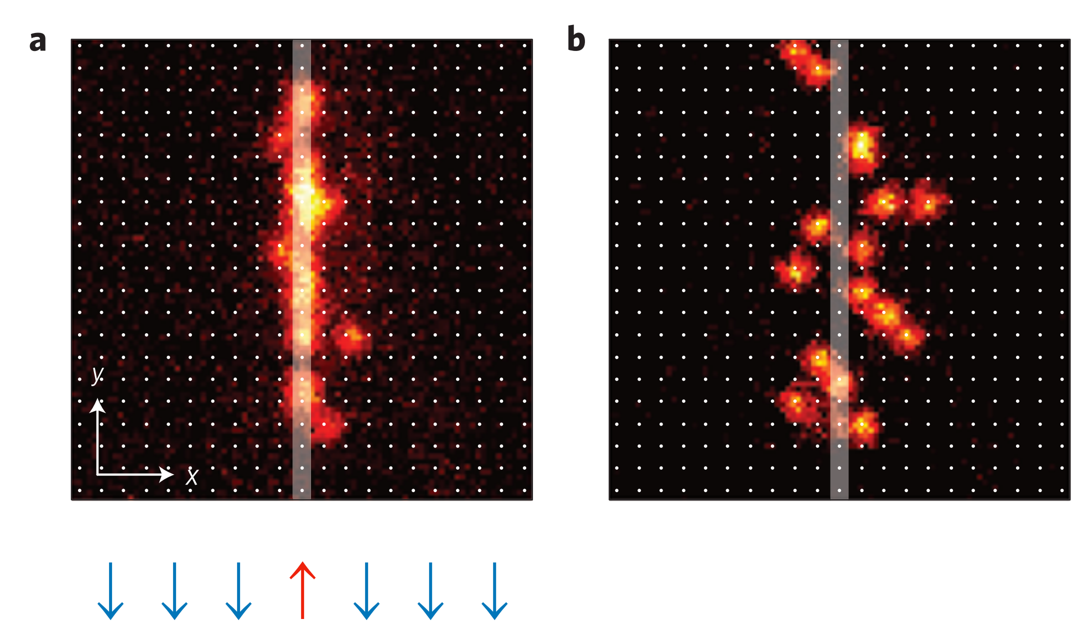
  Spin-resolved manipulation & detection

1. Spin diffusion w/o lattice



Sommer et al.,(MIT) Nature (2011)

2. Spin impurity on lattice



Fukuhara et al.,(MPI) Nat. Phys. (2013)

Spin dynamics with cold atoms

- Ideal experimental grounds to study spin dynamics

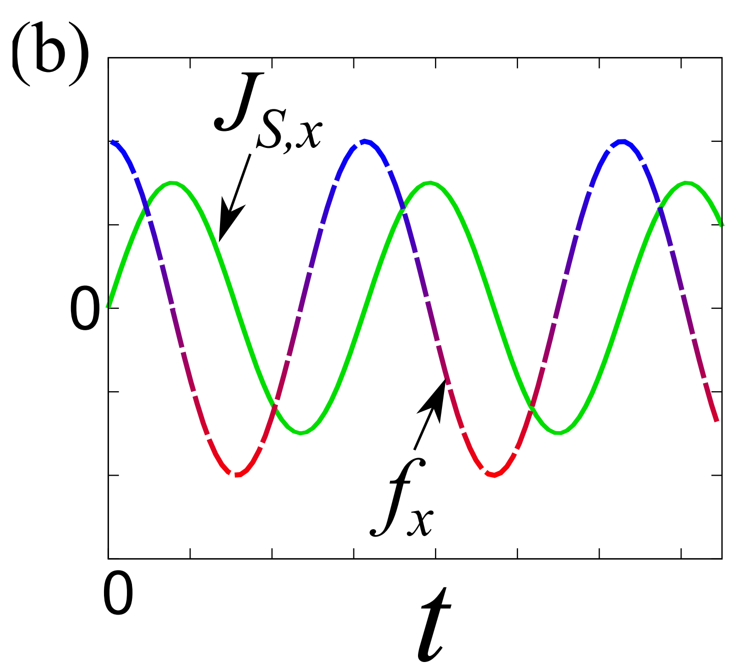
  Spin-resolved manipulation & detection

1. Spin diffusion w/o lattice

2. Spin impurity on lattice

3. AC spin transport (Our proposal)

YS, Tajima, & Uchino, accepted by PRResearch (2022)



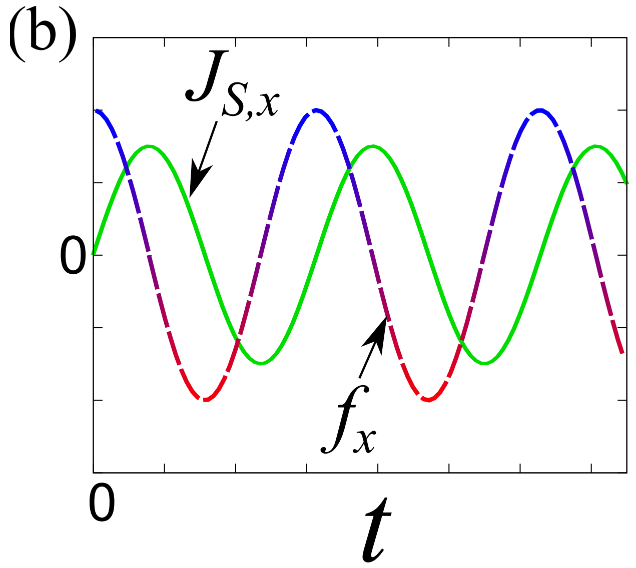
Driven by magnetic or optical fields

Today's topic

- Optical (AC) spin conductivity

$$\sigma^{(S)}_{\alpha,\beta}(\omega) = \tilde{J}_{S,\alpha}(\omega)/\tilde{f}_\beta(\omega)$$

$(\alpha, \beta = x, y, z)$



Measurable in cold-atom experiments

YS, Tajima, & Uchino, accepted by PRResearch (2022)

- Significance of $\sigma^{(S)}_{\alpha\beta}(\omega)$

YS, Tajima, & Uchino, accepted by PRResearch (2022)
Tajima, YS, & Uchino, PRB (2022)

1. Elusive in solid-state systems
2. Powerful probe for quantum many-body states

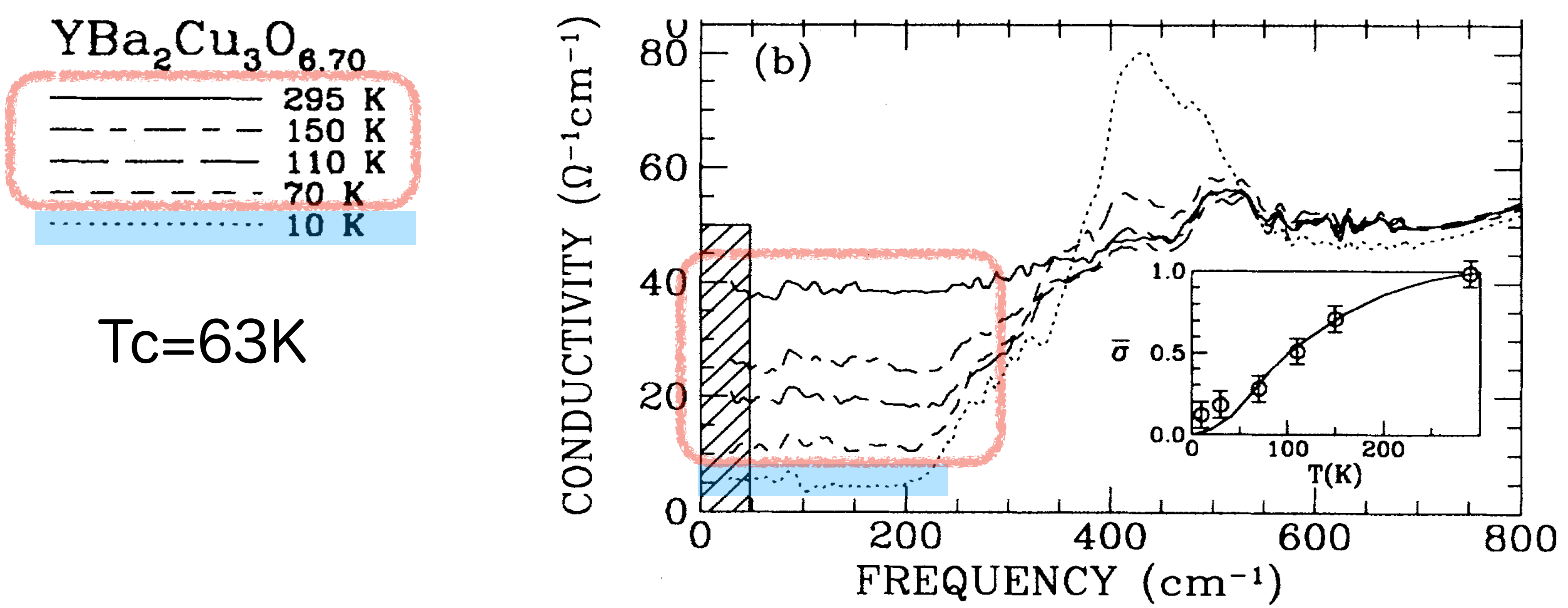
   BCS-BEC crossover, Tomonaga-Luttinger liquid(TLL), Spinor BEC,⋯
3. Widely applicable probe for clean systems

2. Optical conductivity for solids

- Powerful probe for exotic electron systems

Superconductor, Pseudogap phase, Non-Fermi liquid, Dirac fermions, …

Homes et al., PRL (1993)



Tc=63K

Optical spin conductivity would also be a useful probe for nontrivial spin dynamics

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1. Introduction
2. Optical spin conductivity
    - Proposal of measurement
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    - Formalism
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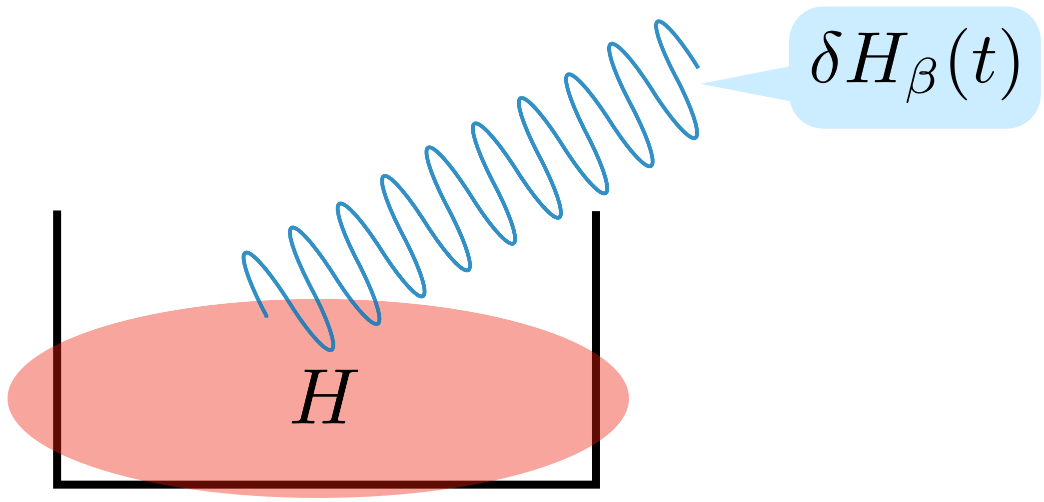
Measurement scheme: set up

