Poster No. 10

Finite-Density Massless Two-Color QCD at Isospin Roberge-Weiss Point and 't Hooft Anomaly

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Furusawa, Tanizaki, Itou, PRResearch 2, 033253 (2020)

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Outline

1. Introduction
   massless 2-color QCD at isospin RW point
2. 't Hooft anomaly at isospin RW point NEW
3. Applications NEW
   Finite-T&-μ phase diagram, Similarity to AF
4. Summary & discussion

Massless 2-color QCD

SU(2) gauge theory w/ 2 massless Dirac fermions (fund. rep.)

$$\frac{1}{2g^2}\mathrm{tr}(F_{\mu\nu}[a])^2 + \bar{u}\gamma^\mu(\partial_\mu + ia_\mu)u + \bar{d}\gamma^\mu(\partial_\mu + ia_\mu)d$$

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Kogut, Stephanov, Toublan (1999), Kogut, Stephanov, Toublan, Verbaarschot, Zhitnitsky (2000)

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Good toy model for 3-color QCD at finite-T &-μ

Shows asymptotic freedom, ChSB, ⋯

No sign problem at finite density

⇒Lattice simulation is available.

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Lattice simulation needs much computational cost.

⇒ 't Hooft anomaly matching

Nakamura (1984), Hands, Kogut, Lombardo, Morrison (1999),
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Anomalies in QCD-like theories

Center symmetry is key for anomaly at finite T.

e.g., Pure Yang-Mills gauge theory w/ theta term

Gaiotto, Kapustin, Komargodski, Seiberg (2017)

Anomaly btw. center and CP symmetries

→ Constraint on deconfinement & CP-breaking temperatures

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Fund. matters usually break the center.

⇒ Twisted b.c. of matter fields

e.g., Zn QCD, QCD w/ imaginary μ @ RW pt., ⋯

Shimizu, Yonekura (2018), Tanizaki, Kikuchi, Misumi, Sakai (2018), Yonekura (2019), ⋯

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→ 2cQCD w/ imaginary isospin chemical potential

Imaginary isospin chemical potential

Fermion action:

$$\mu_I = \theta_I/\beta$$

$$\bar{u}\gamma^\mu(\partial_\mu + ia_\mu)u + \bar{d}\gamma^\mu(\partial_\mu + ia_\mu)d + \underline{i\mu_I(\bar{u}\gamma^0 u - \bar{d}\gamma^0 d)}$$

Boundary condition:

$$u(\tau+\beta) = -\,u(\tau), \qquad d(\tau+\beta) = -\,d(\tau),$$

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Absorbed into B.C.

Boundary condition:

$$u(\tau+\beta) = -e^{i\theta_I}u(\tau), \qquad d(\tau+\beta) = -e^{-i\theta_I}d(\tau),$$

Field redefinition:

$$u(\tau) \to e^{i\theta_I\tau/\beta}u(\tau) \quad d(\tau) \to e^{-i\theta_I\tau/\beta}d(\tau)$$

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Boundary condition:

$$u(\tau+\beta) = -\,e^{i\theta_I}u(\tau), \qquad d(\tau+\beta) = -\,e^{-i\theta_I}d(\tau),$$

$\Rightarrow \theta_I$ is periodic.

SU(2) gauge invariance

$$\theta_I \sim \theta_I + \pi \qquad u(x) \sim -u(x),\ d(x) \sim -d(x),$$

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Isospin Roberge-Weiss Point

$\theta_I = \pi/2$ is special (isospin RW point).

The boundary condition is invariant under $u(x) \leftrightarrow d(x)$ up to SU(2) gauge transformation.

$$\theta_I = \pi/2 \rightarrow -\pi/2 \sim \pi/2 \qquad \left(\theta_I \sim \theta_I + \pi \right)$$

Emergent symmetry $[(\mathbb{Z}_2)_{\text{center}}]$ at the isospin RW point

$$u(x) \leftrightarrow d(x)$$

Gauging flavor symmetry

To find anomaly, let's gauge flavor subgroup at T, μ, & $\theta_I = \pi/2$:

$$\mathrm{U(1)_B} \times \mathrm{U(1)_{L,3}} \subset G_{\mu,\theta_I}$$

$\mathrm{U(1)_B}$: $\mathrm{U(1)}$ baryon symmetry ⇐ Gauged by A_B

$\mathrm{U(1)_{L,3}}$: left-handed isospin $\mathrm{U(1)}$ symmetry ⇐ Gauged by $A_\mathrm{L,3}$

$$u_L \to e^{i\lambda_3/2} u_L,\ d_L \to e^{-i\lambda_3/2} d_L,$$

[See our paper for complete discussion.]

