高温・高密度QCD物質中の相転移に伴う輸送係数の異常

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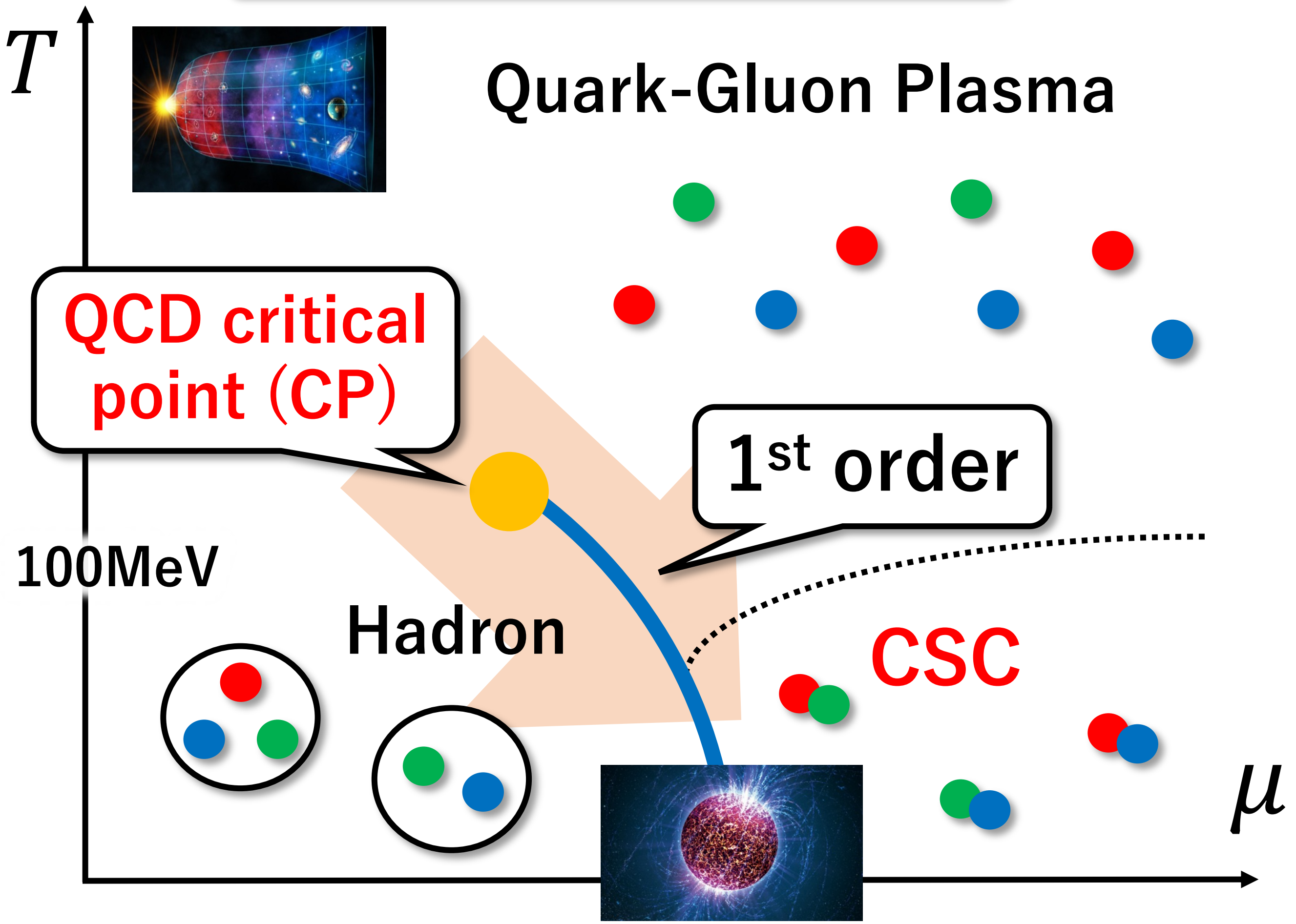
熱場の量子論　9/20-22 （京大基研）

Outline

* ＊Introduction
* ＊Formalism : Review of the previous study
* ＊Results : Conductivity & Relaxation time
* ＊Summary

High density region in QCD phase diagram

QCD phase diagram



Experiments for high density region with high statistics

Ongoing

- **BES II** at RHIC
- **NA61/SHINE** at LHC
- **HADES** at GSI

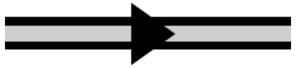
Future

- **FAIR** at GSI
- **NICA** at JINR
- **J-PARC-HI** (planned)

We focus on QCD CP & CSC.

Soft modes & Dileptons

Soft modes



They develop around critical points from 2nd phase transition.

QCD CP : mesonic fluctuations (Fujii and Ohtani (2004))

CSC : diquark fluctuations (Kitazawa, Koide, Kunihiro, Nemoto (2005))

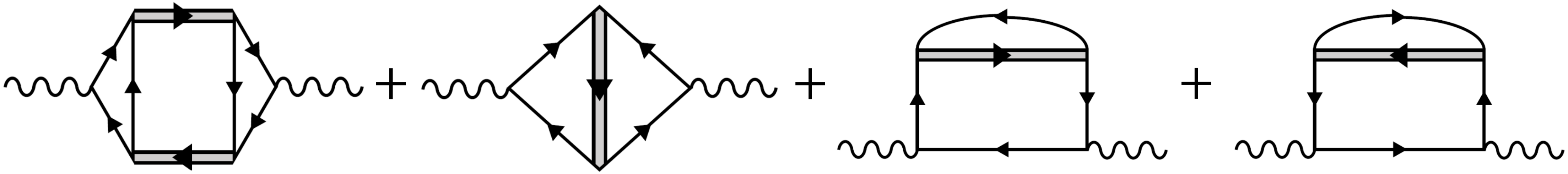
Dileptons

$$\frac{d^4\Gamma}{dk^4} = -\frac{\alpha}{12\pi^4}\frac{1}{k^2}\frac{1}{e^{\beta\omega}-1}g_{\mu\nu}\mathrm{Im}\,\Pi^{R\mu\nu}(k)$$

They don't interact strongly.

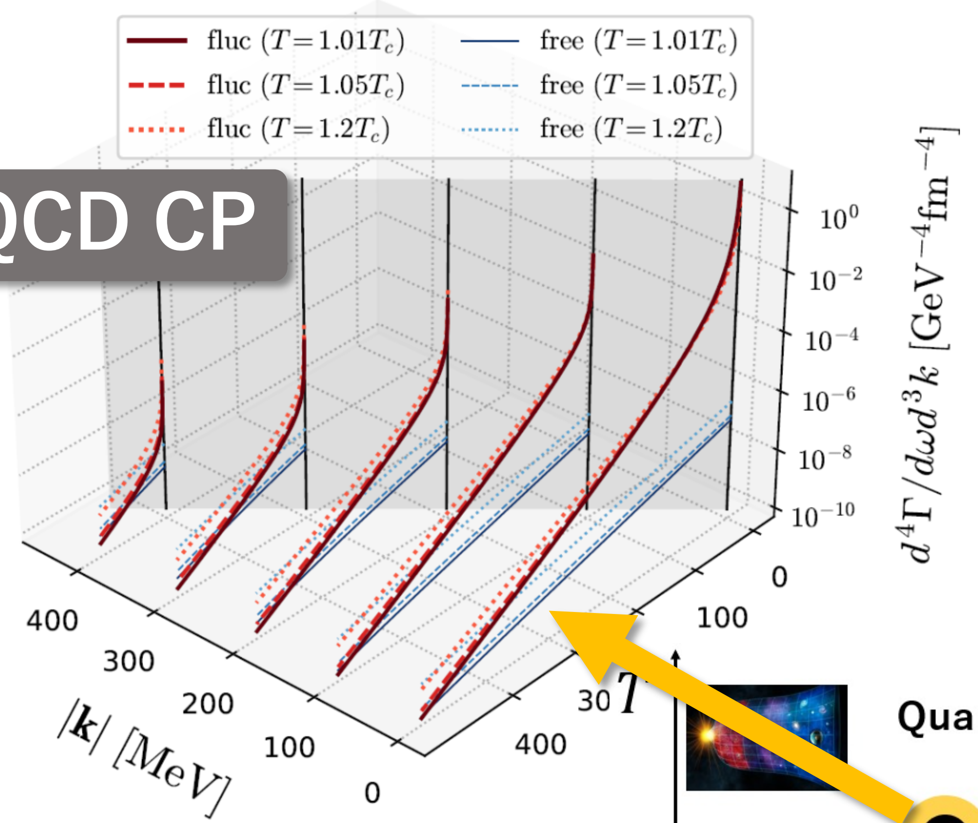
Photon self-energies $\Pi^{\mu\nu}$ are needed for dilepton production rates.

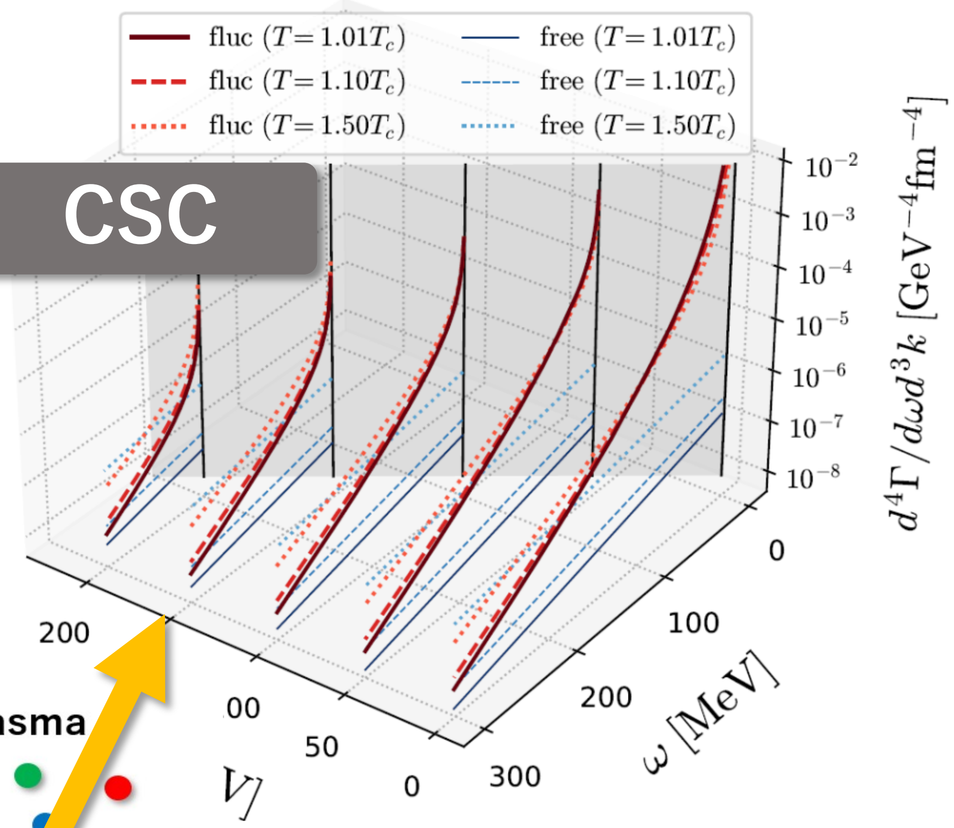
We have calculated how soft modes affect dilepton productions.

