クォーク図を用いた
ハドロン有効模型の構築方法および、
$(3,\bar{3}) \oplus (\bar{3},3)$と$(8,1) \oplus (1,8)$表現を
用いた3フレーバーパリティ二重項模型

Hadronic effective model considering quark flow diagrams, and a 3-flavor parity doublet model with (3,3*)+(3*,3) and (8,1)+(1,8) representations

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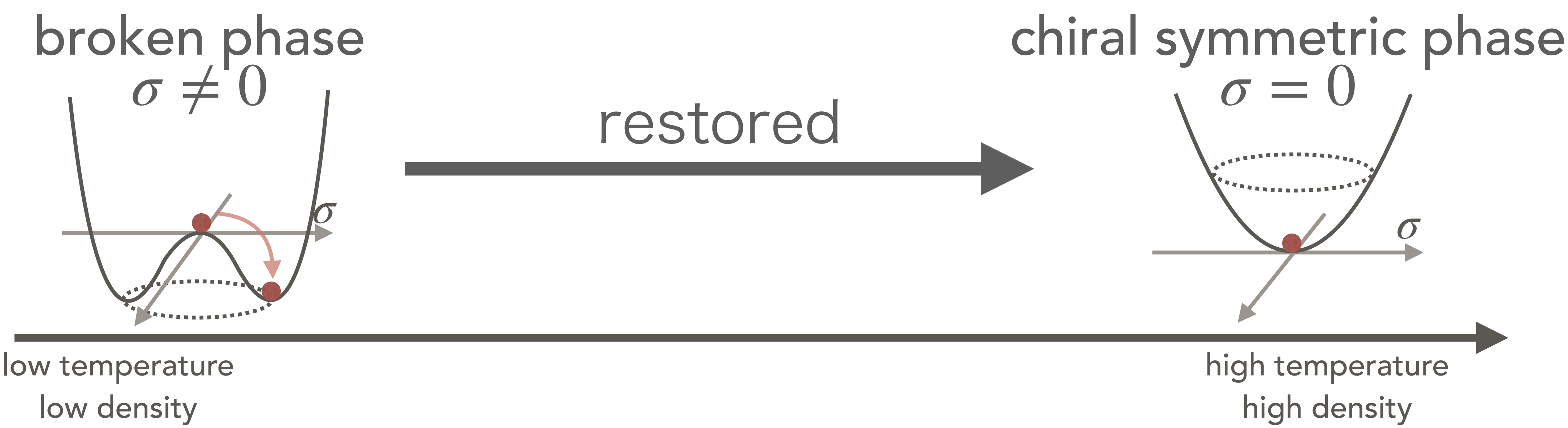
collaborators: M. Harada (Nagoya Univ.) and T. Kojo (Tohoku Univ.)

TQFT, Sep 21 2022

Fate of Nucleon Mass?



- In effective models, e.g. the linear-sigma model, $M_N \propto \sigma$

$$M_N \neq 0 \xrightarrow{\sigma \to 0} M_N \to 0\ ?$$

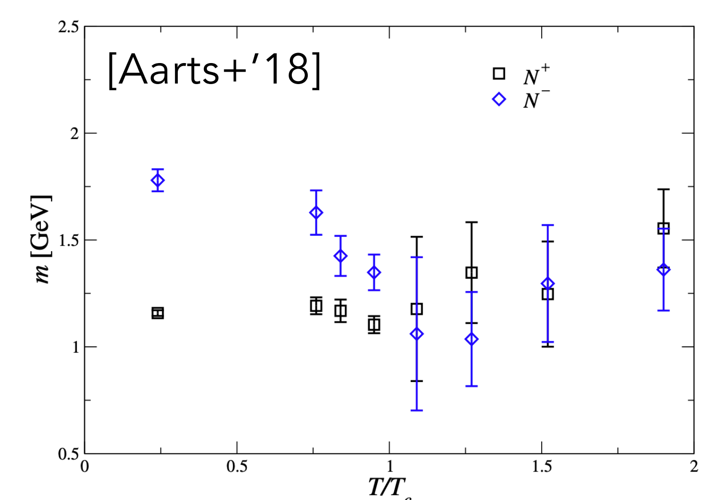
- However, lattice QCD at finite T (e.g.Aarts+'15, '18):

$T > T_c$ but $\underline{M_N \text{ remains } \sim 1\,\text{GeV}}$

chiral "invariant" mass?

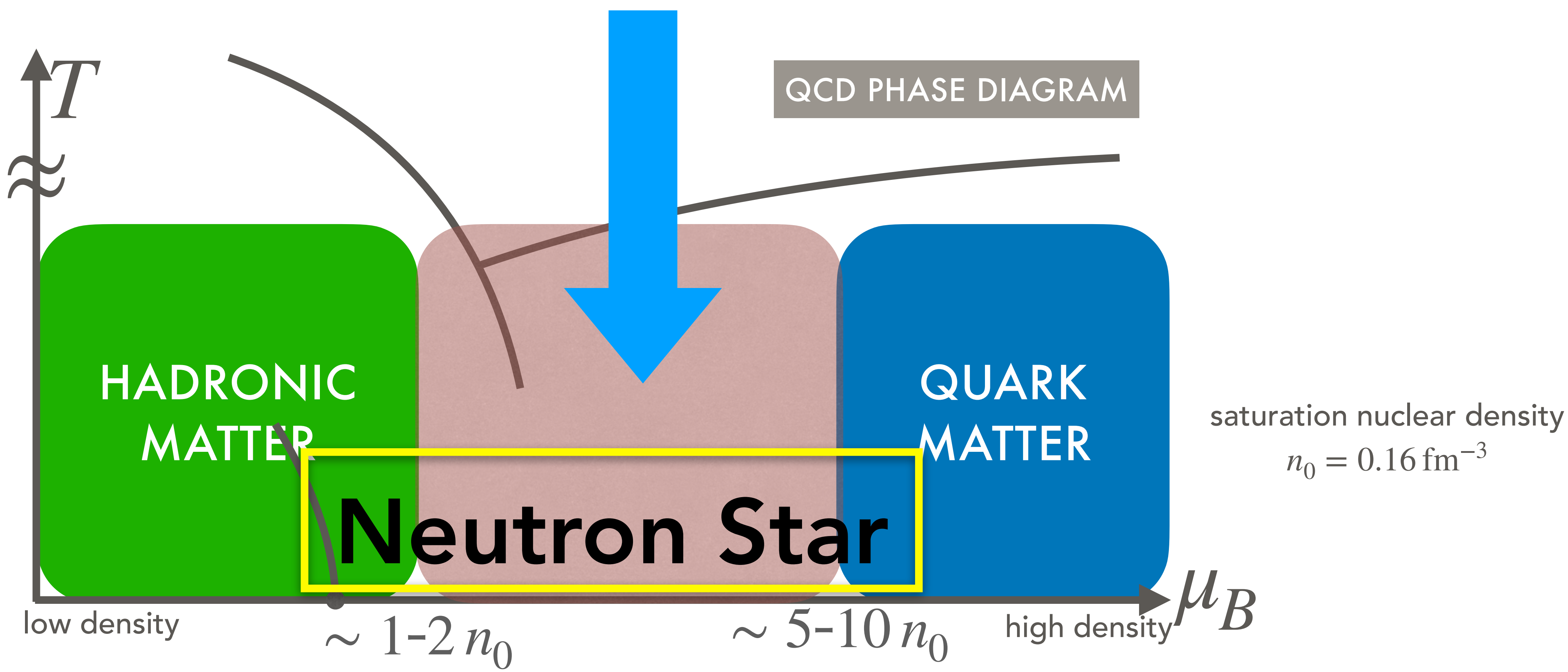
- M_N **are not very sensitive to the environment ?**

relevant for the physics of heavy ion collisions and Neutron Stars (NSs)



NSs as Cosmic Laboratories

fate of nucleons? chiral?



EOS $\underset{}{\overset{\text{1-to-1}}{\rightleftarrows}}$ NS M-R

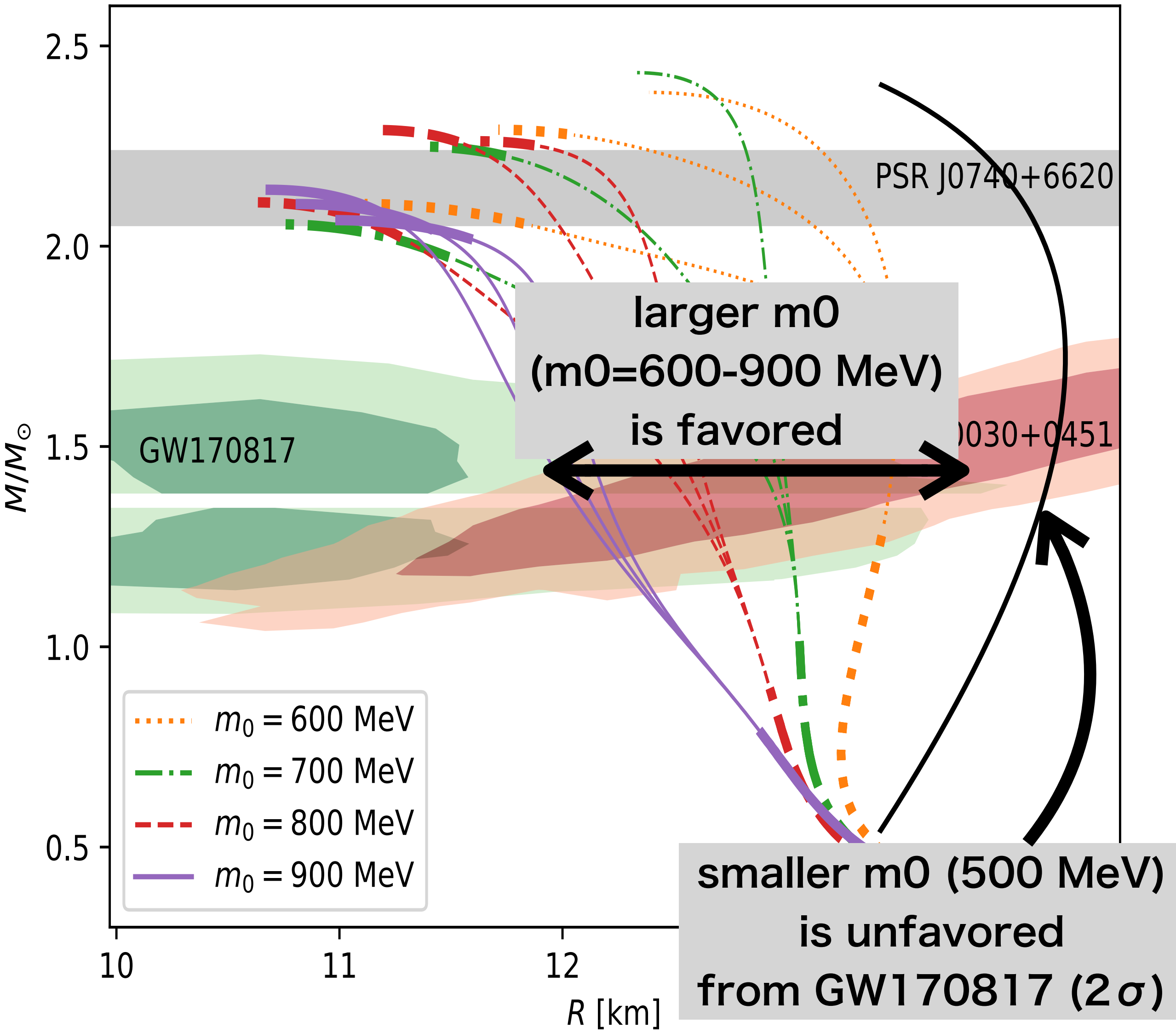
└→ constraints on the microphysics

Our Previous Works

using 2-flavor chiral hadronic model with “chiral invariant mass”

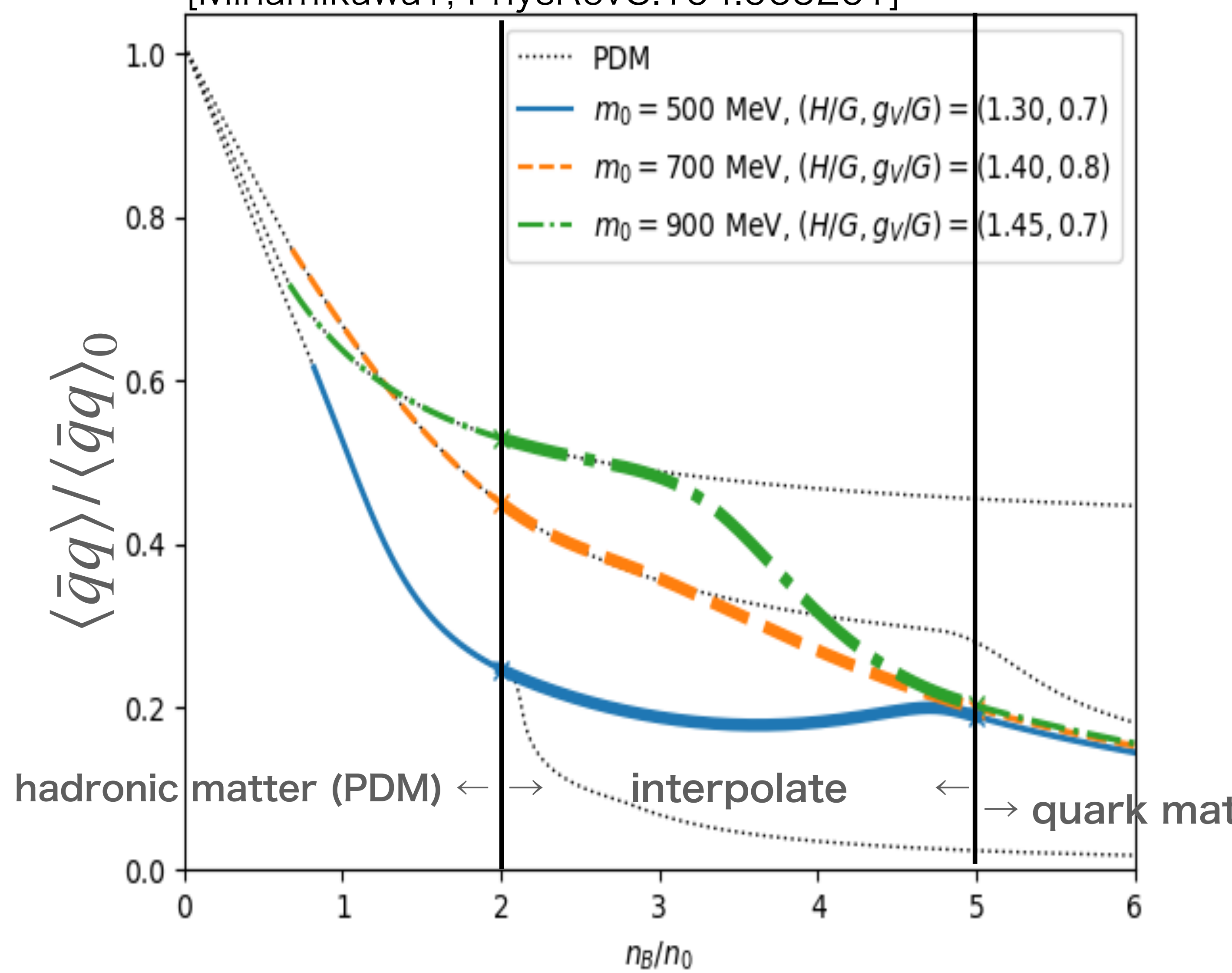
① NS data constrain m0 through MR relations

[Minamikawa+; PhysRevC.103.045205]



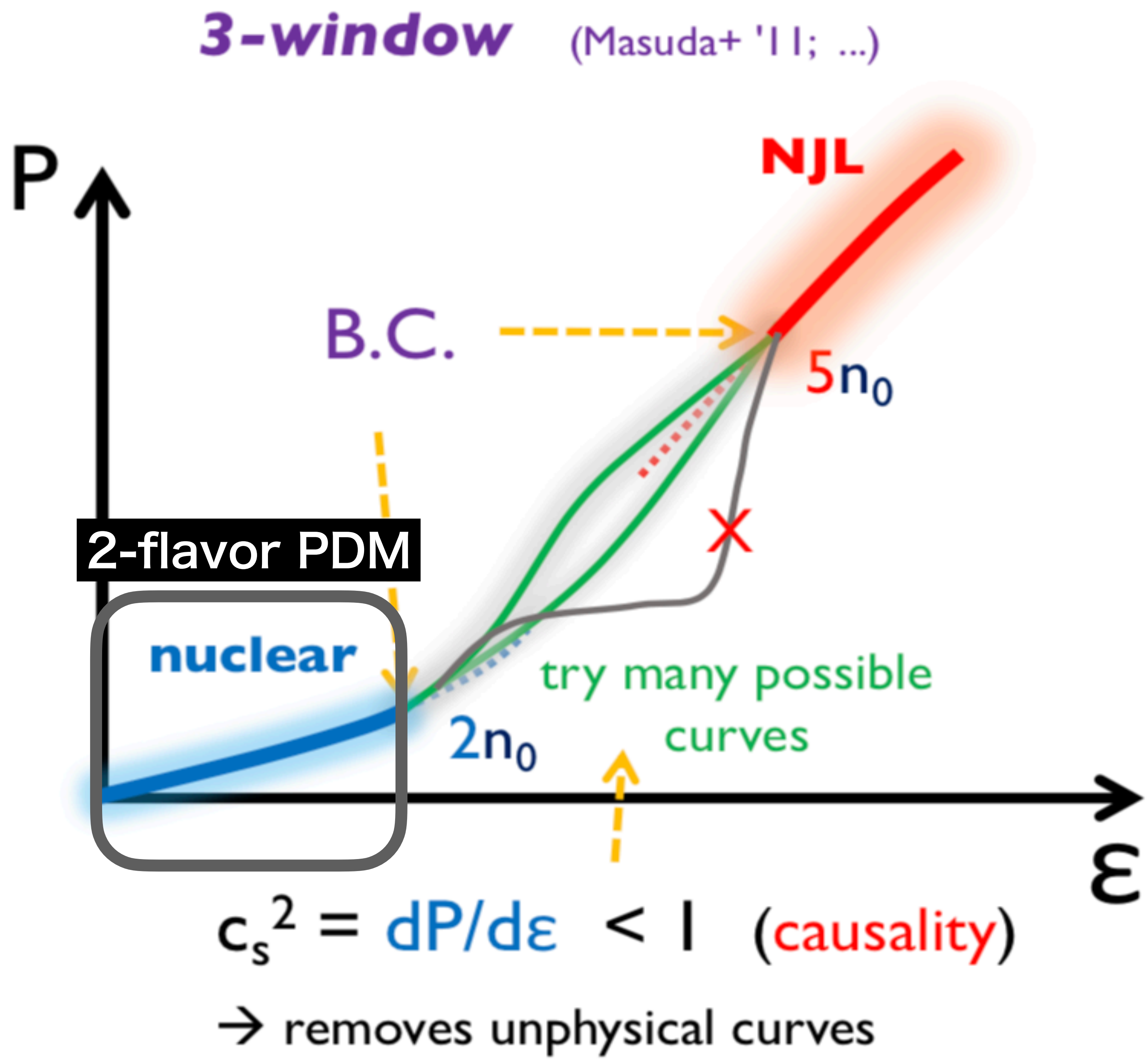
② chiral condensates in crossover

[Minamikawa+; PhysRevC.104.065201]

