Effects of quark chemical potential on analytic structure of Landau-gauge gluon propagator

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Introduction

- The analytic structure of a propagator contains information on the spectrum and in-medium behavior.
- Massive Yang-Mills model: an effective model of the Landau-gauge pure Yang-Mills theory [Tissier and Wschebor 2011] or QCD [Peláez et al. 2014].

→ We study the analytic structure of the gluon propagator in dense QCD using the massive Yang-Mills model.

Complex poles of in-medium propagators

- In-medium propagator with complex poles: an analytically continued propagator $D(z,\vec{k})$ from Matsubara frequencies $D(z=i\omega_n,\vec{k})$ has the following form

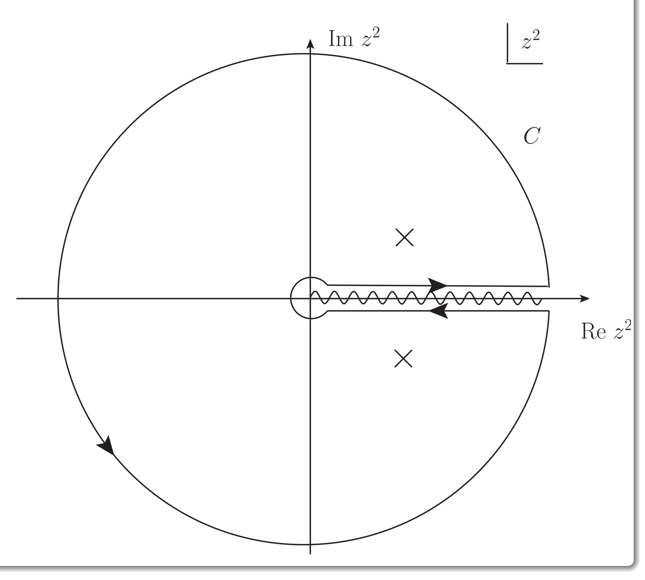
$$D(z,\vec{k}) = \int_0^\infty d\sigma^2 \frac{\rho(\sigma,\vec{k})}{\sigma^2 - z^2} + \sum_{\ell=1}^{n} \frac{Z_\ell(\vec{k})}{w_\ell(\vec{k}) - z^2},$$
$$\rho(\sigma,\vec{k}) = \frac{1}{\pi}\operatorname{Im} D(\sigma + i\epsilon, \vec{k}).$$

- In the vacuum case, such complex poles in the Landau-gauge gluon propagator have been widely discussed. Complex poles represent deviation from an observable particle and could be relevant to confinement, since they invalidate the Källén-Lehmann spectral representation.
- The analytic continuation is in principle not unique. However, as a straightforward generalization of the well-known theorem [Baym and Mermin 1961], the two conditions
  1. $D(z) \to 0$ as $|z| \to \infty$,
  2. $D(z)$ is holomorphic except for the real axis *and a finite number of poles.*

  suffice to determine the correct continuation.
- Counting complex poles: Argument principle

$$N_W(C) := \frac{1}{2\pi i}\oint_C dz^2 \frac{D'(z)}{D(z)} = N_Z - N_P.$$

- Some relations to spectral function: (under suitable assumptions)
  - positive spectral function: $N_W(C) = 0 \Rightarrow N_P = N_Z$.
  - negative spectral function: $N_W(C) = -2 \Rightarrow N_P = N_Z + 2 > 0$.

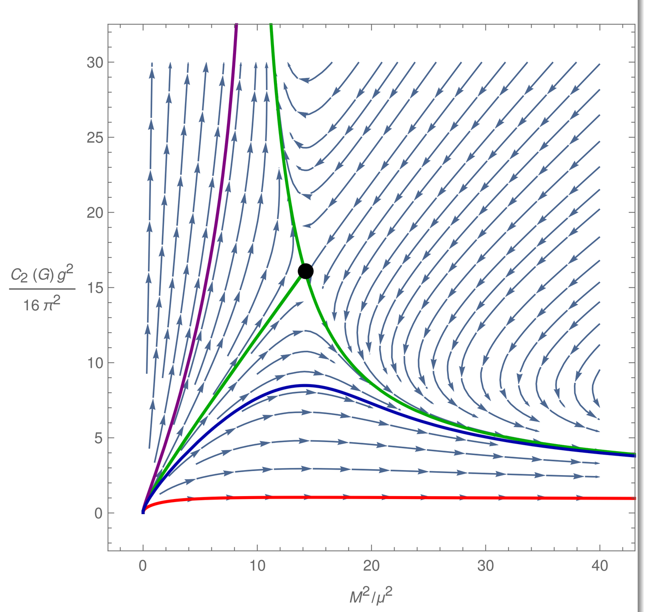


Massive Yang-Mills model

The Landau gauge Yang-Mills theory ($\alpha \to 0$) + gluon mass term

$$\mathcal{L}_{mYM} = \frac{1}{4}F^A_{\mu\nu}F^A_{\mu\nu} + \frac{1}{2\alpha}(\partial_\mu A^A_\mu)^2 + \bar{c}^A\partial_\mu \mathcal{D}_\mu[A]^{AB}c^B + \frac{1}{2}M^2 A^A_\mu A^A_\mu$$