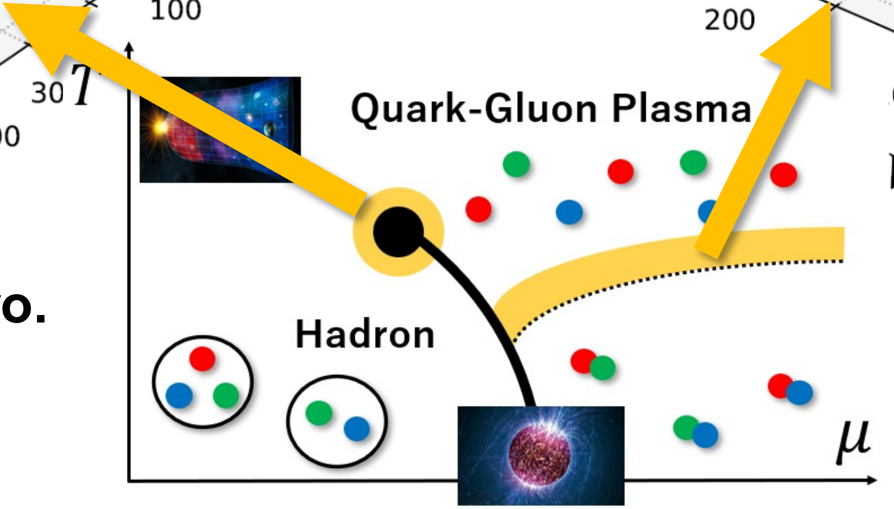
$\Pi^{\mu\nu} =$ 

Dilepton production rates from soft modes

Red : Contribution of soft modes & Blue : Results of free quark gases







TN, Kitazawa, and Kunihiro.
In preparation

TN, Kitazawa, and Kunihiro.
In press (PTEP 2022)

Purpose of this study

Difference of dilepton production in the two systems

Kubo formula

Kadanoff & Martin (1963)

$$\rho^{\mu\nu}(\boldsymbol{k},\omega) = -\frac{1}{\pi}\mathrm{Im}\Pi^{R\mu\nu}(\boldsymbol{k},\omega) \Rightarrow \rho_L(\boldsymbol{k},\omega) = \frac{\sigma\omega^3}{(\tau\omega^2 - D\boldsymbol{k}^2)^2 + \omega^2}$$

Electrical conductivity "$\boldsymbol{\sigma}$" and relaxation time "$\boldsymbol{\tau}$"

$$\sigma = \frac{\partial}{\partial\omega}\rho_L(\mathbf{0},\omega)\Big|_{\omega=0},\ \tau = \sqrt{-\frac{1}{3!\,\sigma}\frac{\partial^3}{\partial^3\omega}\rho_L(\mathbf{0},\omega)\Big|_{\omega=0}}$$

Model & soft modes

TN, Kitazawa, and Kunihiro. In press (PTEP 2022)

2-flavor NJL model

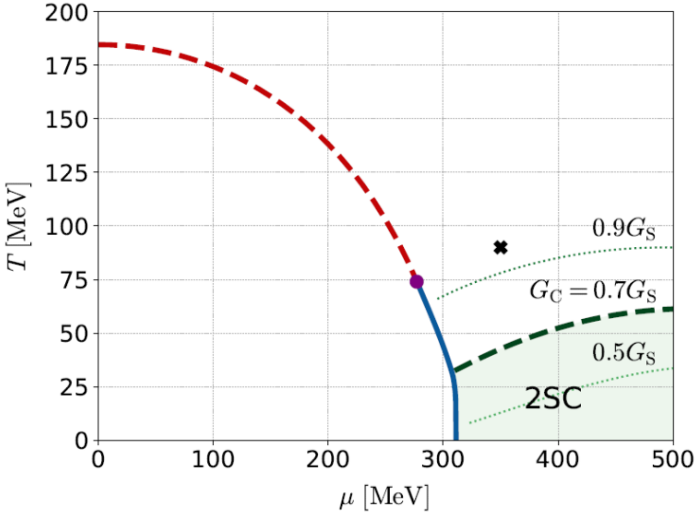
$$\mathcal{L} = \bar{\psi} i\gamma^{\mu}\partial_{\mu}\psi + \mathcal{L}_S + \mathcal{L}_C$$

$$\mathcal{L}_S = G_S[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \boldsymbol{\tau}\psi)^2]$$

$$\mathcal{L}_C = G_C(\bar{\psi} i\gamma_5\tau_2\lambda_A\psi^C)(\bar{\psi}^C i\gamma_5\tau_2\lambda_A\psi)$$

$$(G_S = 5.01\mathrm{MeV}, \Lambda = 650\mathrm{MeV})$$

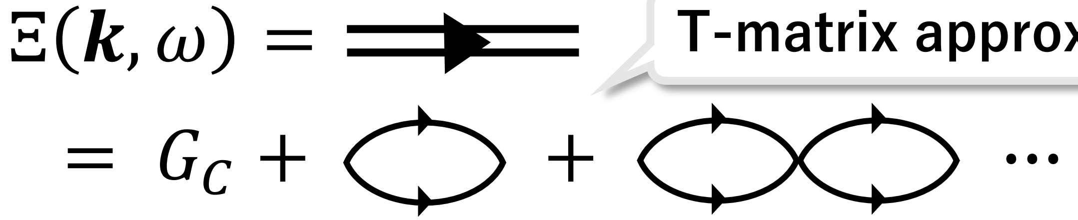
MFA

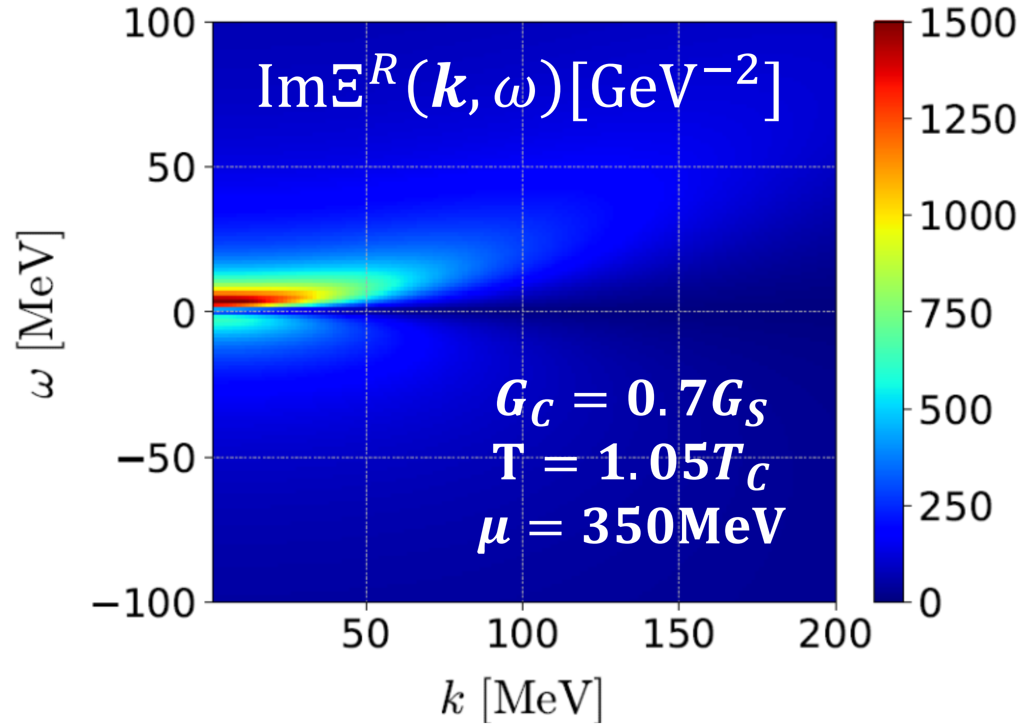


Kitazawa, Koide, Kunihiro, Nemoto (2002)

Propagator of soft modes

T-matrix approx.

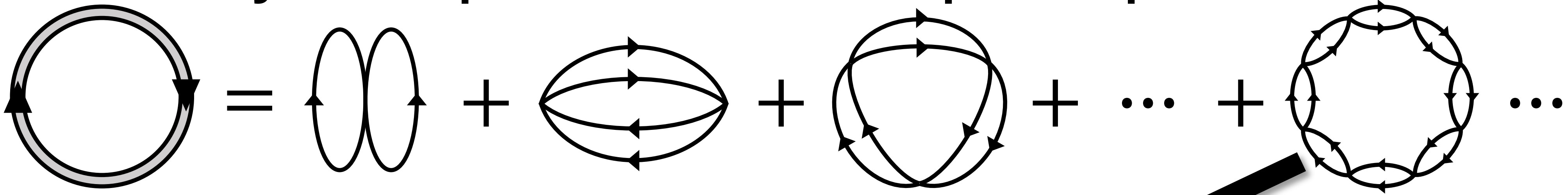




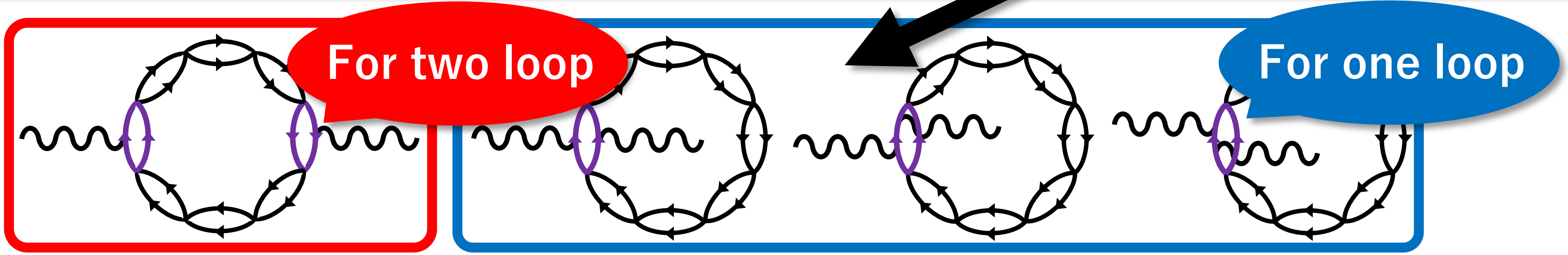
Kitazawa, Koide, Kunihiro, Nemoto (2005)

Construction of photon self-energy

Thermodynamic potential : One loop of diquark fluctuations

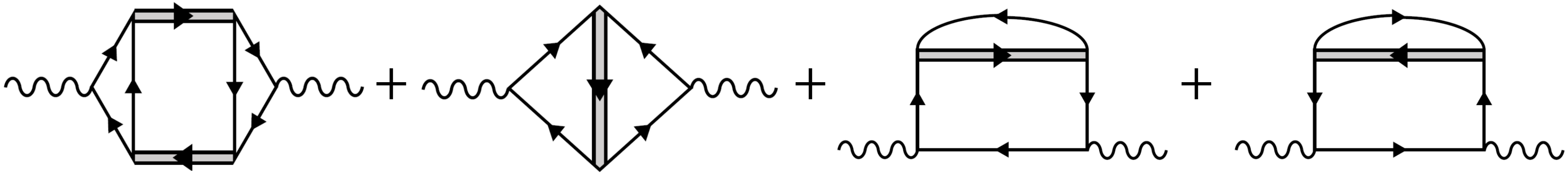


Attach two photon lines



Aslamazov-Larkin (AL) term | **Maki-Thompson (MT) term** | **Density of states (DOS) term**

$\Pi^{\mu\nu} =$



Ward identity of $\Pi^{\mu\nu}$ is satisfied.

