熱場の量子論とその応用 2020年8月24日～26日

双対超伝導描像に基づく高次元クォークの閉じ込め・非閉じ込め相転移

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❑ Introduction

Dual superconductivity is a promising mechanism for quark confinement. [Y.Nambu (1974). G.'t Hooft, (1975). S.Mandelstam, (1976) A.M. Polyakov (1975)]

In this scenario, the QCD vacuum is considered as a dual super conductor.

superconductor	dual superconductor
➢Condensation of electric charges (Cooper pairs)	➢Condensation of magnetic monopoles
➢Meissner effect: Abrikosov string (magnetic flux tube) connecting monopole and anti-monopole	➢Dual Meissner effect: formation of a hadron string (chromo-electric flux tube) connecting quark and antiquark
➢Linear potential between monopoles	➢Linear potential between quarks

➢To establish dual superconductivity, we must show evidences in various situations.

➢ There exist many preceding studies for the fundamental representations at zero temperature by using Abelian projection in the MA gauge

- Abelian dominance in the string tension.
  SU(2) Suzuki-Yotsuyanagi (1990); Stack-Tucker-Wensley (2002) ;
  SU(3) Shiba-Suzuki (1994); Sakumichi-Suganuma (2016) (perfect dominance);
- Magnetic monopole (DeGrant-Toussaint) dominance in the string tension.
  SU(2), SU(3) Stack-Tucker-Wensley (2002)

➔Problem:
Color (global) symmetry and gauge symmetry is broken.

❑A new formulation of Yang-Mills theory (on a lattice) for fundamental representation

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For SU(N) YM gauge Decomposition of SU(N) gauge links

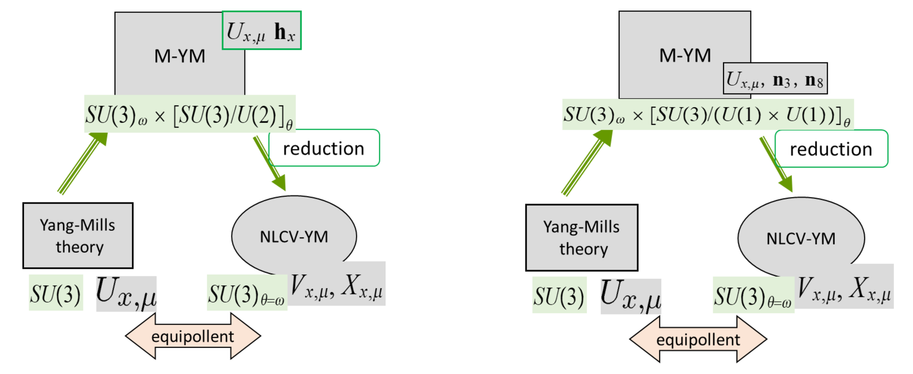
link, there are several possible options of decomposition *discriminated by its stability groups*:

$$W_C[U] := \mathrm{Tr}\left[P \prod_{\langle x,x+\mu\rangle\in C} U_{x,\mu}\right]/\mathrm{Tr}(\mathbf{1}) \qquad U_{x,\mu} = X_{x,\mu}V_{x,\mu}$$

$$\begin{aligned} U_{x,\mu} &\to U'_{x,\mu} = \Omega_x U_{x,\mu}\Omega^\dagger_{x+\mu} \\ V_{x,\mu} &\to V'_{x,\mu} = \Omega_x V_{x,\mu}\Omega^\dagger_{x+\mu} \\ X_{x,\mu} &\to X'_{x,\mu} = \Omega_x X_{x,\mu}\Omega^\dagger_{x} \end{aligned} \qquad \Omega_x \in G = SU(N)$$

$$W_C[V] := \mathrm{Tr}\left[P \prod_{\langle x,x+\mu\rangle\in C} V_{x,\mu}\right]/\mathrm{Tr}(\mathbf{1})$$

