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Quantum phase transition in 3dim SQED and Lefschetz thimble analysis (work in progress)

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Abstract

- Quantum phase transition in *3d* $\mathcal{N}=4$ SQED is understood from the viewpoint of Lefschetz thimble analysis
- Resurgence structure for large-flavor expansion is discussed
- A class of Borel singularities found in this model should appear universally in large-flavor gauge theories

1. Resurgence and aim of this work

Resurgence

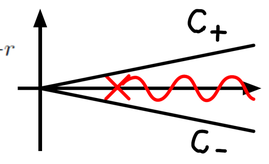
In a wide range of QFTs, it is expected that

Perturbation theory		Non-perturbative effects
$\sim \sum_{n=0}^{\infty} c_n g^n$		Phase transitions $\sim e^{-1/g}$
↓		↑
Borel resummation Ambiguities caused by Borel singularities	Resurgence structure ⟷ Cancel	Lefschetz thimble analysis Ambiguities caused by stokes phenomena

Borel resummation

- A resummation technique to handle divergent formal series
- Singularities or branch cuts hinder the resum, yielding ambiguities
- Considering the Borel resum of partition functions, singularities are associated with saddles

$$\Phi(g) = \sum_{n=0}^{\infty} c_n g^{n+r}$$



$$\mathcal{S}\Phi(g) = \int_C \mathrm{d}t\ e^{-t/g}\mathcal{B}\Phi(t)$$

$$\mathcal{B}\Phi(t) = \sum_{n=0}^{\infty} \frac{c_n}{\Gamma(n+r)} t^{n+r-1}$$

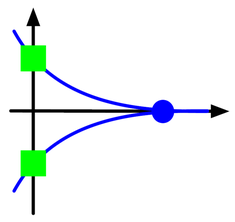
$$\mathcal{B}Z(t) \simeq \sum_{n=0}^{\infty} \left(\frac{t}{\tilde{S}(\sigma_*)}\right)^n = \frac{1}{1-\frac{t}{\tilde{S}(\sigma_*)}}$$

Lefschetz thimble

- "Steepest descents" in configuration space
- Changing the phase of parameters, dominant saddles may jump, yielding ambiguities
- The jumps may cause "discontinuity" of free energy

$$\mathcal{J}_n : \frac{\mathrm{d}\sigma(t)}{\mathrm{d}t} = \overline{\frac{\mathrm{d}S}{\mathrm{d}\sigma}}, \quad \sigma(-\infty) = \sigma_n$$

$$\mathcal{K}_n : \frac{\mathrm{d}\sigma(t)}{\mathrm{d}t} = -\overline{\frac{\mathrm{d}S}{\mathrm{d}\sigma}}, \quad \sigma(-\infty) = \sigma_n$$



Aim of this work

To describe a phase transition in QFT in terms of resurgence

2. SQED3 and its quantum phase transition

Setups

Model: *3d* $\mathcal{N}=4$ *U(1)* SUSY gauge theory + *2N* hypermultiplets with charge *1*

Parameters: FI parameter η and flavor mass m

Free energy

- Partition function on S^3 can be computed exactly by the SUSY localization method [1,2,3]
- The result depends only on η, m
- In the limit $N\to\infty$ w/ λ=fixed, the second derivative of the free energy jumps [4]

$$Z = \int_{-\infty}^{\infty} \mathrm{d}\sigma \frac{e^{-i\eta\sigma}}{\left[2\cosh\frac{\sigma+m}{2}\cdot 2\cosh\frac{\sigma-m}{2}\right]^N}$$

$$\frac{\mathrm{d}^2F}{\mathrm{d}\lambda^2} = \begin{cases} \frac{N}{1+\lambda^2}\left(1+\frac{\cosh m}{\sqrt{1-\lambda^2\sinh^2 m}}\right) & \lambda<\lambda_c \\ \frac{N}{1+\lambda^2} & \lambda\geq\lambda_c \end{cases}$$