Time-depending Ginzburg-Landau (TDGL) approx.

W-I of $\Pi^{\mu\nu}$ is satisfied with this approximation!!

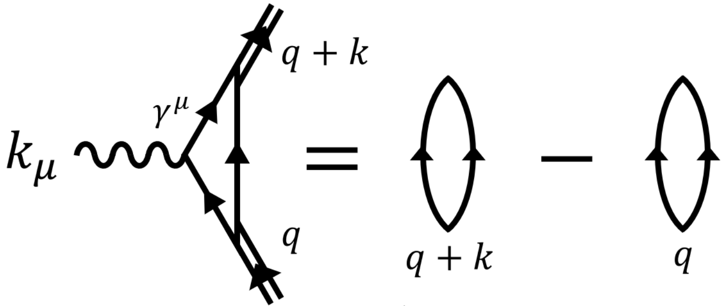
Propagator of soft modes

$$\Xi^R(\boldsymbol{k},\omega) = \frac{G_C}{1+G_C Q^R(\boldsymbol{k},\omega)} = \frac{1}{A + B\boldsymbol{q}^2 + C\omega}$$

Thouless criterion : $1 + G_C Q^R(\boldsymbol{0},0) = 0$ at $T = T_C$

Vertex of AL term

W-I of vertex

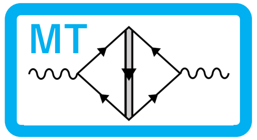


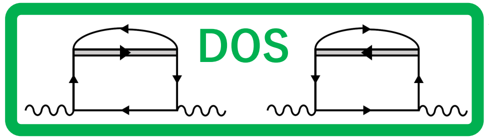
$$k_\mu \Gamma^\mu(q, q+k) = \Xi^{-1}(q+k) - \Xi^{-1}(q)$$

Compare the lowest order terms of $\boldsymbol{k}$ and $\boldsymbol{\omega}$.

$$\Gamma^0(q,q+k) = C,\ \Gamma^i(q,q+k) = 2B(2q^i + k^i)$$

Imaginary part of MT and DOS term cancels.

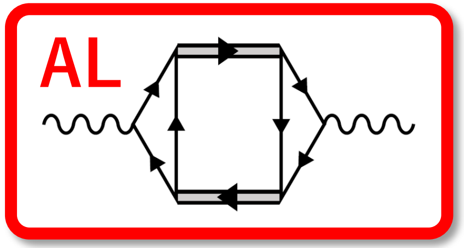
Im(

+ 

) = 0

Consistent with metallic SC !!!

Only AL term is necessary to calculate the DPR.



Dilepton production rates from soft modes

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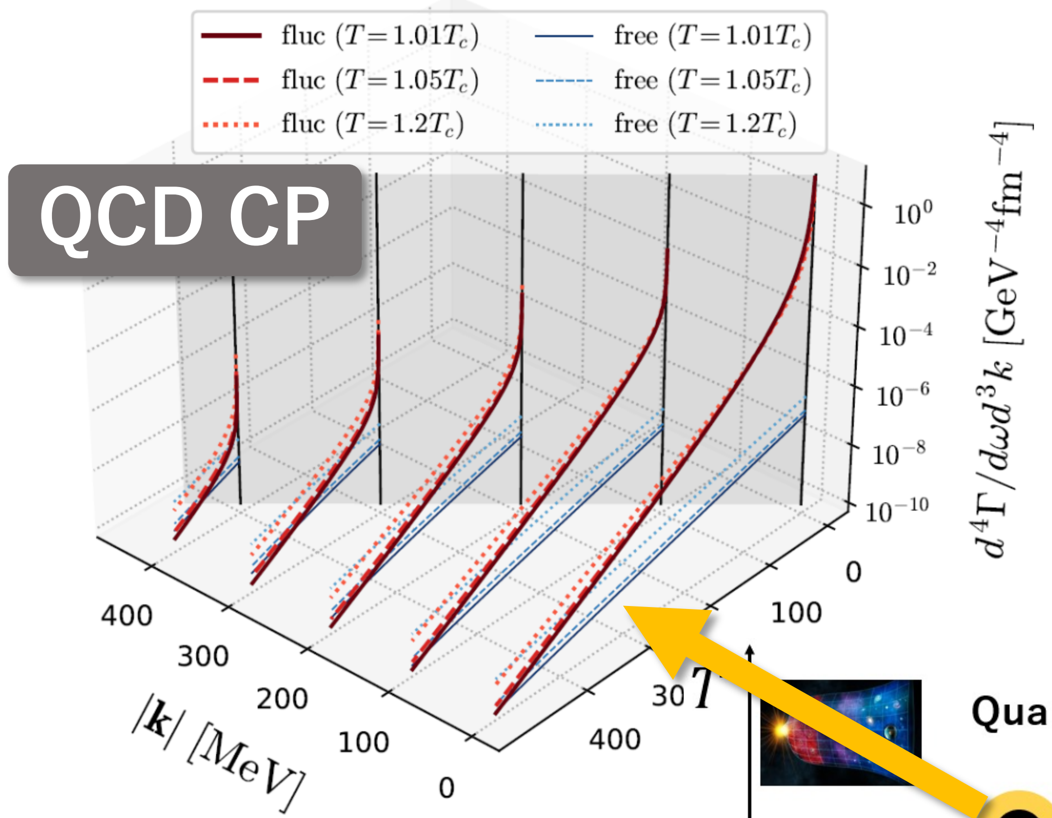

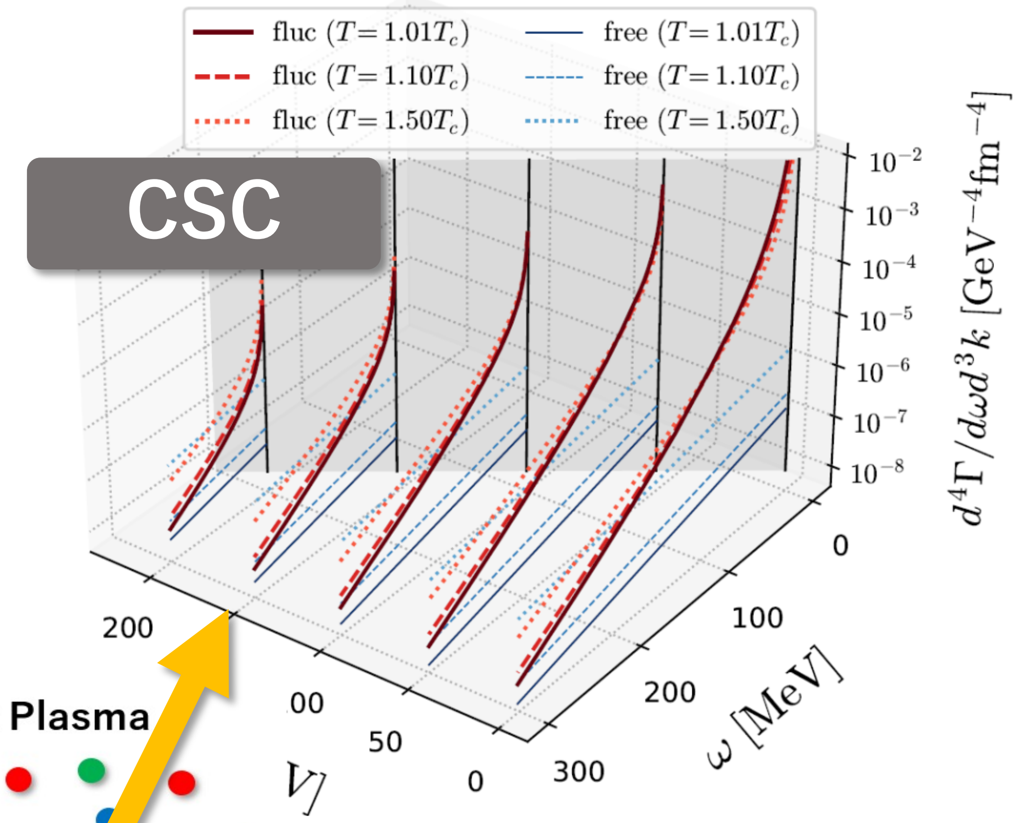

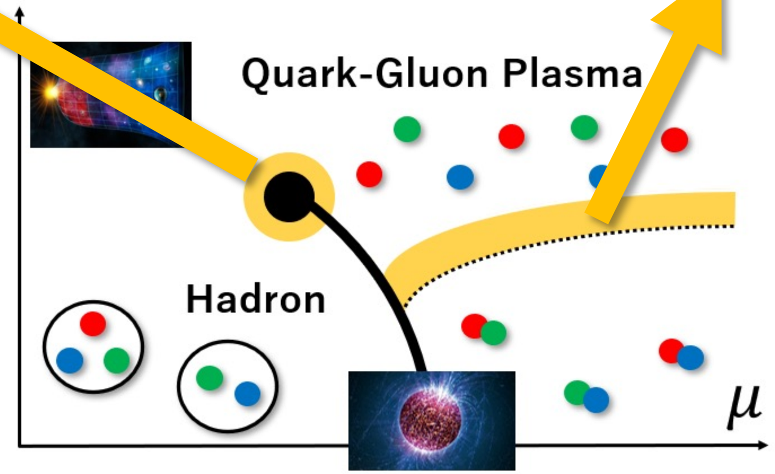

TN, Kitazawa, and Kunihiro.
In preparation

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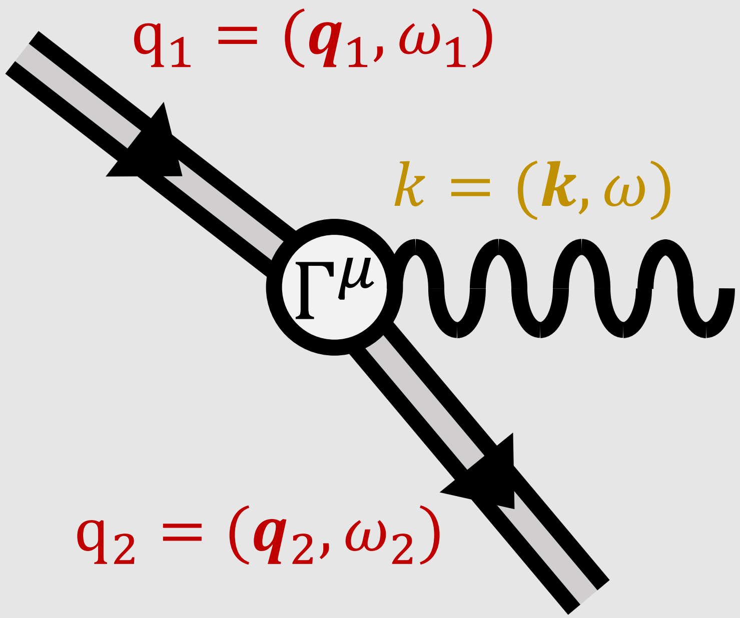
In press (PTEP 2022)

