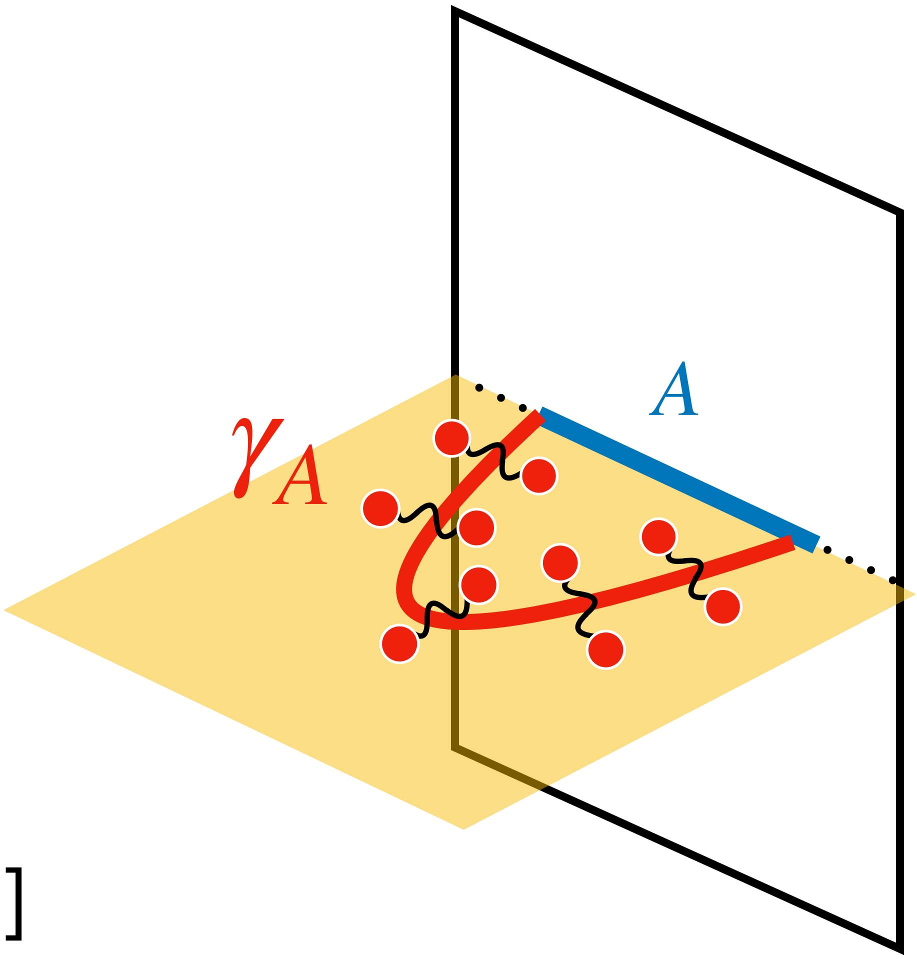
Entanglement distillation in tensor networks

Takato Mori

KEK
SOKENDAI



Based on arXiv:2205.06633 [hep-th]
with H. Matsueda (Tohoku U) and H. Manabe (Kyoto U)

熱場の量子論とその応用, 2022年9月21日（オンライン参加）

Take-home message

Tensor network defines a set of
reduced *transition* matrices
They describe entanglement distillation via geometry

The method works for arbitrary tensor networks;
a systematic, quantitative study of states vs. geometry
in tensor networks is now possible

Outline

1. Introduction — AdS/CFT, tensor networks as toy models

2. Entanglement distillation in MERA

3. Numerical results for random MERA

4. Entanglement distillation in MPS

5. Summary

Motivation: Understanding AdS/CFT from entanglement

(d+1)-dim. AdS spacetime ⇆ d-dim. quantum field theory (CFT) [Maldacena]

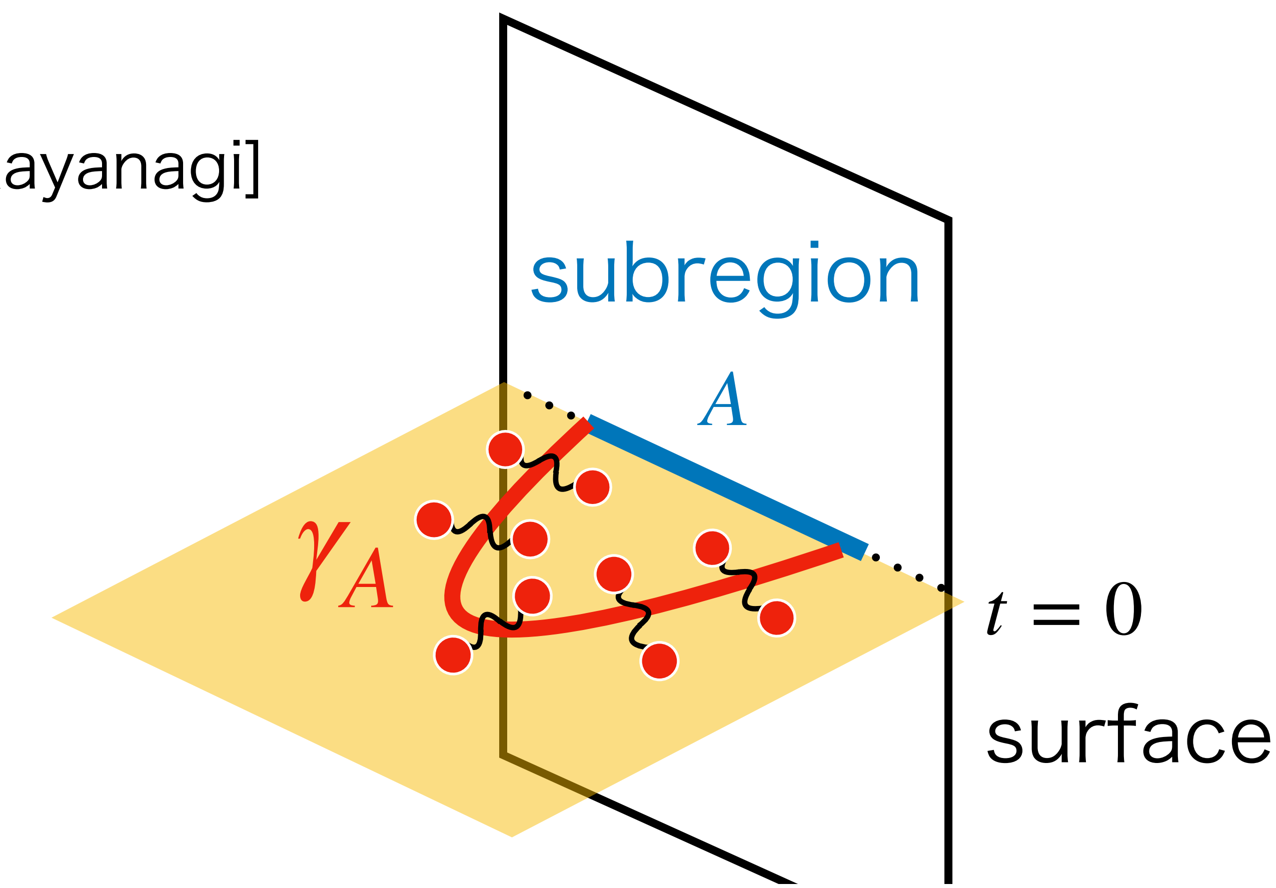
geometry quantum information

Ryu-Takayanagi formula [Ryu-Takayanagi]

$$S_A = -\mathrm{tr}\rho_A \log \rho_A, \quad \rho_A = \mathrm{tr}_{\bar{A}}\rho$$

$$\parallel$$

$$S_{HEE}(A) = \min_{\gamma_A} \frac{\mathrm{Area}(\gamma_A)}{4G_N}$$



cf. [Freedman-Headrick]

🤔 **Beyond AdS (hyperbolic) / CFT (critical)?**

Can we really see EPR pairs across γ_A?

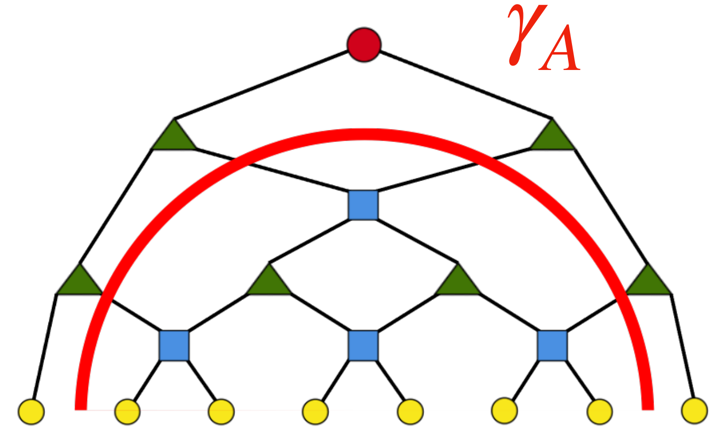
How does the geometry arise from a wave function?

Tensor networks as toy models of holography

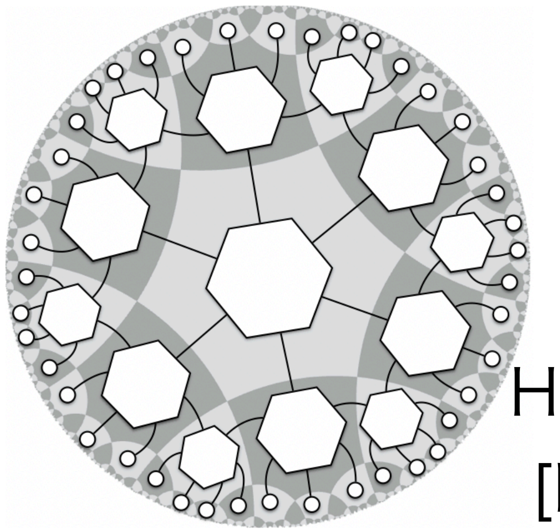
- Tensor networks (=variational wave function) provide a *qualitative* picture [Swingle]

$S_{TN}(A) \lesssim \min_{\gamma_A} (\#\ \text{bond cut by}\ \gamma_A) \times \log \chi$ ∼RT formula?

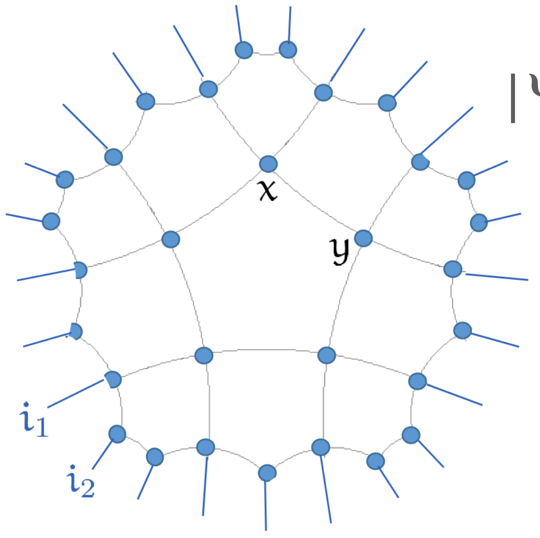
Multi-scale entanglement renormalization ansatz (MERA) [Vidal]



- Some proposals try to mimic holography (esp. RT formula)



Holographic state [Pastawski et al.]



$|\Psi\rangle = \left(\otimes_{\langle x,y\rangle} \langle \mathrm{MES}|_{xy} \right) \left(\otimes_x U_{random} |0_x\rangle \right)$

Random TN with large bond dim. [Hayden et al.] [Qi-Yang][Apel et al.]

CONS: Lack of expressivity; TN state ≠ conformally invariant

Entanglement distillation in holographic tensor networks

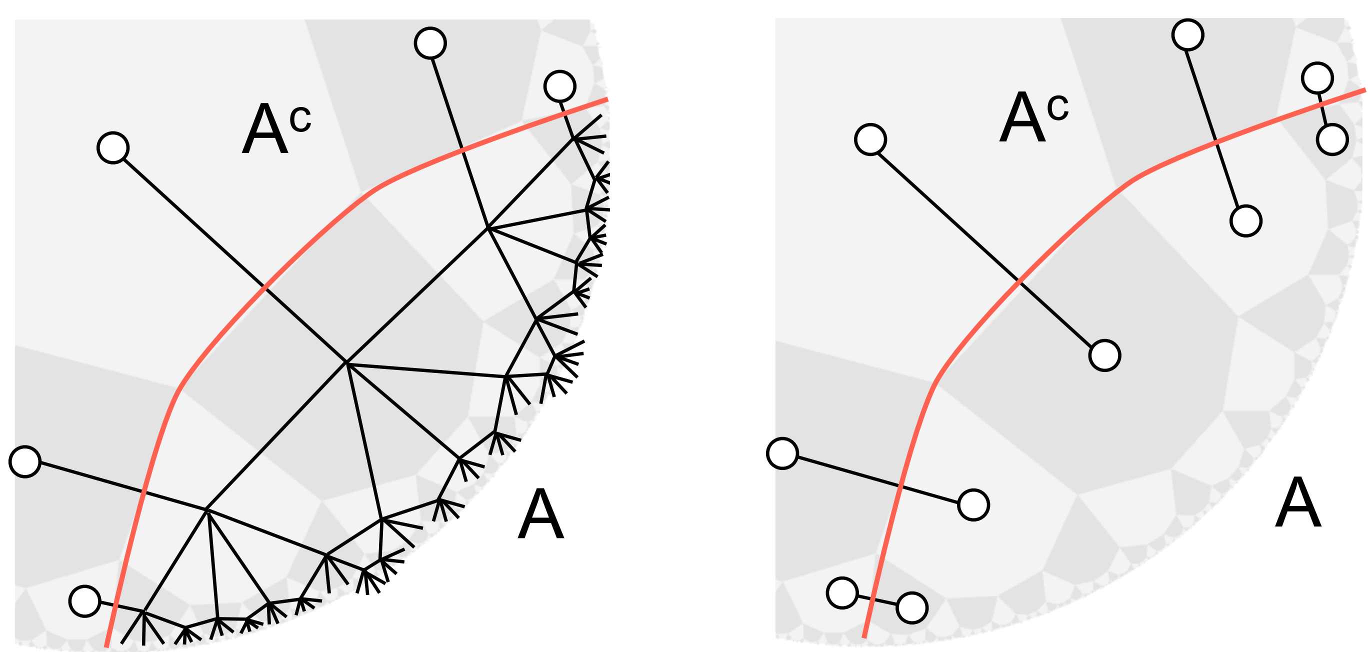
[Pastawski et al.]

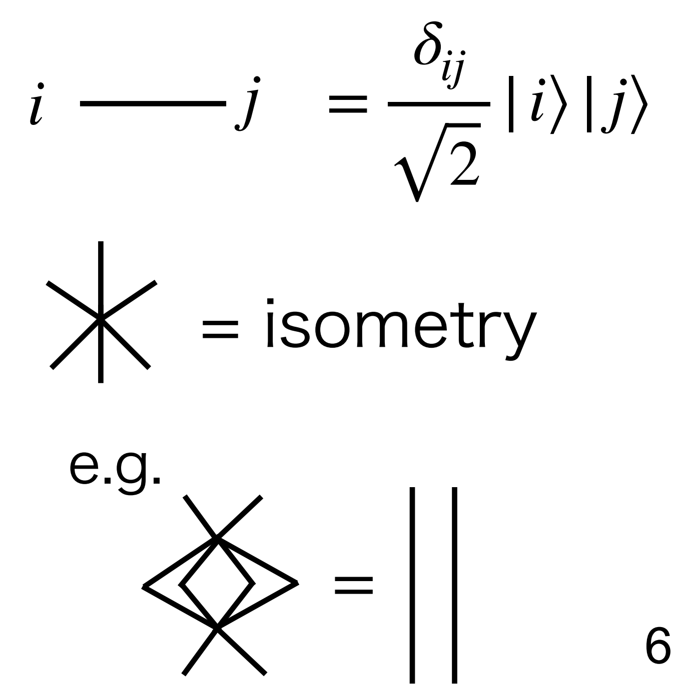
(Similar work: [Bao-Penington-Sorce-Wall], [Lin-Sun-Sun])

- A holographic state (or isometric tensor networks in general) is known to geometrize entanglement distillation

  = extracting S_A bits of EPR pairs from the state

- Removing tensors, we obtain EPR pairs across the minimal surface $\because S_A(V|\Psi\rangle) = S_A(|\Psi\rangle)$





Entanglement distillation in holographic tensor networks

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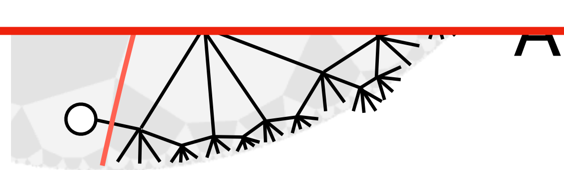
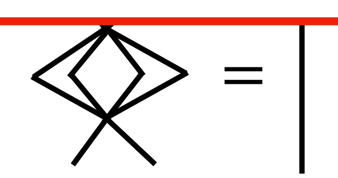
So far this manipulation is limited to isometric TNs.

Geometrization of entanglement distillation in other types of TNs?

➜ Clarify the operational role of internal d.o.f. after optimization!





Entanglement distillation in tensor networks

The important aspect for entanglement distillation is

1. Conservation of entanglement (entropy)

2. Extracting strongly entangled pairs

Entanglement distillation in tensor networks

The important aspect for entanglement distillation is

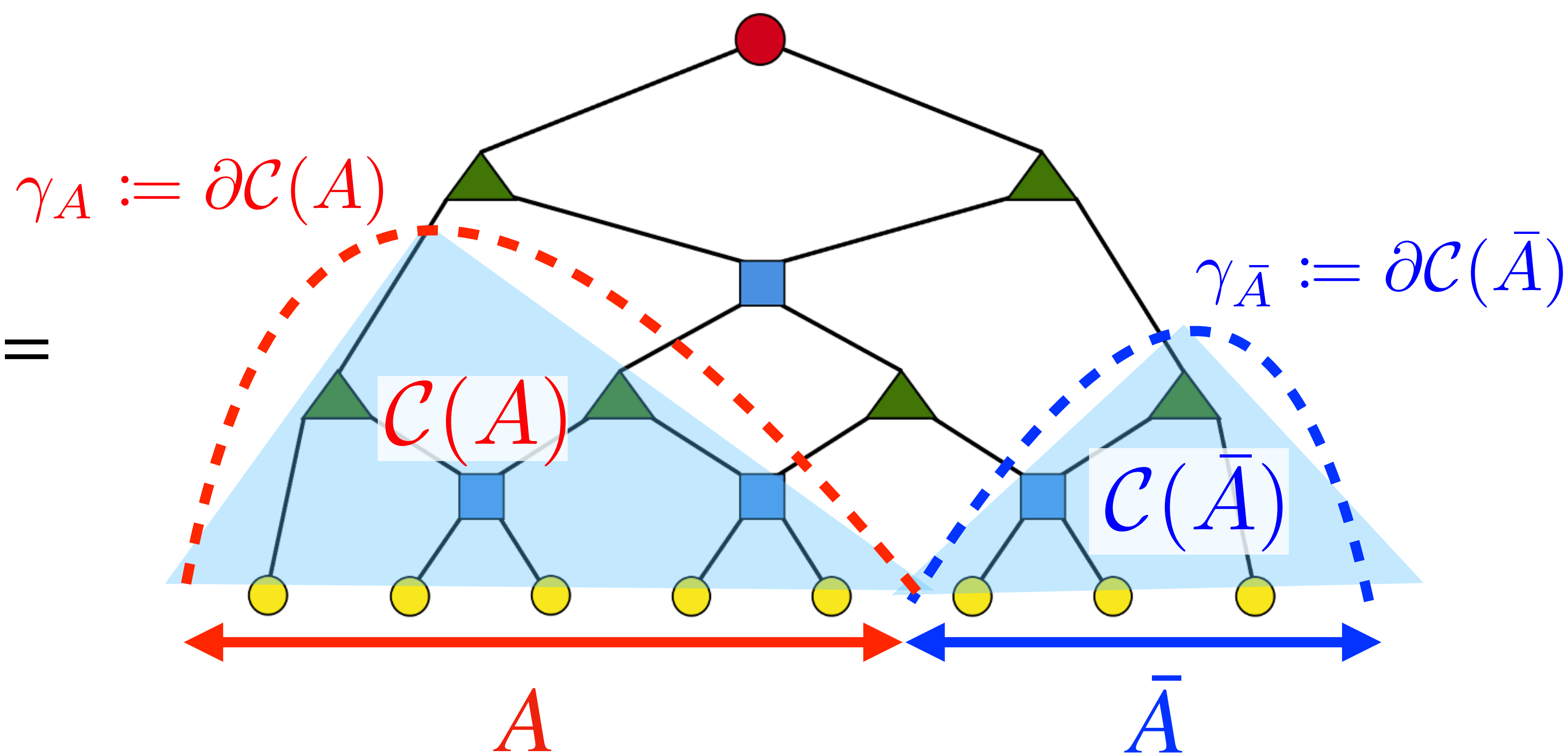
1. Conservation of entanglement (entropy)
   ✔ Reduced transition matrix instead of reduced density matrix
2. Extracting strongly entangled pairs
   ✔ Nontrivial for non-isometic TNs but can be systematically studied