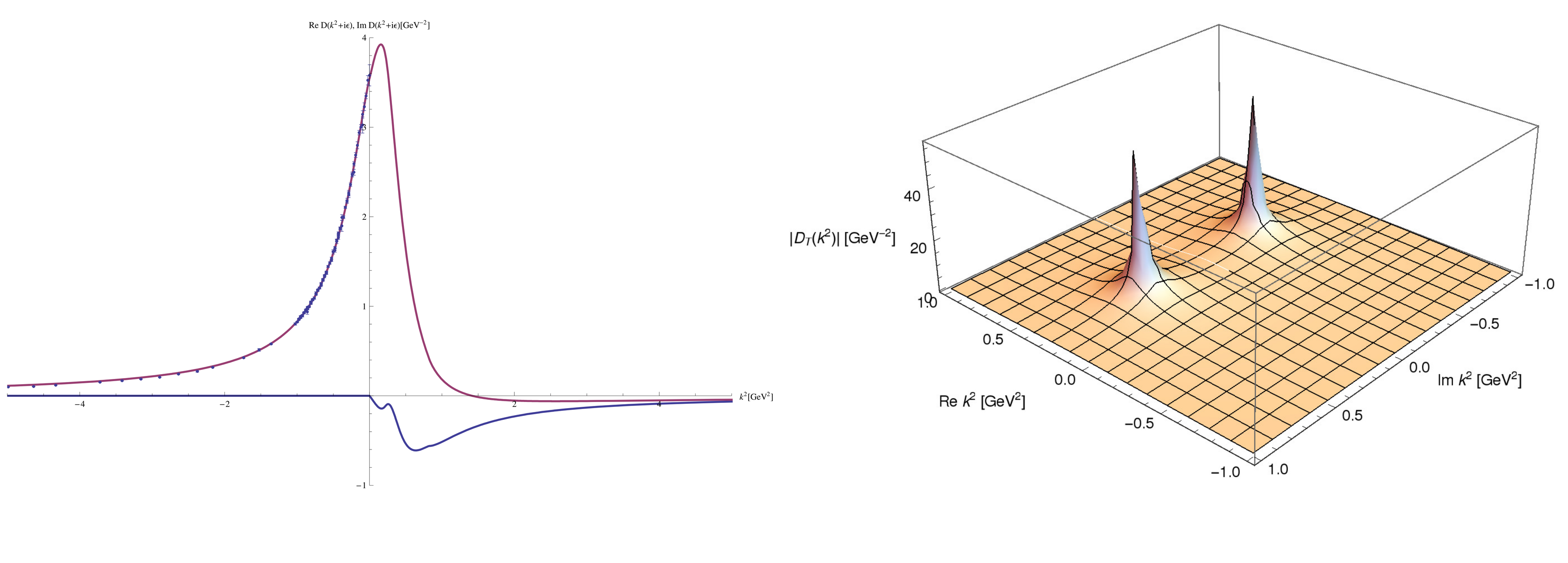
- The gluon and ghost propagator agree strikingly with the lattice results even in the strict one-loop level.
- For some renormalization conditions and parameters, the running coupling has **no Landau pole in all scales.** [Tissier and Wschebor 2011]
- This model with dynamical quarks reproduces the unquenched lattice gluon and ghost propagators as well. [Peláez et al. 2014].



- A similar model in the Landau-deWitt gauge predicts a sensible deconfinement temperature. [Reinosa et al. 2014]
- At finite μ_q, the gluon propagator has been compared to the lattice results in QC$_2$D. [Suenaga and Kojo 2019] (Suenaga-san's poster)

In the vacuum case ($T = \mu_q = 0$), we find:

- the gluon propagator has a negative spectral function and therefore two complex poles ($N_P = 2$) for any parameters (g^2, M^2).
- With $N_F = 2$ dynamical quarks, the gluon propagator has two complex poles ($N_P = 2$) at the best-fit parameter.