Mechanism of enhancement at low ω

One of production processes caused by soft modes

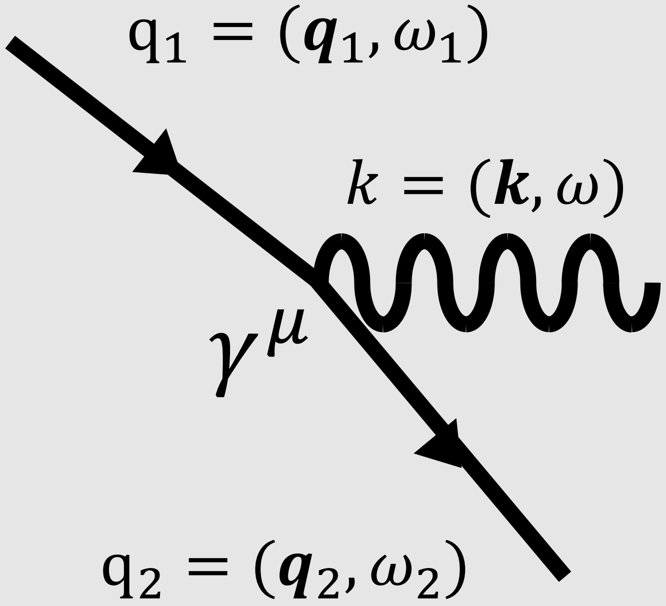
$$\omega = \omega_1 - \omega_2$$
$$|\boldsymbol{k}| = |\boldsymbol{q}_1 - \boldsymbol{q}_2|$$

$$|\boldsymbol{q}_1| > \omega_1$$
$$|\boldsymbol{q}_2| > \omega_2$$
$$\omega > |\boldsymbol{k}|$$



* Soft modes have strong support in space-like.
* Dileptons are produced in time-like.
* MT&DOS cancel. → Scattering in AL.

Scattering process In free quark gases



This occurs only in space-like.
→ $q\bar{q}$ annihilation contributes dilepton production in time-like

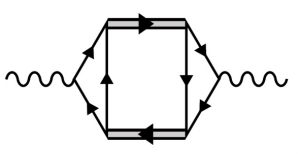
Conductivity “σ” and Relaxation time “τ”

Kubo formula

$$\rho^{\mu\nu}(\boldsymbol{k},\omega) = -\frac{1}{\pi}\mathrm{Im}\Pi^{R^{\mu\nu}}(\boldsymbol{k},\omega) \Rightarrow \sigma = \frac{\partial}{\partial\omega}\rho_L(\boldsymbol{0},\omega)\Big|_{\omega=0},\ \tau = \sqrt{-\frac{1}{3!\,\sigma}\frac{\partial^3}{\partial^3\omega}\rho_L(\boldsymbol{0},\omega)\Big|_{\omega=0}}$$

AL term

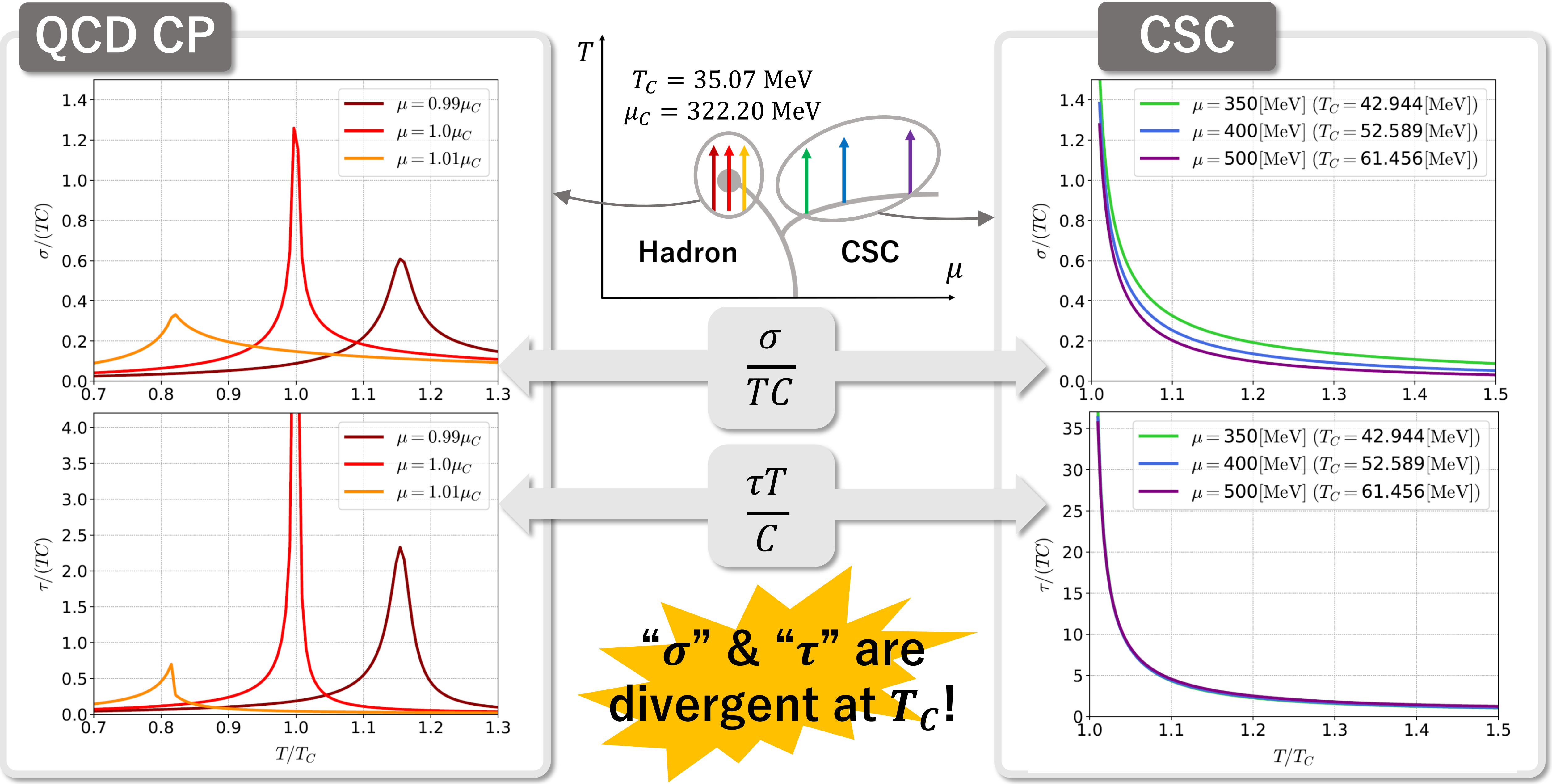
$$\Pi^{\mu\nu}_{AL}(\boldsymbol{k},\omega) =$$



$$= N\int\frac{d^4q}{(2\pi)^4}\tilde{\Gamma}^{\mu}(q,q+k)\Xi(\mathrm{q}+\mathrm{k})\tilde{\Gamma}^{\nu}(q+k,q)\Xi(\mathrm{q})$$

	QCD CP	CSC
Soft mode $\Xi^{\mathrm{R}}(\mathbf{q},\boldsymbol{\omega}')$	$\dfrac{1}{A+B\boldsymbol{q}^2+\frac{C}{\lvert\boldsymbol{q}\rvert}\omega'}$, $A\propto\lvert T-T_C\rvert^{\frac{2}{3}}$ Fujii & Ohtani (2004)	$\dfrac{1}{A+B\boldsymbol{q}^2+C\omega'}$, $A\propto\lvert T-T_C\rvert^{1}$ Kitazawa & Kunihiro et al (2005)

Results of "σ" & "τ"

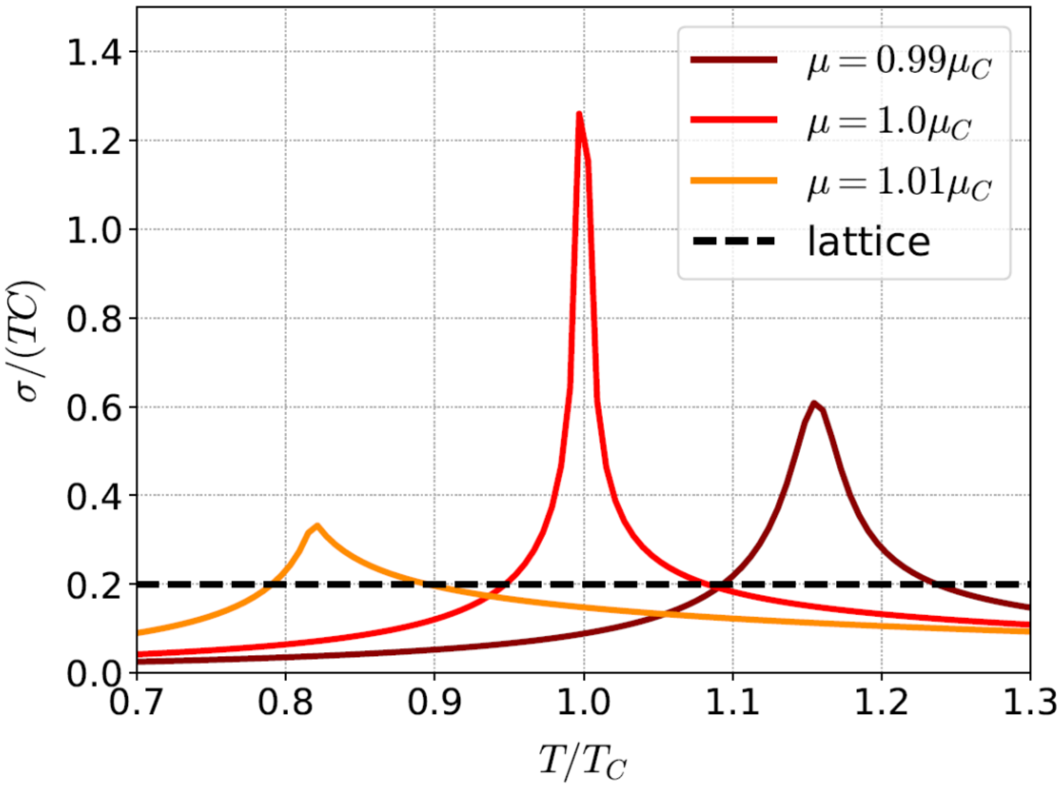


"σ" & "τ" are divergent at T_C!

