Inhomogeneous phases in 1+1 dimensional chiral Gross-Neveu model on the lattice

ChudenCTI Co.,Ltd.[A],
Hiroshima University[B]
Kobayashi Maskawa Institute, Nagoya University[C],
Department of Physics, Nagoya University[D]
Keita Horie[A] and Chiho Nonaka[B,C,D]

Contents

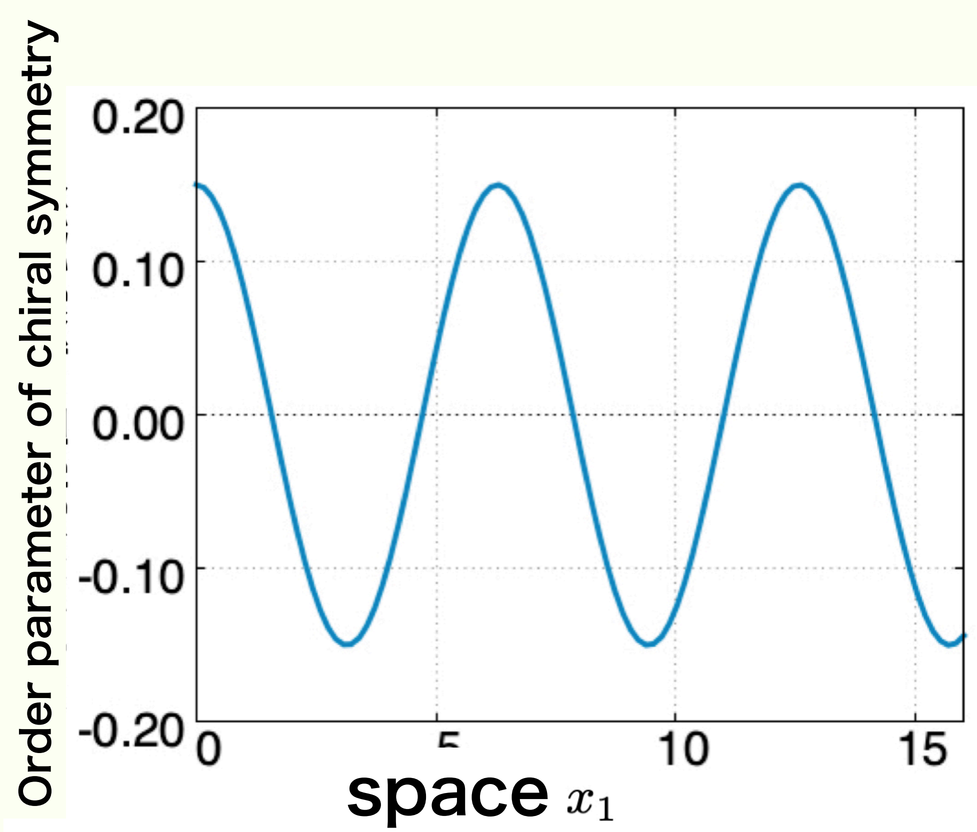
Motivation: QCD Phase diagram at finite μ

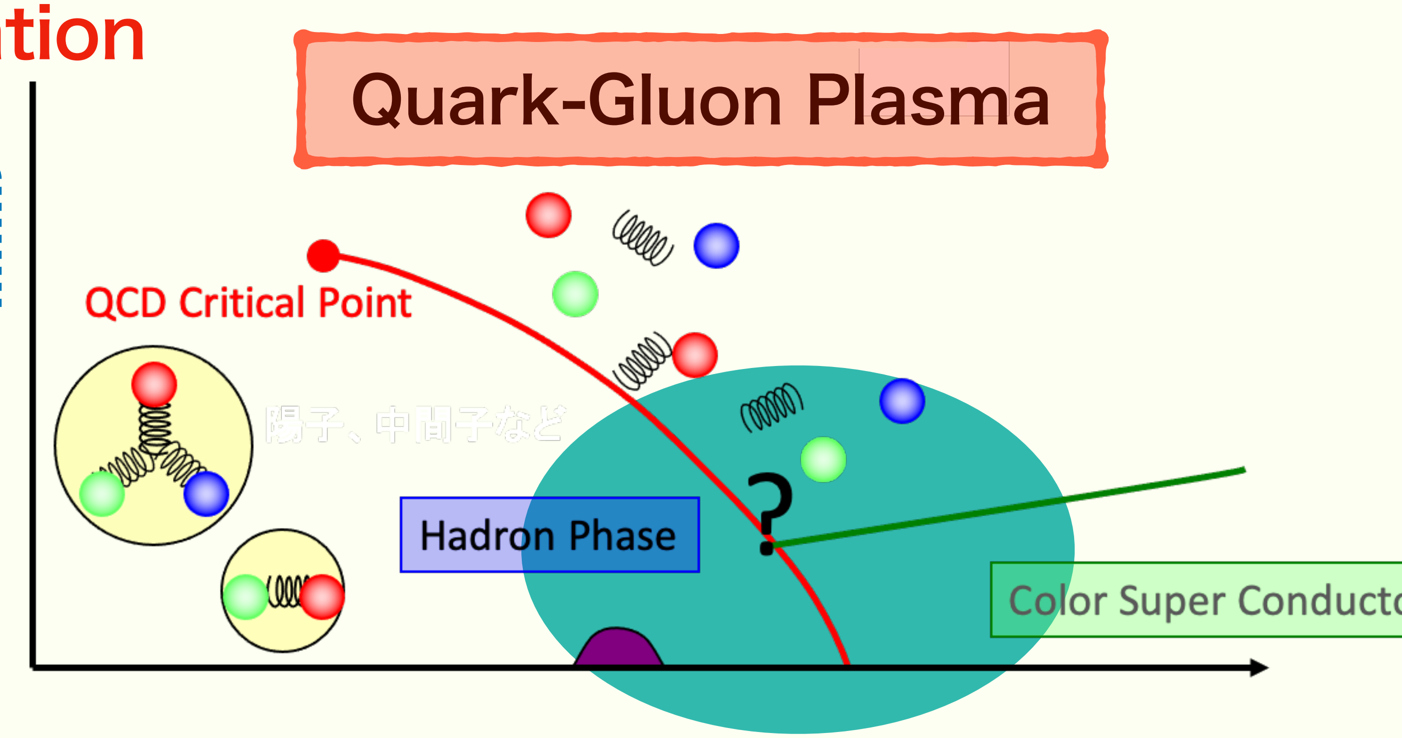
✣ Possible interesting phases at high μ

↓ **Effective field theory**

inhomogeneous chiral condensation

spatial dependence of chiral condensate σ





※ Space-dependent condensates

→ Specific Ansatz
ex. constant, chiral density wave

Model: (1+1)d chiral Gross–Neveu model

✣ Lagrangian

$$\mathcal{L} = \bar{\psi} i \gamma^{\nu} \partial_{\nu} \psi + \frac{g^2}{2N} \left[\left(\bar{\psi}\psi \right)^2 + \left(\bar{\psi} i \gamma^5 \psi \right)^2 \right]$$

$\sigma \sim \langle \bar{\psi}\psi \rangle$ $\pi \sim \langle \bar{\psi} i \gamma^5 \psi \rangle$

➤ Important features from comparison with QCD

- ✔ Asymptotic freedom
- ✔ Spontaneous symmetry breaking of discreet chiral symmetry
- ✔ No sign problem : Monte Carlo simulation
- ✔ Imhomogeneous chiral condensate in $N_f \to \infty$

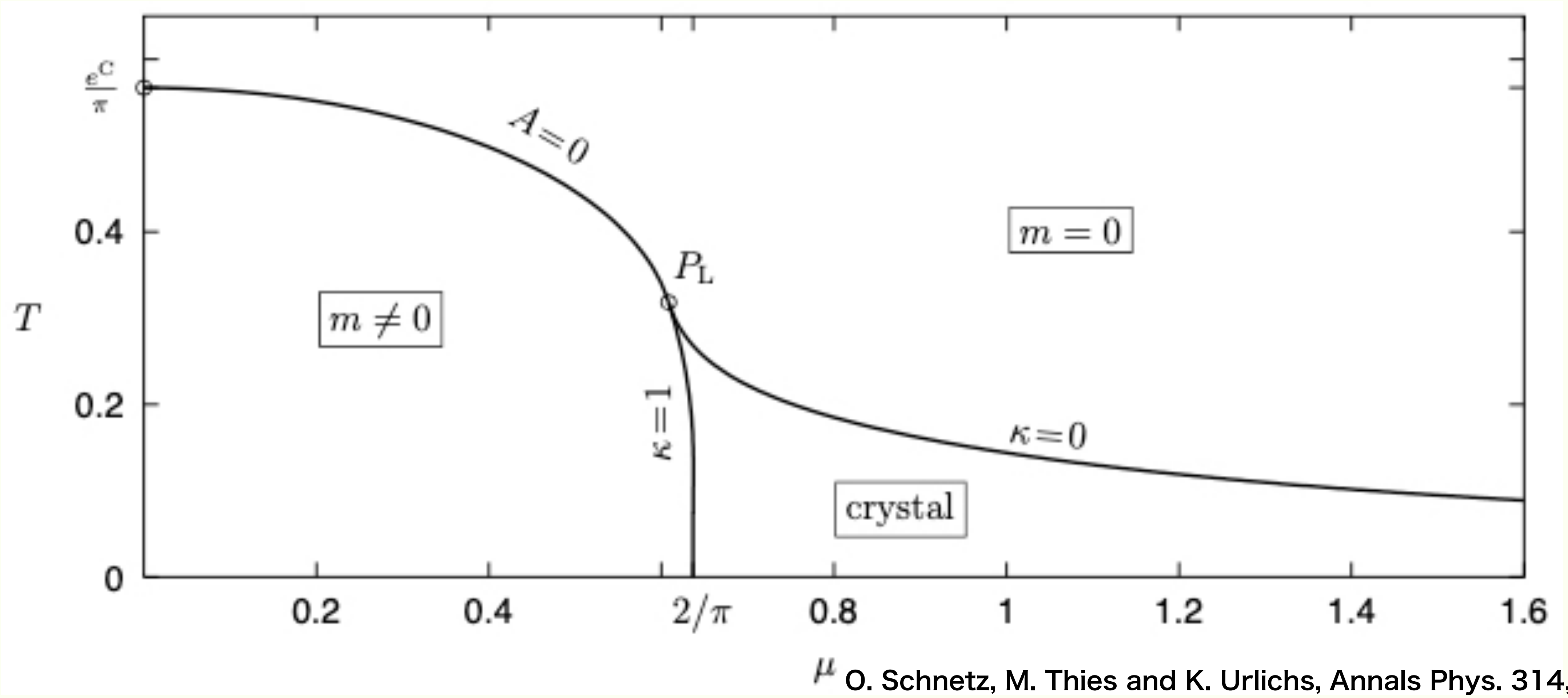
@ continuous theory

→ 1st step: (1+1)d Gross-Neveu model

$$\mathcal{L} = \bar{\psi} i \gamma^{\nu} \partial_{\nu} \psi + \frac{g^2}{2N} \left(\bar{\psi}\psi \right)^2$$