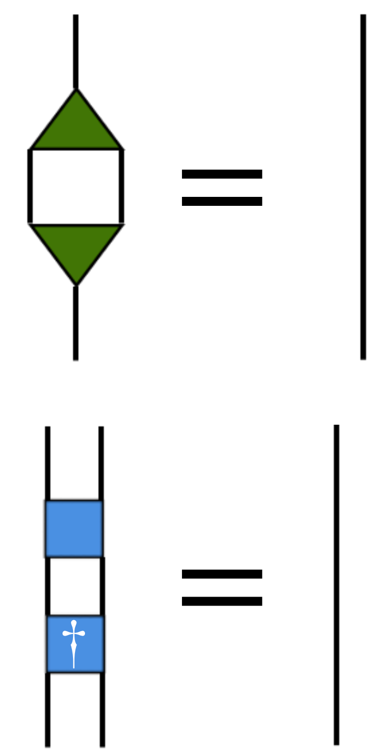
Multi-scale entanglement renormalization ansatz (MERA)

[Vidal]

- MERA has minimal bond cut surface(s) $\gamma_* = \min(\gamma_A, \gamma_{\bar{A}})$
- What we want to see: A state on $\gamma_* \overset{?}{=}$ EPR pairs
- Need to provide a method to properly define a state on a bond cut surface γ

$|\Psi\rangle_{A\bar{A}} =$





Entanglement distillation in MERA

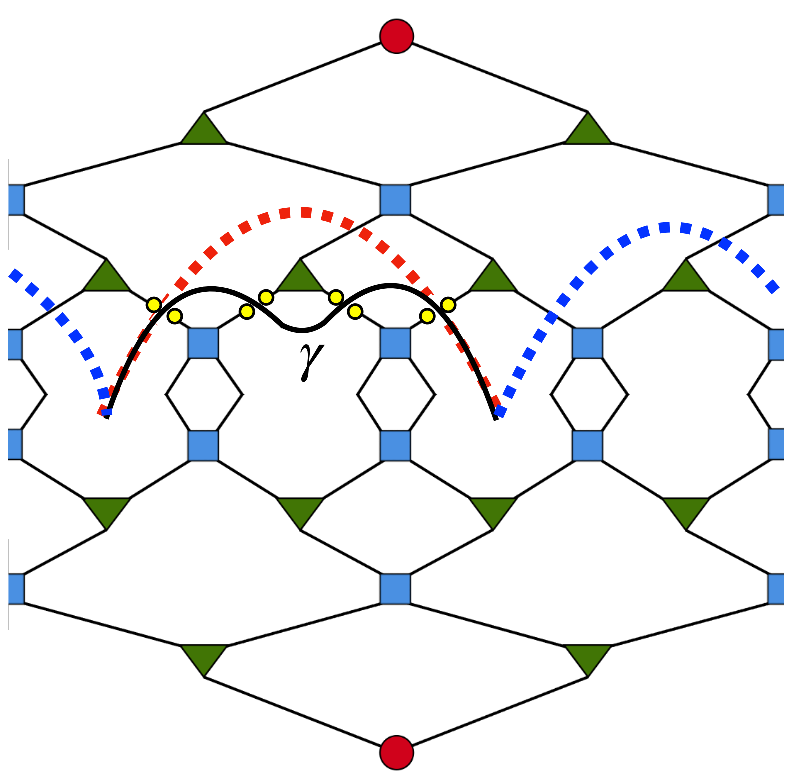
[**TM**-Manabe-Matsueda]

- Cut internal bonds across a bond cut surface γ instead of removing tensors from

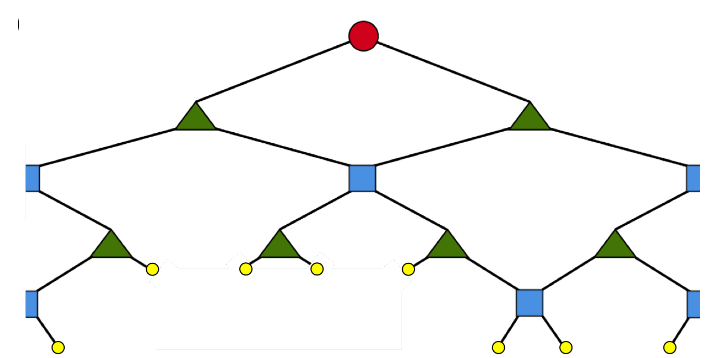
  ➡This defines a reduced transition matrix $\rho_\gamma = \mathrm{tr}_{\bar{A}} \left(|\Psi(\gamma)\rangle\langle\Phi(\gamma)| \right)$ on $\mathscr{H}_\gamma$

- To relate it with entanglement distillation, we consider foliations $\{\gamma\}$ s.t. $\partial\gamma = \partial A$.

$\rho_\gamma =$

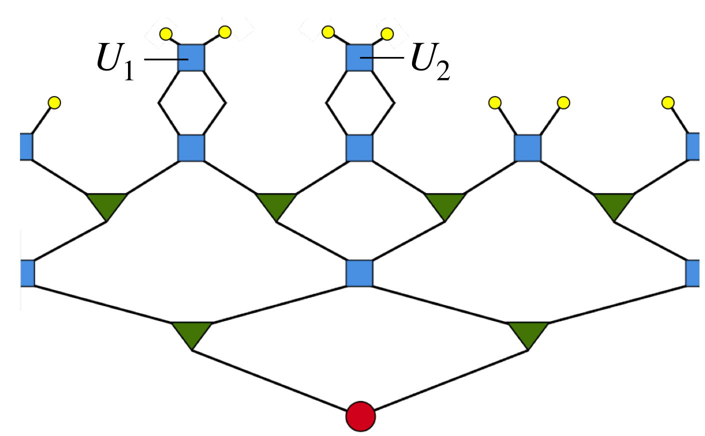


$|\Psi(\gamma)\rangle =$



$\in \mathscr{H}_\gamma \otimes \mathscr{H}_{\bar{A}}$

$\langle\Phi(\gamma)| =$



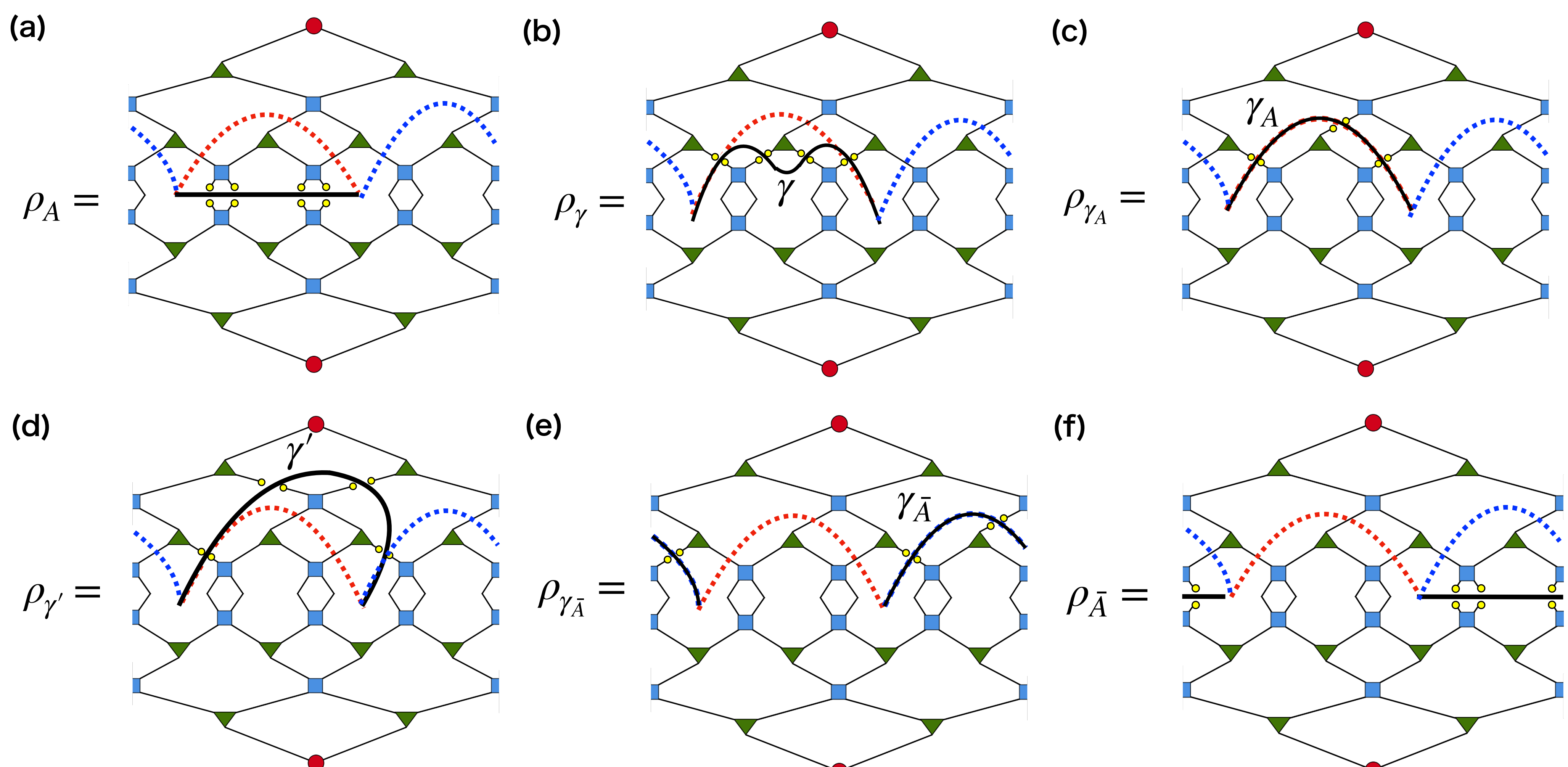
$\in \mathscr{H}^*_\gamma \otimes \mathscr{H}^*_{\bar{A}}$

$\equiv \langle\Psi| M_\gamma$

Entanglement distillation in MERA

[**TM**-Manabe-Matsueda]

- Changing the location of foliations γ, we obtain a family of states on various bond cut surfaces (e.g. ρ_A for $\gamma = A$, ρ_{γ_*} for $\gamma = \gamma_*$)
- Entanglement conservation w.r.t. $\forall\gamma$: $S(\rho_\gamma) = S(\rho_A)$, where $S(\rho_\gamma)$ is the pseudo entropy [Nakata-Takayanagi et al.] $S(\rho_\gamma) = -\mathrm{tr}\rho_\gamma \log \rho_\gamma$



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  ⬅ This is owing to the common eigenvalue distribution between ρ_γ and ρ_A ; the same entanglement spectrum!

- Furthermore, when M_γ is isometric, we can show $\langle \Phi(\gamma) | = \langle \Psi(\gamma) |$

  $\Rightarrow$ For isometric TNs, we obtain EPR pairs across the minimal bond cut surface (i.e. $\rho_{\gamma_*} \propto \mathbf{1}$) when $\gamma = \gamma_*$

Entanglement distillation in MERA

[**TM**-Manabe-Matsueda]

- To go beyond the isometric case, we need to define a state from a reduced transition matrix.

We use the purification technique (a.k.a. channel-state duality)

$$|\rho_\gamma^{1/2}\rangle \equiv \mathcal{N}_\gamma \sqrt{\dim \mathscr{H}_\gamma}(\rho_\gamma^{1/2} \otimes \mathbf{1})|\mathrm{EPR}_\gamma\rangle,$$

where $\mathcal{N}_\gamma = \left[\mathrm{tr}(\rho_\gamma^{\dagger\,1/2}\rho_\gamma^{1/2})\right]^{-1/2}$ and $|\mathrm{EPR}_\gamma\rangle = (\dim \mathscr{H}_\gamma)^{-1/2} \sum_{i=1}^{\dim \mathscr{H}_\gamma} |i\rangle \otimes |i\rangle$.

This will be regarded as a geometrically distilled state up to γ by TN.

Now we can make a quantitative comparison w.r.t. EPR pairs!

➡ Trace distance from the EPR pair: $D_\gamma \equiv \sqrt{1 - \left|\langle \mathrm{EPR}_\gamma | \rho_\gamma^{1/2}\rangle\right|^2}$

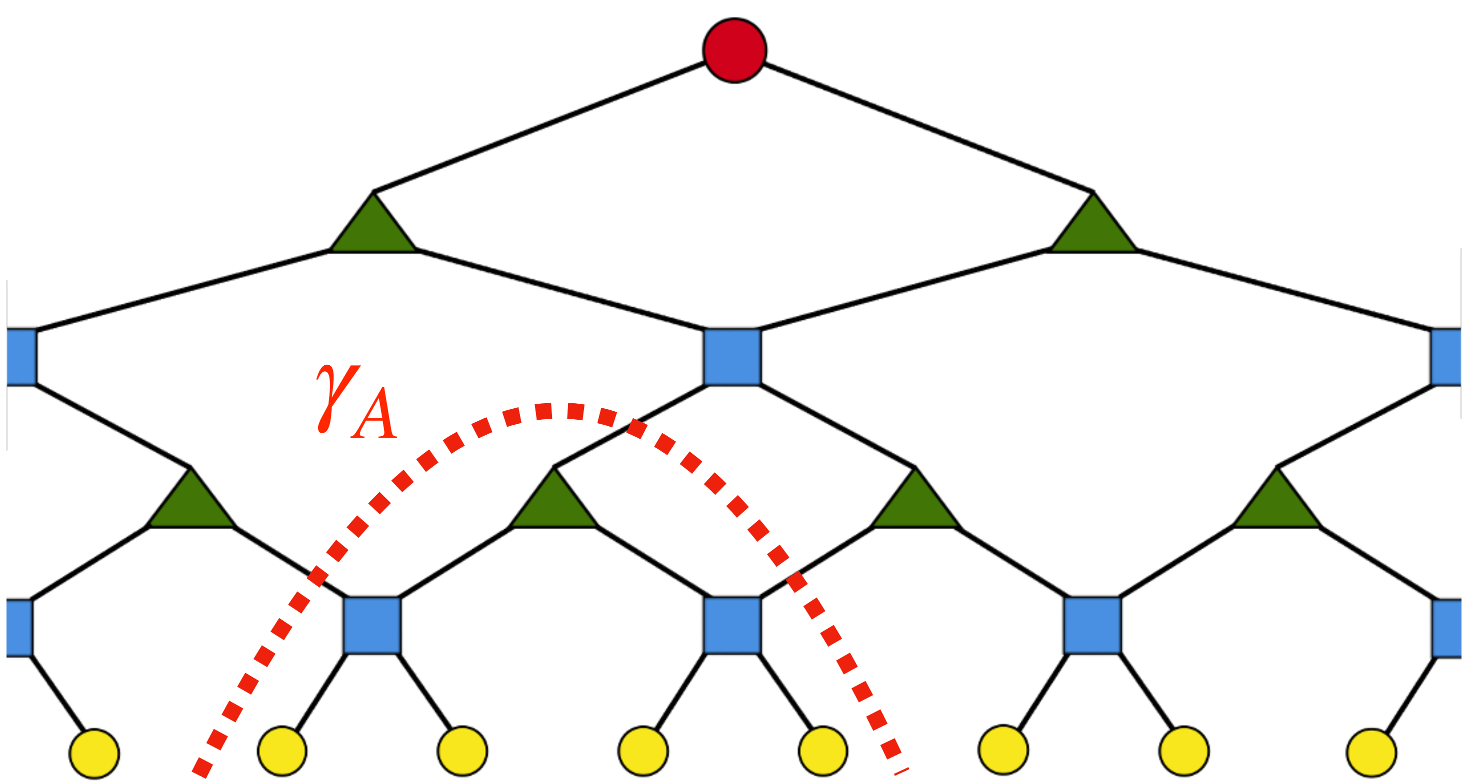
(related to Rényi-1/2 entropy $\left|\langle \mathrm{EPR}_\gamma | \rho_\gamma^{1/2}\rangle\right|^2 = \frac{\mathcal{N}_\gamma^2}{\dim \mathscr{H}_\gamma} e^{S_{1/2}}$)

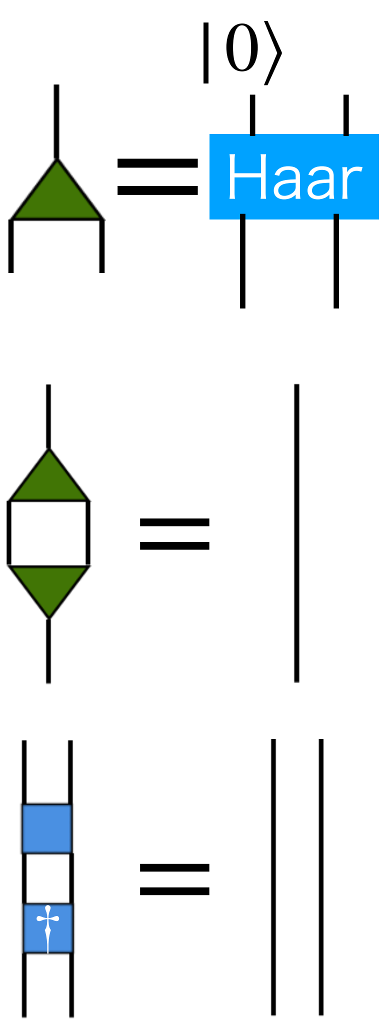
Random MERA

- Random TN is expected to reproduce RT formula in the large bond dimension limit [Hayden et al.]

🤔 Why is the minimal bond cut surface is special?
(RT formula only tells us about entanglement entropy)

🤔 The large bond dimension limit is essential?



