Anomalous Hydrodynamics of Charge Fluctuations

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We study the fluctuating hydrodynamics of a vector charge and an axial charge with a damping rate under an external magnetic field. We find that charge fluctuation leads to a nontrivial magnetic-field dependence of the conductivity beyond the bare anomalous hydrodynamics. We discuss its possible relevance to the longitudinal negative magnetoresistance observed in the table-top experiments as a consequence of the chiral magnetic effect.

Introduction

- Chiral Matter
  - QGP, Electroweak plasma, Weyl semimetals, Core-collapse supernovae
  - Novel transport phenomena (chiral Magnetic effect [1], etc.)
- Fluctuating hydrodynamics
  - Coarse graining description & nonlinearity
    ⟶ Inherent fluctuation effects
  - Possible importance in heavy-ion collisions (small # of particles)

[1] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, PRD ('08)

This study

- Fluctuating hydrodynamics of chiral charges: $n^i = n^{\rm V}, n^{\rm A}$
  - External fields: $\langle \mu^i \rangle = \langle \mu^{\rm V} \rangle, \langle \mu^{\rm A} \rangle$ and $\boldsymbol{B}_{\rm ex}$
  - Relaxation time for $n^{\rm A}$: τ
- Finite hydrodynamic loop corrections on the (static) conductivity

$\delta\sigma \propto -\boldsymbol{B}_{\rm ex}^2\sqrt{\tau^{-1}}$ At finite $\langle \mu^{\rm V} \rangle$ C.f., Long-time tail [2,3]

$\delta\sigma \propto |\boldsymbol{B}_{\rm ex}|^3 \ (\tau \to \infty)$ At finite $\langle \mu^{\rm A} \rangle$ $\sigma(\omega) \sim \omega^{\frac{1}{2}}$

[2] Y. Pomeau and P. Resibois, Physics Reports (1975),
[3] P. Kovtun, J. Phys. ('12)

Setup

- Stochastic equation of motion:

$$\partial_t n^{\rm V} = -\nabla\cdot\boldsymbol{j}^{\rm V} + \xi^{\rm V}$$
$$\partial_t n^{\rm A} = -\nabla\cdot\boldsymbol{j}^{\rm A} - \frac{n^{\rm A}}{\tau} + \xi^{\rm A}$$

- Currents:

$$\boldsymbol{j}^{\rm V} = C\mu^{\rm A}\boldsymbol{B}_{\rm ex} - \sigma_{\rm VV}\nabla\mu^{\rm V}$$
$$\boldsymbol{j}^{\rm A} = C\mu^{\rm V}\boldsymbol{B}_{\rm ex} - \sigma_{\rm AA}\nabla\mu^{\rm A}$$

- Fluctuations:

$$\mu^i = \frac{1}{\chi}\left(n^i - \frac{1}{2\chi}\chi_{i,jk}n^j n^k + \mathcal{O}(n^3)\right) \qquad \left(\chi_{i,jk} \equiv \frac{1}{\chi}\frac{\partial n^i}{\partial\mu^j\partial\mu^k}\right)$$

$$\sigma_{ij} = \sigma\left(\delta_{ij} + \sigma_{i,jk}\mu^k + \mathcal{O}(\mu^2)\right)$$

Beyond Gaussian noises

Effective Field Theory Approach

- Non-equilibrium low-energy effective theory [4]
- Symmetries:
  - $\mathrm{U_V}(1)\times\mathrm{U_A}(1)$
  - Dynamical KMS symmetry [4,5]
  - CPT
- Gaussian terms (without anomaly)

$$\mathcal{L}_2 = \chi_{i,j}\mathcal{A}^i_{\rm a0}\mathcal{A}^j_{\rm r0} - \frac{\sigma_{i,j}}{2}\left[\boldsymbol{\mathcal{A}}^i_{\rm a}\cdot\partial_t\boldsymbol{\mathcal{A}}^j_{\rm r} - {\rm i}T\boldsymbol{\mathcal{A}}^i_{\rm a}\cdot\boldsymbol{\mathcal{A}}^j_{\rm a} + (i\leftrightarrow j)\right]$$

$$\mathcal{A}^i_{\sigma\mu} = A^i_{\sigma\mu} + \partial_\mu\psi^i_\sigma \qquad i = \mathrm{V,A}, \quad \sigma = \mathrm{r,a}$$
$$\mu^i = \mathcal{A}^i_{\rm r0}$$

Dynamical KMS symmetry (classical):

$$\mathcal{A}^i_{\rm r\mu} \to \Theta\mathcal{A}^i_{\rm r\mu},$$
$$\mathcal{A}^i_{\rm a\mu} \to \Theta\mathcal{A}^i_{\rm a\mu} + {\rm i}\beta\Theta\partial_t\mathcal{A}^i_{\rm r\mu}$$