YS, Tajima, & Uchino, arXiv:2103.02418

- Total Hamiltonian:

$$H(t) = H + \delta H_\beta(t)$$



Cold atoms with spin (at least S_z) conserved

Spin: S=1/2, 1, 3/2, ⋯

Zeeman field, synthetic gauge field, ⋯

trap & lattice potentials, ⋯

BCS-BEC crossover, Spinor BEC, ferromagnets/antiferromagnets, ⋯

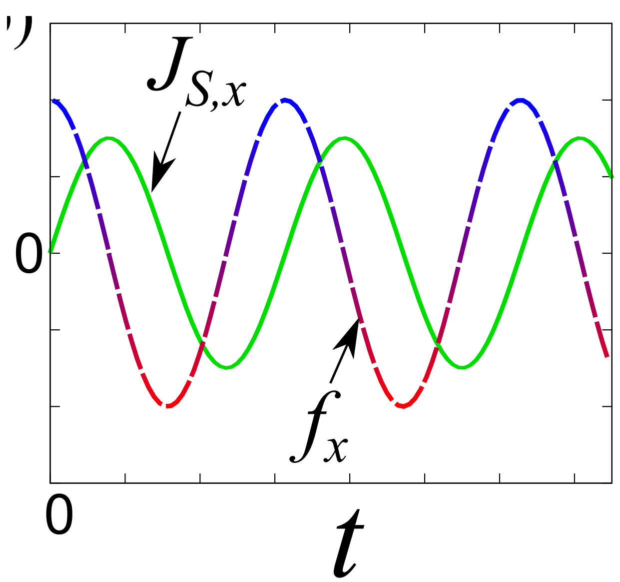
(Extension to spin-nonconserving & nonequilibrium systems is possible)

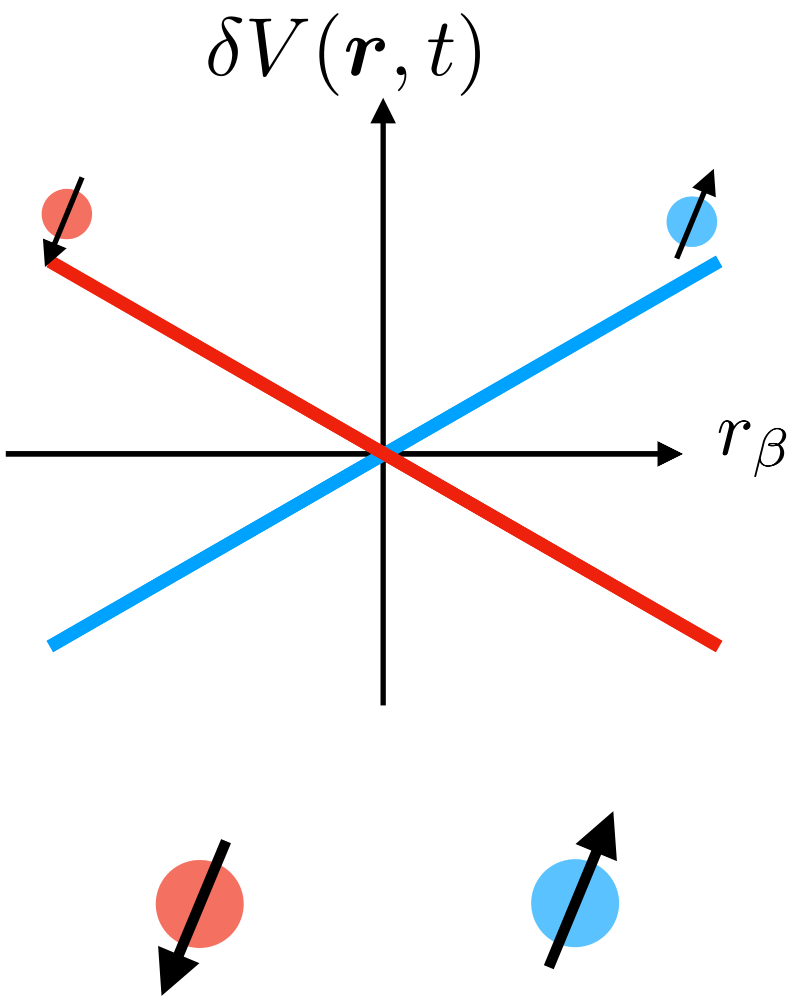
How to induce AC spin current

YS, Tajima, & Uchino, arXiv:2103.02418

- Time-dependent force coupled to spin density S_z

$$\delta H_\beta(t) = -\int d\boldsymbol{r}\, f_\beta(t) r_\beta S_z(\boldsymbol{r}), \qquad (\beta = x, y, z)$$





How to induce AC spin current

YS, Tajima, & Uchino, arXiv:2103.02418

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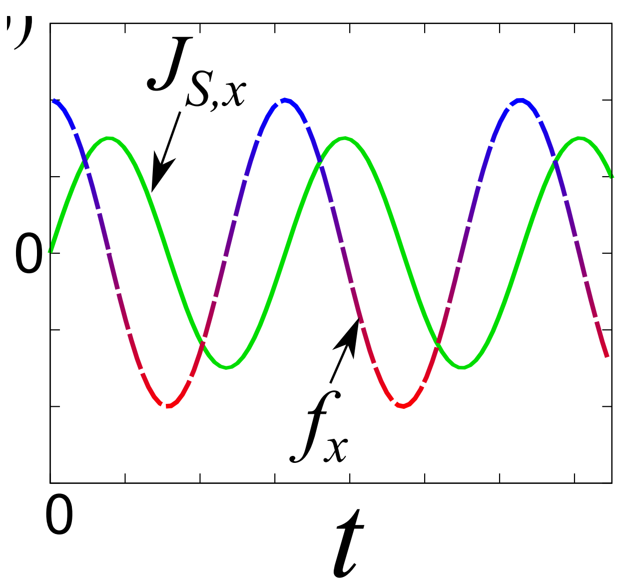


Single-frequency driving force toward $\beta = x, y, z$

$$f_\beta(t) = F_\beta \cos(\omega_0 t)$$

1. Magnetic field gradient
   Medley et al., (MIT) PRL (2011); Jotzu et al., (ETH) PRL (2015)

2. Optical Stern-Gerlach effect
   Taie et.al., (Kyoto) PRL (2010)

How to extract spin conductivity

YS, Tajima, & Uchino, arXiv:2103.02418

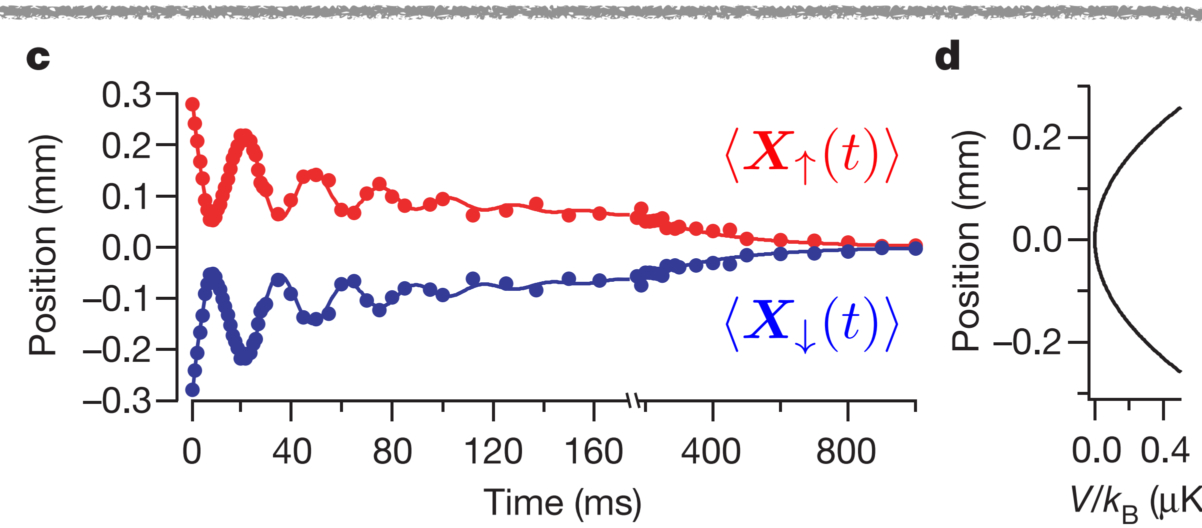
1. Spin current: $\langle \boldsymbol{J}_S(t)\rangle = \frac{d}{dt}\left\langle \int d\boldsymbol{r}\,\boldsymbol{r}S_z(\boldsymbol{r},t)\right\rangle \equiv \frac{d}{dt}\langle \boldsymbol{X}_S(t)\rangle$ (Spin conservation)
2. Spin conductivity: $\langle \tilde{J}_{S,\alpha}(\omega)\rangle = \sigma^{(S)}_{\alpha\beta}(\omega)\tilde{f}_\beta(\omega)$ (Ohm's law in frequency space)
3. Driving force: $f_\beta(t) = F_\beta \cos(\omega_0 t)$

$$\frac{\langle \delta X_{S,\alpha}(t)\rangle}{F_\beta} = -\frac{\mathrm{Im}\,\sigma^{(S)}_{\alpha\beta}(\omega_0)}{\omega_0}\cos(\omega_0 t) + \frac{\mathrm{Re}\,\sigma^{(S)}_{\alpha\beta}(\omega_0)}{\omega_0}\sin(\omega_0 t)$$

Measurement of $\langle X_{S,\alpha}(t)\rangle$ → $\sigma^{(S)}_{\alpha\beta}(\omega=\omega_0)$

e.g. Experiment on spin diffusion (w/o $f_\beta(t)$)

$$\langle \boldsymbol{X}_S(t)\rangle = \langle \boldsymbol{X}_\uparrow(t)\rangle - \langle \boldsymbol{X}_\downarrow(t)\rangle$$



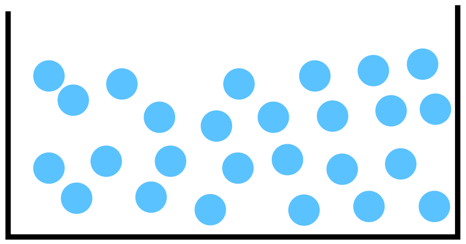
Sommer et al.,(MIT) Nature (2011)

Outline of this talk

Theoretical studies

▶ Our works on optical spin conductivity in homogeneous gases

YS, Tajima, & Uchino, arXiv:2103.02418
Tajima, YS, & Uchino, PRB (2022)



1. S=1/2 superfluid Fermi gas with spin gap
2. 1D p-wave Fermi superfluid with topological phase transition
3. Tomonaga-Luttinger liquid
4. S=1 polar BEC with gapped or gapless spin modes

General relations

Enss & Haussmann PRL (2012)   Enss, Euro. Phys. J. Special Topics (2013)

YS, Tajima, & Uchino, arXiv:2103.02418 $(\hbar = k_{\rm B} = 1)$

1. Kubo formula

$$\sigma^{(S)}_{\alpha\beta}(\omega) = \frac{i}{\omega^+}\left(\delta_{\alpha\beta}\sum_{s_z}\frac{s_z^2 N_{s_z}}{m} + \chi_{\alpha\beta}(\omega)\right)$$

$\alpha, \beta \in \{x, y, z\}$ $\omega^+ \equiv \omega + 0^+$

Magnetic quantum #: $s_z = -S, -S+1, \cdots, S$

Spin-current response func.: $\chi_{\alpha\beta}(\omega) = -i\int_{-\infty}^{\infty} dt\, e^{i\omega^+ t}\theta(t)\langle[J_{S,\alpha}(t), J_{S,\beta}(0)]\rangle_{\rm eq}$