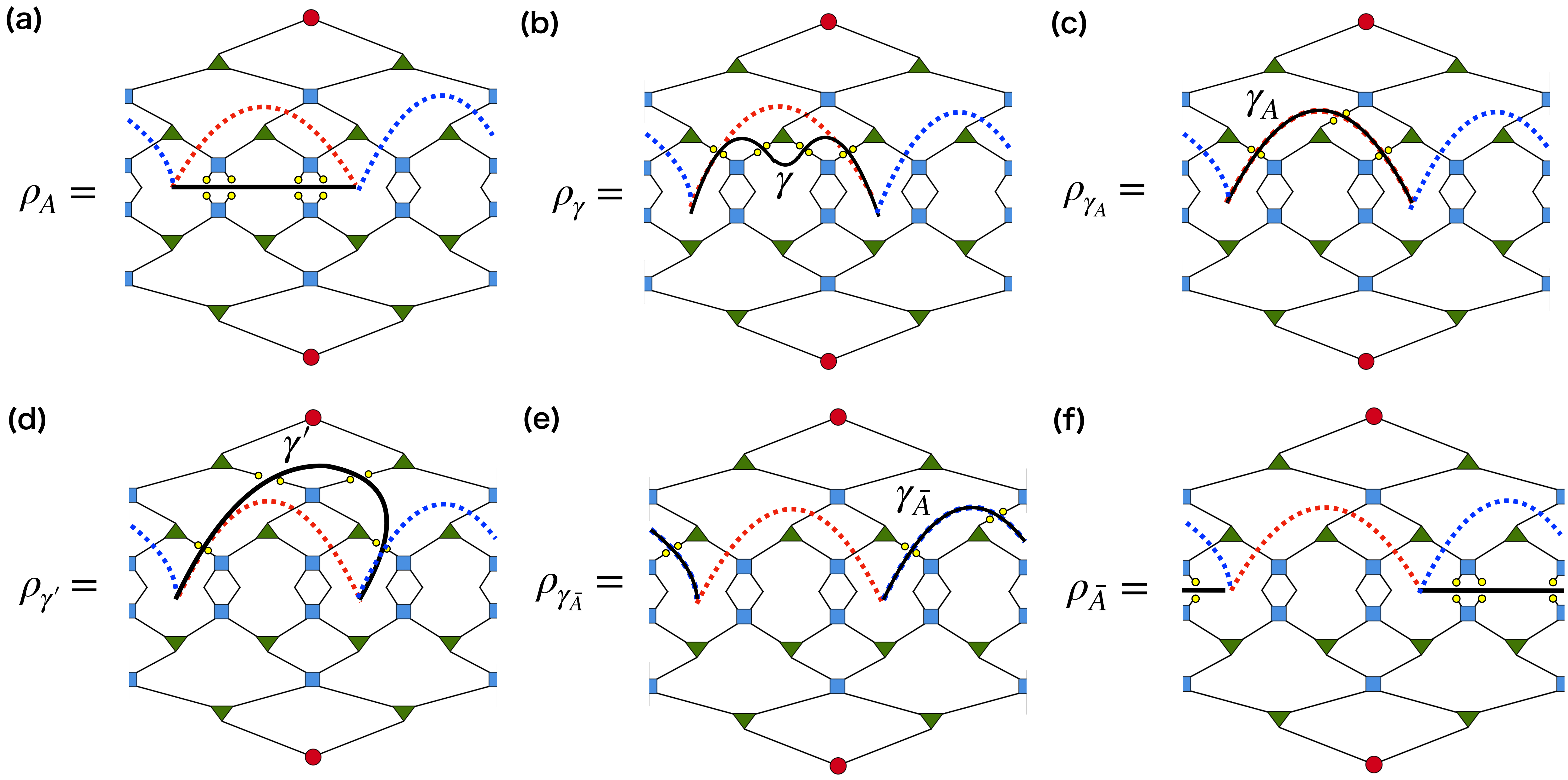


(All tensors are Haar random)

Numerical results for random MERA

[**TM**-Manabe-Matsueda]

Choice of foliations:

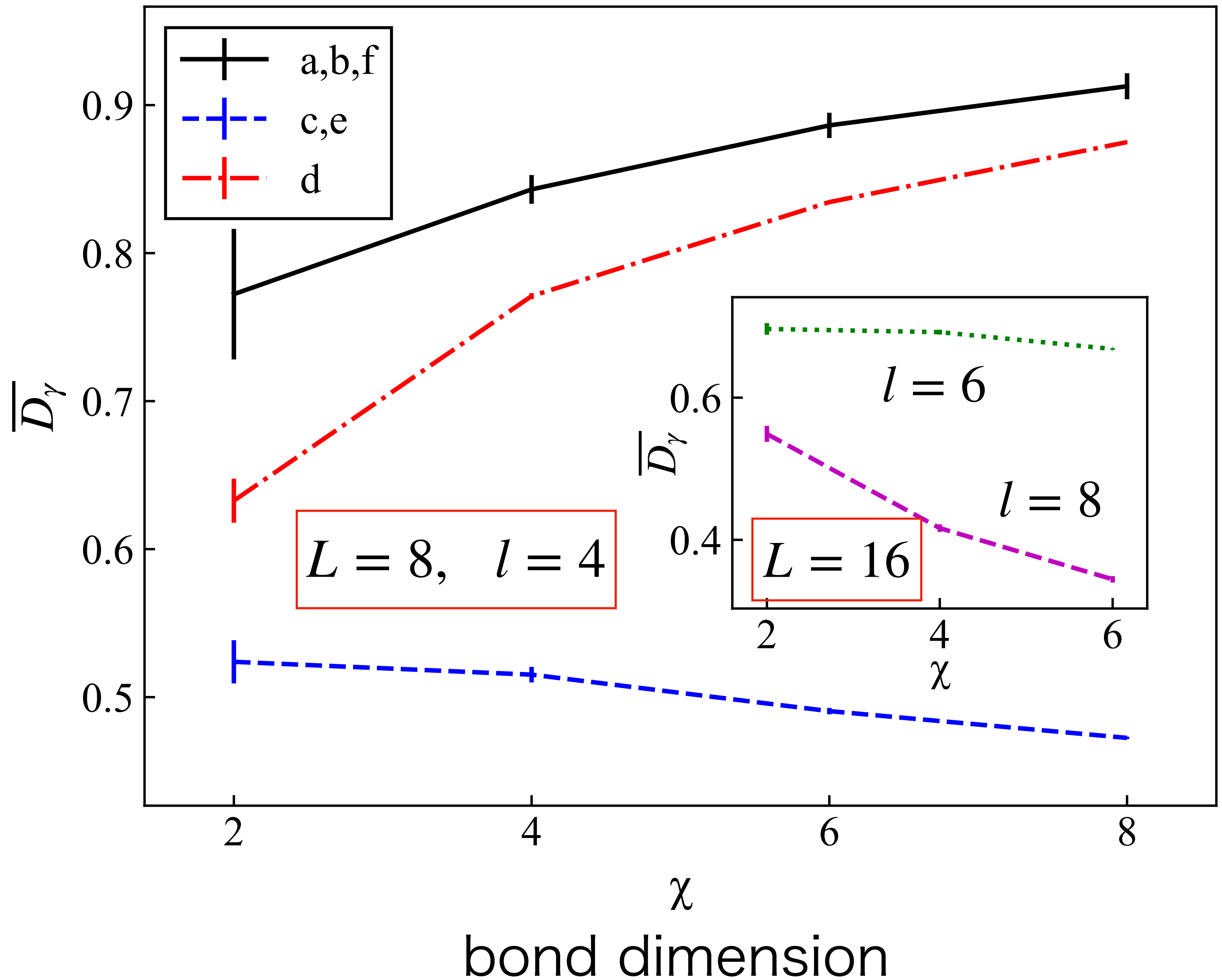


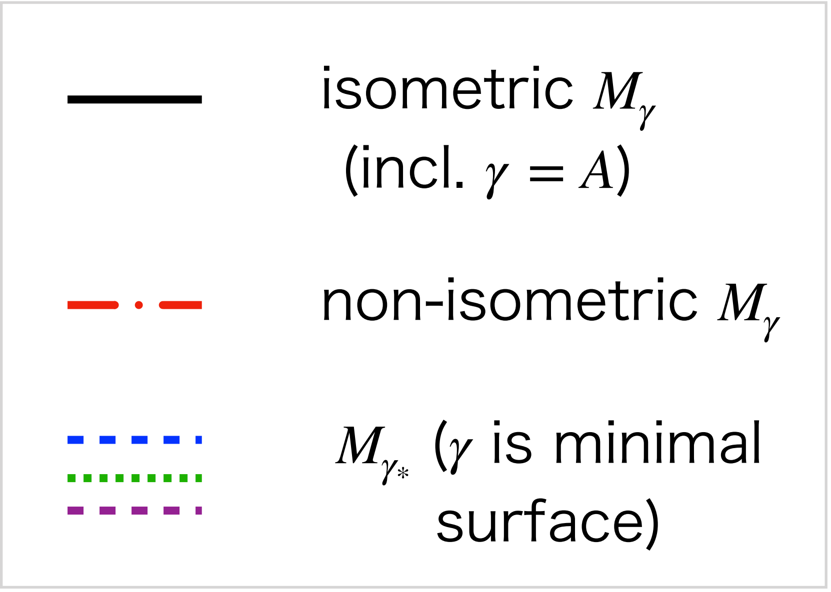
Numerical results for random MERA

[**TM**-Manabe-Matsueda]

We compared the (averaged) trace distance $D_\gamma \equiv \sqrt{1 - \left|\langle \mathrm{EPR}_\gamma | \rho_\gamma^{1/2} \rangle\right|^2}$

between $|\rho_\gamma^{1/2}\rangle$ and the EPR pair $|\mathrm{EPR}_\gamma\rangle$.





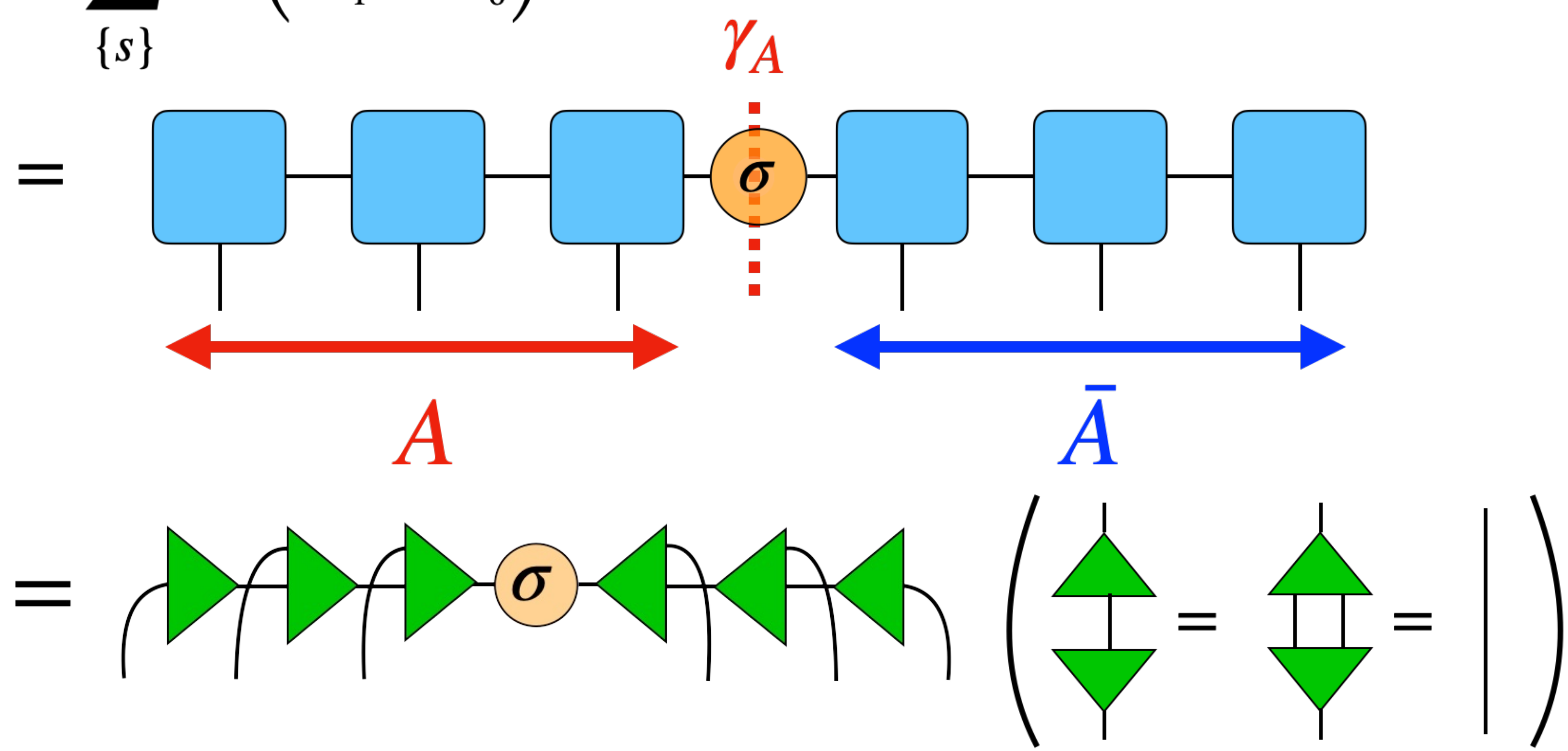
Entanglement distillation in MPS

[**TM**-Manabe-Matsueda]

Similarly consider pushing the foliation towards the minimal bond cut surface in matrix product states (MPS)

Let us focus on the MPS in a mixed canonical form

$$|\Psi\rangle = \sum_{\{s\}} \mathrm{Tr}\left(A_{s_1} \cdots A_{s_6}\right) |s_1 \cdots s_6\rangle$$

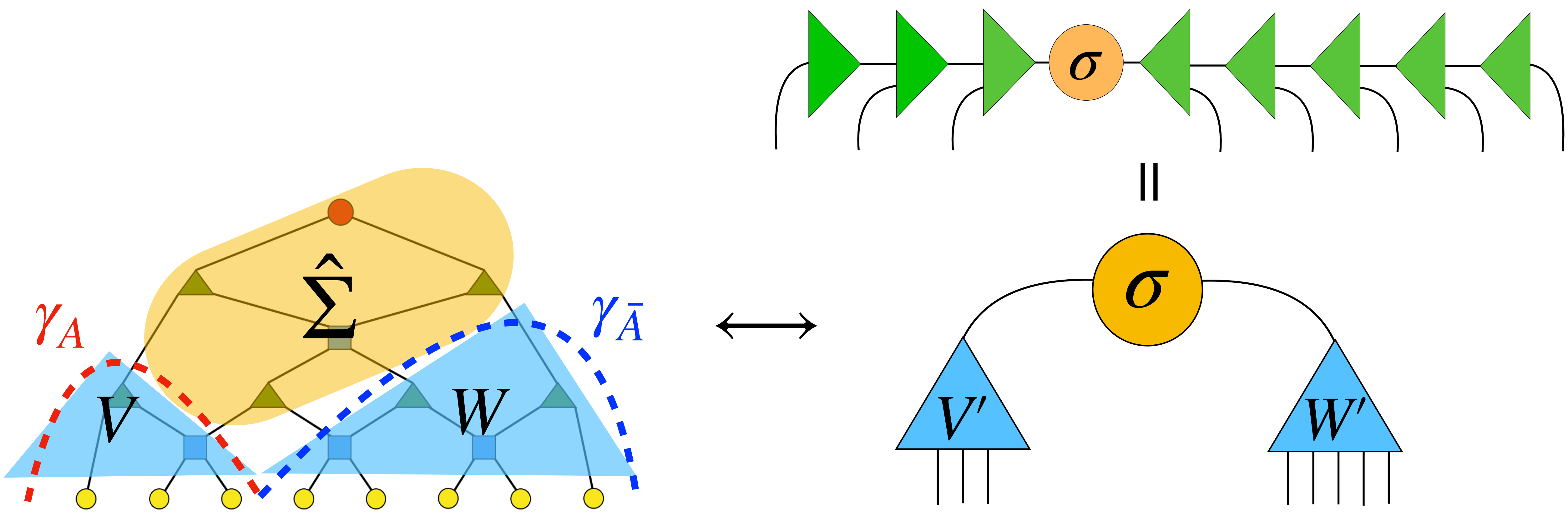


Entanglement distillation in MPS

[**TM**-Manabe-Matsueda]

Note:

An MPS in a mixed canonical form is an analog of MERA (regarding its structure)

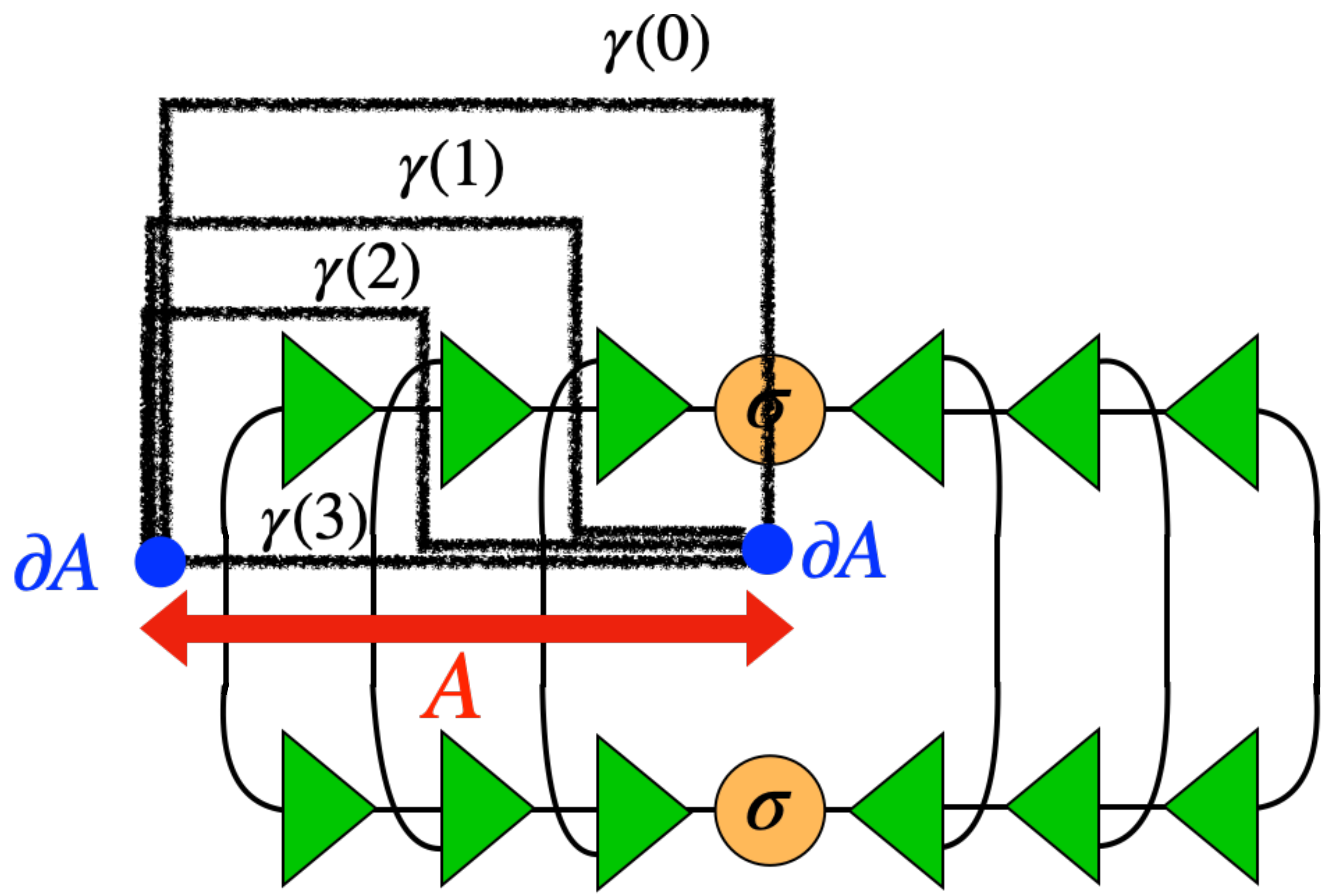


Entanglement distillation in MPS

[**TM**-Manabe-Matsueda]

We can consider the following foliations $\gamma = \gamma(\tau), \quad \tau = 0,1,2,3$

($\tau \sim$ distance from the minimal bond cut surface)

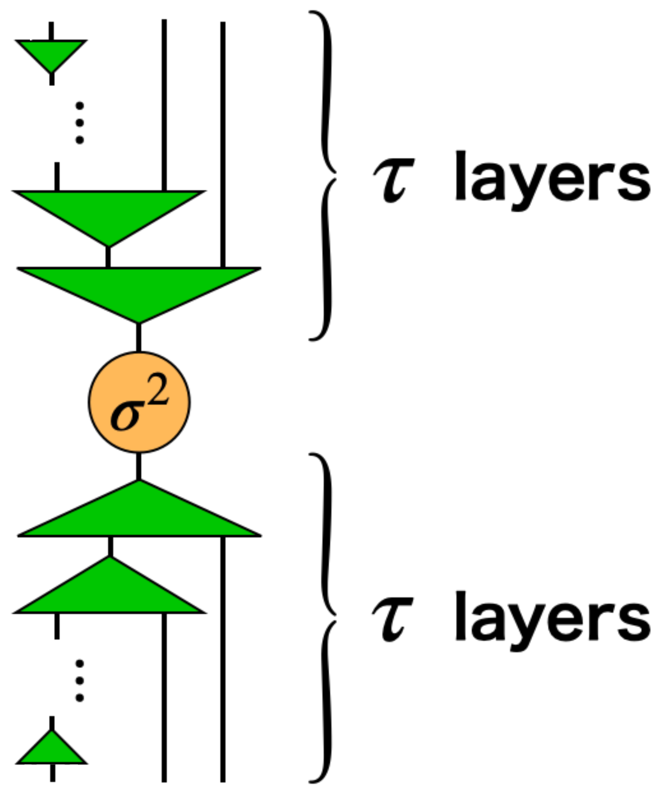


Entanglement distillation in MPS

[**TM**-Manabe-Matsueda]

The reduced transition matrix and the trace distance are

$$\rho_{\gamma(\tau)} =$$



$$D_{\gamma(\tau)} = \sqrt{1 - \frac{e^{S_{1/2}}}{\chi^{\tau+1}}}, \quad \tau = 0,1,2$$

$$D_{\gamma(3)} = D_{\gamma(2)}$$

The distillation by pushing the foliation equals removing redundant tensors.