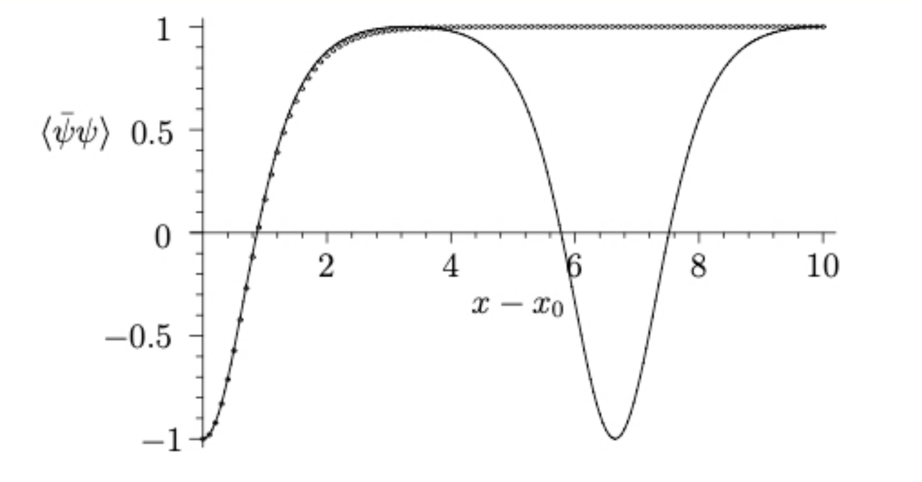
D. J. Gross and A. Neveu, Phys. Rev. D 10, 3235 (1974)

Phase Diagram of (1+1)d Gross-Neveu model



O. Schnetz, M. Thies and K. Urlichs, Annals Phys. 314, 425-447 (2004)

Spatial dependent chiral condensate



Ansatz

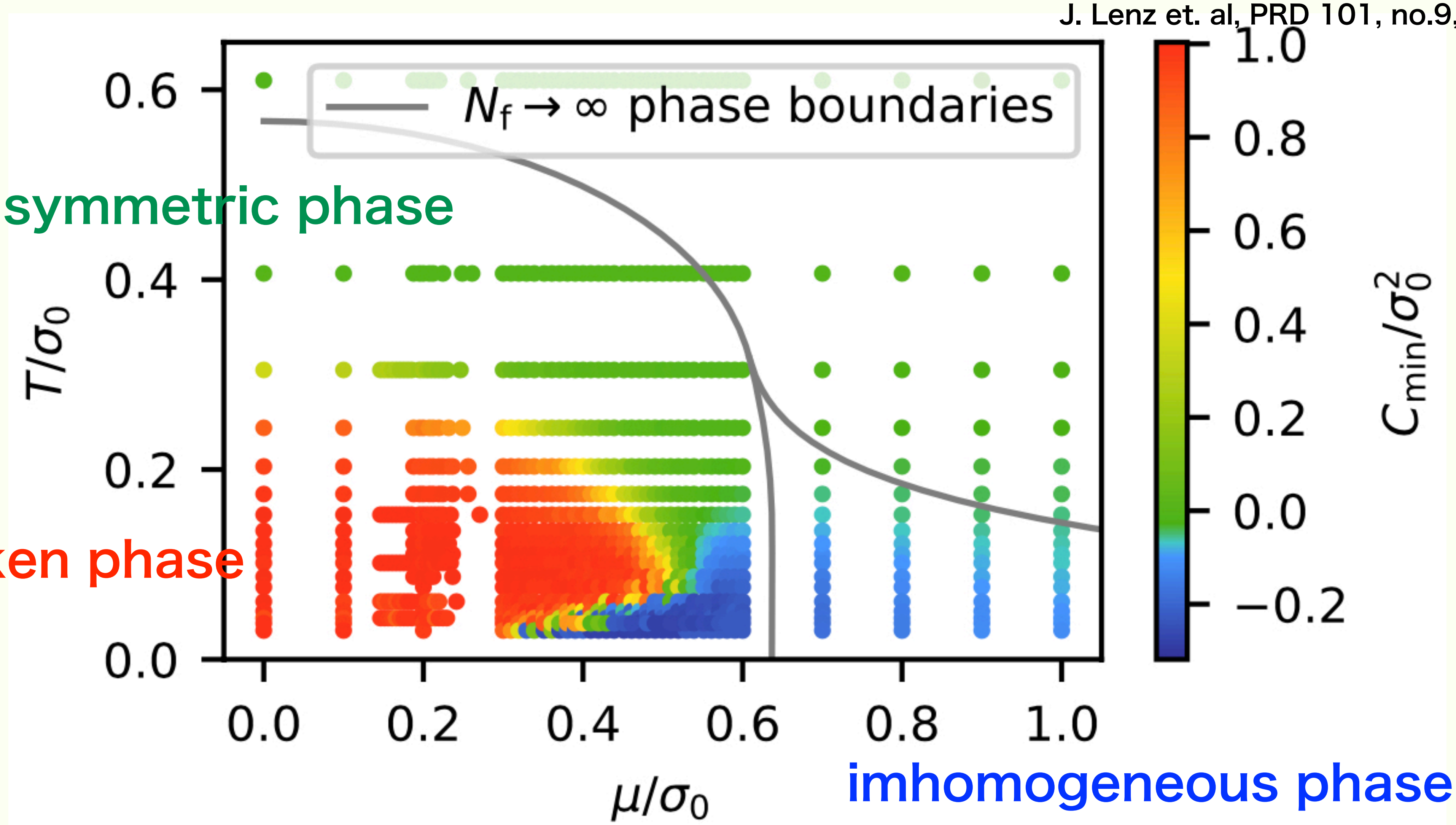
V. Schon and M. Thies, Phys. Rev. D 62, 096002 (2000)

→ Without ansatz ?
Lattice Gauge simulation

J. Lenz et. al, Phys. Rev. D 101, no.9, 094512 (2020)

(1+1)d Gross-Neveu model on the Lattice

J. Lenz et. al, PRD 101, no.9, 094512 (2020)



➤ They confirmed the following: J. Lenz et. al, PRD 101, no.9, 094512 (2020)

- ✔ Existence of inhomogeneous phase at low T and high μ
- ✔ Naive fermions and SLAC fermions: same results
- ✔ lattice spacing dependence
- ✔ volume dependence:
- ✔ flavor number dependence:

} GN model calculation

Simulation setup for GN model

➤ Preparation for analysis of chiral GN model on the lattice

- First we carry out GN model calculation based on J. Lenz et. al, PRD 101, no.9, 094512 (2020)
- We choose naive fermion.
- Hybrid Monte Carlo
- Parameters

fermion	N_f	$N_s = L/a$	$N_t = 1/Ta$	g^2	$a\sigma_0$	μ/σ_0
naive	8	32	2, 4, 6, . . ., 64	1.9132	0.4190(1)	0.0, 0.1, . . .

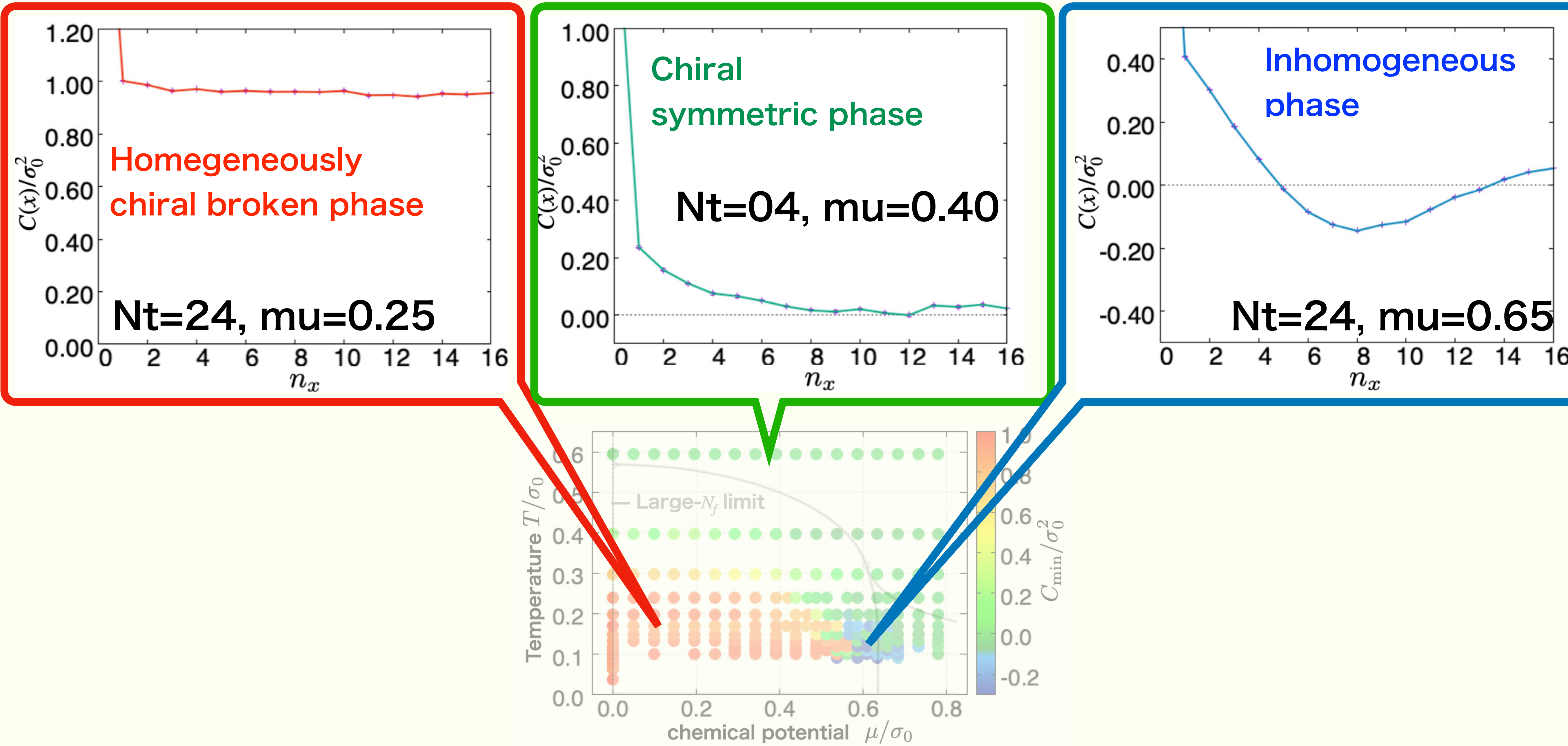
➤ Investigation of phase diagram of GN model

- Spatial dependent chiral condensate → inhomogeneous phase
- Correlator: $C(x) = \frac{1}{N_t N_x} \sum_{t,y} \langle \sigma(t, y+x)\sigma(t,y) \rangle$