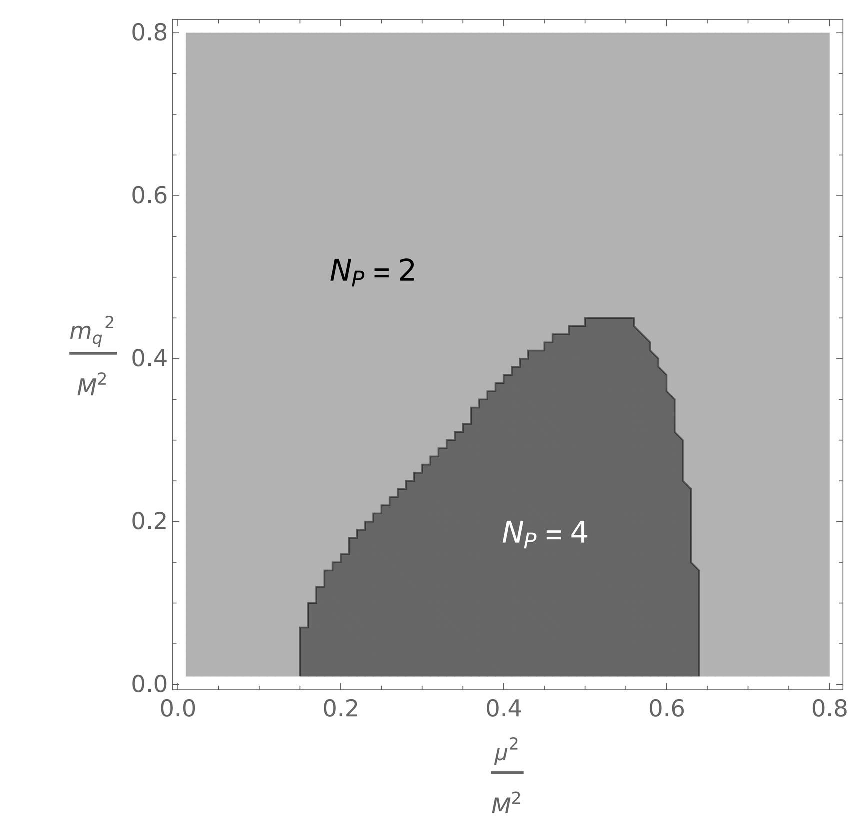


The gluon propagator at $g = 4.1, M = 0.45$ GeV, $G = SU(3)$ in a suitable renormalization condition (used in [Tissier and Wschebor 2011]) in the pure Yang-Mills case. It has one pair of complex poles at $k^2 = 0.23 \pm 0.42i$ GeV^2

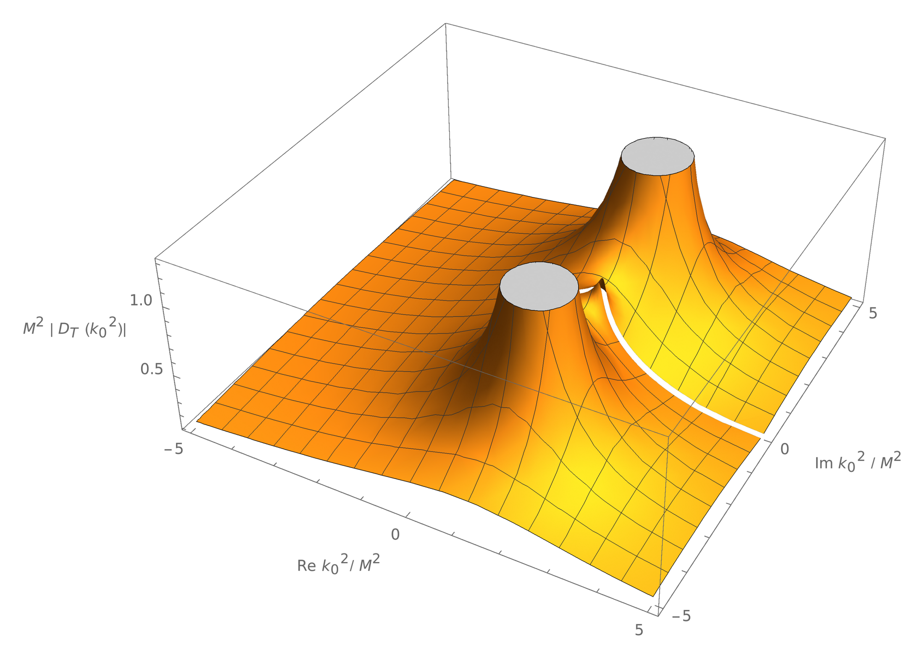
Gluon in the cold quark matter: massive YM at finite μ_q

We investigate the analytic structure of the gluon propagator $D(k_0, \vec{k} \to 0)$ at $T \to 0$, $\mu_q > 0$ and $N_F = 2$ quarks of mass m_q at the best-fit parameter $g = 4.5,\ M = 0.42$ GeV with $G = SU(3)$.

