エネルギー運動量テンソルを用いたキンクの力学的構造の量子論的解析

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エネルギー運動量テンソル (EMT)

- $T_{\mu\nu}$ ：並進対称性に付随したネーターカレント

エネルギー密度　運動量密度

$$T_{\mu\nu} = \begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{01} & T_{11} & T_{12} & T_{13} \\ T_{02} & T_{21} & T_{22} & T_{23} \\ T_{03} & T_{23} & T_{32} & T_{33} \end{pmatrix}$$

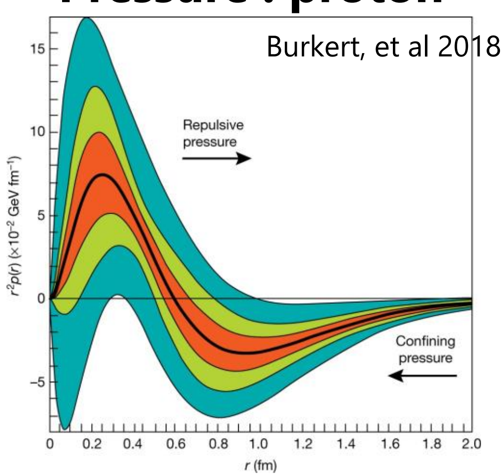
応力

EMTの良いところ

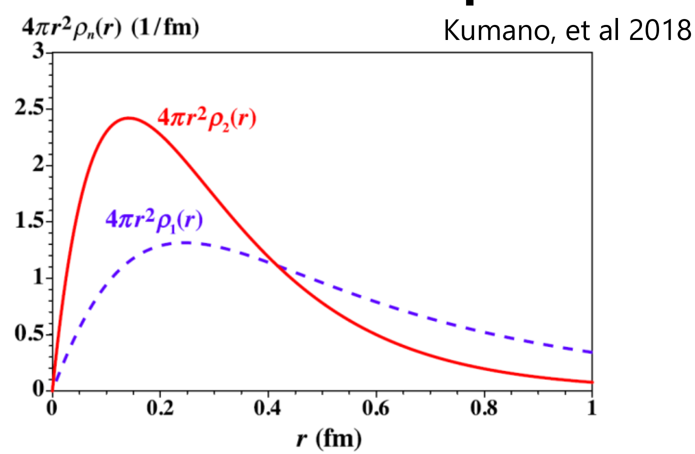
- ゲージ不変・保存量
- 物理的意味が明瞭
- 近接作用なので直観的

- ハドロン内部のEMT分布の実験的測定

Pressure : proton



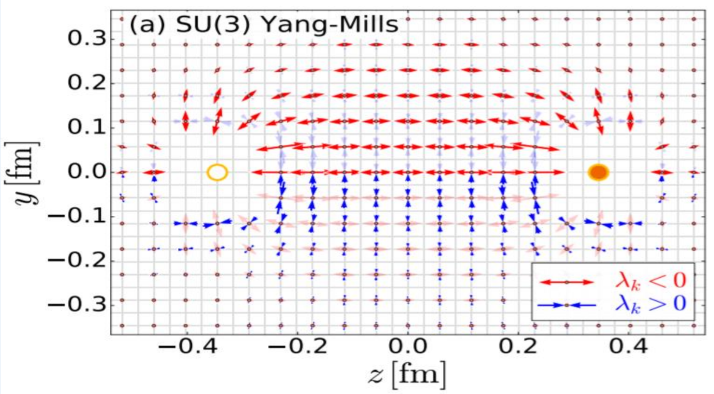
EMT distribution : pion



Quantum fluctuation of EMT

- **Distribution of EMT** in $Q\bar{Q}$

Stress distribution around $Q\bar{Q}$ on Lattice QCD



Yanagihara, et al. 2019

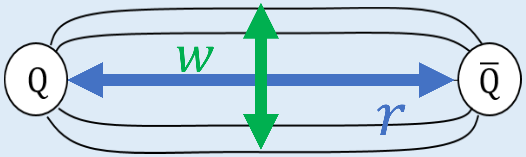
Pulling force (parallel to field)

