非エルミート系における分数量子ホール状態

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TY-Kudo-Katsura-Hatsugai arXiv: 2005.12635

Workshop: 熱場の量子論
2020/08/26 @Kyoto U

Motivation

	Short-range entangled	Long-range entangled (topo. + U)
$H = H^\dagger$	▪ Topological insulators ▪ Weyl semimetals	▪ Fractional quantum Hall ▪ Z2 spin liquid
$H \neq H^\dagger$	▪ nH topo. ins. ▪ exceptional points	???

Question

Topological ordered phases for non-Hermitian systems?

Yes !

FQH states with 1/3 filling can emerge for non-Hermitian systems.

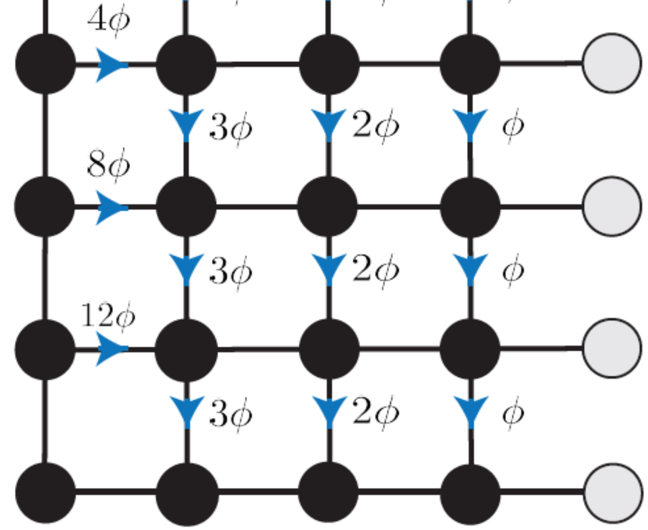
Hamiltonian

~Spinless fermions in magnetic flux~

$$H_{\mathrm{eff}} = H_{\mathrm{kin}} + H_{\mathrm{int}}$$

$$H_{\mathrm{kin}} = -\sum_{\langle i,j \rangle} t_0 e^{i\phi_{ij}} c_i^\dagger c_j$$