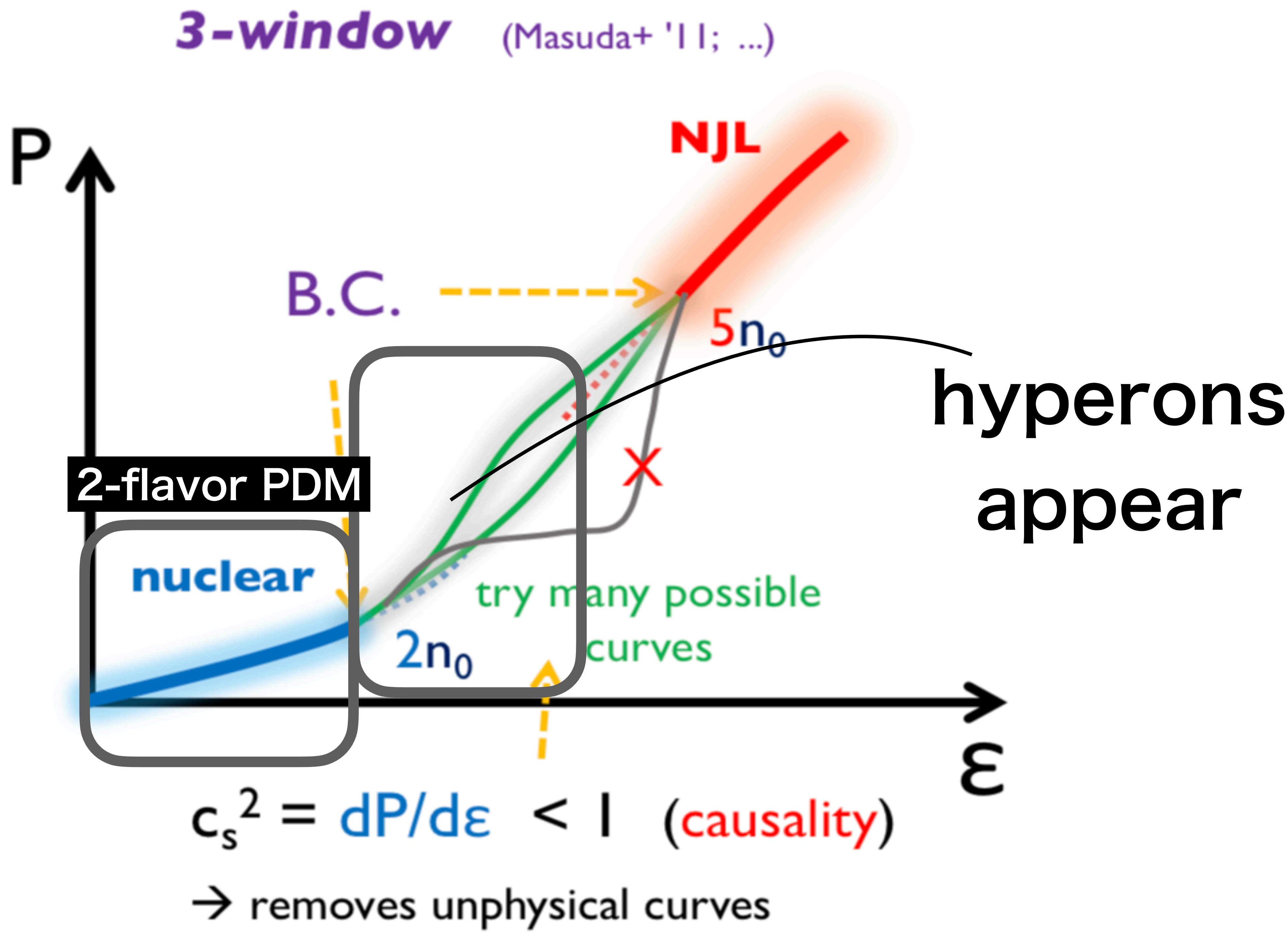


Skip these works today

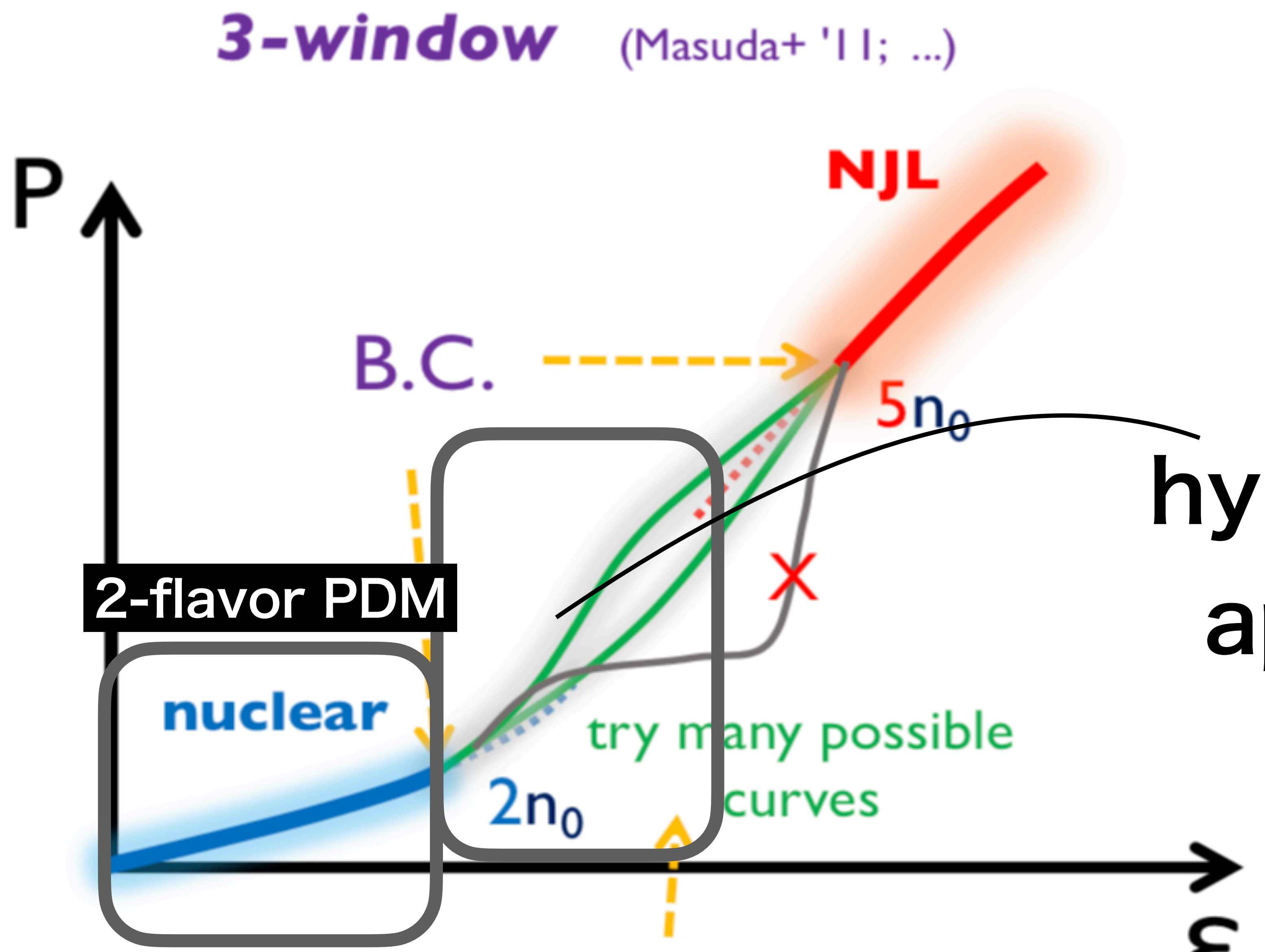
Hyperon in a NS



Hyperon in a NS



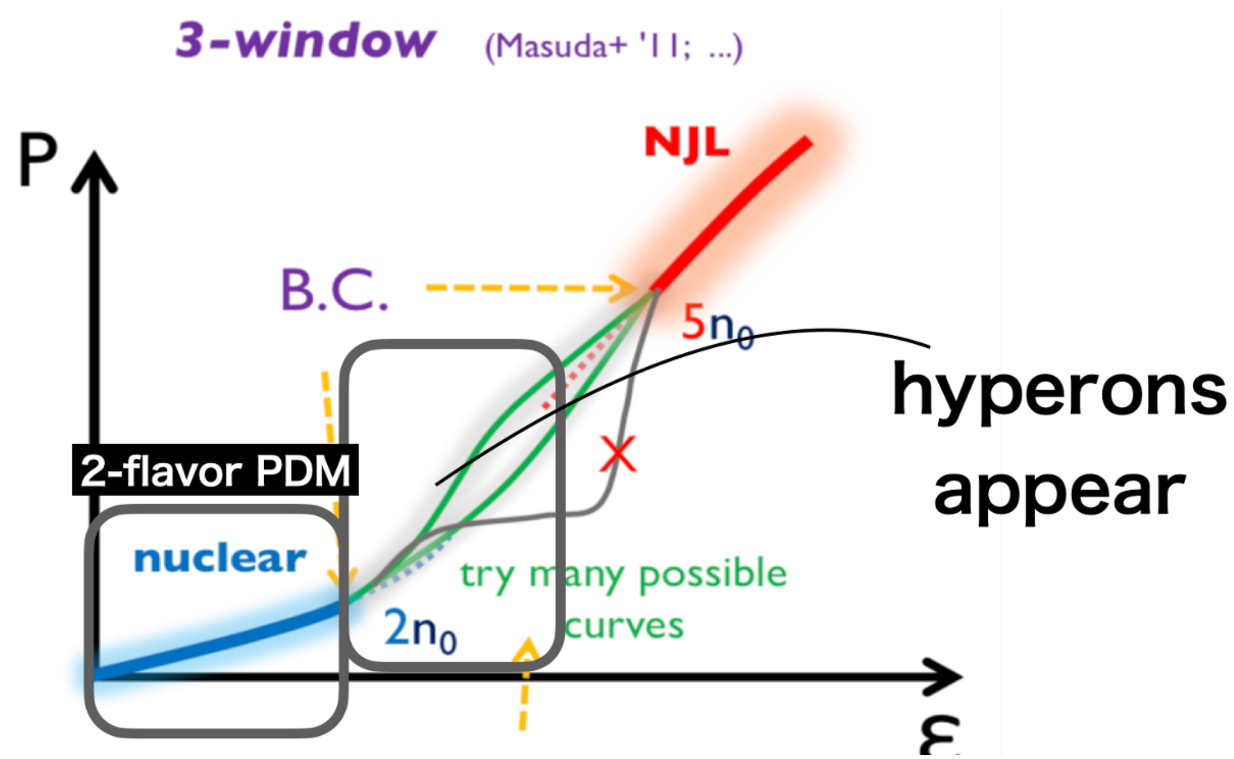
Hyperon in a NS



<u>Need to extend chiral hadronic model (PDM) with hyperons</u>

(Additional parameters may be constrained from NS data)

Motivations (in this work)

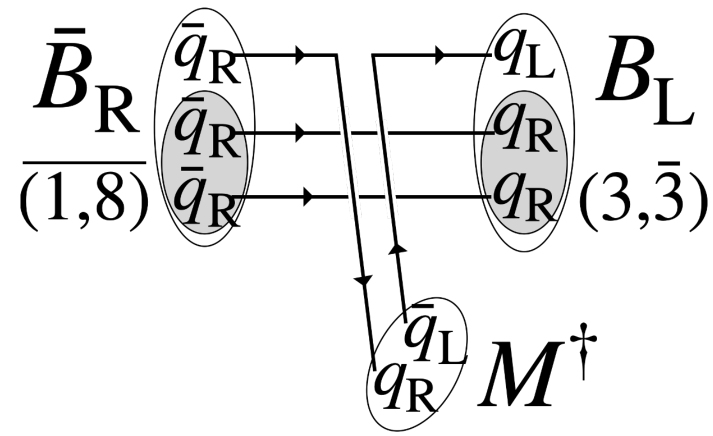


- build a chiral model for hyperons
  → chiral condensate $\langle \bar{q}q \rangle$,$\langle \bar{s}s \rangle$, hyperons in higher density

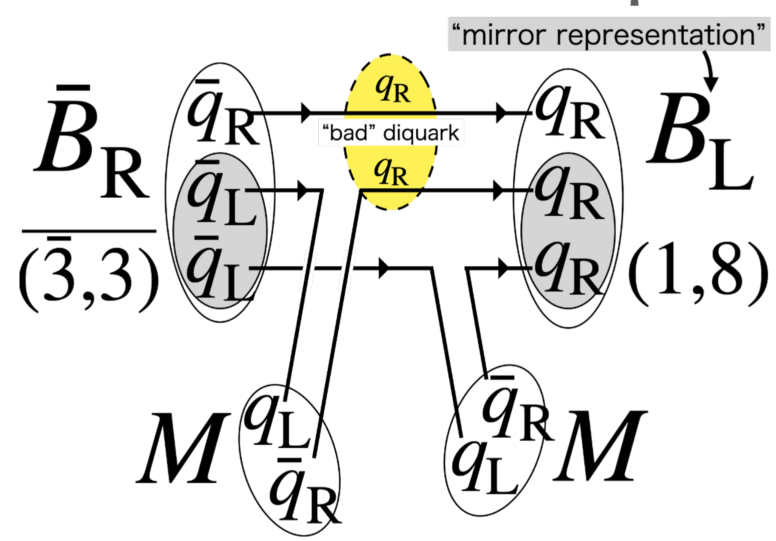
- HADRONIC effective model considering QUARK picture
  → hadron quark crossover ?? (in the future??)

What I Show

- 1st-order (ordinal) Yukawa is not sufficient.