Pushing force (perpendicular to field)

Describing stress

Viewing fluxtube

- **Quantum effect** in $Q\bar{Q}$



Lüscher, Münster, and Weisz 1981

Theoretical model

Fattening due to string vibration

$$w^2 \sim \log r$$

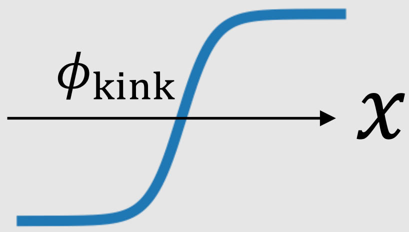
Motivate theoretical analysis of quantum correction to EMT distribution in $Q\bar{Q}$ system

キンクの量子振動

目的

1+1次元ソリトン近傍のEMT分布の量子補正を解析的に計算する。

1+1次元ϕ^4模型のキンク



- □ 場の方程式の安定な古典解
- □ 局所的で非自明な構造

Classical EMT: Trivial

Quantum EMT: Nontrivial, **First trial**

今日話すこと

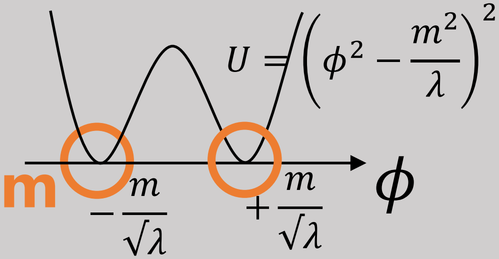
1 Introduction

2 ϕ^4模型のEMT

3 Sine-Gordon模型のEMT

ϕ^4 Model in 1+1 d

$$S = \int dx^2 \left\{ \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{\lambda}{4}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2 \right\}$$

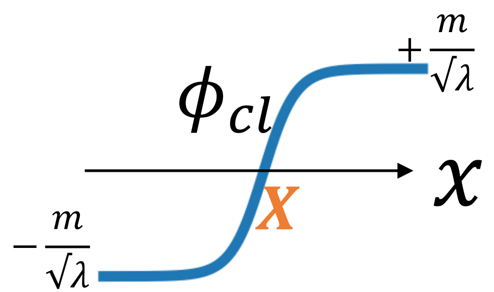


Classical

- Kink solution of EOM

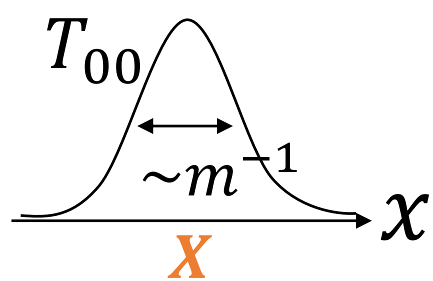
$$\phi_{cl} = +\frac{m}{\sqrt{\lambda}} \tanh\left(\frac{m(x-X)}{\sqrt{2}}\right)$$

X: location of kink



- T_{00} under kink solution

$$T_{00} = \frac{m^4}{\lambda}\cosh^{-4}\left(\frac{m(x-X)}{\sqrt{2}}\right)$$



1-loop $E_{LO} \sim O(\lambda^0)$ $T_{00}^{LO} \sim O(\lambda^0)$

Dashen, et al 1974

Goldhaber, et al. 2003 ?

We calculate $T_{00}^{LO}(x)$ and $T_{11}^{LO}(x)$ in $O(\lambda^0)$!
(Comparison with previous studies ➡ last slide)

Eigenmode of quantum fluctuation

Dashen, Hasslacher and Neveu 1974

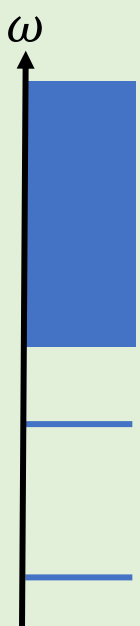
Expanding ϕ^4 action $S[\phi]$ around a kink