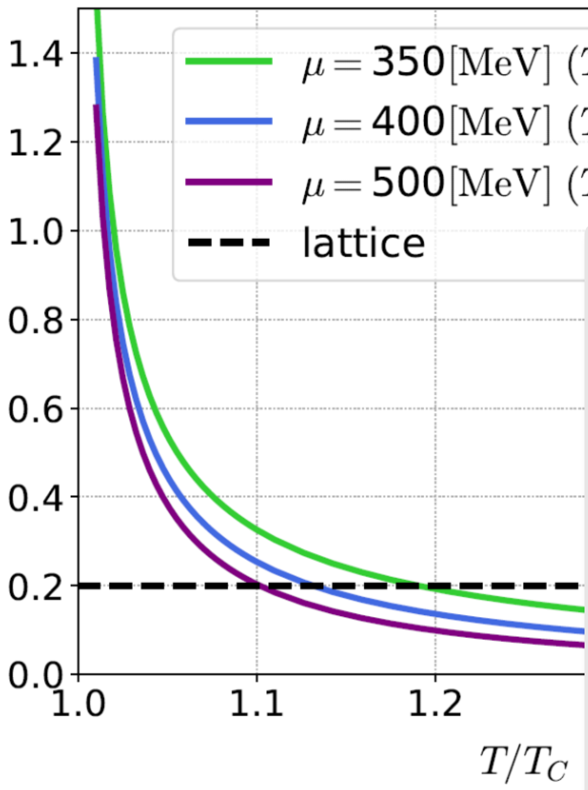
Conductivity "σ"

QCD CP

$T_C = 35.07$ MeV
$\mu_C = 322.20$ MeV



CSC



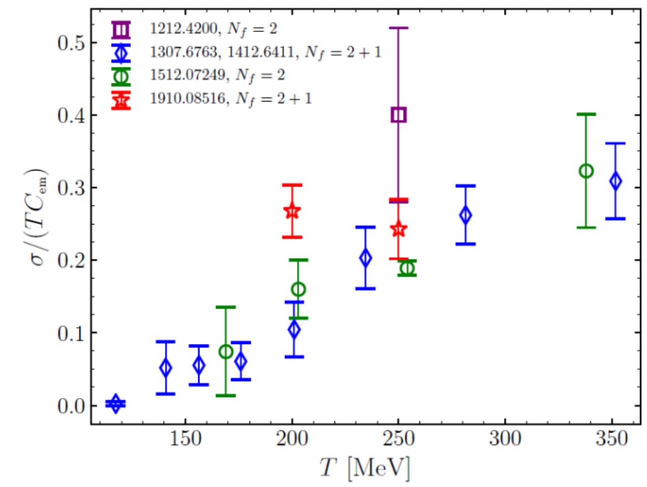


Fig. 3. Results for electrical conductivity, normalised as $\sigma/(TC_{em})$, in QCD with $N_f = 2$ and $2+1$ dynamical flavours as a function of temperature in MeV.

$$\frac{\sigma}{TC_{\mathrm{em}}} \sim 0.2$$

Aarts & Nikolaev (2021)

Contributions from soft modes are bigger than $\sigma/(TC_{\mathrm{cm}}) = 0.2$ in the range of $|T - T_C|/T_C < 0.1 \sim 0.2$!!

Divergences of “σ” & “τ”

	QCD CP	CSC
σ	$\lvert T-T_C\rvert^{-\frac{2}{3}}$	$\lvert T-T_C\rvert^{-\frac{1}{2}}$
τ	$\lvert T-T_C\rvert^{-1}$	$\lvert T-T_C\rvert^{-\frac{3}{2}}$

$$\sigma = \frac{\partial}{\partial\omega}\rho_L(\mathbf{0},\omega)\Big|_{\omega=0} = \frac{1}{\pi}\sum_i \frac{\partial}{\partial\omega}\mathrm{Im}\Pi^{ii}_{AL}(\mathbf{0},\omega)\Big|_{\omega=0} \propto \int q^4 dq \int \coth\frac{\omega'}{2T}\, d\omega'\, \mathrm{Im}\Xi^R(\boldsymbol{q},\omega')\frac{\partial}{\partial\omega'}\mathrm{Im}\Xi^R(\boldsymbol{q},\omega')$$

- ✓ TDGL approximation for Ξ^R
- ✓ T dependences of A

QCD CP
$$\sigma \sim \int q^4 dq \int d\omega' \left(\frac{1}{[A+Bq^2]^2q^2+[C\omega']^2}\right)^2 \sim \frac{1}{A} \propto |T-T_C|^{-\frac{2}{3}}$$

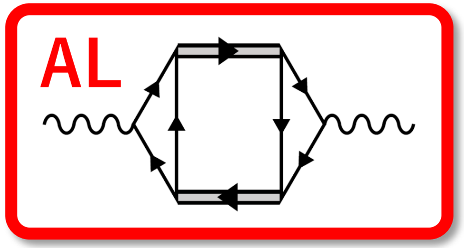
CSC
$$\sigma \sim \int q^4 dq \int d\omega' \left(\frac{1}{[A+Bq^2+C_{\mathrm{Im}}\omega']^2+[C_{\mathrm{Re}}\omega']^2}\right)^2 \sim \frac{1}{\sqrt{A}} \propto |T-T_C|^{-\frac{1}{2}}$$

Divergences of “σ” & “τ” are caused by soft modes.

Summary

We calculated how soft modes affect σ and τ through AL terms.

* σ & τ diverge around QCD CP and CSC phase transition.
* The behavior of divergence are different in the two systems.



Outlook

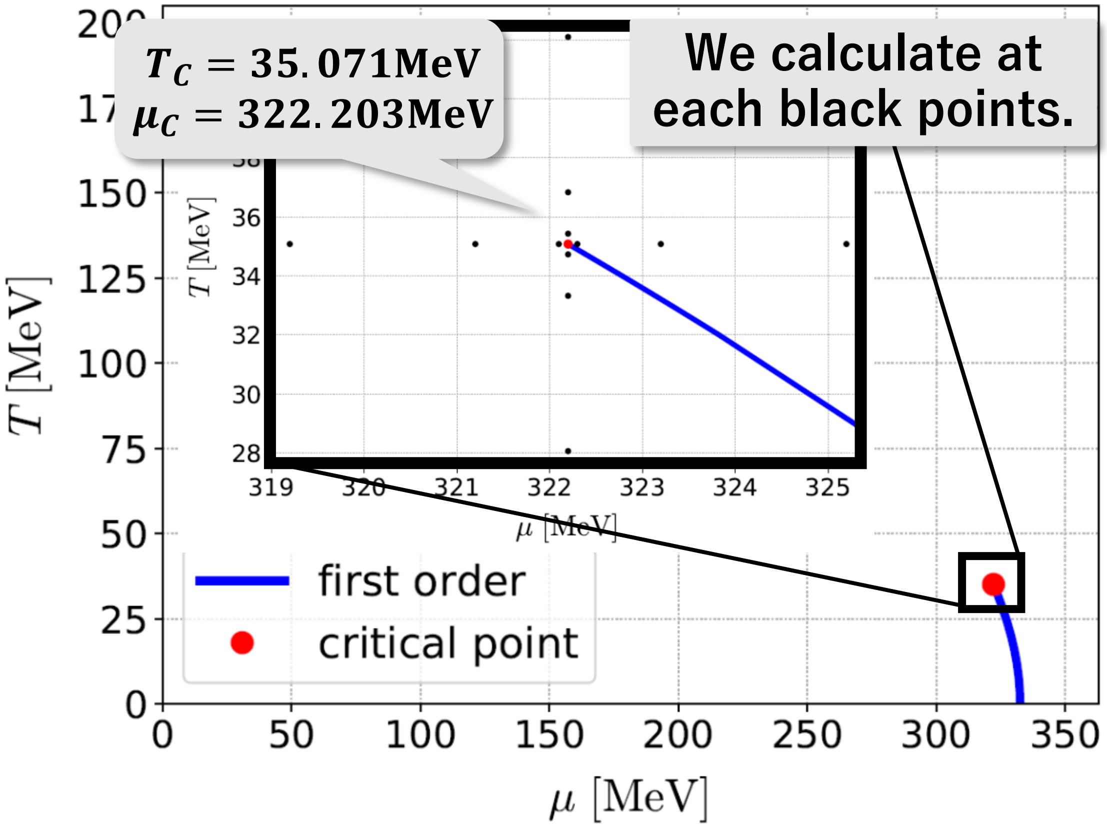
* Causality (quark number susceptibility)
* Higher order contribution of soft modes
* Dependence on the position of QCD CP

Back up

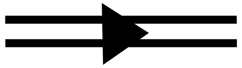
Phase diagram

$$\mathcal{L} = \bar{\psi} i(\gamma^{\mu}\partial_{\mu} - m)\psi$$
$$+G_S[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \boldsymbol{\tau}\psi)^2]$$

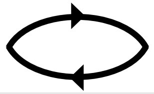
$G_S = 5.01\mathrm{MeV}, \Lambda = 650\mathrm{MeV}, m = 4\mathrm{MeV}$