Analysis on chiral GN model

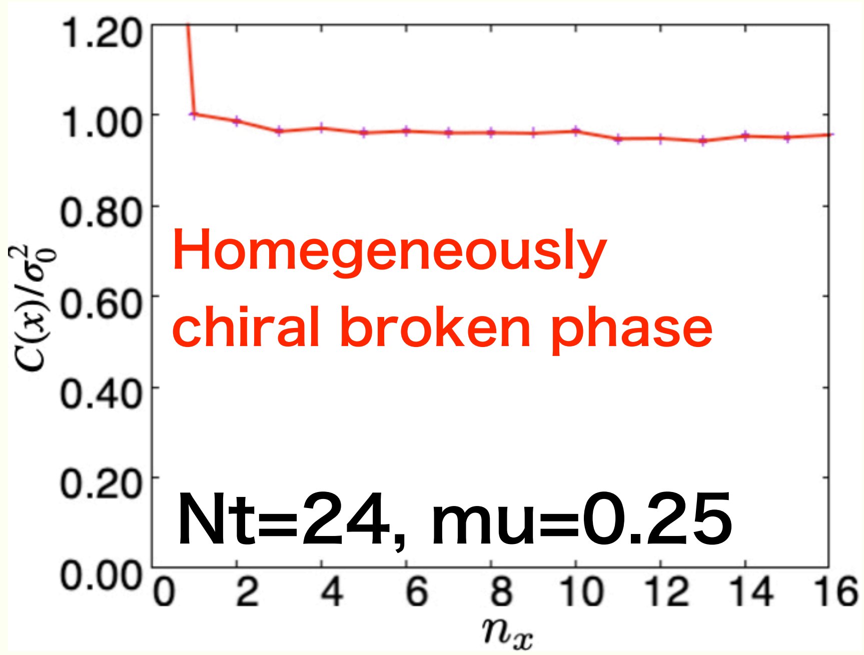
Phase Diagram of GN model

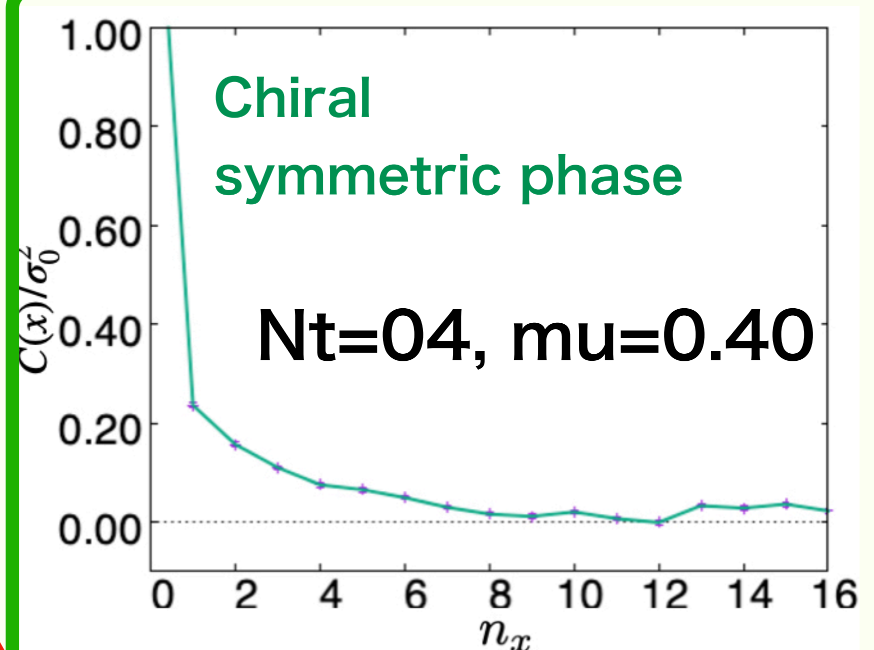
Correlator: $C(x) = \frac{1}{N_t N_x} \sum_{t,y} \langle \sigma(t, y+x)\sigma(t,y) \rangle$

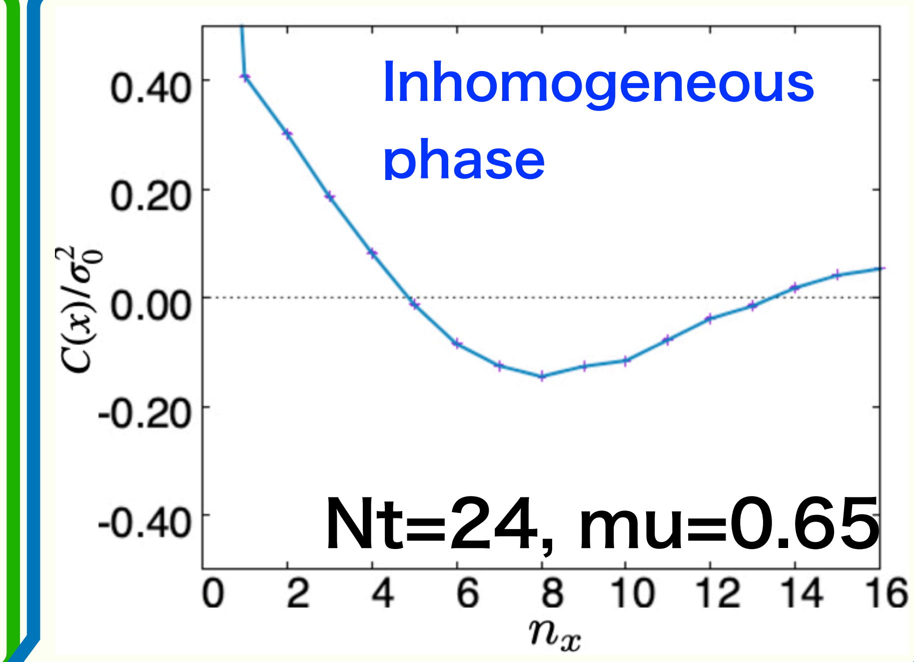


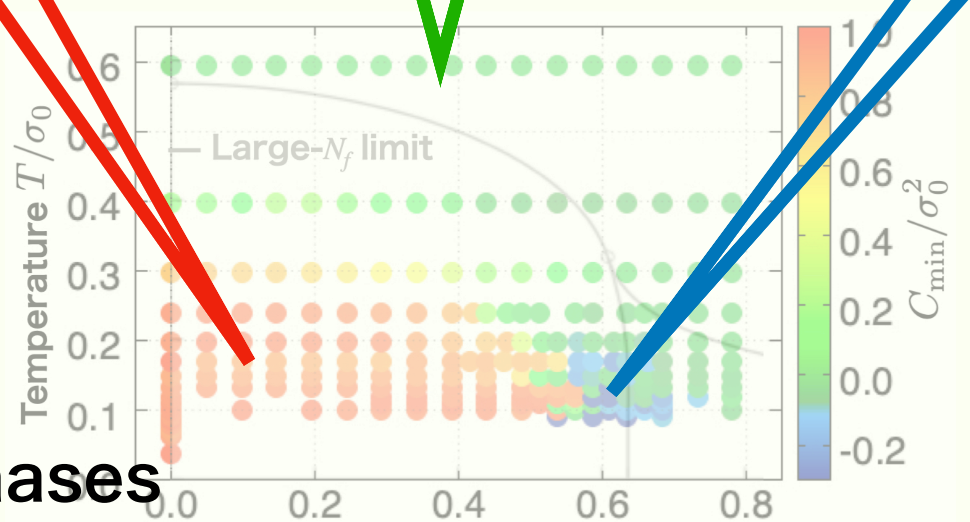
Phase Diagram of GN model

$$C(x) = \frac{1}{N_t N_x} \sum_{t,y} \langle \sigma(t, y+x) \sigma(t, y) \rangle$$









Classification of three phases

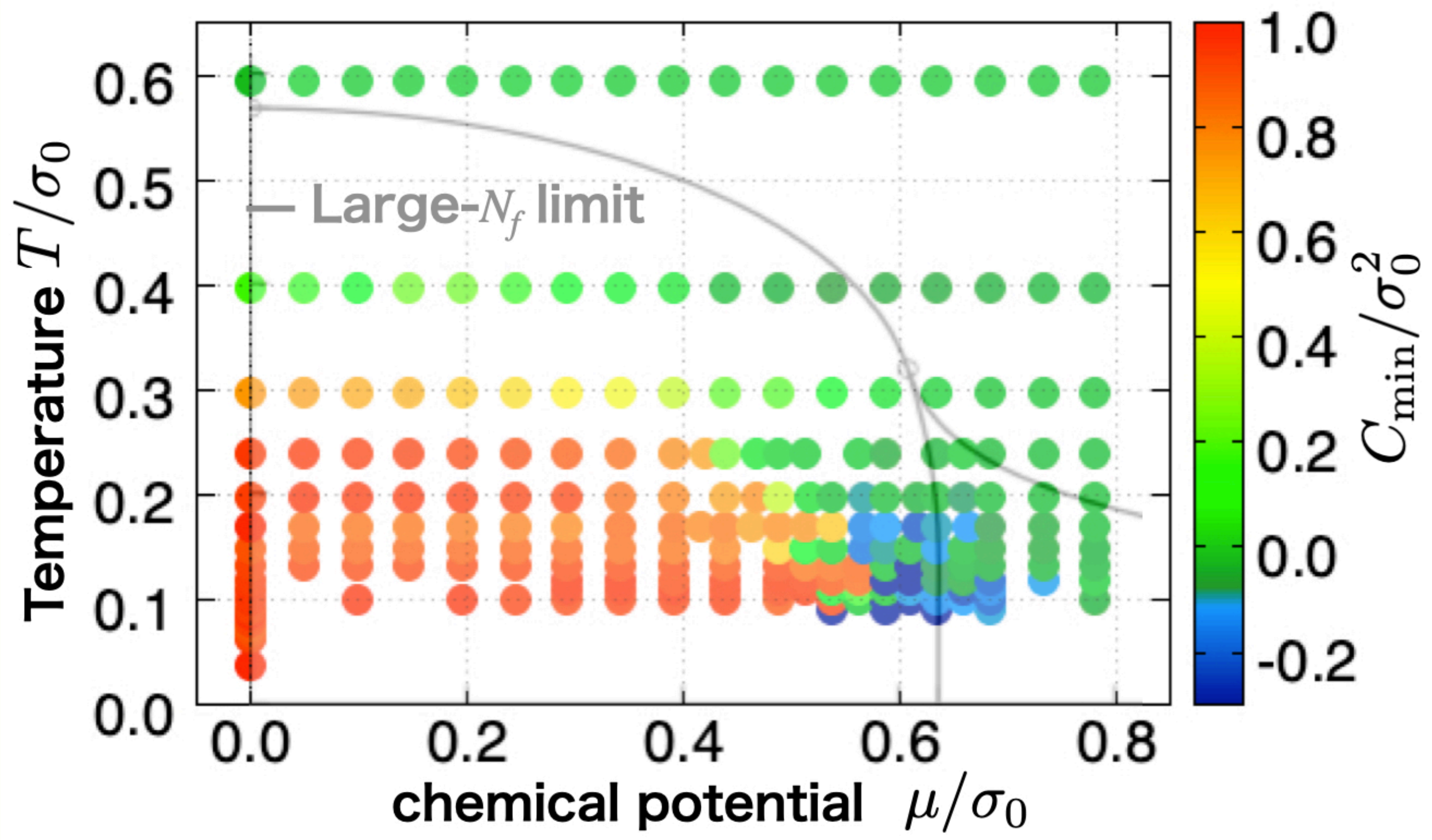
$$C_{\min} = \min_x C(x) \begin{cases} \gg 0 & \text{Homegeneously chiral broken phase} \\ \approx 0 & \text{Chiral symmetric phase} \\ < 0 & \text{Inhomogeneous phase} \end{cases}$$