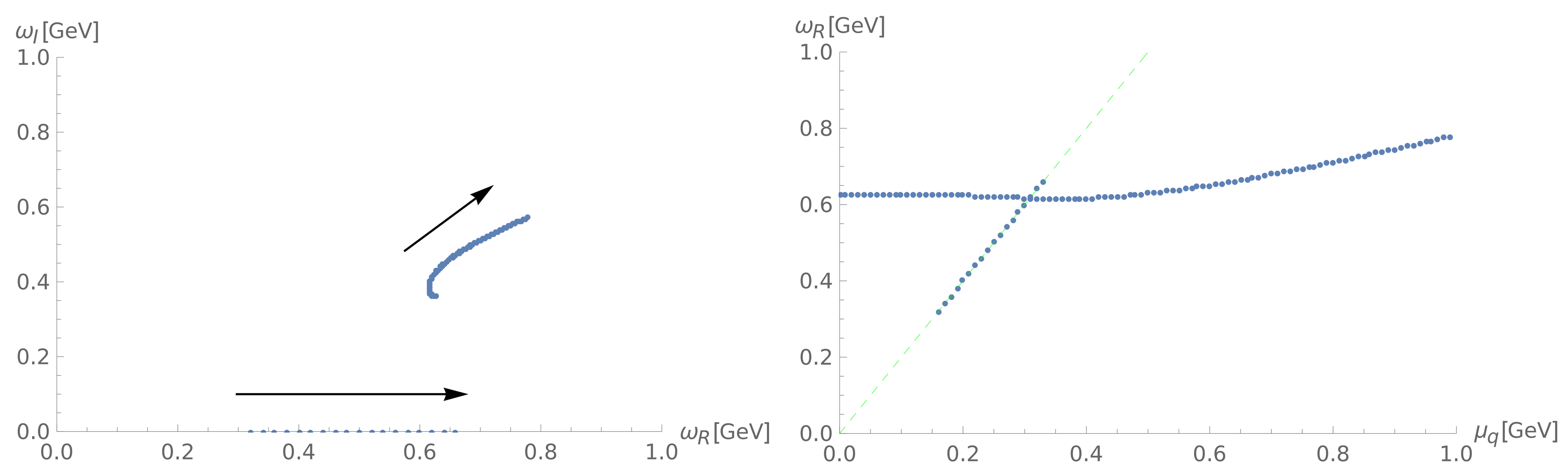
- An $N_P = 4$ region appears between $m_q \lesssim \mu_q \lesssim 0.8M \approx 0.33$ GeV. In this region, the gluon propagator has two pairs of complex conjugate poles with respect to k_0^2.
- In the $N_P = 4$ region, the gluon propagator has almost real complex poles at $\operatorname{Re} k_0 \approx 2\mu_q$.
- With almost real poles, the real part and imaginary part (to be identified with the spectral function) of the gluon propagator $D(k_0^2 + i\epsilon)$ on the real axis have narrow peaks at $k_0 \approx 2\mu_q$.
- The ratio ω_I/ω_R of a complex pole $k_0 = \omega_R + i\omega_I$, $(\omega_R > 0,\ \omega_I > 0)$ tends to increase as μ_q increases, except for the almost real poles. ("less particlelike")



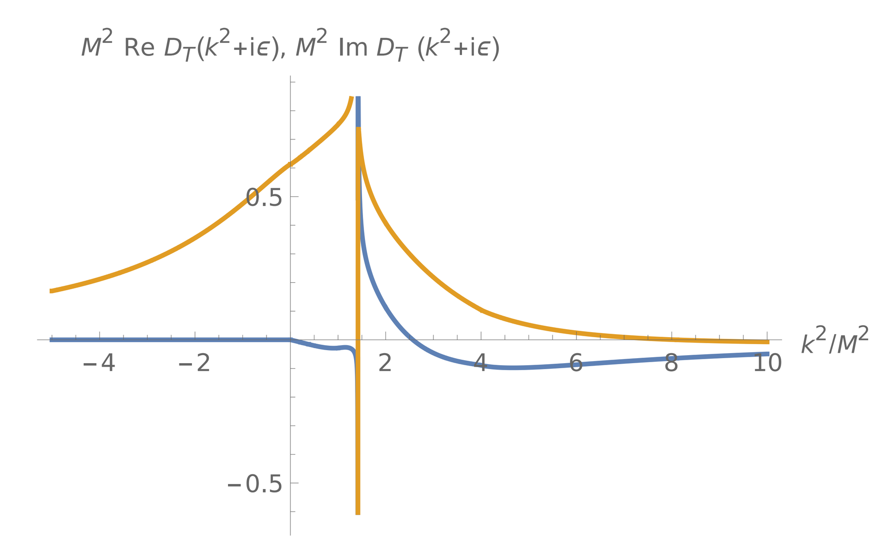
The number of complex poles $N_P = -N_W(C)$ computed numerically