Furusawa, Tanizaki, Itou, PRResearch 2, 033253 (2020)

’t Hooft anomaly at isospin RW point

$A_{\rm B}$ & $A_{\rm L,3}$ violates $(\mathbb{Z}_2)_{\rm center}$ symmetry:

$$Z_{\rm QC_2D}[A_B, A_{L,3}] = Z^{\rm Sym.}_{\rm QC_2D}[A_B, A_{L,3}]$$

μ_I in action ↑ μ_I in boundary condition

$$\begin{cases} u(\tau) \to e^{i\theta_I \tau/\beta} u(\tau) \\ d(\tau) \to e^{-i\theta_I \tau/\beta} d(\tau) \end{cases}$$

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$$Z_{\mathrm{QC_2D}}[A_B, A_{L,3}] = Z^{\mathrm{Sym.}}_{\mathrm{QC_2D}}[A_B, A_{L,3}] \times \exp\left[-\frac{i}{4\pi}\int A_B \wedge dA_{\mathrm{L},3}\right]$$

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↑ Redefinition generates additional phase!

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't Hooft anomaly at isospin RW point

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$(\mathbb{Z}_2)_{\mathrm{center}}$ w/ background gauge fields:

$$Z_{\mathrm{QC_2D}}[A_B, A_{L,3}] \to Z_{\mathrm{QC_2D}}[A_B, A_{L,3}] \times \exp\left[\frac{i}{2\pi}\int A_B \wedge dA_{\mathrm{L},3}\right]$$

Mixed 't Hooft anomaly btw. $(\mathbb{Z}_2)_{\mathrm{center}} \ltimes \mathrm{U}(1)_{\mathrm{B}} \times \mathrm{U}(1)_{\mathrm{L},3}$

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’t Hooft anomaly matching

’t Hooft anomaly is invariant under RG flow.

⇒2cQCD must be nontrivial at IR:

○ SSB, CFT, Topological order, ✕ unique gapped

’t Hooft (1980), Frishman, Schwimmer, Banks, Yankielowicz (1981), Coleman, Grossman (1982)

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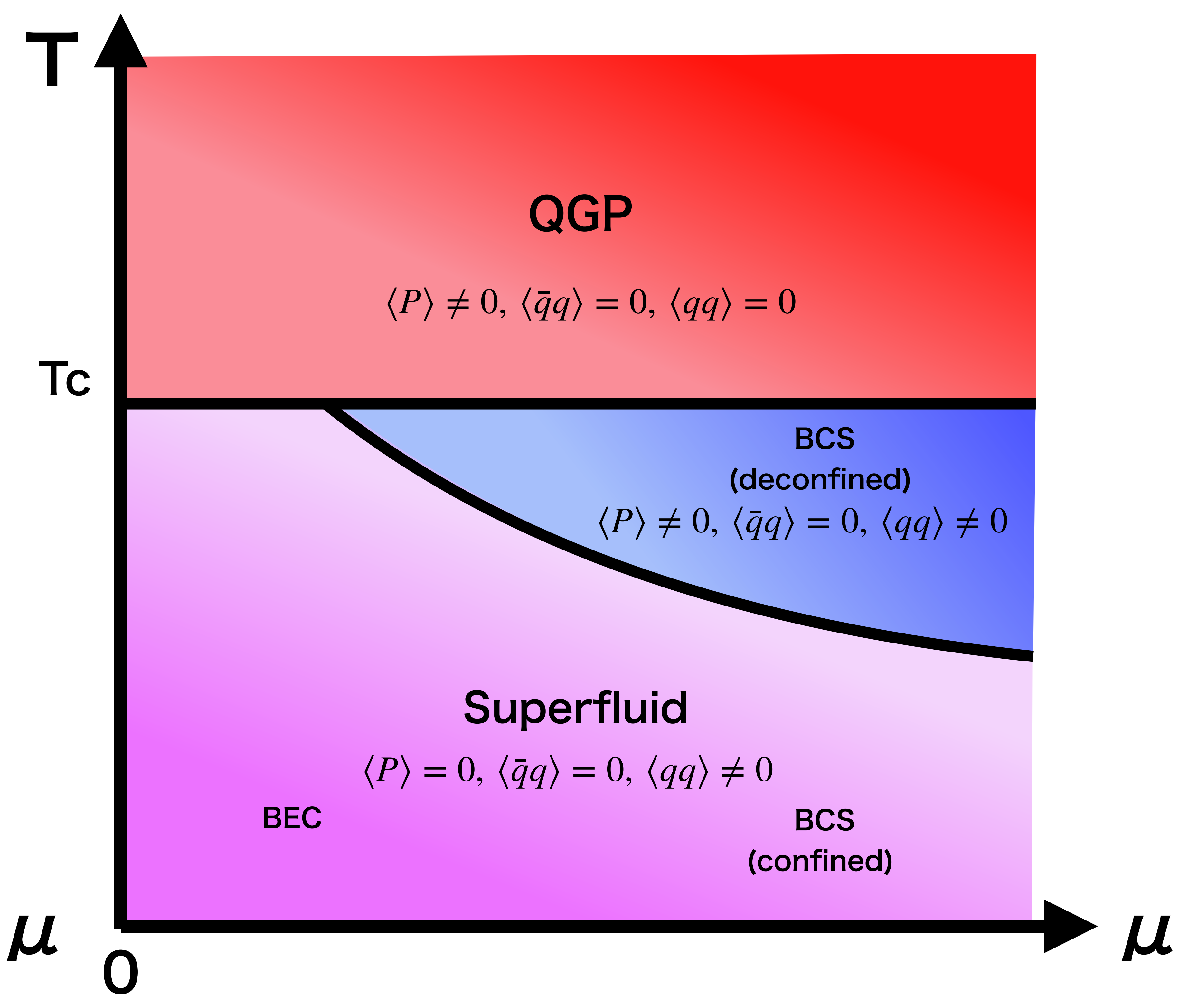
One of $(\mathbb{Z}_2)_{\text{center}} \ltimes U(1)_B \times U(1)_{L,3}$ must be broken:

