Generalized crossover in strongly interacting fermions with finite effective range

Paper: HT, J. Phys. Soc. Jpn. **88**, 093001 (2019).; Phys. Rev. A **97**, 043613 (2018).

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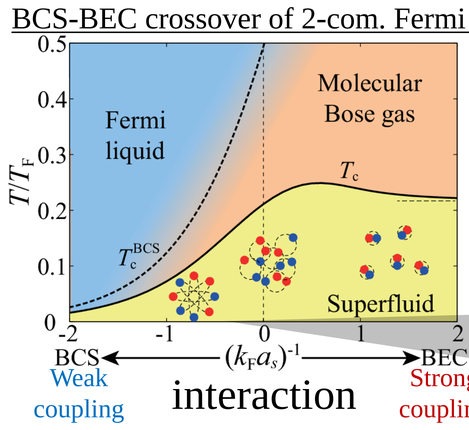
—The purpose of this poster—

We generalize BCS-BEC crossover physics with zero-range interactions to the case with arbitrary effective range and scattering length. Using a Nozieres-Schmitt-Rink approach, we proposed a universal phase diagram of two-component fermions with respect to low-energy constants. A novel crossover induced by the change of effective ranges is found at the unitarility limit. Our results cover an ultracold Fermi gas as well as dilute neutron matter.

Introduction

"Similarity" between ultracold Fermi gas and pure neutron matter

BCS-BEC crossover of 2-com. Fermi gas

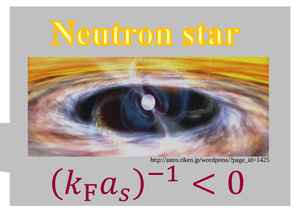


S-wave scattering is dominant even in a low density nuclear matter

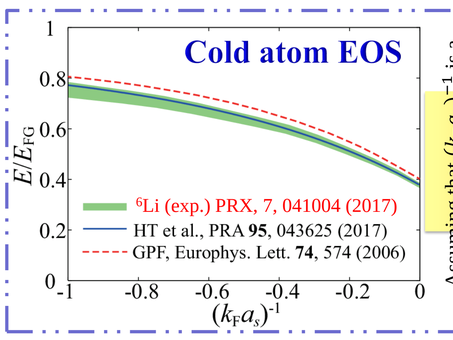
S-wave phase shift

$$k\cot\delta_s = -\frac{1}{a_s} + \frac{1}{2} r_e k^2$$

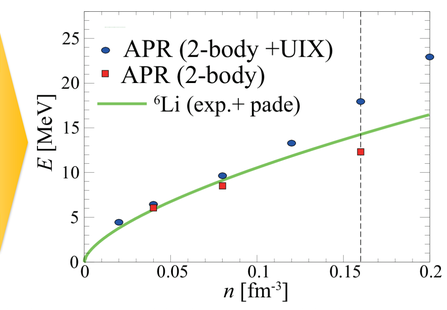
a_s: scattering length
r_e: effective range



	Cold atom	Neutrons
a_s	$-\infty\sim\infty$	-18.5 fm
r_e	~0 (broad res.)	2.8 fm
density	$\sim 10^{15}$ cm^{-3}	$\sim$0.16 fm^{-3}
$(k_F a_s)^{-1}$	$-\infty\sim\infty$	$-\infty\sim 0$



Assuming that $(k_F a_s)^{-1}$ is a function of the density at fixed a_s



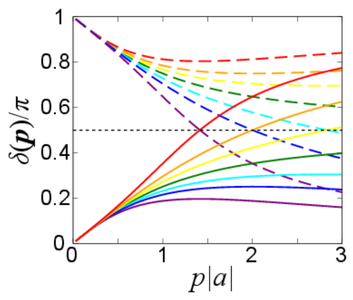
We can access neutron star physics on Earth via cold atom experiments

APR: PRC, **58**, 1804 (1998)
Simulation for neutron matter

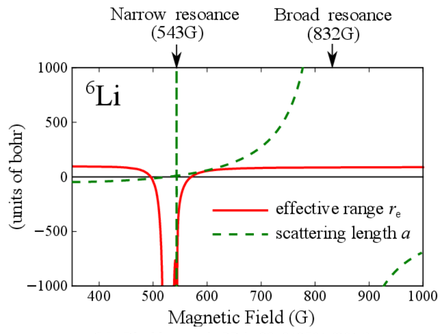
"Difference" between ultracold Fermi gas and pure neutron matter