Particle # in the s_z channel: N_{s_z}

2. f-sum rule

$$\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\mathrm{Re}\,\sigma^{(S)}_{\alpha\beta}(\omega) = \delta_{\alpha\beta}\sum_{s_z}\frac{s_z^2 N_{s_z}}{m}.$$

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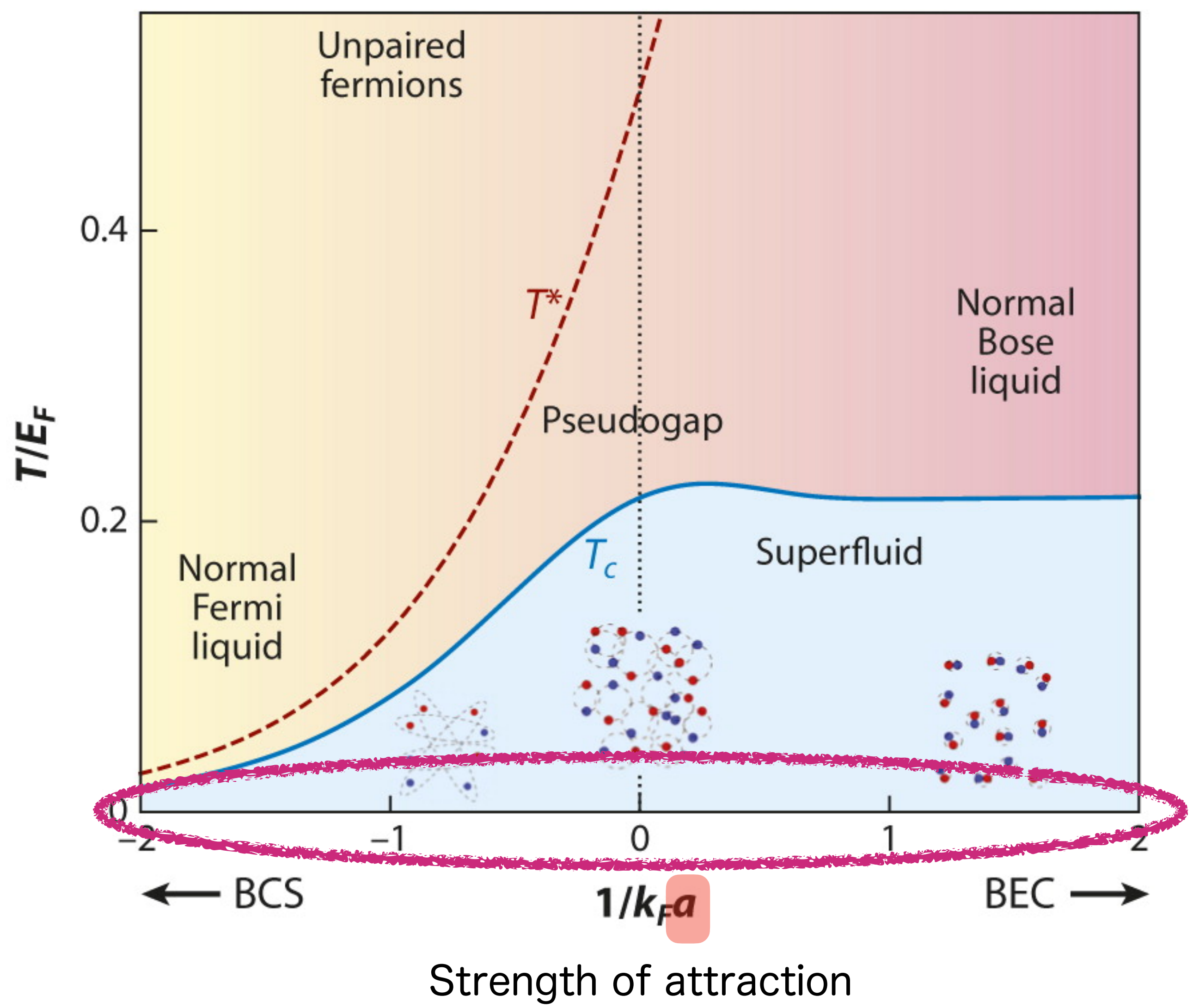
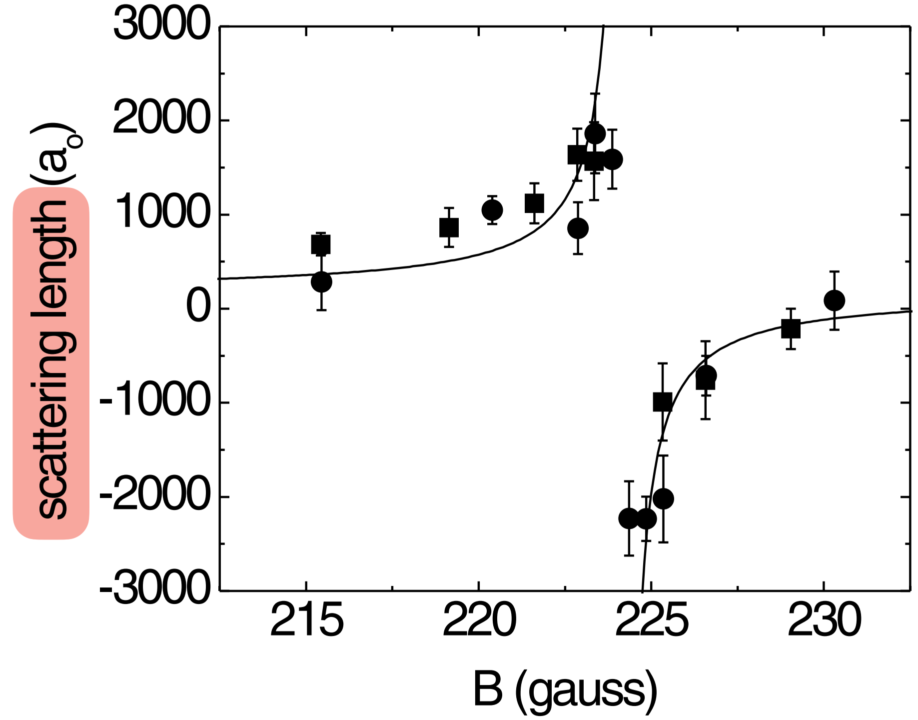
Superfluid Fermi gas

$$H = \int d\boldsymbol{x} \left[\sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \left(-\frac{\boldsymbol{\nabla}^2}{2m} - \mu \right) \psi_\sigma - g \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow \right]$$

$g > 0$: S-wave attraction



Randeria & Taylor (2014)



Regal & Jin (JILA), PRL(2003)

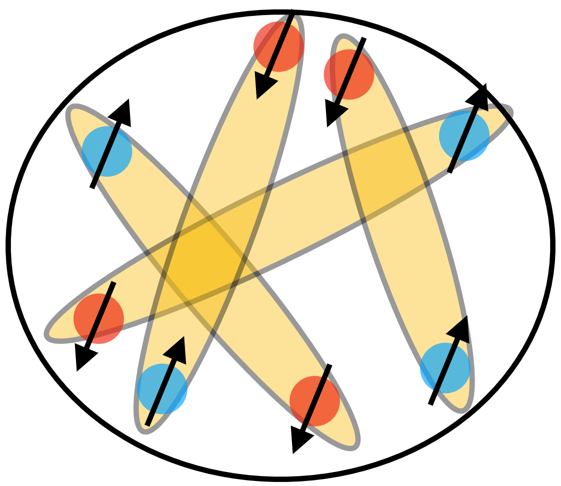
BCS-Leggett mean-field theory

Eagles (1969); Leggett (1980)