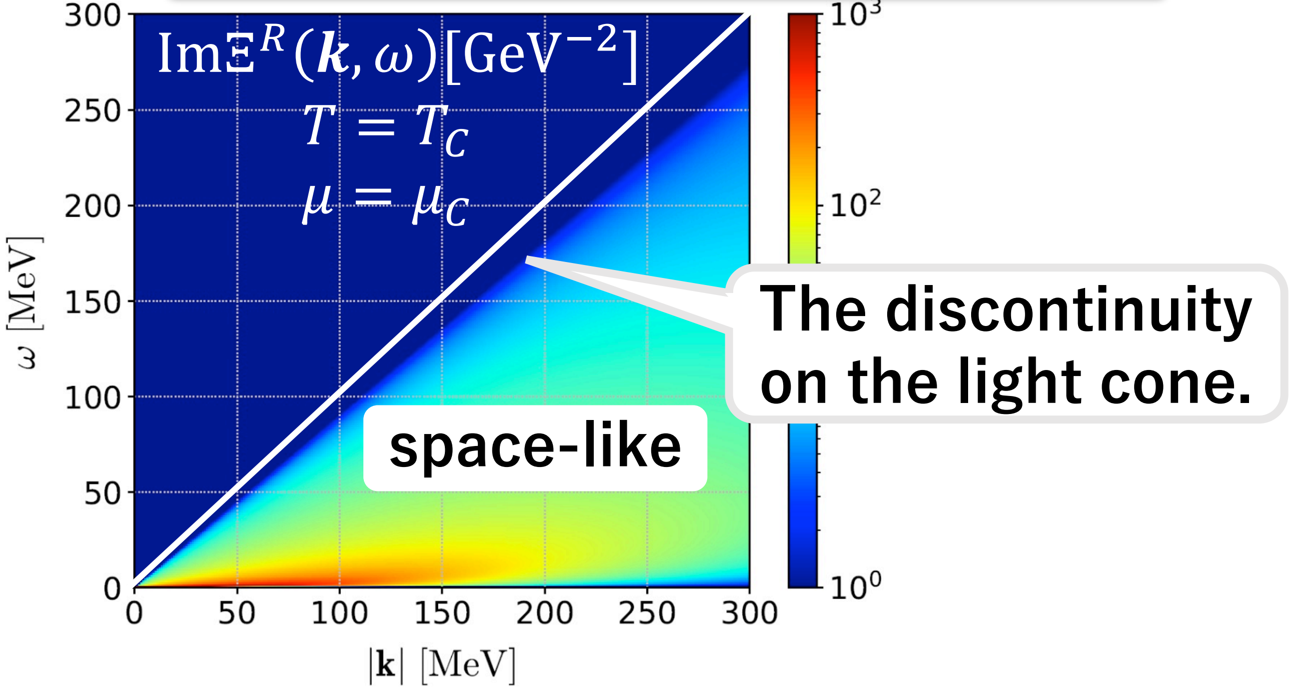
Soft modes

$\Xi(\boldsymbol{k},\omega) =$  T-matrix approx.

$$= \frac{G_C}{1 + G_C Q^R(\boldsymbol{k},\omega)}$$

$\bar{q}q$ correlation 

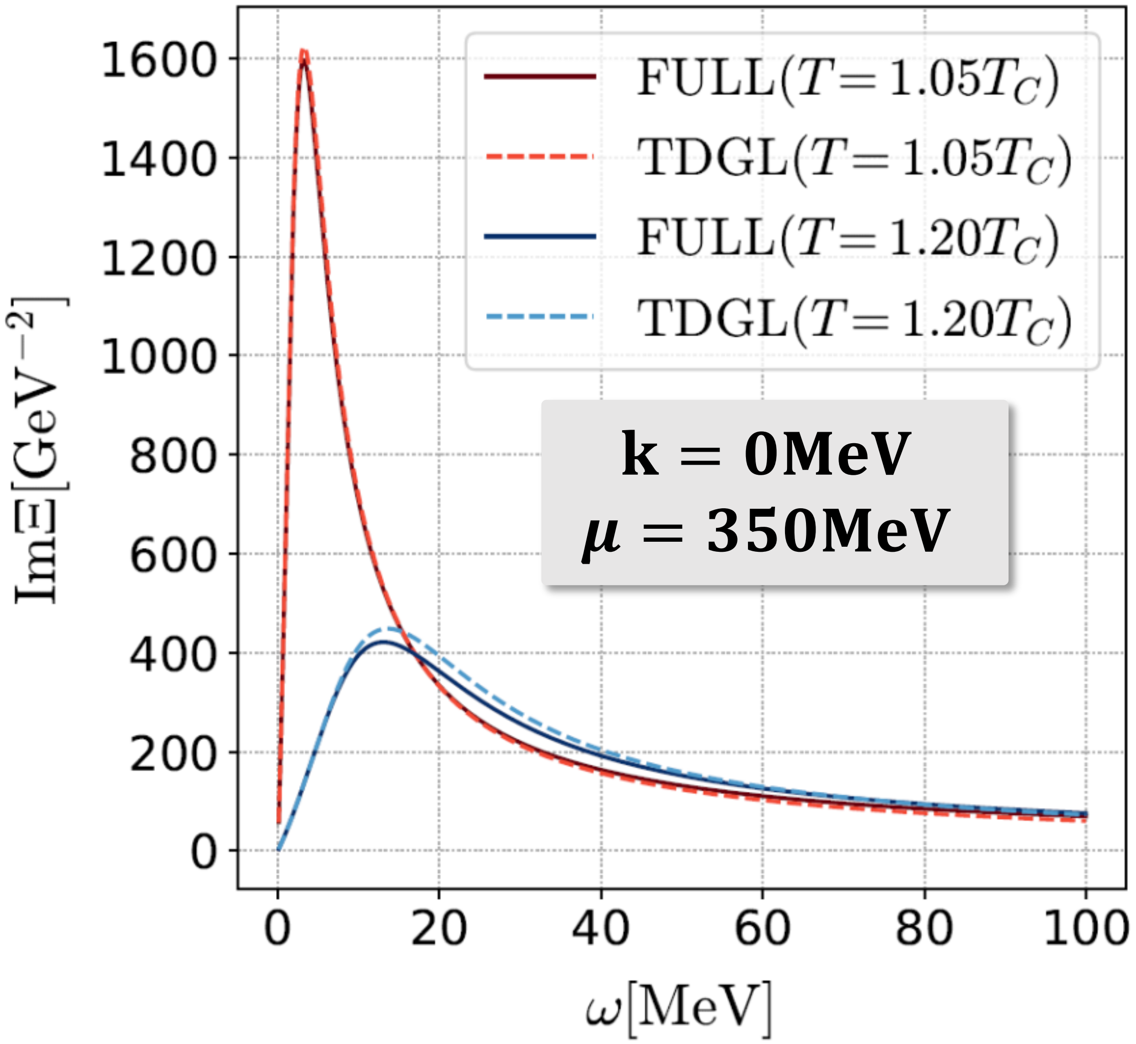
$\mathrm{Im}\Xi^R$ in energy-momentum plane

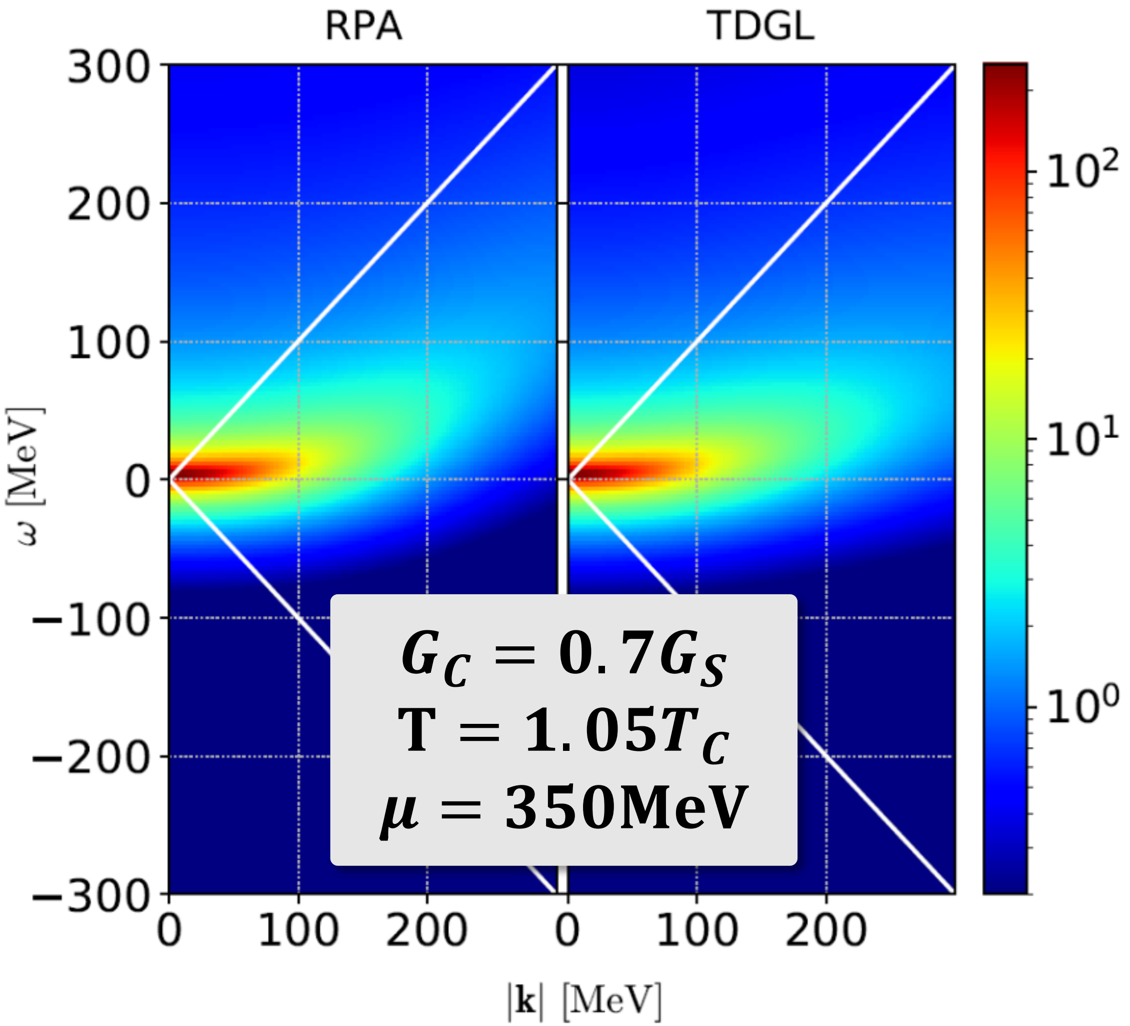


The soft mode in space-like region is the p-h mode.