The entanglement spectrum σ remains unchanged.

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Summary

- By pushing towards the minimal surface γ_*, strongly entangled pairs are geometrically distilled in tensor networks while retaining the entanglement spectrum
- It is essential to consider reduced transition matrices rather than a reduced density matrix
- Our method works for non-holographic TNs: This suggests geometry of TN is intimately related to distillation for generic TNs
  ➡ Holography beyond AdS/CFT

Emergent geometry from distillation

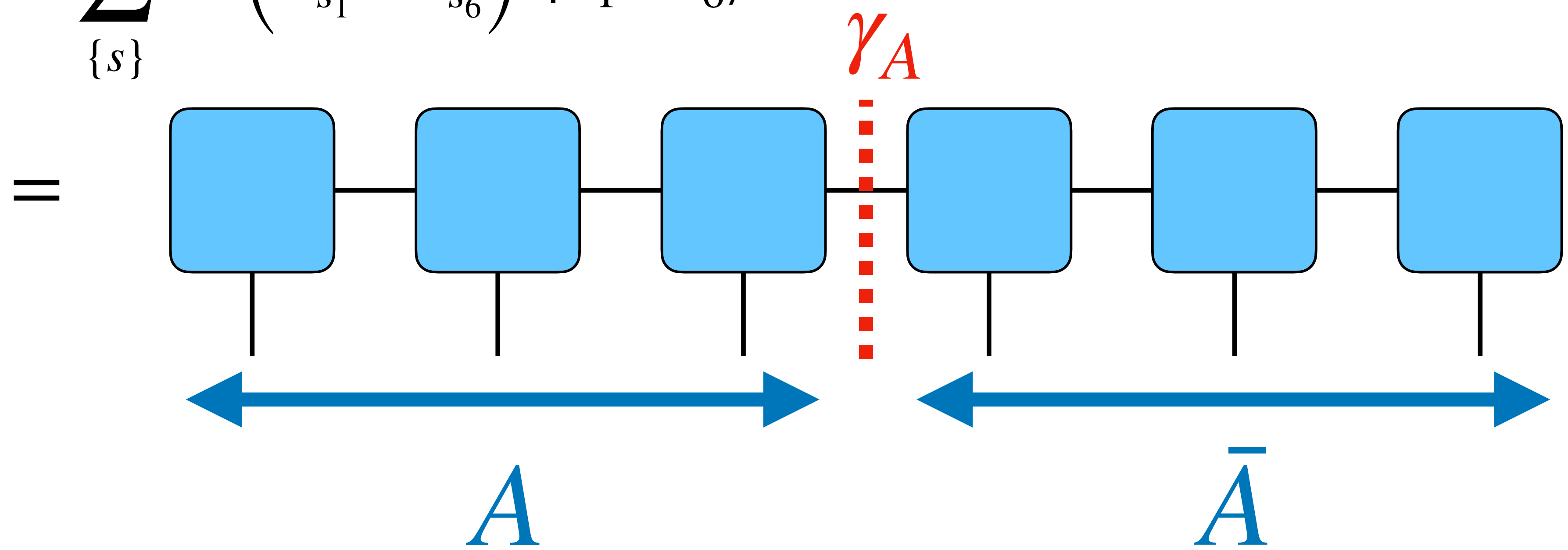
Future directions

- Operational interpretation of geometric distillation: local conf. trf., corner transfer matrix, modular flow? [Milsted-Vidal; Nishino-Okunishi; Okunishi-Seki]
- Analytic "proof" beyond MPS using analytic MERA rep. [Evenbly-White]
- CFT realization? Quant. adiabatic comp. (~annealing) with $T\bar{T}$ [McGough-Mezei-Verlinde]

Appendix

Example: Matrix Product States

$$|\Psi\rangle = \sum_{\{s\}} \mathrm{Tr}\left(A_{s_1}\cdots A_{s_6}\right) |s_1\cdots s_6\rangle$$

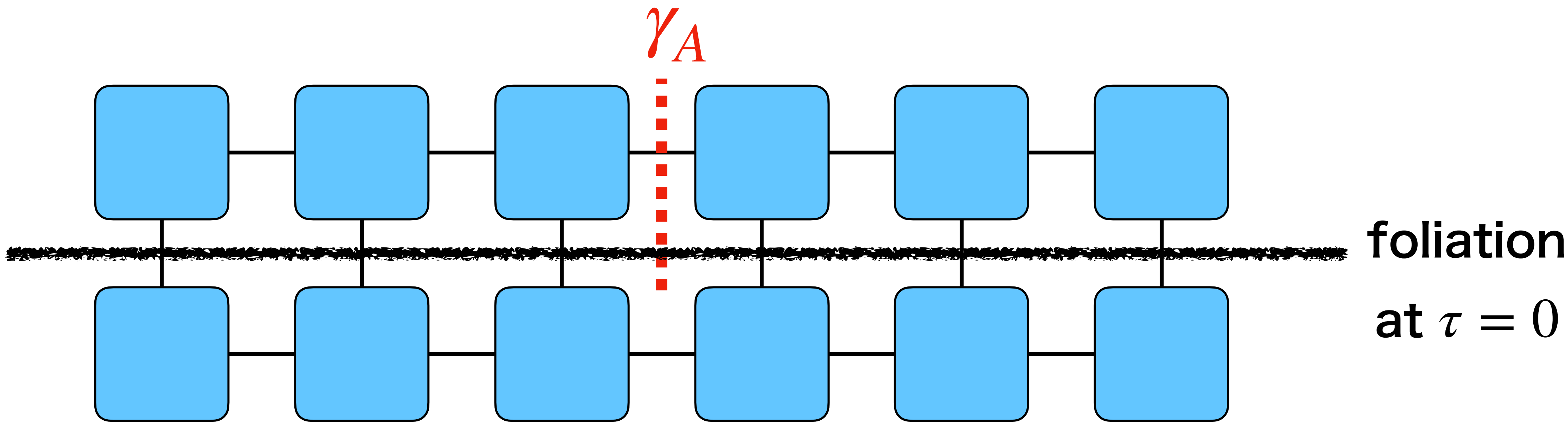


<u>Proposal: Holographic entanglement distillation</u> [Manabe-Matsueda-TM wip]

1. Push the boundaries for each A and $\bar{A}$ towards the RT (min bond cut) surface (cf. surface/state correspondence [Miyaji-Takayanagi])
2. Define a new state on a pushed boundary ("foliation") by removing the unreached part of TN and taking an inner product with the original TN

(Note: We will use the word RT surface and min. bond cut surface interchangeably)

Example: Matrix Product States



(τ: flow time)

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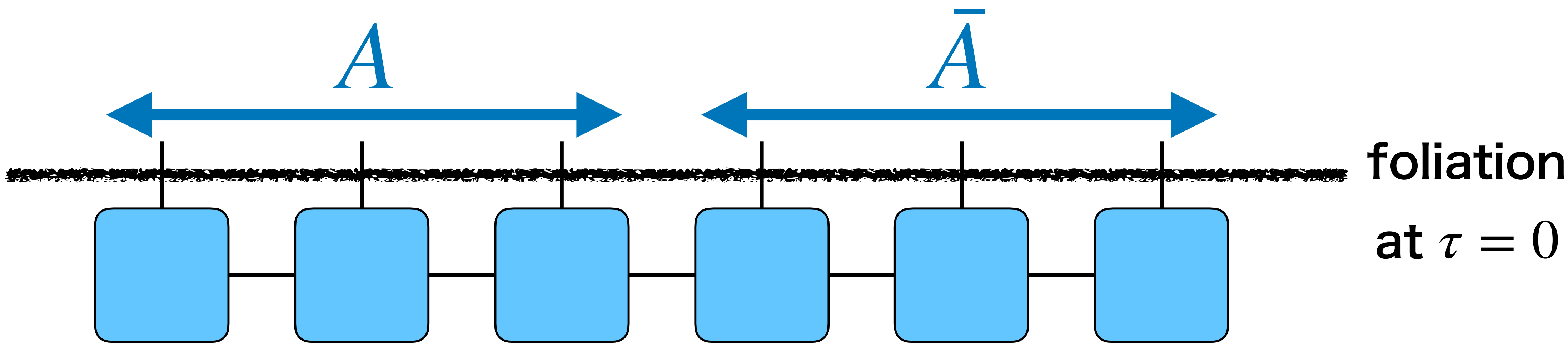

$= |\Psi\rangle$ (original state)

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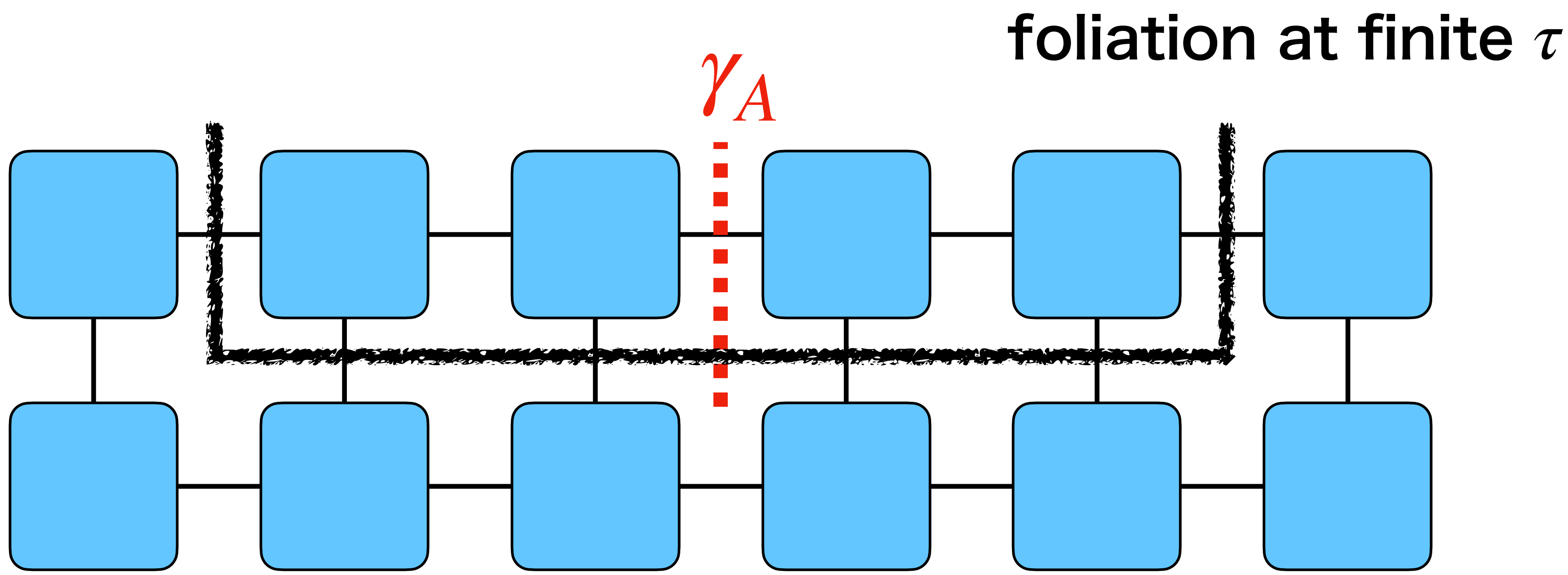

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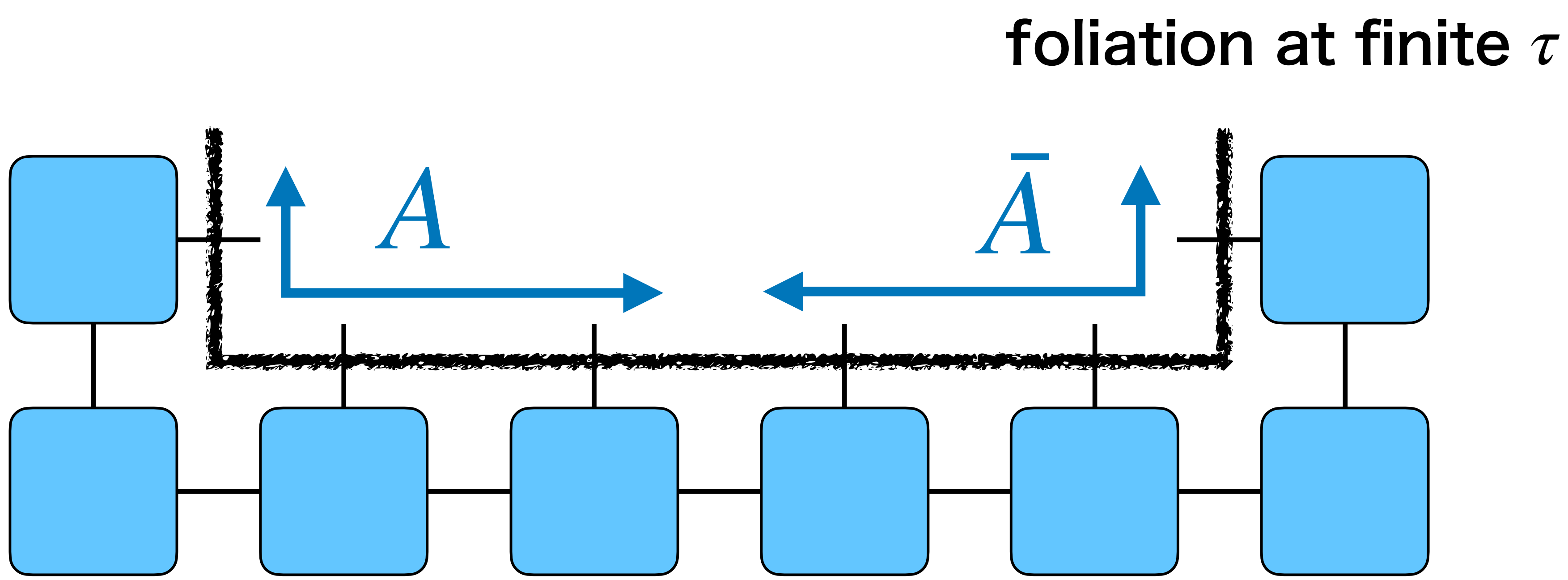

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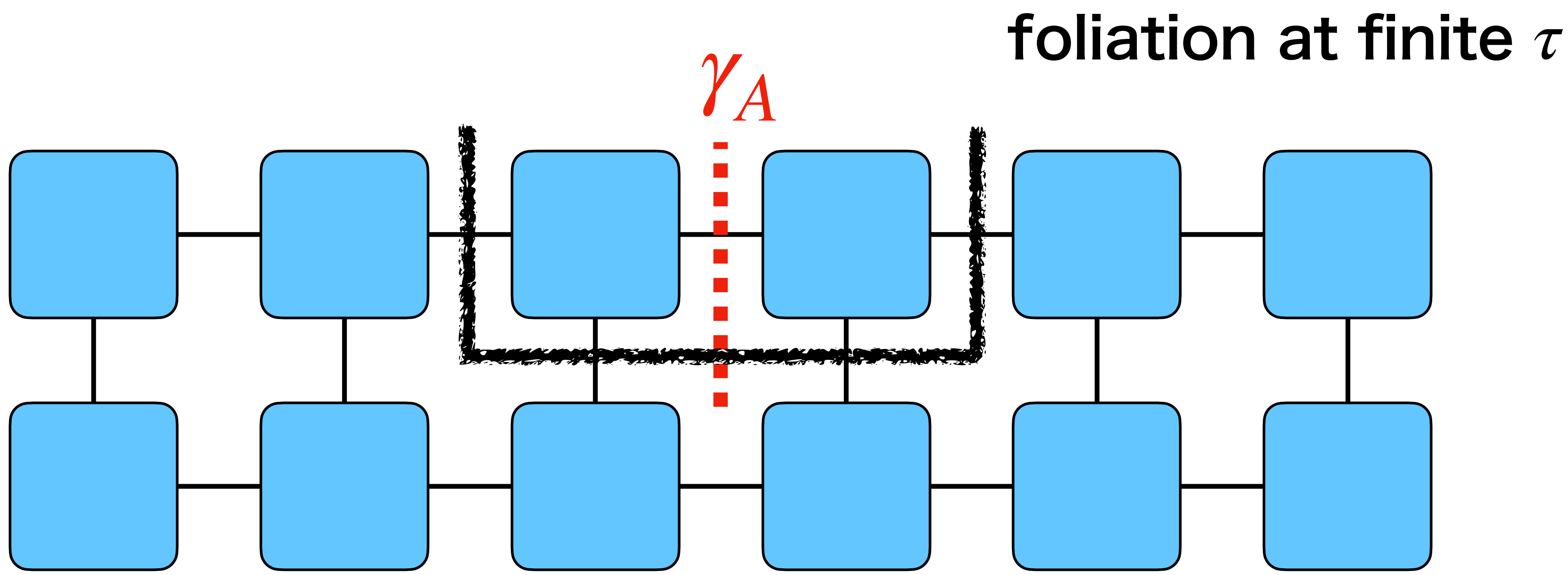

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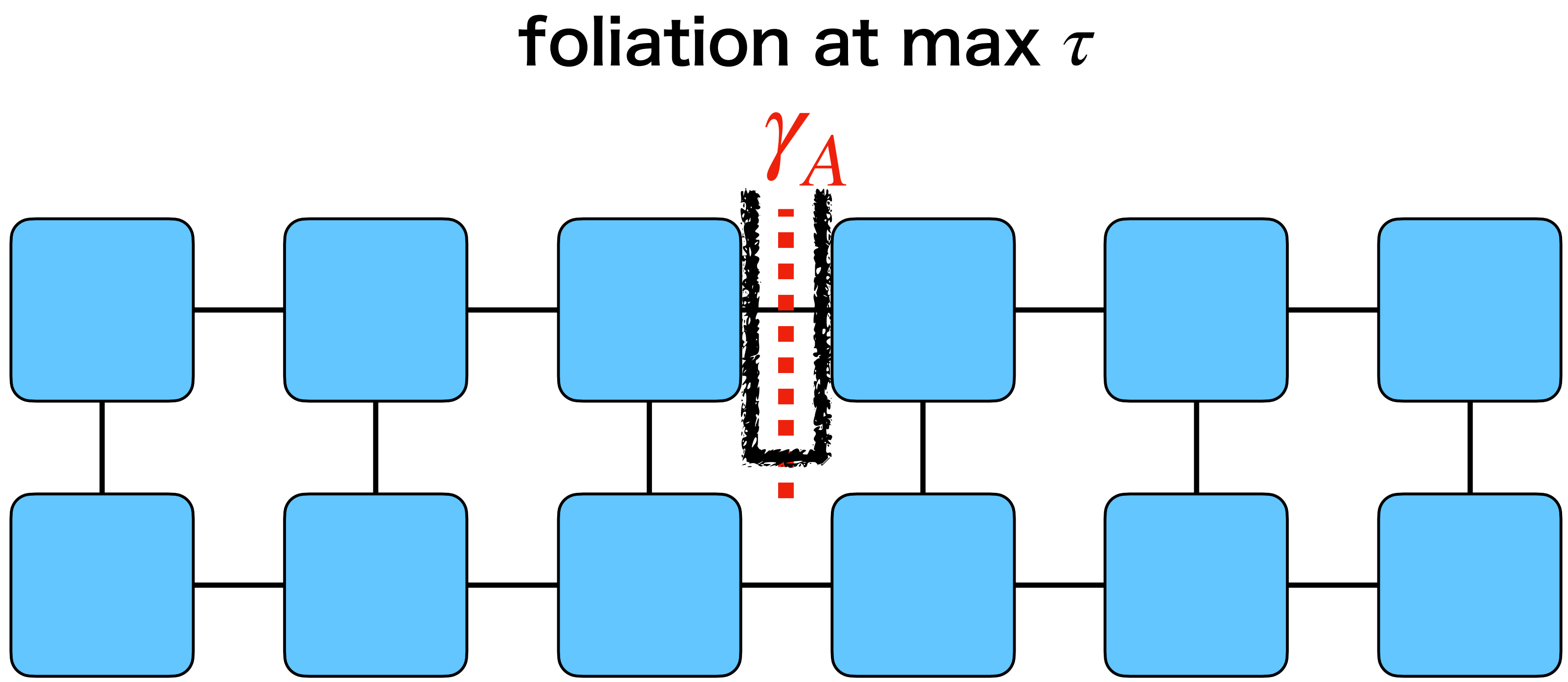