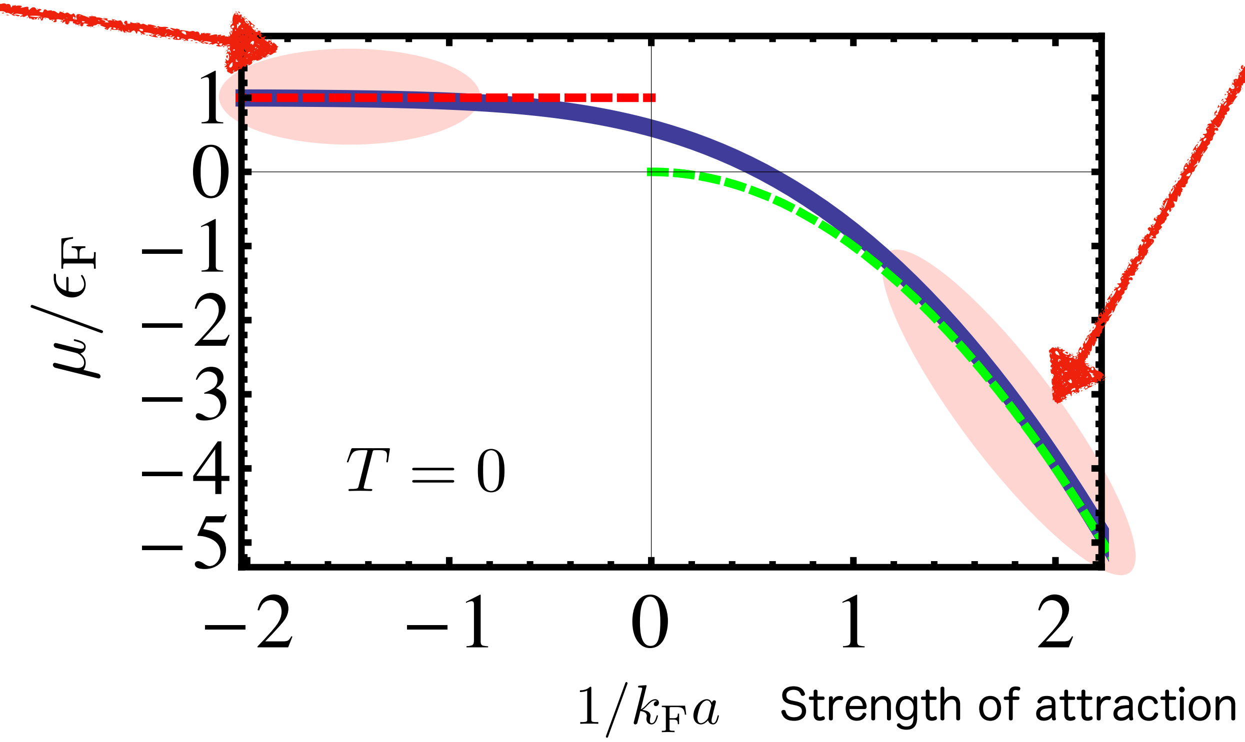
BCS-BEC crossover

Chemical potential @ T=0

BCS state

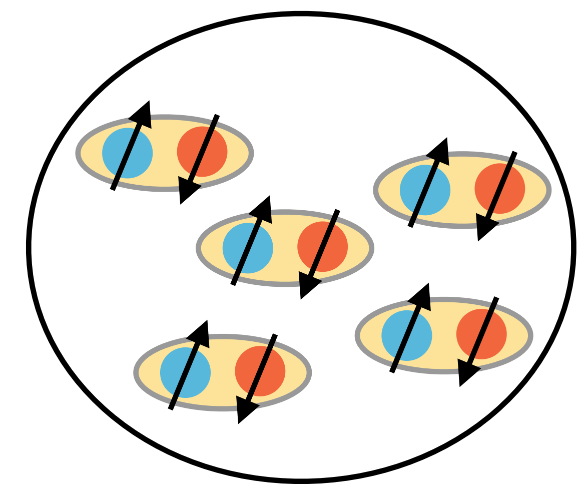


$\mu \simeq \epsilon_F$



Strength of attraction

BEC of diatomic molecules

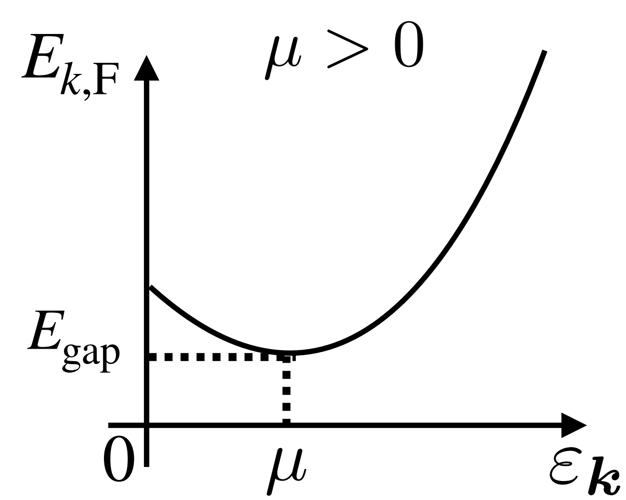


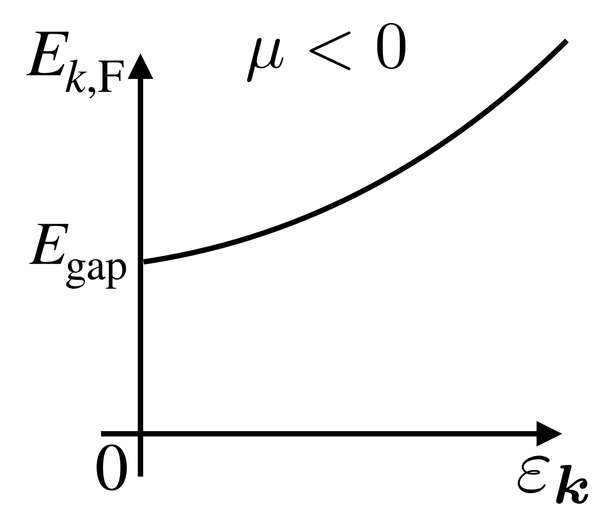
$\mu \to -E_{\text{molecule}}/2$

Energy dispersion

$$E_{\boldsymbol{k},\mathrm{F}} = \sqrt{(\varepsilon_{\boldsymbol{k}} - \mu)^2 + \Delta^2}$$

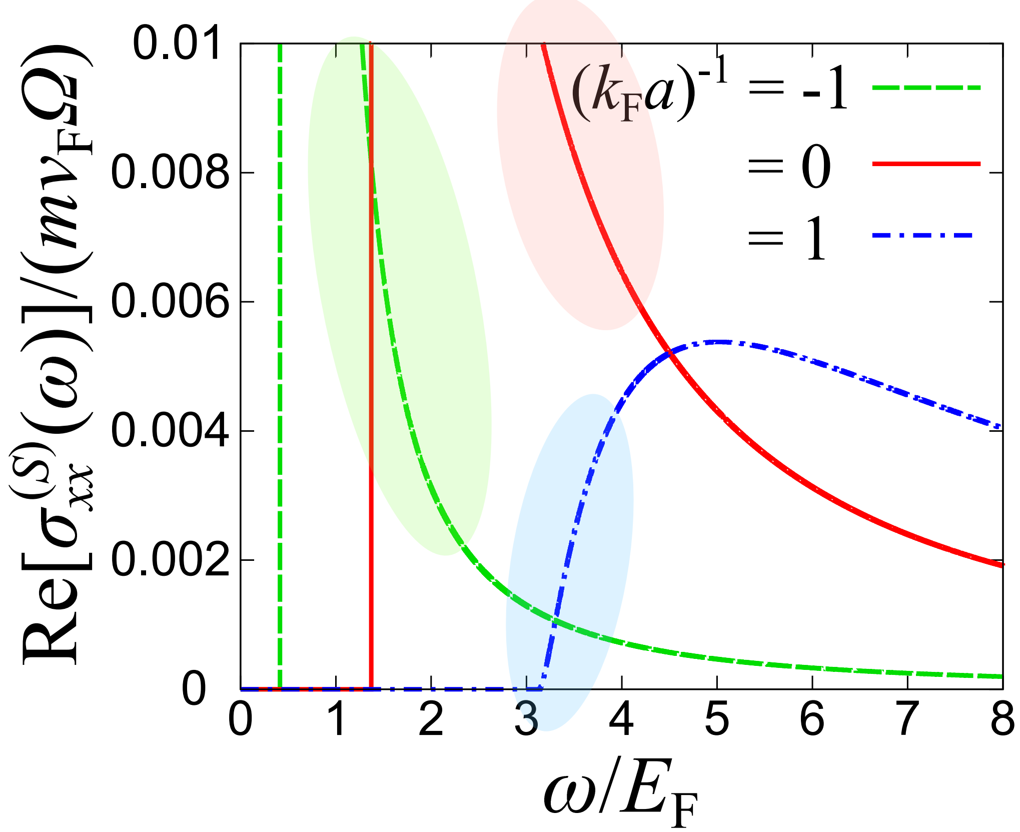
$$(\varepsilon_{\boldsymbol{k}} = \boldsymbol{k}^2/2m)$$





Result for a Fermi superfluid

YS, Tajima, & Uchino, arXiv:2103.02418



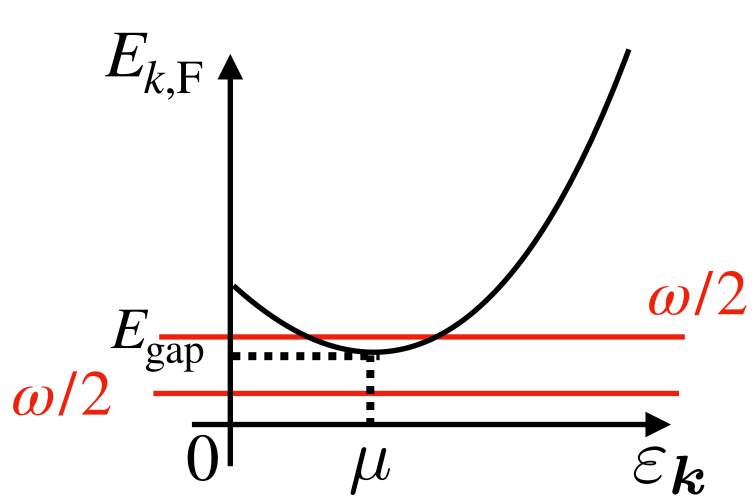
$$\mathrm{Re}\,\sigma_{xx}^{(S)}(\omega) \propto \sum_{\boldsymbol{k}} k_x^2 \delta(|\omega| - 2E_{\boldsymbol{k},\mathrm{F}})$$

1. Spin is insulated for small ω

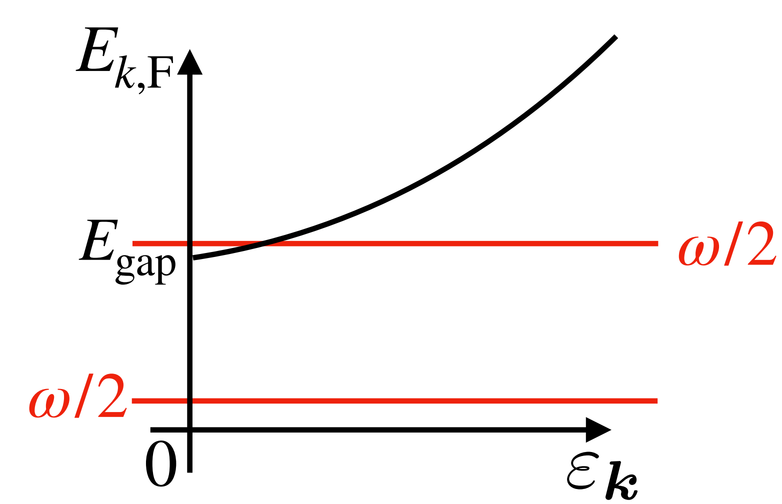
2. Behaviors for $\omega \to 2E_{\mathrm{gap}} + 0$

   μ > 0 [$(k_{\mathrm{F}}a)^{-1}$= -1,0] → coherence peak

   μ < 0 [$(k_{\mathrm{F}}a)^{-1}$= 1] → decay



$(k_{\mathrm{F}}a)^{-1}$= -1,0



$(k_{\mathrm{F}}a)^{-1}$= 1

Topological Fermi superfluid

Tajima, YS, & Uchino, PRB (2022)

- Fermi atoms in quasi 1D

$$H = \sum_{k,\sigma} \xi_k c^\dagger_{k,\sigma} c_{k,\sigma} + V, \qquad \xi_k = k^2/(2m) - \mu$$

- P-wave Feshbach resonance in the ↑ - ↓ channel:

$$V = -U \sum_{k,k',q} kk' c^\dagger_{k+q/2,\uparrow} c^\dagger_{-k+q/2,\downarrow} c_{-k'+q/2,\downarrow} c_{k'+q/2,\uparrow}$$

Triplet paring

$$S = 1,\ S_z = 0$$

- BdG Hamiltonian

$$H_{\mathrm{MF}} = \sum_k \Psi^\dagger_k H_{\mathrm{BdG}}(k) \Psi_k \qquad \Psi_k = \begin{pmatrix} c_{k,\uparrow} \\ c^\dagger_{-k,\downarrow} \end{pmatrix}$$

$$H_{\mathrm{BdG}}(k) = \boldsymbol{\sigma} \cdot \boldsymbol{R}(k) = -\sigma_x \Delta(k) + \sigma_z \xi_k,$$

$$\Delta(k) = kD \qquad (D > 0)$$

Class BDI with winding # $\nu \in \mathbb{Z}$

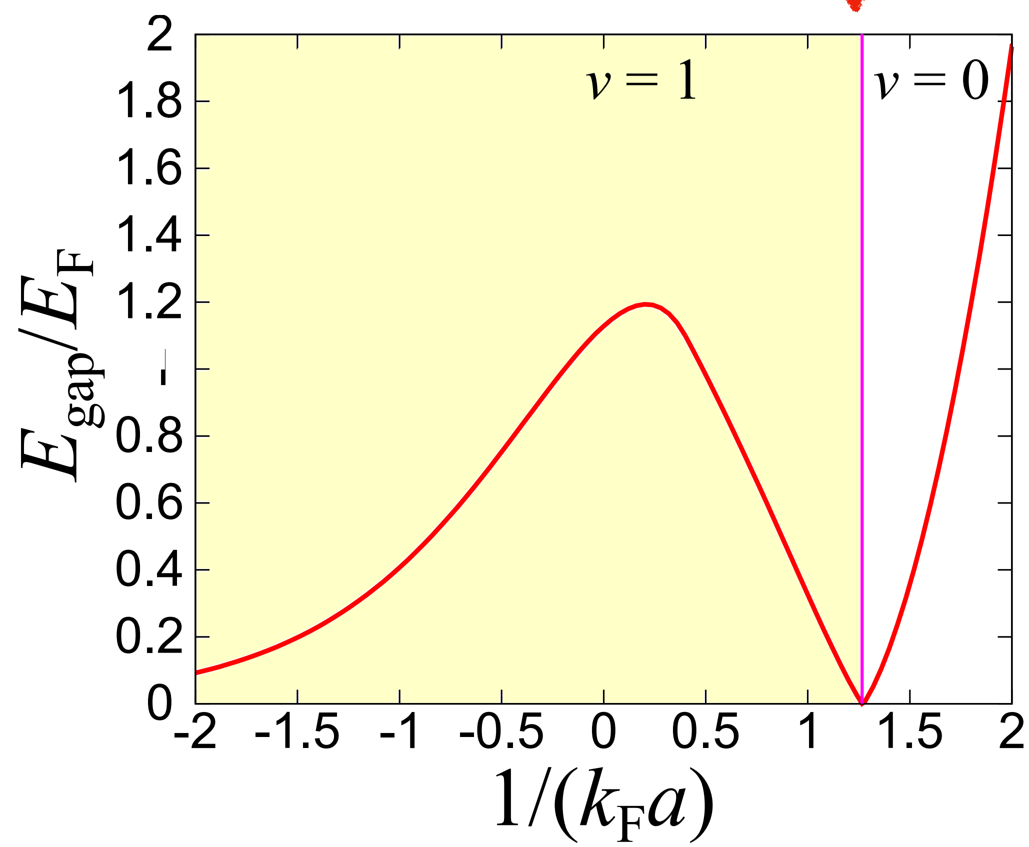
		TRS	PHS	SLS	$d=1$
Standard (Wigner-Dyson)	A (unitary)	0	0	0	-
	AI (orthogonal)	+1	0	0	-
	AII (symplectic)	−1	0	0	-
Chiral (sublattice)	AIII (chiral unitary)	0	0	1	$\mathbb{Z}$
	BDI (chiral orthogonal)	+1	+1	1	$\mathbb{Z}$
	CII (chiral symplectic)	−1	−1	1	$\mathbb{Z}$
BdG	D	0	+1	0	$\mathbb{Z}_2$
	C	0	−1	0	-
	DIII	−1	+1	1	$\mathbb{Z}_2$
	CI	+1	−1	1	-