$$\lambda_c \equiv \frac{1}{\sinh m}$$

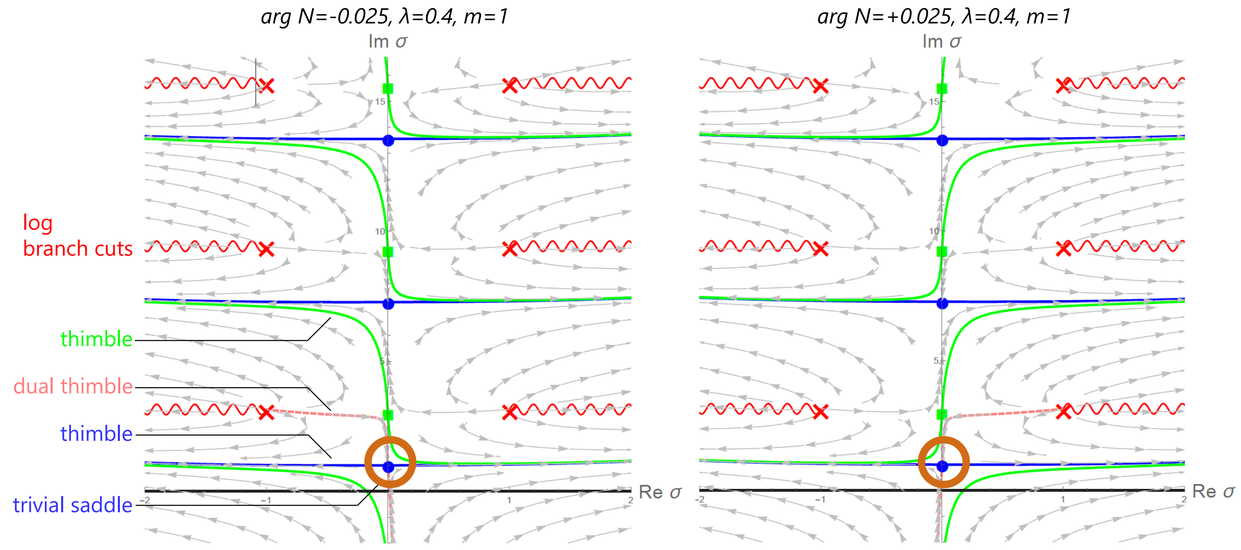
3. Quantum phase transition and Lefschetz thimbles

Results

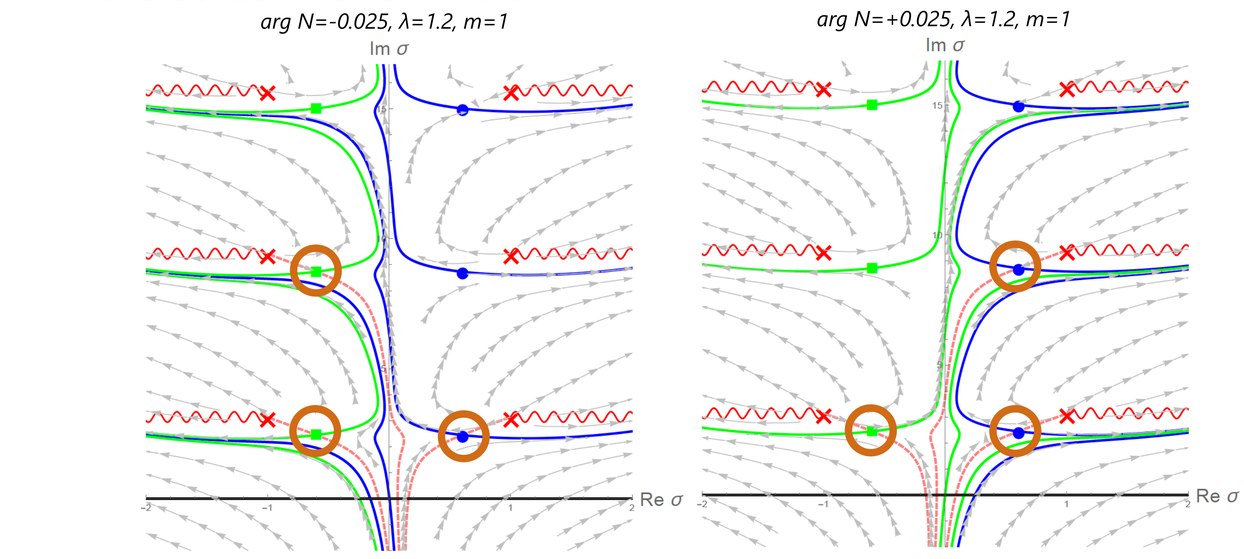
- The quantum phase transition is understood as

trivial saddle only w/o stokes phenomenon ⟷ infinite number of saddles w/ stokes phenomenon