- SU(2) Yang-Mills link variables: unique U(1)⊂SU(2)
- SU(3) Yang-Mills link variables: **Two options**
  minimal option : U(2)≅SU(2)×U(1)⊂SU(3)
  ✓Minimal case is derived for the Wilson loop, defined for quark in the fundamental representation, which follows from the non-Abelian Stokes' theorem
  maximal option : U(1)×U(1)⊂SU(3)
  ✓Maximal case is a gauge invariant version of Abelian projection in the maximal Abelian (MA) gauge. (the maximal torus group)



- Defining equation : maximal option

By introducing color fields $\mathbf{n}_x^{(3)} = \Theta_x(\lambda^3/2)\Theta^\dagger$, $\mathbf{n}_x^{(8)} = \Theta_x(\lambda^8/2)\Theta^\dagger$ $\in SU(3)_\omega \times [SU(3)/(U(1)\times U(1))]_\theta$, a set of the defining equation for the decomposition $U_{x,\mu} = X_{x,\mu}V_{x,\mu}$ is given by

$$D^\varepsilon_\mu[V]n_x^{(k)} = \frac{1}{\varepsilon}(V_{x,\mu}n^{(k)}_{x+\mu} - n^{(k)}_x V_{x,\mu}) = 0, \ (k = 3,8)$$

$$g_x = \exp(2\pi i n/N)\exp(i\sum_{j=3,8} a^{(j)}n_x^{(j)}) = 1$$

Coressponding to the continuum version of the dexomposition $\mathcal{A}_\mu(x) = V_\mu(x) + \mathcal{X}_\mu(x)$

$$D_\mu[V_\mu]\mathbf{n}^{(k)}(x) = 0, \quad tr(\mathbf{n}^{(k)}(x)\mathcal{X}_\mu(x)) = 0, \ (k = 3,8)$$

- Decomposed fields

$$X_{x,\mu} = \hat{K}^\dagger_{x,\mu}\det(K_{x,\mu})^{1/3}g_x^{-1}, \quad V_{x,\mu} = g_x\hat{K}_{x,\mu}\det(K_{x,\mu})^{-1/3}$$

where

$$\hat{K}_{x,\mu} := \left(\sqrt{K_{x,\mu}K^\dagger_{x,\mu}}\right)^{-1}K_{x,\mu}, \quad \hat{K}^\dagger_{x,\mu} = K^\dagger_{x,\mu}\left(\sqrt{K_{x,\mu}K^\dagger_{x,\mu}}\right)^{-1}$$

$$K_{x,\mu} = 1 + 6\mathbf{n}_x^{(3)}U_{x,\mu}\mathbf{n}^{(3)}_{x+\mu}U^\dagger_{x,\mu} + 6\mathbf{n}_x^{(8)}U_{x,\mu}\mathbf{n}^{(8)}_{x+\mu}U^\dagger_{x,\mu}$$

❑Non-Abelian Stokes theorem

Kondo and Matsudo RRD92 125083 (2015)

➢Non-Abelian theorem in the presentation R is given by

$$W_C[A] = \int[d\mu(g)]_C \exp\left(ig\oint\langle\Lambda|A^U|\Lambda\rangle\right) = \int[d\mu(g)]_\Sigma \exp\left(ig\int_{\Sigma:\partial\Sigma=C} d(\langle\Lambda|A^U|\Lambda\rangle)\right)$$

where $[d\mu(g)]_C$ and $[d\mu(g)]_\Sigma$ are the product of the Haar measure over the loop and a surface, respectively. $A^{U^\dagger} := UAU^\dagger + ig^{-1}UdU$, and $|\Lambda\rangle$ the highest weight state of the representation R.

Area low

$$W_C[\mathscr{A}] = \int[d\mu(g)]_\Sigma \exp\left[-ig_{\mathrm{YM}}\int_{\Sigma:\partial\Sigma=C} F^g\right] \qquad F^g := \frac{1}{2}f^g_{\mu\nu}(x)dx^\mu\wedge dx^\nu$$

$$F^g_{\mu\nu}(x) = \Lambda_j\{\partial_\mu[n^A_j(x)\mathscr{A}^A_\nu(x)] - \partial_\nu[n^A_j(x)\mathscr{A}^A_\mu(x)] - g^{-1}_{\mathrm{YM}}f^{ABC}n^A_j(x)\partial_\mu n^B_k(x)\partial_\nu n^C_k(x)\},$$

$$\boldsymbol{n}_j(x) = g(x)H_jg^\dagger(x) = n^A_j(x)T_A(j = 1,\dots,r).$$

❑**Wilson loop operator** in the representation R

Matsudo, Shibata, Kato, and Kondo, PRD 100, 014505 (2019)

Written by using decomposed gauge field V (for fundamental representation) and color fields, $\mathbf{n}^{(3)}$, $\mathbf{n}^{(8)}$.