C.Nonaka

J. Lenz et. al, PRD 101, no.9, 094512 (2020)

Phase Diagram of GN Model

$$C_{\min} = \min_x C(x) \begin{cases} \gg 0 & \text{Homegeneously chiral broken phase} \\ \approx 0 & \text{Chiral symmetric phase} \\ < 0 & \text{Inhomogeneous phase} \end{cases}$$



Existence of inhomogeneous phase, consistent with previous study

J. Lenz et. al, PRD 101, no.9, 094512 (2020)

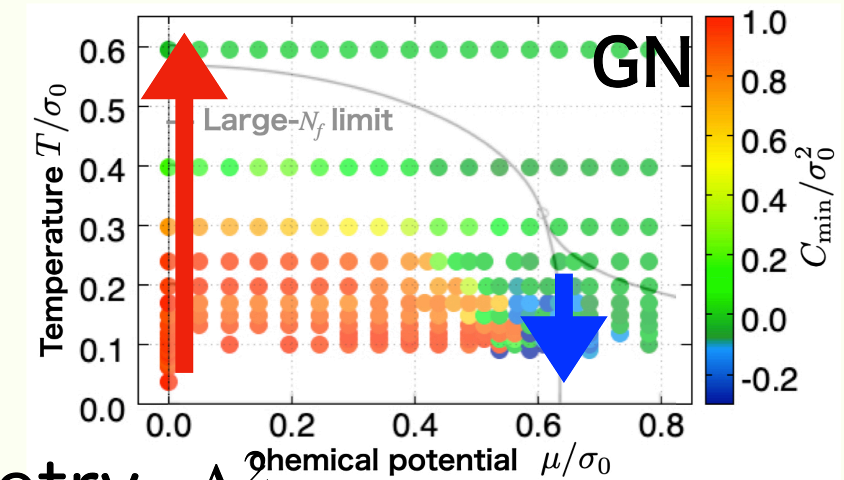
Chiral Gross-Neveu model

$$\mathcal{L} = \bar{\psi} i \gamma^\nu \partial_\nu \psi + \frac{g^2}{2N}\left[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma^5 \psi)^2\right]$$

$\sigma \sim \langle \bar{\psi}\psi \rangle$ $\pi \sim \langle \bar{\psi} i \gamma^5 \psi \rangle$

- Parameters

fermion	N_f	$N_s = L/a$	$N_t = 1/Ta$	g^2	$a\Lambda_0$	μ/Λ_0
naive	8	32	4, 6, ..., 32	1.9332	0.4153(3)	0.0, 0.1, ..., 0.9
				1.8132	0.3791(2)	0.0, 0.1, ..., 0.9
				1.7132	0.3436(2)	0.0, 0.1, ..., 0.9



- Numerical results

1. Analysis on homogeneous phase

The order parameter of discrete chiral symmetry Λ^2

@ $T/\Lambda_0 \neq 0,\ \mu/\Lambda_0 = 0$

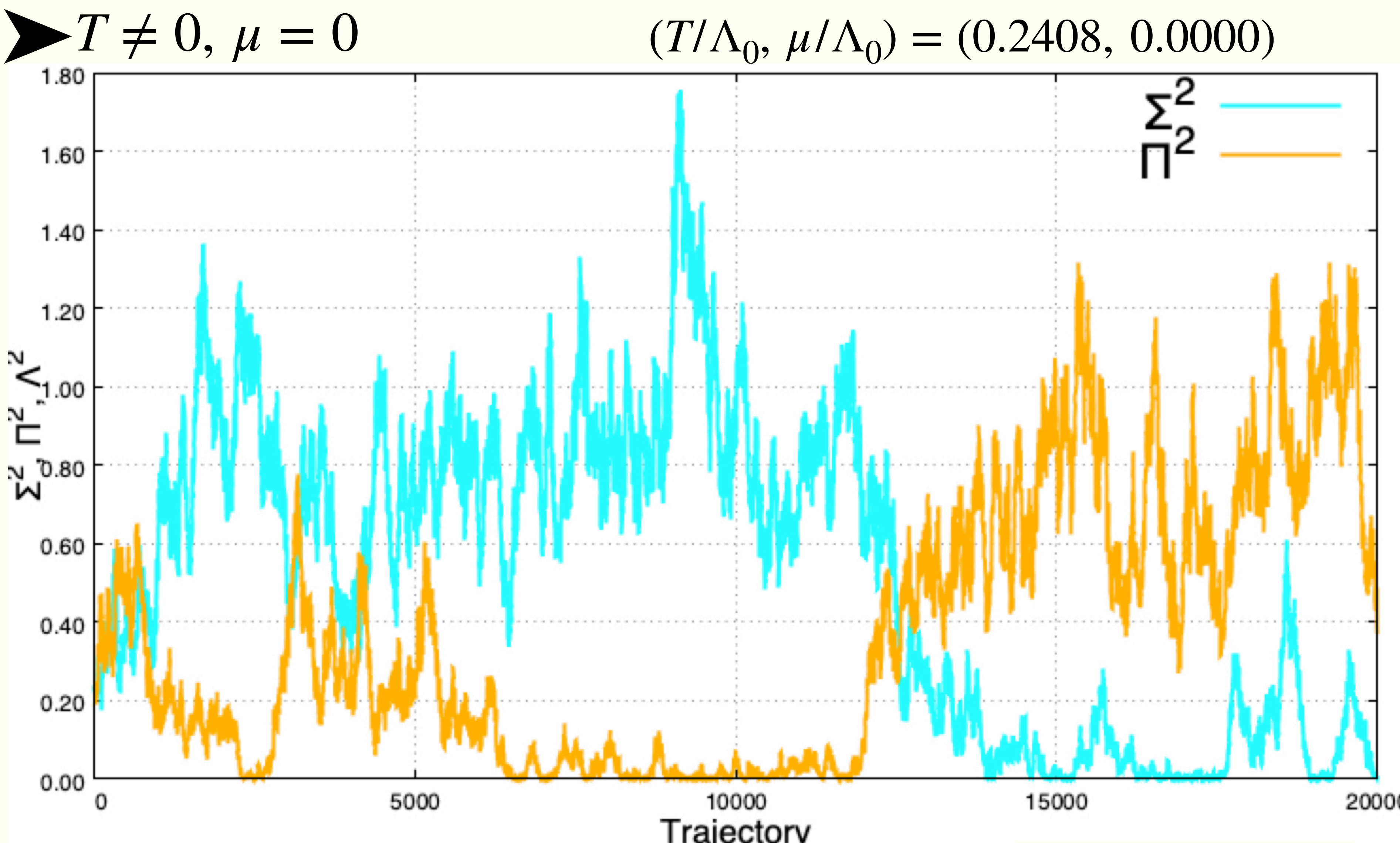
2. Analysis on inhomogeneous phase

Using the minimum value of spatial correlation function $C_\sigma(x)$, $C_\pi(x)$

@ $T/\Lambda_0 \neq 0,\ \mu/\Lambda_0 \neq 0$

1. Analysis on Homogeneous Phase

➤ $T \neq 0,\ \mu = 0$



$$\Sigma^2 = \frac{\langle \bar{\sigma}^2 \rangle}{\Lambda_0^2}$$

$$\Pi^2 = \frac{\langle \bar{\pi}^2 \rangle}{\Lambda_0^2}$$

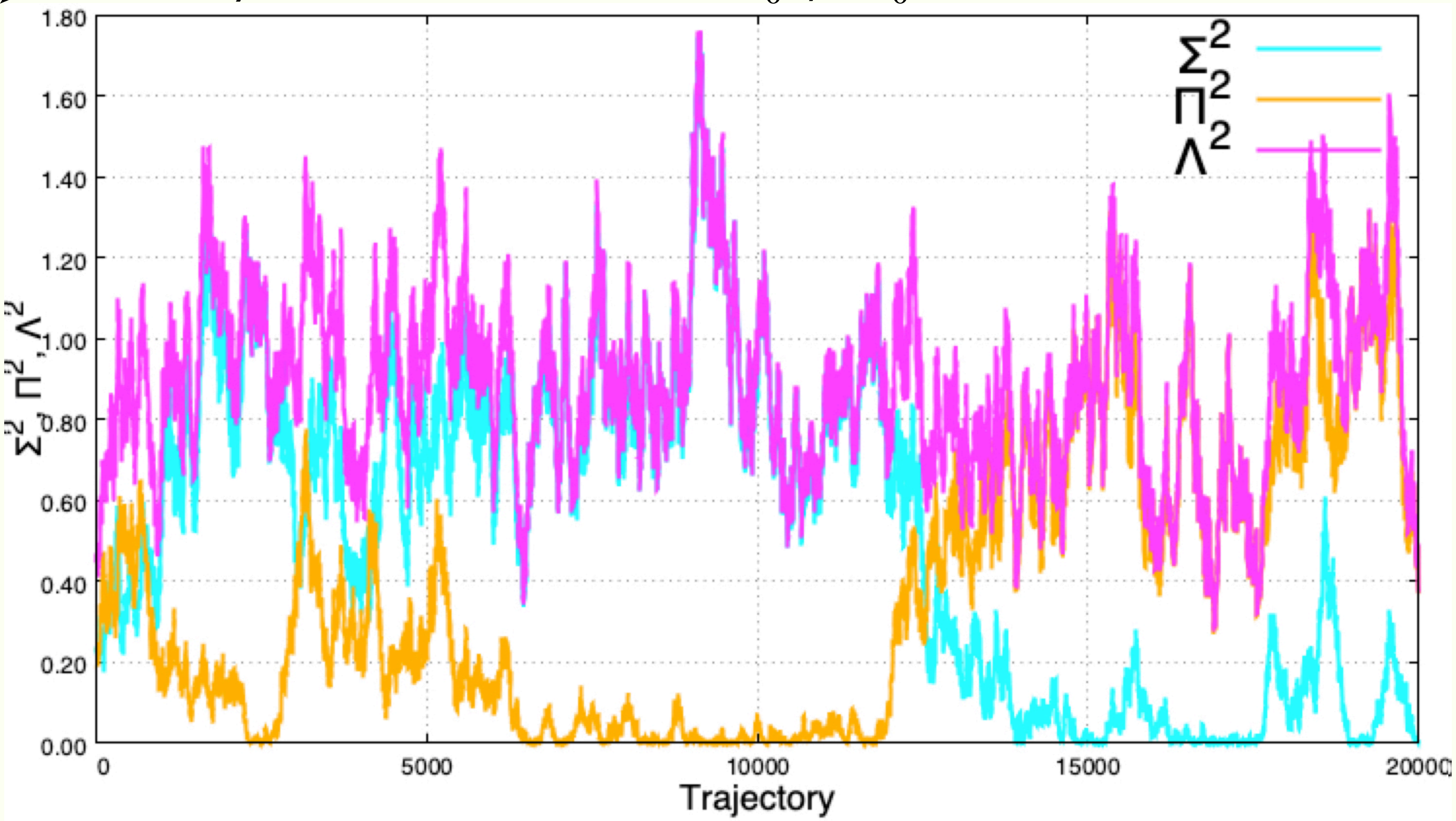
- Π^2 and Σ^2 are fluctuating as a function of time trajectory.
- First, the values of Π^2 and Σ^2 are almost the same.
- Σ^2 increases and Π^2 decreases.
- Around 12500 Π^2 becomes larger than Σ^2.