- We introduce 2nd-order Yukawa-like with integrating out "bad" diquark



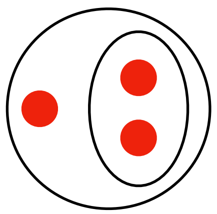
moreover, parity doubling structure appears naturally.

chiral invariant mass

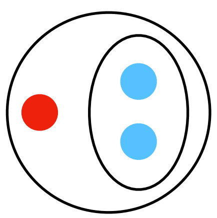
Chiral Representations

left-handed quark $q_L \sim (3,1) \sim 3_{\mathrm{F}}$ $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$

right-handed quark $q_R \sim (1,3) \sim 3_{\mathrm{F}}$ $\mathrm{SU}(3)_{\mathrm{F}}$



$$B_L \sim q_L \otimes q_L \otimes q_L \sim (1,1) \oplus (8,1) \oplus (10,1)$$

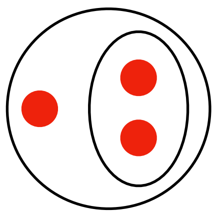


$$B_L \sim q_L \otimes q_R \otimes q_R \sim (3,\bar{3}) \oplus (3,6)$$

Chiral Representations

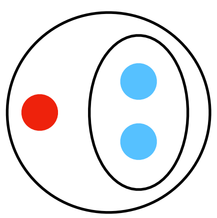
left-handed quark $q_L \sim (3,1) \sim 3_{\mathrm{F}}$ — $SU(3)_L \times SU(3)_R$

right-handed quark $q_R \sim (1,3) \sim 3_{\mathrm{F}}$ — $SU(3)_{\mathrm{F}}$



$$B_L \sim q_L \otimes q_L \otimes q_L \sim (1,1) \oplus (8,1) \oplus (10,1)$$

and $B_R \sim q_R \otimes q_R \otimes q_R$



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Chiral Representations

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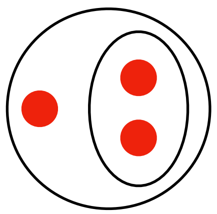

$$B_L \sim q_L \otimes q_L \otimes q_L \sim (1,1) \oplus \underset{8_F}{\underline{(8,1)}} \oplus (10,1)$$

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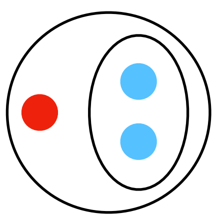

$$B_L \sim q_L \otimes q_R \otimes q_R \sim \underset{1_F \oplus 8_F}{\underline{(3,\bar{3})}} \oplus \underset{8_F \oplus 10_F}{\underline{(3,6)}}$$

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Chiral Representations

left-handed quark $q_L \sim (3,1) \sim 3_F$

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$SU(3)_L \times SU(3)_R$

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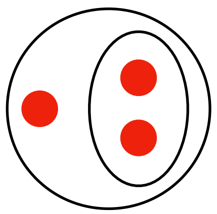

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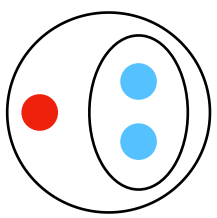
$(8,1)$: 8_F — octet baryon



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$(3,\bar{3})$: $1_F \oplus 8_F$ — octet baryon

$(3,6)$: $8_F \oplus 10_F$ — octet baryon including “bad” diquark

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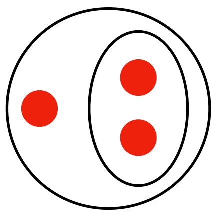

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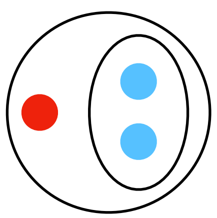
$(8,1) \to 8_F$ — octet baryon



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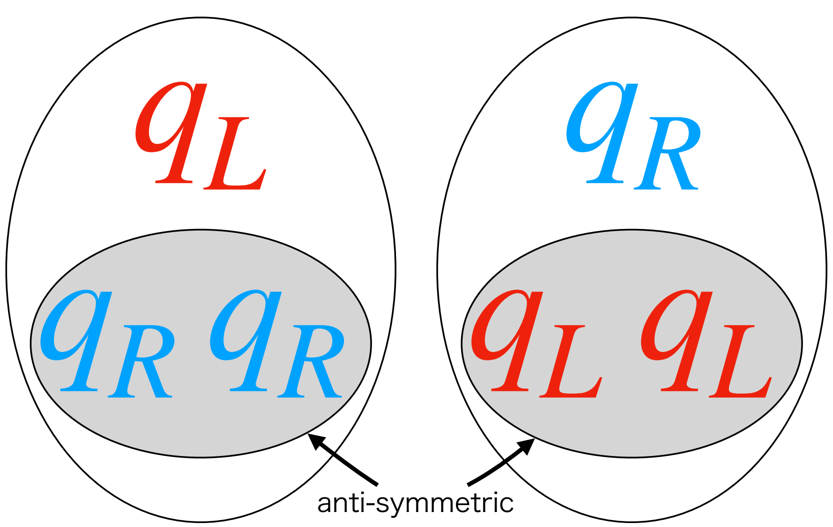
$(3,\bar{3}) \to 1_F \oplus 8_F$ — octet baryon (8_F)

$(3,6) \to 8_F \oplus 10_F$ — octet baryon including “bad” diquark (8_F)



$\psi_L \sim (3,\bar{3})$ $\psi_R \sim (\bar{3},3)$

Chiral Representations

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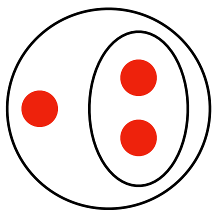

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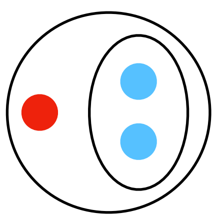

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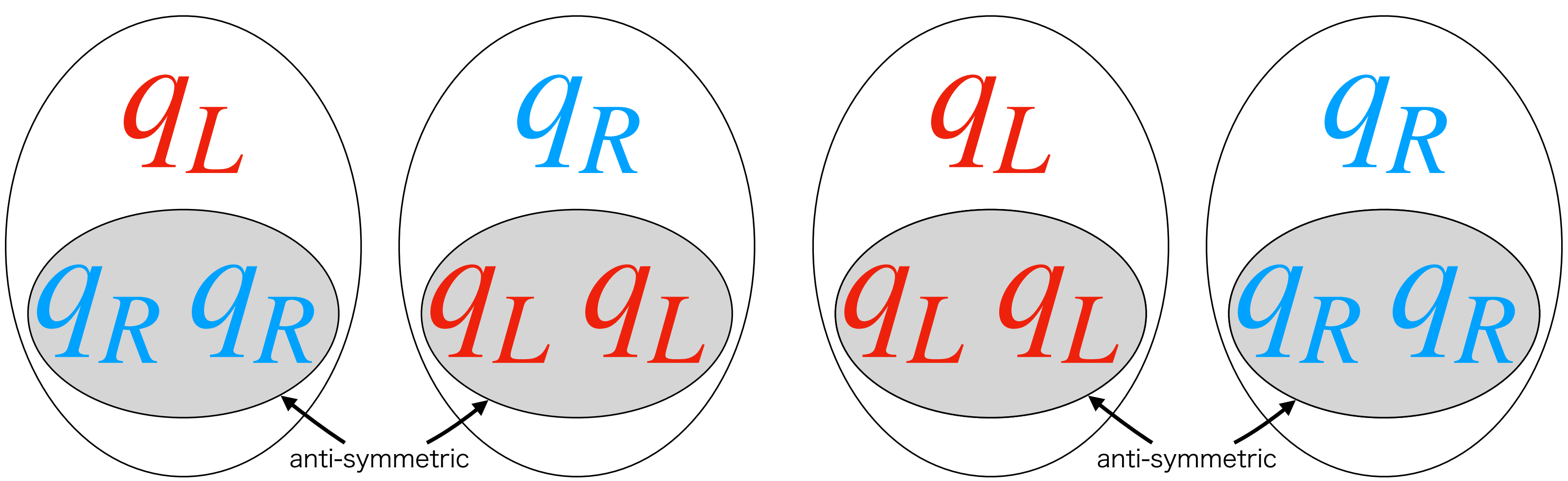

$\psi_L \sim (3,\bar{3})$ $\psi_R \sim (\bar{3},3)$ $\chi_L \sim (8,1)$ $\chi_R \sim (1,8)$

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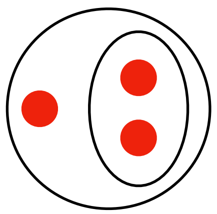

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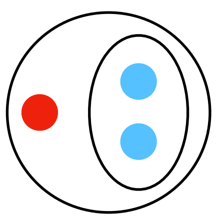

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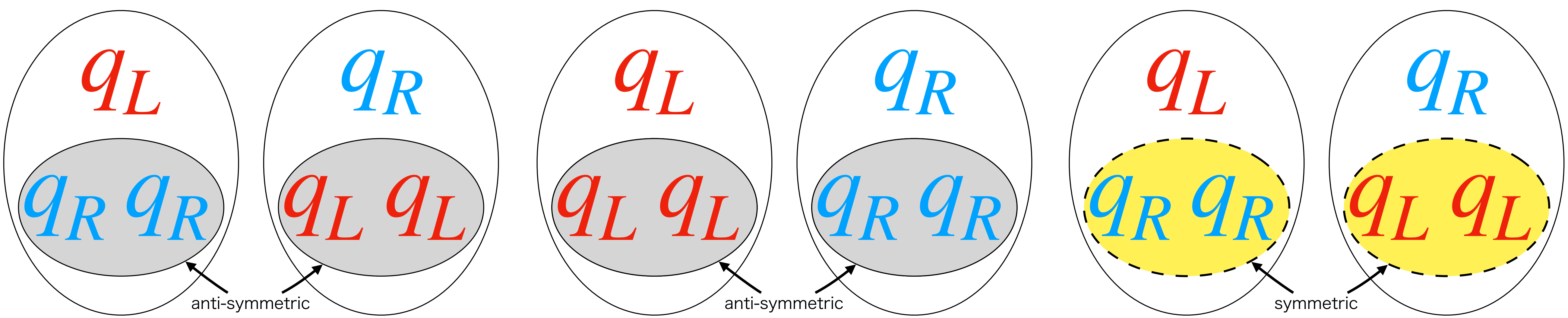

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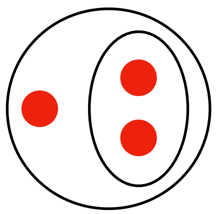

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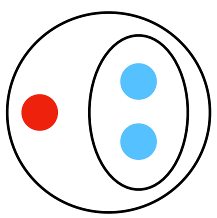

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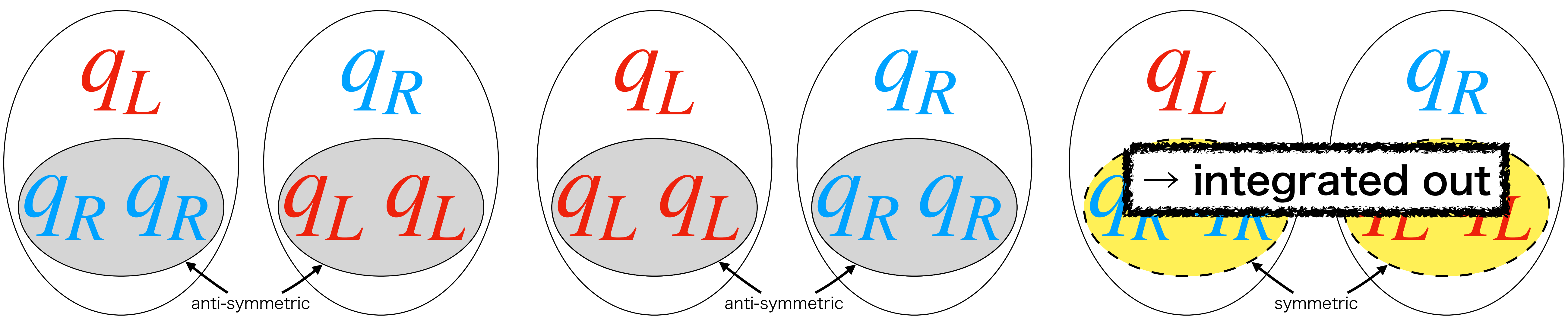

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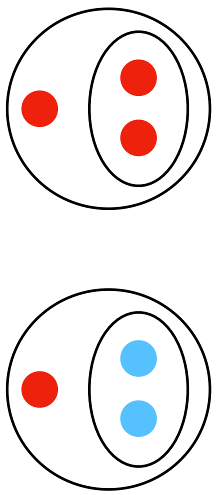


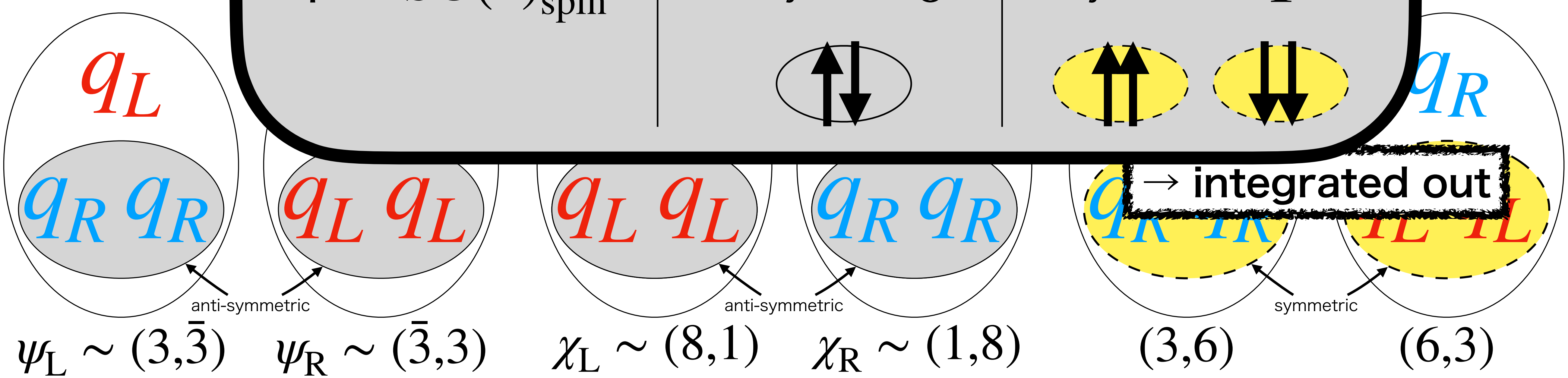
$B_L \sim q \otimes q \otimes q \sim (1,1) \oplus (8,1) \oplus (10,1)$

aryon

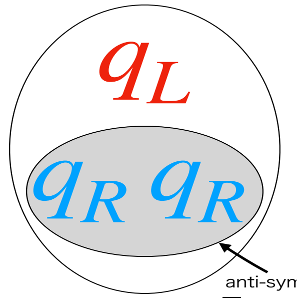
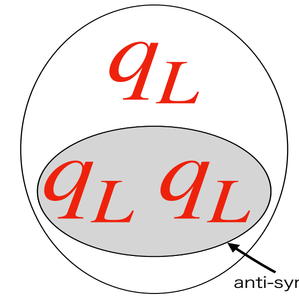
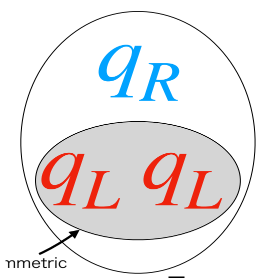
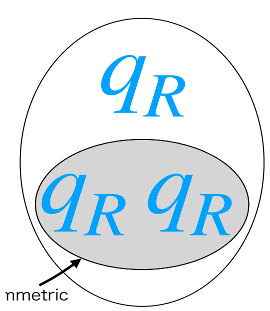
B_L

d” diquark

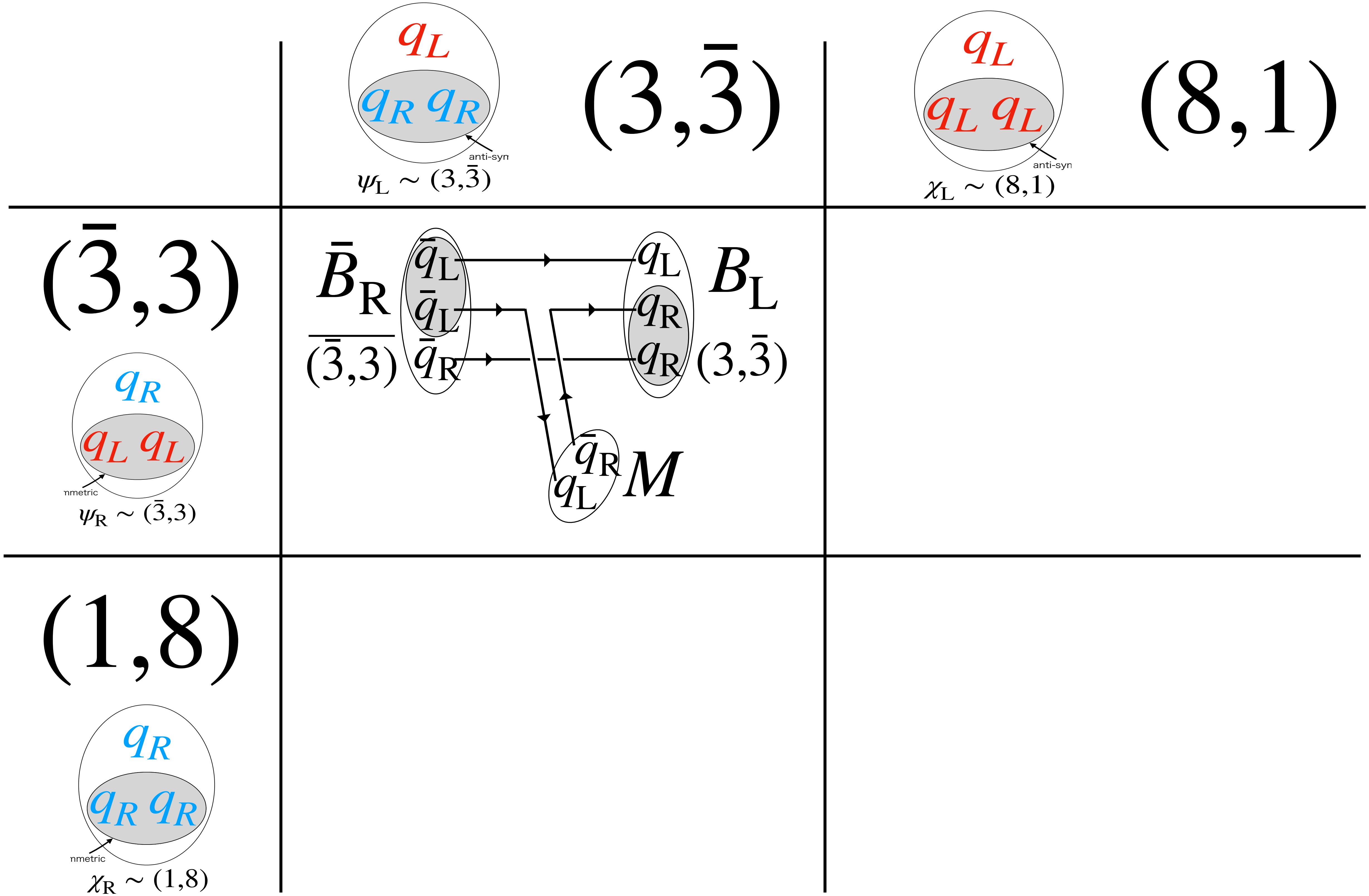
	“good” diquark		“bad” diquark	
color $SU(3)_c$	anti-sym	$\bar{3}$	anti-sym	$\bar{3}$
flavor $SU(3)_F$	anti-sym	$\bar{3}$	sym	6
spin $SU(2)_{spin}$	anti-sym	0	sym	1
	↑↓		↑↑ ↓↓	