Fujii & Ohtani (2004)

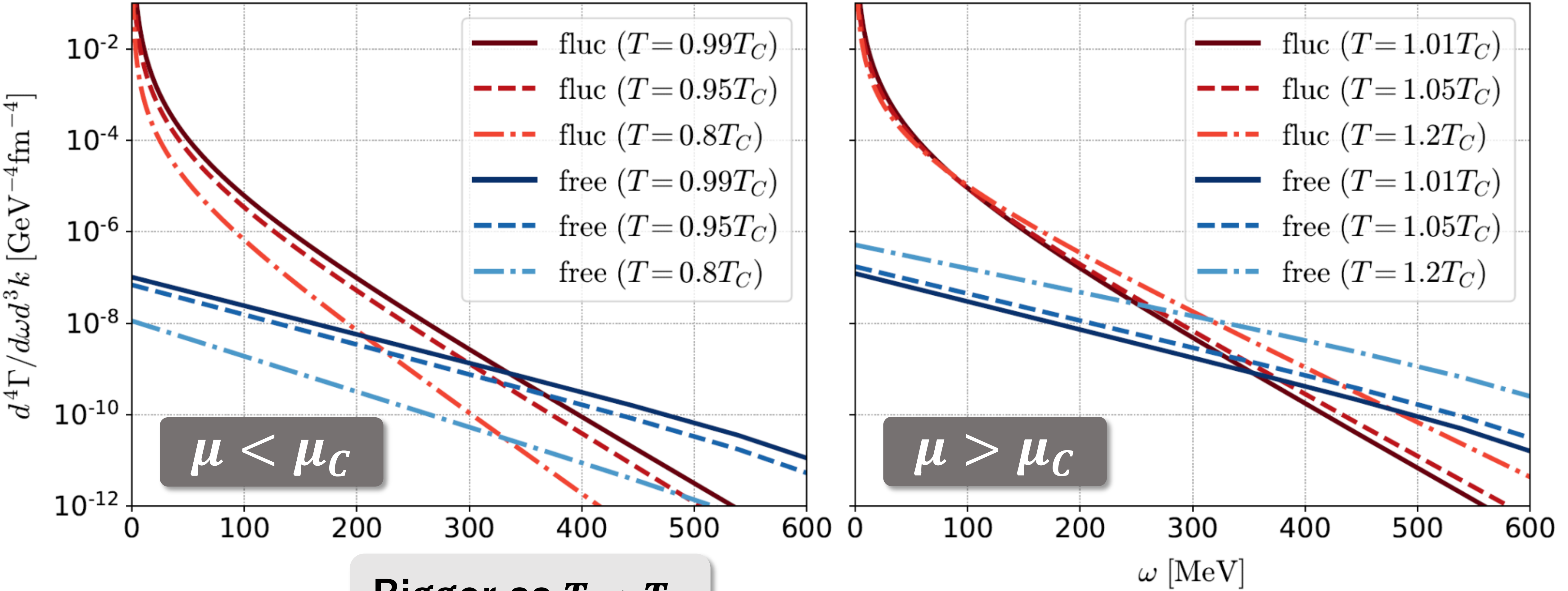
Validity of TDGL approximation (diquark)





TDGL approx. can reproduce $\Xi^R(\mathbf{k},\omega)$ in low $\mathbf{k}$-ω region near T_C

T-dependence of DPR from mesonic modes at $k = 0$

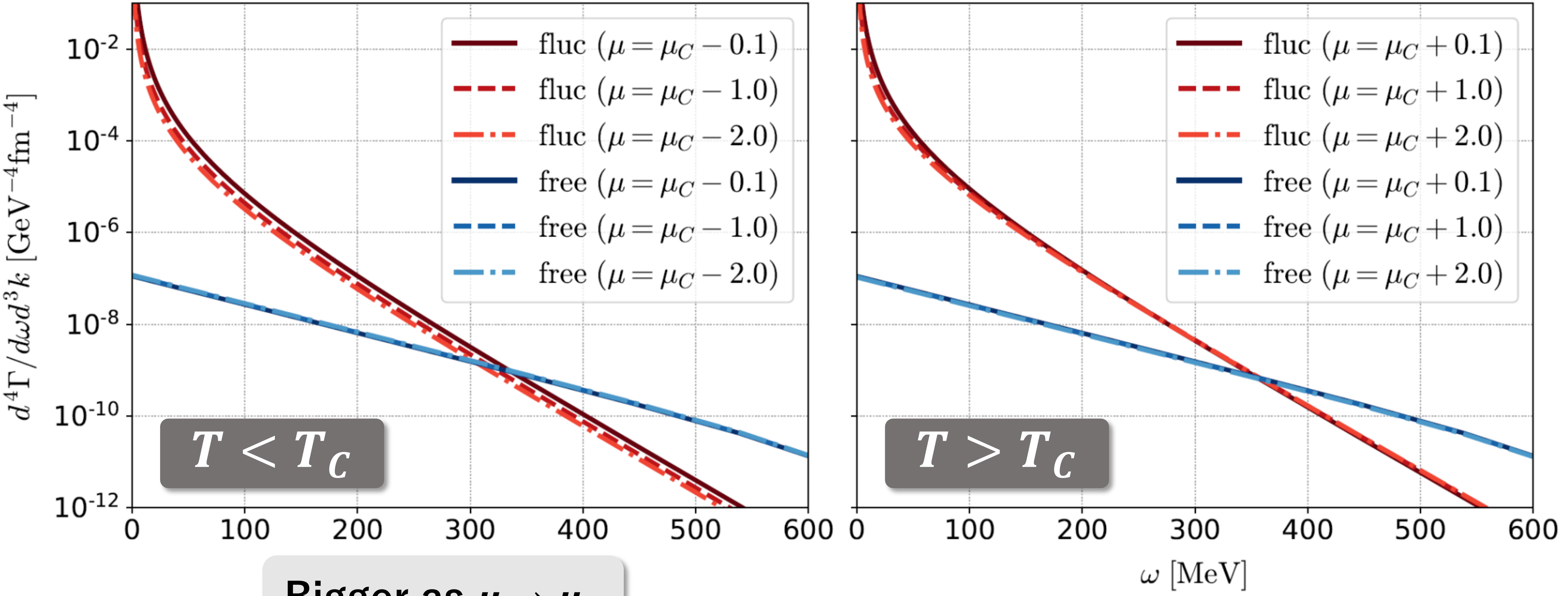


Bigger as $T \to T_C$

Bigger as T is bigger

Competition between contributions of soft modes and kinematical (temperature) effects

μ-dependence of DPR from mesonic modes at $k = 0$

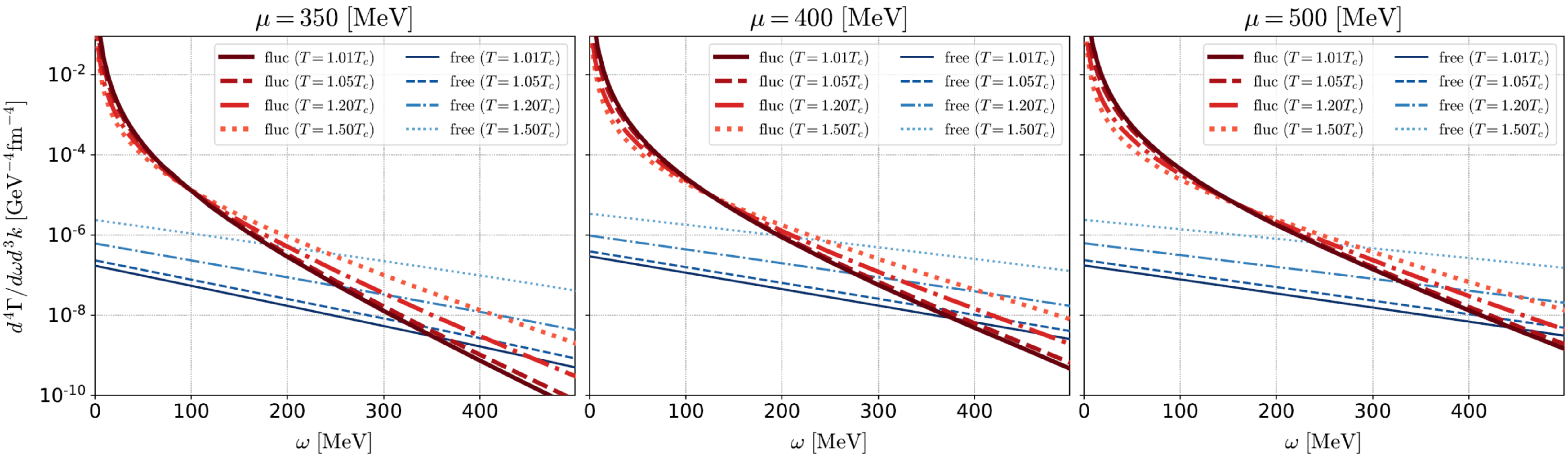


Bigger as $\mu \to \mu_C$

Bigger as μ is bigger

Competition between
contributions of soft modes and
enhances of interaction between γ & Ξ

DPR from diquark soft modes at $k = 0$



T-dependence: Enhanced as $T \to T_C$

… This behavior is expected from the property of soft modes.

