Quantum Nucleation of Topological Solitons

熱場の量子論
@ YITP
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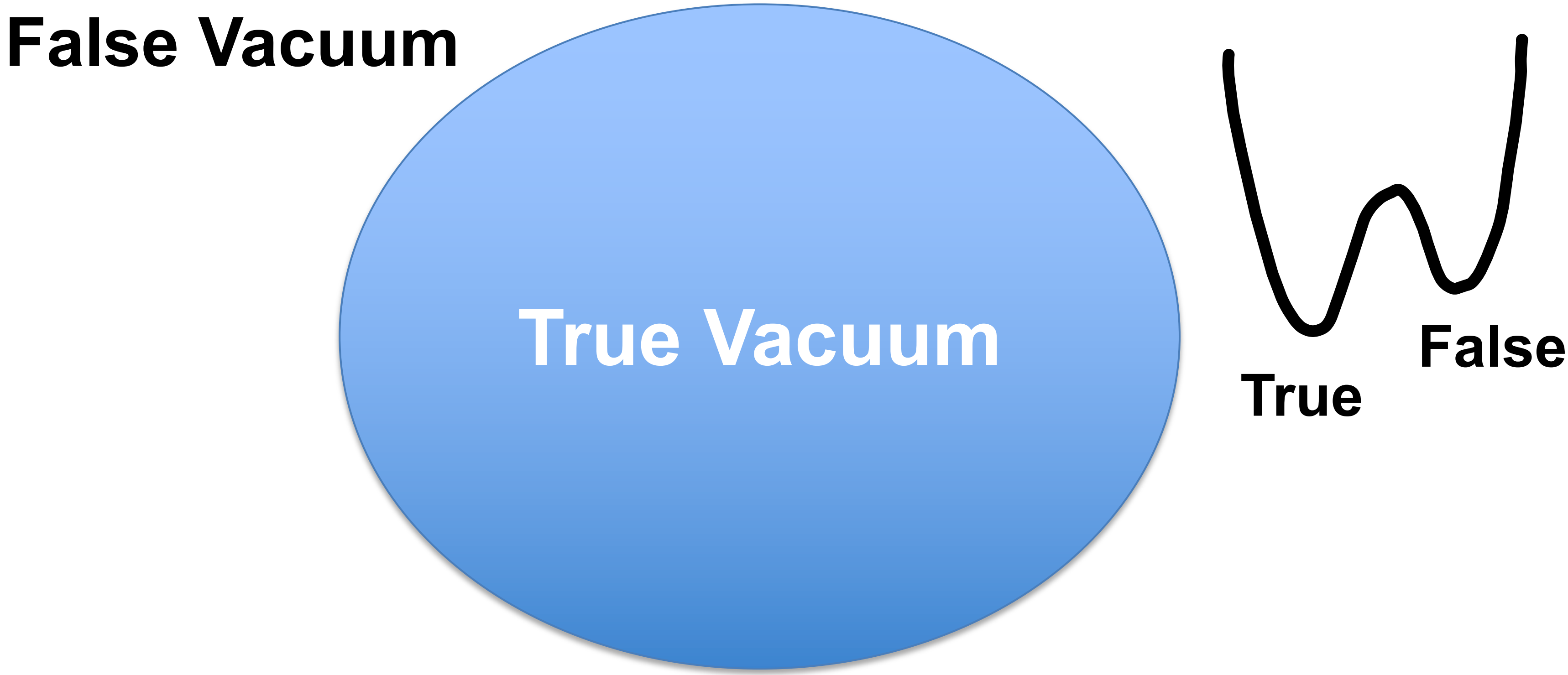
JHEP 09 (2022) 077 [2207.00211 [hep-th]]

See also Higaki, Kamada, Nishimura, 2207.00212 [hep-th]

False vacuum decay **Coleman('77)**
= Quantum nucleation of a bubble



False vacuum decay **Coleman('77)**
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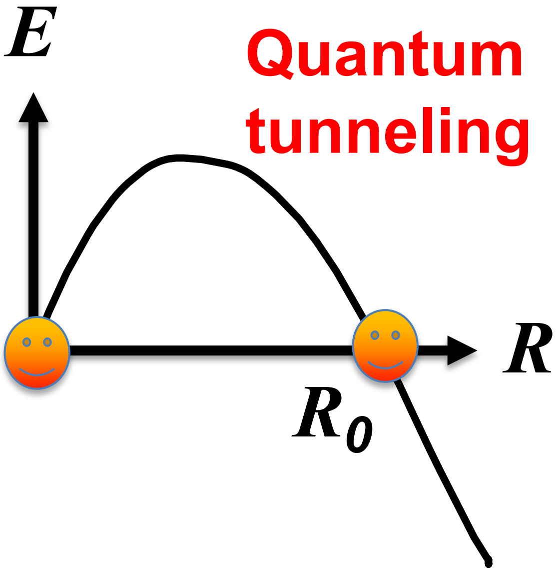
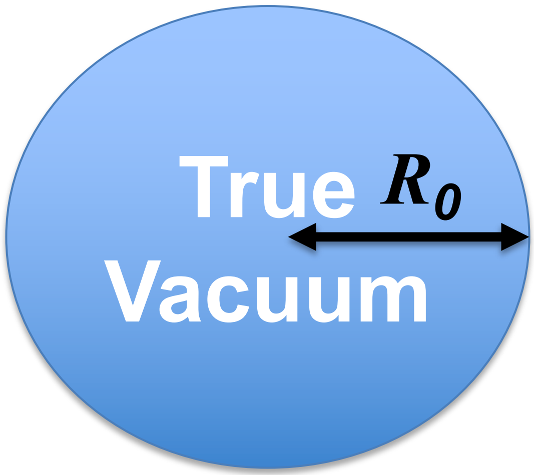
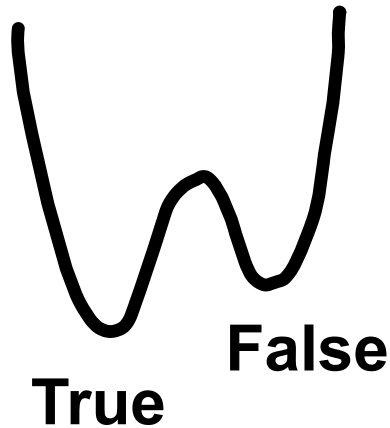
False Vacuum



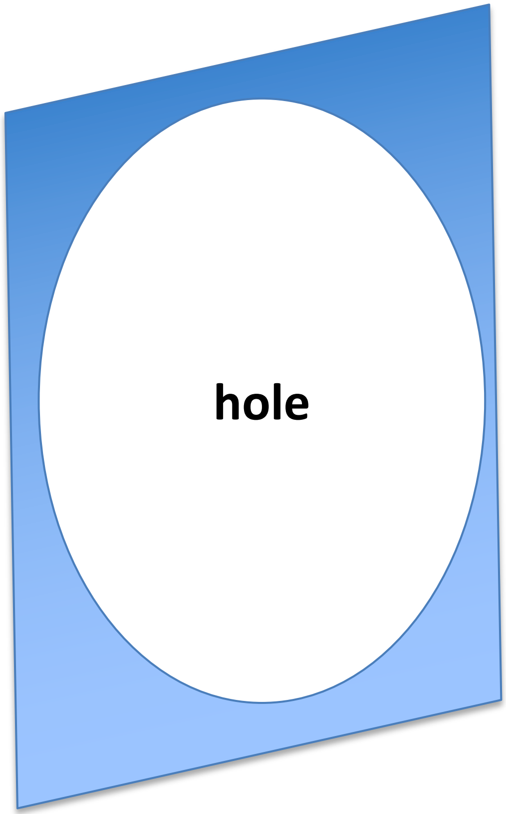




Domain wall tension @surface

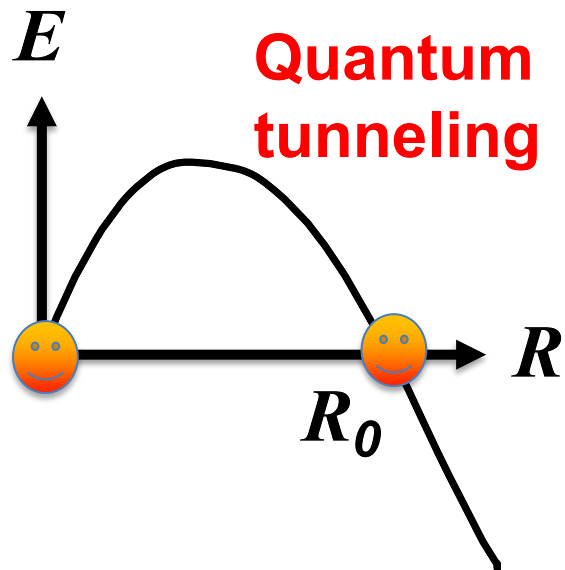
Topological soliton decay Preskill & Vilenkin('93)
= Quantum nucleation of a hole