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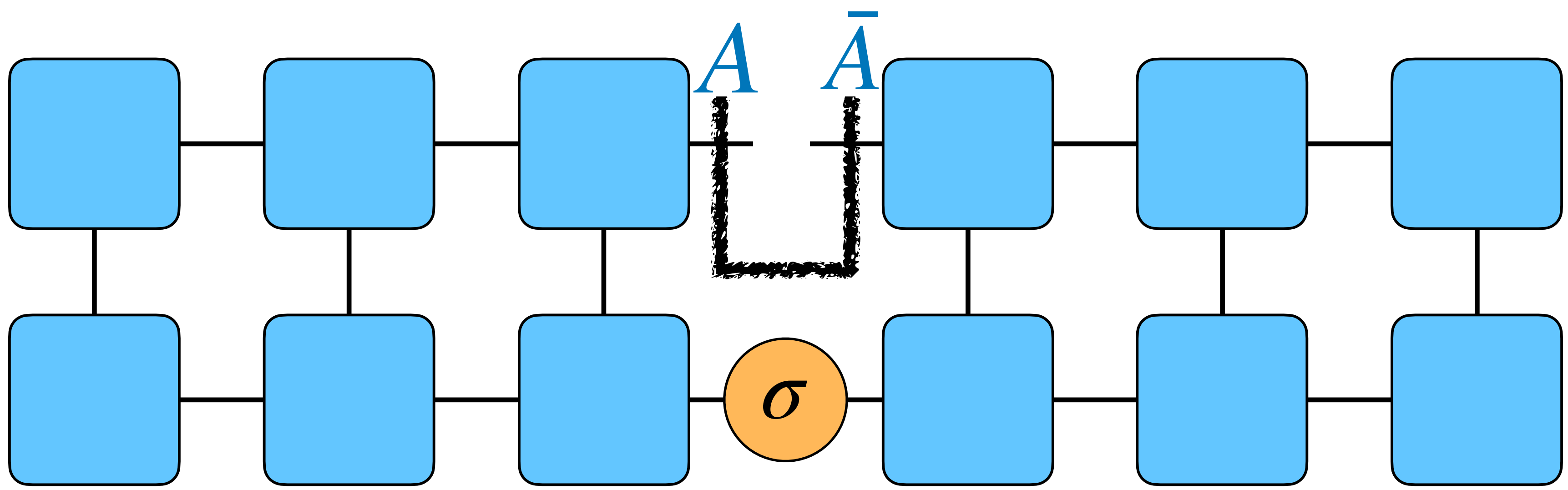
Example: Matrix Product States

foliation at max τ



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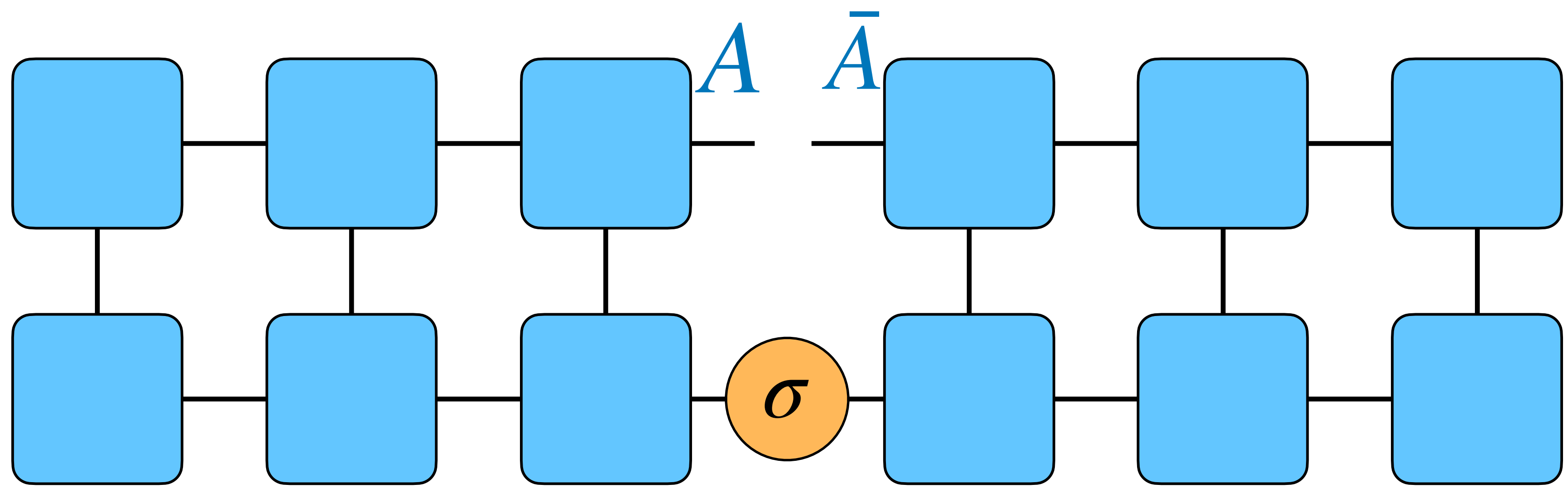
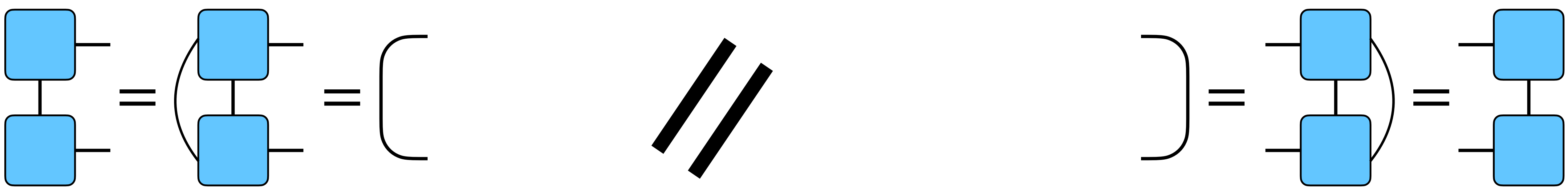
2. Define a new state on a pushed boundary (“foliation”) by removing the unreached part of TN and taking an inner product with the original TN
3. To retain the amount of entanglement, remove singular value matrix σ_A at γ_A

Example: Matrix Product States

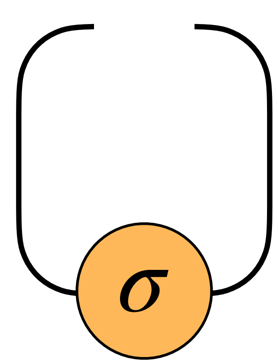
foliation at max τ



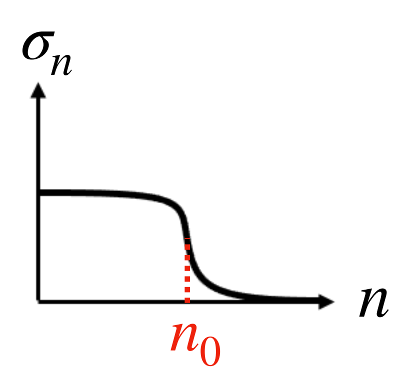


(∵ mixed canonical form)

$$\sum_{i}^{\chi} \sigma_i |i\rangle_A |i\rangle_{\bar{A}} =$$



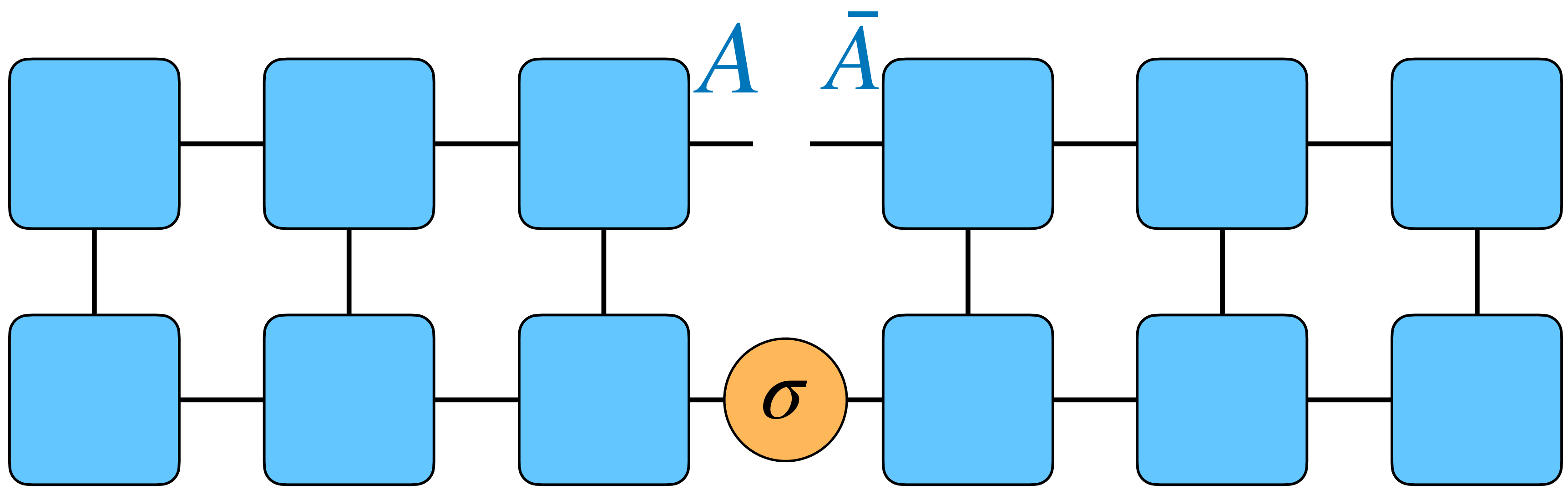
When ES is like a plateau,



σ can be considered as nearly uniform by taking $\chi = n_0$, which well approximate the original state

Example: Matrix Product States

foliation at max τ



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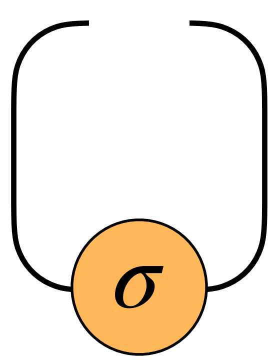
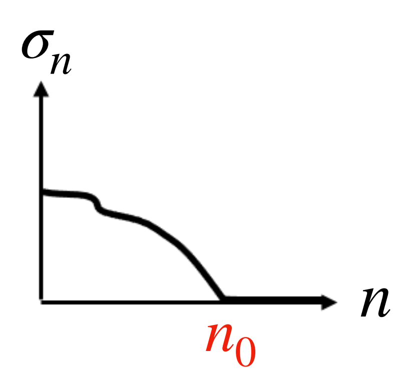
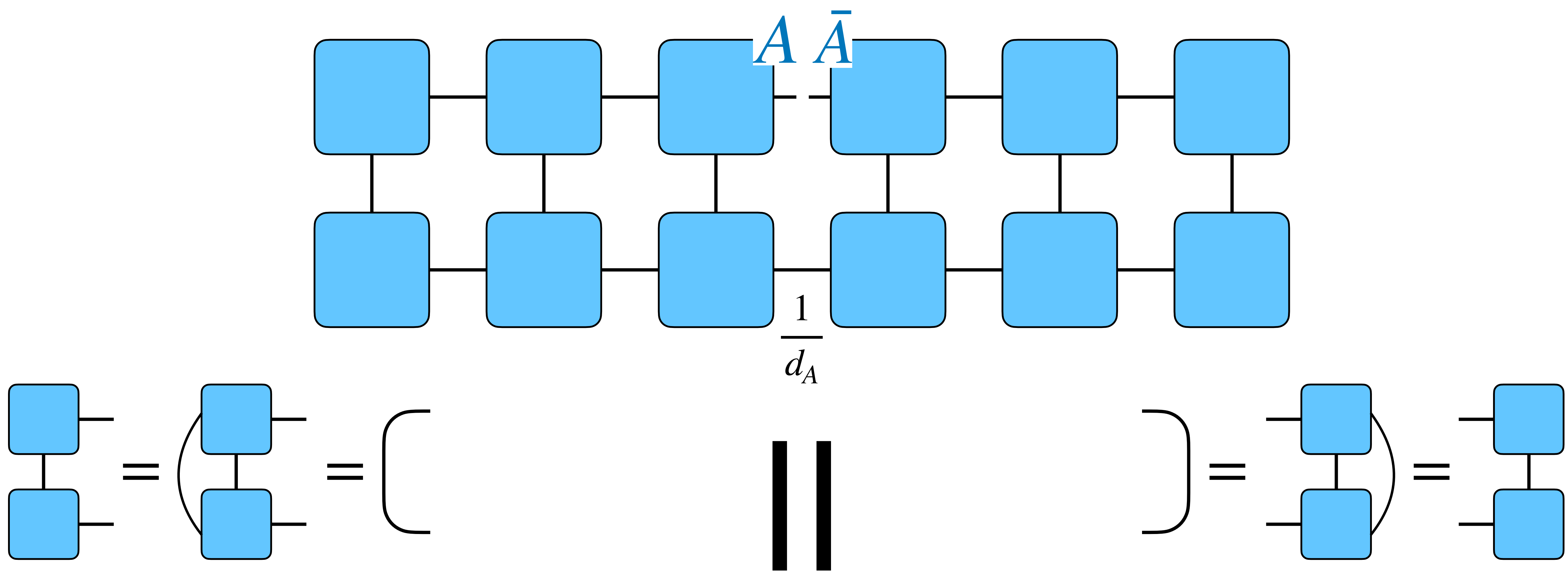




If the ES is like the left figure,
σ cannot be uniform although
$\chi = n_0$ gives a good approx.
(We try to approx. the state by geometry not with each tensors as much as possible)

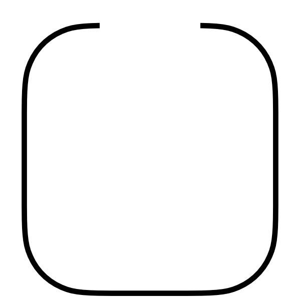
Example: Matrix Product States

foliation at max τ



(∵ mixed canonical form)

Flat ES (leading order in holography)
~ iid [Bao et al. '19]

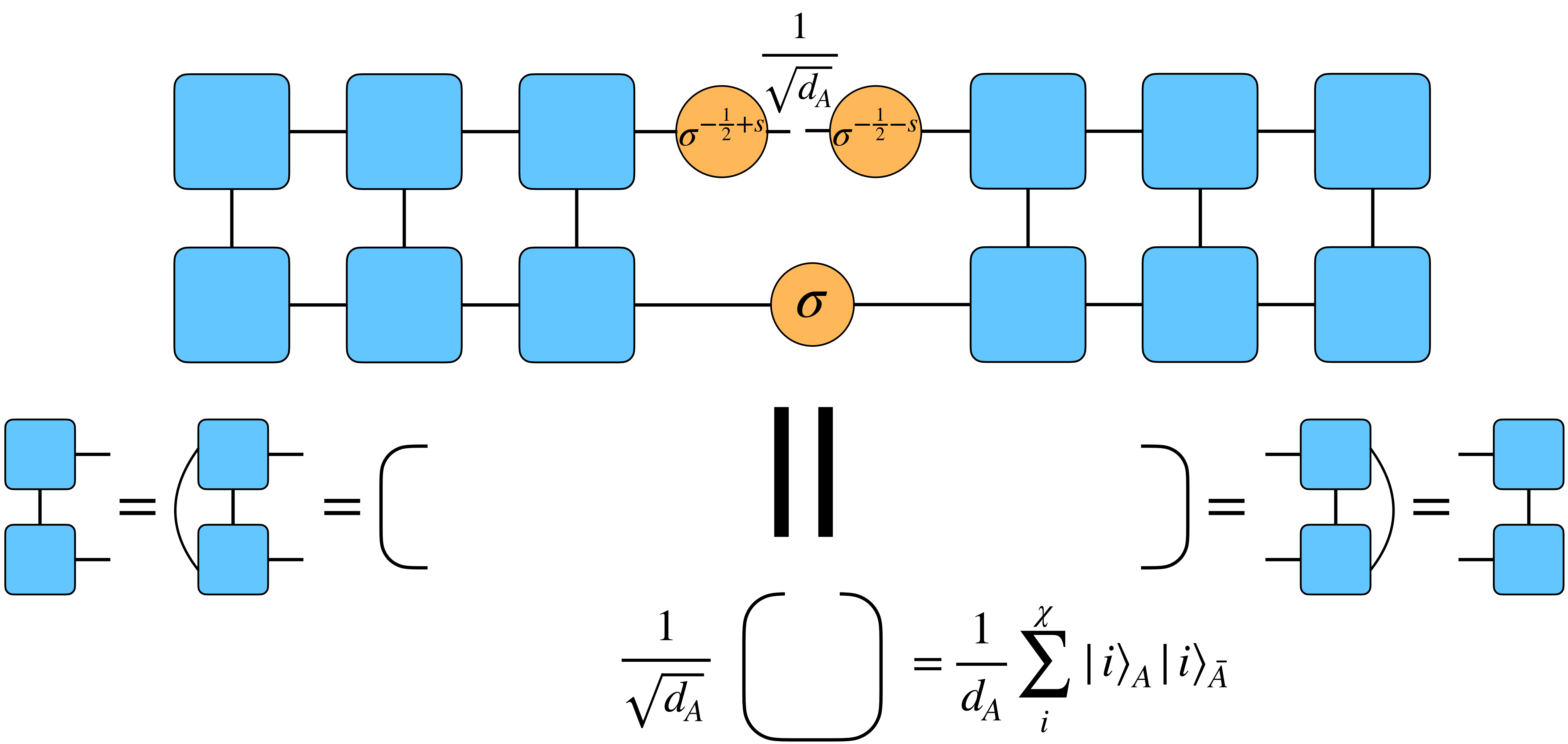
$\frac{1}{d_A}$  $= \frac{1}{d_A} \sum_{i}^{\chi} |i\rangle_A |i\rangle_{\bar{A}}$

Maximally entangled state is distilled!