➢ SU(2)

$$W^{(j)}[U](C) = \frac{1}{2j+1}\sum_{n=\lfloor j\rfloor}^{|j|}\mathrm{tr}\left(U_C^{2j-2n)}\right) \qquad U_C := \prod_{<x.\mu>\in C} U_{x,\mu}$$

$$W^{(j)}[V](C) = \frac{1}{2}\mathrm{tr}\left(V_C^{2j}\right) \qquad V_C := \prod_{<x.\mu>\in C} V_{x,\mu}$$

➢SU(3)

$$W_{(m_1,m_2)}[V](C) = \frac{1}{6}\left(\mathrm{tr}(V_c^{m_1})\mathrm{tr}(V_c^{\dagger m_2}) - \mathrm{tr}(V_c^{m_1}V_c^{\dagger m_2})\right)$$

$$V_c := \prod_{<x.\mu>\in C} V_{x,\mu}$$

- Where V-field must be decomposed by using the maximal option.
- Invariant integration measure Dμ[ξ] is dropped by using the reduction condition, **i.e.,** V-field is obtained by using the color field determined from the reduction condition.
- Note that we have arbitrariness in choice of the reduction condition

❑ Lattice data

➢SU(2) case ::

- standard Wilson action 24^4 lattice β=2.5
- hyper-blocking smearing

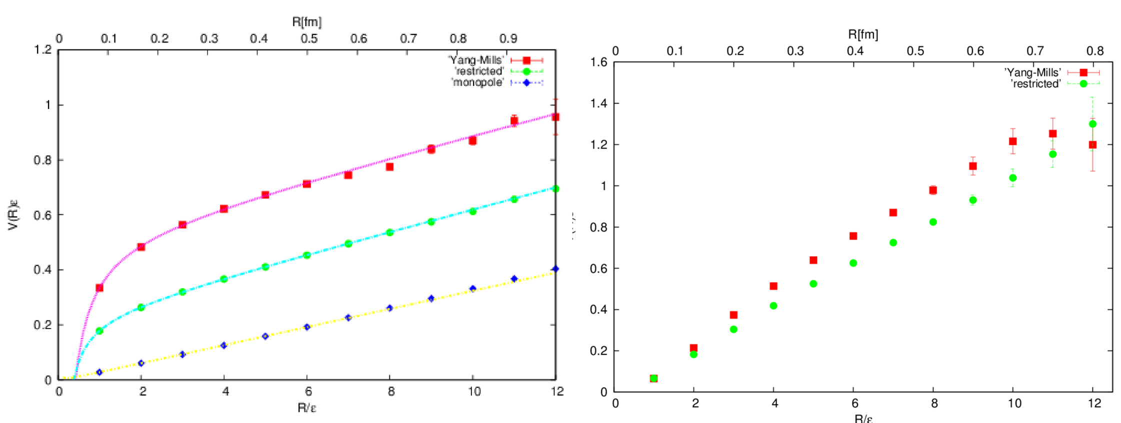
➢Fundamental representation (Left)

➢Adjoint representation (j=1) (Right)

- V dominance for the static potential
- (Right) the string breaking for YM field starts at R = 0.7fm, but not for V (Abelian) field.

$$W^{(j=1)}[U](C) = \frac{1}{3}[\mathrm{tr}(U^2) + 1]$$

$$W^{(j=1)}[V](C) = \frac{1}{2}\int D\mu[\xi]\mathrm{tr}(V^2) \approx \frac{1}{2}\mathrm{tr}(V(\mathbf{n})^2)$$



➢SU(3) case ::

- standard Wilson action 24^4 lattice β=6.2, β=6.0
- APE smearing
- various reduction condition (MA, n3, n8)