Domain wall



A hole bound by a string loop

$$E = -\pi R^2 T_{wall} + 2\pi R T_{string}$$



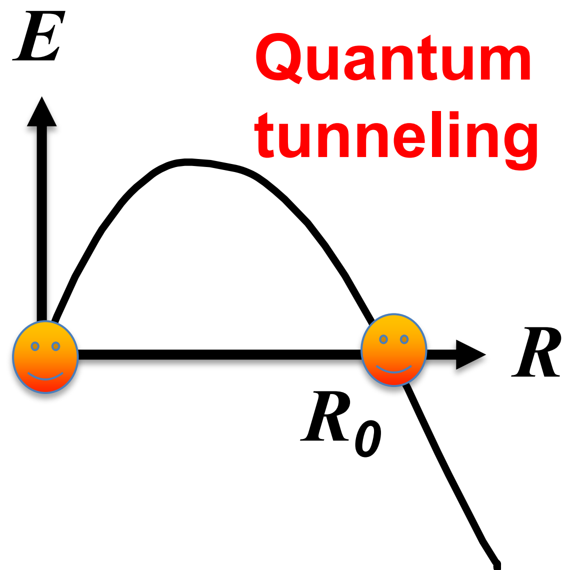
Topological soliton creation Ours ('22)
= Quantum nucleation of a soliton

$$E = +\pi R^2 T_{wall} + 2\pi R T_{string}$$

Possible if $T_{wall} < 0$

Vacuum

A soliton disk bound by a string loop



Topological soliton creation Ours ('22)
= Quantum nucleation of a soliton

Is it possible?

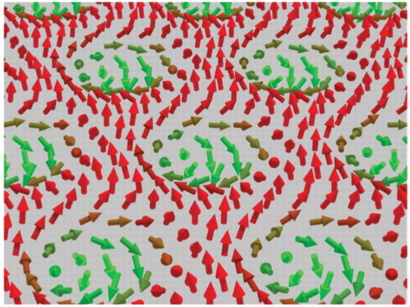
$$E = +\pi R^2 T_{wall} + 2\pi R T_{string}$$

YES!!

Possible if $T_{wall} < 0$

Solitonic ground states

c



1. Chiral soliton lattices in chiral magnets
2. Skyrmion lattices in chiral magnets
3. Chiral soliton lattices in QCD under
   - strong magnetic field Son-Stephanov('07), Brauner-Yamamoto ('16)
   - rapid rotation Huang-Nishimura-Yamamoto('17)

Both relevant for QGP

Other situation: de Sitter space Basu-Guth-Vilenkin('91)

The model at IR (common for chiral magnets & QCD)
= sine-Gordon model + topological term

$$\mathcal{L}_{\mathrm{IR}} = v^2\left[(\partial_\mu\theta)^2 + 2m^2(\cos\theta - 1) + \underline{c\boldsymbol{B}\cdot\nabla\theta}\right]$$

CME for magnetic field
CVE for rotation

$$\mathcal{H}_{\mathrm{IR}} = v^2\left[\dot{\theta}^2 + (\nabla\theta)^2 - 2m^2(\cos\theta - 1) - c\boldsymbol{B}\cdot\nabla\theta\right]$$

The UV theory = axion (Goldstone) model + topo

$$\mathcal{L}_{\mathrm{UV}} = |\partial_\mu\phi|^2 - \frac{\lambda}{4}\left(|\phi|^2 - v^2\right)^2 + vm^2(\phi + \phi^*) + c\boldsymbol{j}\cdot\boldsymbol{B}$$

$$j^\mu = -\frac{i}{2}(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*) = |\phi|^2\partial^\mu\theta\,, \qquad \phi = |\phi|e^{i\theta}$$

For $m = 0$ (Goldstone model)

Nambu-Goldstone (NG) mode + Higgs mode $m_h = v\sqrt{\lambda}$

Global string $\delta_{\rm st} \sim m_h^{-1}$, $\mu|_{m\to 0} \sim \pi v^2 \log(m_h L)$

thickness tension

L: system size (IR cutoff)

For $m \neq 0$, pseudo NG mode

We consider $m_h \gg m \quad \Leftrightarrow \quad m^2 \ll \lambda v^2$

For simplicity $m_h \to \infty$

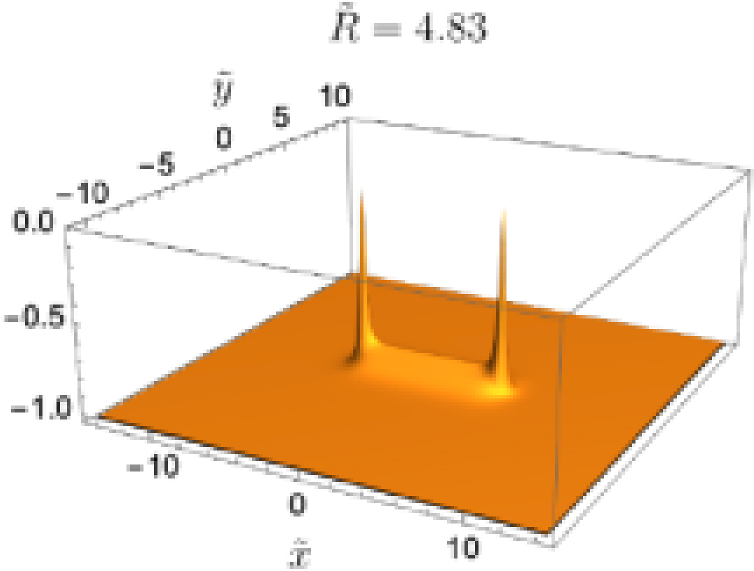
Sine-Gordon(SG) soliton

$\theta = 4 \tan^{-1} e^{mz}$ $\qquad \delta_{\mathrm{dw}} = m^{-1},$ $\qquad \sigma|_{\lambda \to \infty} = 16 m v^2$

thickness $\qquad$ tension

For finite m_h,

1) SG soliton is metastable
2) SG-soliton can be bound by a string
3) String has a ***finite*** **tension** $\mu\big|_{m>0} = \text{const.}$



Missing in Preskill-Vilenkin

Without the topological term,
the decay probability is (Preskill-Vilenkin)C

$$P_{\rm decay} = Ae^{-S}, \qquad S = \frac{16\pi\mu^3}{3\sigma^2}, \qquad R = \frac{2\mu}{\sigma}$$

Nucleation probability with the topological term

$$\tilde{x}^{\mu} = mx, \qquad \tilde{\phi} = v^{-1}\phi, \qquad \tilde{\lambda} = \frac{m_h^2}{m^2}, \qquad \tilde{\boldsymbol{B}} = m^{-1}c\boldsymbol{B}.$$

$$\mathcal{L}_{\rm UV} = m^2 v^2 \left[|\tilde{\partial}_\mu \tilde{\phi}|^2 - \frac{\tilde{\lambda}}{4}\left(|\tilde{\phi}|^2 - 1\right)^2 + \tilde{\phi} + \tilde{\phi}^* + \tilde{\boldsymbol{j}} \cdot \tilde{\boldsymbol{B}} \right]$$

$$\tilde{\boldsymbol{j}} \cdot \tilde{\boldsymbol{B}} = \tilde{B}\tilde{j}_z \cos\alpha$$