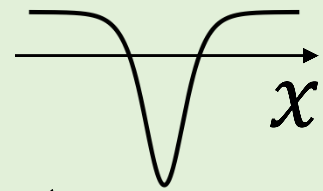
Substituting $\phi(x) = \phi_{cl} + \eta(x)$

η:quantum fluctuation

$$S[\eta] = S_{cl} + \int dx^2 [\frac{1}{2}(\partial_0 \eta)^2 - \frac{1}{2}\eta\left(-\frac{\partial^2}{\partial x^2} - m^2 + \frac{3\phi_{cl}^2}{\lambda}\right)\eta - \lambda\phi_{cl}\eta^3 - \frac{\lambda}{4}\eta^4]$$

$$\left(-\frac{\partial^2}{\partial x^2} - m^2 + \frac{3\phi_{cl}^2}{\lambda}\right)\eta_n = \omega_n^2\, \eta_n$$





Eigenvalues

$\omega_q^2 = q^2 + 2m^2$

$\omega_1^2 = \frac{3}{2}m^2$

$\boldsymbol{\omega_0^2 = 0}$

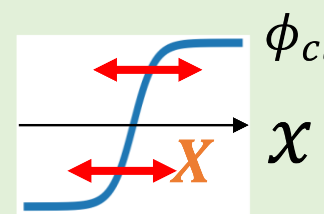
Eigenfunctions

$\eta_q(x) \xrightarrow{x\to\pm\infty} \exp[i\left(qx \pm \frac{1}{2}\delta(q)\right)]$

Phase shift

$\eta_1(x)$

$\boldsymbol{\eta_0(x) = \partial_x \phi_{cl}}$



Translational mode➡IR divergence

集団座標法

ラグランジアンの書き換え

$$X \to X(t)$$

$$L[\pi, \phi] \to L[\tilde{\pi}, \tilde{\eta}, X, P]$$

$\tilde{\pi}, \tilde{\eta}$:without translational mode

||

場の並進モードの自由度 →ソリトンの重心Xの運動の自由度

〇
- 並進対称性
- ローレンツ対称性

Gervais, Jevicki, Sakita 1975
Goldstone and Jackiw 1975
Tomboulis 1975

計算する物理量

$$\langle T^{\mu}{}_{\nu} \rangle \equiv \frac{1}{Z} \int \mathcal{D}\tilde{\pi}\mathcal{D}\tilde{\eta}\; T^{\mu}{}_{\nu}[\tilde{\pi}, \tilde{\eta}]\; e^{-i\int dt\, L[\tilde{\pi},\tilde{\eta}]}$$

in $O(\lambda^0)$

Expectation value of EMT distribution

Diagram

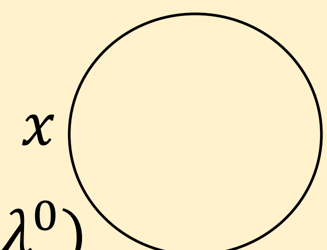
$$\langle T_{00}\rangle = T_{00}(\phi_{cl}) + \frac{1}{2}\langle(\partial_0\eta)^2\rangle + \frac{1}{2}\langle(\partial_1\eta)^2\rangle + \underbrace{(\partial_1\phi_{cl})}_{O(\lambda^{-\frac{1}{2}})}\langle\partial_1\eta\rangle + \underbrace{\lambda\phi_{cl}\left(\phi_{cl}^2 - \frac{m^2}{\lambda}\right)}_{O(\lambda^{-\frac{1}{2}})}\langle\eta\rangle$$

$$+ \underbrace{\frac{\lambda}{2}\left(3\phi_{cl}^2 - \frac{m^2}{\lambda}\right)}_{O(\lambda^0)}\langle\eta^2\rangle + O(\lambda^1)$$

$$\langle\eta^2\rangle = \langle\eta(x)\eta(x)\rangle$$

$$= G(x,x)$$

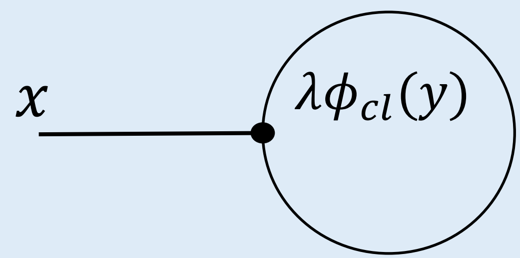
$$=$$



$$\sim O(\lambda^0)$$

UV divergent!

$$\langle\eta\rangle =$$



$$= \int dy\, \lambda\phi_{cl}(y) G(x,y) G(y,y)$$

$$\sim O(\lambda^{\frac{1}{2}})$$

UV divergent!