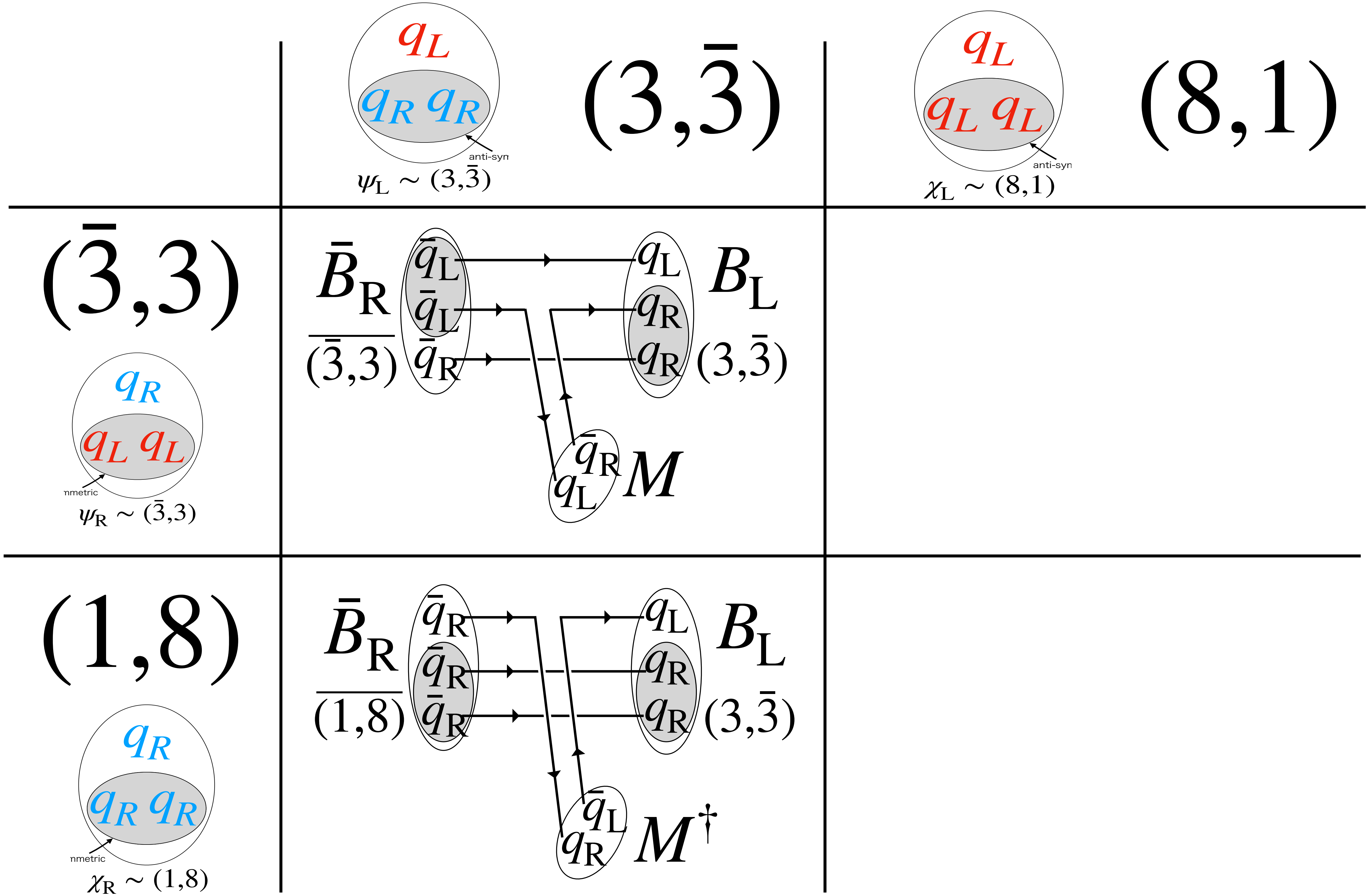
Chiral Yukawa Interactions

	$(3,\bar{3})$  $\psi_L \sim (3,\bar{3})$	$(8,1)$  $\chi_L \sim (8,1)$
$(\bar{3},3)$  $\psi_R \sim (\bar{3},3)$		
$(1,8)$  $\chi_R \sim (1,8)$		

Chiral Yukawa Interactions



Chiral Yukawa Interactions



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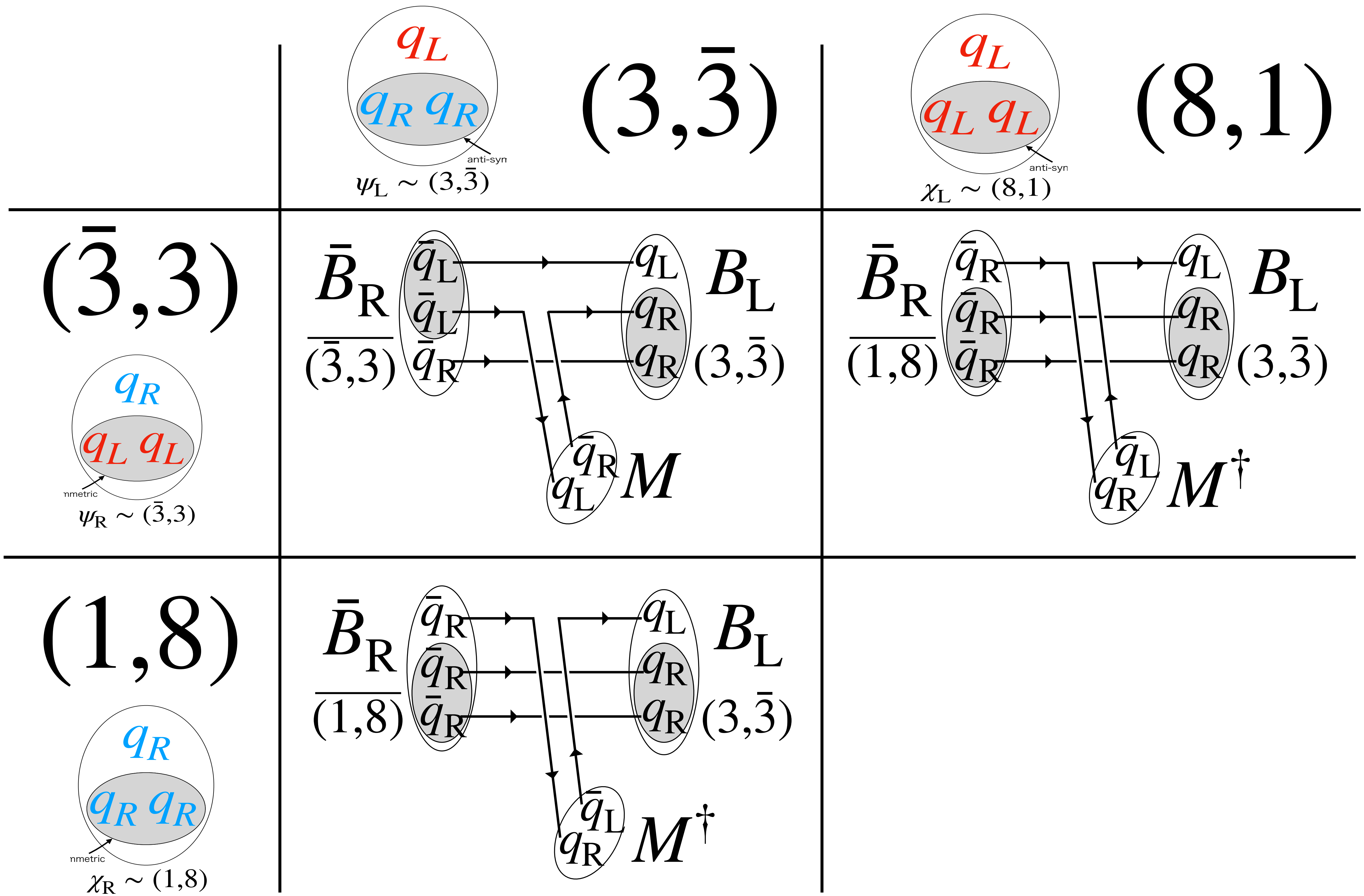

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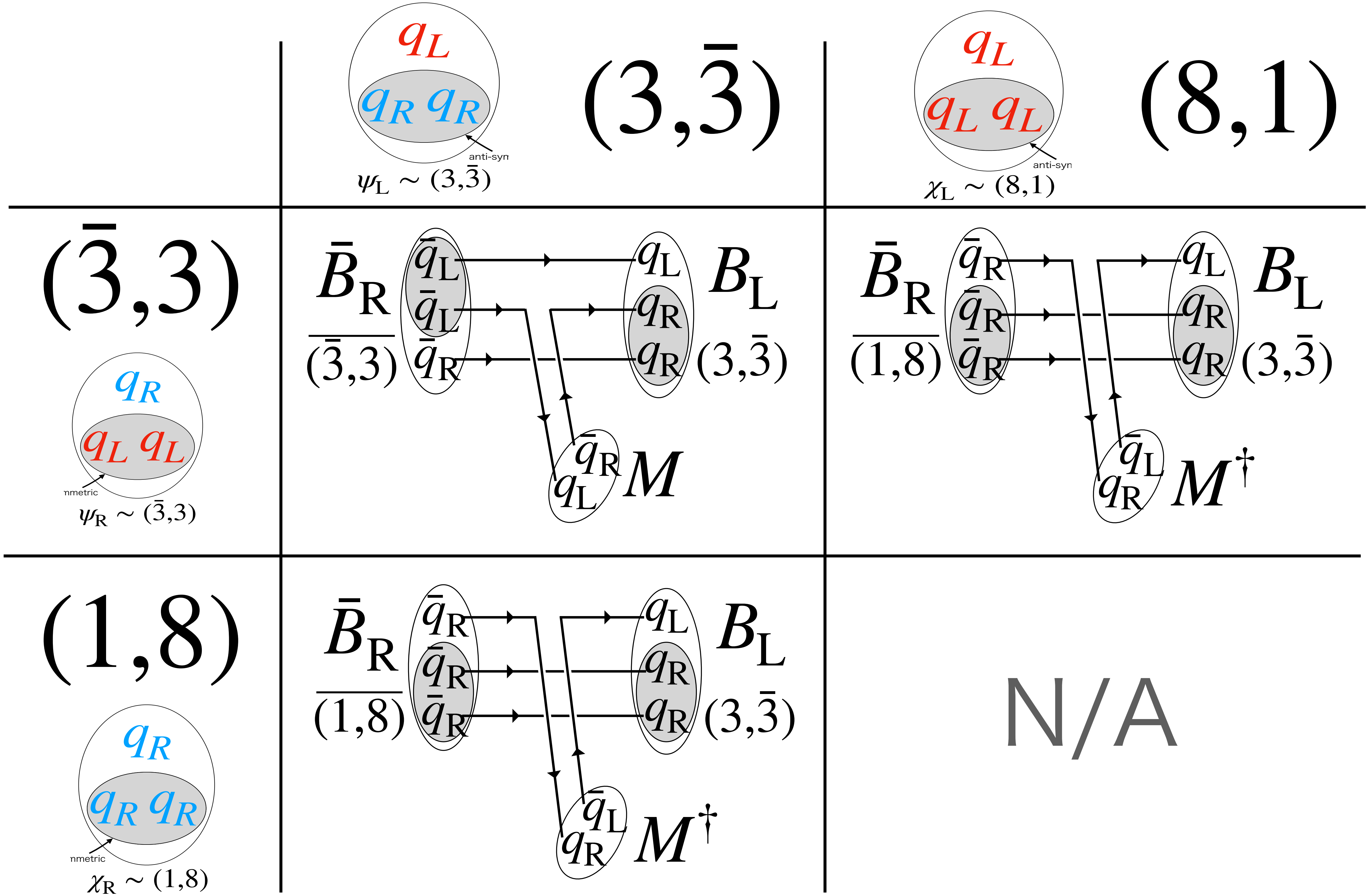
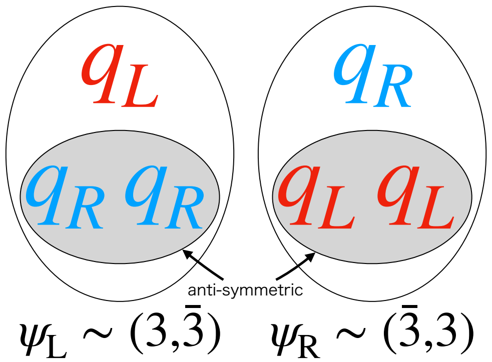
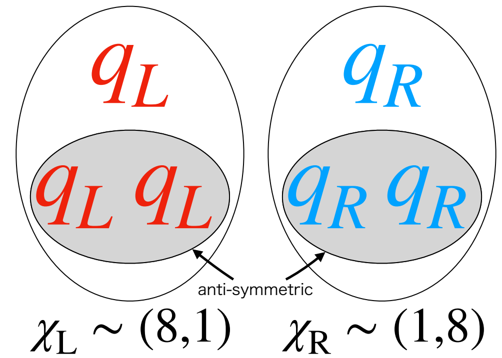
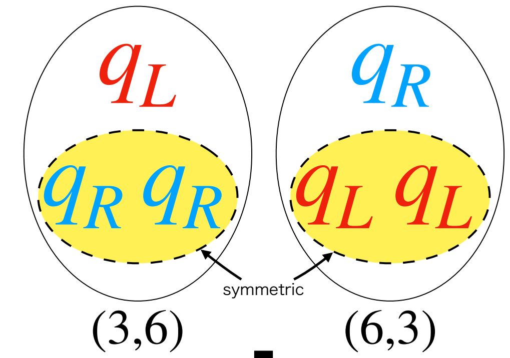


Case (1): Simplest, Only ψ

$\psi \sim (3,\bar{3}) + (\bar{3},3)$
$\chi \sim (8,1) + (1,8)$







no Yukawa b/w (8,1) & (1,8)

may be too heavy
because of “bad” diquark

PROBLEM: $m[N] = m[\Xi]$ in this model

Case (1): Simplest, Only ψ

$\psi \sim (3,\bar{3}) + (\bar{3},3)$
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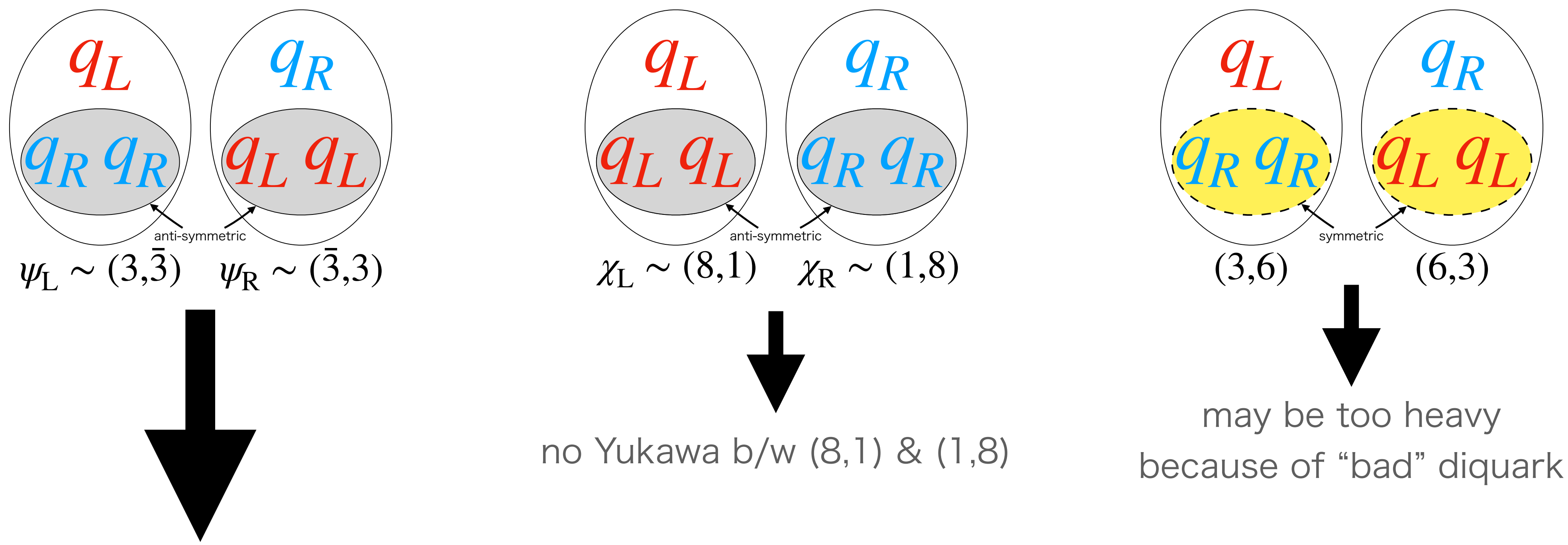

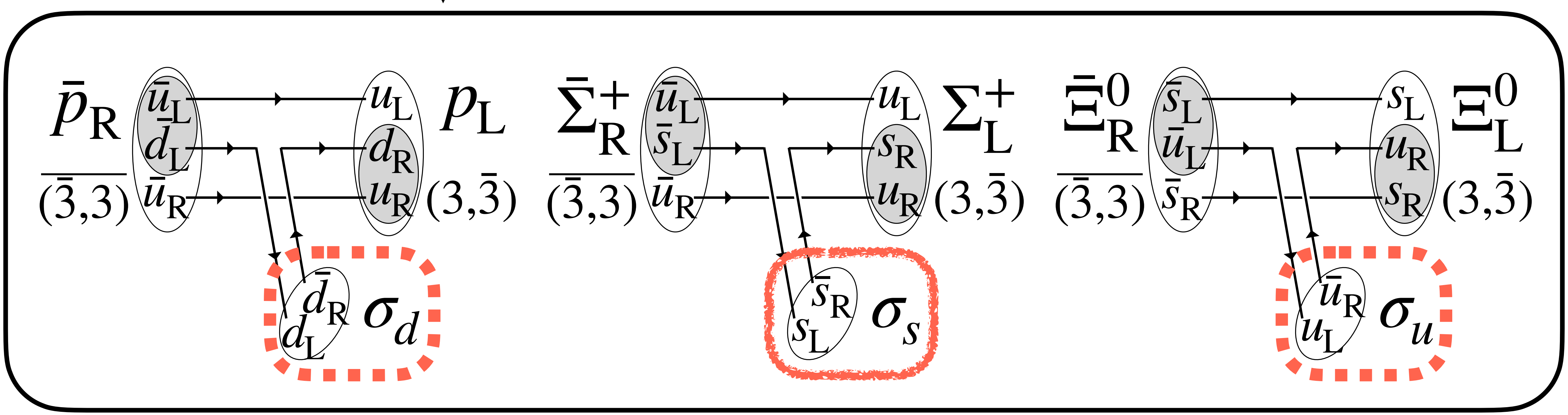

PROBLEM: $m[N] = m[\Xi]$ in this model

Case (2): ψ & χ

$\psi \sim (3,\bar{3}) + (\bar{3},3)$
$\chi \sim (8,1) + (1,8)$

scalar meson $\langle M \rangle = \begin{pmatrix} \alpha & & \\ & \beta & \\ & & \gamma \end{pmatrix} \sim \begin{pmatrix} 93\,\mathrm{MeV} & & \\ & 93\,\mathrm{MeV} & \\ & & 127\,\mathrm{MeV} \end{pmatrix}$