($\chi \sim e^{S_A}$ for exact-MPS state, e.g. VBS)

Example: Matrix Product States

Another candidate for HED: **foliation at max τ**



✓ This is nothing but a distillation by bulk modular flow

✓ The amount of entanglement is not preserved; it maximizes EE

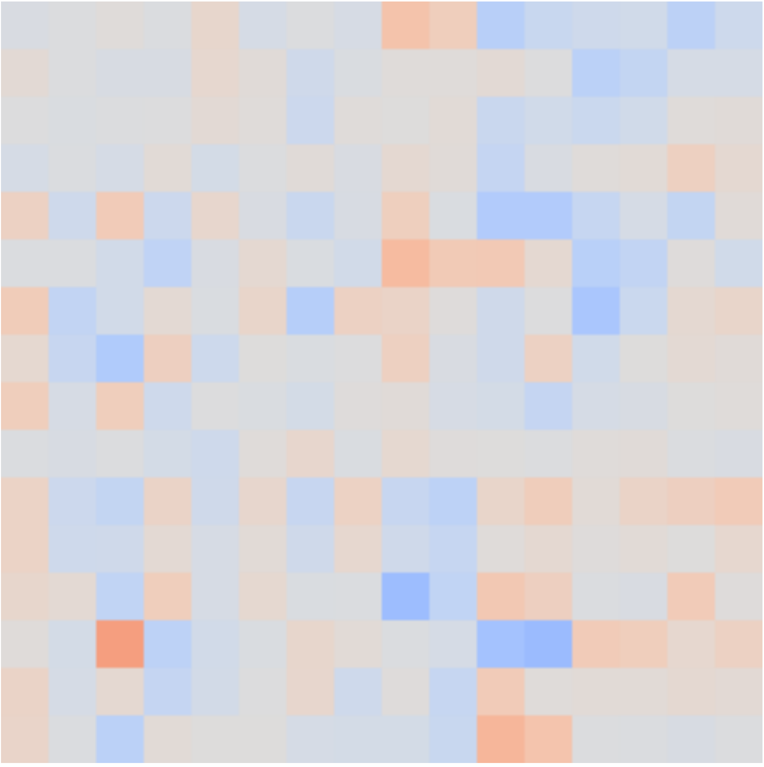
✓ This distillation is consistent with iid limit (inverse needs ∞-ly many copies)

[Quintino, et al. '19][Yoshida, et al. '21]

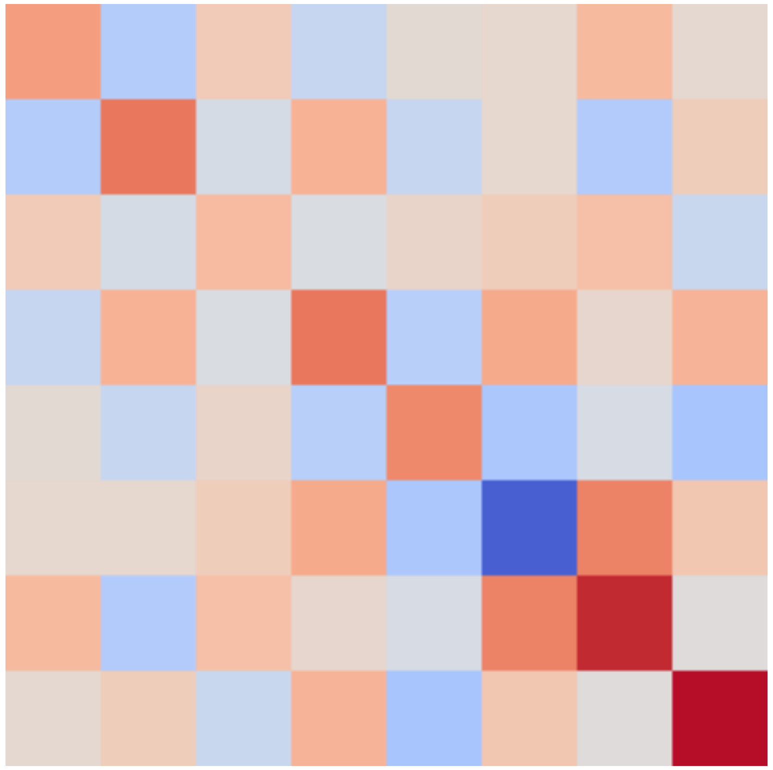
Example 2: Random MERA

- The distribution of the real part of coefficients (wave function) in $|\Psi\rangle$ (original state) and $|\Psi(\tau_{fin})\rangle$ (distilled one) for $\mathscr{H}_A$ and $\mathscr{H}_{\bar{A}}$:
  (larger: red, lower: blue, almost zero: white)

$\chi = 2$



$|\Psi\rangle$



$|\Psi(\tau_{fin})\rangle$

(Images are coarse-grained by 48×48 for visualization)

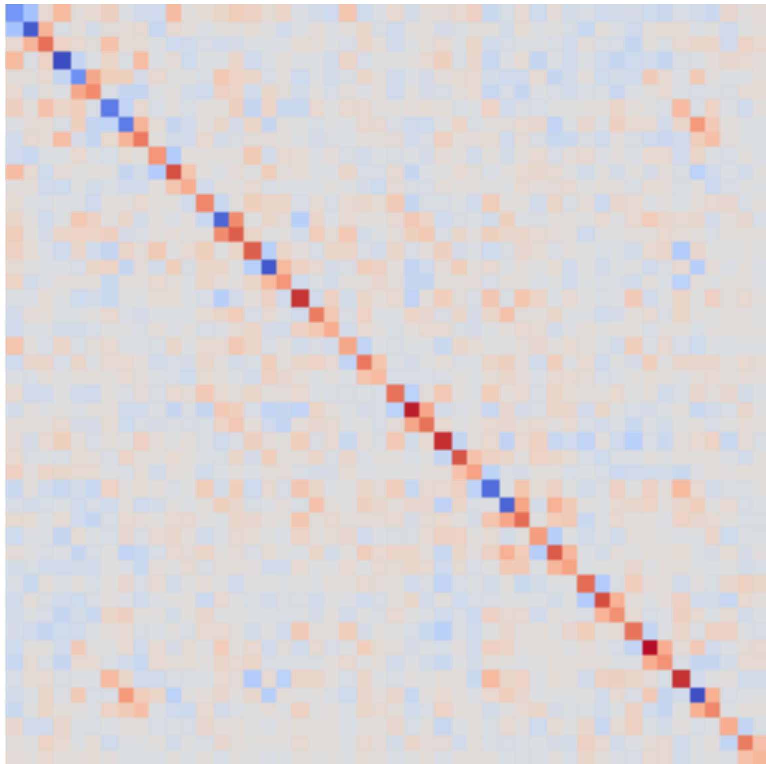
Example 2: Random MERA

- The distribution of the real part of coefficients (wave function) in $|\Psi\rangle$ (original state) and $|\Psi(\tau_{fin})\rangle$ (distilled one) for $\mathscr{H}_A$ and $\mathscr{H}_{\bar{A}}$:

  (larger: red, lower: blue, almost zero: white)

$\chi = 4$

$|\Psi\rangle$ (too scattered)



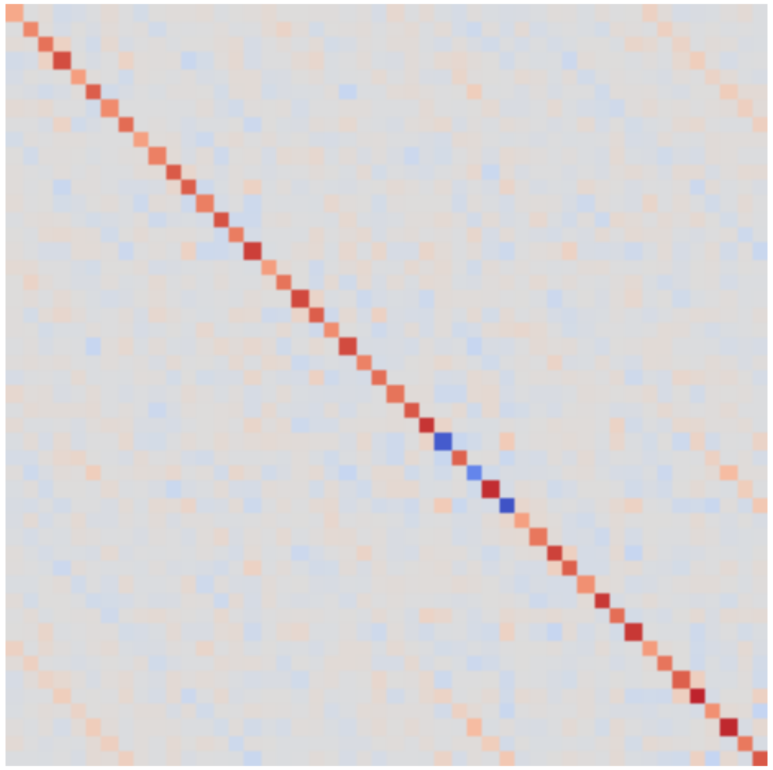
$|\Psi(\tau_{fin})\rangle$

(Images are coarse-grained by 48×48 for visualization)

Example 2: Random MERA

- The distribution of the real part of coefficients (wave function) in $|\Psi\rangle$ (original state) and $|\Psi(\tau_{fin})\rangle$ (distilled one) for $\mathscr{H}_A$ and $\mathscr{H}_{\bar{A}}$:
  (larger: red, lower: blue, almost zero: white)

$\chi = 6$



$|\Psi(\tau_{fin})\rangle$

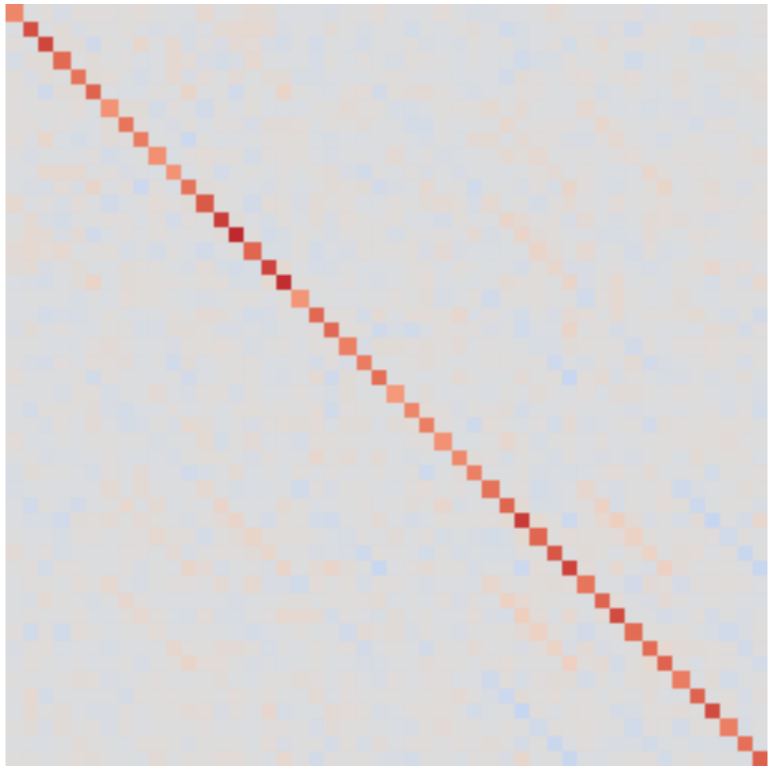
(Images are coarse-grained by 48×48 for visualization)

Example 2: Random MERA

- The distribution of the real part of coefficients (wave function) in $|\Psi\rangle$ (original state) and $|\Psi(\tau_{fin})\rangle$ (distilled one) for $\mathscr{H}_A$ and $\mathscr{H}_{\bar{A}}$:

  (larger: red, lower: blue, almost zero: white)

$\chi = 8$



$|\Psi(\tau_{fin})\rangle$

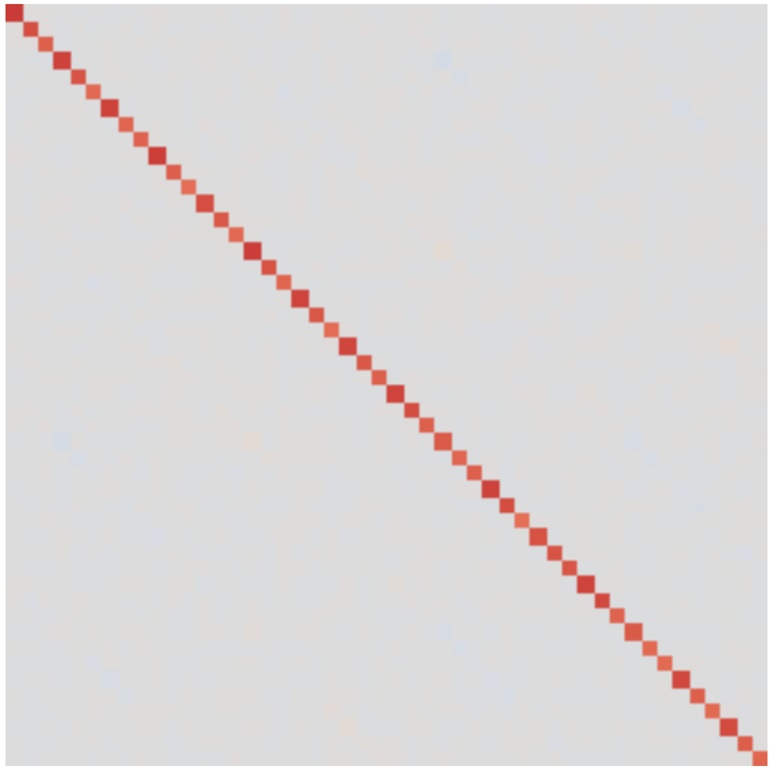
(Images are coarse-grained by 48×48 for visualization)

Example 2: Random MERA

- The distribution of the real part of coefficients (wave function) in $|\Psi\rangle$ (original state) and $|\Psi(\tau_{fin})\rangle$ (distilled one) for $\mathscr{H}_A$ and $\mathscr{H}_{\bar{A}}$:

  (larger: red, lower: blue, almost zero: white)

$$\chi = 20$$

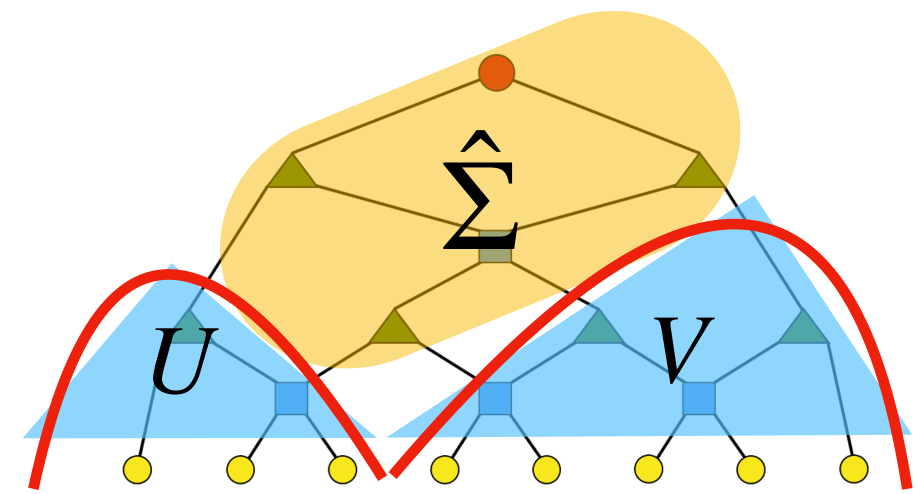


$|\Psi(\tau_{fin})\rangle$

(Images are coarse-grained by 48×48 for visualization)

Beyond MPS: MERA with pure BH analogy

- What's important here: RT surface (min bond cut surface) in TN is NOT unique



Why does this happen and can we see this in the AdS/CFT?

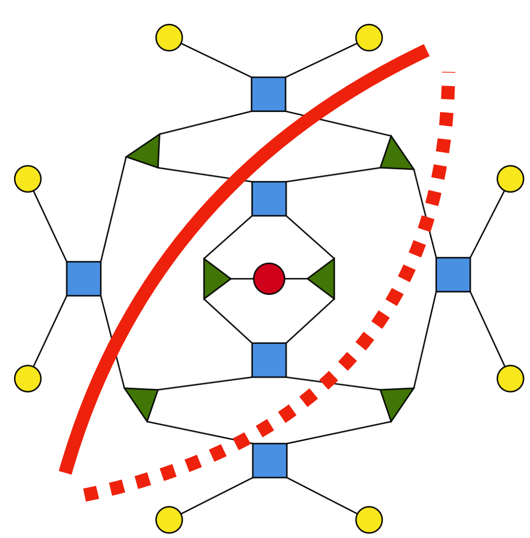
Identifying this phenomenon must be extremely important as $\hat{\Sigma}$ includes all the information about singular values

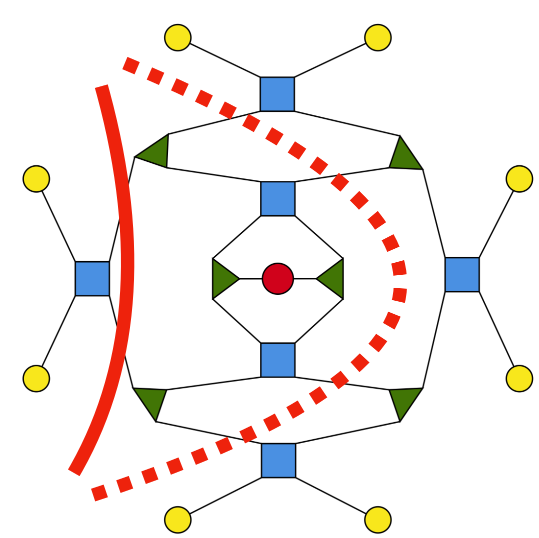
✓ The enclosed region by each boundary and each RT surface is isometric; beyond that (even in EW) is NOT isometric (in the direction from one boundary to the other)

✓ $\hat{\Sigma}$ region necessarily contains the top tensor

✓ By taking the infinite volume and continuum limit, two RT surfaces should get closer and each slope at boundary should becomes orthogonal; But the each RT surface is bounded by the location of the top tensor

✓ By smoothly changing the subregion, other possible candidates fail to give minimum

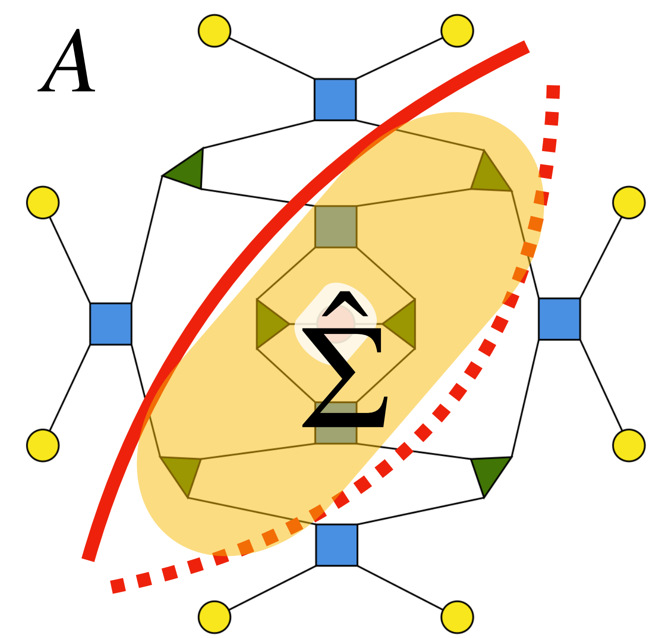




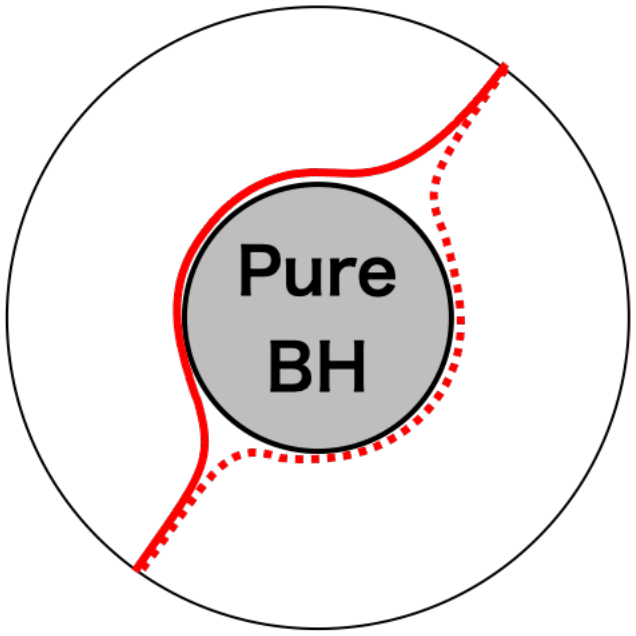
Beyond MPS: MERA with pure BH analogy

- This picture quite resembles to the **pure** BH. An RT surface can continuously move from one side to the other without any obstructions (because the state is pure)

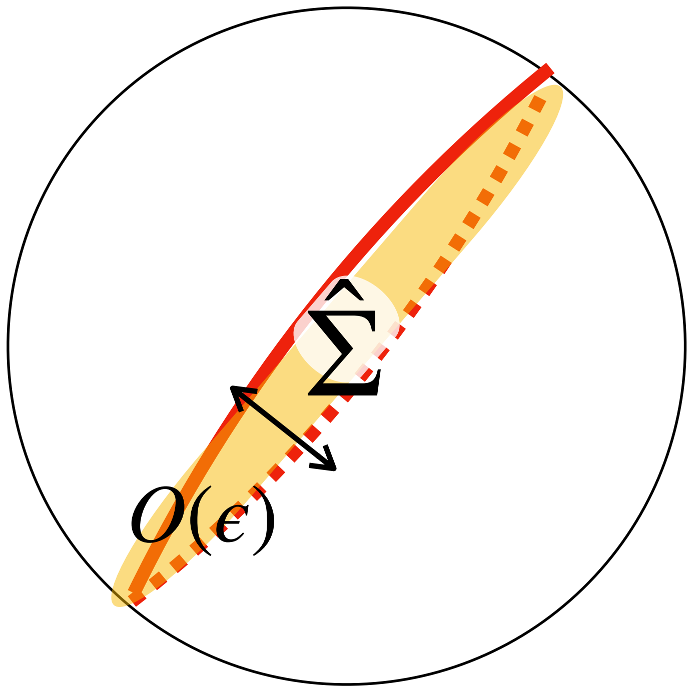
- TN suggests $\hat{\Sigma}$ region can exists at the origin of the bulk even for pure AdS as a point-like region.