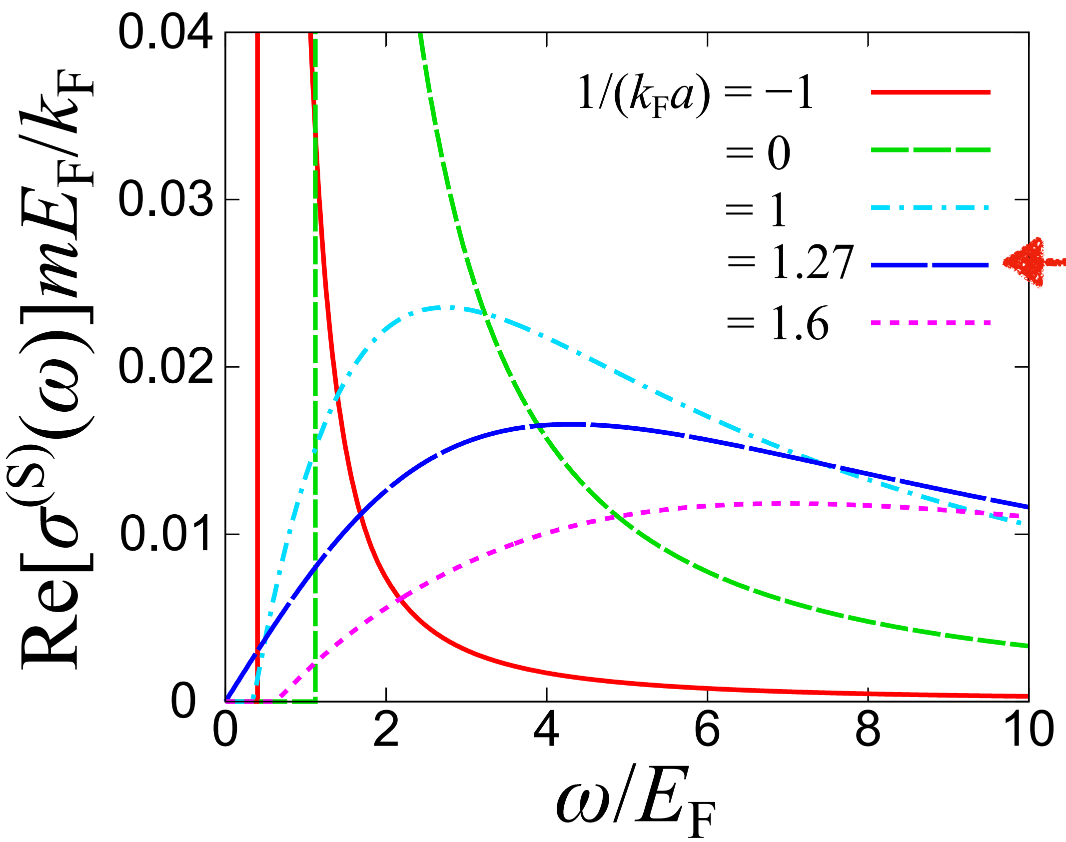
Schnyder, Ryu, Furusaki & Ludwig, PRB (2008)

Spectrum of spin conductivity

Tajima, YS, & Uchino, PRB (2022)

Topological phase transition ⟷ Closing of the spectral gap



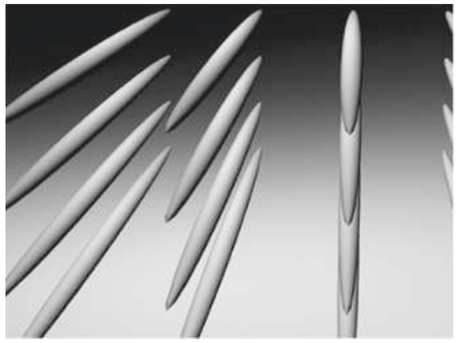


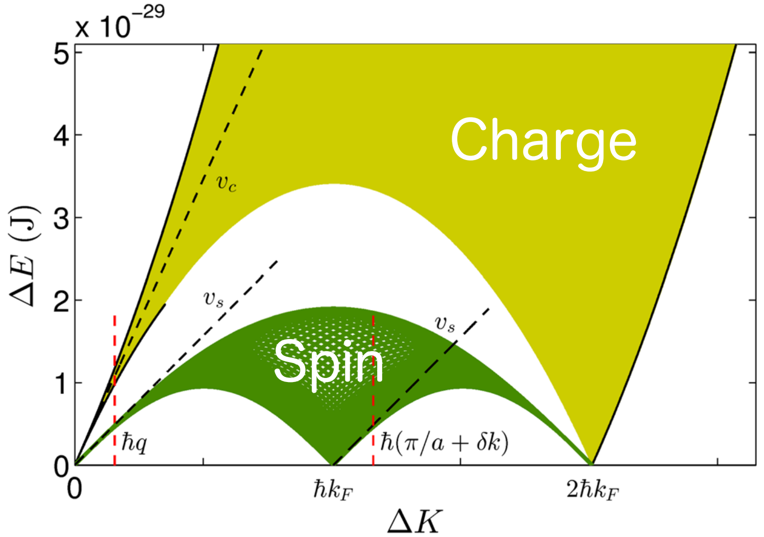
$$\mathrm{Re}\,\sigma^{(S)}(\omega) \propto \sum_k k^2 \delta(|\omega| - 2E_k)$$

Tomonaga-Luttinger liquid

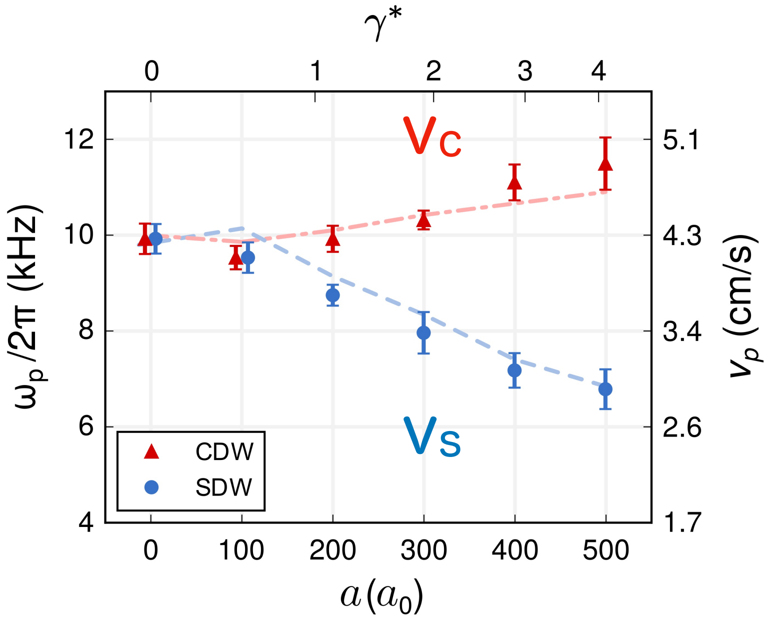
- One-dimensional systems with spin-charge separation

Described by 4 parameters v_c, v_s, K_c, K_s





He et al., PRL (2020)



Measurement of v_c & v_s

Senaratne et al. (Rice), to be published in Science (2022)

K_s can be experimentally determined by spin conductivity at low frequency

YS, Tajima, & Uchino, arXiv:2103.02418

$$\mathrm{Re}\,\sigma^{(S)}(\omega) \propto \omega^{4K_S-5}$$

(Memory function method)

cf. Charge conductivity

Giamarchi, PRB (1991); (1992)

Outline of this talk

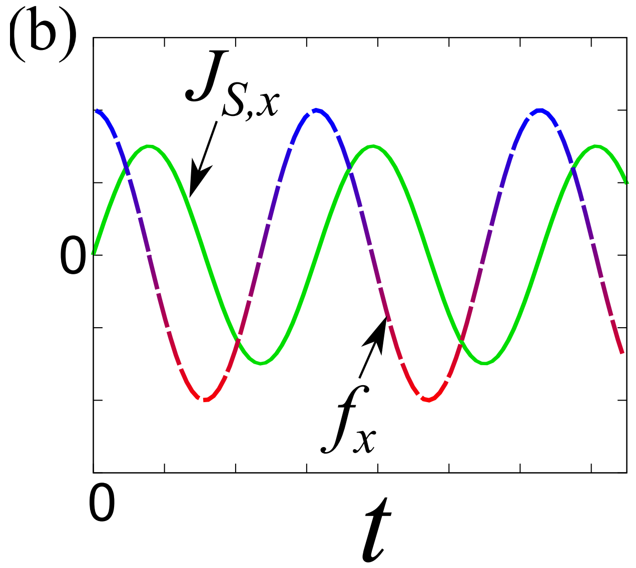
Summary of this talk

- Optical (AC) spin conductivity

  Measurable in cold-atom experiments

$$\sigma^{(S)}_{\alpha,\beta}(\omega) = \tilde{J}_{S,\alpha}(\omega)/\tilde{f}_{\beta}(\omega)$$

$(\alpha, \beta = x, y, z)$



- Significance of $\sigma^{(S)}_{\alpha\beta}(\omega)$

  YS, Tajima, & Uchino, accepted by PRResearch (2022)
  Tajima, YS, & Uchino, PRB (2022)

1. Elusive in solid-state systems
2. Powerful probe for quantum many-body states

   BCS-BEC crossover, Tomonaga-Luttinger liquid(TLL), Spinor BEC,⋯
3. Widely applicable probe for clean systems

- Future perspective: Pseudogap of the unitary Fermi gas ?