EMTの正則化

全エネルギーの正則化

Dashen, et al 1974

$$E_{\mathrm{phys}} = E_{\mathrm{soliton}} - E_{\mathrm{vac}} + (\delta E_{\mathrm{soliton}} - \delta E_{\mathrm{vac}})$$

Counter terms

- Vacuum subtraction
- Mass renormalization

EMTの正則化

$$\langle T_{\mu\nu}\rangle_{\mathrm{phys}} = \langle T_{\mu\nu}\rangle_{\mathrm{soliton}} - \langle T_{\mu\nu}\rangle_{\mathrm{vac}} + (\langle \delta T_{\mu\nu}\rangle_{\mathrm{soliton}} - \langle \delta T_{\mu\nu}\rangle_{\mathrm{vac}})$$

Vacuum subtraction in ϕ^4 model

Rebhan and Nieuwenhuizen 1997

$$\langle T_{\mu\nu}\rangle_{\text{soliton}} - \langle T_{\mu\nu}\rangle_{\text{vac}} + \left(\langle \delta T_{\mu\nu}\rangle_{\text{soliton}} - \langle \delta T_{\mu\nu}\rangle_{\text{vac}}\right) = \text{finite}$$

Mode number cutoff (MNC)

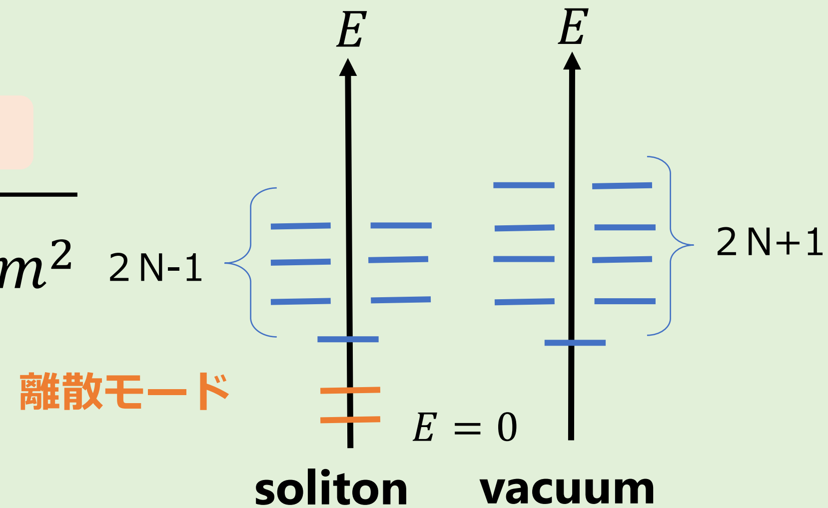
✓ 有限サイズ系、離散運動量のもとで計算

✓ カットオフ数を有限に取って(有限) – (有限)に

-**ソリトンと真空の総モード数が同じになるように カットオフ数を決める**

Soliton continuous mode　　vacuum

$$\sum_{-(N-1)}^{N-1} \sqrt{q_n^2 + 2m^2} - \sum_{n=-N}^{N} \sqrt{k_n^2 + 2m^2}$$



反周期境界でのモードの様子

✓ 引き算の後、カットオフ数を無限に

Mass renormalization

Dashen, Hasslacher and Neveu 1974

EMT regularization

Counter terms

$$\langle T_{\mu\nu}\rangle_{\text{soliton}} - \langle T_{\mu\nu}\rangle_{\text{vac}} + \left(\langle \delta T_{\mu\nu}\rangle_{\text{soliton}} - \langle \delta T_{\mu\nu}\rangle_{\text{vac}}\right) = \text{finite}$$

