

格子QCDによるカラー超伝導の解明に向けて - 摂動論の予言と複素ランジュバン法 -

Towards color superconductivity on the lattice - perturbative predictions and the complex Langevin method -

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- Motivation
 - QCD phase diagram and sign problem
 - Possibility of investigating color superconductivity (CSC) on the lattice
- Perturbative predictions
 - Gap equation
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 - Phase diagram on (μ_q, β) -plane
- Current status of our analysis by the complex Langevin method
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Methods to overcome sign problems

Lefschetz thimble, tensor renormaliztion group, complex Langevin method (CLM)…



Application of CLM to lattice QCD at finite density

Sexty '14 Fodor, Katz, Sexty, Török '15 Nagata, Nishimura, Shimasaki '18 Tsutsui, Ito, Matsufuru, Nishimura, Shimasaki, Tsuchiya '18 Ito, Matsufuru, Nishimura, Shimasaki, Tsuchiya, Tsutsui '18 Sexty '19 Kogut, Sinclair '19 Sinclair, Kogut '19 Tsutsui, Ito, Matsufuru, Nishimura, Shimasaki, Tsuchiya '19 Scherzer, Sexty, Stamatescu '20 Ito, Matsufuru, Namekawa, Nishimura, Shimasaki, Tsuchiya, Tsutsui '20



aryon Chemical Potential $\mu_{\rm B}$

Color superconductivity (CSC)



Cooper instability

CSC with various symmetries of Cooper pairs has been predicted (2SC, CFL···)

CSC on the lattice?

Analysis of CSC on the lattice by firstprinciple methods such as CLM?

Reasonable starting point: high β region (small volume) $\beta = 2N_c/g_0^2$

Asymptotic freedom \rightarrow weak coupling



But the region in which the CSC occurs has not been predicted so far based on lattice QCD.

c.f.) Calculation in effective models (e.g. NJL) in finite box Hands, Walters, PLB (2002) Amore, Birse, McGovern, Walet, PRD (2002)

Aim

We give the **perturbative prediction of the parameter region in Lattice QCD** for the first time.

- Derive the gap equation for Cooper pair with general shape and symmetry.
- Derive the condition to determine the critical β
 - The shape and symmetry of Cooper pairs also can be determined with our calculation method.
- Calculate critical β and give the prediction for the phase diagram on (μ_q, β) -plane.

Such a calculation in general setup is possible because **only finite number of coupled equations appear on the lattice**.

 $(\leftrightarrow functional equation in continuum case)$

We also present some preliminary results of the CLM in the large β regime.

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Nambu-Gorkov formalism

Action for $N_{\rm f} = 4$ Staggered fermion

$$S = S_{\text{fermi}} + S_{\text{int}} + S_{\text{gluon}} \qquad S_{\text{fermi}} = \sum_{N,N',\rho,\rho',a,a'} \overline{\chi}_{\rho}^{a}(N) D_{NN',\rho\rho'}^{aa'}(\mu_{q}, m_{q}) \chi_{\rho'}^{a'}(N')$$
Staggered fermion $\chi_{\rho}^{a}(N) = \chi^{a}(2N+\rho)$
 $[\rho_{\mu} = 0 \text{ or } 1 (\mu = 1 \sim 4), a: \text{ color index}]$
Nambu spinor
$$\Psi_{\rho}^{a}(N) = \begin{pmatrix} \chi_{\rho}^{a}(N) \\ \overline{\chi}_{\rho}^{a}(N) \end{pmatrix} \qquad S_{\text{fermi}} = \frac{1}{2} \sum_{N,N',\rho,\rho',a,a'} \overline{\Psi}_{\rho}^{a}(N) D_{NN',\rho\rho'}^{aa'} \Psi_{\rho'}^{a'}(N')$$

$$D_{NN',\rho\rho'}^{aa'} = \begin{pmatrix} \sum_{11,NN',\rho\rho'}^{aa'} \sum_{12,NN',\rho\rho'}^{2a'} \sum_{22,NN',\rho\rho'}^{2a'} \sum_{22,NN',\rho\rho'}^{2$$

In the case of continuum QCD: Bailin, Love, PR (1984) \cdots

Gap equation

Gap equation: self-consistent equation to determine $\Sigma_{12}(21)$

We ignore $\Sigma_{11(22)}$ (higher-order contribution in the gap eq.) $\Sigma_{NN',\rho\rho'}^{aa'} \approx \begin{pmatrix} 0 & \Sigma_{12,NN',\rho\rho'}^{aa'} \\ \Sigma_{21,NN',\rho\rho'}^{aa'} & 0 \end{pmatrix}$

Gap equation (lowest-order, momentum space)



If this eq. has $\Sigma_{12(21)} \neq 0$ solution, CSC emerges.

Transition point

At the transition point, $S_{12,q'q,\rho'\rho}^{a'a} \approx -[\tilde{D}_{11}^{-1}\tilde{\Sigma}_{12}\tilde{D}_{22}^{-1}]_{q'q,\rho'\rho}^{a'a} + \mathcal{O}((\Sigma_{12(21)})^2)$ since $\Sigma_{12(21)} \rightarrow 0$

Gap eq. is reduced to linear equation

zero eigenvalue.