The effective range correction plays an important role in pairing properties

Phase shift with different r_e

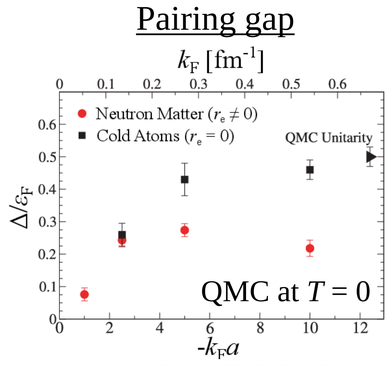


Feshbach resonance in ^{6}Li atoms



C.L. Blackley, *et al.*, PRA, **89**, 042701 (2014).

Pairing gap



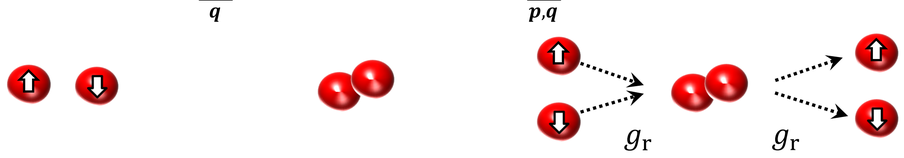
A. Gezerlis and J. Carlson, PRC, 77, 032801(R) (2008).

We investigate pairing properties with finite effective range and show the superfluid critical temperature at arbitrary scattering length and effective range.

Formulation

●*Hamiltonian*

$$H = \sum_{k,\sigma} (\varepsilon_k - \mu) c^\dagger_{k,\sigma} c_{k,\sigma} + \sum_q (\varepsilon_q/2 + \nu - 2\mu) b^\dagger_q b_q + \sum_{p,q} \left(g_p b^\dagger_q c_{p+q/2,\uparrow} c_{-p+q/2,\downarrow} + \text{H. c.} \right)$$



Two-component Fermi atoms closed-channel molecule Feshbach coupling

- kinetic energy: $\varepsilon_k = \frac{k^2}{2m}$
- atomic annihilation operator: $c_{k,\sigma}$
- atom-dimer coupling: $g_p = \frac{g}{\sqrt{1 + (p/p_c)^2}}$
- pseudospin: $\sigma = \uparrow, \downarrow$
- diatomic annihilation operator: b_q
- chemical potential: μ
- threshold energy of a closed-channel molecule: ν
- scattering length: $a = \left[-\frac{4\pi\nu}{mg^2} + p_c \right]^{-1}$
- effective range: $r_e = -\frac{8\pi}{m^2 g^2} + \frac{2}{p_c}\left(1 - \frac{1}{p_c a}\right)$

●*Nozieres-Schmidt-Rink approach*

- Particle number equation:

$$N = 2\sum_p f(\varepsilon_p - \mu) + 2\sum_q b(\varepsilon_q + \nu - 2\mu) - T\sum_{q, i\zeta_\ell} \frac{\partial}{\partial\mu} \ln[1 - D(q, i\zeta_\ell)\Pi(q, i\zeta_\ell)]$$

$f(b)$: Fermi (Bose) distribution function D: dimer propagator ζ_ℓ: fermion (boson) Matsubara frequency

- Thouless criterion (condition for T_c):

$$D(q = 0, i\zeta_l = 0) = \infty$$

(gapless bosonic excitation)

- Particle-particle bubble for Cooper channel

$$\Pi(q, i\nu_\ell) = \sum_p g_p^2 \frac{1 - f(\varepsilon_{p+q/2} - \mu) - f(\varepsilon_{-p+q/2} - \mu)}{i\nu_\ell - \varepsilon_{p+q/2} - \varepsilon_{-p+q/2} + 2\mu}$$

Results

● Generalized phase diagram

BD: the region where effective range expansion breaks down

Cutoff parameter

$$p_c = \frac{1 + \sqrt{1 - 2r_e/a}}{r_e}$$

Two-body binding energy

$$E_b = \frac{1}{ma^2} \frac{1}{[1 - 1/(ap_c)]^2}$$

Bound for effective range
(≃ Wigner's causality bound)