$$H_{\mathrm{int}} = V \sum_{\langle i,j \rangle} n_i n_j$$



nearest neighbor interactions:
non-Hermitian

$$\boxed{V = V_R - i\frac{\gamma}{2}}$$

$$V_R \geq 0$$

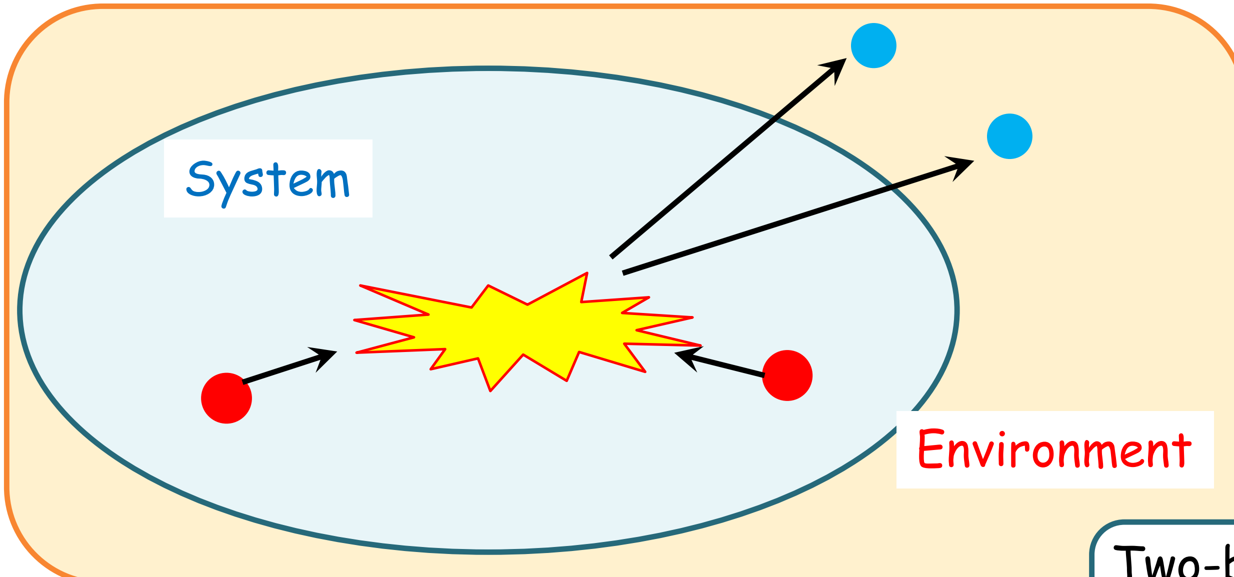
$$\gamma > 0$$

~Relevance with cold atoms~

Y. Ashida *et al.*, PRA (2016)
M. Nakagawa *et al.*, PRL (2018)
K. Yamamoto *et al.*, PRL (2019)

Inelastic collision (Feshbach resonance)

Atoms may release their internal energy as kinetic energy



Two-body loss:

$$L_\alpha \to \sqrt{\gamma} c_i c_{i+\mathbf{e}_{x(y)}}$$

Time-evolution of open quantum systems

$$\partial \rho = -i[H_0, \rho] + \sum_\alpha \left(L_\alpha \rho L_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\} \right)$$

$\rho(t)$:density matrix of the system

~Relevance with cold atoms~

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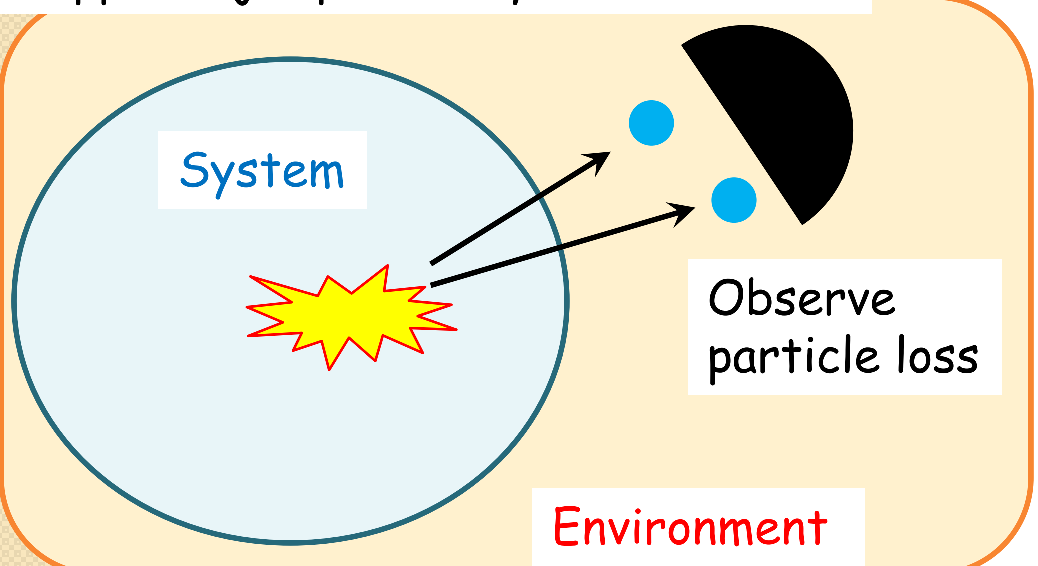
Suppress jump term by measurement



~Relevance with cold atoms~

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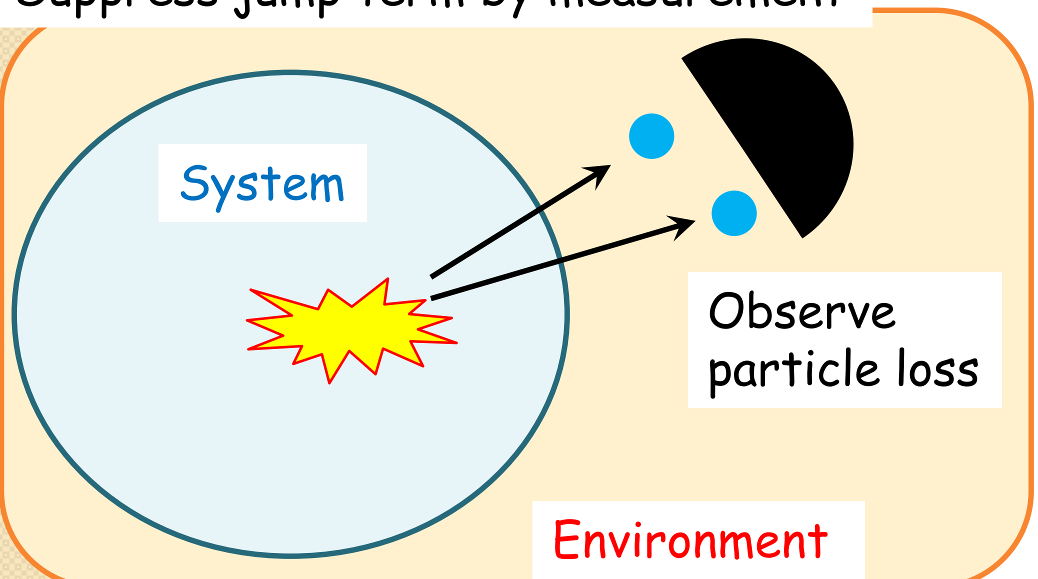