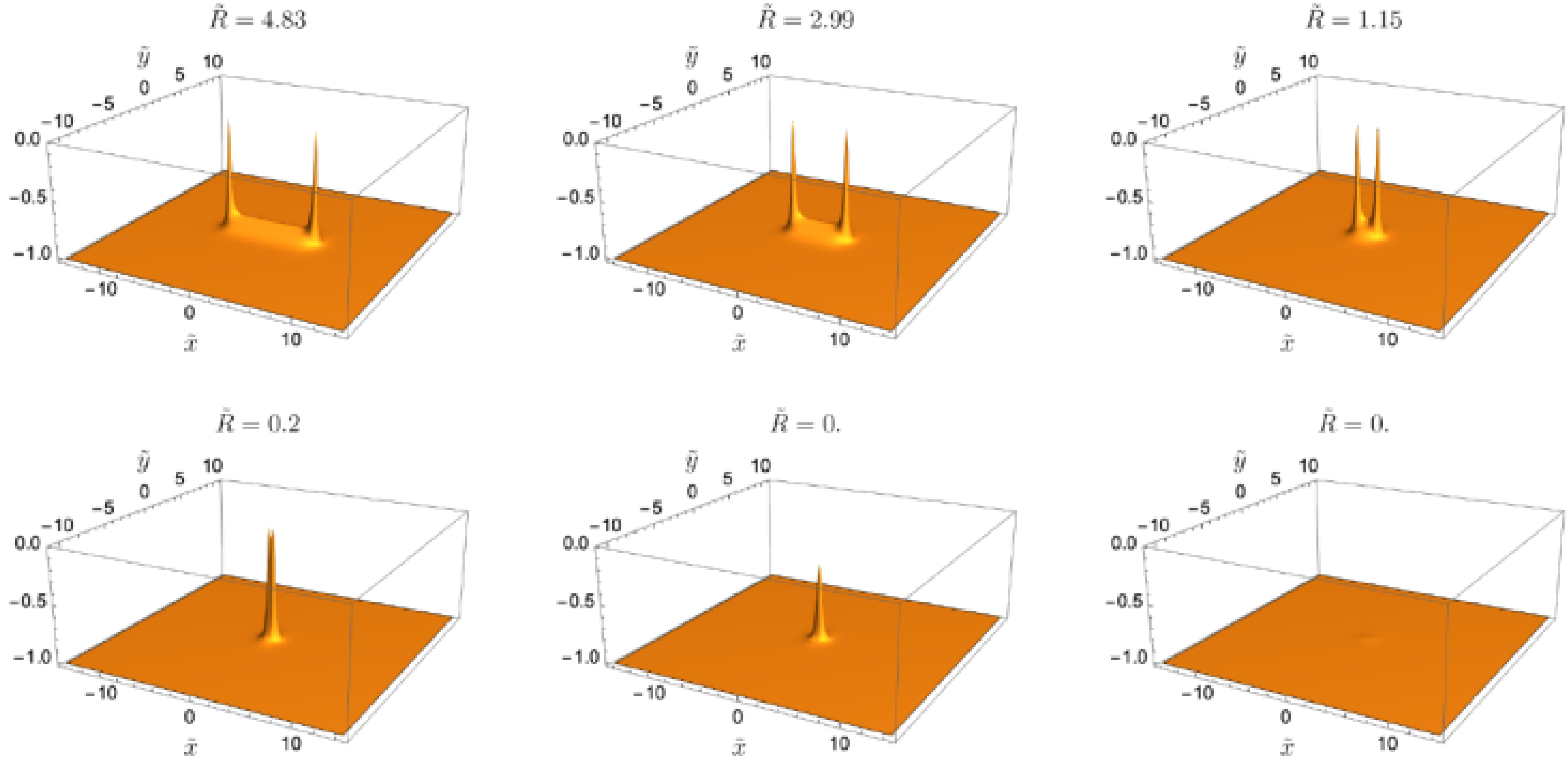
α: Angle between soliton and *B*

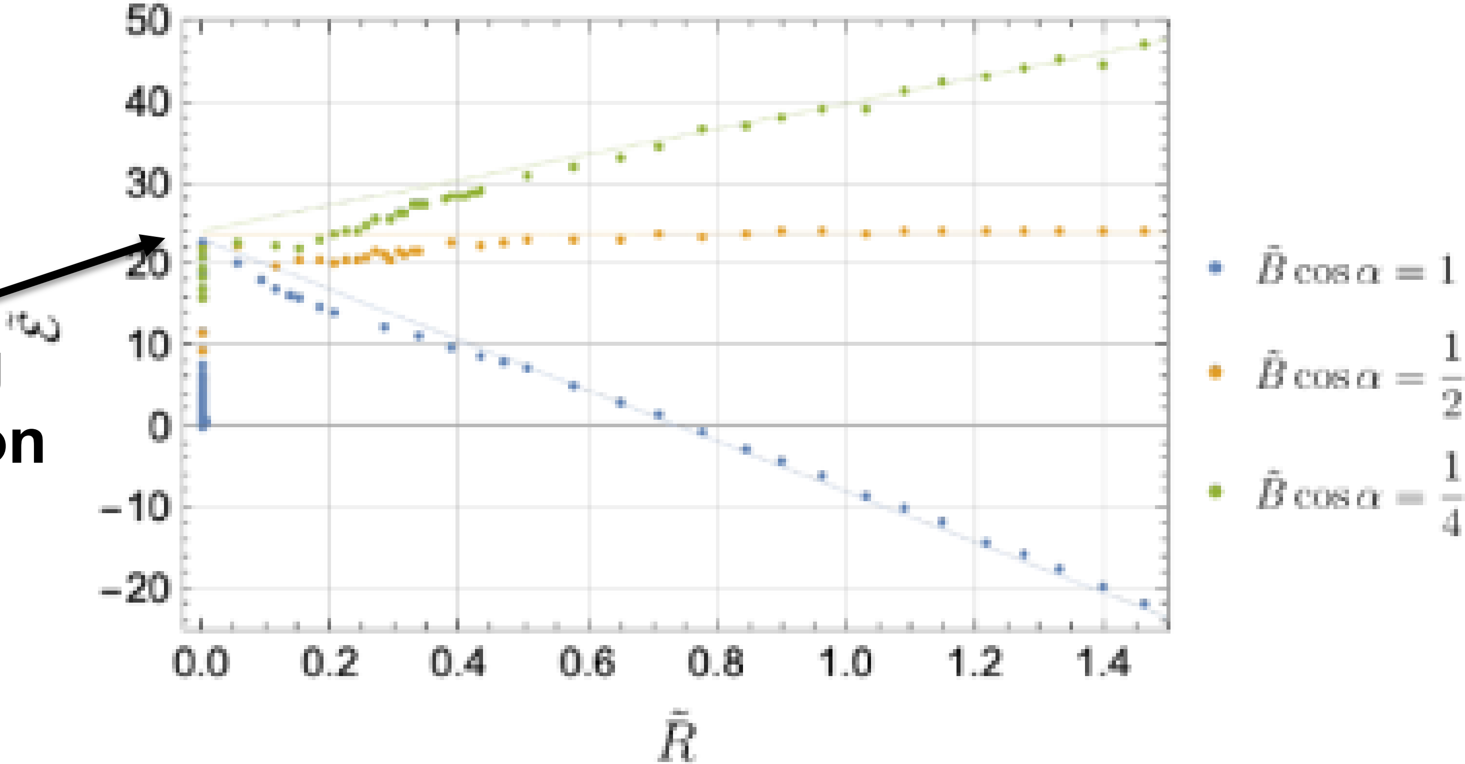
2+1 dim

Thin-defect approx

$$S = 2\pi R\mu + \pi R^2 \sigma. \qquad R_0 = \frac{\mu}{-\sigma}, \qquad S_0 = \frac{\pi\mu^2}{-\sigma}$$