$\Lambda^2 = \Pi^2 + \Sigma^2$ is constant.

1. Analysis on Homogeneous Phase

➤ $T \neq 0,\ \mu = 0$

$(T/\Lambda_0,\ \mu/\Lambda_0) = (0.2408,\ 0.0000)$



$$\Sigma^2 = \frac{\langle \bar{\sigma}^2 \rangle}{\Lambda_0^2}$$

$$\Pi^2 = \frac{\langle \bar{\pi}^2 \rangle}{\Lambda_0^2}$$

$$\Lambda^2 = \Sigma^2 + \Pi^2$$

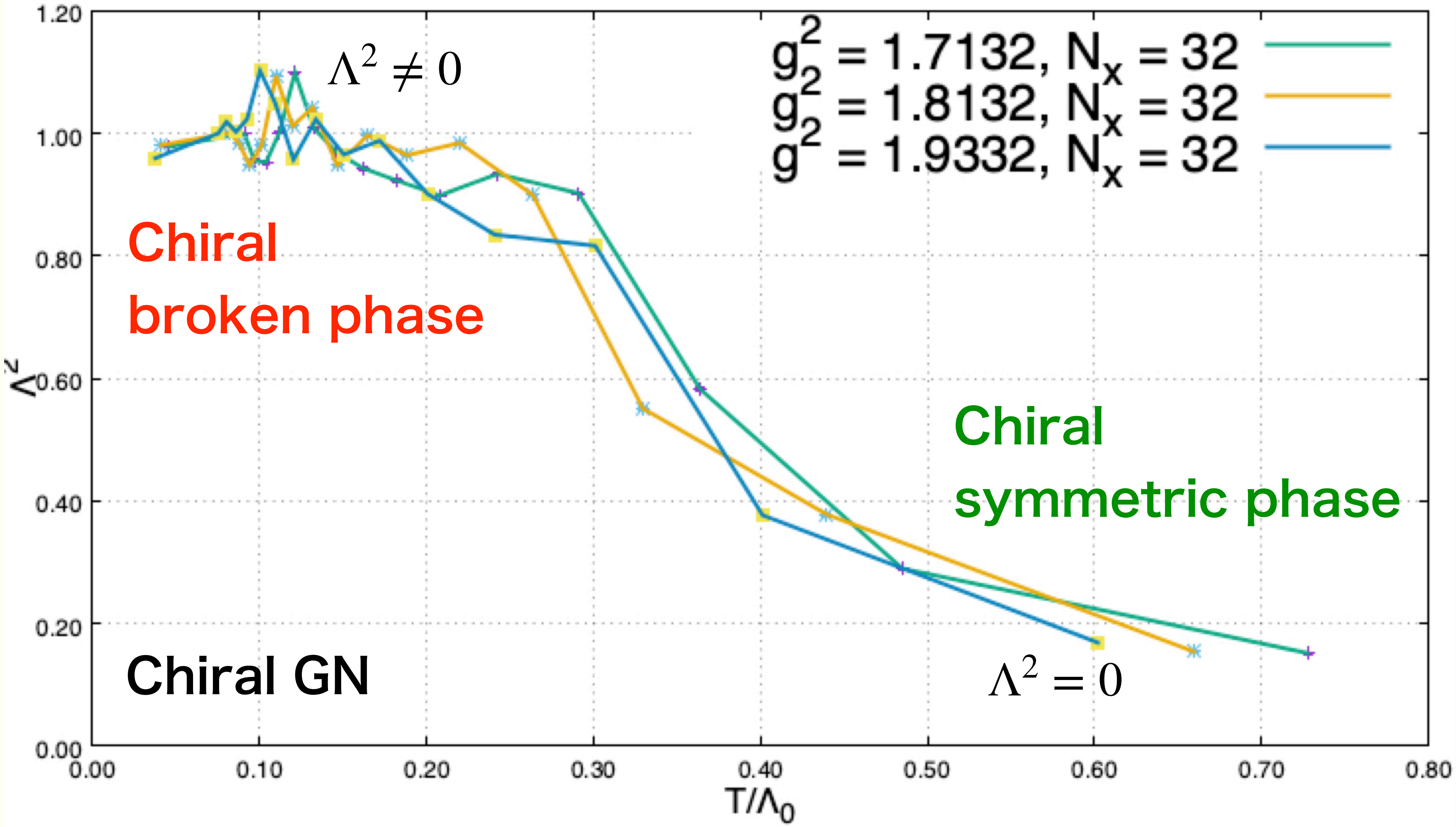
- Π^2 and Σ^2 are fluctuating as a function of time trajectory.
- First, the values of Π^2 and Σ^2 are almost the same.
- Σ^2 increases and Π^2 decreases.
- Around 12500 Π^2 becomes larger than Σ^2.

$\Lambda^2 = \Pi^2 + \Sigma^2$ is constant.

Λ^2 is an order parameter for chiral symmetry.

1. Analysis on homogeneous phase

➤ Temperature dependence of Λ^2 at $\mu = 0$.

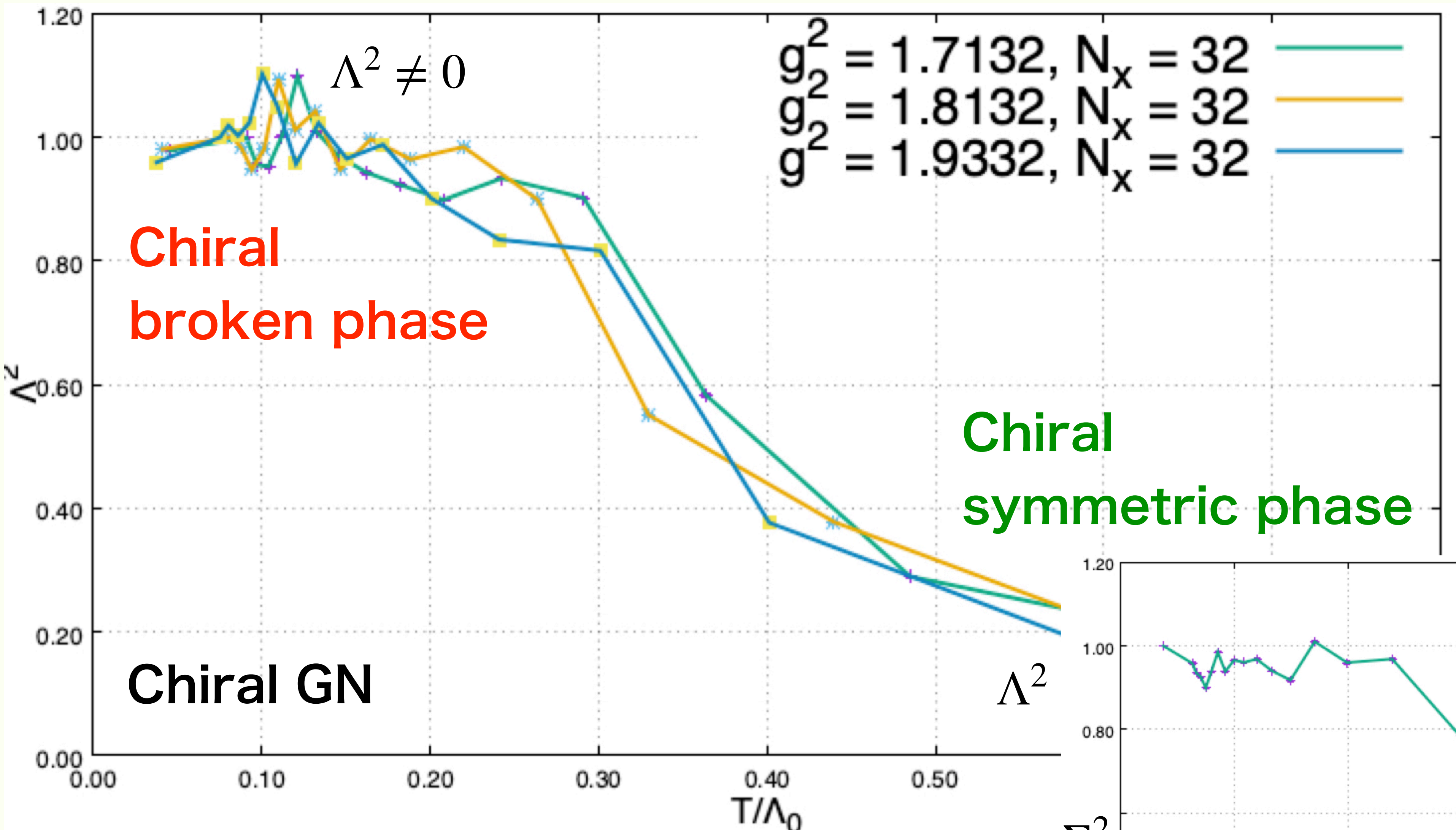


coupling	lattice
1.7132	0.3436
1.8132	0.3791
1.9332	0.4153

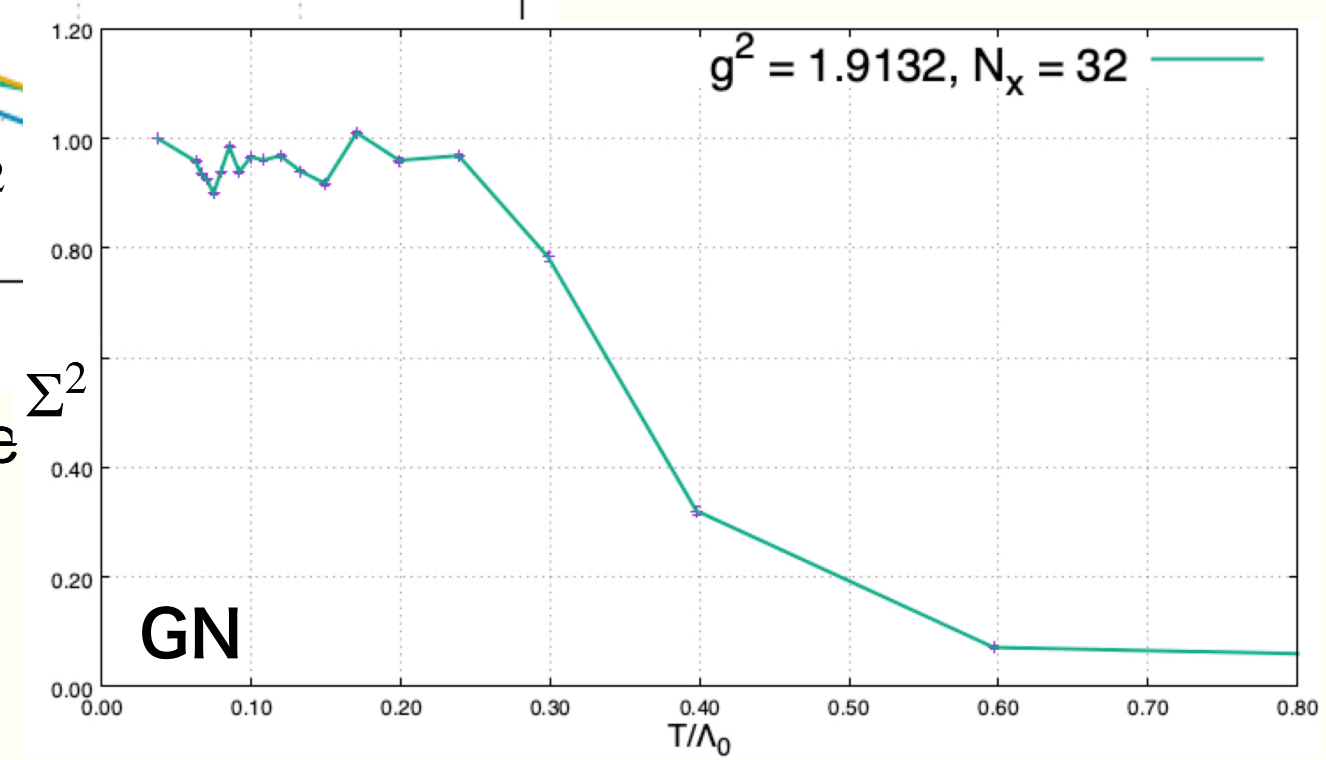
- Λ^2 is an order parameter for chiral symmetry. ex. GN model

1. Analysis on homogeneous phase

➤ Temperature dependence of Λ^2 at $\mu = 0$.