❖Reduction condition (determination of color fields)

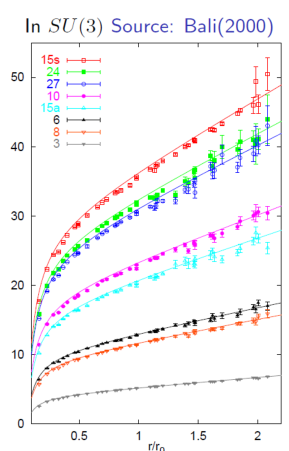
- The decomposition is uniquely determined for a given set of link variables $U_{x,\mu}$ and color fields which is given by minimizing the reduction condition.
- The reduction condition is introduced such that the theory in terms of new variables is equipollent to the original Yang-Mills theory, i.e., defining an effective gauge-Higgs model whose kinetic term is given by the reduction condition

for given $U_{x,\mu}$

$$F[\Theta;U] = \begin{cases} \sum_{x,\mu}\mathrm{tr}\left[\sum_{j=3,8}(D^\epsilon_\mu[U]\mathbf{n}^{(j)})^\dagger(D^\epsilon_\mu[U]\mathbf{n}^{(j)})\right] & \text{represented MA} \\ \sum_{x,\mu}\mathrm{tr}\left[\sum_j(D^\epsilon_\mu[U]\mathbf{n}^{(8)})^\dagger(D^\epsilon_\mu[U]\mathbf{n}^{(8)})\right] & \text{representied n8} \\ \sum_{x,\mu}\mathrm{tr}\left[\sum_j(D^\epsilon_\mu[U]\mathbf{n}^{(3)})^\dagger(D^\epsilon_\mu[U]\mathbf{n}^{(3)})\right] & \text{represented n3} \end{cases}$$

$\mathbf{n}^{(3)}$where $\mathbf{n}_j := \Theta^\dagger H_j\Theta$, H_j Cartan generators, and $D^\epsilon_\mu[U]\mathbf{n}^{(j)} := U_{x,\mu}\mathbf{n}^{(j)}_{x+\mu} - \mathbf{n}^{(j)}_x U_{x,\mu}$

- Wilson Loop operator for higher representation:

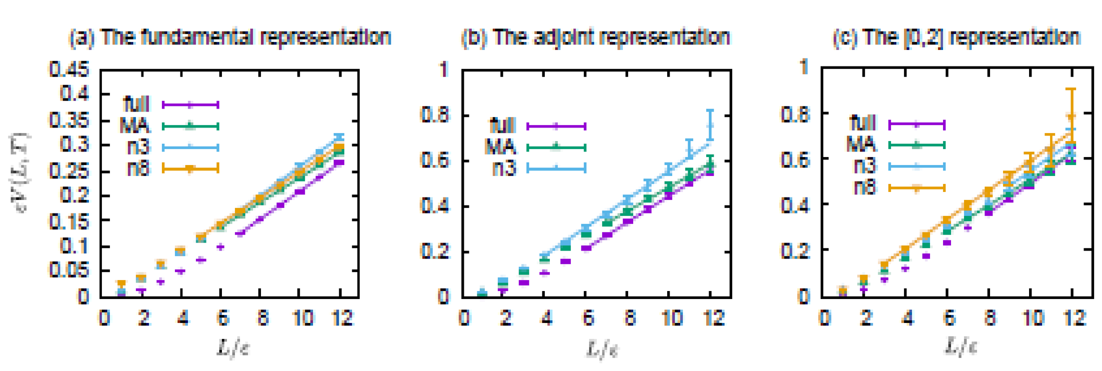


$$U = \prod_{<x,\mu>\in C} U_{x,\mu}$$

$$\begin{aligned} \mathrm{tr}(U_{[1,0]}) &= \mathrm{tr}(U) \\ \mathrm{tr}(U_{[1,1]}) &= |\mathrm{tr}(U)|^2 - 1 \\ \mathrm{tr}(U_{[2,0]}) &= \frac{1}{2}(\mathrm{tr}(U)^2 + \mathrm{tr}(U^2)) \\ \mathrm{tr}(U_{[2,1]}) &= \mathrm{tr}(U^*)\mathrm{tr}(U_{[2,0]}) - \mathrm{tr}(U) \\ \mathrm{tr}(U_{[3,0]}) &= \frac{1}{6}(\mathrm{tr}(U)^3 + 3\mathrm{tr}(U)\mathrm{tr}(U^2) + 2\mathrm{tr}(U^3)) \end{aligned}$$