Numerical simulation in 2+1 dim: relaxation





String tension

Decay prob

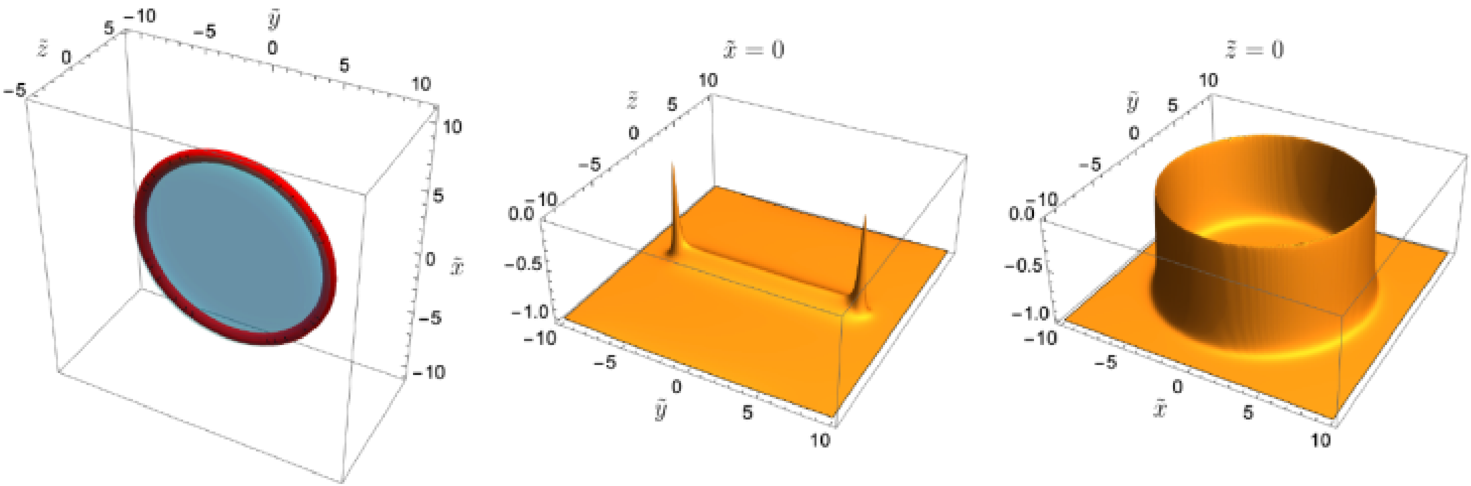
Consistent with thin-defect approx

$$P_{\text{nucleation}} = A \exp\left(-\alpha_1 \frac{v^2}{m} \times 9.0\right)$$

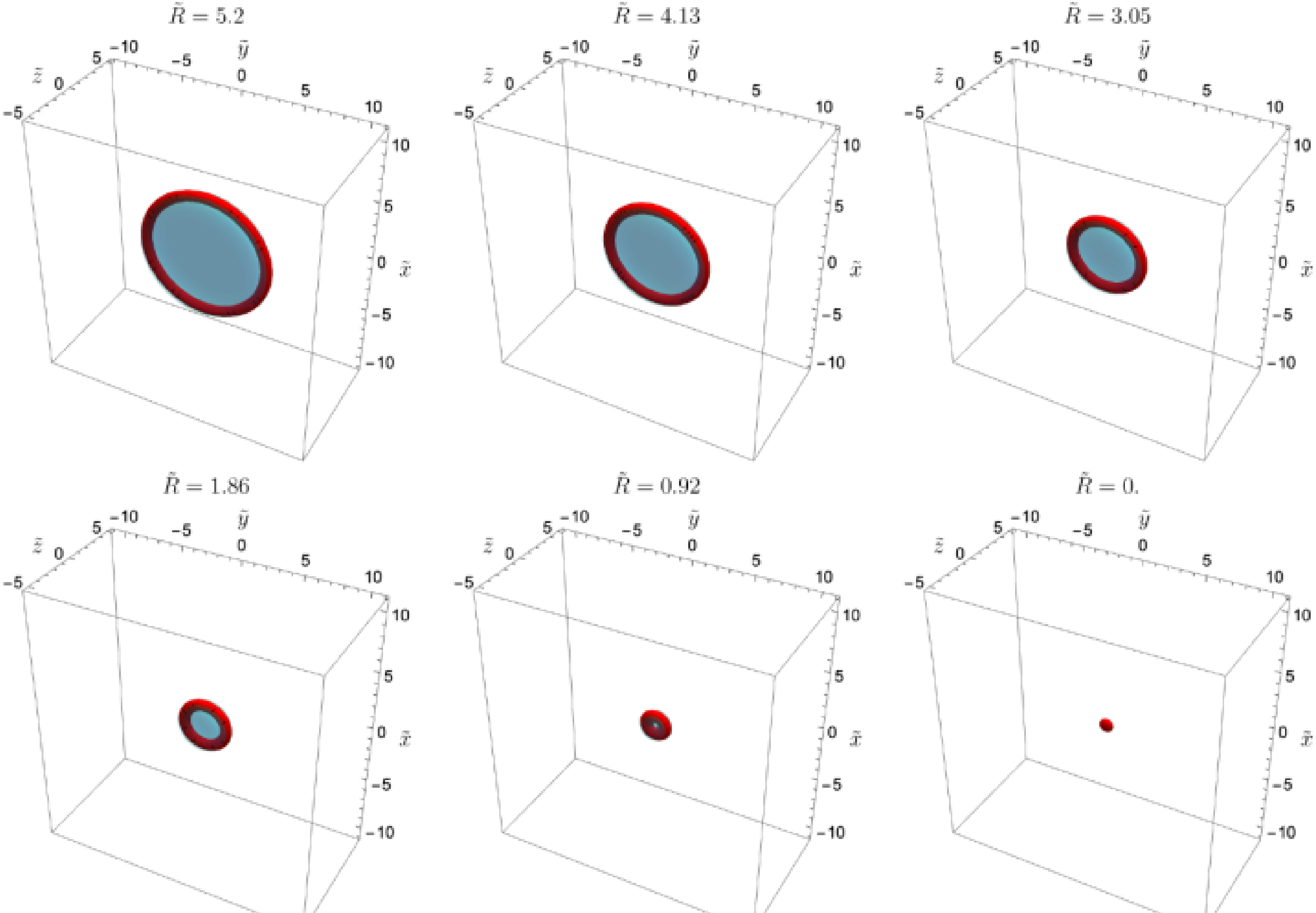
3+1 dim

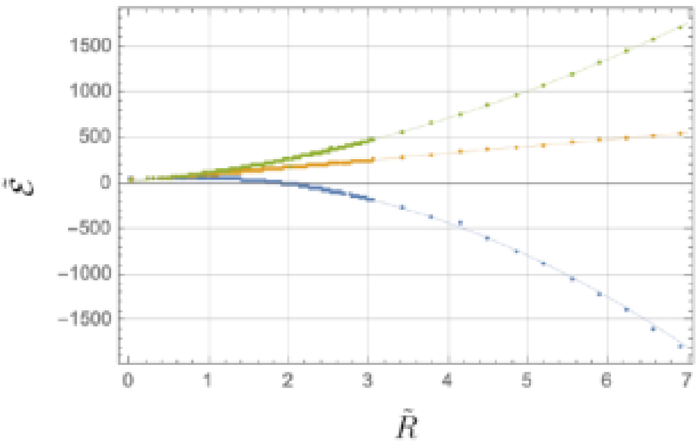
Thin-defect approx

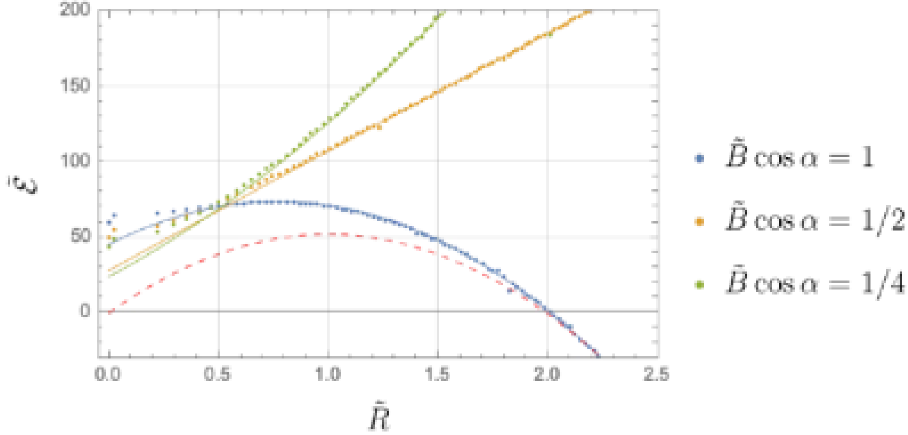
$$S = \pi R^2 \mu + \frac{4\pi}{3} R^3 \sigma \qquad R_0 = \frac{2\mu}{-\sigma}, \qquad S_0 = \frac{16\pi\mu^3}{3\sigma^2}$$



