The finite N origin of the Bardeen-Moshe-Bander phenomenon and its extension at N=∞ by singular fixed points

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arXiv:2001.07682

O(N) models

- They have played an important role in our understanding of second order phase transitions.

- N-component vector order parameter
  N=1…Ising, N=2…XY, N=3…Heisenberg Model

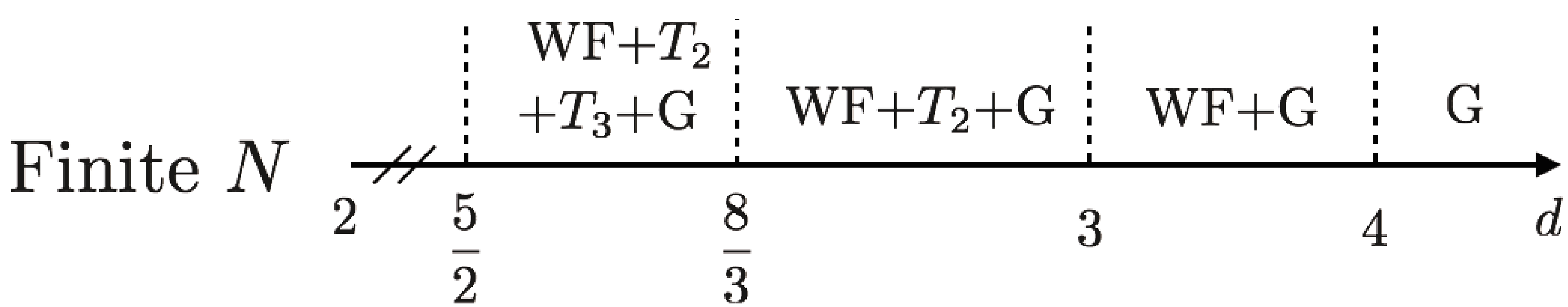
- The playground of almost all the theoretical approaches …Exact solution (2d Ising), Renormalization group（d=4-ε, 2+ε expansion), conformal bootstrap,…

Common wisdom on the criticality of O(N) models (finite N case)

GLW Hamiltonian $$H[\phi] = \frac{1}{2}\int_x (\nabla\phi_i)^2 + U(\phi)$$

$$U(\phi) = a_2(\phi_i)^2 + a_4(\phi_i)^4 + a_6(\phi_i)^6 + \cdots$$

Below the critical dimension $d_n = 2 + 2/n$, the ϕ^{2n} term becomes relevant around the Gaussian FP (G).



A nontrivial fixed point T_n with n relevant (unstable) directions branches from G at d_n. (Wilson-Fisher FP, which describes second order phase transition, at d=4 and the tricritical FP T_2 at d=3….)

Common wisdom on the criticality of O(N) models at $N = \infty$

- At $N = \infty$, in generic dimensions 2<d<4, only Gaussian (G) and Wilson-Fisher (WF) fixed points (FPs) have been found.

- Exceptional case: At $d_n = 2 + 2/n$, there exists a line of FPs starting from G and it terminates at BMB (Bardeen-Moshe-Bander) FP… A finite-N counterpart exists??

Spontaneous Breaking of Scale Invariance and the Ultraviolet Fixed Point in O(N)-Symmetric (φ_3^6) Theory

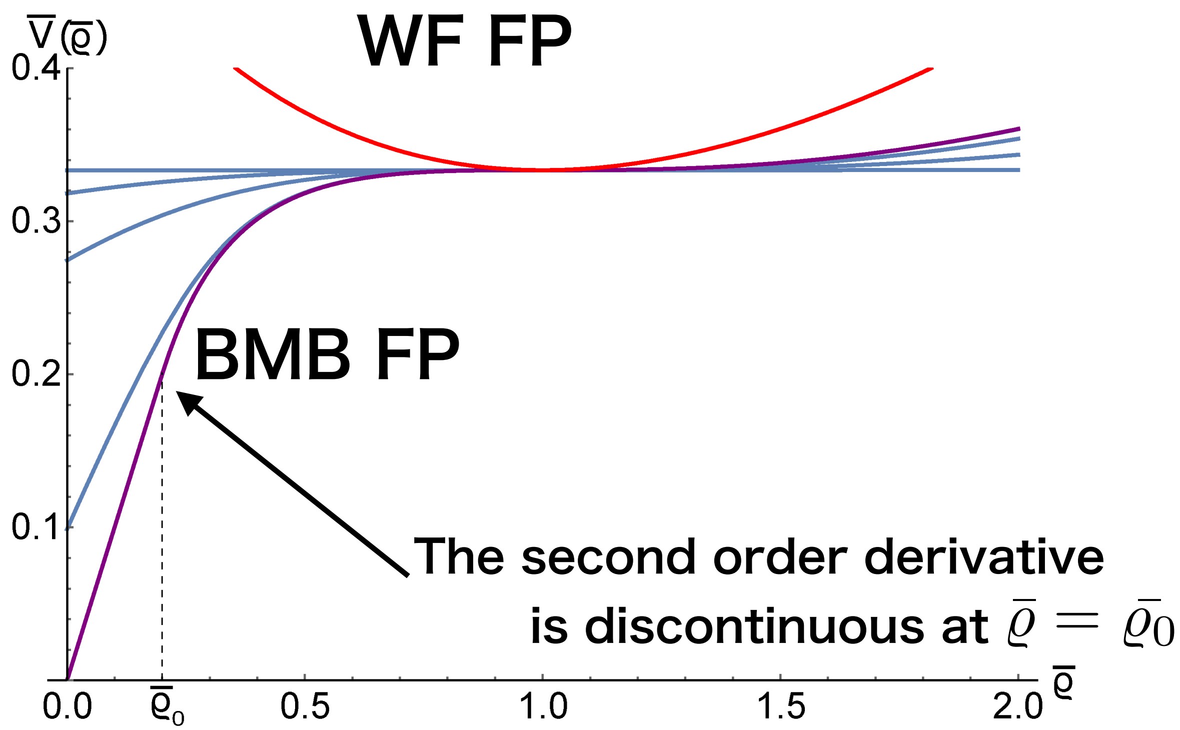
William A. Bardeen, Moshe Moshe, and Myron Bander