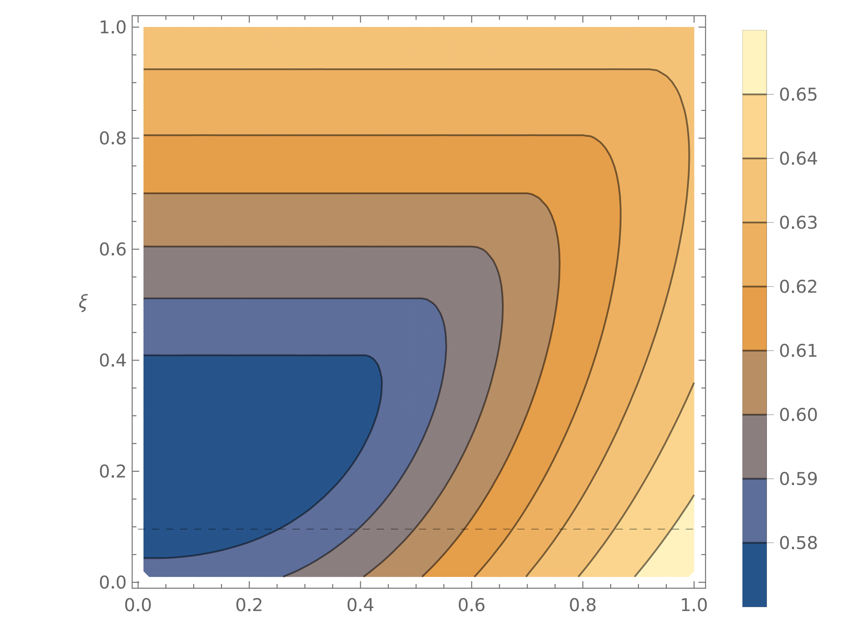
Modulus of the gluon propagator at $m_q = 0.13$ GeV and $\mu_q = 0.25$ GeV on the complex k_0^2 plane.



Location of poles $k_0 = \omega_R + i\omega_I$ at $m_q = 0.13$ GeV. (Left) plots of (ω_R, ω_I) (Right) ω_R vs μ_q



Real and imaginary parts of the gluon propagator at $m_q = 0.13$ GeV and $\mu_q = 0.25$ GeV



Contour plot of $\max \omega_I/\omega_R$ of pole $k_0 = \omega_R + i\omega_I$

- The quark chemical potential significantly affects the gluon propagator around $k_0 \approx 2\mu_q$, which is the least energy for the quark-pair production at $\vec{k} = 0$.
- The appearance of the new pair of almost real poles suggests a transition in the confined dynamics or, if the almost real pole are artifacts, would correspond to a long-lived quasi-particle at $\omega_R \approx 2\mu_q$.

Summary

- The uniqueness of analytic continuation of the Matsubara propagator holds in a class of functions that vanish at infinity and are holomorphic except for at most a finite number of complex poles and singularities on the real axis.
- At $T = \mu_q = 0$, in the massive Yang-Mills model (with $N_F = 0$ or with $N_F = 2$ quarks at the best-fit parameter), an effective theory of the Landau gauge Yang-Mills theory, **the gluon propagator has one pair of complex conjugate poles** in the one-loop level.
- The quark chemical potential significantly affects the gluon propagator around $k_0 \approx 2\mu_q$. In particular, for $m_q \lesssim \mu_q \lesssim 0.8M \approx 0.33$ GeV, the gluon propagator has **a new pair of complex conjugate poles near the real axis** at $\operatorname{Re} k_0 \approx 2\mu_q$ and $\operatorname{Im} k_0 \approx 0$. This suggests a transition of confined degrees of freedom or appearance of quasi-particle at $\omega_R \approx 2\mu_q$.