~Relevance with cold atoms~

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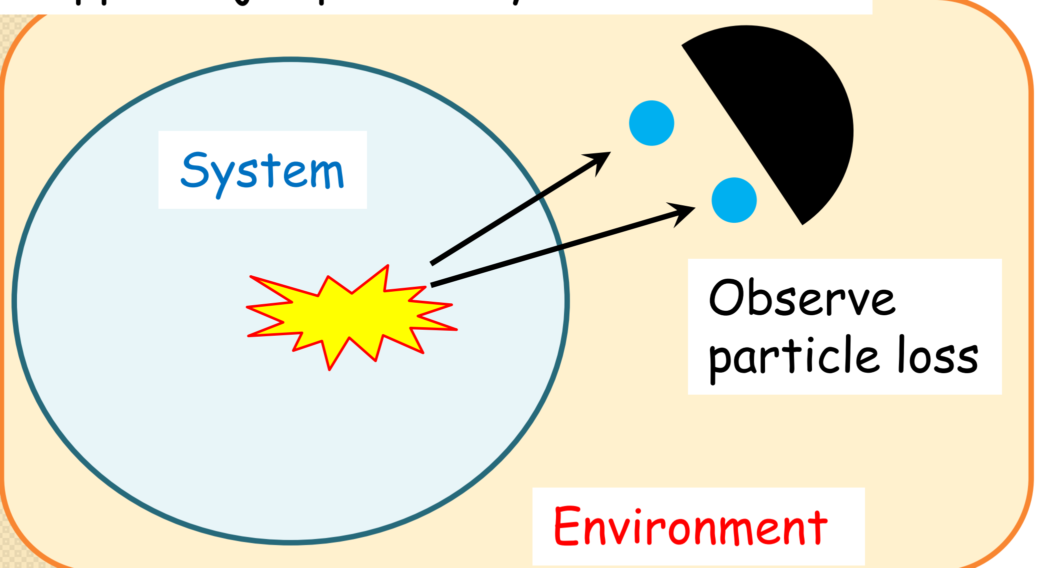


$$\partial_t \rho(t) = -i(H_{\rm eff}\rho(t) - \rho(t) H_{\rm eff}^\dagger)$$

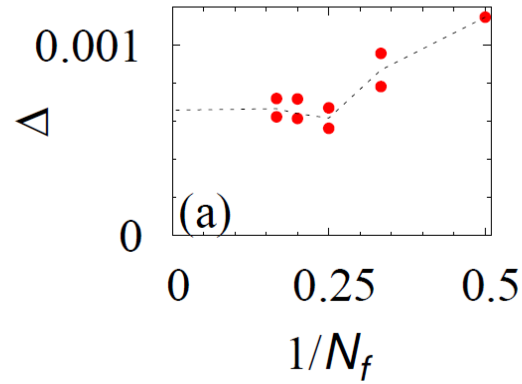
$$H_{\rm eff} = H_0 + \frac{i}{2}\sum_\alpha L_\alpha^\dagger L_\alpha$$

$$H_{\rm int} = V \sum_{\langle i,j \rangle} n_i n_j$$

$$\boxed{V = V_R - i\frac{\gamma}{2}}$$

～Warm up: Hermitian case～

- Presence of the bulk gap

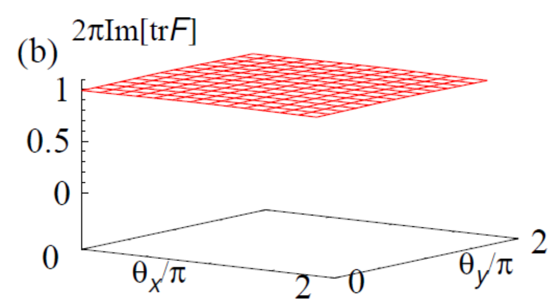


- 3-fold degeneracy for ground states

Haldane PRL (1983)