- $(\mathbb{Z}_2)_{\text{center}}$: Quark gluon plasma
- $U(1)_B$: Baryon superfluidity
- $U(1)_{L,3}$: Chiral symmetry breaking

Possible phase diagram

$T_{\mathrm{QGP}} = T_{\mathrm{BSF}}$



T_{BSF}

Coexisting phase is allowed.

T_{QGP}

$T_{\mathrm{QGP}} \leq T_{\mathrm{ChSB}}$ or T_{BSF}

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Similarity to antiferromagnet

2+1D CP1 model (QFT of antiferromagnets):

$$|(\partial_\mu - ib_\mu)\phi|^2 + r|\phi|^2 + \lambda|\phi|^4 + \lambda_{EP}|\phi^\dagger\sigma^z\phi|^2$$

ϕ: 2-comp. complex scalar, b_μ: U(1) dynamical gauge field

Spin vector: $S_\alpha \sim \phi^\dagger\sigma^\alpha\phi$

Easy-plane potential breaks

$\mathrm{SU}(2)_{\mathrm{spin}} \to \mathrm{O}(2)_{\mathrm{spin}}$.

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“Hidden” global symmetry: $\mathrm{U}(1)_{\mathrm{magnetic}}$

Current: $J_\mu = \epsilon^{\mu\nu\rho}\partial_\nu b_\rho/(2\pi)$ Charged object: monopole

Metlitski,Thorngren (2018), Komargodski, Sulejmanpasic, Unsal (2018), Komargodski, Sharon, Thorngren, Zhou, (2019).

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$\mathrm{O(2)_{spin}} \times \mathrm{U(1)_{magnetic}}$ has the same anomaly as 2cQCD.

Identification: $\left(\begin{array}{l} \mathrm{O(2)_{spin}} \leftrightarrow (\mathbb{Z}_2)_{\mathrm{center}} \ltimes \mathrm{U(1)_{L,3}} \\ \mathrm{U(1)_{magnetic}} \leftrightarrow \mathrm{U(1)_B} \end{array}\right.$

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Novel quantum criticality: SO(5) at $\lambda_{\mathrm{EP}} = 0$ & $r = r_{\mathrm{c}}$

$\Rightarrow$ SO(5) symmetry in 2cQCD?

Wang, Nahum, Metlitski, Xu, Senthil (2017)

Summary & Discussion

2 color QCD with imaginary isospin chemical potential.

- $(\mathbb{Z}_2)_{\rm center}$ symmetry at isospin RW point
- $(\mathbb{Z}_2)_{\rm center} \ltimes U(1)_B \times U(1)_{L,3}$ anomaly at finite T & μ

Nonperturbative constraint from anomaly matching

- Finite-T &-μ phase diagram $T_{\rm QGP} \leq T_{\rm ChSB}$ or $T_{\rm BSF}$
- The same anomaly as 2+1D antiferromagnets
  ⇒ Unconventional quantum critical point at finite T?
  [See our paper for details. PRResearch 2, 033253 (2020)]

Sign problem free ⇒ testable by lattice simulation!