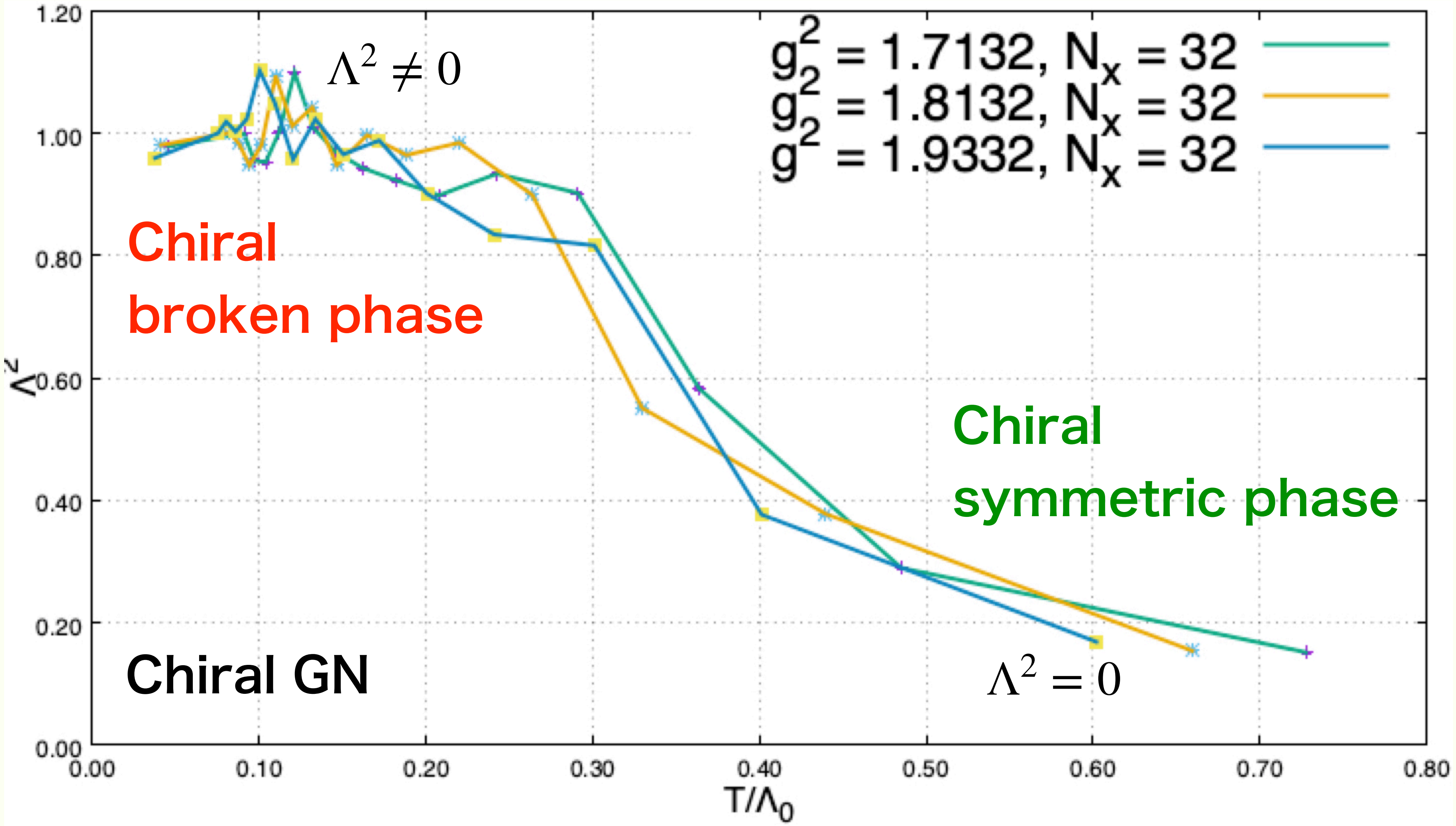
coupling	lattice
1.7132	0.3436
1.8132	0.3791
1.9332	0.4153



- Λ^2 is an order parameter for chiral symme

1. Analysis on homogeneous phase

➤ Temperature dependence of Λ^2 at $\mu = 0$.



coupling	lattice
1.7132	0.3436
1.8132	0.3791
1.9332	0.4153

- Λ^2 is an order parameter for chiral symmetry. ex. Σ^2 in GN model.
- No lattice spacing dependence.
- Consistent with mean field theory analysis.

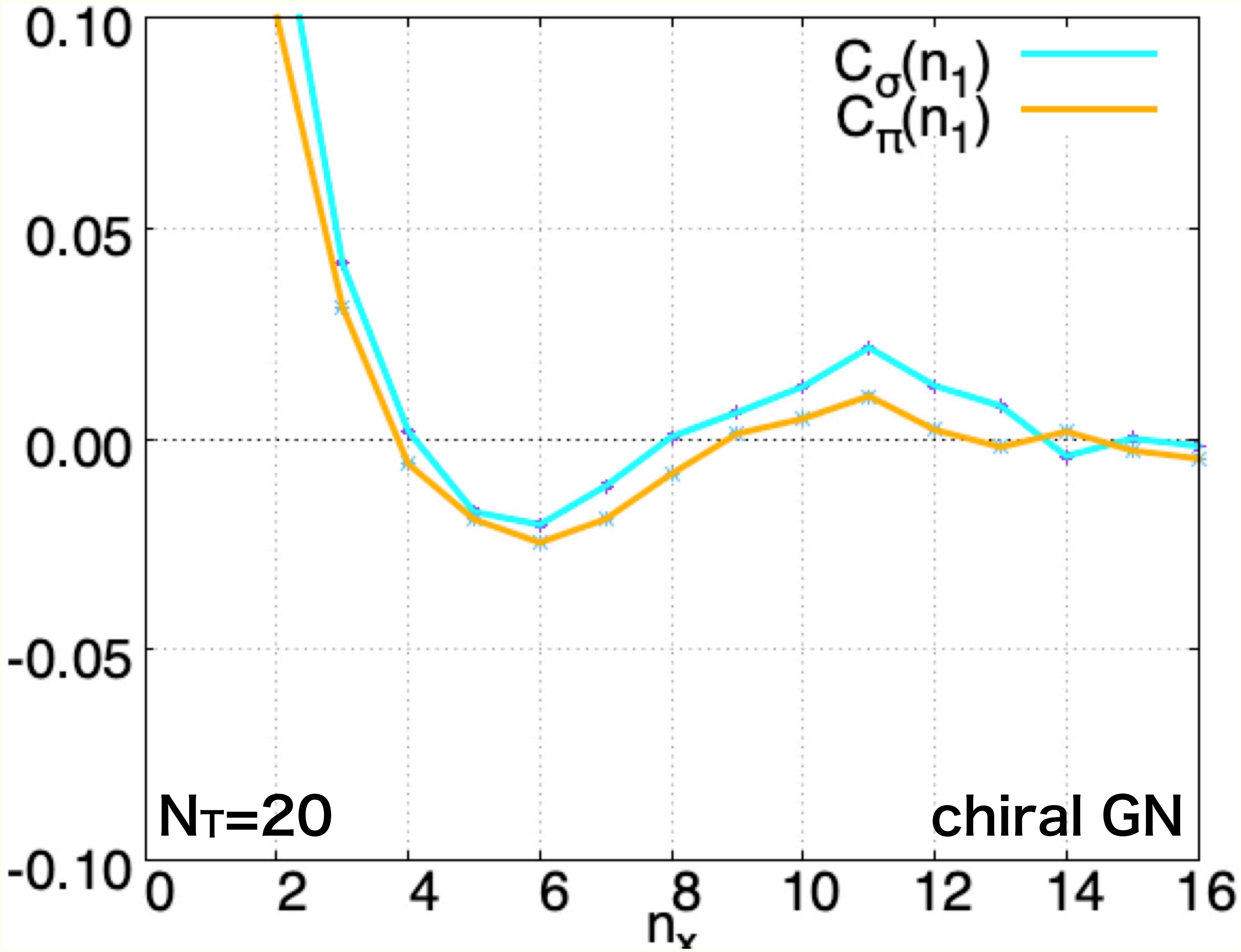
Restoration of chiral symmetry at high T. No lattice spacing dependence.

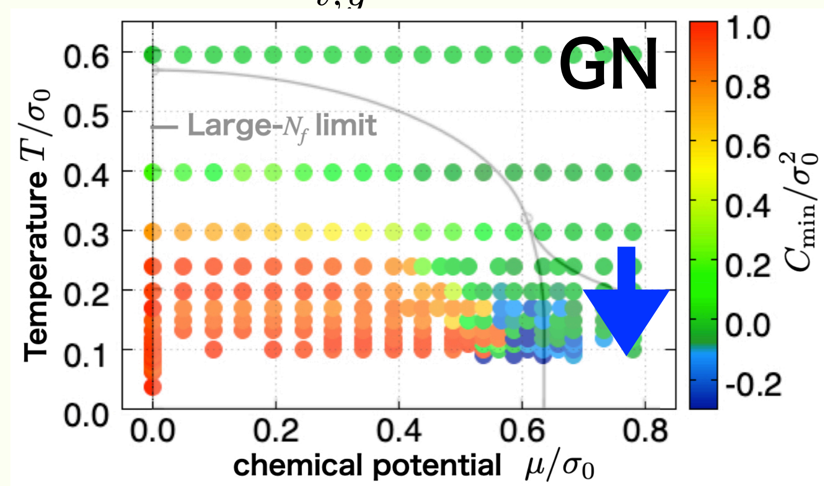
2. Analysis on inhomogeneous phase

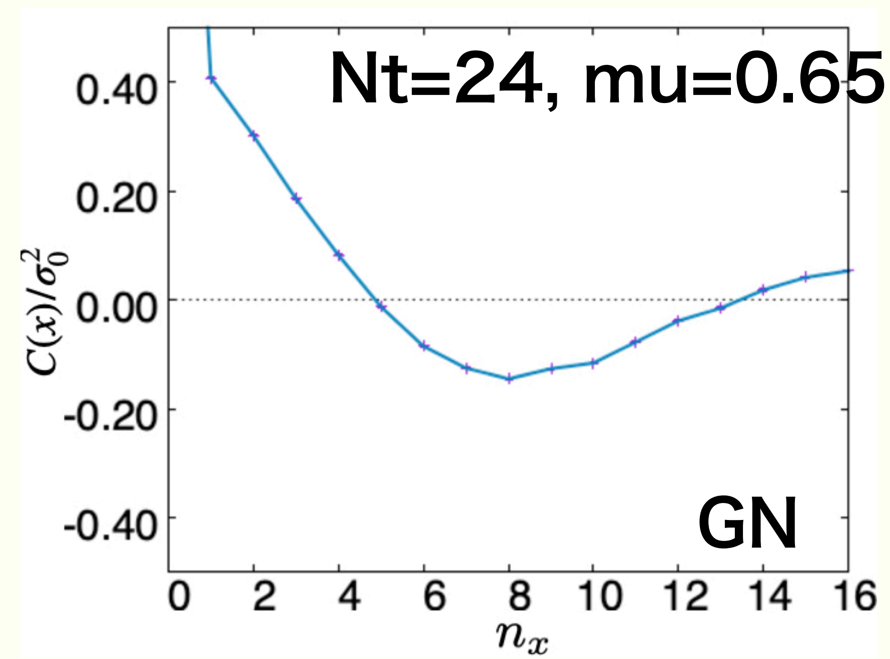
➤ Correlators of σ and π

$$C_\sigma(x) = \frac{1}{N_t N_x} \sum_{t,y} \langle \sigma(t, y+x)\sigma(t,y) \rangle$$