Numerical simulation in 3+1 dim: relaxation







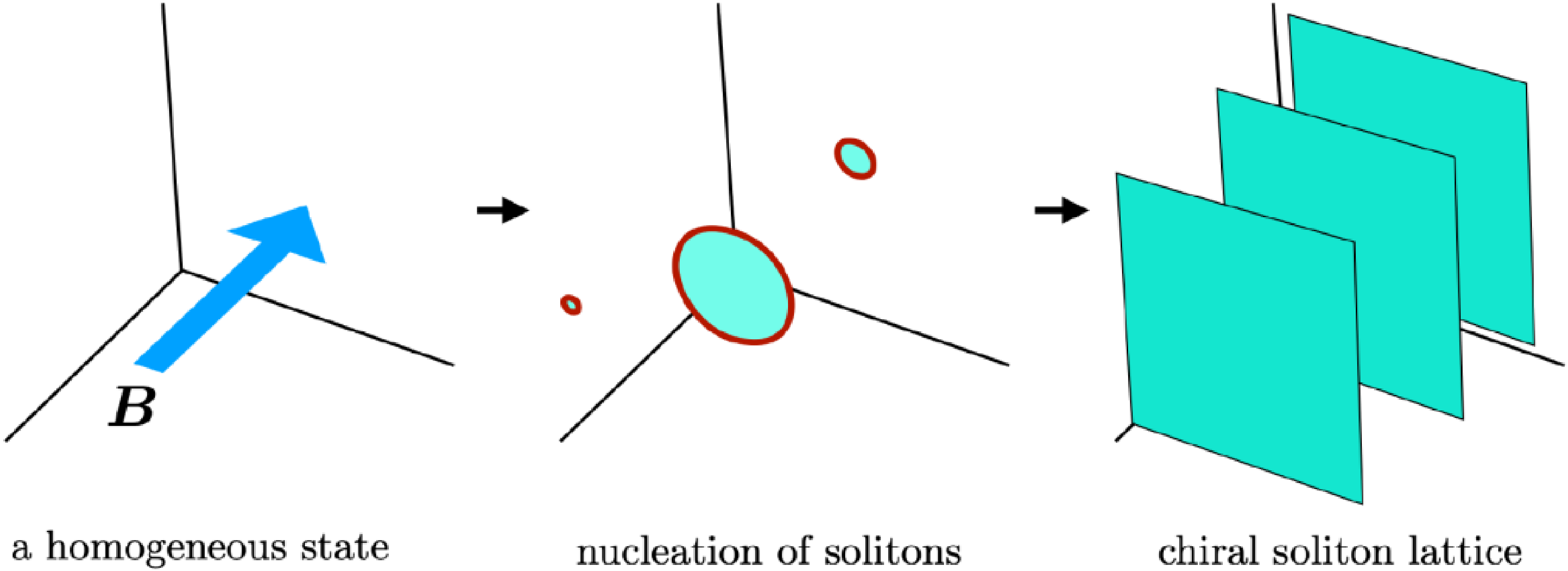
Nucleation probability

$$P_{\text{nucleation}} = A\exp\left(-111\alpha_2\frac{v^2}{m^2}\right)$$

$$\tilde{\mathcal{E}} = \pi\tilde{R}^2 a + 2\pi\tilde{R}b + c.$$

We found a remnant energy *c*
giving a correction to the thin-defect approx

Formation of chiral soliton lattice



Summary

Topological soliton creation Ours ('22)
= Quantum nucleation of a soliton

M.Eto & MN, *JHEP* 09 (2022) 077
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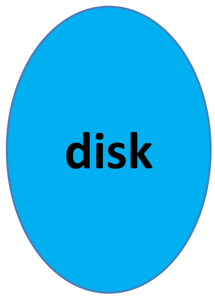
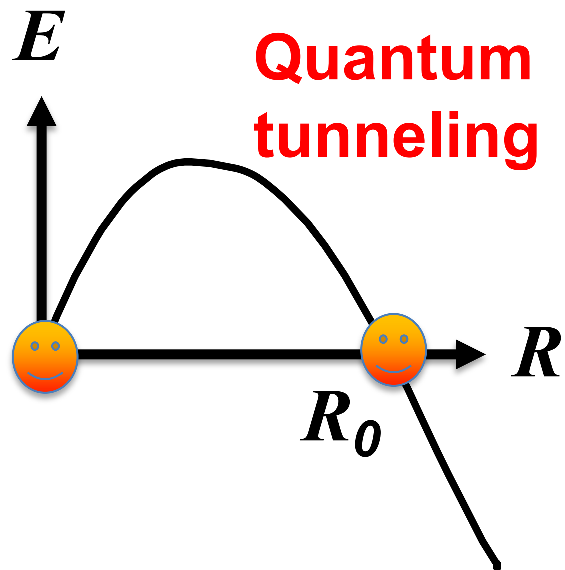
Possible if $T_{wall} < 0$

Vacuum



A soliton disk bound by a string loop



Classification of topological solitons: 3 types

d	Defects		Textures		Gauge Structure	
1	Domain wall, Kink	π_0	Sine-Gordon soliton	π_1		
2	Vortex, Cosmic string	π_1	Lumps, Baby Skyrmion	π_2		
3	Monopole	π_2	Skyrmion, Hopfion	π_3		
4					YM instanton	π_3

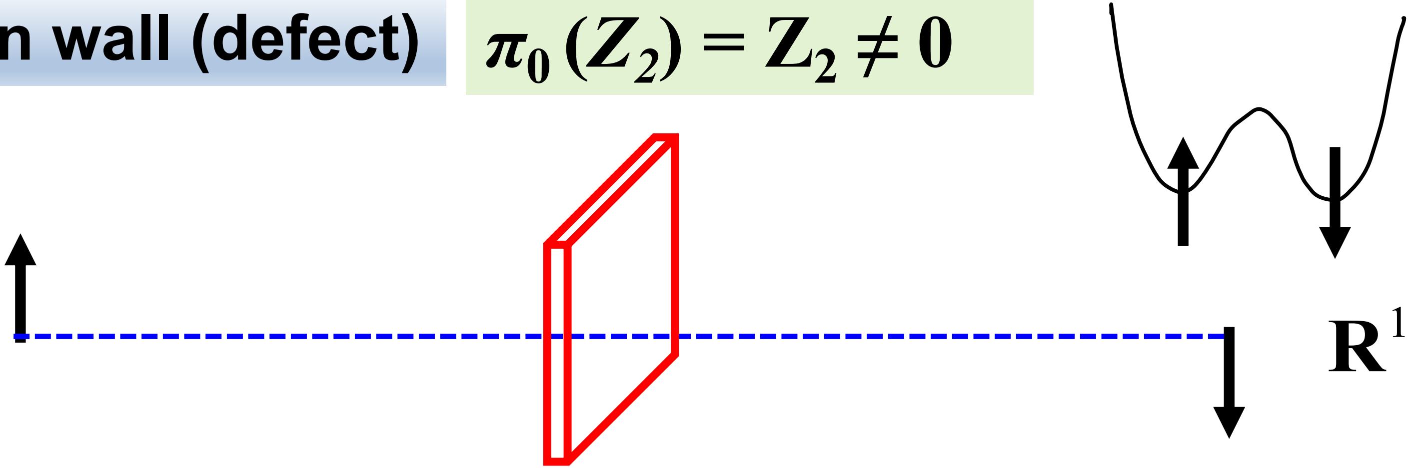
$\partial R^d \cong S^{d-1} \to G/H$ | $R^d + \{\infty\} = S^d \to G/H$ | $\partial R^d \cong S^{d-1} \to G$