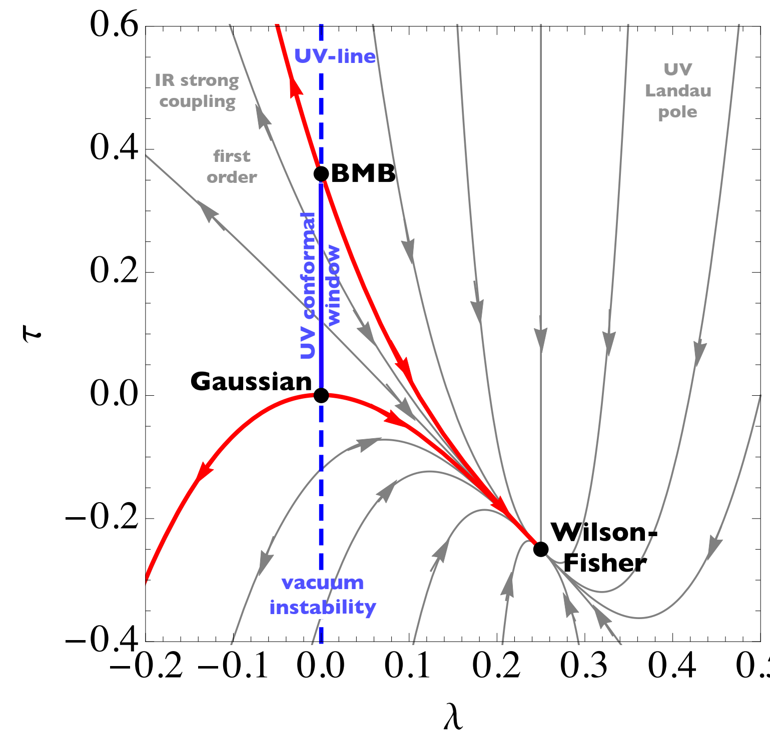
Phys. Rev. Lett. **52**, 1188 – Published 2 April 1984

ABSTRACT —

At large N, the $\eta\vec{\varphi}^6$ theory is shown to possess a nontrivial ultraviolet fixed point. A new phase is found where asymptotic scale invariance is spontaneously broken and a dynamical mass is generated through dimensional transmutation. At the tricritical limit, the spontaneous breaking of an exact scale invariance at leading N results in the formation of a massless composite Goldstone mode, the dilaton. We compare these results to standard $\frac{1}{N}$ expansion and emphasize the nonperturbative nature of these phenomena.