Backup Slides

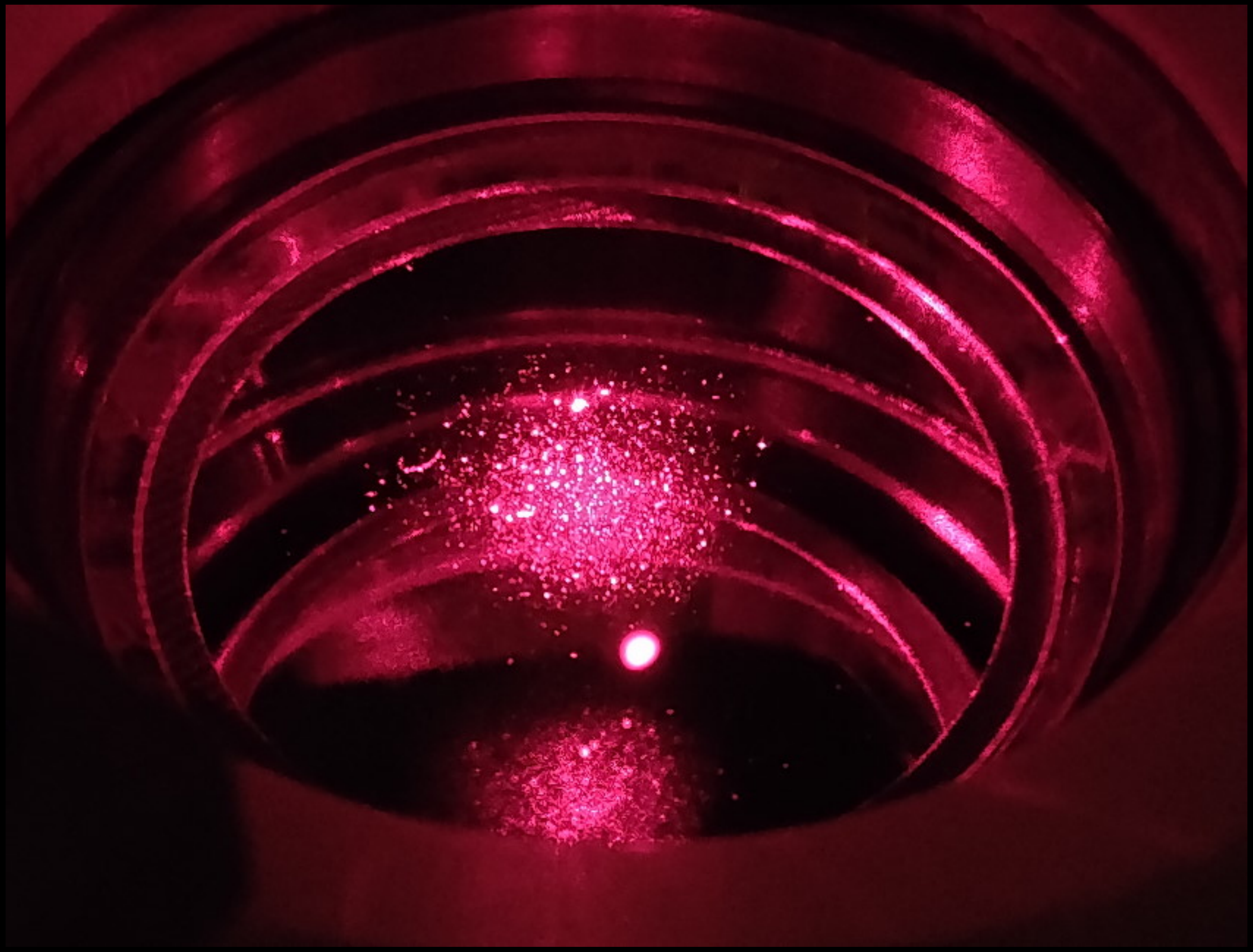
Ultracold atoms

- Very pure & highly controllable atomic gases

$n = 10^{13}$ —10^{15} cm^{-3}, $T = 10^{-6}$ —10^{-8} K (μK—nK)

Coldest in the Universe !!

(cf. O_2 in a room: $n = 10^{19}$ cm^{-3}, $T = 10^3$ K)

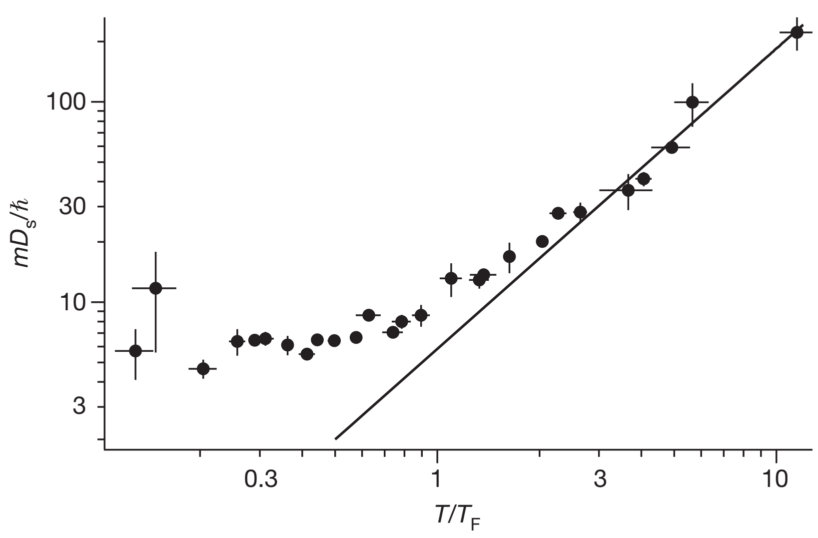


From Schauss Group, University of Virginia (https://ultracold.phys.virginia.edu/public_html/)

Transport phenomena

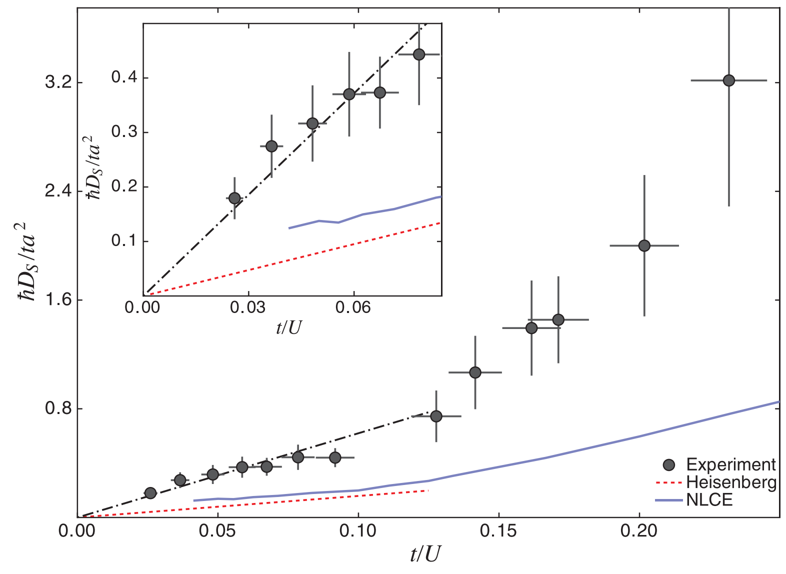
1. Spin diffusivity

Sommer et al.,(MIT) Nature (2011); …



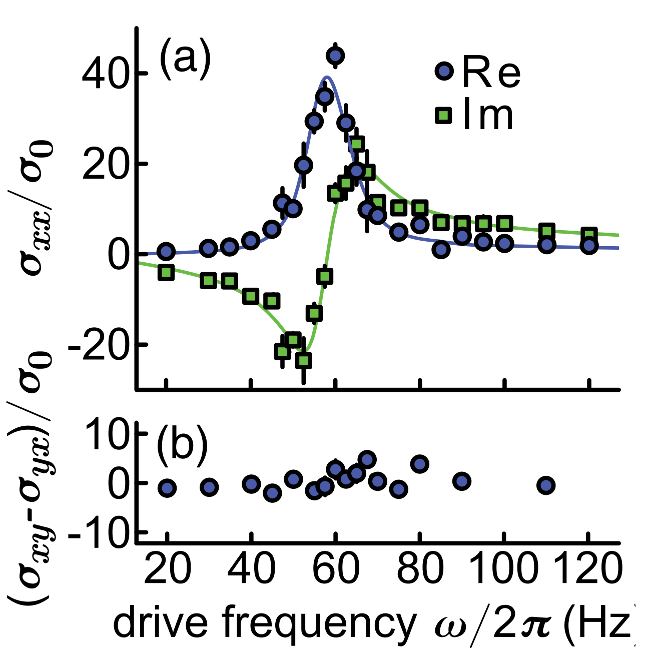
2. DC spin conductivity

Nicholos et al.,(MIT) Nature (2019); …



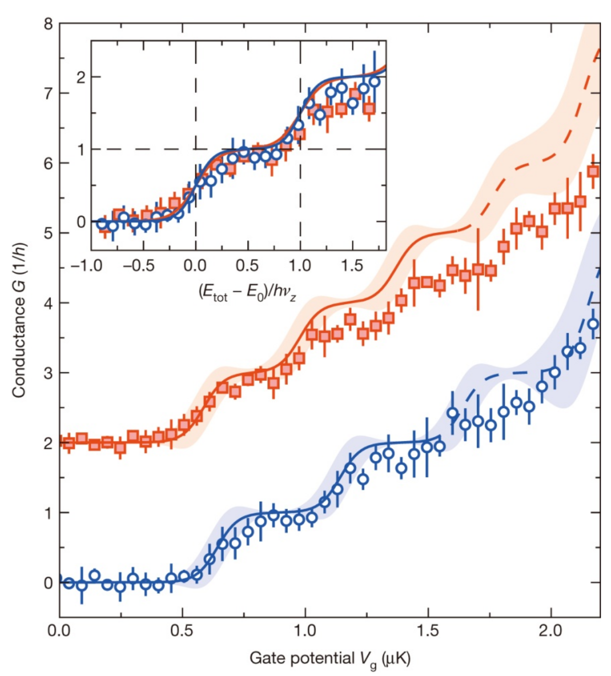
3. Optical (AC) conductivity

Anderson et. al.,[Toronto group] PRL (2019)



4. Quantized conductance

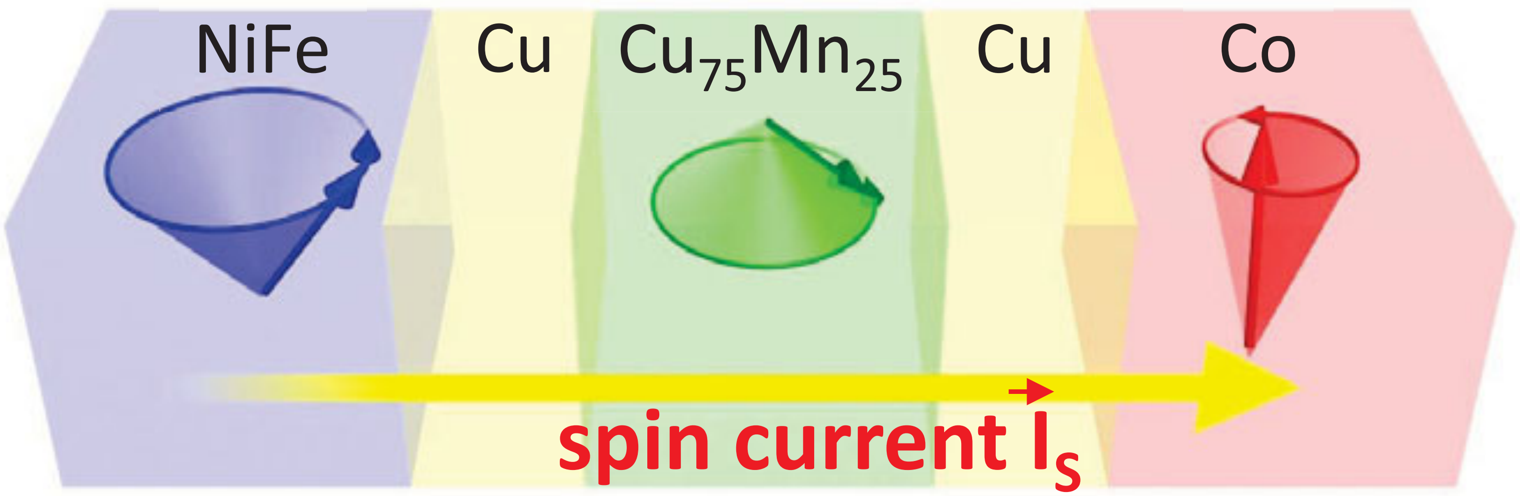
Krinner et al.[EHT], Nature(2015)



1. AC spin transport

- Hot topic in spintronics

  Multilayer systems are considered



Ferromagnetic resonance in Multilayer systems   Li et al., PRL (2016)

Bulk transport property $\sigma^{(S)}_{\alpha\beta}(\omega)$ is elusive !!