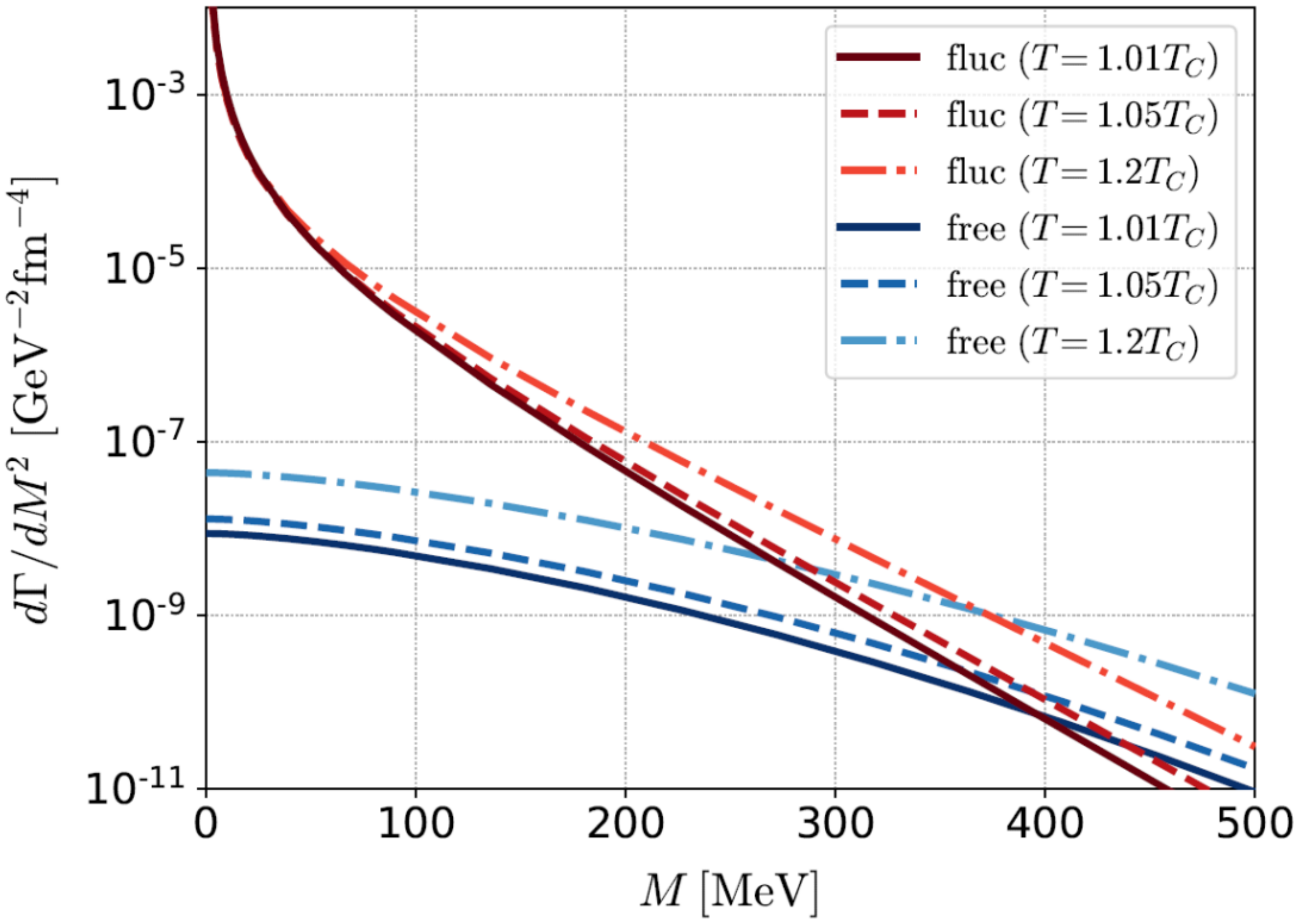
μ-dependence: Enhanced as μ is bigger

… Interactions between soft modes and photons increase.

Invariant mass spectrum

QCD CP

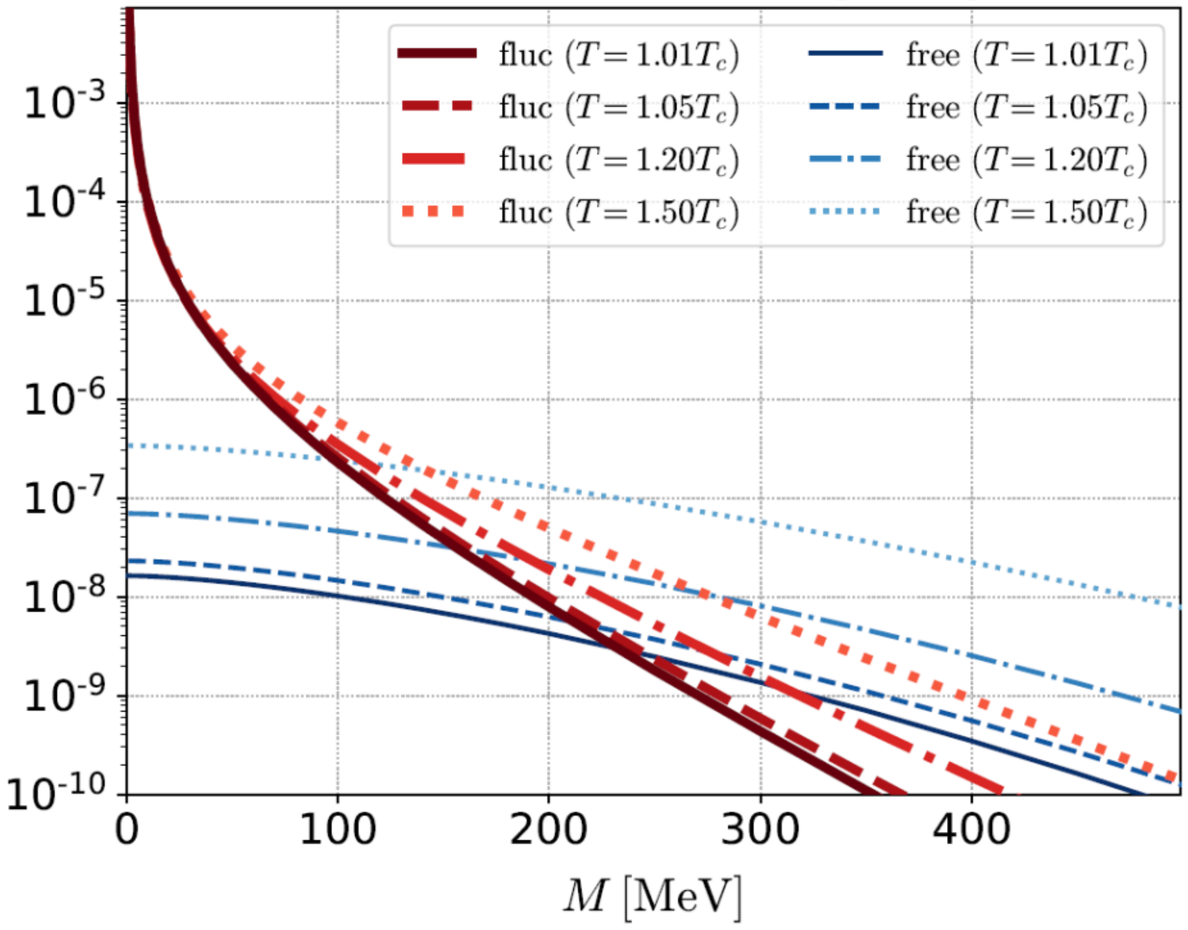
$\mu = \mu_C = 332$ MeV



$M < 200$ MeV

CSC

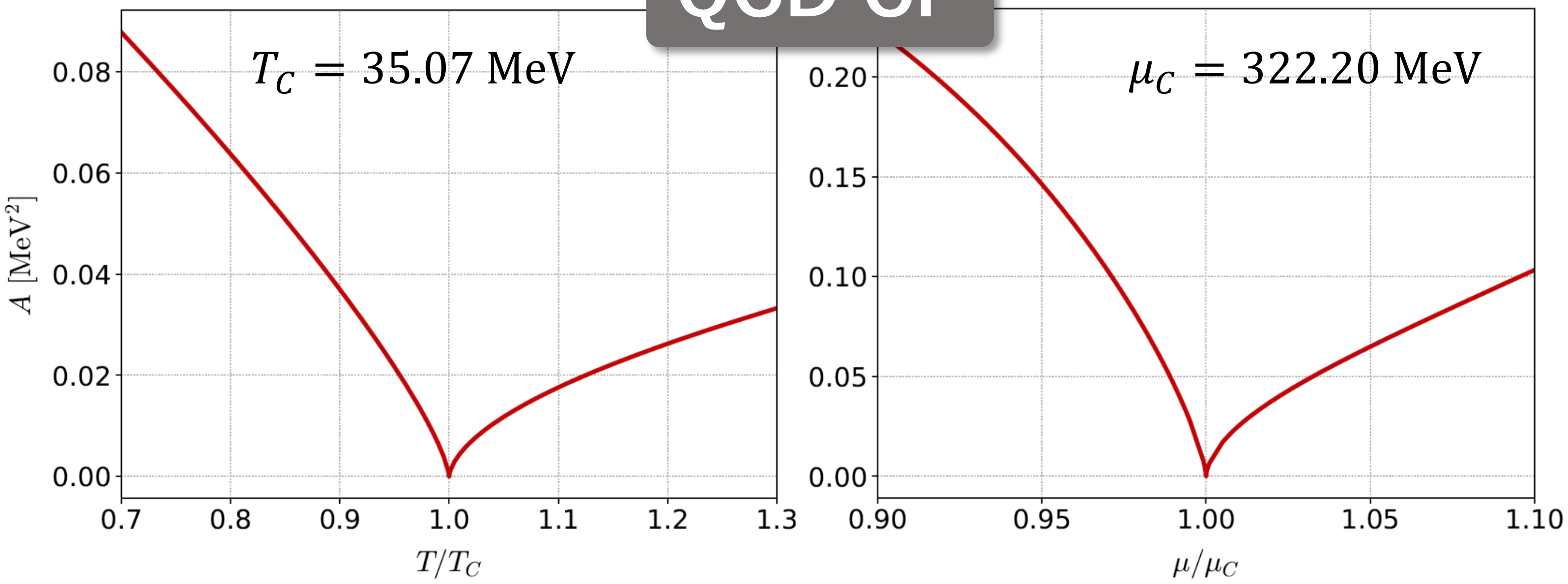
$G_C = 0.7G_S, \mu = 350$ [MeV]



$M < 400$ MeV

Parameter dependence of A

QCD CP

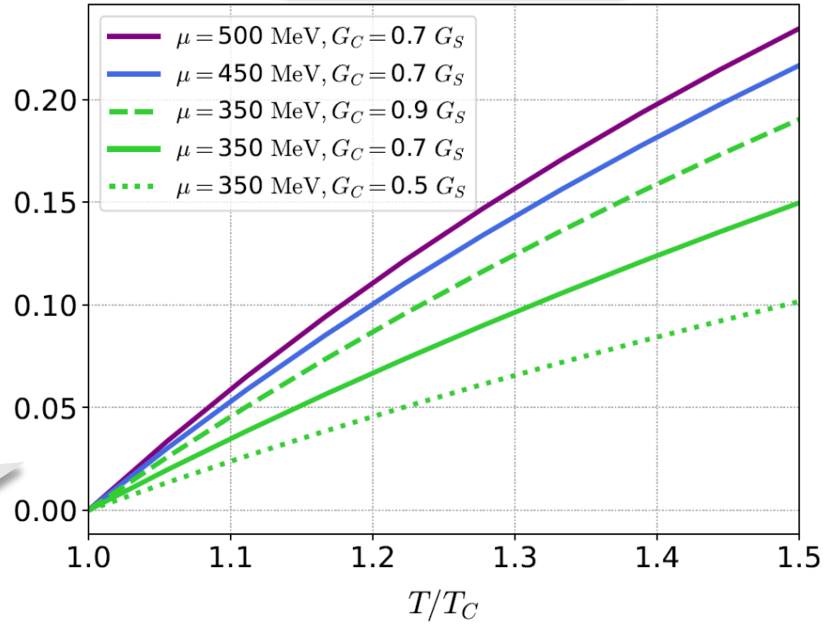


$T < T_C$	$T > T_C$
$\lvert T - T_C\rvert^{0.672}$	$\lvert T - T_C\rvert^{0.664}$

$\mu < \mu_C$	$\mu > \mu_C$
$\lvert T - T_C\rvert^{0.675}$	$\lvert T - T_C\rvert^{0.658}$

$$\Xi^R(\boldsymbol{k},\omega) = \frac{G_C}{1 + G_C Q^R(\boldsymbol{k},\omega)} = \frac{1}{A + B\boldsymbol{q}^2 + C\omega}$$

CSC



A of "CSC" is almost independent of μ & G_C
($A \propto \lvert T - T_C\rvert^{0.997\cong 1}$)