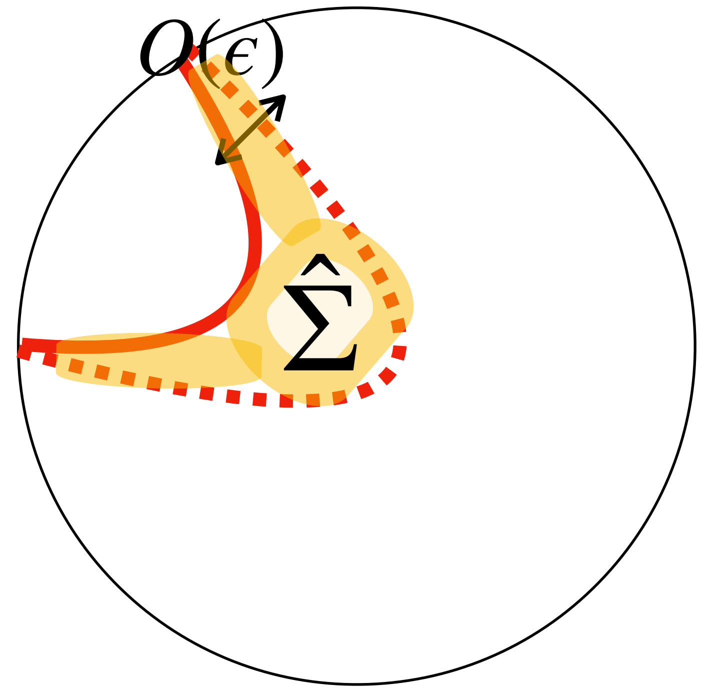


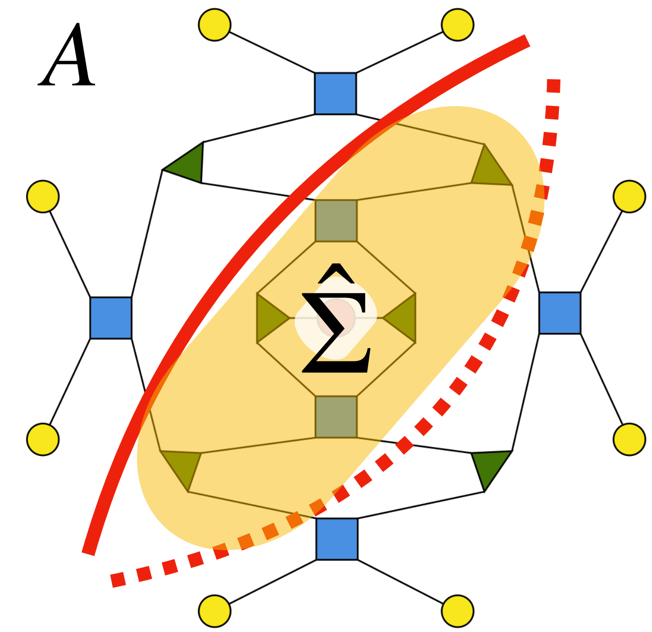
Dashed: causal cone for A



Beyond MPS: MERA with pure BH analogy



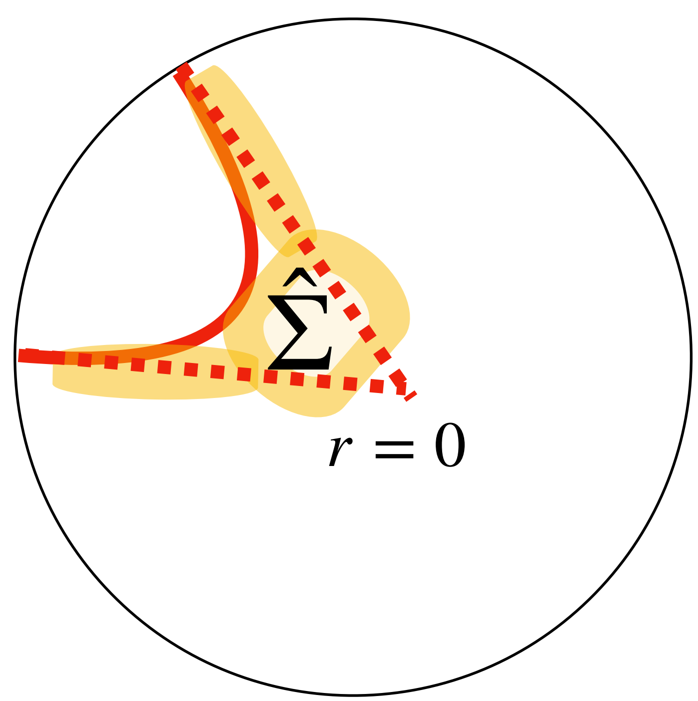


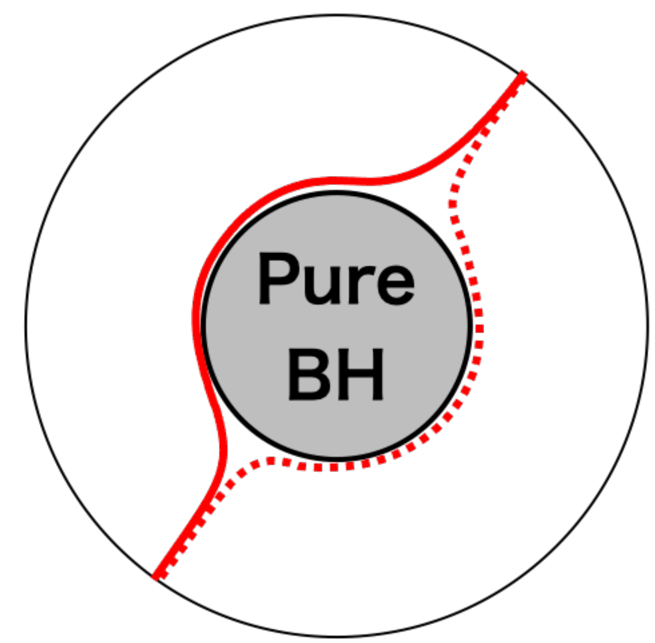


Dashed: causal cone for A

Python's lunch (region sandwiched by extremal surfaces) or entanglement shadow appear due to discretization

cont. limit





This is very close to EWCS=EoP setup.
The important differences are:

- This exists even in the pure state
- If you take sufficiently small subregion, the RT surface is get disconnected for minimization condition whereas $\hat{\Sigma}$ remains finite

Entanglement distillation (concentration)

Suppose we want to extract maximally entangled pairs from the following state: $|\Psi\rangle_{AB}^{\otimes n} = \left[\cos\theta|00\rangle + \sin\theta|11\rangle\right]_{AB}^{\otimes n}$

Expanding terms: $|00\cdots 00\rangle, \cdots, |11\cdots 11\rangle$ n+1 different coefficients. Regard them as n+1 different orthogonal states.

Within each state, there are $\binom{n}{k}$ basis.

The probability to extract k-th density matrix (via projective measurement $|\phi_k\rangle = \sum_{i=1}^{\binom{n}{k}} |i\rangle|i'\rangle$): $p_k = \binom{n}{k}\cos^{2(n-k)}\theta\sin^{2k}\theta$

Then the averaged # of EPRs (ebits) are

$$\sum_{k=0}^{n} p_k \log\binom{n}{k} = \sum_{k=0}^{n}\exp\left[\log\binom{n}{k} + (n-k)\log\cos^2\theta + k\log\sin^2\theta\right]\log\binom{n}{k}$$

(Stirling's formula $\log n! \sim n\log n - n \Rightarrow \log\binom{n}{k} \sim n\log n - k\log k - (n-k)\log(n-k) = k\log\frac{n}{k} + (n-k)\log\frac{n}{n-k}$)

$$= \sum_{k=0}^{n}\exp\left[k\log\left(\frac{n}{k}\sin^2\theta\right) + (n-k)\log\left(\frac{n}{n-k}\cos^2\theta\right)\right]\log\binom{n}{k}$$

The saddle point approx. for the blue part: $k = n\sin^2\theta$

$$\sim -n\cos^2\theta\log\cos^2\theta - n\sin^2\theta\log\sin^2\theta = nS_A$$

Entanglement distillation in holography

- In contrast, holography (or tensor network), we only have a single state. Then, why can one argue distillation?

First, the distillation or modular flow is not state-independent. (TTbar might offer a state-independent (but Hamiltonian-dependent) construction.)

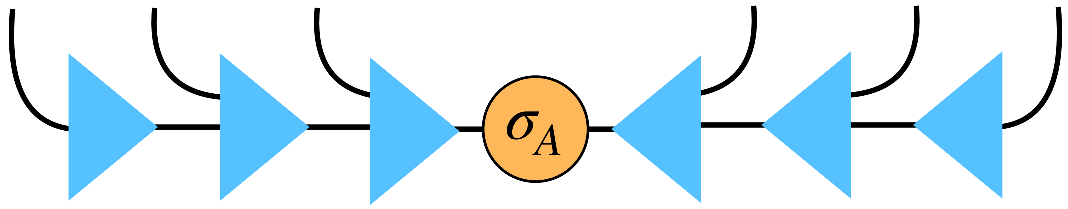
Second, it has been argued that a holographic theory has a flat entanglement spectrum, i.e. $\partial_n S_n = 0$, (to leading order). This is equivalent to preparing iid. [Bao, et al.]

(In contrast, when we consider back reaction flat ES is false because gravitational Renyi entropy is affected by backreacting cosmic brane.)

Finally, holographic toy models like HaPPY code, random TN have flat ES [Dong-Harlow-Marolf]. Thus it is convincing that they offer a complete distillation. In general, we expect quasi randomness or quasi perfectness (~approx. QEC); such cases might result decrease in the success prob. for distillation.

Beyond MPS: MERA with pure BH analogy

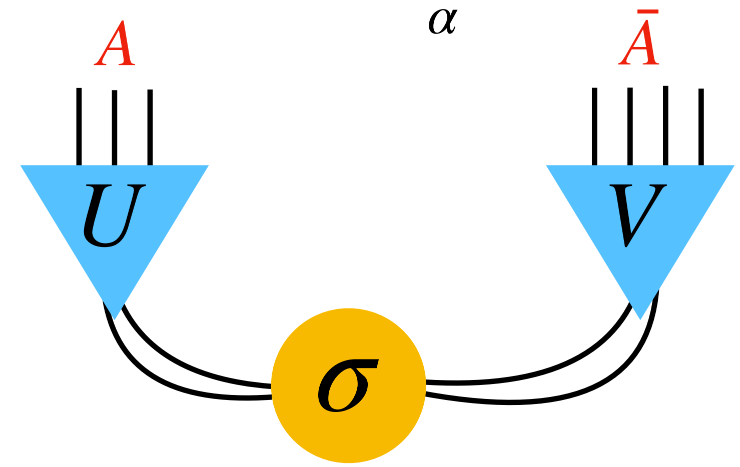
- So far this works for MPS,



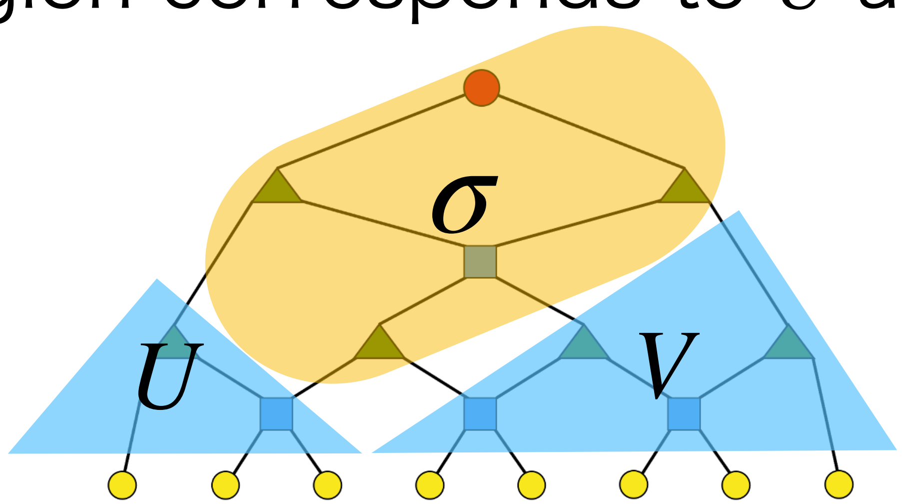
or more generally the state of the type $|\psi\rangle = \left(U_A \otimes V_{\bar{A}}\right) \sum_\alpha \sigma_\alpha |\alpha\alpha\rangle_{A\bar{A}}$

- Q. Can we extend our analysis for MERA?

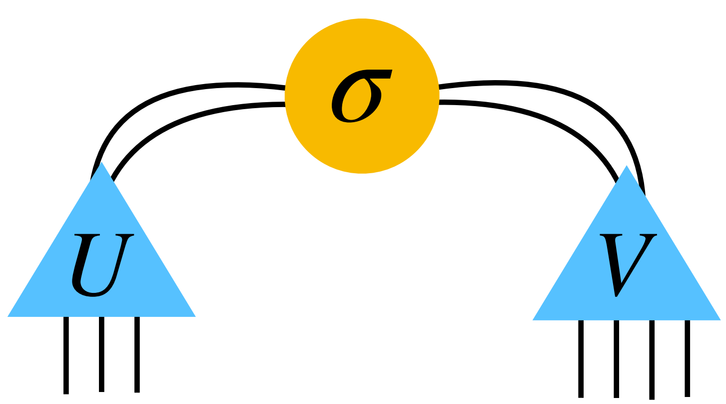
A. At least we can make a guess!



- The domain of dependence (≠ entanglement wedge) for each region corresponds to U and V for MERA

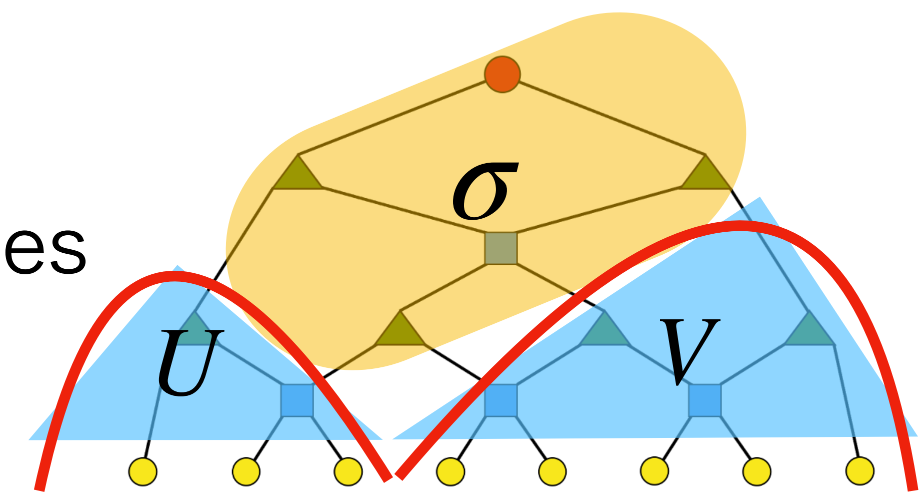


$\Longleftrightarrow$

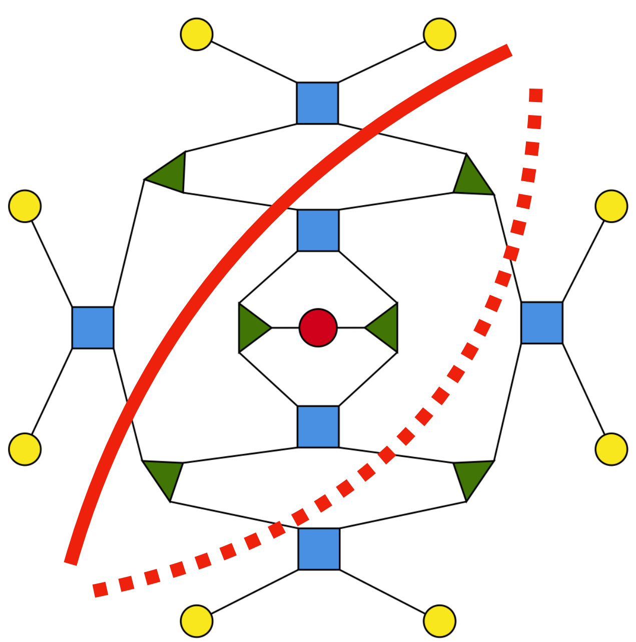


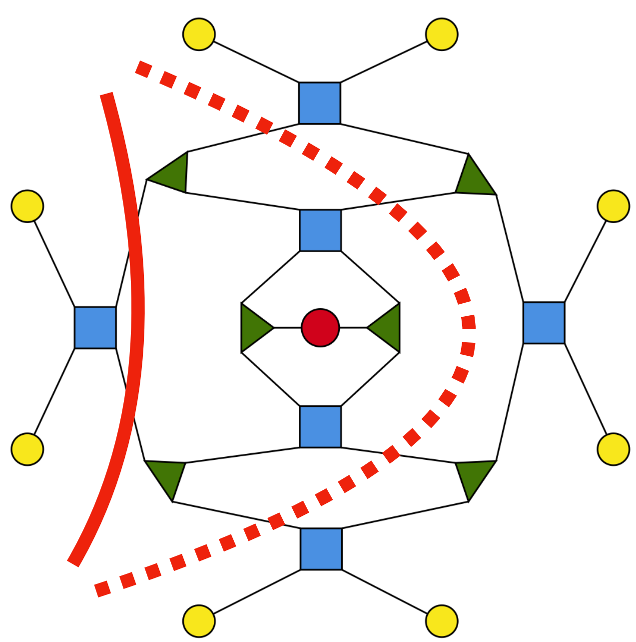
Beyond MPS: MERA with pure BH analogy

- The region σ can appear because we take $D(A), D(\bar{A})$, not EW
- RT surface is not unique in TN;
  Even within the pure state 2 RT surfaces
  seemingly does not match



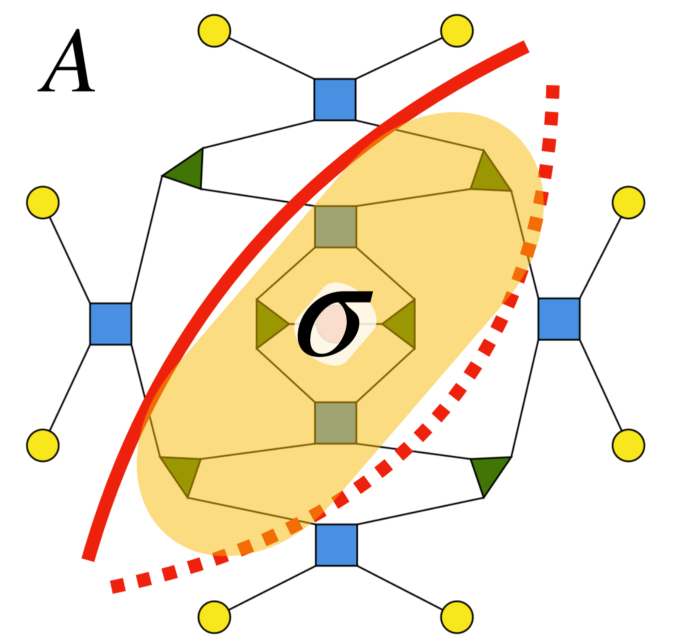
- But by smoothly changing the subregion, other possible candidates fail to give minimum