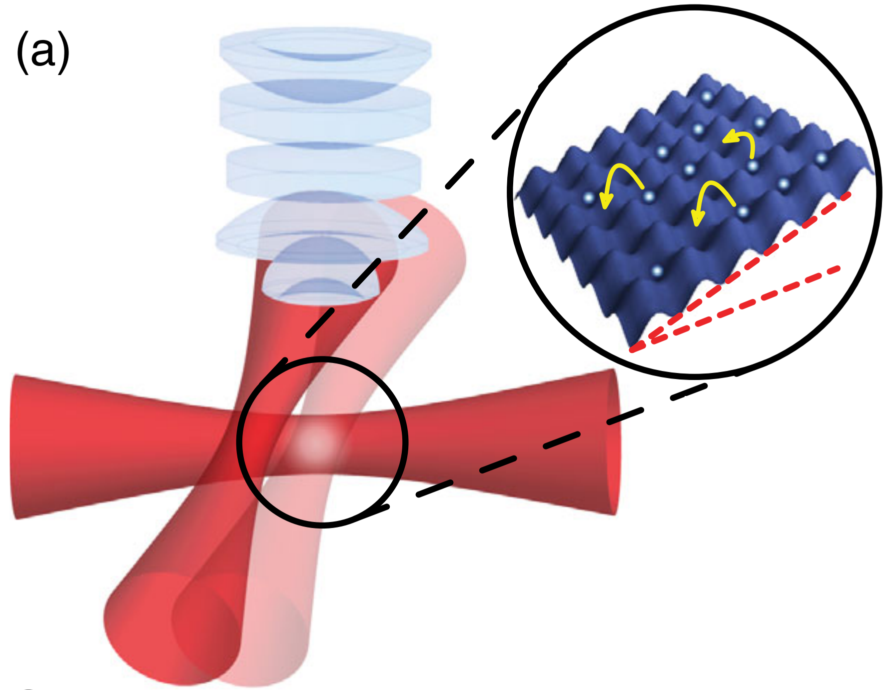
AC spin transport within bulk is accessible with cold atoms

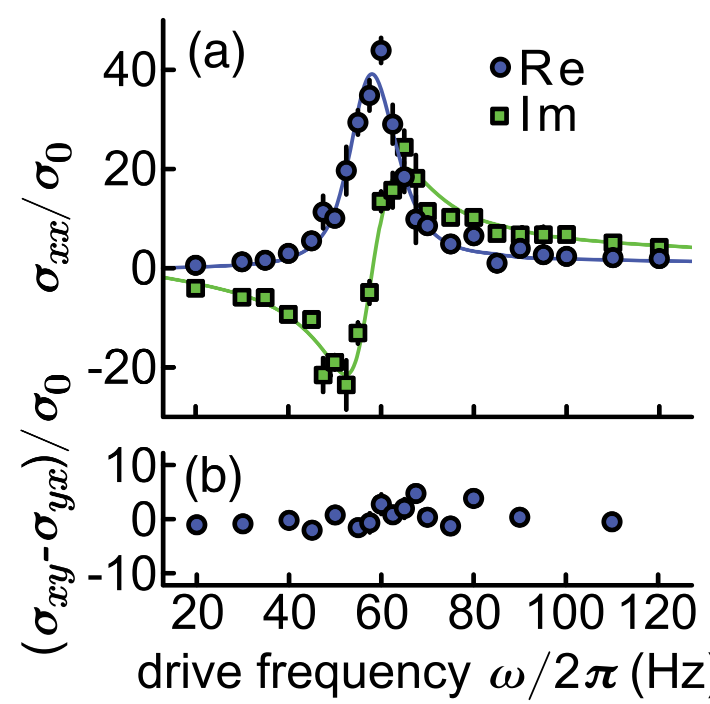
3. Probe for clean gases

- Optical mass conductivity for cold atoms

Proposal: Tokuno & Giamarchi, PRL (2011)
Wu, Taylor, & Zaremba, EPR (2015)

Experiment: Anderson et. al.,[Toronto group] PRL (2019)



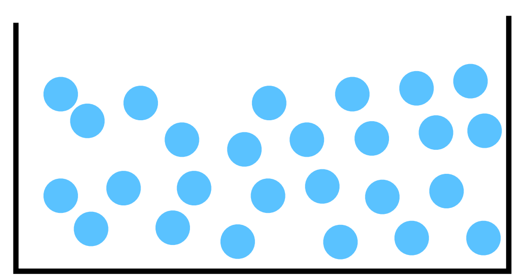
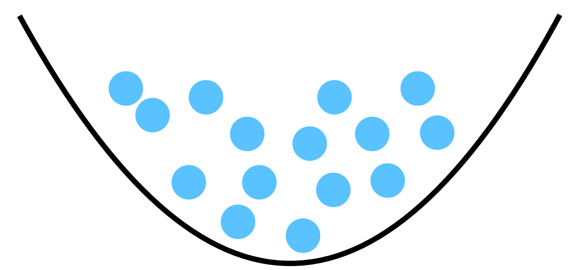
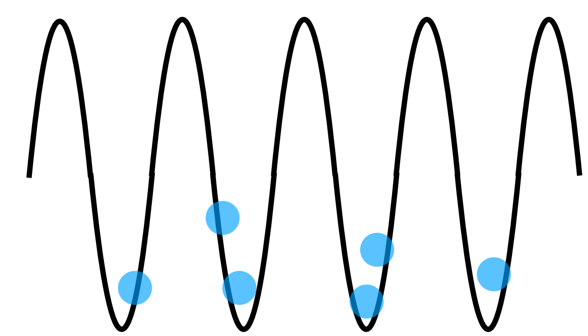


Optical lattice is essential !!!

3. Probe for clean gases

- Generalized Kohn's theorem: Kohn (1961); Brey et al. (1989); Li et al. (1991)

  Strong constraint on mass conductivity of clean gases

	Homogeneous	Harmonically trapped	On lattice
			
Mass conductivity	$\frac{N}{m}\delta(\omega)$	$\frac{N}{m}\delta(\omega - \omega_{\mathrm{trap}})$	Sensitive to quantum states

Spin conductivity is never constrained and works as probes w/o lattice

How to extract spin conductivity

YS, Tajima, & Uchino, arXiv:2103.02418

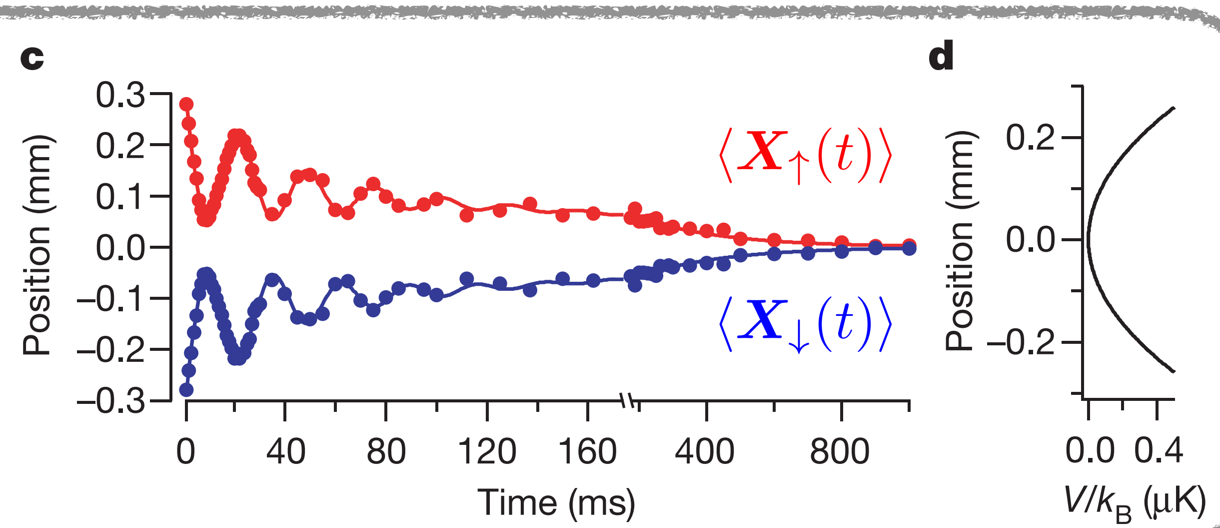
1. Spin current: $\langle \boldsymbol{J}_S(t)\rangle = \frac{d}{dt}\left\langle \int d\boldsymbol{r}\, \boldsymbol{r} S_z(\boldsymbol{r},t)\right\rangle \equiv \frac{d}{dt}\langle \boldsymbol{X}_S(t)\rangle$ (Spin conservation)
2. Spin conductivity: $\langle \tilde{J}_{S,\alpha}(\omega)\rangle = \sigma^{(S)}_{\alpha\beta}(\omega)\tilde{f}_\beta(\omega)$ (Ohm's law in frequency space)
3. Driving force: $f_\beta(t) = F_\beta \cos(\omega_0 t)$

$$\frac{\langle \delta X_{S,\alpha}(t)\rangle}{F_\beta} = -\frac{\mathrm{Im}\,\sigma^{(S)}_{\alpha\beta}(\omega_0)}{\omega_0}\cos(\omega_0 t) + \frac{\mathrm{Re}\,\sigma^{(S)}_{\alpha\beta}(\omega_0)}{\omega_0}\sin(\omega_0 t)$$

Measurement of $\langle X_{S,\alpha}(t)\rangle$ → $\sigma^{(S)}_{\alpha\beta}(\omega=\omega_0)$

e.g. Experiment on spin diffusion (w/o $f_\beta(t)$)

$$\langle \boldsymbol{X}_S(t)\rangle = \langle \boldsymbol{X}_\uparrow(t)\rangle - \langle \boldsymbol{X}_\downarrow(t)\rangle$$