Bardeen-Moshe-Bander FP



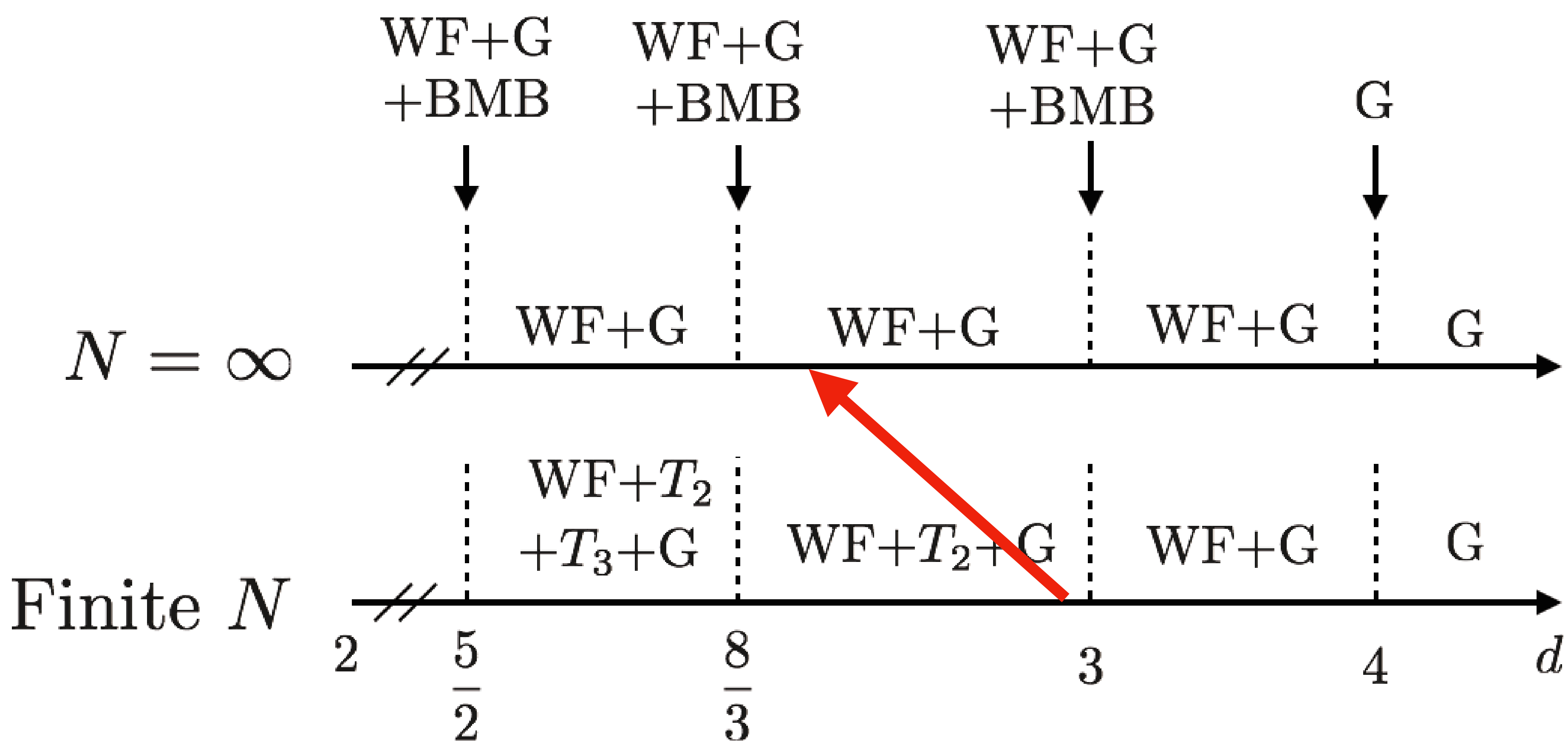


D. F. Litim and M. J. Trott, PRD (2018)

- d=3, N=∞ there exists a line of trictritilal FPs (UV stable) that starts with Gaussian FP and ends with a singular FP that we call BMB FP.

- The BMB FP has small field singularity and scale invariance breaks down at the BMB FP.

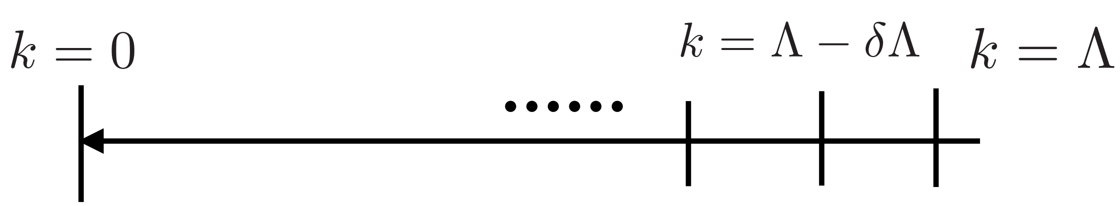
Summary of common wisdom and a simple paradox (S. Yabunaka and B. Delamotte PRL 2017)

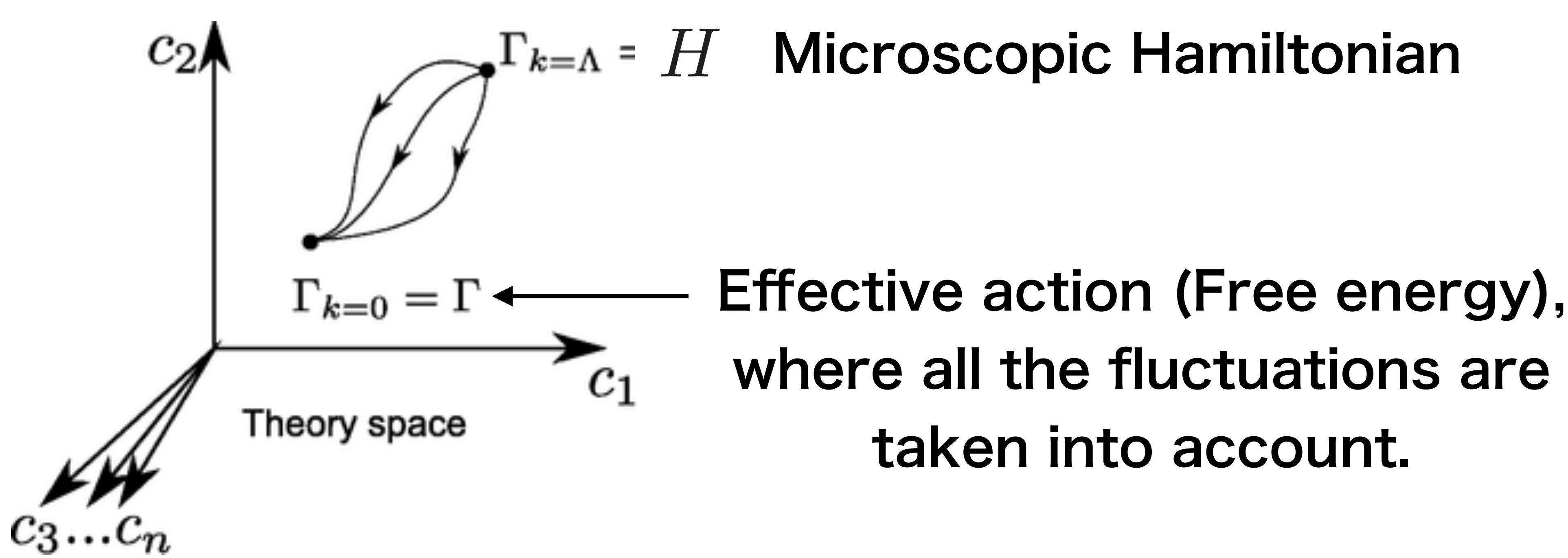


- What occurs if we follow T2 from $(d=3^-, N=1)$ to $(d=2.8, N=\infty)$ continuously as a function of (d,N)?