$$C_\pi(x) = \frac{1}{N_t N_x} \sum_{t,y} \langle \pi(t, y+x)\pi(t,y) \rangle$$







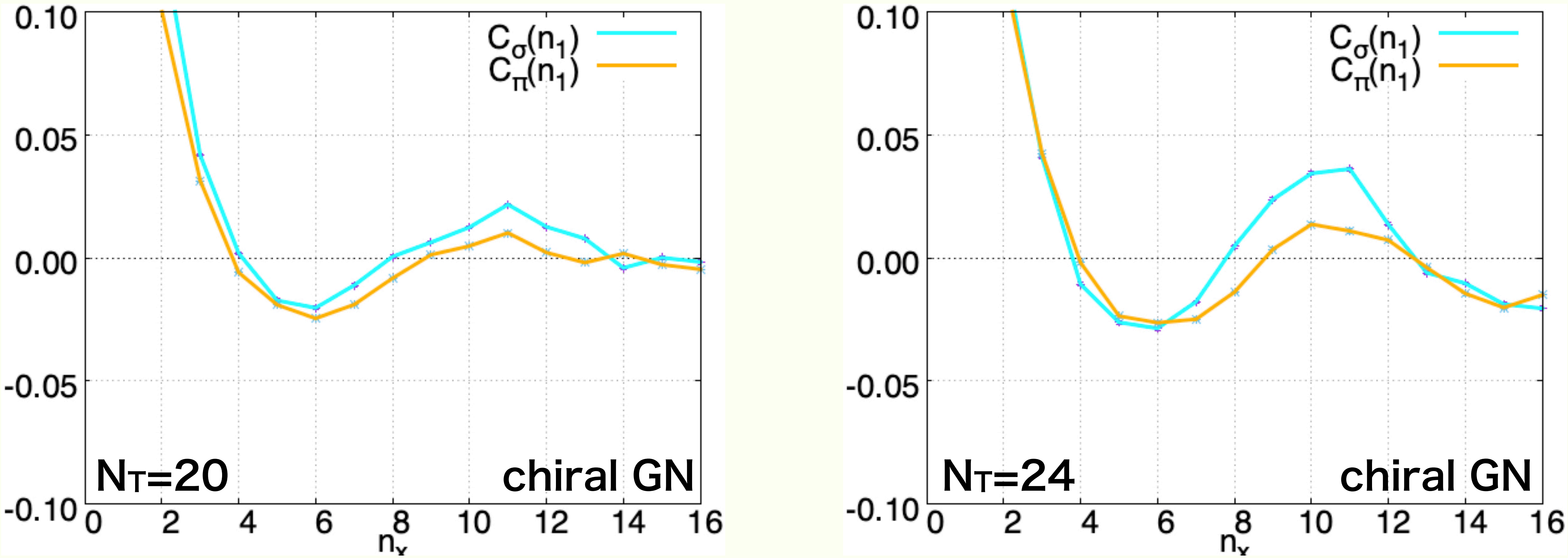
$(T/\Lambda_0, \mu/\Lambda_0) = (0.1204, 0.8886)$

Existence of inhomogeneous phase

- C_σ and C_π are fluctuating
- Amplitude of correlates of chiral GN is smaller than that of GN.

  → The fluctuation is decomposed into C_σ and C_π.

2. Analysis on inhomogeneous phase



$(T/\Lambda_0,\ \mu/\Lambda_0) = (0.1204,\ 0.8886)$ $(T/\Lambda_0,\ \mu/\Lambda_0) = (0.1003,\ 0.8886)$

- Existence of inhomogeneous phase at lower T
- Amplitude at lower T becomes larger. The tendency is also found in GN model.
- More detailed analysis: correlator of Λ, cross term of σ and π, phase difference between σ and π

Existence of inhomogeneous phase, no lattice spacing dependence

Analysis Method for Phases

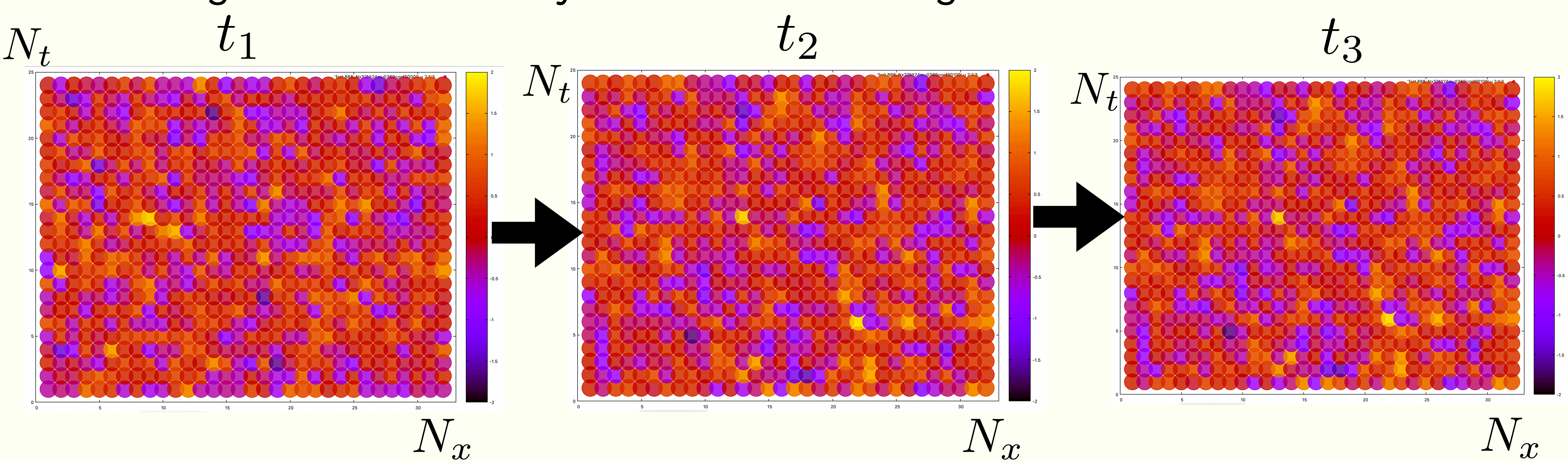
✣ Characteristic structure in configurations

- Correlators:

$$C_{\min} = \min_x C(x) \begin{cases} \gg 0 & \text{Homegeneously chiral broken phase} \\ \approx 0 & \text{Chiral symmetric phase} \\ < 0 & \text{Inhomogeneous phase} \end{cases}$$

✣ Is it possible to extract a pattern without any assumption?

- Configurations of σ in Hybrid Monte Carlo algorithm



K-shape: Analysis Method for Phases

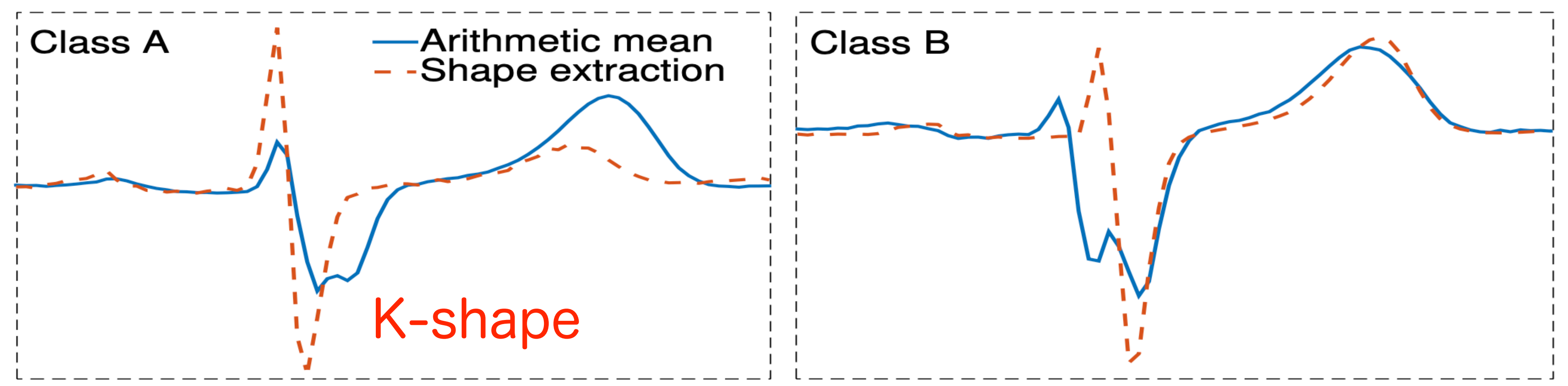
Clustering: partitioning n observables into k clusters

- Model-based: a finite combination of component models
- Feature-based: picking up characteristic structure with reduced dimensional vectors
- Shape-based: similarity search in clusters