$\alpha = \beta \sim \sigma \sim \langle \bar{u}u + \bar{d}d \rangle$
$\gamma \sim \sigma_s \sim \langle \bar{s}s \rangle$

mass matrix for nucleons

for Sigma baryons

for Xi baryons

$$\begin{array}{c} \quad \psi_N \quad \chi_N \\ \begin{pmatrix} -g\alpha & h\alpha \\ h\alpha & 0 \end{pmatrix} \begin{array}{c} \psi_N \\ \chi_N \end{array} \end{array}$$

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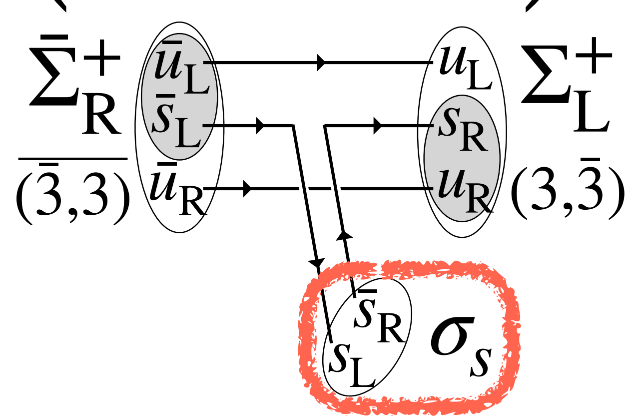

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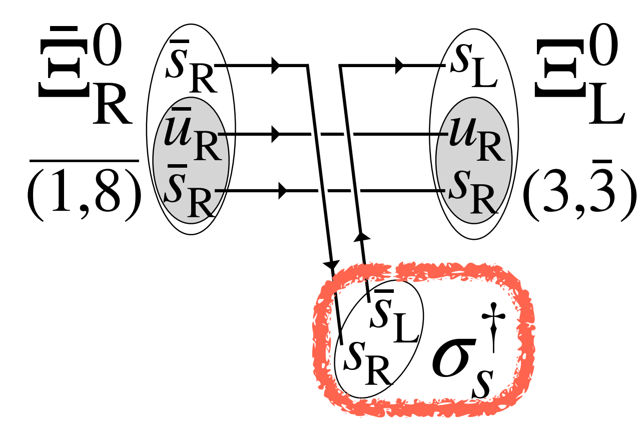

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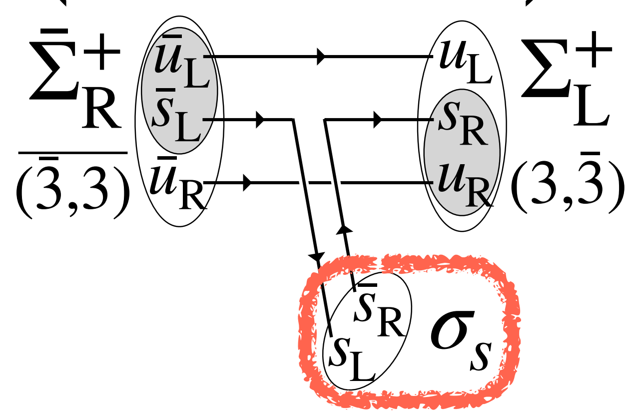

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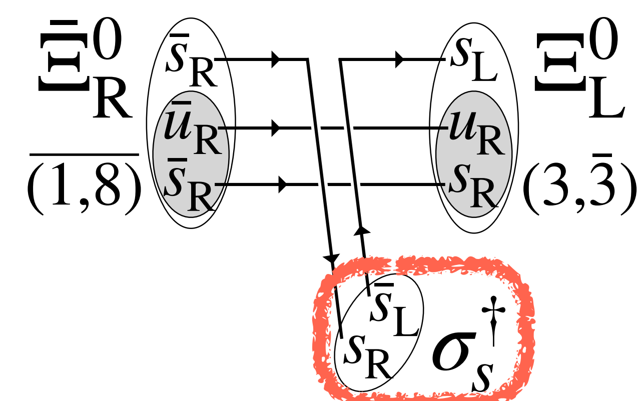

mass eigenvalues for <u>ground-state</u> octet baryons

$$m[N] = m(|g\alpha|, |h\alpha|)$$
$$m[\Sigma] = m(|g\gamma|, |h\alpha|)$$
$$m[\Xi] = m(|g\alpha|, |h\gamma|)$$

eigenvalue with the smallest absolute value of $\begin{pmatrix} x & y \\ y & 0 \end{pmatrix}$

$$m(x, y) := \sqrt{(x/2)^2 + y^2} - x/2$$

PROBLEM: $m[N] > m[\Sigma]$ in this model

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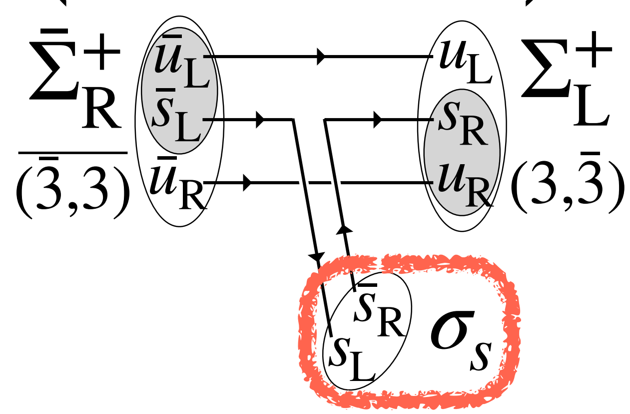

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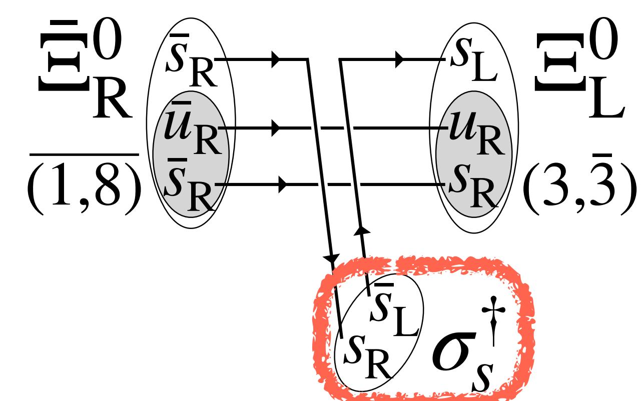

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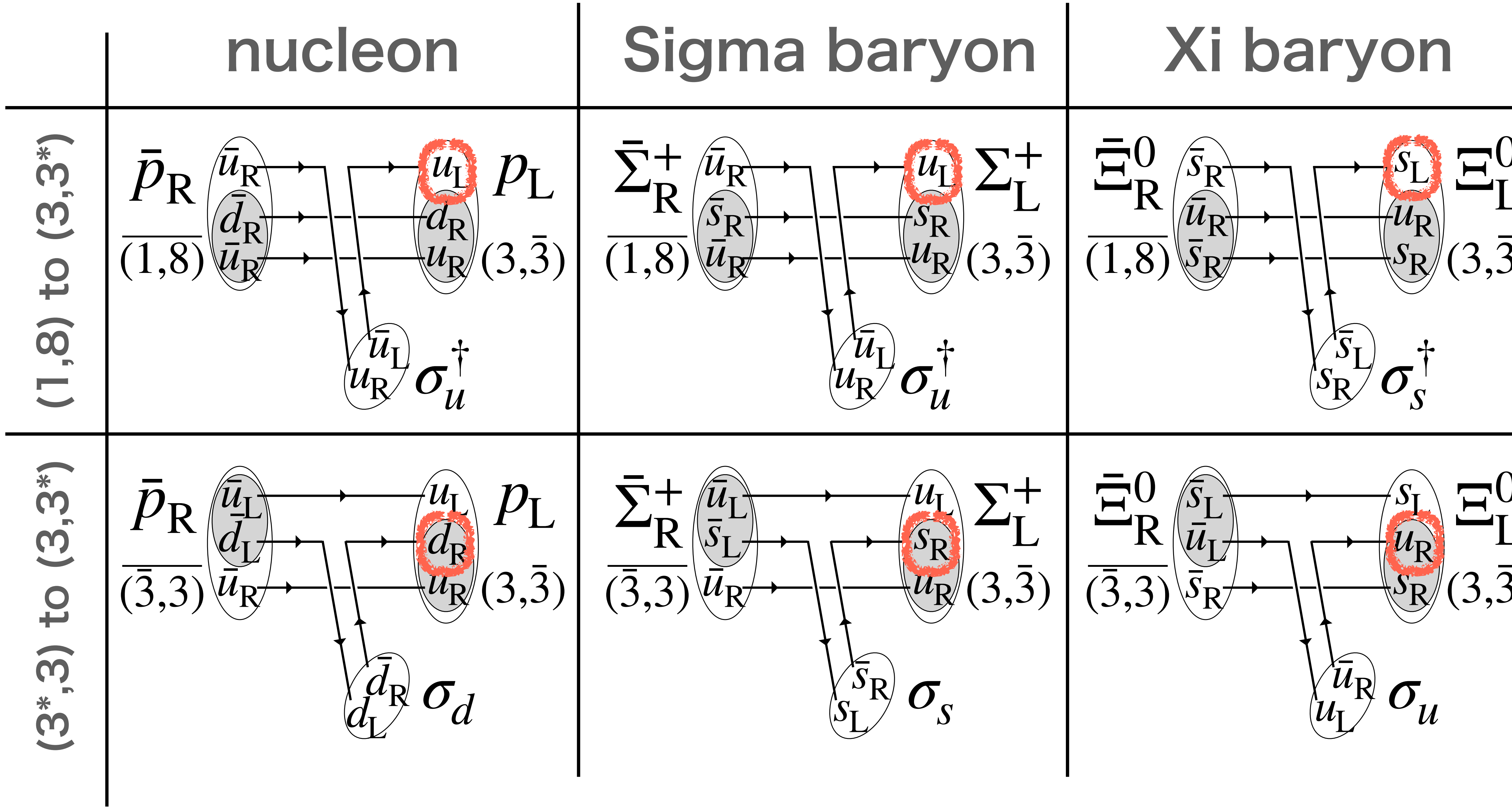
eigenvalue with the smallest absolute value of $\begin{pmatrix} x & y \\ y & 0 \end{pmatrix}$

$$m(x, y) := \sqrt{(x/2)^2 + y^2} - x/2$$

$\partial_x m(x, y) < 0$ **therefore m[N]>m[Σ]**

PROBLEM: $m[N] > m[\Sigma]$ in this model

Why Not Enough?

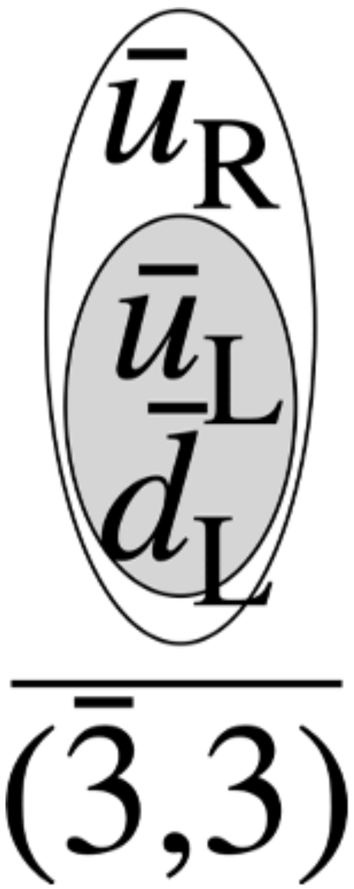


For Xi baryon, there are no interactions of s quark in the diquark.

(3,6)+(6,3) rep. is needed. We integrate out it for simplicity.

2nd-Order Yukawa (1)

Let us make a diagram starting from $(\bar{3},3)$



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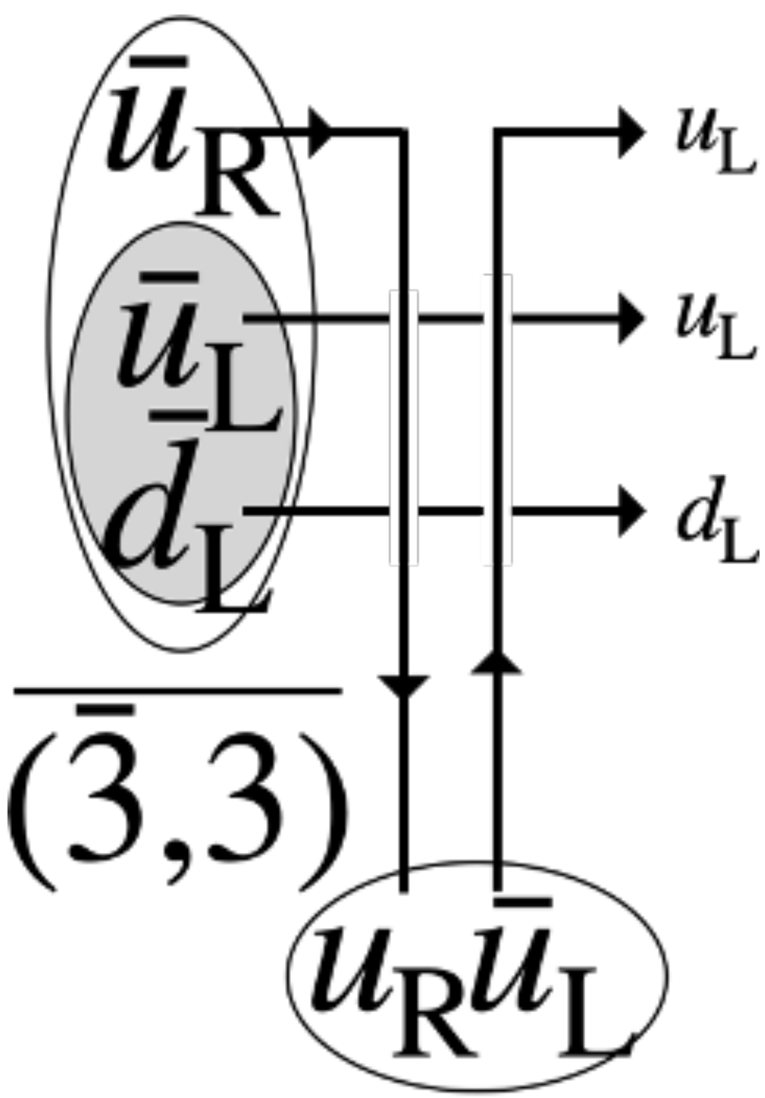

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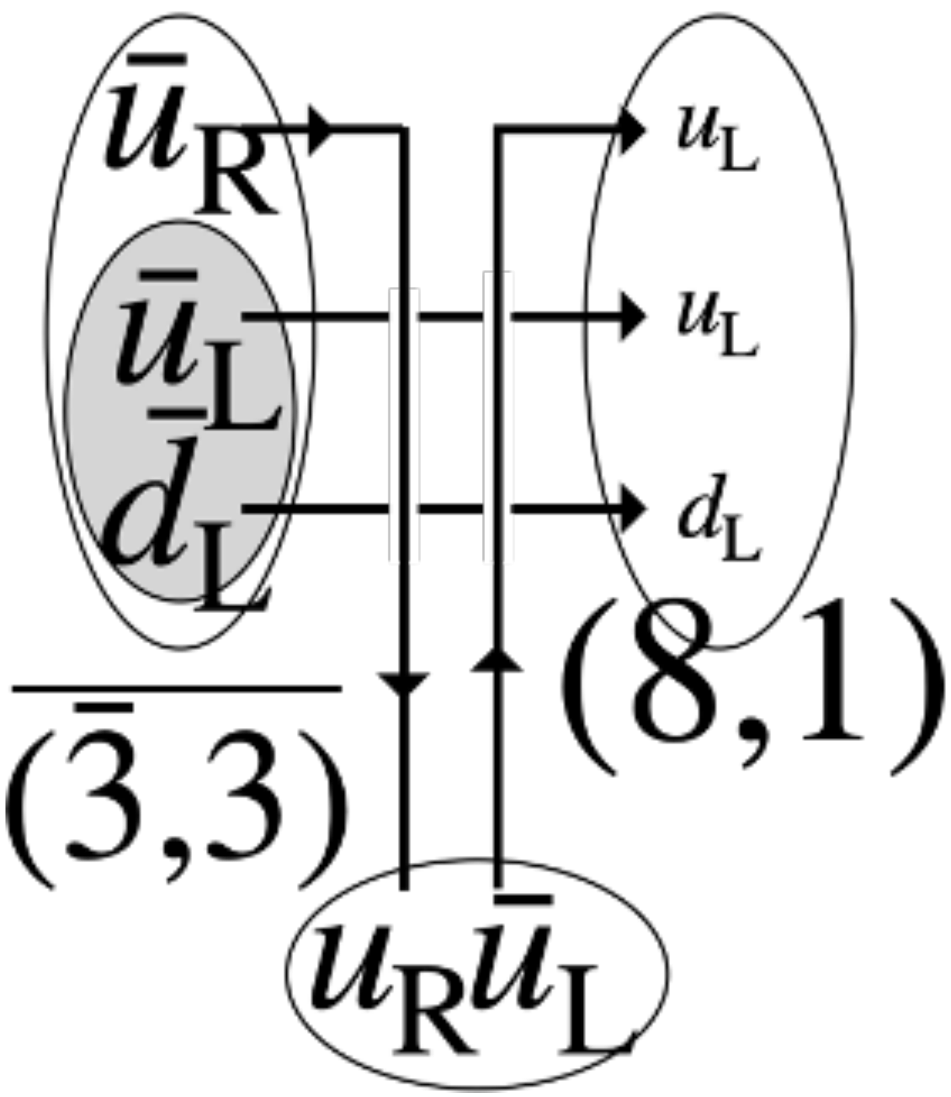