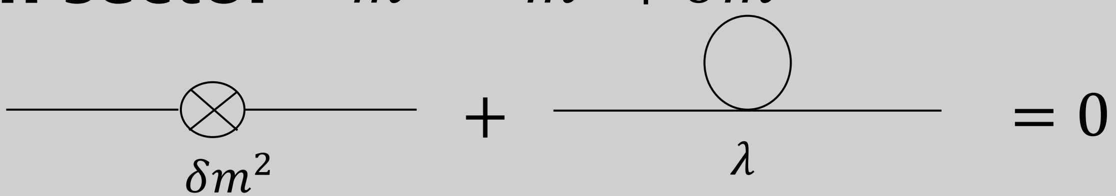
Mass renormalization

✓ Only 1-loop mass renormalization

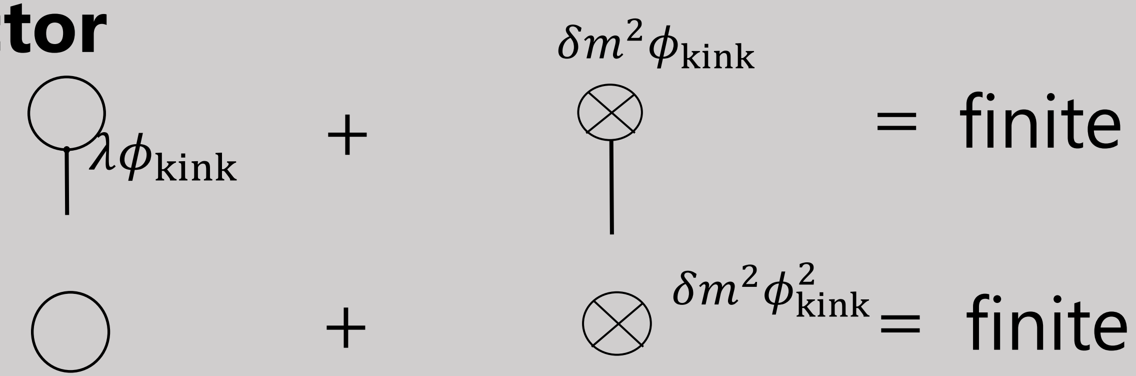
✓ Mass renormalization in **vacuum** sector