Thimble structure for $\lambda<\lambda c$



Thimble structure for $\lambda>\lambda c$

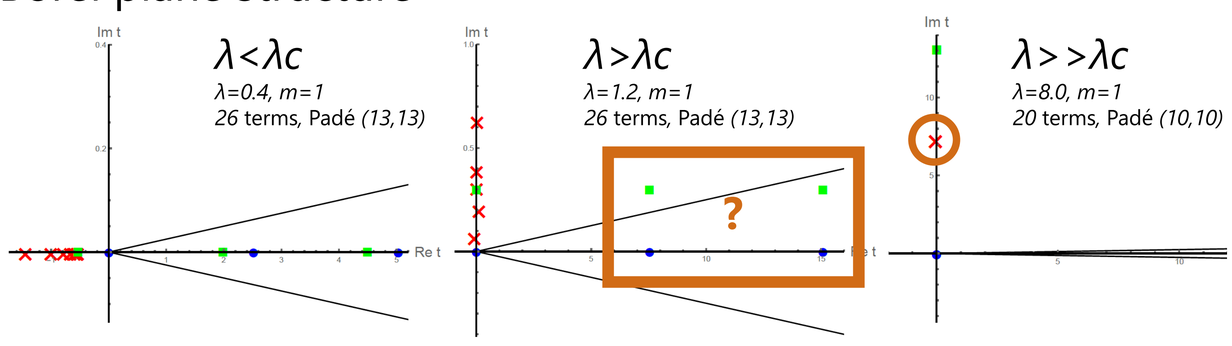


4. Borel transformation in large-flavor expansion

Results

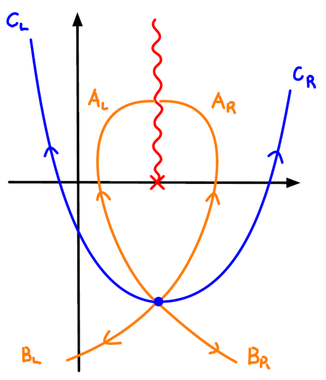
- A class of the Borel singularities should appear universally in large-flavor gauge theories

Borel plane structure



Toy model

- Singularities found for $\lambda>>\lambda c$ are captured by a toy model: $S(\sigma) = N\left[-i\lambda\sigma + \ln(\sigma - c)\right]$
- There are infinite number of Borel singularities associated with saddles on infinite number of Riemann surfaces
- Such a structure should appear at least in SUSY observables of large-flavor gauge theories w/
  - *3*dim $\mathcal{N}=2$ SUSY on S^3, rank(*G*)=*1*
  - *2*dim $\mathcal{N}=(2,2)$ SUSY on S^2, rank(*G*)=*1*



5. Conclusion and future works

- A quantum phase transition in *3d* $\mathcal{N}=4$ SQED is discussed in terms of resurgence
  - the phase transition is interpreted as a stokes phenomenon
  - A class of Borel singularities found in this model should appear universally in large-flavor gauge theories
- Description in terms of resurgence is not yet completed
  - Borel singularities should be found on the positive real axis

References

[1] Anton Kapustin, Brian Willett, and Itamar Yaakov. Exact Results for Wilson Loops in Superconformal Chern-Simons Theories with Matter. JHEP, 03:089, 2010.

[2] N. Hama, K. Hosomichi, and S. Lee, "Notes on SUSY Gauge Theories on Three-Sphere," *JHEP* **1103** (2011) 127.

[3] Daniel L. Jafferis. The Exact Superconformal R-Symmetry Extremizes Z. JHEP, 05:159, 2012.

[4] J. G. Russo and M. Tierz, "Quantum phase transition in many-flavor supersymmetric QED3," *Phys. Rev. D* **95** no. 3, (2017) 031901.