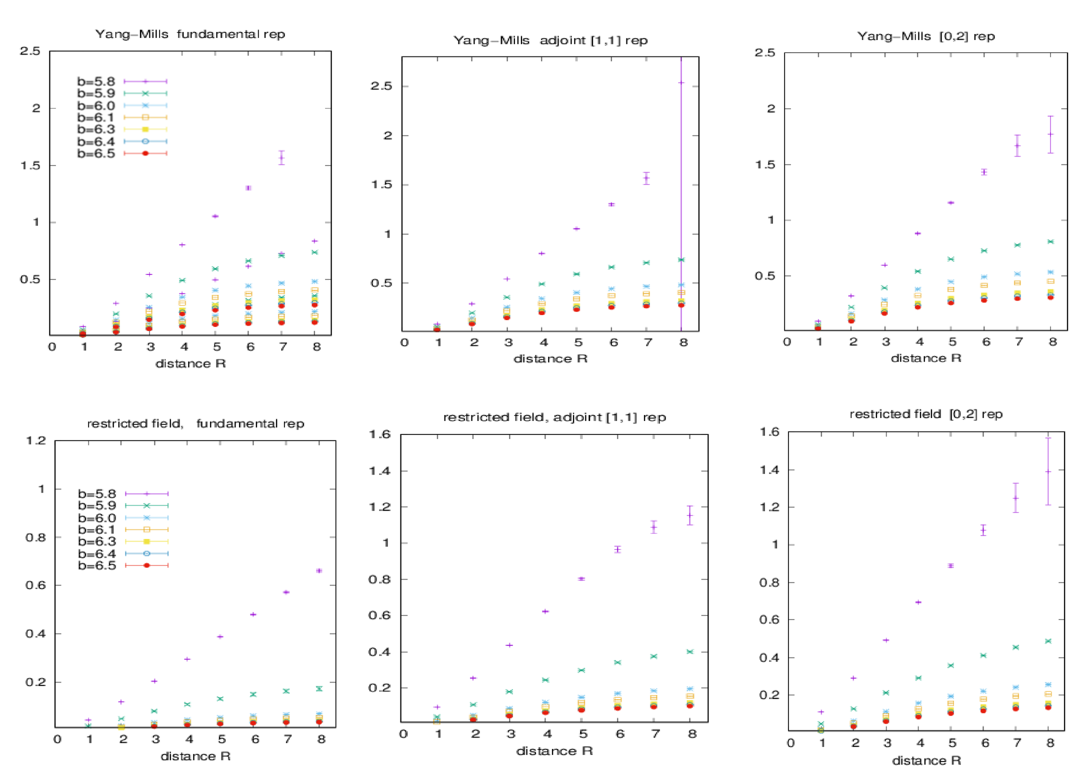
$3 = [1,0]$
$8 = [1,1]$
$6 = [2,0]$
$15a = [2,1]$
$10 = [3,0]$
....