$\pi_{d\text{-}1}(G/H) \neq 0$ | $\pi_d(G/H) \neq 0$ | $\pi_{d\text{-}1}(G) \neq 0$

d: codimensions (in which solitons are particles, or on which solitons depend)

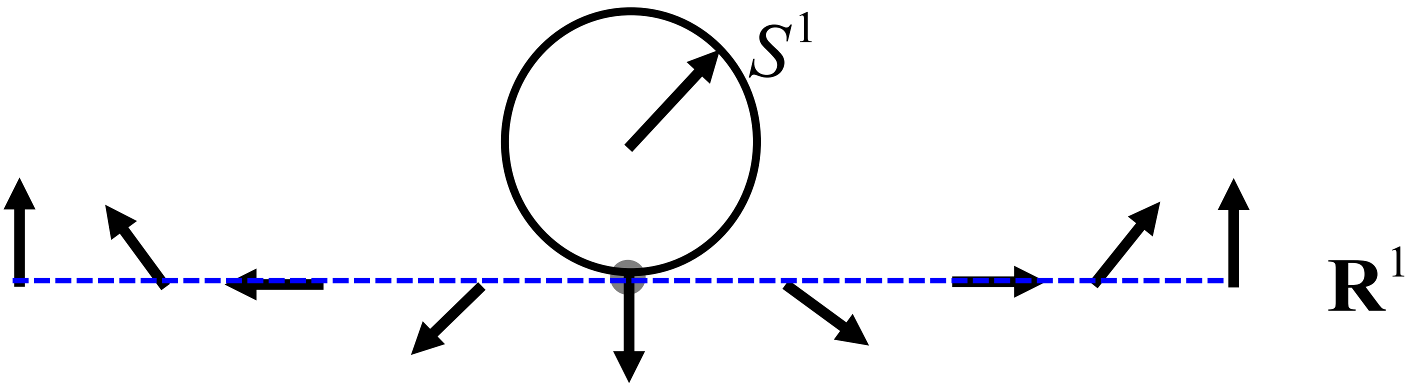
Domain wall (defect)

$$\pi_0(Z_2) = Z_2 \neq 0$$



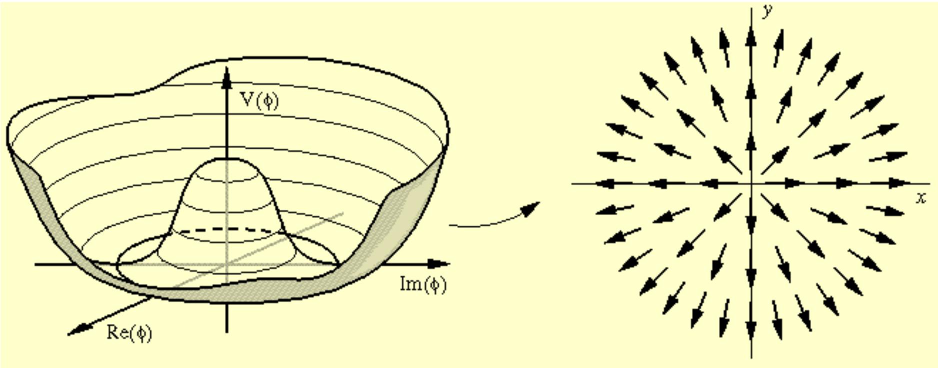
Sine-Gordon soliton (texture)

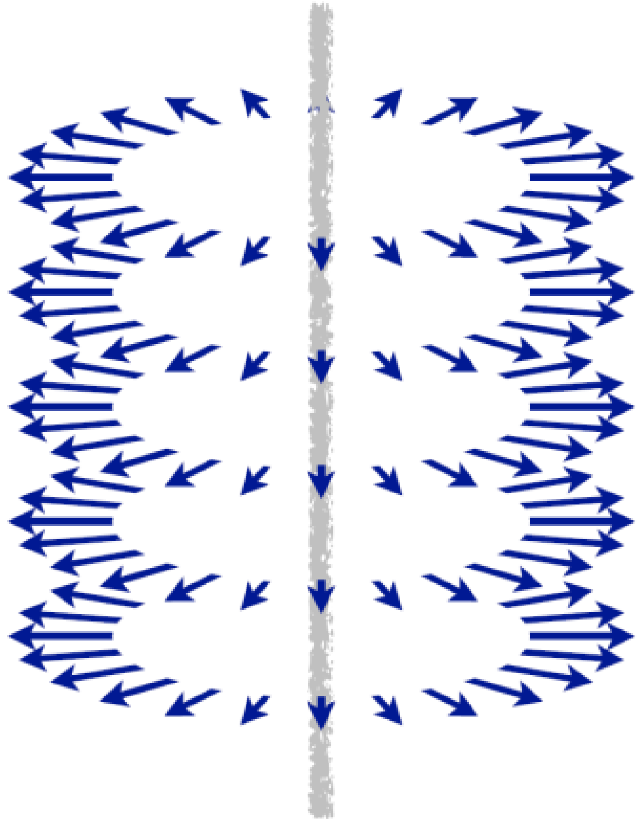
$$\pi_1(S^1) = Z \neq 0$$



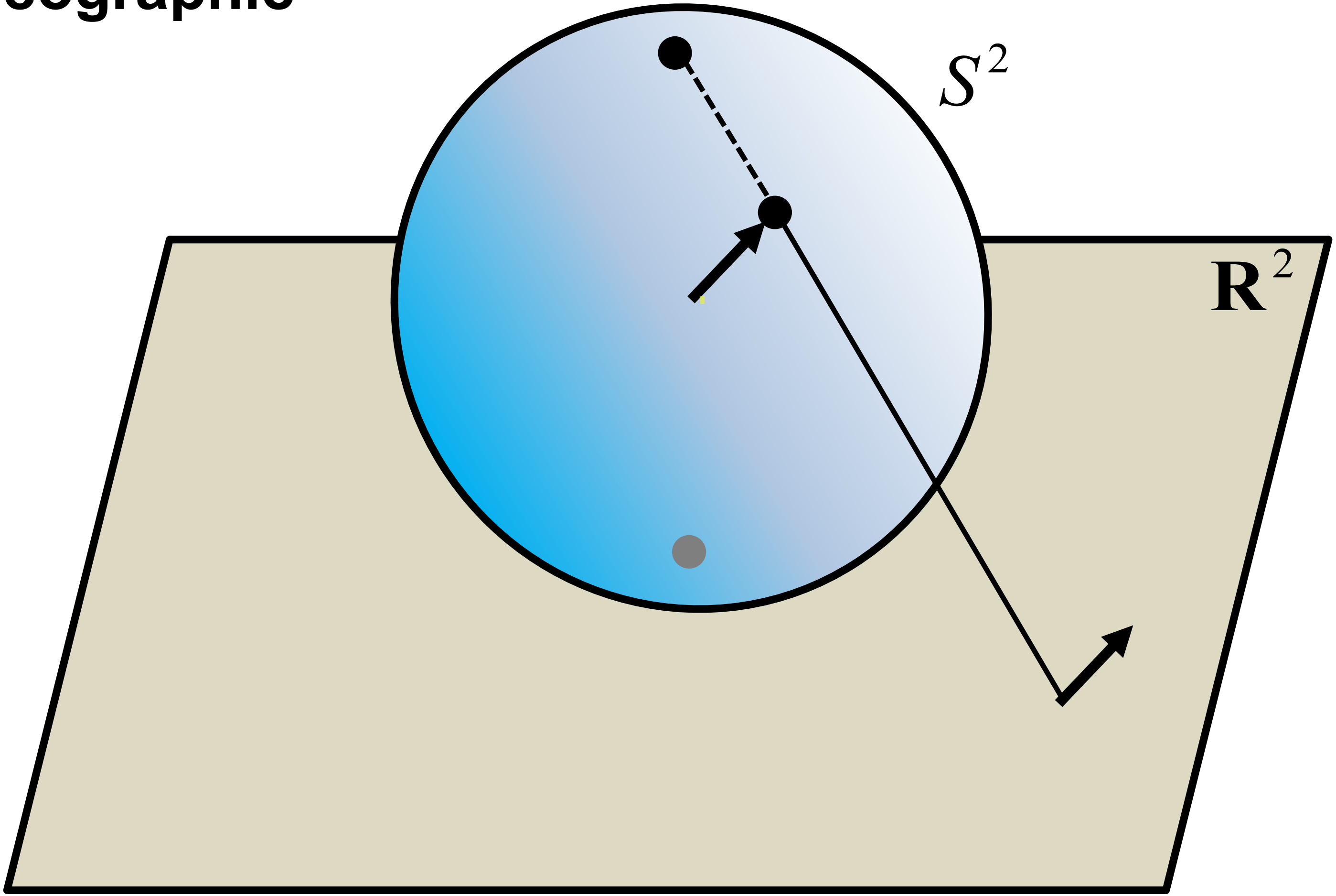
Vortex, cosmic string (defect)