Non perturbative renormalization group (NPRG)

- Modern implementation of Wilson's RG that takes the fluctuation into account step by step in lowering the cut-off wavenumber k, in terms of wavenumber-dependent effective action Γ_k





NPRG equation

NPRG equation (Wetterich, Phys. Lett. B, 1993) is

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}[\partial_t R_k(q^2)(\Gamma_k^{(2)}[q,-q;\phi] + R_k(q))^{-1}]$$

$$t = \ln(k/\Lambda)$$

Derivative expansion(DE2)

- It is impossible to solve the NPRG equation exactly and we have recourse to approximations,

$$\Gamma_k[\phi] = \int_x \left(\frac{1}{2} Z_k(\rho)(\nabla \phi_i)^2 + \frac{1}{4} Y_k(\rho)(\phi_i \nabla \phi_i)^2 + U_k(\rho) + O(\nabla^4) \right).$$

$$\rho = \phi_i \phi_i / 2$$

- Simpler approximations…LPA(η=0), LPA' approximation

$$Y_k(\rho) = 0$$

$$Z_k(\rho) = \bar{Z}_k$$

↓

$$\eta_t = -\partial_t \log \bar{Z}_k$$

Scaled NPRG equation

- Fixed point is found by nondimensionalized renormalized field

$$\tilde{\phi} = \sqrt{Z_k} k^{\frac{2-d}{2}} \phi \qquad \tilde{\rho} = Z_k k^{2-d} \rho \qquad \tilde{U}_t(\tilde{\rho}) = k^{-d} U_k(\rho)$$

Litim cutoff $y = \dfrac{q^2}{k^2}$ $\quad R_k(q^2) = Z_k k^2 y r(y) \quad r(y) = (1/y - 1)\theta(1-y)$

Under LPA,

$$\partial_t \tilde{U}_t(\tilde{\phi}) = -d\,\tilde{U}_t(\tilde{\phi}) + \frac{1}{2}(d-2)\tilde{\phi}\,\tilde{U}_t'(\tilde{\phi}) + (N-1)\,\frac{\tilde{\phi}}{\tilde{\phi} + \tilde{U}_t'(\tilde{\phi})} + \frac{1}{1 + \tilde{U}_t''(\tilde{\phi})}.$$

Rescaled finite N equation $\quad \tilde{U}_t = N\bar{U}_t \qquad \tilde{\phi} = \sqrt{N}\bar{\phi}$

$$\partial_t \bar{U}_t(\bar{\phi}) = -d\,\bar{U}_t(\bar{\phi}) + \frac{1}{2}(d-2)\bar{\phi}\,\bar{U}_t'(\bar{\phi}) + \left(1 - \frac{1}{N}\right)\frac{\bar{\phi}}{\bar{\phi} + \bar{U}_t'(\bar{\phi})} + \frac{1}{N}\frac{1}{1 + \bar{U}_t''(\bar{\phi})}$$

Polchinski's parametrization

$$\tilde{V}(\tilde{\varrho}) = \tilde{U}(\tilde{\rho}) + \left(\tilde{\phi}_i - \tilde{\Phi}_i\right)^2 /2$$

$$\tilde{\varrho} = \tilde{\Phi}_i \tilde{\Phi}_i /2 = \tilde{\Phi}^2/2 \qquad \tilde{\phi}_i - \tilde{\Phi}_i = -\tilde{\Phi}_i \tilde{V}'(\tilde{\varrho}) = -\tilde{\phi}_i \tilde{U}'(\tilde{\rho})$$

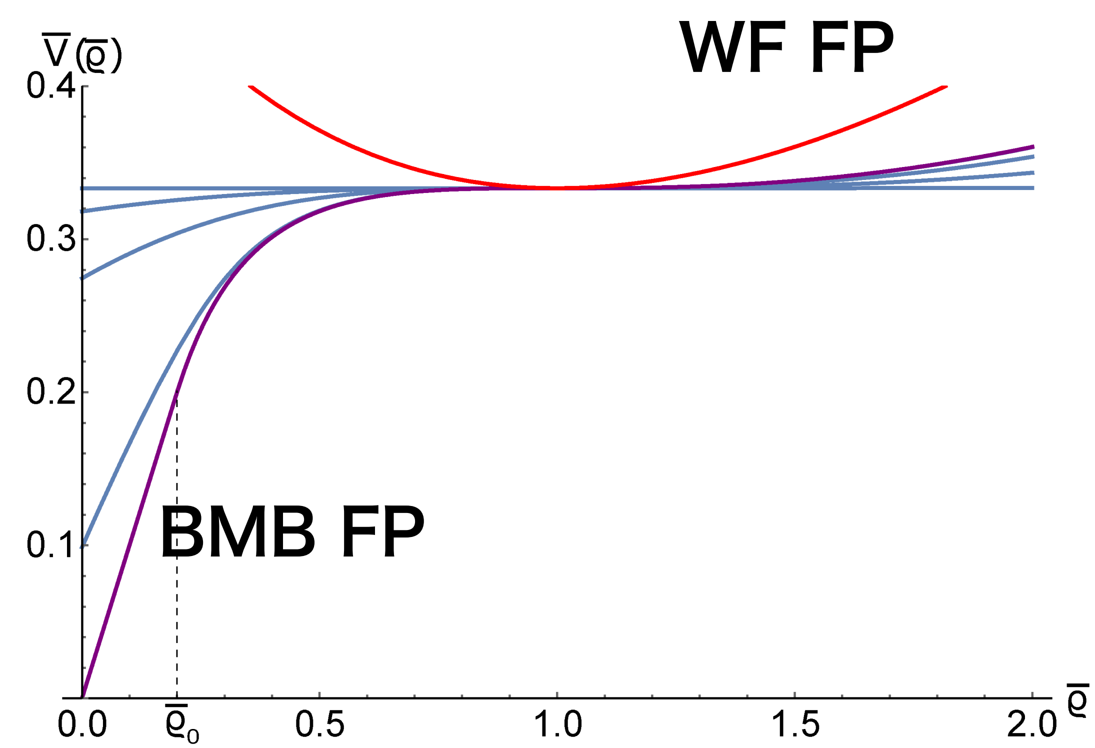
- With rescaling in terms of N $\bar{\varrho} = \tilde{\varrho}/N,\ \bar{V} = \tilde{V}/N$