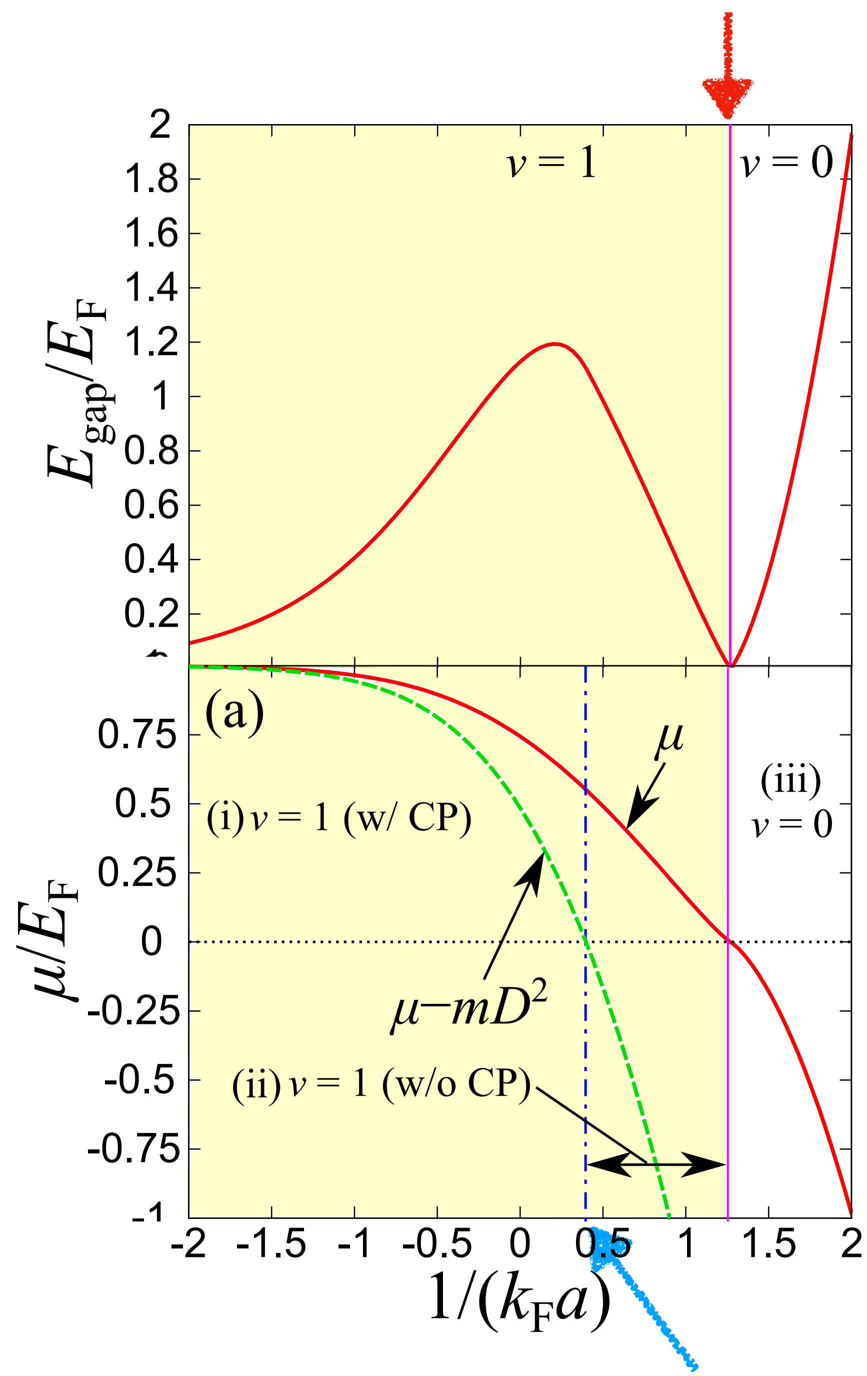


Sommer et al.,(MIT) Nature (2011)

Topological phase transition

Tajima, YS, & Uchino, PRB (2022)

Topological phase transition

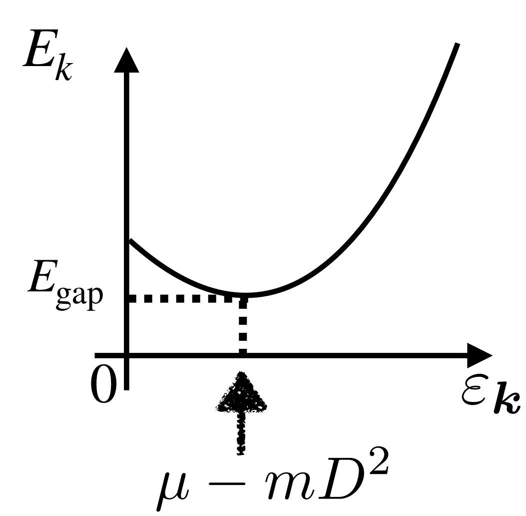


Coherence Peak (CP) disappears

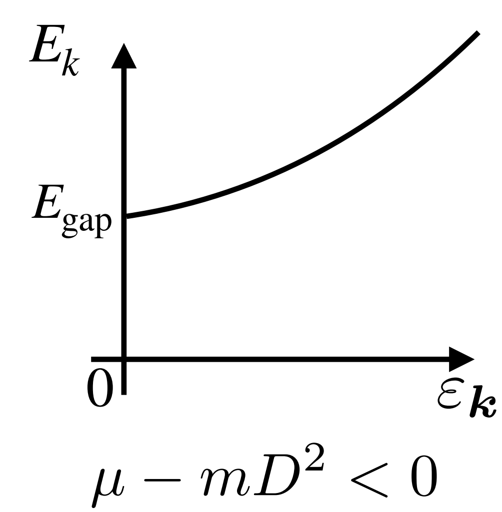
Topological phase transition at μ = 0

$$\nu = 1 \quad \rightarrow \quad \nu = 0$$

Quasiparticle energy $E_k = \sqrt{(\varepsilon_k - \mu)^2 + D^2 k^2}$



w/ CP



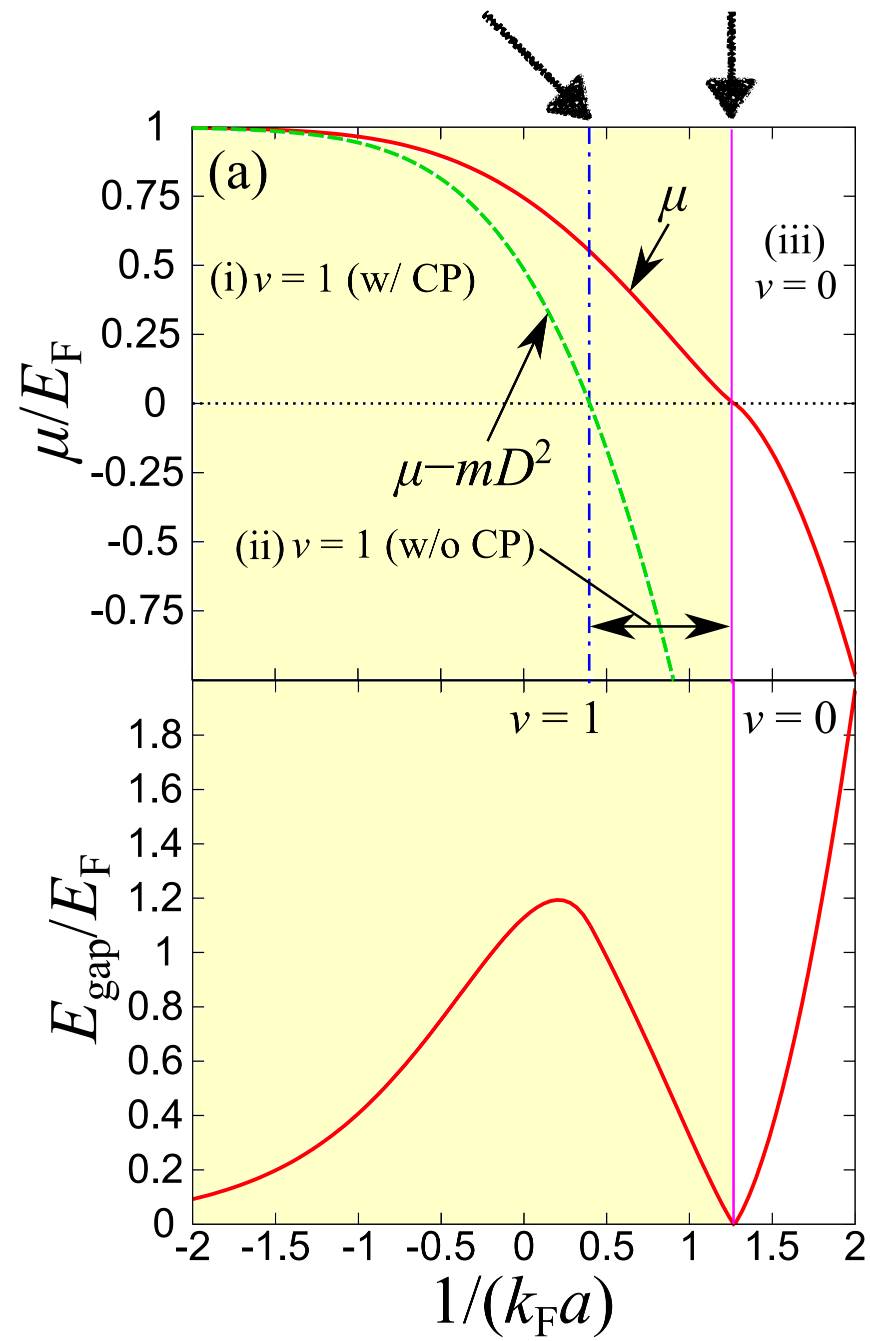
w/o CP

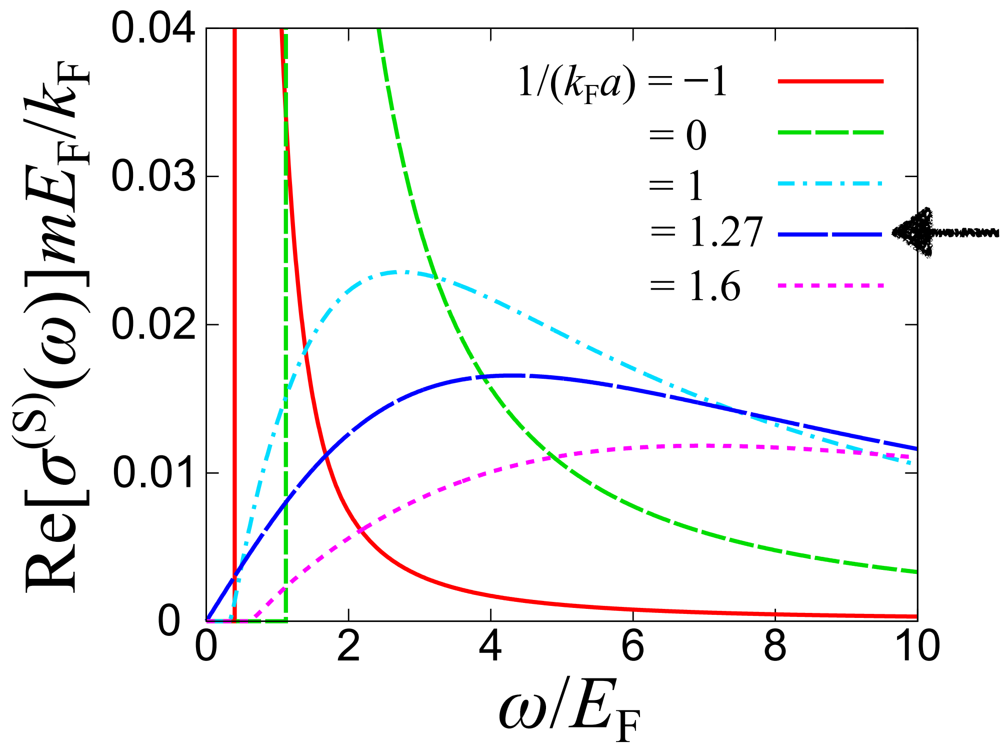
Spectrum of spin conductivity

Tajima, YS, & Uchino, PRB (2022)

Coherence peak (CP) disappears

Topological phase transition point





Topological phase transition point

$$\mathrm{Re}\,\sigma^{(S)}(\omega) \propto \sum_{k} k^2 \delta(|\omega| - 2E_k)$$

Closing of the spectral gap of $\mathrm{Re}\,\sigma^{(S)}(\omega)$

⟺ Topological phase transition