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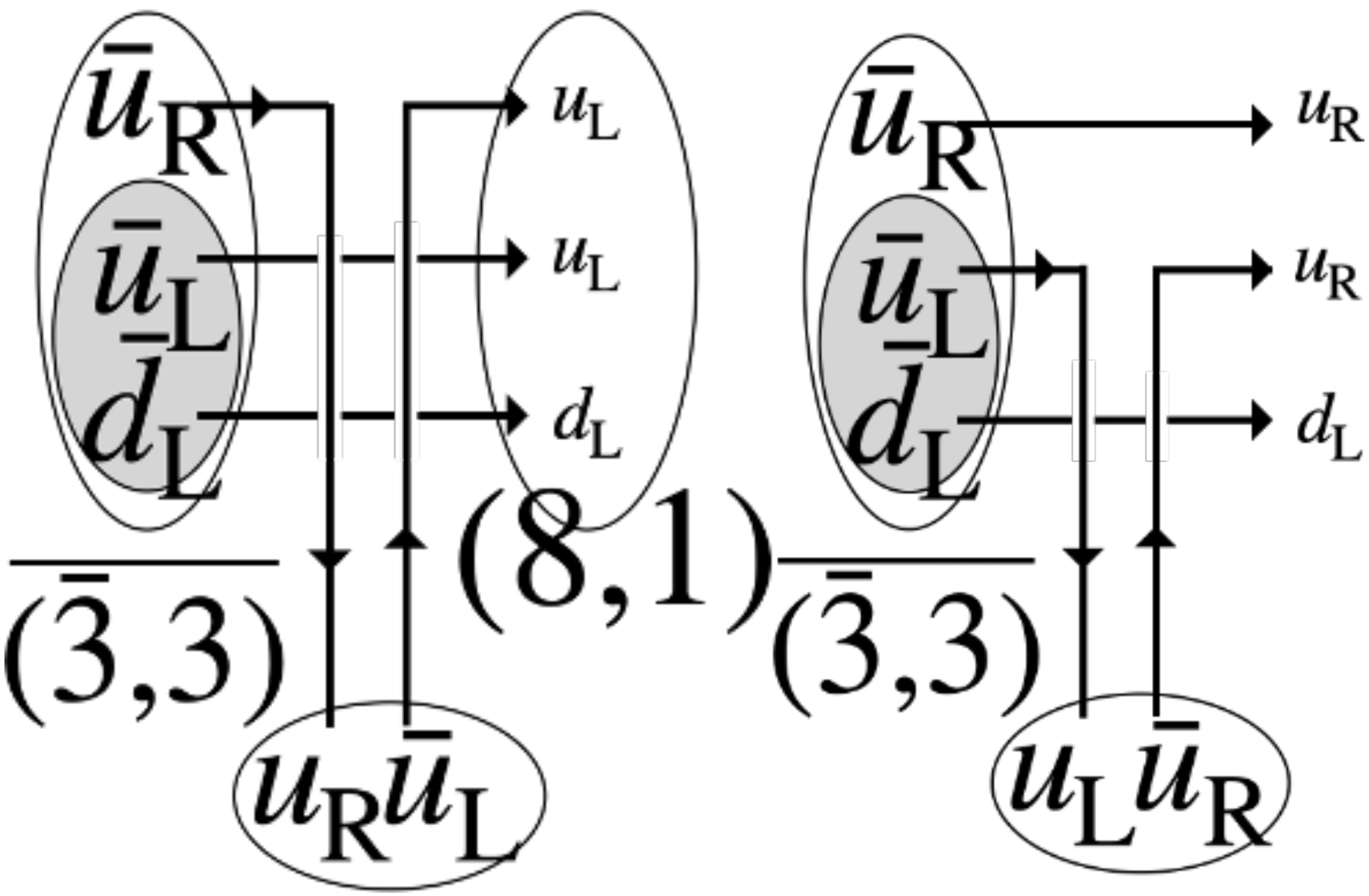

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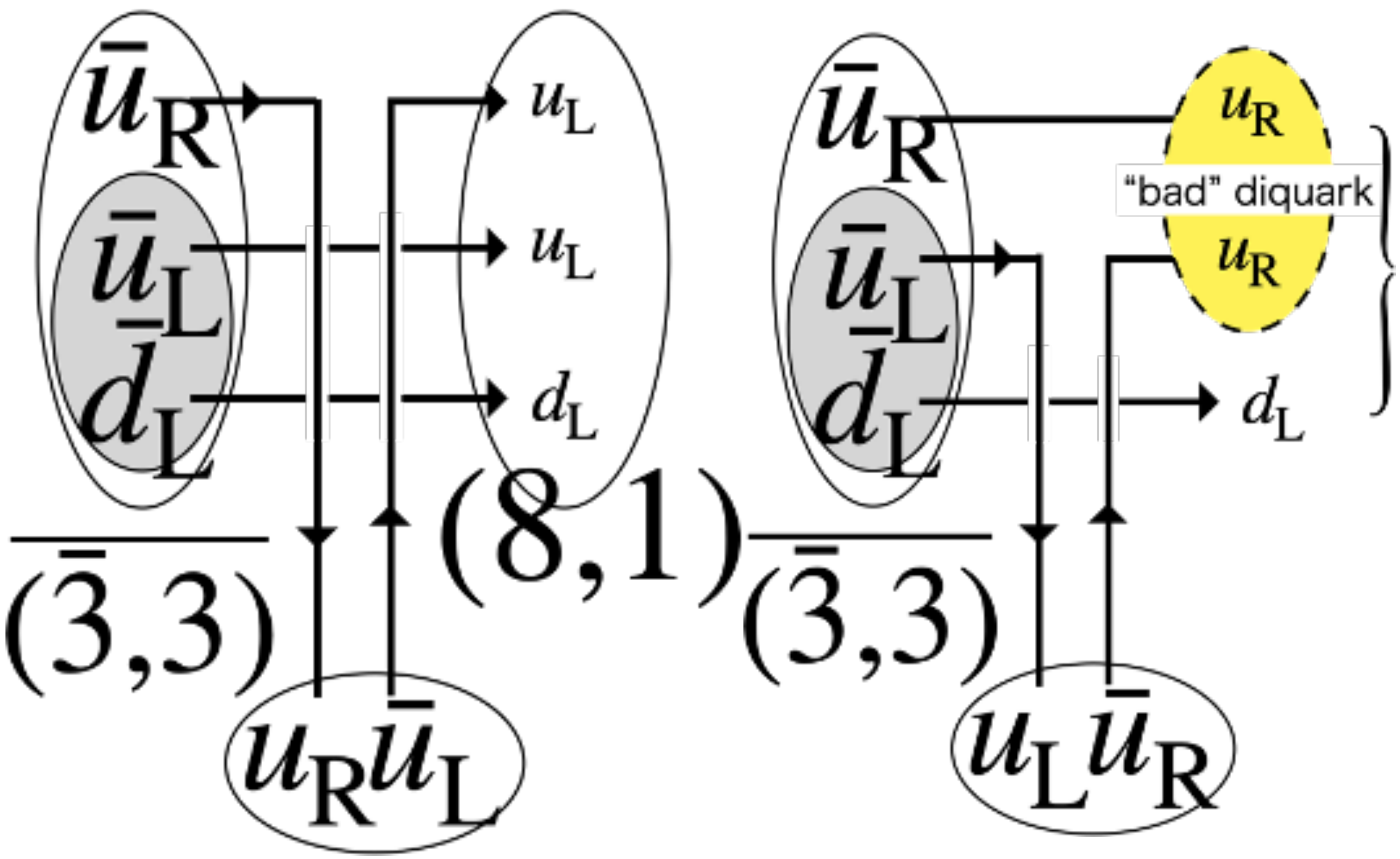

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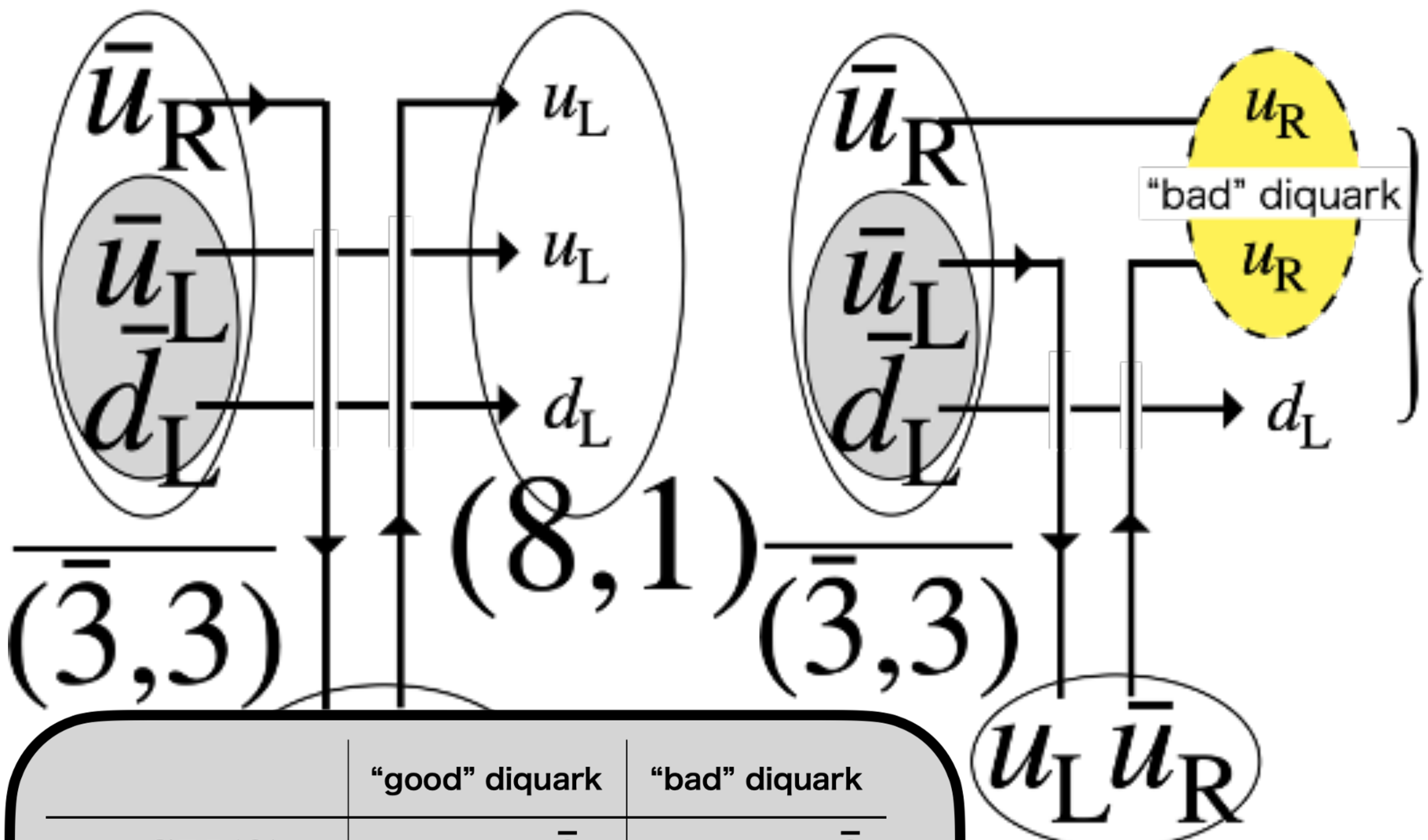

	"good" diquark		"bad" diquark	
color $SU(3)_c$	anti-sym	$\bar{3}$	anti-sym	$\bar{3}$
flavor $SU(3)_F$	anti-sym	$\bar{3}$	sym	6
spin $SU(2)_{spin}$	anti-sym	0	sym	1
	↑↓		↑↑	↓↓

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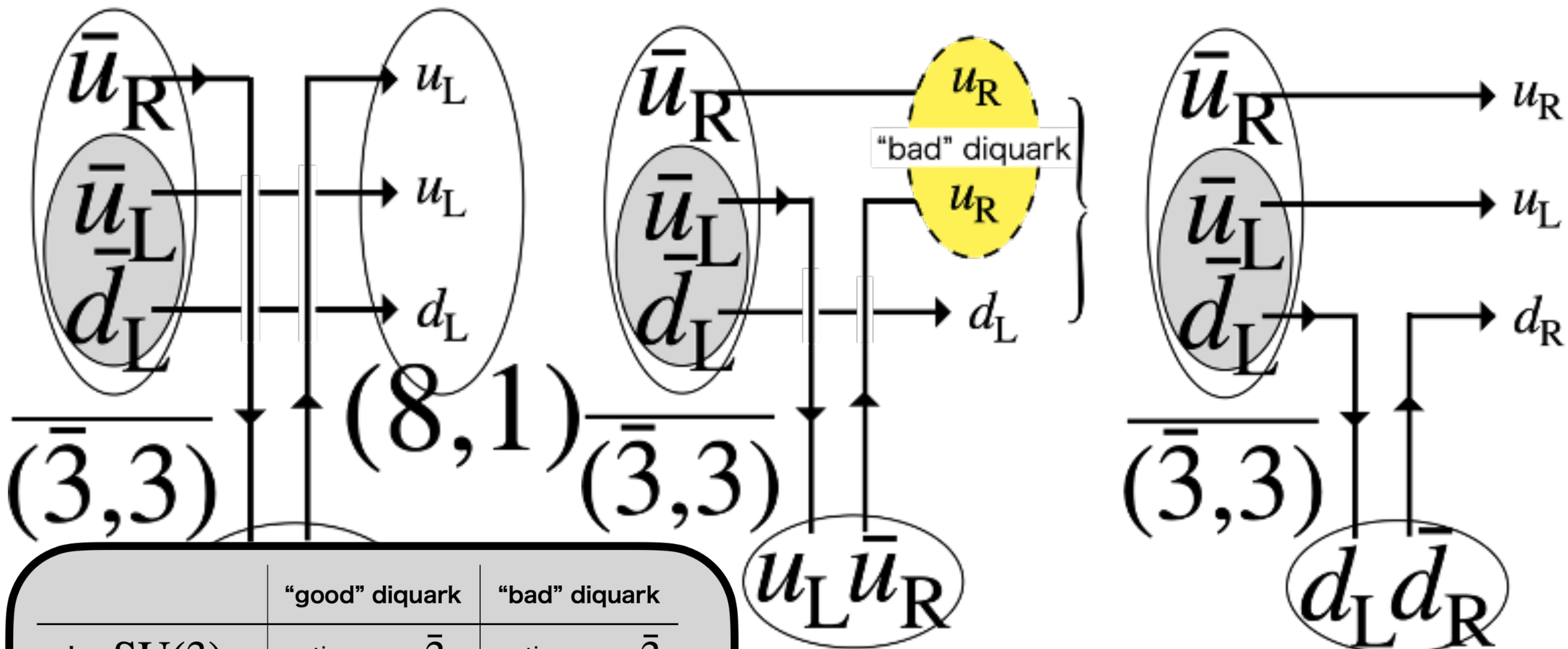

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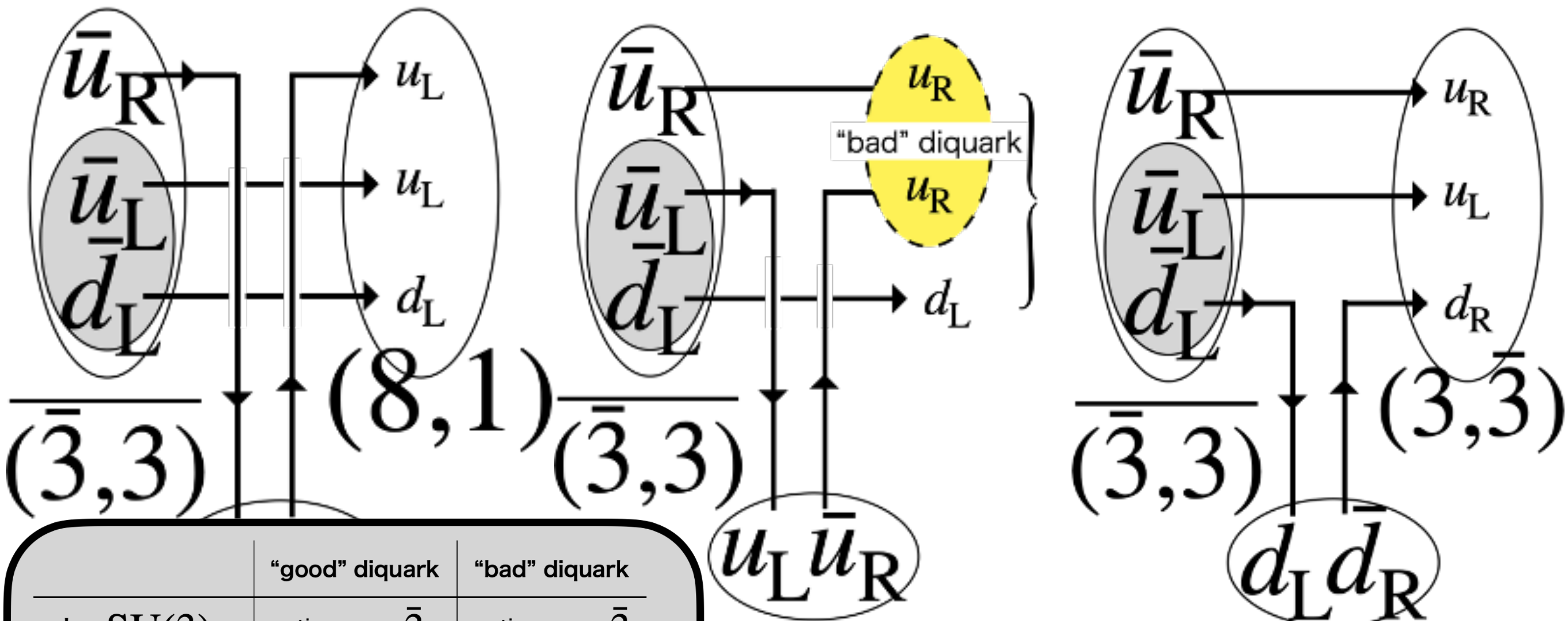