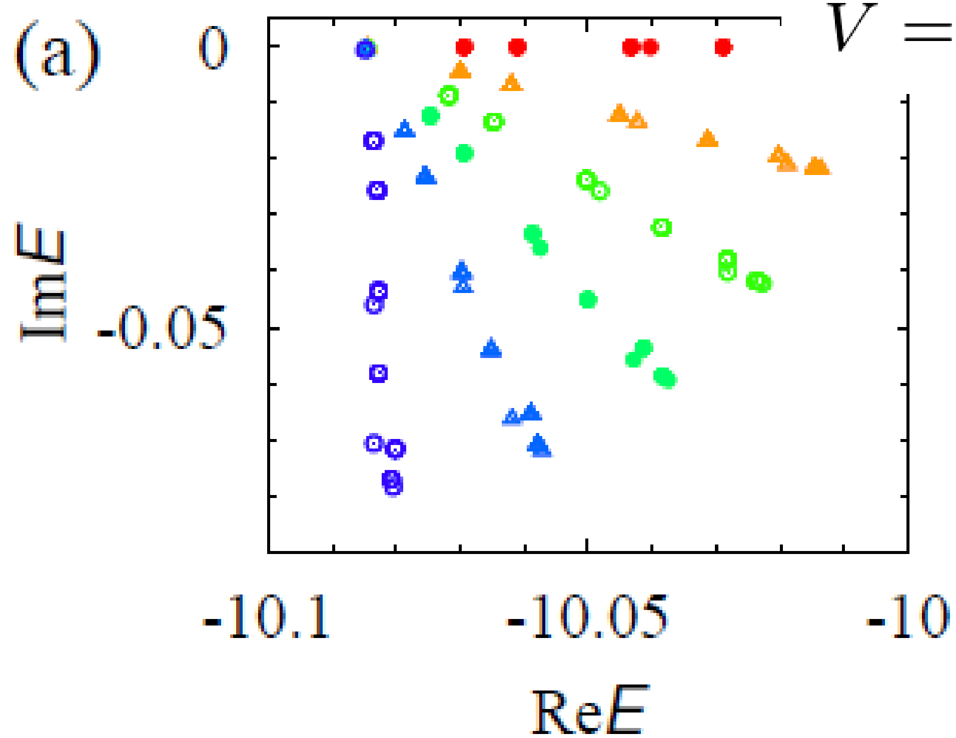
- Ctot=1 for the ground state triplet

$$\boxed{\sigma_{xy} = 1/3}$$



~non-Hermitian case~

$V = V_0 e^{i\theta}$



For non-Hermitian systems, eigenvalues take complex numbers…

Q:definition of ground states?

State with
- minimum $\mathrm{Re}E$??
- longest lifetime?? $\sim 1/\mathrm{Im}E$

A:

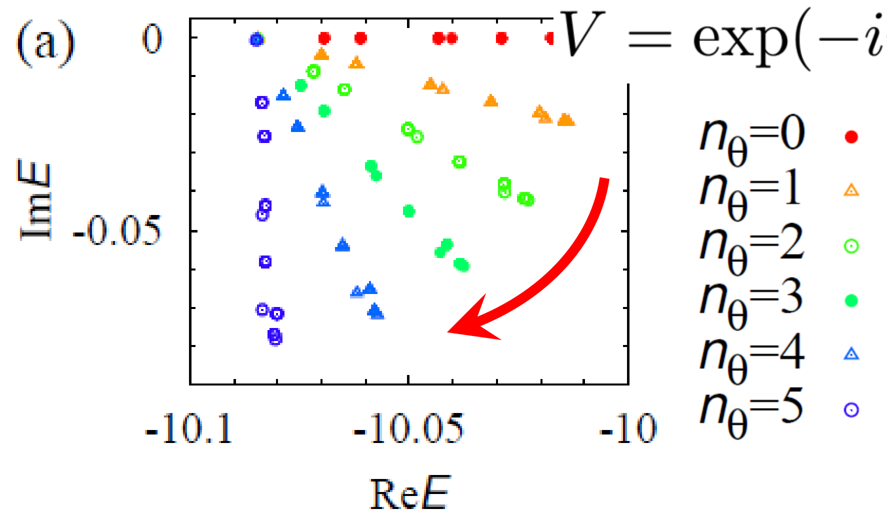
We define ground states with minimum $\mathrm{Re}E$