$$\pi_1(S^1) = Z \neq 0$$



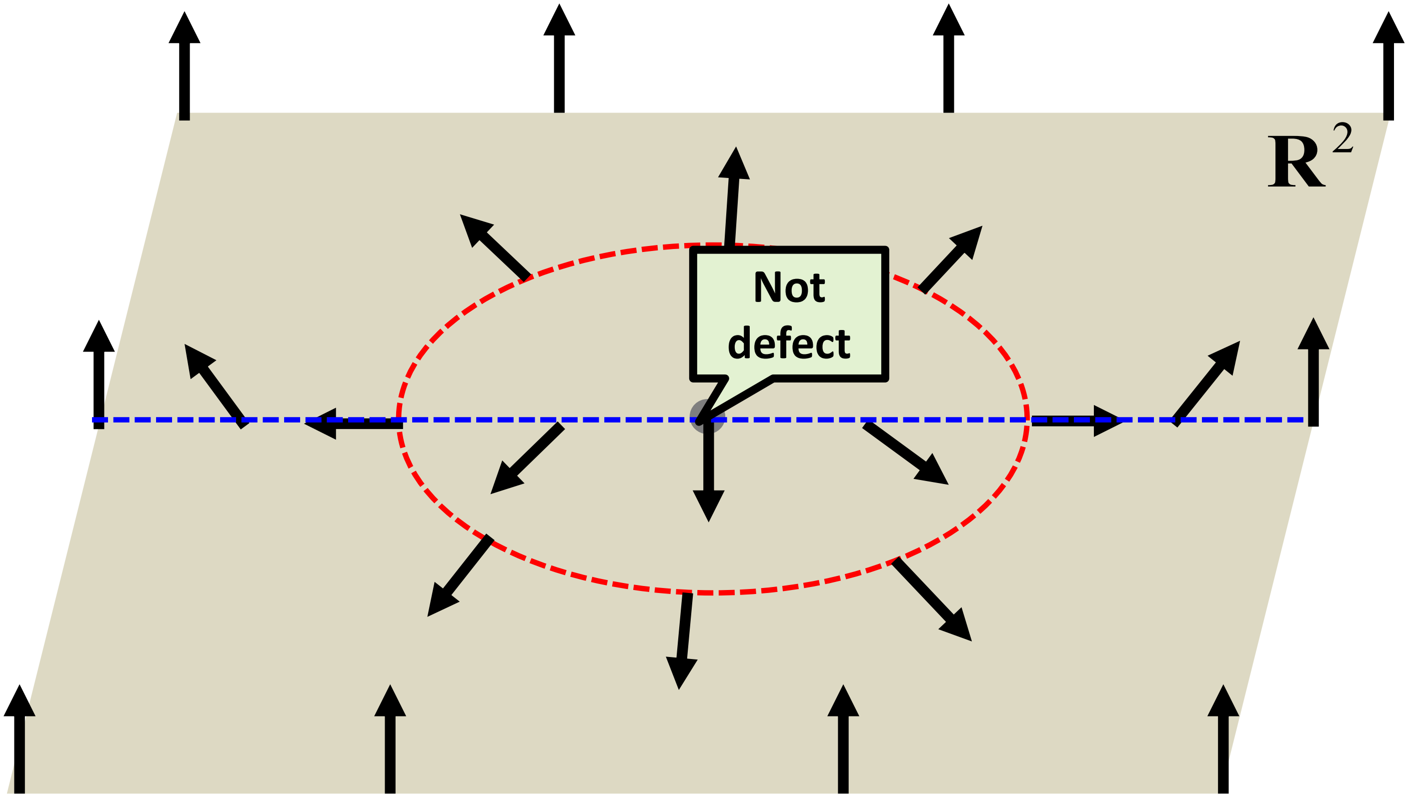


stereographic



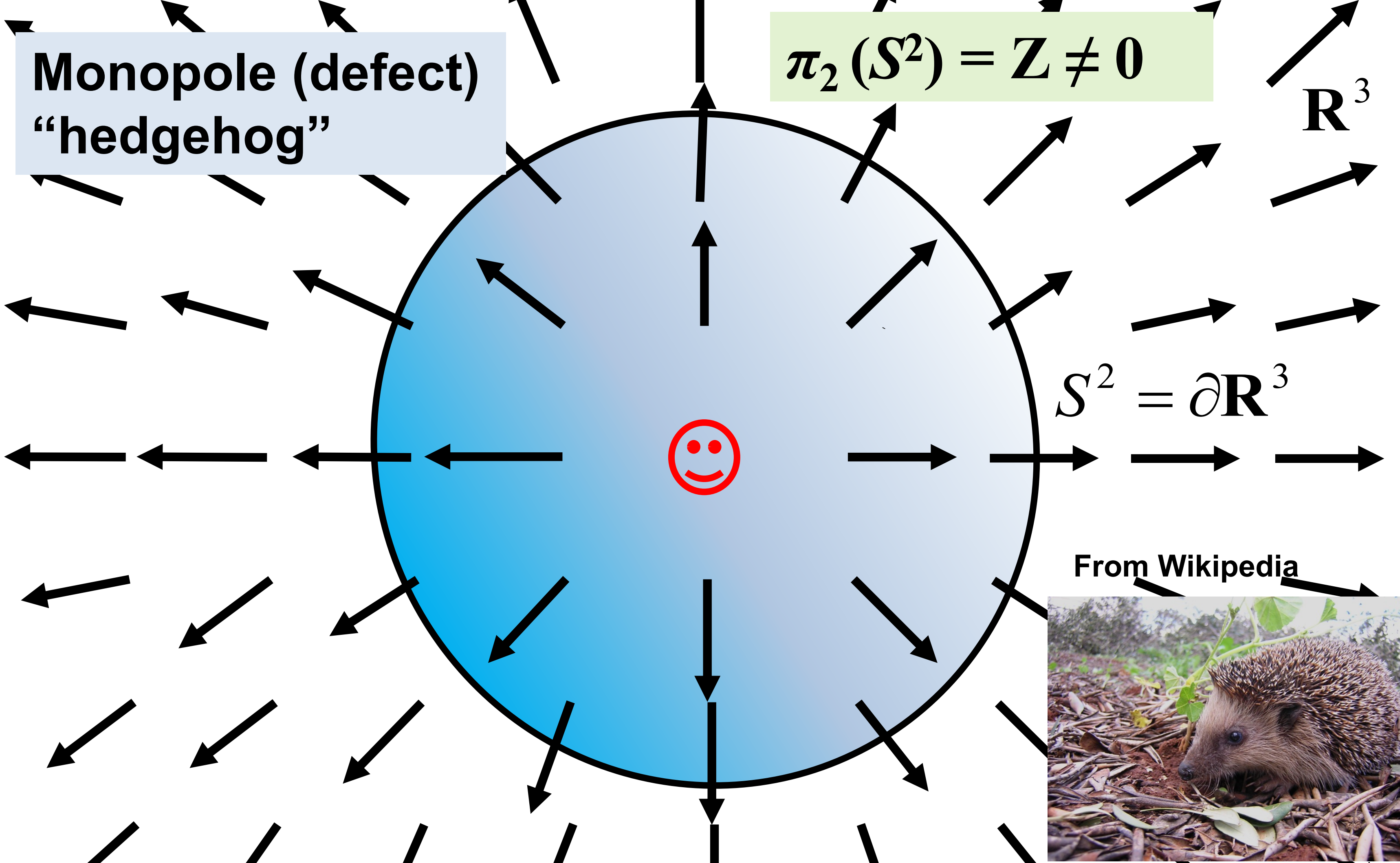
Lump, baby Skyrmion (texture)