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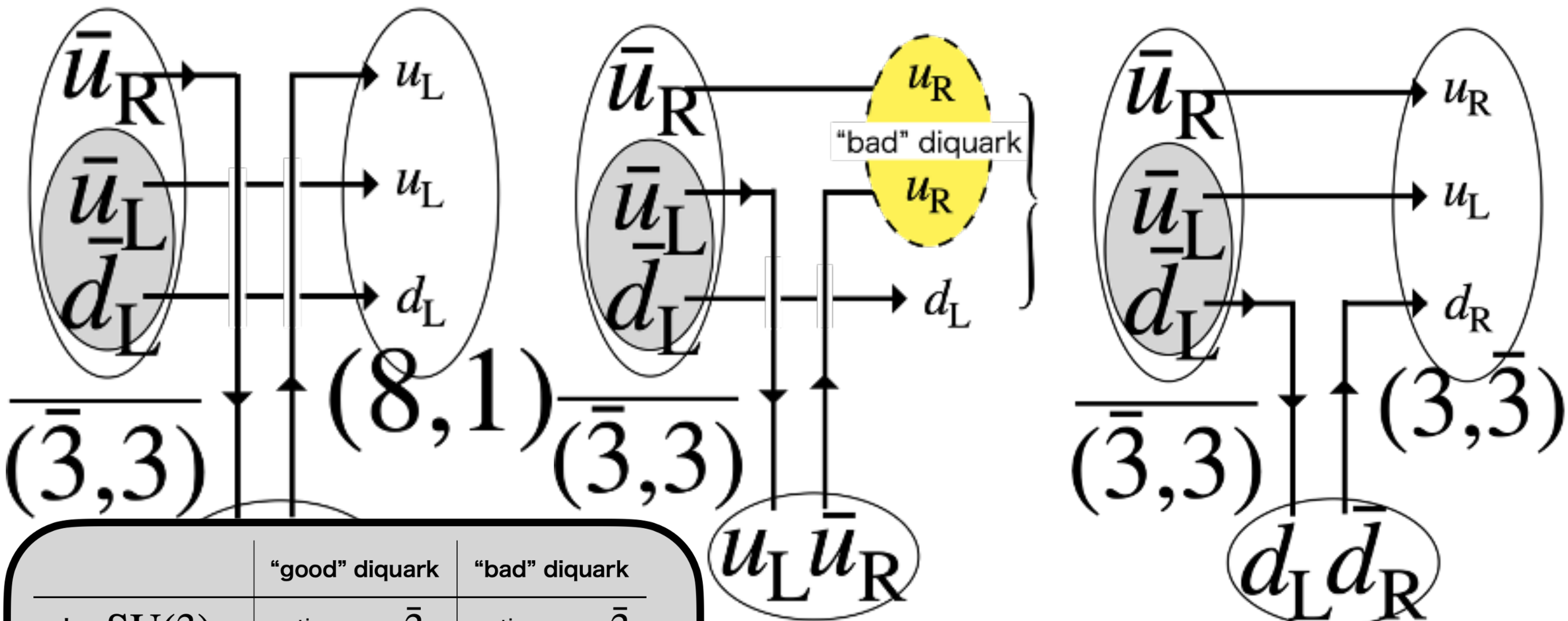

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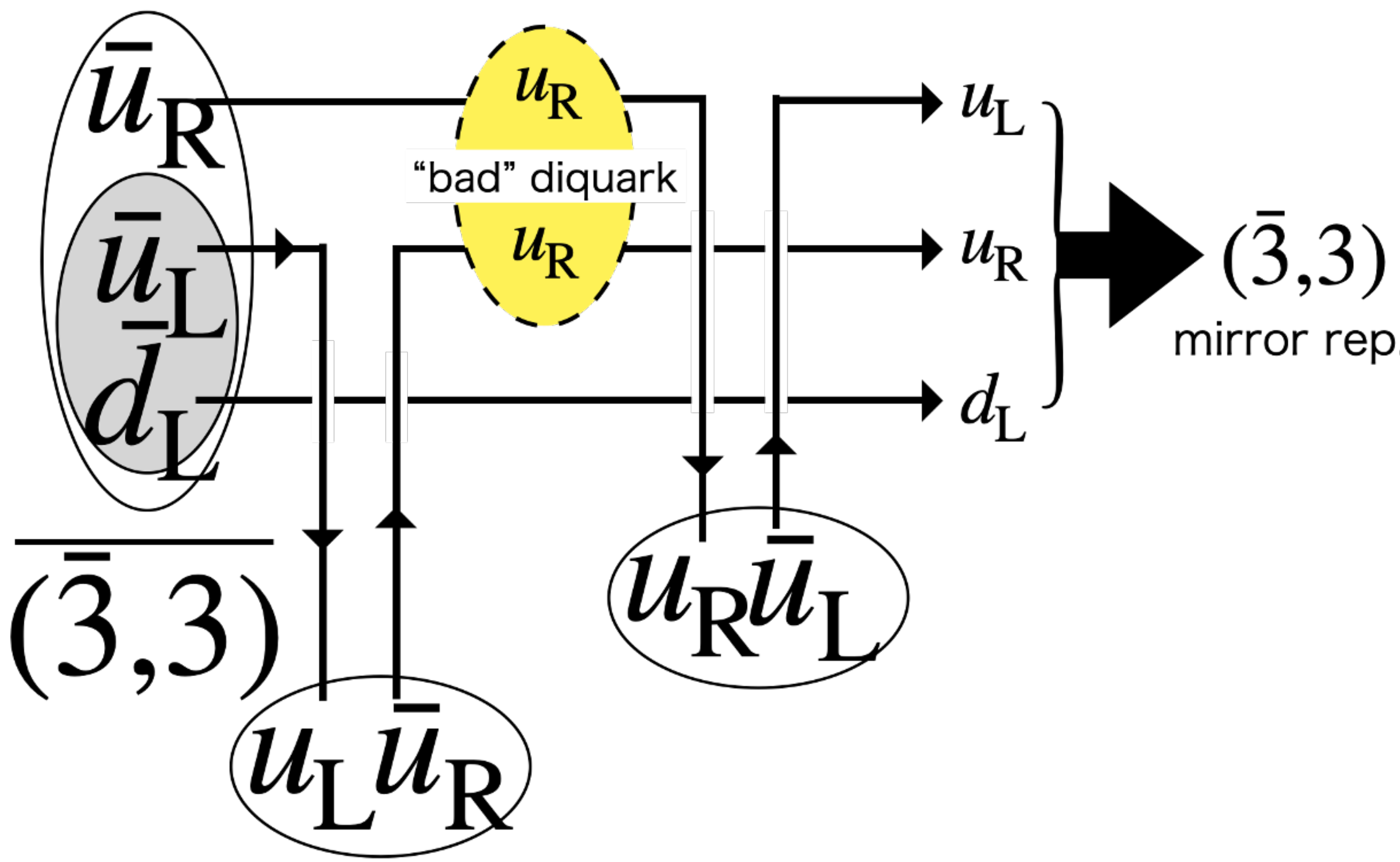
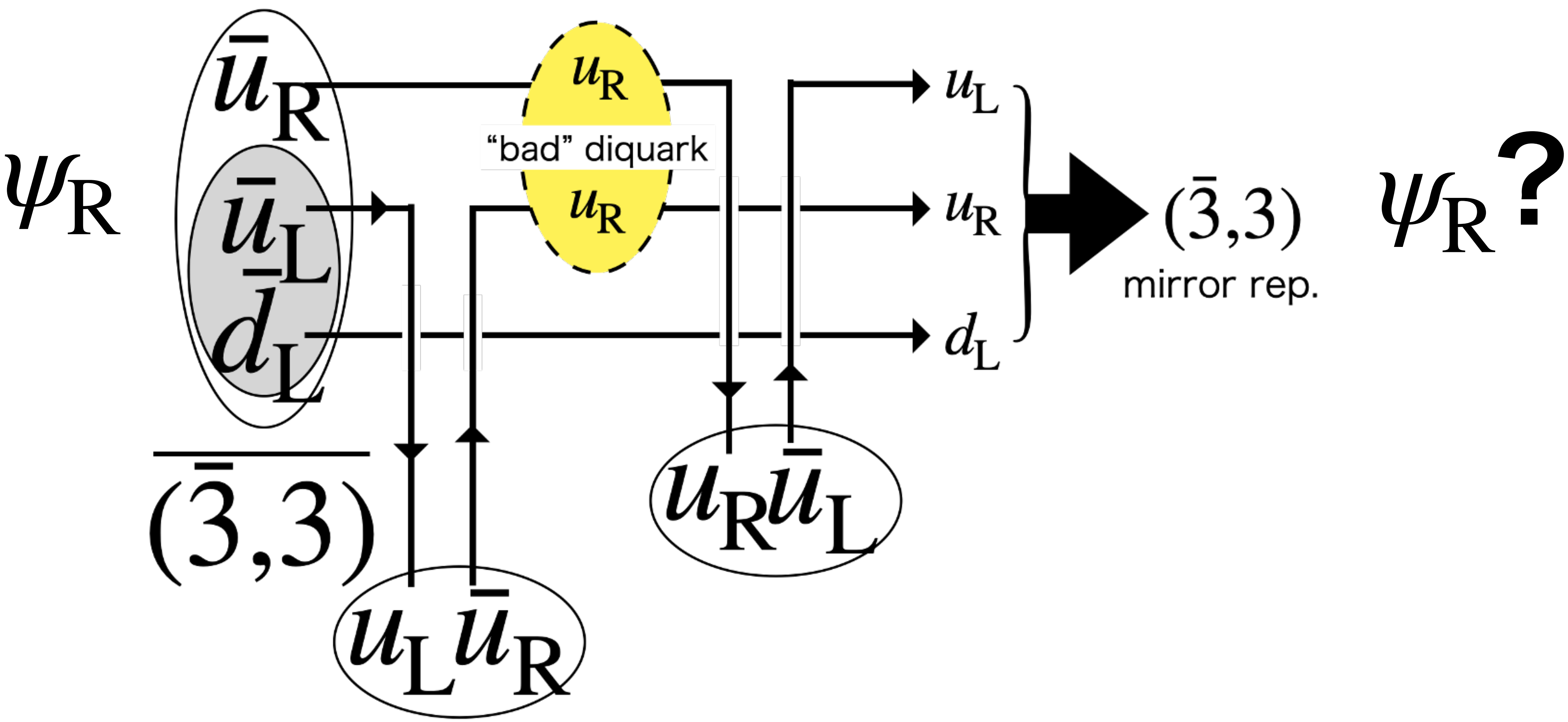
	↑↓		↑↑	↓↓

one more quark exchange

2nd-Order Yukawa (2)



2nd-Order Yukawa (2)



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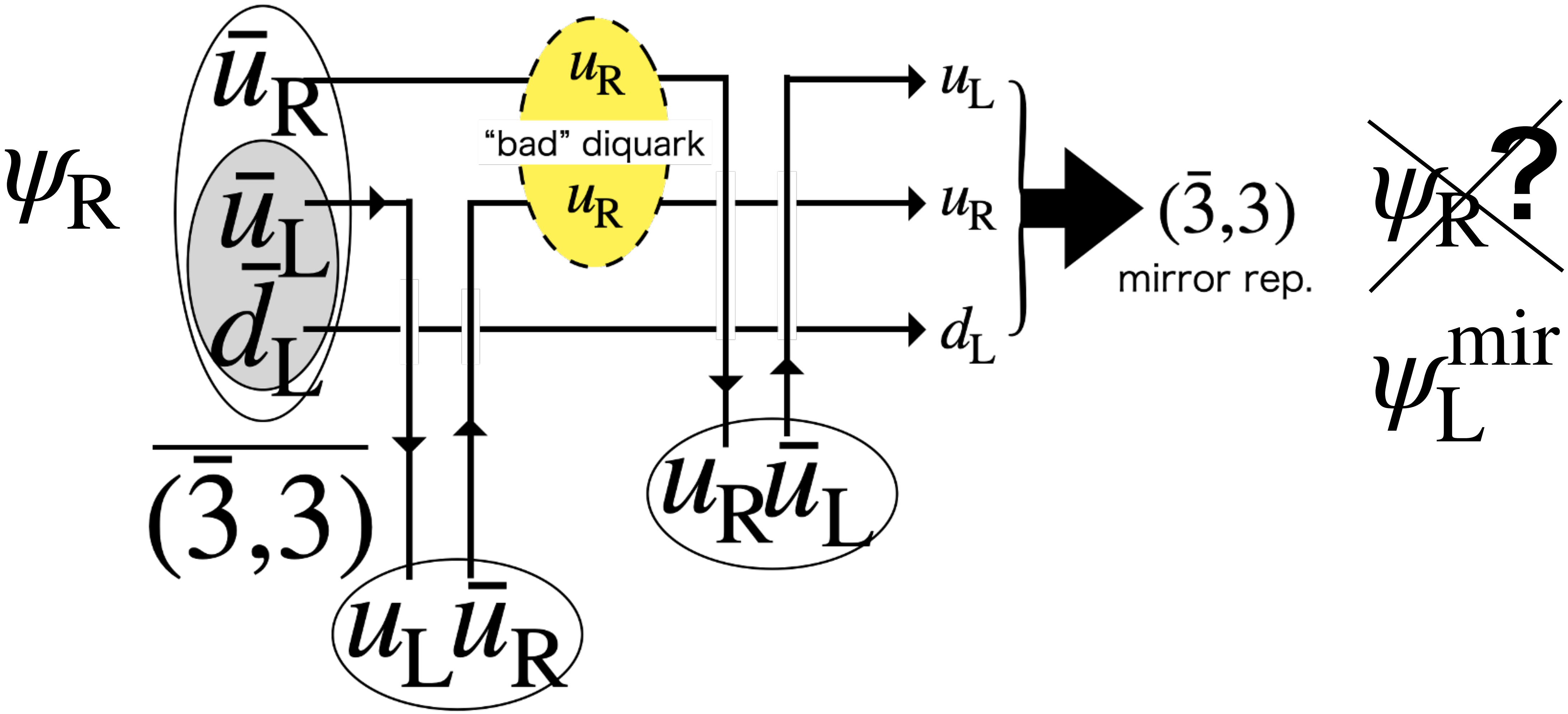

2nd-Order Yukawa (2)

Parity doubling multiplet

$$\psi_L \sim (3,\bar{3}),\ \psi_R \sim (\bar{3},3)$$

$$\psi_L^{\mathrm{mir}} \sim (\bar{3},3),\ \psi_R^{\mathrm{mir}} \sim (3,\bar{3})$$

$$\mathscr{L}_{\mathrm{CIM}} = \underline{m_0}(\bar{\psi}\gamma_5\psi^{\mathrm{mir}} - \bar{\psi}^{\mathrm{mir}}\gamma_5\psi)$$

m_0: chiral invariant mass

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m_0: chiral invariant mass

Interpolating field for mirror rep.

$$\psi_L \sim q_L(q_R^t C\gamma_5 q_R) = P_L q d_R$$

$$\psi_R^{\rm mir} \sim \not\partial(q_L)(q_R^t C\gamma_5 q_R) = P_R(\not\partial q) d_R$$

minimization function

$\psi \sim (3,\bar{3}) + (\bar{3},3)$
$\chi \sim (8,1) + (1,8)$
$\psi^{\text{mir}} \sim (\bar{3},3) + (3,\bar{3})$
$\chi^{\text{mir}} \sim (1,8) + (8,1)$

$$\mathscr{L} = \mathscr{L}(\psi, \chi, \psi^{\text{mir}}, \chi^{\text{mir}}, M)$$

input mass [MeV]	N	Σ	Ξ
Ground-state	939	1193	1318
excited ($J^P = 1/2^+$)	1440	1660	1790?
excited ($J^P = 1/2^-$)	1535	2203.67?	1989?
excited ($J^P = 1/2^-$)	1650	1750	1925?