- Static potential at zero temperature

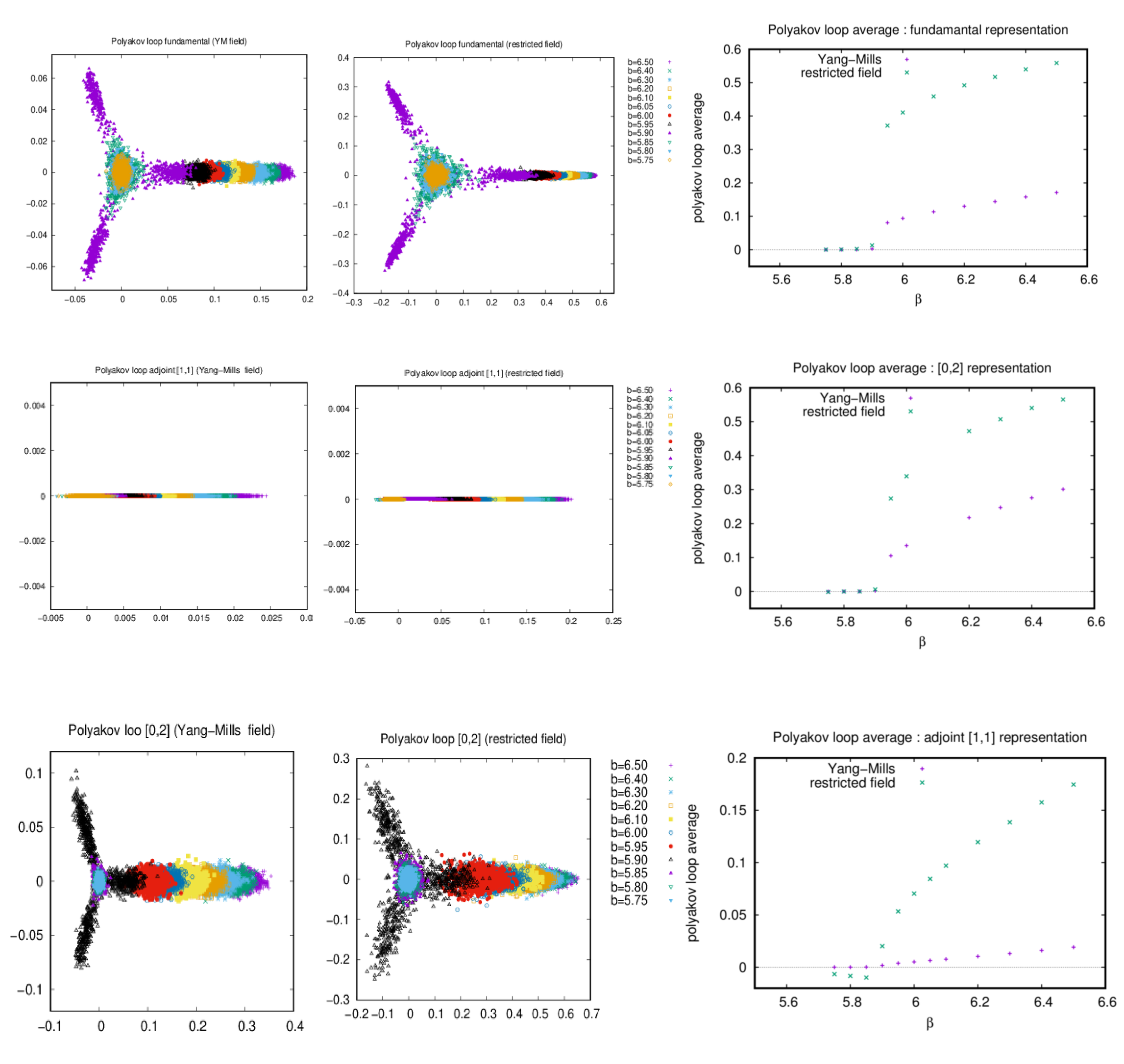


The ristricted field (V-field) dominance for (left) fundamental [0,1] (middle) adjoint [1,1] (right) [0,2] representations.

- Static potential for various temperature (β)



Polyakov loop value distribution and the Poliyakov loop average for various temperature



❑Summary

- Through the non-Abelian Stokes theorem (NAST), we have obtained the Wilson loop operator in the representation R for the restricted field.
- We have investigated Wilson loop average in the higher representation by using lattice simulation, and obtained correct behavior, i.e., restricted field (V) dominance in the string tension for the higher representation:
  - ➢ the adjoint representation for SU(2) yang-Mills theory
  - ➢ the adjoint and 6-dimansional representation for SU(3) YM theory.
- We have extended the formula for Wilson loop at the zero temperature to that for Poliyakov loop at nonzero temperature.
- We have investigated Polyakov loop average in the higher representations.
  - ➢ We find V-field dominance in the Polyakov loop.
  - ➢ V-field dominance in the string tension for the higher representation at finite temperature.