∵) it also has longest lifetime

We define energy gap as

$$\Delta = \mathrm{Re}E_e - \mathrm{Re}E_g$$

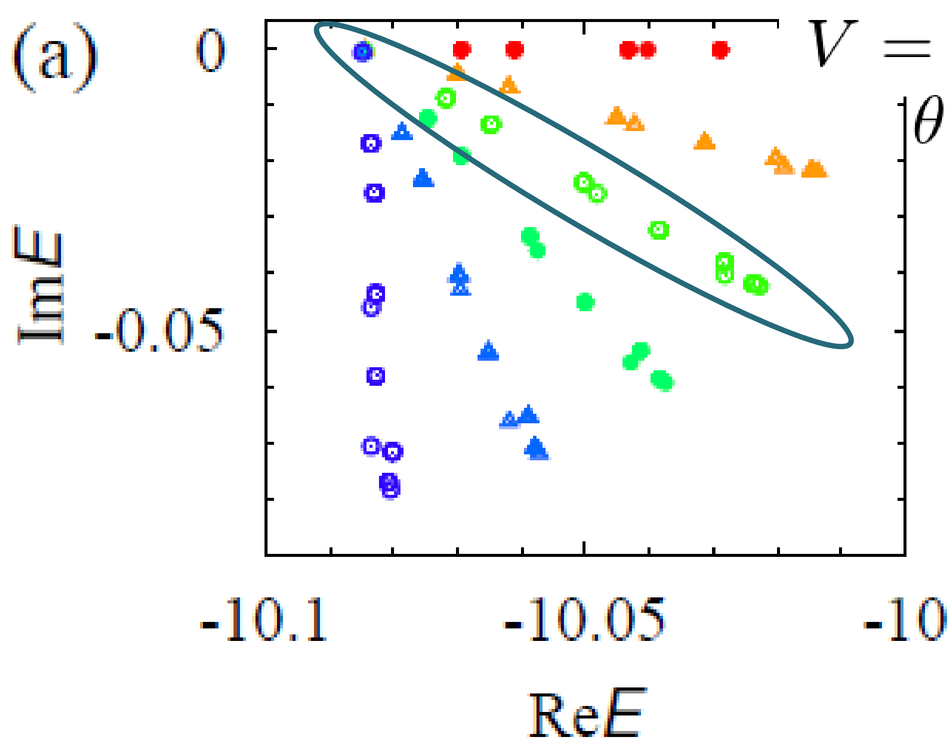
$V = \exp(-in_\theta \pi/10)$



We conclude emergence of non-Hermitian FQH states (1/3 filling)

based on

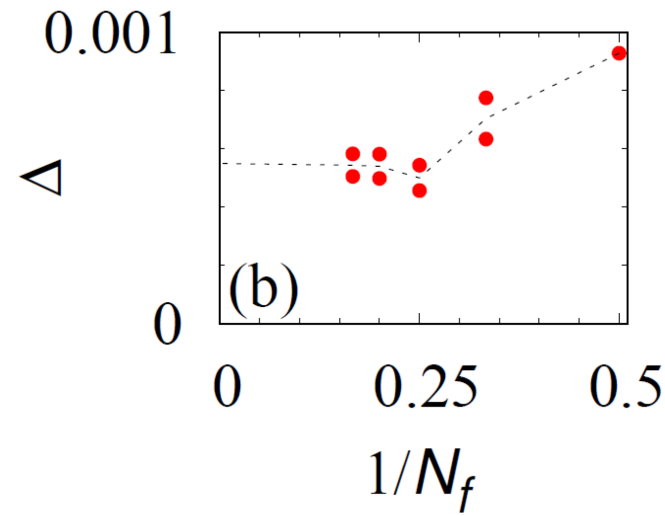
- 3-fold degeneracy for PBC (topological degeneracy)
- $C_{tot}=1$ for 3-fold degenerated ground states