minimization function

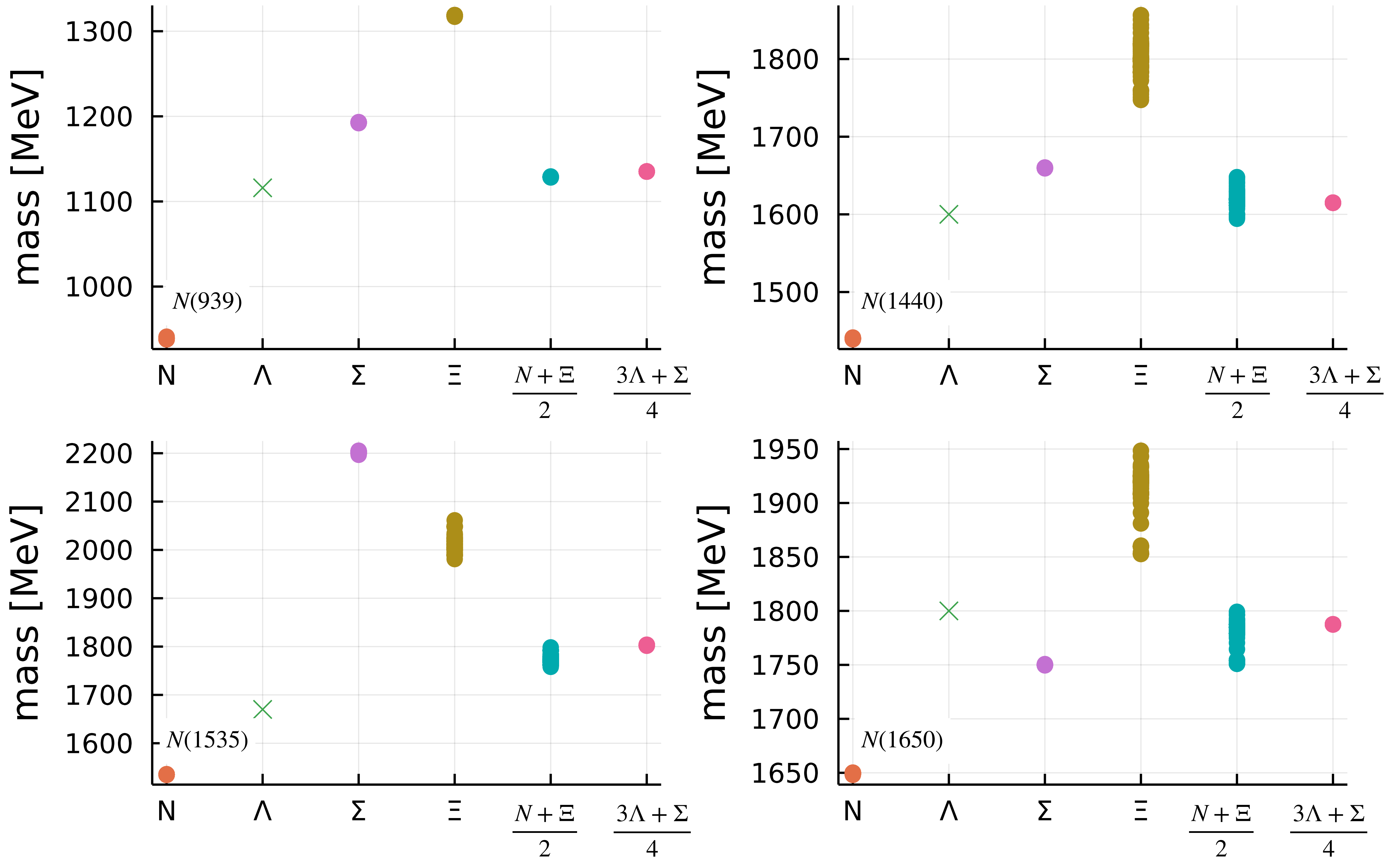
$$f(\text{couplings}) = \sum_{i=1}^{8} \left(\frac{m_i^{\text{theory}} - m_i^{\text{input}}}{10\,\text{MeV}} \right)^2 + \sum_{i=9}^{12} \left(\frac{m_i^{\text{theory}} - m_i^{\text{input}}}{100\,\text{MeV}} \right)^2$$

Numerical Result

Gell-Mann—Okubo mass formula

$$\frac{m[N]+m[\Xi]}{2}=\frac{3m[\Lambda]+m[\Sigma]}{4}$$

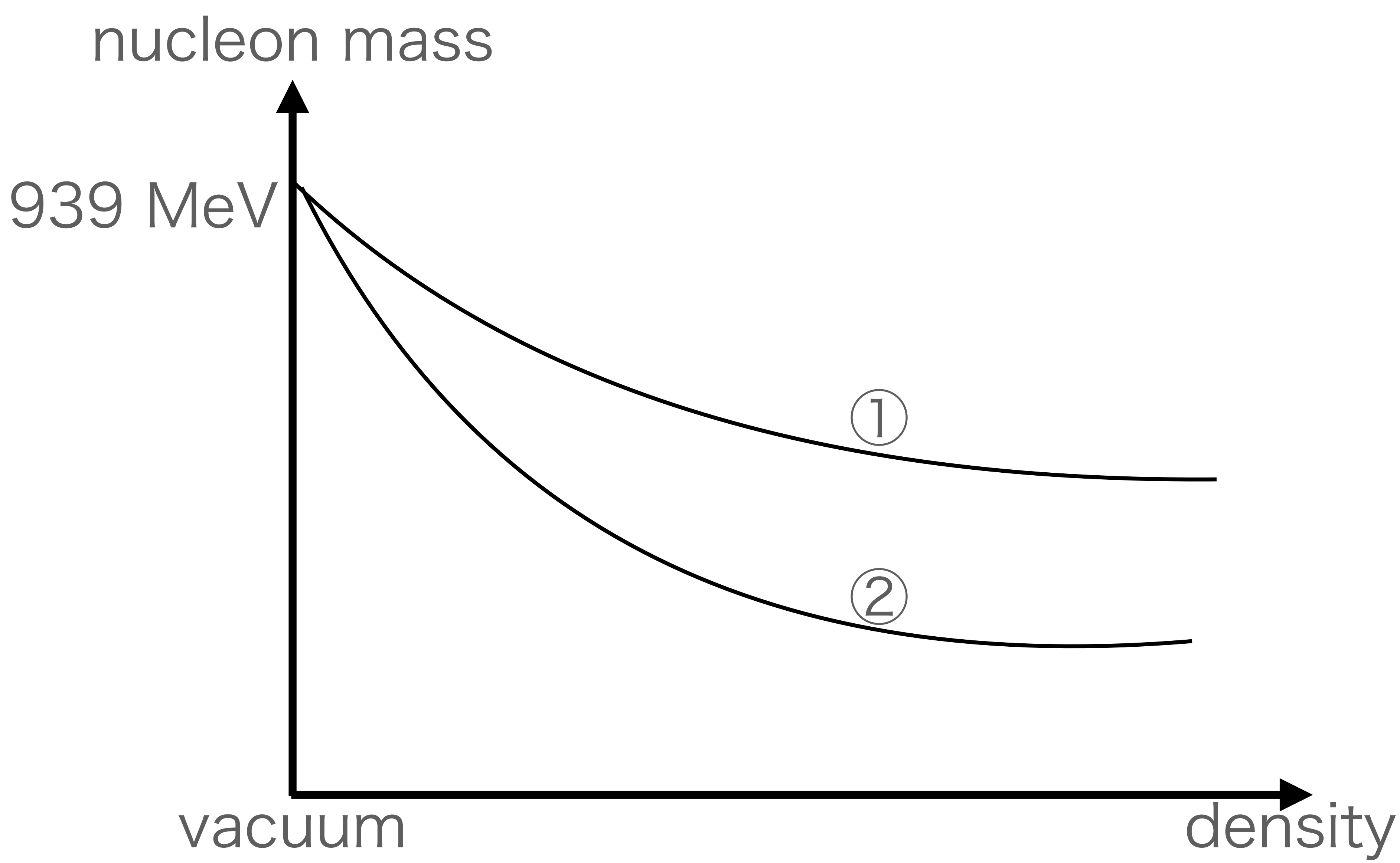
m_0=800MeV



Discussion

① Ground-state nucleon consists mainly of ψ & χ

② Ground-state nucleon consists mainly of ψ^{mir} & χ or ψ & χ^{mir}



Summary

- カイラル対称な3フレーバーハドロン模型について、クォーク図を用いて考察。
- カイラル(3,3*)+(3*3)表現と(8,1)+(1,8)表現を用いた模型を構築。
- 通常の湯川型相互作用(メソン1次の湯川)だけではHyperonの質量を再現しない。
- 高次の湯川型相互作用(メソン2次の湯川)を入れることでHyperonの質量を再現でき、またParity doubling構造が自然に出てくることを見た。Parity doubling構造はカイラル不変質量と密接に関わる。

Outlook

- 基底状態における(3,3*)+(3*3)と(8,1)+(1,8)の成分比
- 質量の密度(or σ)依存性
- 有限密度や中性子星における解析

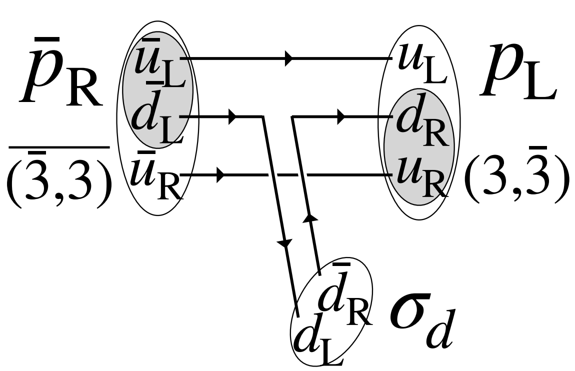
backup

Diagram vs Eff. Interaction

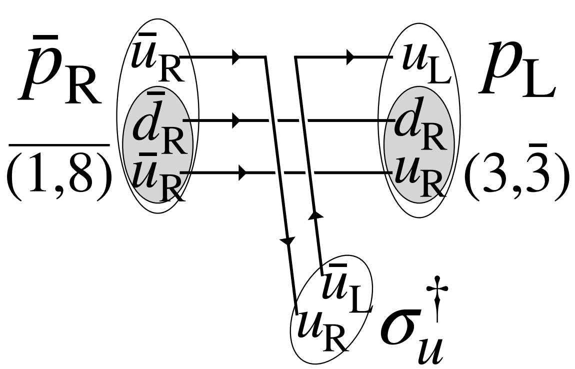
$$\psi \sim (3,\bar{3}) + (\bar{3},3)$$
$$\chi \sim (8,1) + (1,8)$$

$$(\psi_L)^{l[r_1 r_2]_{\mathrm{AS}}} := \varepsilon^{r r_1 r_2} (\psi_L)^l_r \quad \leftrightarrow \quad (\psi_L)^l_r = \frac{1}{2} \varepsilon_{r r_1 r_2} (\psi_L)^{l[r_1 r_2]_{\mathrm{AS}}}$$

$$\psi \sim \frac{1}{\sqrt{3}} \Lambda^{\text{singlet}} + \begin{bmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{bmatrix} \qquad \chi \sim \begin{bmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{bmatrix}$$



$$\Longleftrightarrow \quad (\bar{\psi}_R)_{r_1[l_2 l_3]} (M)^{l_2}_{r_2} (\psi_L)^{l_3[r_1 r_2]} \propto \varepsilon_{l_1 l_2 l_3} \varepsilon^{r_1 r_2 r_3} (\bar{\psi}_{1R})^{l_1}_{r_1} (M)^{l_2}_{r_2} (\psi_{1L})^{l_3}_{r_3}$$
$$=: \det{}'(\bar{\psi}_{1R}, M, \psi_{1L})$$



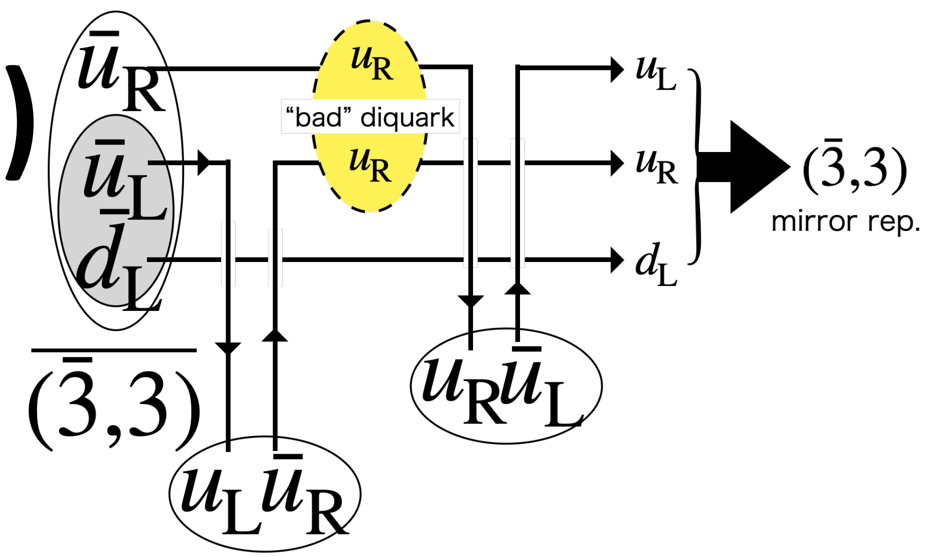
$$\Longleftrightarrow \quad (\bar{\chi}_R)_{r[r_1 r_2]} (M^\dagger)^r_l (\psi_L)^{l[r_1 r_2]} \propto (\bar{\chi}_R)^{r'}_r (M^\dagger)^r_l (\psi_L)^l_{r'}$$
$$= \mathrm{tr}(\bar{\chi}_{1R} M^\dagger \psi_L)$$

$\psi \sim (3,\bar{3}) + (\bar{3},3)$

$\chi \sim (8,1) + (1,8)$

$$(\psi_L)^{l[r_1 r_2]_{\rm AS}} := \varepsilon^{r r_1 r_2} (\psi_L)^l_r \quad \leftrightarrow \quad (\psi_L)^l_r = \frac{1}{2} \varepsilon_{r r_1 r_2} (\psi_L)^{l[r_1 r_2]_{\rm AS}}$$

(b-1)



$$(\bar{\psi}^{\rm mir}_R)_{l_2[r_3 r_2]} (M^\dagger)^{r_3}_{l_1} (M)^{l_2}_{r_1} (\psi_L)^{l_1[r_1 r_2]} \propto \varepsilon_{r r_3 r_2} \varepsilon^{r' r_1 r_2} (\bar{\psi}^{\rm mir}_R)^r_{l_2} (M^\dagger)^{r_3}_{l_1} (M)^{l_2}_{r_1} (\psi_L)^{l_1}_{r'}$$
$$= {\rm tr}(\bar{\psi}^{\rm mir}_R M M^\dagger \psi_L) - {\rm tr}(\bar{\psi}^{\rm mir}_R M) {\rm tr}(M^\dagger \psi_L)$$

mass input

Ξ(1790)
(to satisfy G.O.)

J^P	(D, L_N^P)	S	Octet members				Singlets
$1/2^+$	$(56,0_0^+)$	$1/2$	$N(939)^{4*}$	$\Lambda(1116)^{4*}$	$\Sigma(1193)^{4*}$	$\Xi(1318)^{4*}$	
$1/2^+$	$(56,0_2^+)$	$1/2$	$N(1440)^{4*}$	$\Lambda(1600)^{4*}$	$\Sigma(1660)^{***}$	~~$\Xi(1690)^{\dagger***}$~~ $J^P=?$	
$1/2^-$	$(70,1_1^-)$	$1/2$	$N(1535)^{4*}$	$\Lambda(1670)^{4*}$	~~$\Sigma(1620)^{*}$~~ ~~$\Sigma(1560)^{\dagger}$~~	~~$\Xi(?)$~~	$\Lambda(1405)^{4*}$
$3/2^-$	$(70,1_1^-)$	$1/2$	~~$N(1520)$~~	~~$\Lambda(1690)$~~	~~$\Sigma(1670)$~~	~~$\Xi(1820)$~~	~~$\Lambda(1520)$~~
$1/2^-$	$(70,1_1^-)$	$3/2$	$N(1650)^{4*}$	$\Lambda(1800)^{***}$	$\Sigma(1750)^{***}$ $\Sigma(1620)^{\dagger}$	~~$\Xi(?)$~~	

Gell-Mann—Okubo mass formula

$$\frac{m[N]+m[\Xi]}{2} = \frac{3m[\Lambda]+m[\Sigma]}{4}$$

Σ(2203.67)
or Σ(1909)
(tr)

Ξ(1925)(G.O.)

Ξ(1989)
(automatically
determined by
the value of trace)

mass matrix

$$\langle M \rangle = \begin{pmatrix} \alpha & & \\ & \beta & \\ & & \gamma \end{pmatrix} \sim \begin{pmatrix} 93\,\mathrm{MeV} & & \\ & 93\,\mathrm{MeV} & \\ & & 127\,\mathrm{MeV} \end{pmatrix} \qquad \begin{matrix} \alpha = \beta \sim \sigma \sim \langle \bar{u}u + \bar{d}d \rangle \\ \gamma \sim \sigma_s \sim \langle \bar{s}s \rangle \end{matrix}$$

$$\begin{pmatrix} g_1^a \sigma^\Sigma & g_1^s \sigma^\Xi & m_0 + g^{\psi\Sigma}(\sigma^\Sigma)^2/f_\pi + g^{\psi\Xi}(\sigma^\Xi)^2/f_\pi & g_1^d 2\sigma\sigma^{\Sigma\Xi}/f_\pi \\ & 0 & g_2^d 2\sigma\sigma^{\Sigma\Xi}/f_\pi & m_0 + g^{\chi\Sigma}(\sigma^\Sigma)^2/f_\pi + g^{\chi\Xi}(\sigma^\Xi)^2/f_\pi \\ & & g_2^a \sigma^\Sigma & g_2^s \sigma^\Xi \\ & & & 0 \end{pmatrix}$$

$$\sigma^\Sigma = \begin{cases} \alpha \text{ for } N \\ \gamma \text{ for } \Sigma \\ \alpha \text{ for } \Xi \end{cases} \qquad \sigma^\Xi = \begin{cases} \alpha \text{ for } N \\ \alpha \text{ for } \Sigma \\ \gamma \text{ for } \Xi \end{cases}$$