➡Counter terms appear in **soliton** sector

Vacuum sector $m^2 \to m^2 + \delta m^2$



Soliton sector

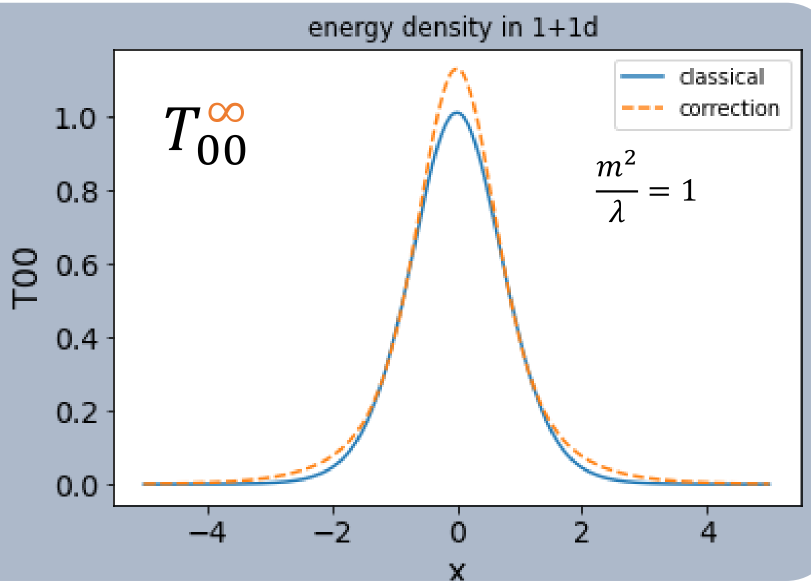


Result in ϕ^4 Model

In finite system whose length is R

$$T_{00}^R(x) = -\frac{3\sqrt{2}m}{2\pi}\frac{1}{R} + T_{00}^{\infty}(x)$$

$$T_{11}^R = \frac{3\sqrt{2}m}{2\pi}\frac{1}{R}$$



- T_{00} distribution

$$\lim_{R\to\infty}\int_{-R/2}^{R/2} dx\, T_{00}^R = E \qquad \times \quad \int_{-\infty}^{\infty} dx \lim_{R\to\infty} T_{00}^R$$

reproduce the E_{MNC} calculated by Dashen et al.

- T_{11} distribution

T_{11}^R is constant ➡ $\partial_0 T^{01} + \partial_1 T^{11} = 0$

EMT conservation law is satisfied

Comparison with Goldhaber et al.

Goldhaber, et al. 2003

Different regularization

Our study: **finite system**

$$T_{00}^{R}(x) = -\frac{3\sqrt{2}m}{2\pi}\frac{1}{R} + T_{00}^{\infty}(x)$$

$$T_{11}^{R} = -\frac{3\sqrt{2}m}{2\pi}\frac{1}{R}$$

Previous study: **infinite system**

$$T_{00}^{Prev}(x) = -\frac{3m^2}{4\pi}\frac{1}{\cosh^2(mx/\sqrt{2})} + T_{00}^{\infty}(x)$$

$$T_{11}^{Prev}(x) = -\frac{3m^2}{4\pi}\frac{1}{\cosh^2(mx/\sqrt{2})}$$

- Different result for **"finite"** and **"infinite"** regularizations
- $\int_{-\infty}^{\infty} dx\, T_{00}^{R} = \int_{-\infty}^{\infty} dx\, T_{00}^{Prev} = E_{\mathrm{Dashen+}}$
- $\partial_x T_{11}^{Prev} \neq 0$

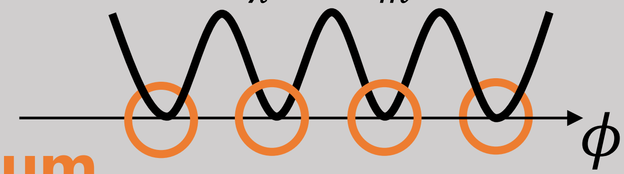
  T_{11}^{Prev} **is inconsistent with EMT conservation law.**

Our result is more reliable.

Sine-Gordon (SG) Model in 1+1 d

$$S = \int dx^2 \left\{ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^4}{\lambda} \left(\cos\left(\frac{\sqrt{\lambda}}{m} \phi \right) - 1 \right) \right\}$$

$$U = \frac{m^4}{\lambda} \left(\cos\left(\frac{\sqrt{\lambda}}{m} \phi \right) - 1 \right)$$

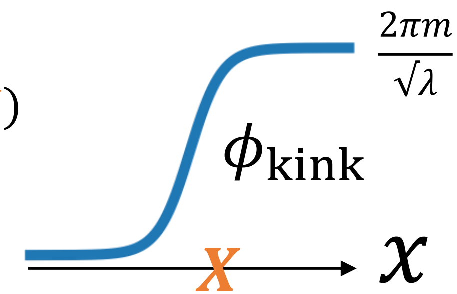


Classical

- Kink solution of EOM

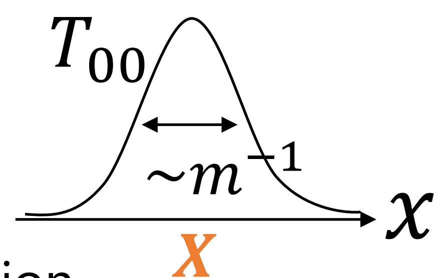
$$\phi_{\text{kink}} = + \frac{4m}{\sqrt{\lambda}} \arctan e^{m(x-X)}$$