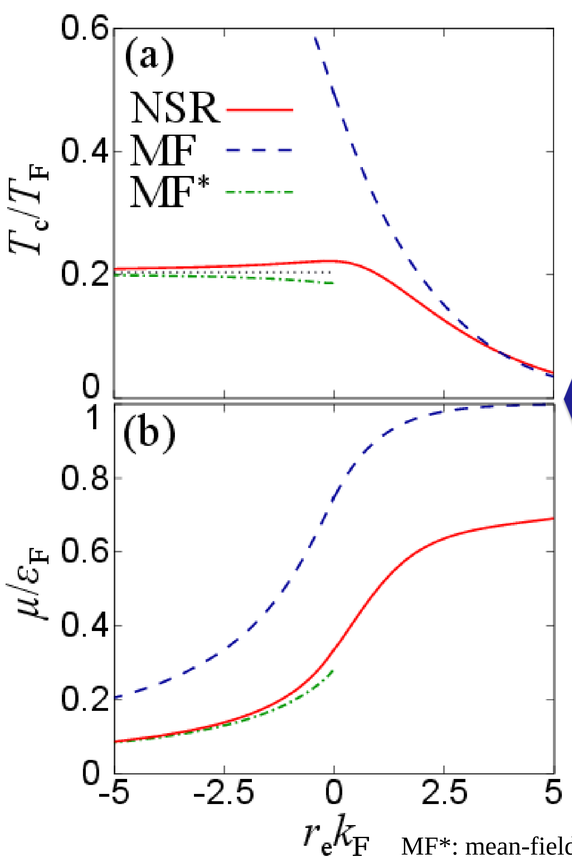
$$r_e \leq a/2$$

● Superfluid critical temperature

● Chemical potential

PNM: pure neutron matter

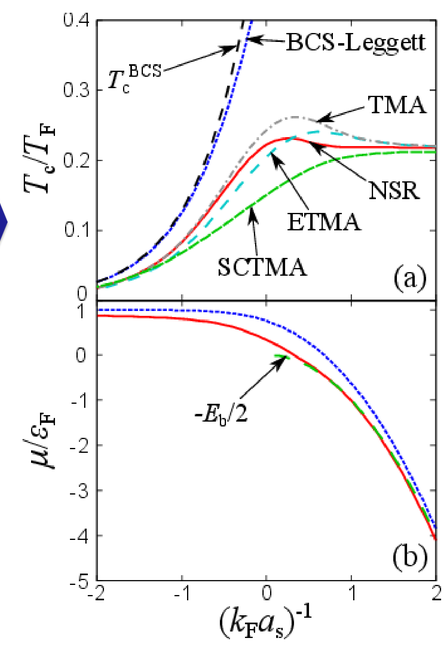
● Effective range induced crossover



MF*: mean-field results with renormalized dimer level

Zero-range crossover with changing the scattering length

Y. Ohashi, HT, P. van Wyk, Prog. Part. Nucl. Phys. **111**, 103739 (2020).



	Effective range induced crossover		Scattering length induced crossover	
Coupling parameters	$k_F r_e \to \infty$ (BCS)	$k_F r_e \to -\infty$ (BEC)	$(k_F a)^{-1} \to -\infty$ (BCS)	$(k_F a)^{-1} \to \infty$ (BEC)
T_c/T_F	$\sim\exp\left(-\frac{\pi}{4} k_F r_e\right)$	$\to 0.204 T_F$	$\sim\exp\left(-\frac{\pi}{2k_F a}\right)$	$\to 0.218 T_F$
μ_c/ε_F	$\to \varepsilon_F$	$\to 0$	$\to \varepsilon_F$	$\to -E_b/2$

Summary

- We have investigated a generalized crossover of superfluid critical temperature in a two-component fermions with arbitrary scattering length and effective range within the Nozieres-Schmitt-Rink approach.
- We have proposed a generalized phase diagram of BCS-BEC crossover and presented the second crossover induced by the effective range.

Future work: low-dimensions, population or mass imbalances, etc…