$$0 = 1 - d\,\bar{V} + (d-2)\bar{\varrho}\bar{V}' + 2\bar{\varrho}\bar{V}'^2 - \bar{V}' - \frac{2}{N}\bar{\varrho}\,\bar{V}''$$

Tricritical FP solutions at N=∞ in LPA

$$\bar{\varrho}_{\pm} = 1 + \frac{\bar{V}'\left(\frac{5}{2} - \bar{V}'\right)}{\left(1-\bar{V}'\right)^2} + \frac{\frac{3}{2}\arcsin\sqrt{\bar{V}'} \pm \sqrt{2/\tau}}{\left(\bar{V}'\right)^{-1/2}\left(1-\bar{V}'\right)^{5/2}}$$

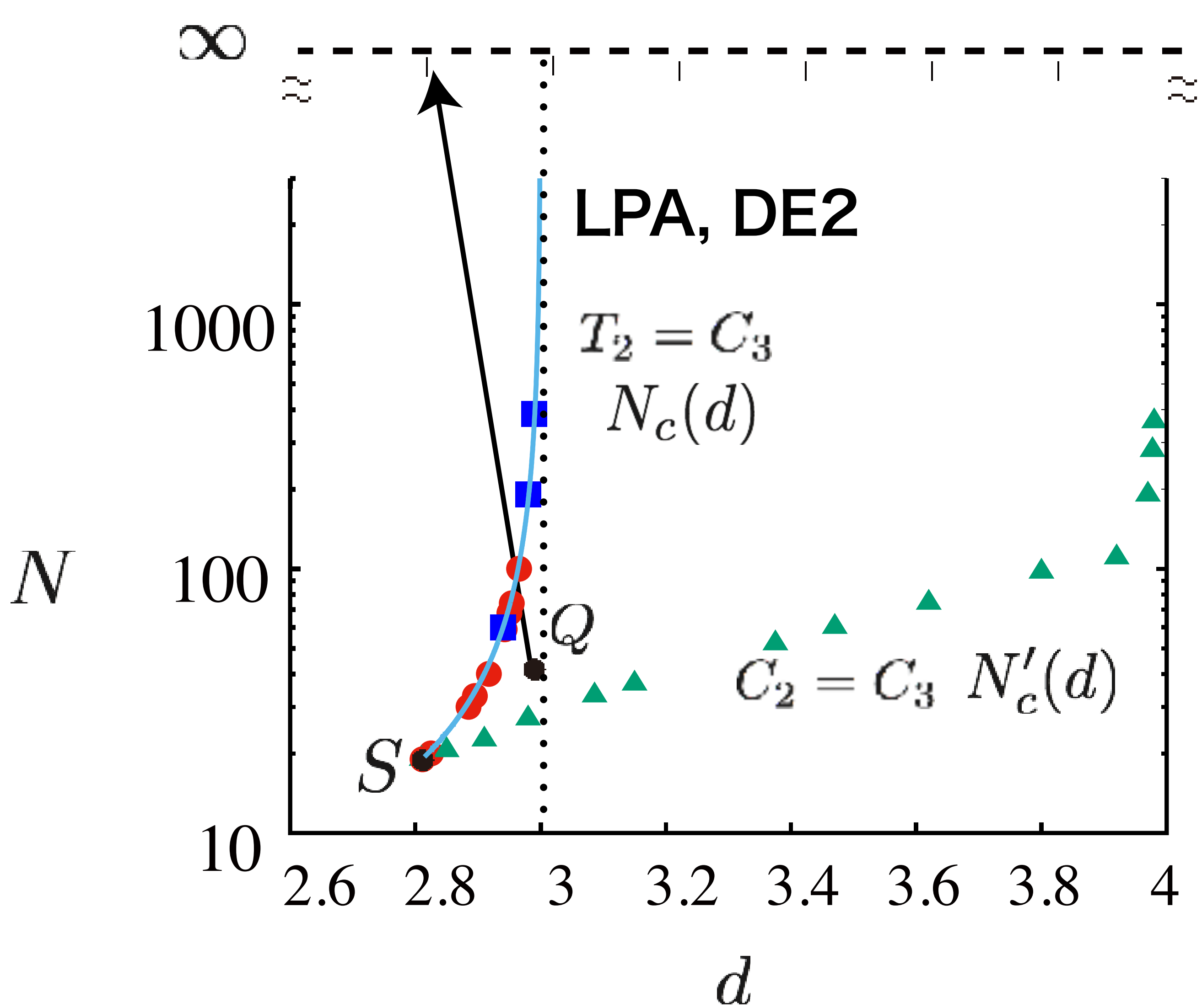
D. F. Litim and M. J. Trott, PRD (2018)



- $\tau = 0$ ⋯Gaussian FP
- $\tau \in [0, \tau_{\rm BMB} = 32/(3\pi)^2]$ ⋯ FPs on the BMB line
- $\tau > \tau_{\rm BMB}$ ⋯No FP defined for all ϱ

FP structure in finite N

We found two nonperturbative fixed points C_2 (two-unstable) and C_3 (three-unstable), which do not coincide with G at any d.



$$N = N_c(d)$$

T_2 and C_3 collide and vanish

$$N = N'_c(d)$$

C_2 and C_3 collide and vanish

The two lines meet at S=(d=2.8,N=19)

The line $N = N_c(d)$

- We can fit this line as $N_C(d)=3.6/(3-d)$.