X: location of kink



- T_{00} under kink solution

$$T_{00} = \frac{2m^4}{\lambda} \cosh^{-2}(m(x-X))$$



Weak interaction
Perturbation

1-loop $E_{LO} \sim O(\lambda^0)$ $T_{00}^{LO} \sim O(\lambda^0)$

Eigenmode of quantum fluctuation

Dashen, Hasslacher and Neveu 1974

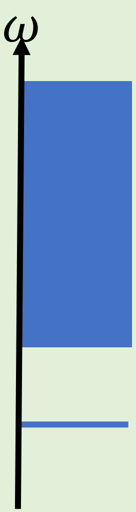
Expanding SG action $S[\phi]$ around a kink

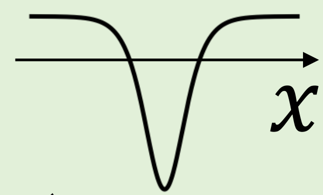
Substituting $\phi(x) = \phi_{\mathrm{kink}} + \eta(x)$

η:quantum fluctuation

$$S[\eta] = S_{cl} + \int dx^2 [\frac{1}{2}(\partial_0 \eta)^2 - \frac{1}{2}\eta \left(-\frac{\partial^2}{\partial x^2} + m^2 \cos(\frac{\sqrt{\lambda}}{m}\phi_{\mathrm{kink}})\right)\eta - \cdots]$$

$$\left(-\frac{\partial^2}{\partial x^2} - m^2 \cos(\frac{\sqrt{\lambda}}{m}\phi_{\mathrm{kink}})\right)\eta_n = \omega_n^2\, \eta_n$$





Eigenvalues

$$\omega_q^2 = q^2 + m^2$$

$$\boldsymbol{\omega_0^2 = 0}$$

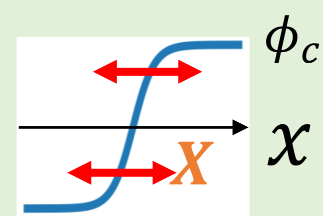
Eigenfunctions

$$\eta_q(x) \xrightarrow{x\to\pm\infty} \exp[i\left(qx \pm \frac{1}{2}\delta(q)\right)]$$