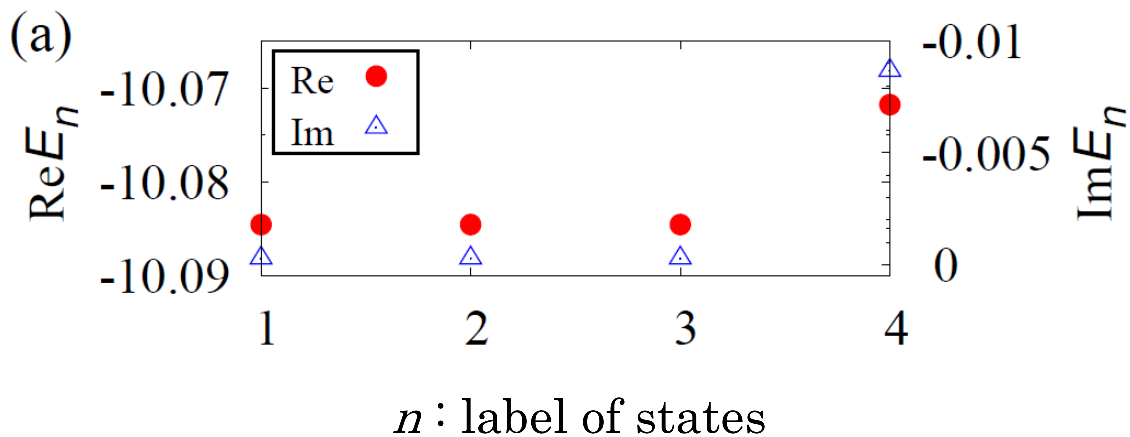


$V = V_0 e^{-i\theta}$

$\theta = \pi/5$

3-fold degeneracy for ground states





Many-body translation symmetry → three-fold degeneracy for the PBC

Hermitian case: Haldane PRL (1983)

We conclude emergence of non-Hermitian FQH states (1/3 filling)

based on

- ✔ 3-fold degeneracy for PBC (topological degeneracy)
- C_{tot}=1 for 3-fold degenerated ground states

total Chern number (Ctot=1) for ground states triplet?

$$C_{\text{tot}} = \int \frac{d\theta_x d\theta_y}{2\pi i} \text{tr}F(\theta_x, \theta_y),$$

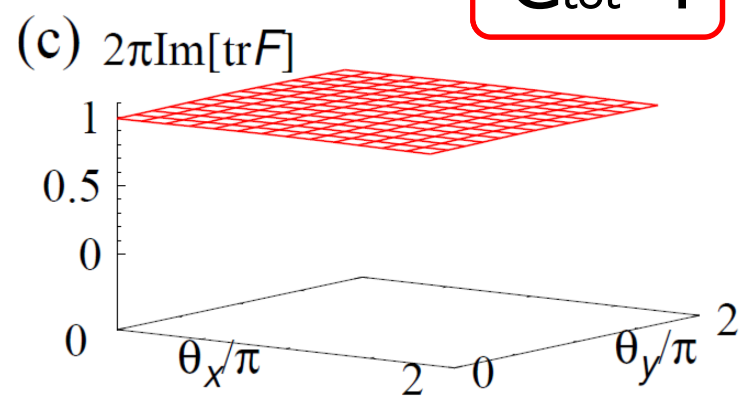
$$F_{nm}(\theta_x, \theta_y) = \epsilon_{\mu\nu\, L}\langle \partial_\mu \Psi_n | \partial_\nu \Psi_m \rangle_R.$$