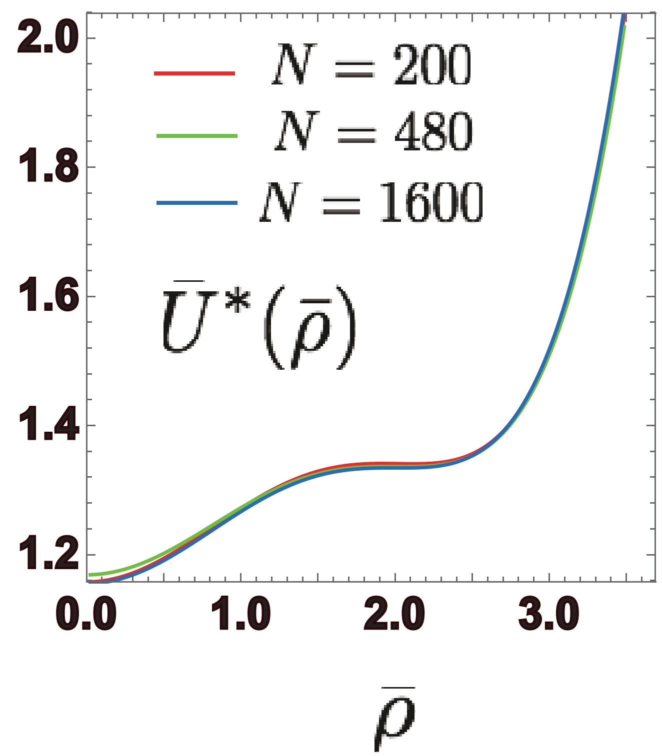
- Pisarski (1982 PRL) and Osborn-Stergiou (2018 JHEP) studied ϕ^6 theory perturbatively (with $\epsilon = 3 - d$ expansion) and showed that T_2 can exist for

$$N \leq N_c^{PT}(d) = \frac{36}{\pi^2(3-d)} \simeq \frac{3.65}{(3-d)}$$

which agrees with our numerical fit within numerical uncertainty.

Shape of effective potential on

$$N = N_c(d)$$



$$T_2 = C_3$$

$$U \to \bar{U} \equiv U/N$$

$$\rho \to \bar{\rho} \equiv \rho/N$$

- Regular function of ρ. Different from the BMB, which shows a cusp.

- T2=C3 approaches a tricritical FP at N=∞ on the BMB line in this limit.

Relation between a path to (d=3, N=∞) and the limiting FP on the BMB line

- Let us consider to follow T2 or C3 on a path toward (d=3,N=∞) : $d = 3 - \alpha/N$

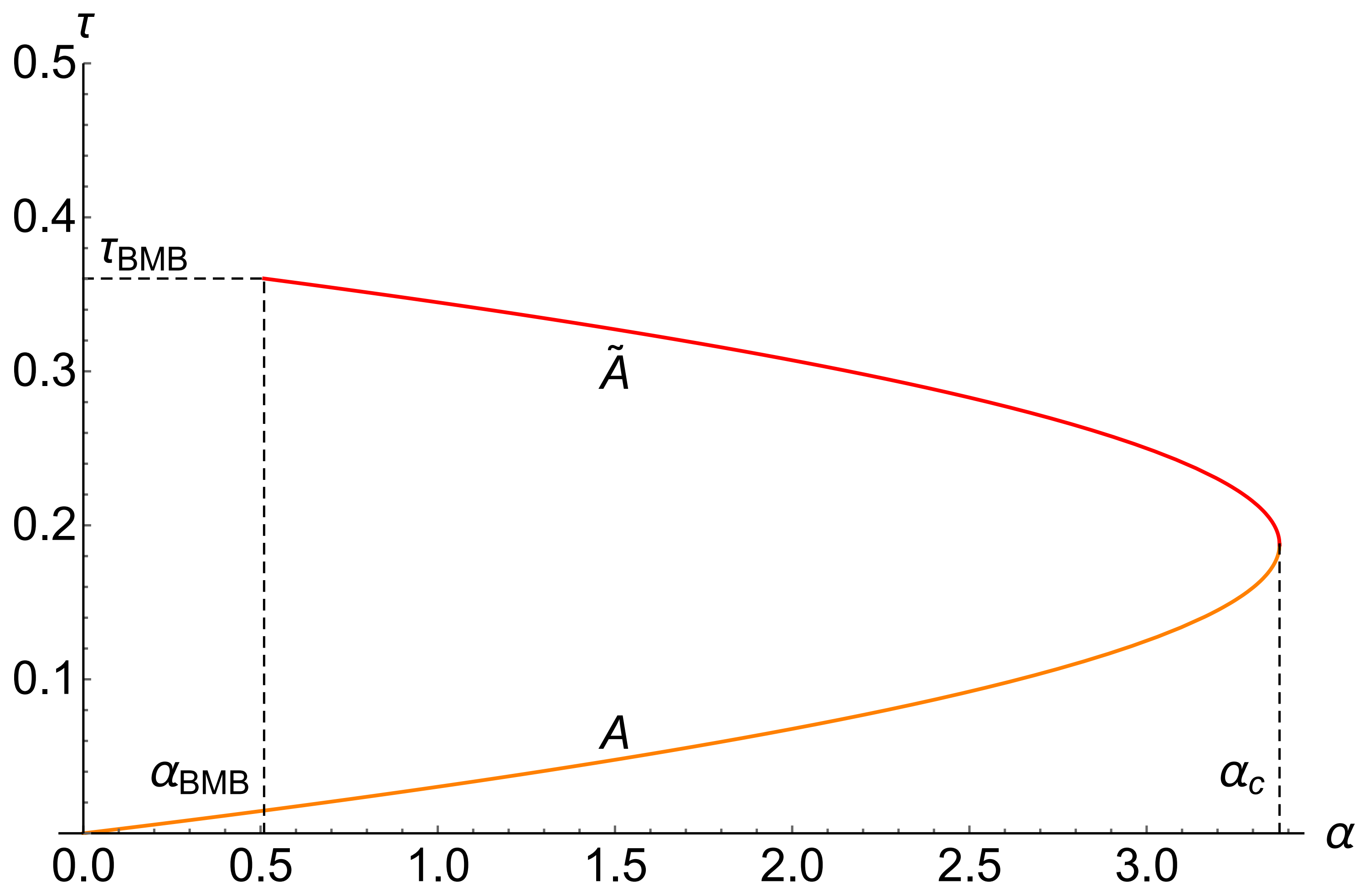
- It approaches a FP on the BMB line and τ is given by
$$\alpha - 36\tau + 96\tau^2 = 0$$