Phase shift

$$\boldsymbol{\eta_0(x) = \partial_x \phi_{cl}}$$

離散モードが一つしか無い！！

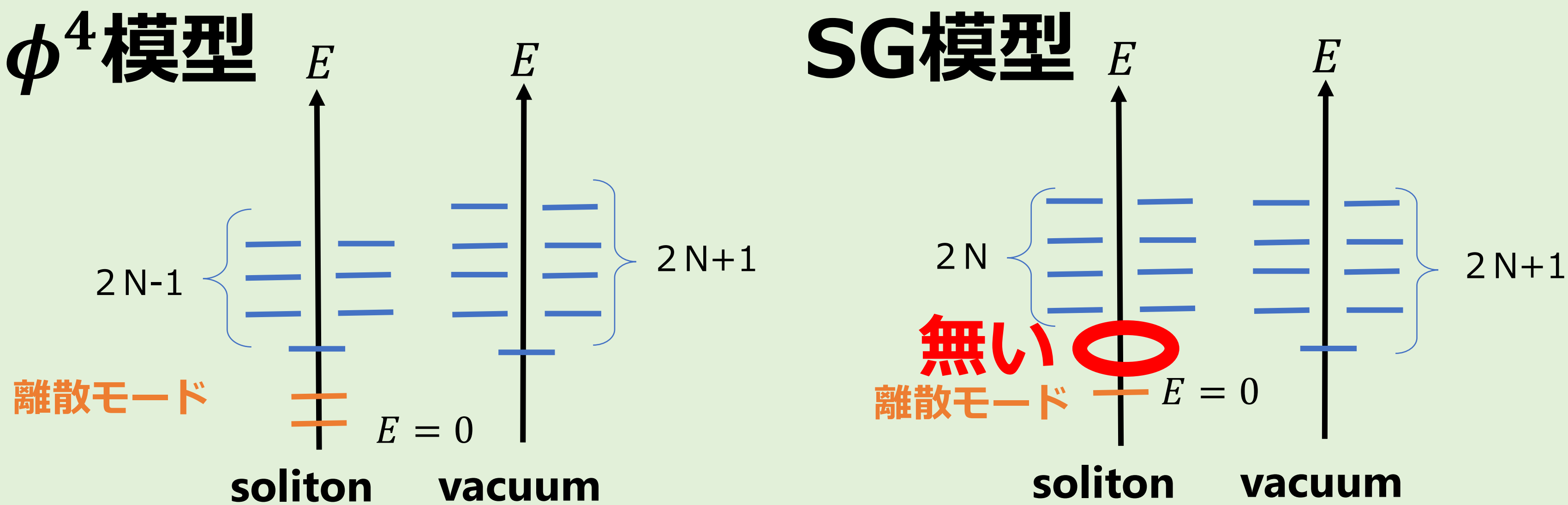


Vacuum subtraction in SG model

Rebhan and Nieuwenhuizen 1997
Dashen, et al 1974

$$\langle T_{\mu\nu}\rangle_{\mathrm{soliton}} - \langle T_{\mu\nu}\rangle_{\mathrm{vac}} + (\langle \delta T_{\mu\nu}\rangle_{\mathrm{soliton}} - \langle \delta T_{\mu\nu}\rangle_{\mathrm{vac}}) = \mathrm{finite}$$

Mode number cutoff (MNC)



反周期境界のモードの様子

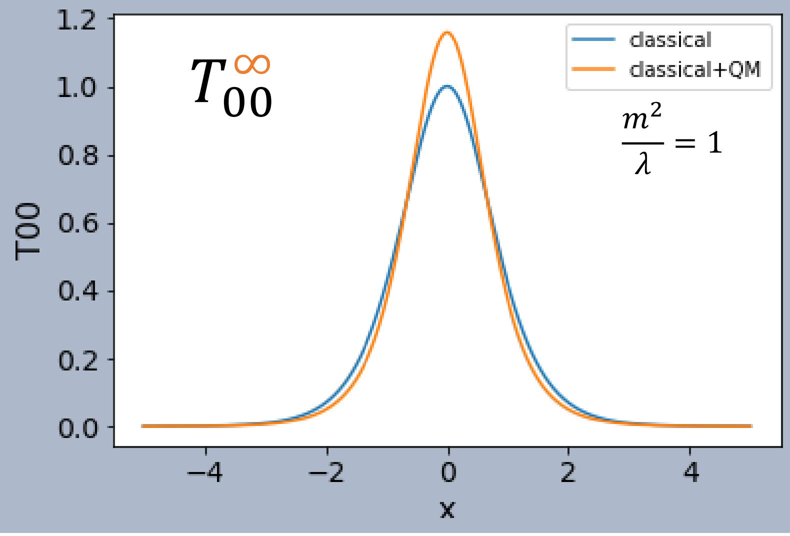
離散モードが一つでもMNCは機能する

Result in SG model

In finite system whose length is R

$$T_{00}^{R}(x) = -\frac{m}{\pi}\frac{1}{R} + T_{00}^{\infty}(x)$$

$$T_{11}^{R} = \frac{m}{\pi}\frac{1}{R}$$



- T_{00} distribution

$$\lim_{R\to\infty}\int_{-R/2}^{R/2} dx\, T_{00}^{R} = E \qquad \times \quad \int_{-\infty}^{\infty} dx \lim_{R\to\infty} T_{00}^{R}$$

reproduce the E_{MNC} calculated by Dashen et al.

- T_{11} distribution

T_{11}^{R} is constant ➡ $\partial_0 T^{01} + \partial_1 T^{11} = 0$

EMT conservation law is satisfied

Summary

1+1d ϕ^4模型・SG模型でEMT分布の1-loop計算

- 集団座標法でIR発散を除去
- Vacuum subtraction(Mode number cutoff)と質量くりこみでUV発散を除去

- 得られたT_{00}の空間積分は既知の全エネルギーを再現
- T_{11}はEMT保存則と無矛盾

Future work:

2+1d ϕ^4model・有限温度・ブリーザー状態