 $(\mathcal{V} \neq 0 \text{ solution})$

cf. another derivation:

Condition for T-matrix

(argument like Thouless criterion) Thouless, AP (1960)

being singular

Gap eq. for color antitriplet channel (Σ_{12}^{-}) $\sum_{q,\tau,\tau'} \left(\mathbf{1} - \frac{1}{\beta_{c}} \mathcal{M}' \right)_{(p\rho\rho'),(q\tau\tau')} \mathcal{V}_{(q\tau\tau')} = 0$ Critical β

 $1 - \frac{1}{\beta_c}\mathcal{M}'$ should have $\not \models \beta_c = \lambda'_{max}$ (maximum eigenvalue of \mathcal{M}')

Corresponding eigenvector $\mathcal V$ determines Σ_{12}^-

(i.e., shape and symmetry of Cooper pairs)

We use power iteration to calculate $\lambda'_{\max} \& \mathcal{V}$.

$$\begin{aligned} \mathcal{M}'_{(p\rho\rho'),(q\tau\tau')} = & \frac{N_{\rm c} + 1}{V} \sum_{k \in \mathrm{BZ}} \frac{\tilde{\delta}_{2(q-p-k)}}{4\sum_{\nu} \sin^2 \frac{k_{\nu}}{2}} \sum_{\mu} \left| \cos\left(\frac{\overline{p}_{\mu} + \overline{q}_{\mu}}{2}\right) \right|^2 \\ & \times \frac{\sum_{\alpha} [\Gamma^{\alpha} \Gamma^{\mu}]_{\rho'\tau'} \sin \overline{p}_{\alpha} + im_{\rm q} \Gamma^{\mu}_{\rho'\tau'}}{\sum_{\nu} \sin^2 \overline{p}_{\nu} + m_{\rm q}^2} \frac{\sum_{\beta} [\Gamma^{\mu} \Gamma^{\beta}]_{\rho\tau} \sin \overline{q}_{\beta}^* - im_{\rm q} \Gamma^{\mu}_{\rho\tau}}{\sum_{\nu} \sin^2 \overline{q}_{\nu}^* + m_{\rm q}^2} \end{aligned}$$

* We omit k = 0 mode by hand to avoid divergence caused by lattice artifact

$$\mathcal{V}_{(q\tau\tau')} = [L^{-1}\tilde{\Sigma}_{12}^{-}]_{q\tau'\tau}$$

Prediction of phase diagram in (μ_q, β) -plane



- Peak structure: CSC phase spreads in perturbative region (large β) at specific μ_q .

Origin of peak structure

The origin of peak is elucidated by comparing with the results of quark number



- Peak positions correspond to μ_q at which quark number changes.
 - At such μ_q , modes of fermions exist on the Fermi surface.

These modes form Cooper pairs.

- Consistent with results in NJL model in small box.
 - Open of the energy gap at discrete μ_q was observed.

Amore, Birse, McGovern, Walet, PRD (2002)

Quark-mass dependence



The peaks are shifted when m_q is changed.

Consistent with the interpretation that the peak positions are determined by the energy spectrum of quark.

Temperature dependence



• β_c is sensitive to the temperature.

Low temperature is preferable to observe CSC in the perturbative region.



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Complex Langevin method (CLM)

Complexification of gauge field

 $U_{x,\mu} \in \mathrm{SU}(3) \to \mathcal{U}_{x,\mu} \in \mathrm{SL}(3,\mathbb{C})$

Parisi '83; Klauder '84; Aarts, Seiler, Stamatescu '09; Aarts, James,Seiler, Stamatescu '11; Seiler, Sexty, Stamatescu '13; Sexty '14; Fodor, Katz, Sexty, Torok '15; Nishimura, Shimasaki '15; Nagata, Nishimura, Shimasaki '15; Sinclair, Kogut '16

Complex Langevin equation

$$\mathcal{U}_{x,\mu}(\tau + \epsilon) = \exp(i(-\epsilon D_{x,\mu}S[\mathcal{U},\mu_q] + \sqrt{\epsilon}\eta_{x,\mu}(\tau)))\mathcal{U}_{x,\mu}(\tau)$$

Drift term Noise term

Condition for justifying
$$\frac{1}{Z} \int dU \mathcal{O}[U] e^{-S[U,\mu]} = \lim_{\tau \to \infty} \langle \mathcal{O}[\mathcal{U}(\tau)] \rangle_{\eta}$$

The probability of the drift term should be suppressed exponentially at large magnitude.

Nagata, Nishimura, Shimasaki, PRD94 (2016)

CLM on the lattice with a small aspect ratio & β

Calculation on $8^3 \times 128$, $\mu_q = 0.66$, $\beta = 20, 30, 40, 50$

• We confirm CLM is justified at these parameters.



Signal of phase transition in the Polyakov line?



- Polyakov line rapidly changes near $1/eta_{
 m c}$
 - Normal phase in $\beta > \beta_c$?
 - Polyakov line may be enhanced due to gapless excitation from Fermi surface.

cf) perturbative calculations on small *S*³ Hands, Hollowood, Myers, JHEP (2010)

- CSC in $\beta < \beta_c$?
 - → Calculation of CSC order parameter is ongoing.

Summary

Based on lattice perturbation for QCD, we explicitly give the prediction for the parameter region in which CSC appears.

- Gap equation is only finite number of coupled equations on the lattice with general setup.
- Phase diagram on (μ_q, β) -plane without assumption on the form and symmetry of Cooper pairs.
- Peak structure is found
 - Peak appears when modes on quark exists on the fermi surface
- Small aspect ratio (e.g. $8^3 \times 128$) is preferable to observe CSC.

Outlook

- Investigation of CSC with complex Langevin method (CLM) is ongoing.
 - CLM works on lattice with a small aspect ratio and β .
 - Signal of phase transition in Polyakov line?
- Determination of the form & symmetry of the Cooper pair from eigenvector.
 - Find better operators for cleaner signals of CSC.