$C_{\text{tot}}=1$

Left and Right eigenvectors

$$H|\Psi_n\rangle_R = |\Psi_n\rangle_R E_n$$

$${}_L\langle \Psi_n | H = E_n \; {}_L\langle \Psi_n |$$



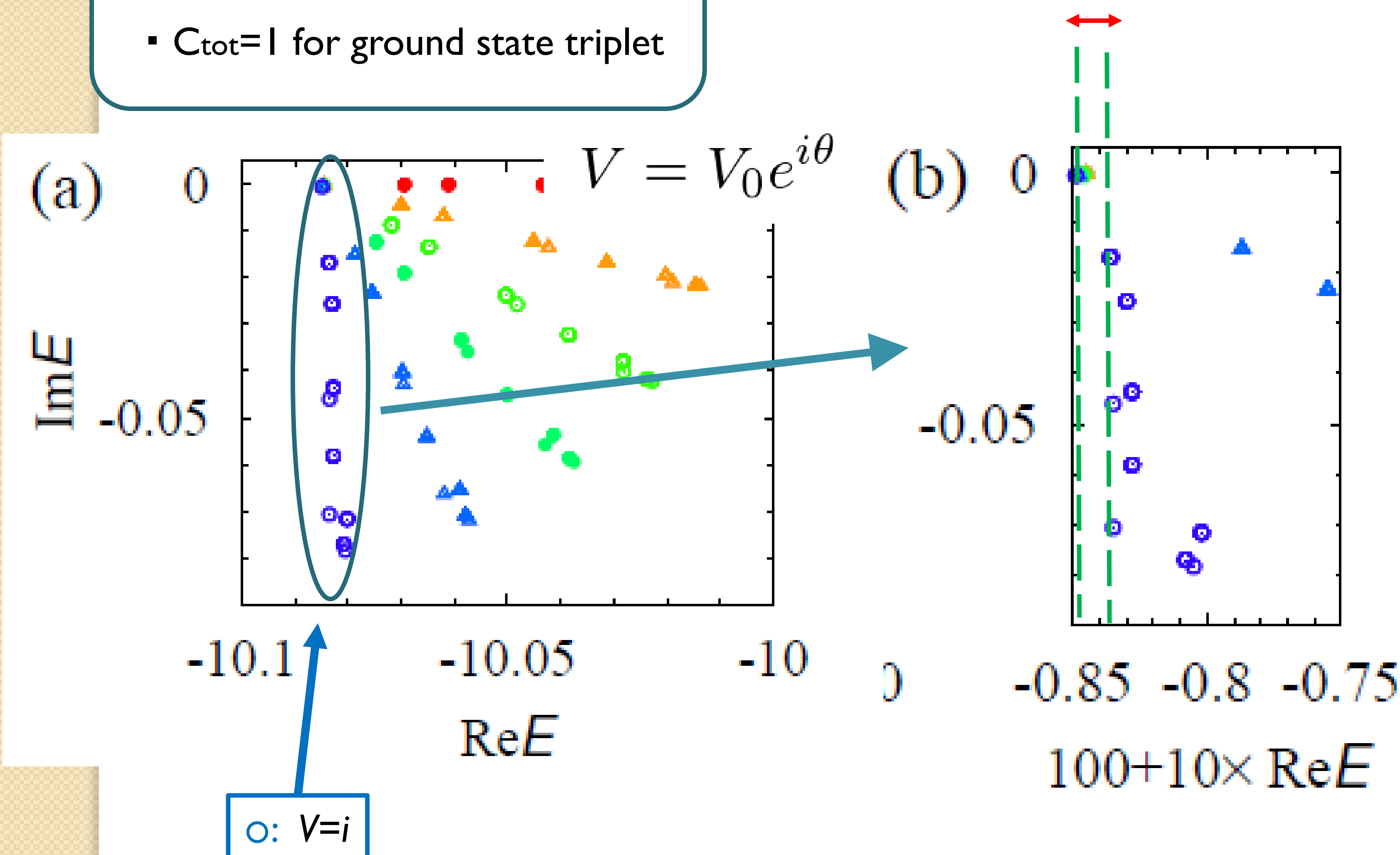
Non-Hermitian FQH states emerge.

But,

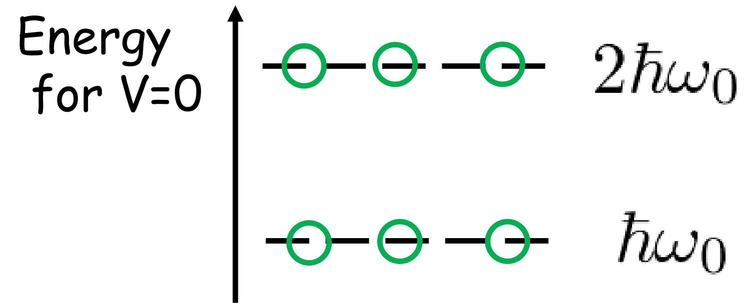
What are unique phenomena induced by non-Hermiticity?