- Derivation: We expand the potential as

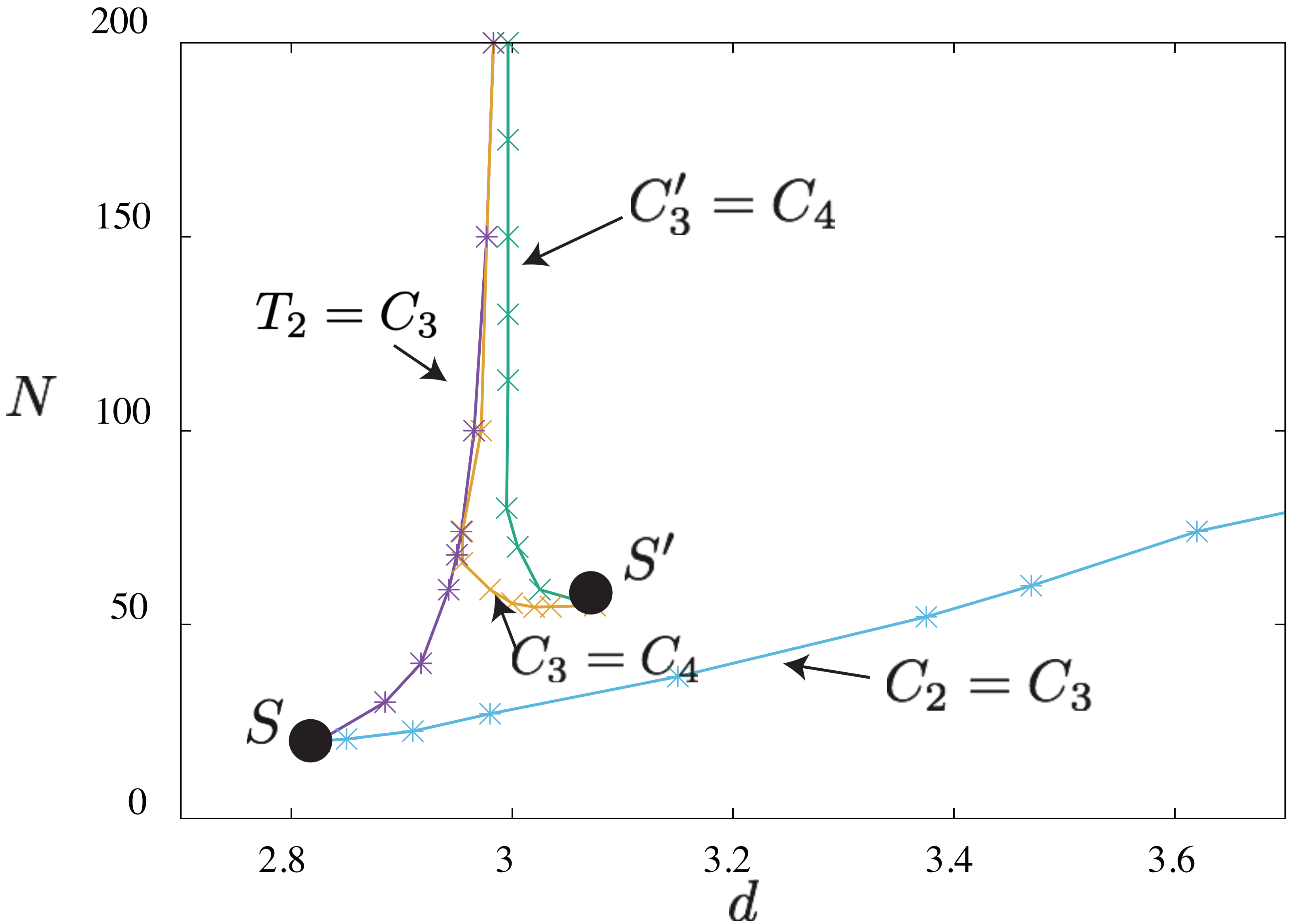
$$\bar{V}_{\alpha,N}(\bar{\varrho}) = \bar{V}_{\alpha,N=\infty}(\bar{\varrho}) + \bar{V}_{1,\alpha}(\bar{\varrho})/N + O(1/N^2).$$

and impose analyticity of $\bar{V}_{1,\alpha}(\bar{\varrho})$ around $\bar{\varrho} = 1$