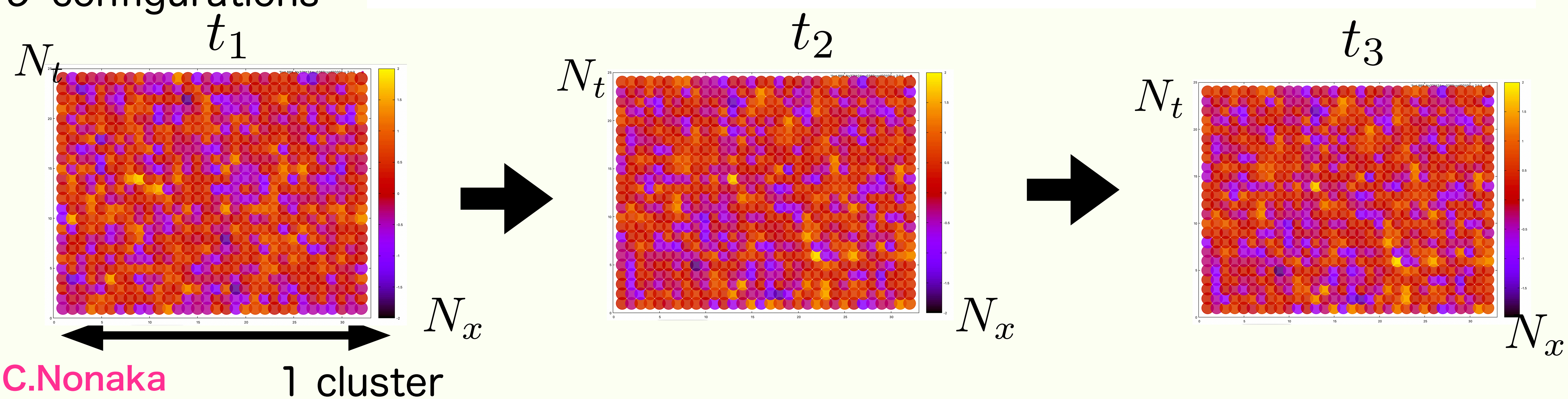
K-shape clustering algorithm: shaped-based clustering

Paparrizos and Gravano, SIGMOD'15

- Shape-based distance (SBD)



σ configurations



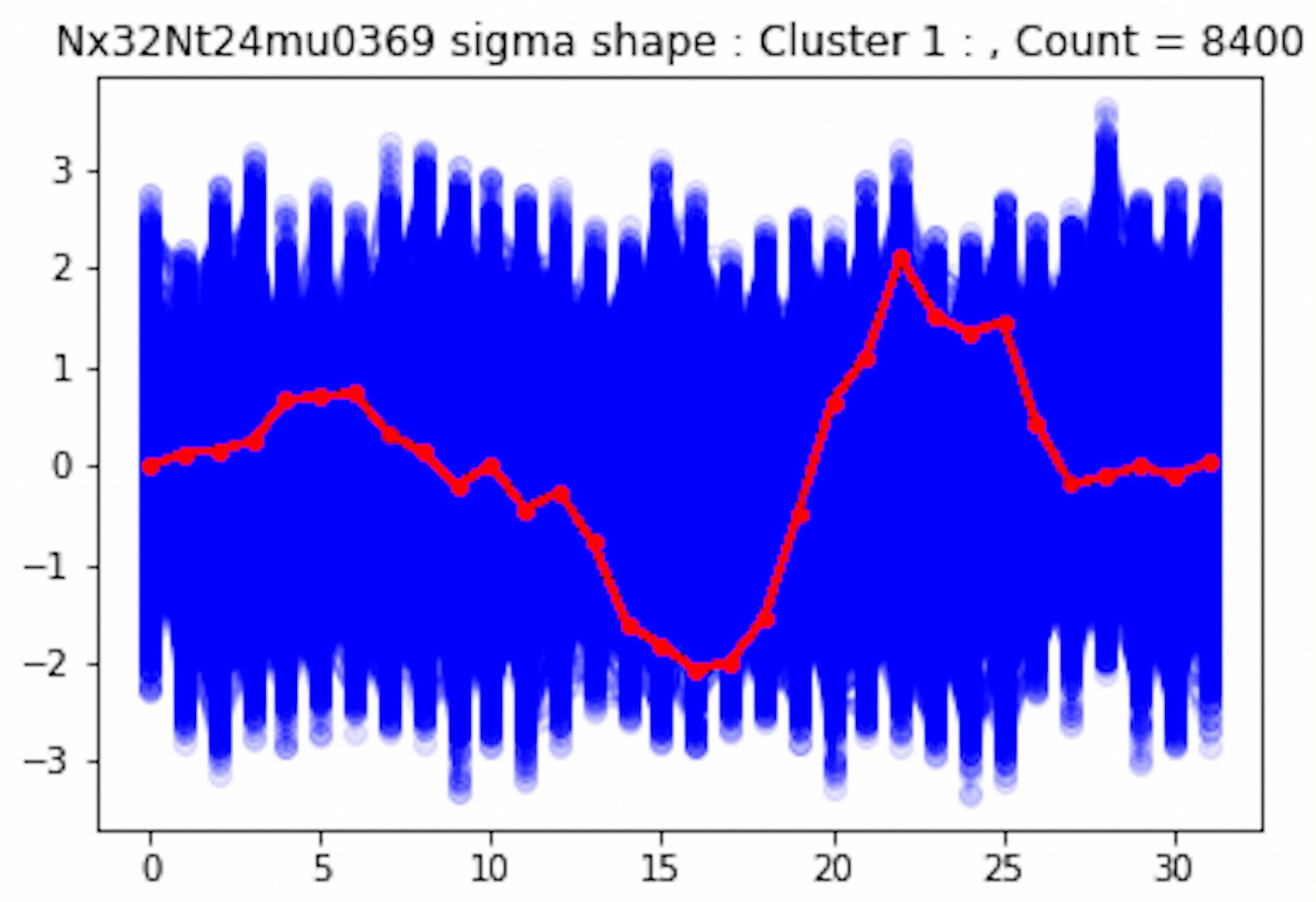
C.Nonaka

K-shape: Analysis Method for Phases

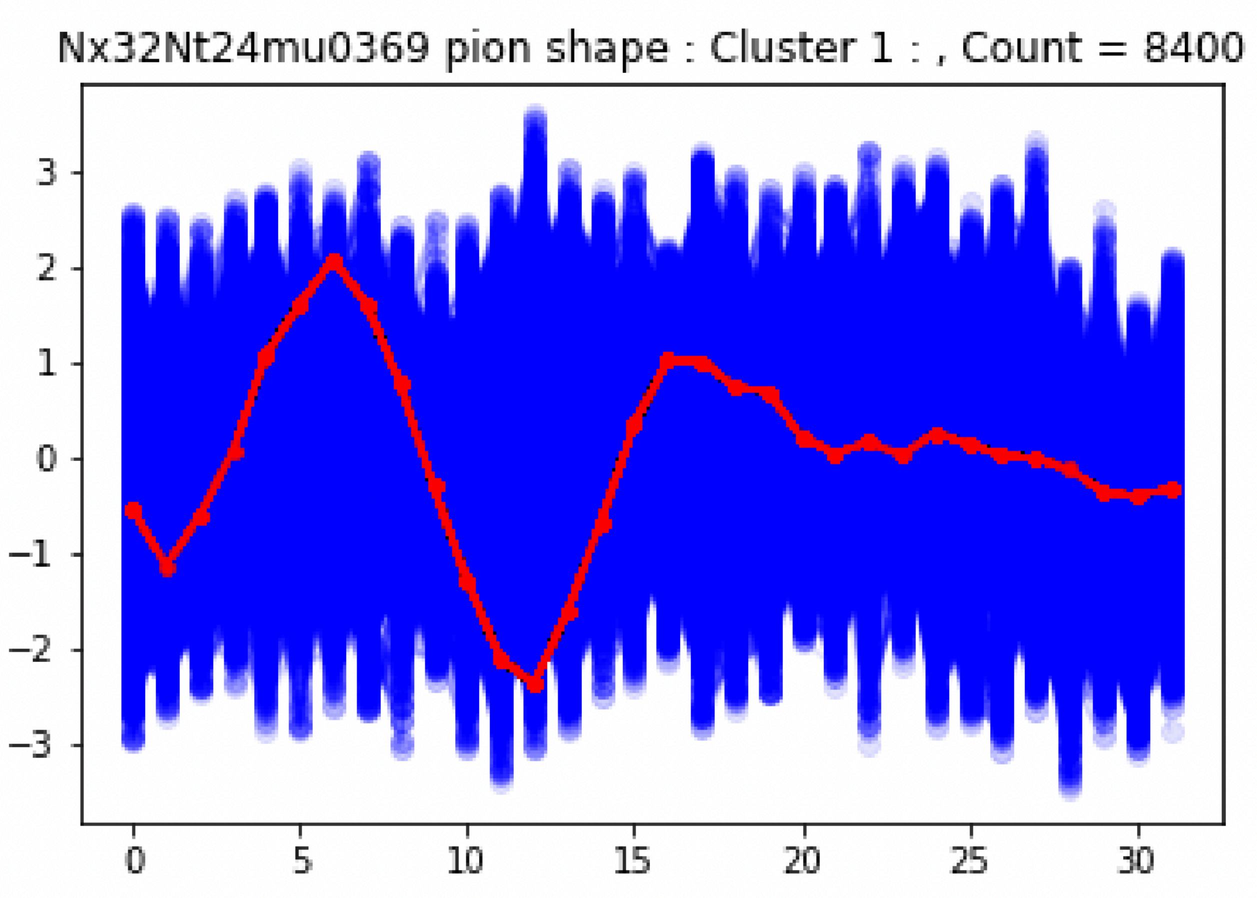
Interesting structure

Preliminary

σ



π



- Interesting structure is observed.
- More statistics
- Larger lattice

Existence of inhomogeneous phase ?

C.Nonaka

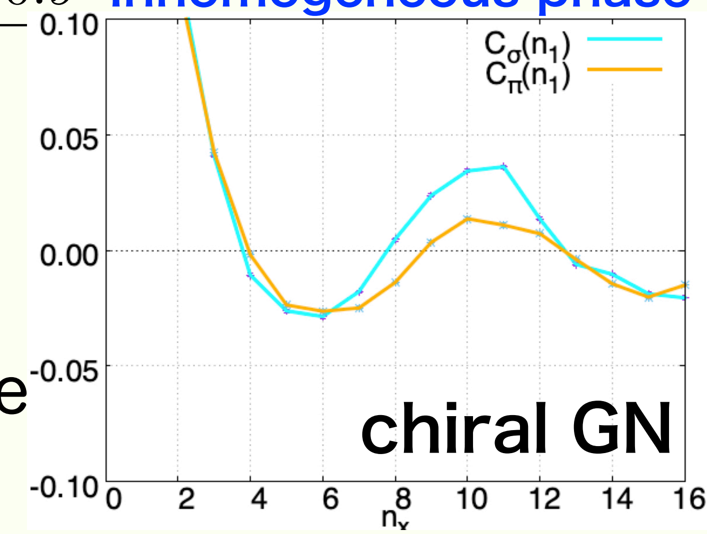
Summary

Existence of inhomogeneous phase in 1+1 dimensional chiral Gross-Neveu model on the lattice.

fermion	N_f	$N_s = L/a$	$N_t = 1/Ta$	g^2	$a\Lambda_0$	μ/Λ_0
naive	8	32	4, 6, ..., 32	1.9332	0.4153(3)	0.0, 0.1, ..., 0.9
				1.8132	0.3791(2)	0.0, 0.1, ..., 0.9
				1.7132	0.3436(2)	0.0, 0.1, ..., 0.9

- $T \neq 0$, $\mu = 0$: restoration of chiral symmetry at high T.
- $T \neq 0$, $\mu \neq 0$: existence of inhomogeneous phase.

Inhomogeneous phase



- Correlator of Λ: cross term of Σ and Λ, phase difference
- Investigation of phase diagram of chiral GN
- long range correlator

- Application of K-shape to finding structures in σ and π configurations

➤ Future task

- Baryon and thermodynamic quantities
- Number of flavor, color degrees of freedom
- Superconducting term