Energy gaps opens for *V=i*

- 3-fold degeneracy for PBC
- Ctot=1 for ground state triplet

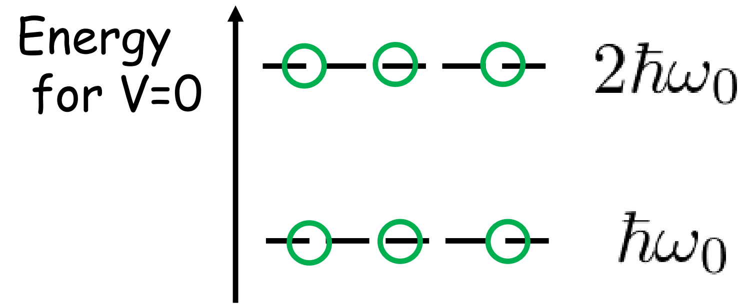


Why purely dissipative interaction opens the gap?



$$H_{\rm ptb} = P_0 H_{\rm int} P_0 + P_0 H_{\rm int} P_1 \frac{1}{E_g^0 - H_{\rm kin}} P_1 H_{\rm int} P_0.$$

Why purely dissipative interaction opens the gap?



$$H_{\rm ptb} = P_0 H_{\rm int} P_0 + \underbrace{P_0 H_{\rm int} P_1 \frac{1}{E_g^0 - H_{\rm kin}} P_1 H_{\rm int} P_0}_{\text{Repulsive interaction due to Landau level mixing}}.$$

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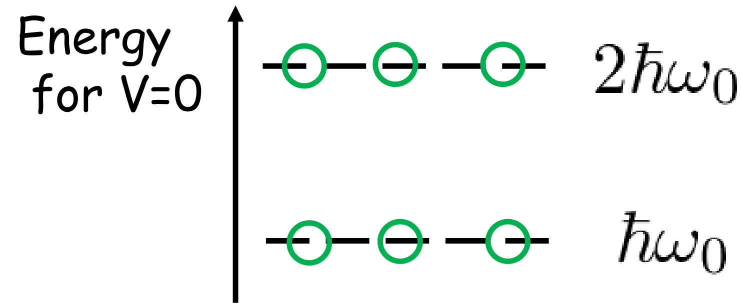


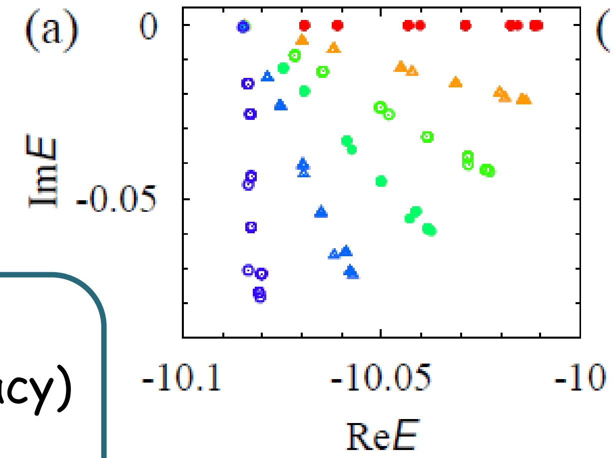
~ $i^2=-1$

$$H_{\rm ptb} = P_0 H_{\rm int} P_0 + \underbrace{\boxed{P_0 H_{\rm int} P_1} \frac{1}{\boxed{E_g^0 - H_{\rm kin}}} \boxed{P_1 H_{\rm int} P_0}}_{\text{Repulsive interaction due to Landau level mixing}}$$

~ $-\hbar\omega_0$

Repulsive interactions induced by dissipations: reminiscent of quantum Zeno effects

Summary



Non-Hermitian FQH states (1/3 filling)

- 3-fold degeneracy for PBC (topological degeneracy)
- $C_{tot}=1$ for 3-fold degenerated ground states

Dissipation induces nHFQH
reminiscent of the Zeno effect

We also find that topology of FQH states is maintained
even in the presence of the jump term

by introducing the pseudo-spin Chern number

$$C_{\sigma\sigma} := \int \frac{d\theta_x d\theta_y}{2\pi} \mathrm{Im} F_{\sigma\sigma}(\theta_x, \theta_y),$$

$$F_{\sigma\sigma} := \epsilon_{\mu\nu} \sum_n {}_L\langle\langle \partial_\mu^\sigma \rho_n | \partial_\nu^\sigma \rho_n \rangle\rangle_R.$$

TY-Kudo-Katsura-Yoshida arXiv:2005.12635

Thank you!