$$\pi_2(S^2) = Z \neq 0$$

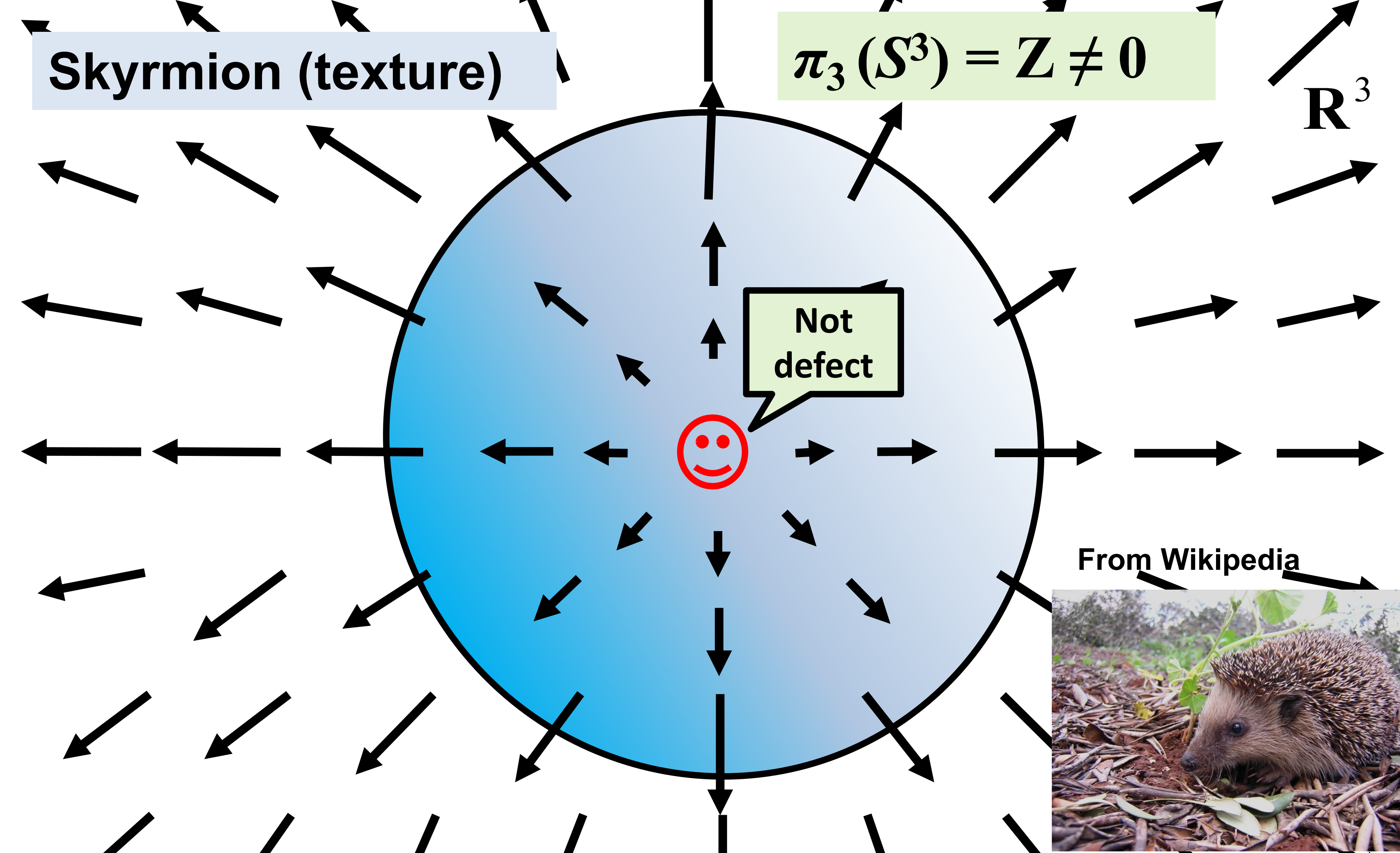


Monopole (defect)
“hedgehog”

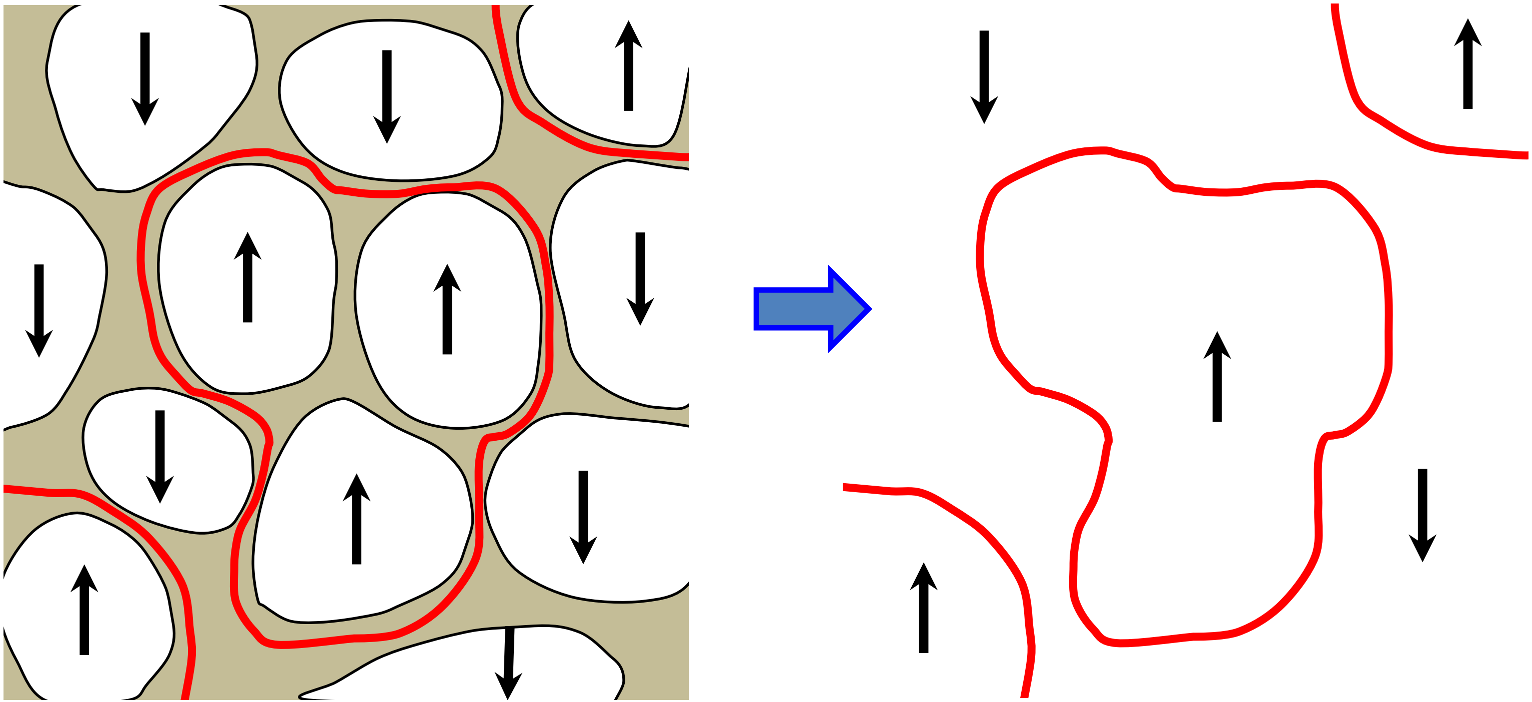
$\pi_2(S^2) = \mathbf{Z} \neq 0$



From Wikipedia


Skyrmion (texture)
$\pi_3(S^3) = Z \neq 0$
R^3
Not defect
From Wikipedia

How are they created?
e.g. Kibble-Zurek mechanism @ phase transition



Domain walls