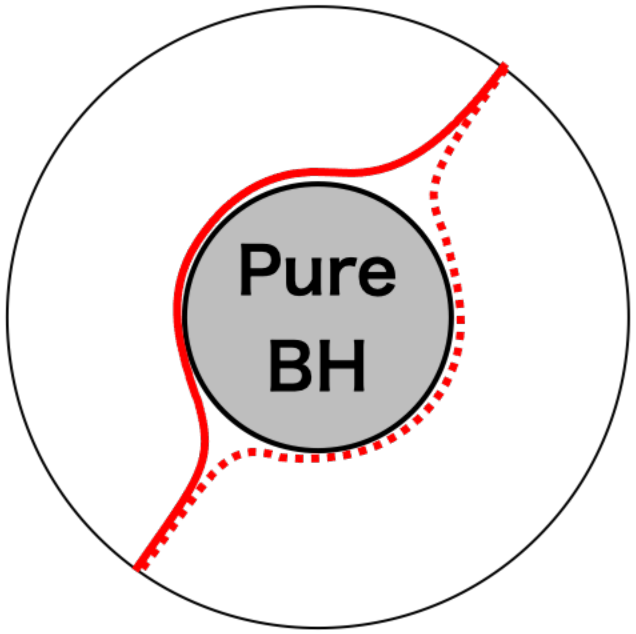


Beyond MPS: MERA with pure BH analogy

- This picture quite resembles to the pure BH. An RT surface can continuously move from one side to the other without any obstructions (because the state is pure)
- But now, the pure BH region is very imp; it accounts for σ. Beyond that region, the channel from the boundary is no more isometry (unless tensors have special properties like perfect, dual-unitary, etc.)
- TN suggests σ region can exists at the origin of the bulk even for pure AdS (but maybe just because of sub-AdS breakdown)

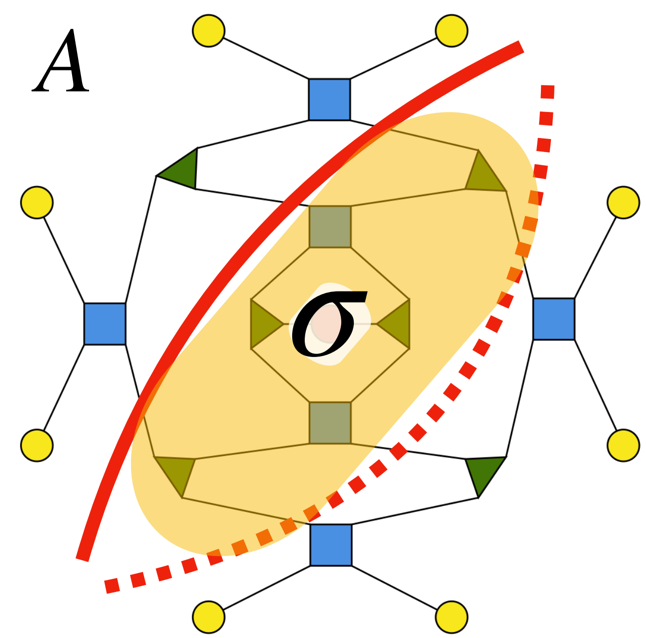


Dashed: causal cone for A



Beyond MPS: MERA with pure BH analogy

- This is special to TN. Since the angle between the minimal bond cut surface and the boundary near the endpoint of the subregion is always bounded by one unit of the isometry. It means the every RT surface sharing a common endpoint looks locally the same near the point.



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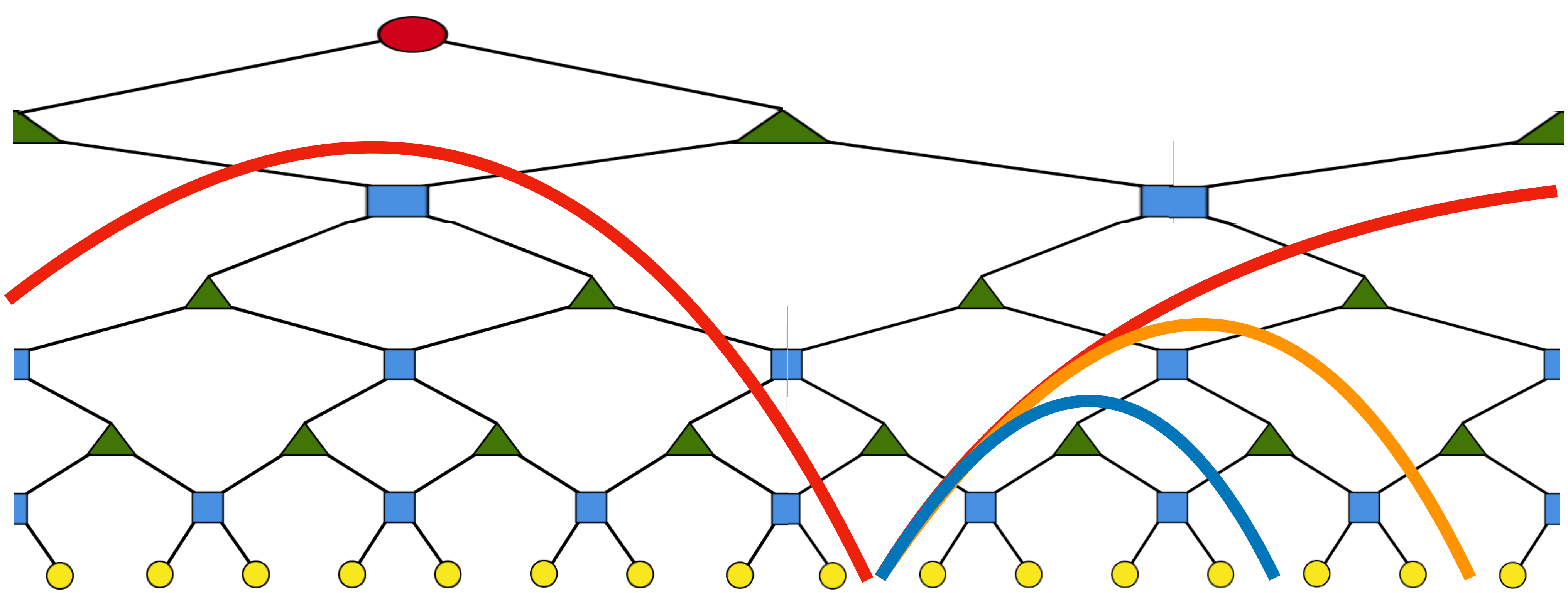


Beyond MPS: MERA with pure BH analogy

- Global coordinates?

Coordinate trf (for a parameter $\forall \alpha \in \mathbb{R}$: $r = 0 \Leftrightarrow z^2 - t^2 = \alpha^2, x = 0$):

$$\sqrt{1+r^2}\cos\tau = \frac{z^2+x^2-t^2+\alpha^2}{2\alpha z}; \quad \sqrt{1+r^2}\sin\tau = \frac{t}{z};$$

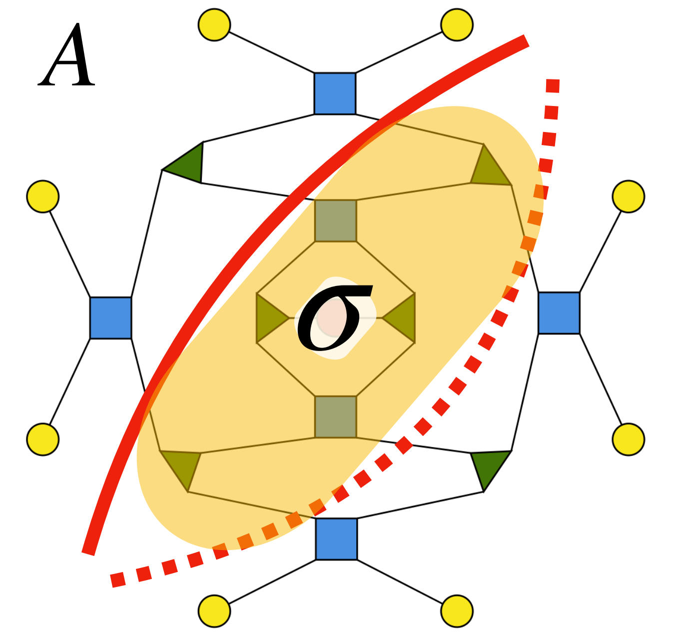
$$-r\cos\theta = \frac{z^2+x^2-t^2-\alpha^2}{2\alpha z}; \quad r\sin\theta = \frac{x}{z};$$

Geodesic in Poincare coordinates: $z^2 + (x - l_0)^2 = (l + l_0)^2$ where $A : [-l, l + 2l_0]$

$\epsilon = 0$ makes the all the slopes of those geodesics same but at the same time the slope becomes orthogonal to the boundary. Furthermore, for a finite ϵ, the slopes are different from each other. This is different from TN.

But isn't this natural from RG? Different lattice regularization leads different theory up to correction vanishing as $\epsilon \to 0$.

→Then, the sigma region becomes thinner and thinner.



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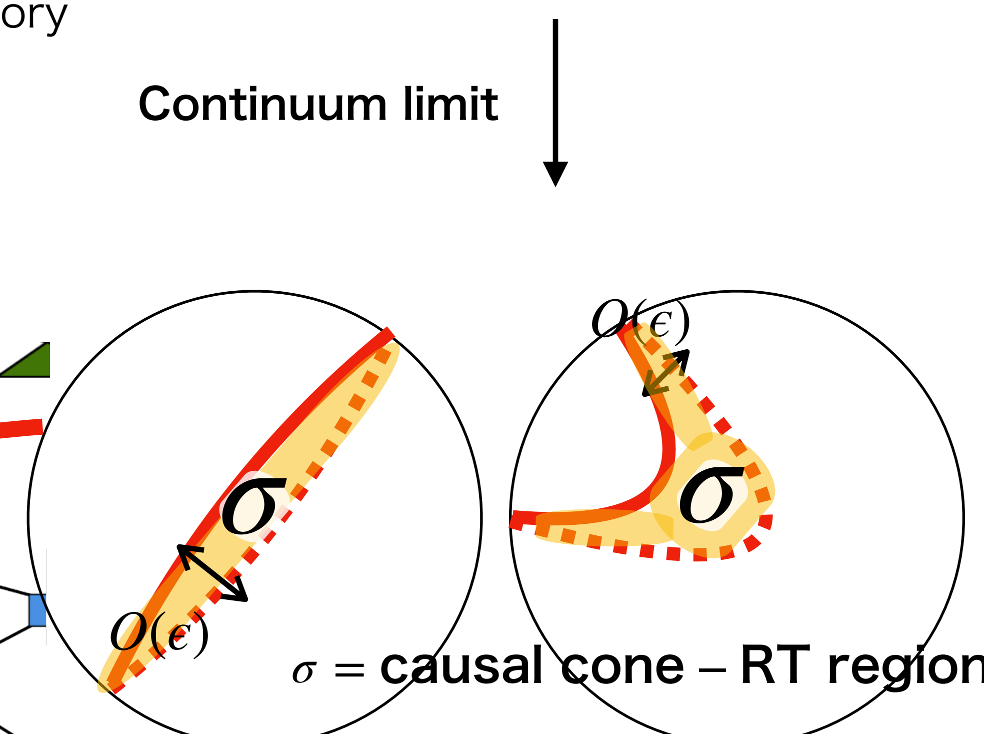
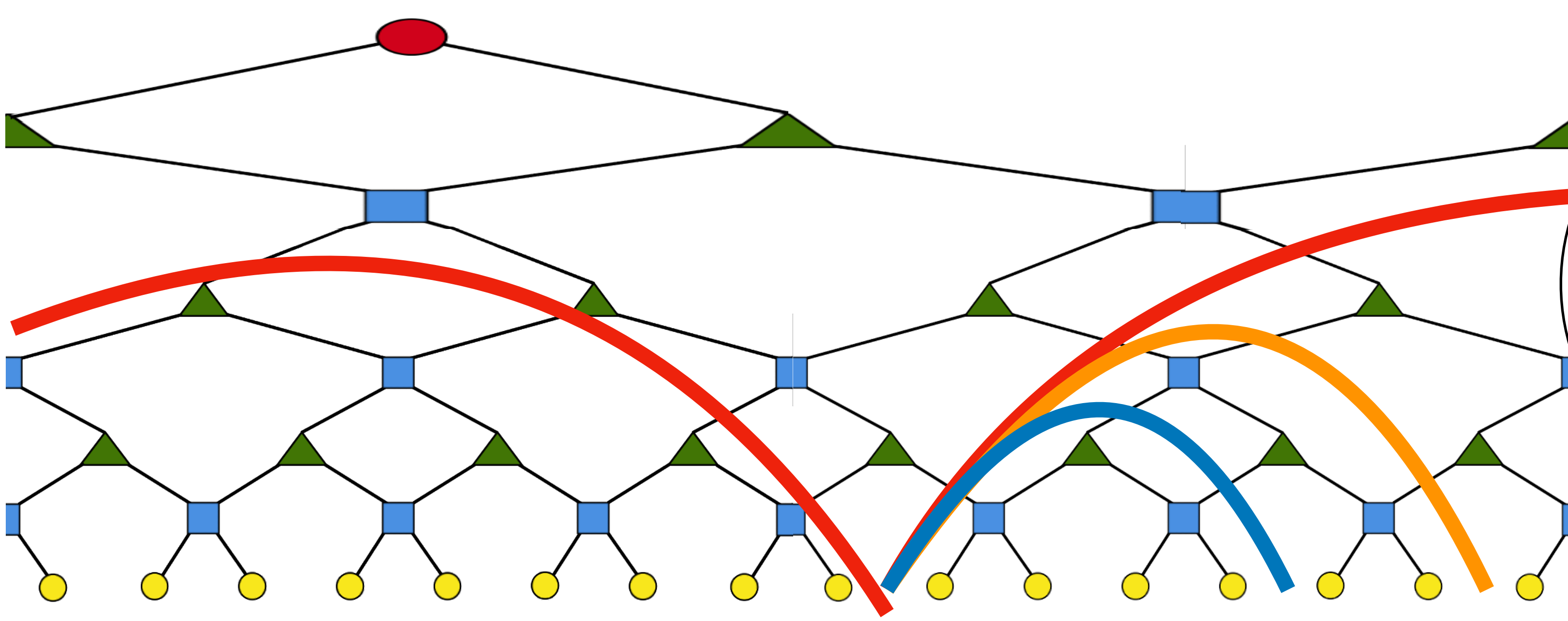
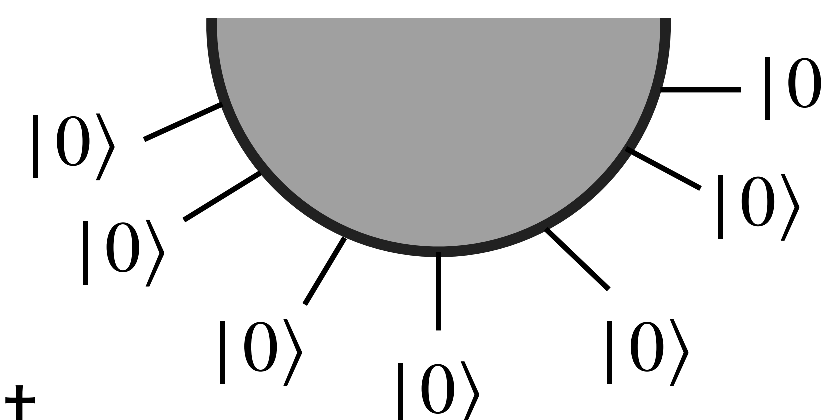




Path integral approach to distillation in MERA

- As we have discussed, modular flow works for a certain circumstance. In such a case, the (open boundary) MERA can be thought as a state prepared by Euclidean path integral on a semi-disc.
- Then, the modular flow=HED
- However, it is a bit different from ordinary BCFT as the boundary is (usually) not conformally invariant. The boundary state is given by many products of ancillae (thus spatially separable).

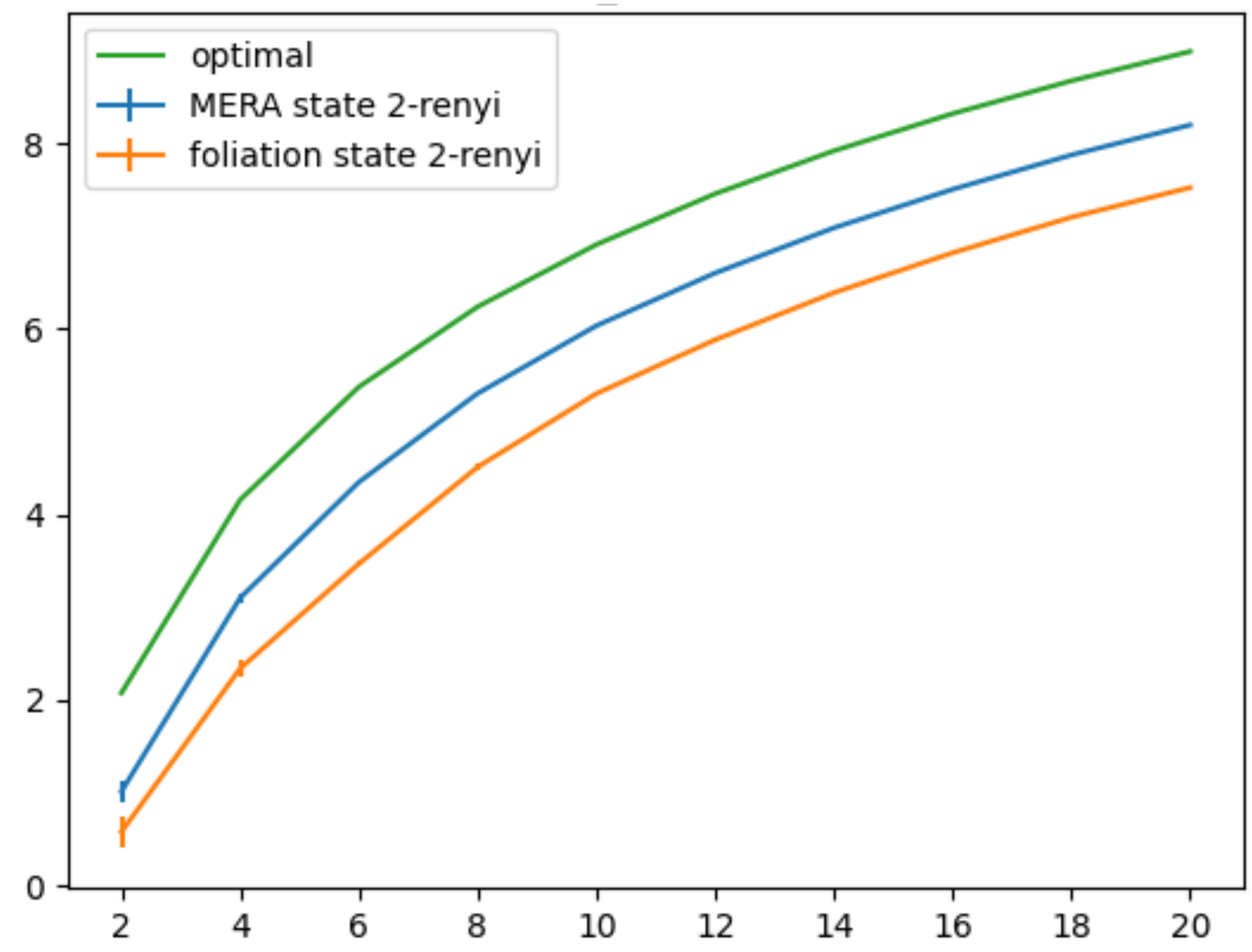


Random MERA: Renyi-2 entropy

- In general, Renyi-2 entropy of a TN state (blue) is less than the expected result from RT formula (green)
- In the large bond dim limit, they are expected to coincide
- The Renyi-2 entropy of holographically distilled state on γ_A (orange) is close to the original one (blue) probably due to doubled singular value

Renyi-2 entropy

$$S_2 = -\log \mathrm{Tr}\rho_A^2$$



Bond dim.

Random MERA: Renyi-2 entropy

- In general, Renyi-2 entropy of a TN state (blue) is less than the expected result from RT formula (green)
- In the large bond dim limit, they are expected to coincide
- The Renyi-2 entropy of holographically distilled state on γ_A (orange) is close to the original one (blue) probably due to doubled singular value