Plot of τ as a function of α



More FP structures in finite N



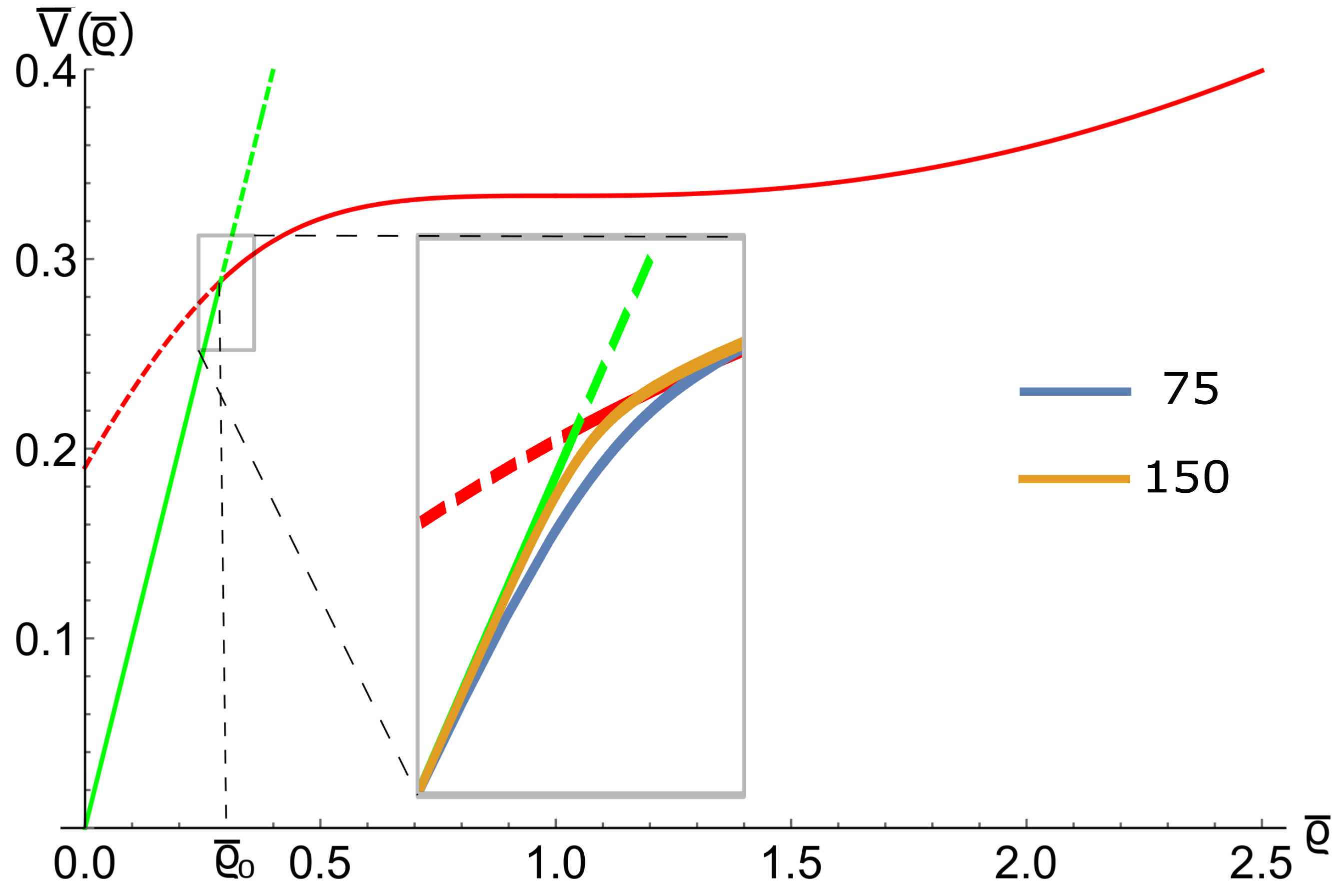
In preparation

Collision between FPs in finite (but large) N

- A (2-unstable) and $\tilde{A}$ (3-unstable) in $d = 3 - \alpha_c/N$
- $\tilde{A}$ and $S\tilde{A}$ (singular and 4-unstable) in $d = 3 - \alpha_{BMB}/N$
- $S\tilde{A}$ and SA (singular and 3-unstable) in $d = 3 - \alpha_c/N$

Construction of singular FPs

$\tau = 0.33$



Boundary layer analysis

- We define the scaled variable $\tilde{\varrho} = N(\bar{\varrho} - \bar{\varrho}_0)$

- At the leading order of 1/N,

$$0 = 1 - 3\bar{V}(\bar{\varrho}_0) + \bar{\varrho}_0 F + 2\bar{\varrho}_0 \left(F^2 - F'\right) - F.$$

- The solution is given as

$$F(\tilde{\varrho}) = V_1 - V_2 \tanh(V_2\, \tilde{\varrho})$$

$$2V_i = V'(\bar{\varrho}_0{}^-) \pm V'(\bar{\varrho}_0{}^+)$$

Summary

- We showed that the BMB line found in d=3 and N=∞ has an intriguing origin at finite N.
- The large N limit in trajectories $d=3-\alpha/N$ allows us to find the BMB line.
- The known BMB line is only the half of the true line of FPs and the other half is made of singular FPs.