Off-shell current: $J^\mu_{ri} \equiv \dfrac{\delta\mathcal{L}}{\delta A^i_{\rm a\mu}}$ (μ=0: conserved charge density)

[4] H. Liu and P. Glorioso, PoS TASI2017 ('18) [5] Y. Hidaka, (Lecture notes)

- Feyman rules:

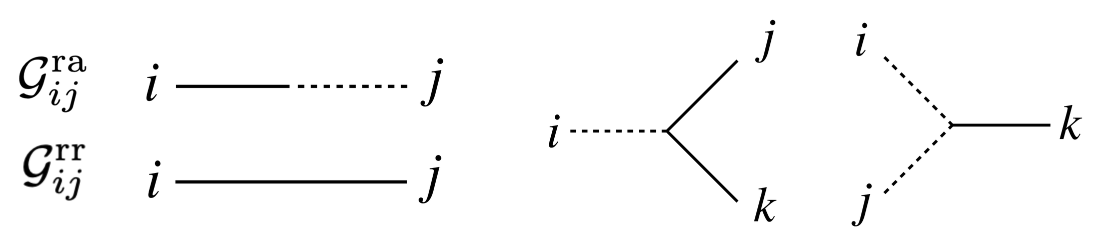
$$\mathcal{G}^{\rm ra}_{ij}(k) \equiv \langle n^i\psi^j_{\rm a}\rangle \qquad \mathcal{G}^{\rm rr}_{ij} \equiv \langle n^i n^j\rangle = \mathcal{G}^{\rm ra}_{ik}\Gamma_{kl}(\mathcal{G}^{\rm ra})^\dagger_{lj}$$

$$= i\begin{pmatrix} i\omega - D\boldsymbol{k}^2 & -iC(\boldsymbol{B}_{\rm ex}\cdot\boldsymbol{k})/\chi \\ -i - iC(\boldsymbol{B}_{\rm ex}\cdot\boldsymbol{k})/\chi & i\omega - \tau^{-1} + D\boldsymbol{k}^2\end{pmatrix}^{-1} \qquad \Gamma_{ij} \equiv \begin{pmatrix} 2\sigma T\boldsymbol{k}^2 & 0 \\ 0 & 2\chi\tau^{-1} + 2\sigma T\boldsymbol{k}^2\end{pmatrix}$$

$$D \equiv \frac{\sigma}{\chi}$$

$$\mathcal{L}_{\rm int} = \psi^i_{\rm a}(\nabla\cdot\delta\boldsymbol{j}^i_{\rm r}),$$

$$\delta\boldsymbol{j}^{\rm V}_{\rm r} = -\frac{C\boldsymbol{B}_{\rm ex}}{2\chi^2}\chi_{{\rm A},jk}n^j n^k - \frac{D}{\chi}(\sigma_{{\rm V},jk} - \chi_{{\rm V},jk})(\nabla n^j)n^k + {\rm i}DT\sigma_{{\rm V},jk}\nabla(\psi^j_{\rm a})n^k$$

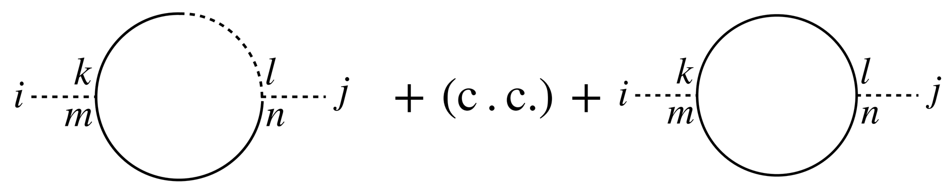


⊃ CME vertex, Diffusive vertex (Relaxation vertex)

- Kubo Formular and loop correction:

$$\sigma_{\rm full} = \sigma + \frac{C^2\boldsymbol{B}_{\rm ex}^2\tau}{\chi} - \beta\frac{\partial^2\Sigma^{\rm aa}_{\rm VV}}{\partial k^2} + \text{(renormalization on } \tau \text{ and } \chi\text{)}$$

$\Sigma^{\rm aa}_{ij}(\omega,\boldsymbol{k}) =$



Results

$$-\beta\frac{\partial^2\Sigma^{\rm aa}_{\rm VV}}{\partial k^2} = \delta\sigma_1(\omega) + \delta\sigma_2(\omega) + \cdots$$

(i) Conductivity at finite τ and long-time tail in $\tau\to\infty$

$$\delta\sigma_1(\omega) = -c_{\rm V}^2\frac{2C^2\boldsymbol{B}_{\rm ex}^2T\sqrt{\tau^{-1} + \sqrt{\tau^{-2} + \omega^2}}}{\pi D^{\frac{3}{2}}} \qquad (c_{\rm V} = \chi_{\rm A,VA} = \chi_{\rm A,AV})$$

Heuristic derivation:

$$\begin{aligned}\delta\sigma^{ij}(t)T &= \int {\rm d}^d\boldsymbol{x}\frac{1}{2}\left\langle\left\{\delta j^i_{\rm V}(t,\boldsymbol{x}), \delta j^j_{\rm V}(0,\boldsymbol{0})\right\}\right\rangle \\ &= \frac{C^2c_{\rm V}^2}{\chi^4}\int {\rm d}^d\boldsymbol{x}\frac{1}{2}\left\langle\left\{n_{\rm V}(t,\boldsymbol{x})n_{\rm A}(t,\boldsymbol{x}), n_{\rm V}(0,\boldsymbol{0})n_{\rm A}(0,\boldsymbol{0})\right\}\right\rangle B^i_{\rm ex}B^j_{\rm ex} \\ &= \frac{C^2c_{\rm V}^2}{\chi^4}\int\frac{{\rm d}^d\boldsymbol{k}}{(2\pi)^d}\mathcal{G}^{\rm rr}_{\rm VV}(t,\boldsymbol{k})\mathcal{G}^{\rm rr}_{\rm AA}(t,\boldsymbol{k})B^i_{\rm ex}B^j_{\rm ex} \\ &= \frac{C^2T^2c_{\rm V}^2}{8\sqrt{2}\chi^2}\frac{e^{-\tau^{-1}|t|}}{(\pi D|t|)^{3/2}}B^i_{\rm ex}B^j_{\rm ex}\end{aligned}$$

(= Fourier transform of $\delta\sigma_1(\omega)$)

- Long-time tail at $\tau\to\infty$ C.f., $\delta j^i = nv^i$ (see [6])

[6] P. Kovtun and L. G. Yaffe, PRD ('03)

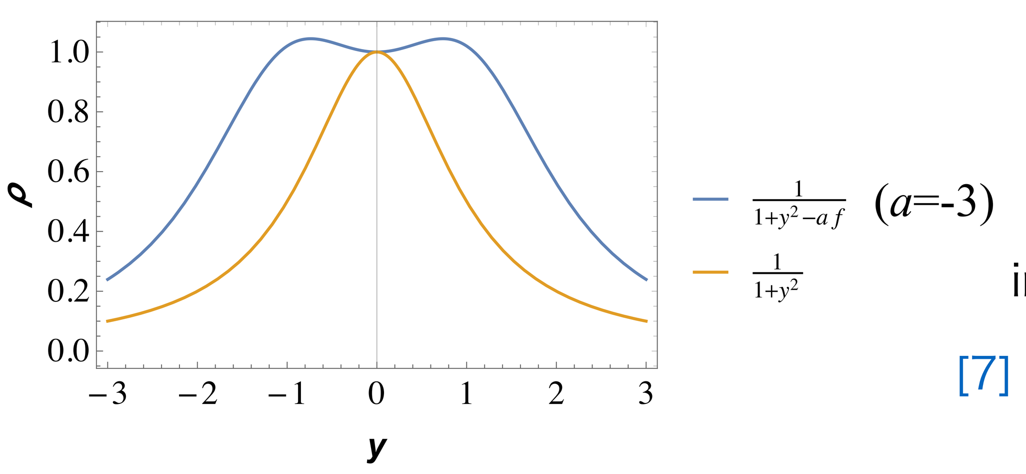
(ii) Finite conductivity at $\tau\to\infty$

$$\delta\sigma_2(0) = -(g_{\rm A2} - g_{\rm A3})^2\frac{5TC^3B_{\rm ex}^3}{48\pi\chi^3D^2} \qquad \left(g_{\rm A2} - g_{\rm A3} = \frac{\sigma_{\rm V,AV} + \sigma_{\rm A,VV}}{2} - \sigma_{\rm V,VA}\right)$$

(iii) Nontrivial magnetic-field dependence

$$\delta\sigma_3(0) = c_{\rm A}(g_{\rm A2} - g_{\rm A3})\frac{9T}{32\pi\sqrt{D}\tau^{3/2}}f(y) \qquad (c_{\rm A} = \chi_{\rm A,VV})$$

$$f(y) = \sqrt{2} - \sqrt{y^2+1} + \frac{1}{y}\tan^{-1}\left(\frac{\sqrt{2}y - y\sqrt{y^2+1}}{y^2-1}\right) \qquad \left(y^2 \equiv \frac{C^2\boldsymbol{B}_{\rm ex}^2\tau}{\sigma\chi}\right)$$



Relevance to negative magnetoresistivity observed in Table-top experiments (e.g., [7])?

[7] Qiang Li, et.al, Nature Phys. (2016)