Poster Preview

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2c QCD w/ imaginary isospin chemical potential μ_I

(sign-problem-free deformation)

Emergent $\mathbb{Z}_2$ symmetry for special μ_I (isospin RW point)

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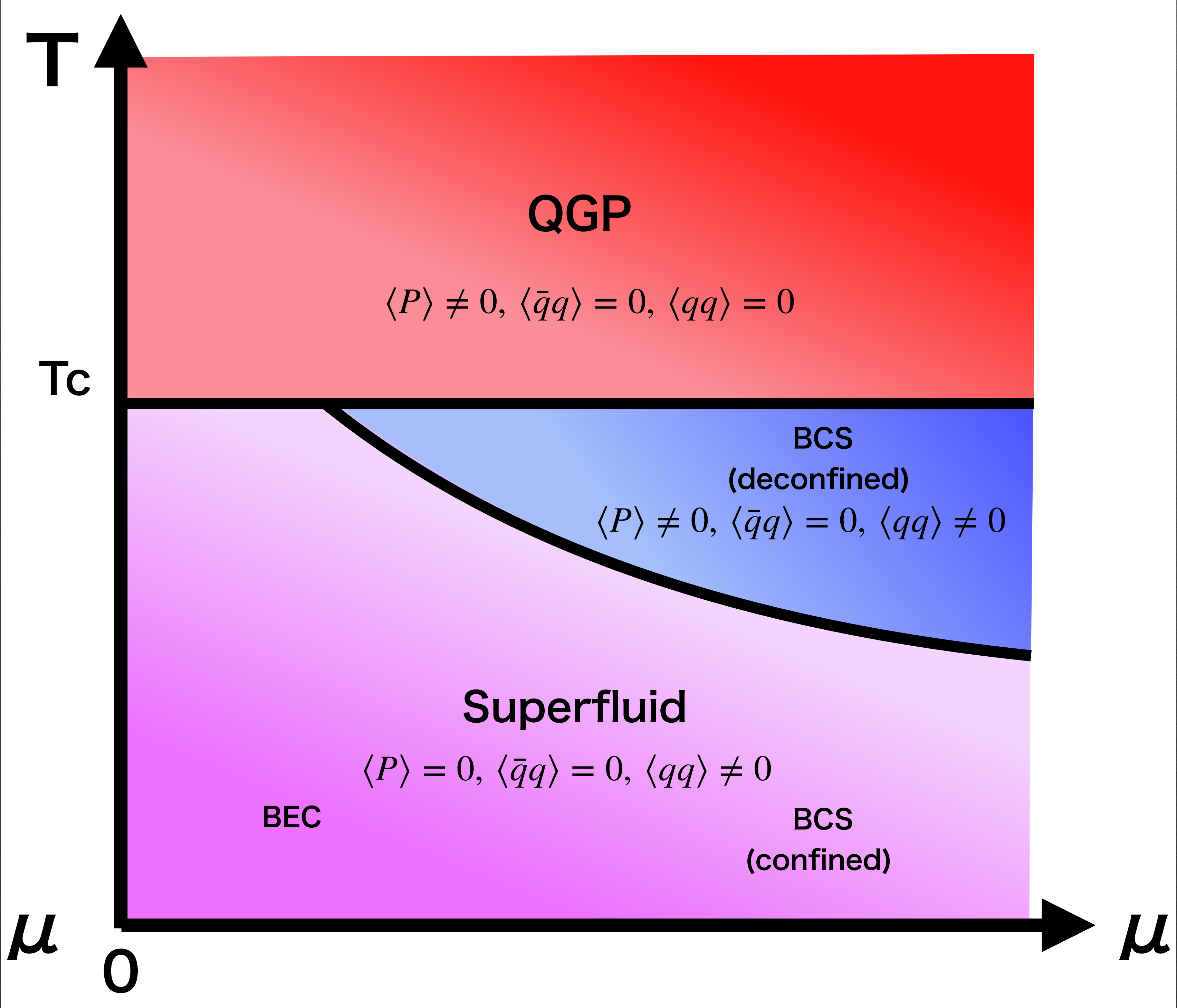
’t Hooft anomaly btw. $\mathbb{Z}_2$ & flavor symmetries

Present even at finite temperature & density

Applications

Finite-T and -μ phase diagram



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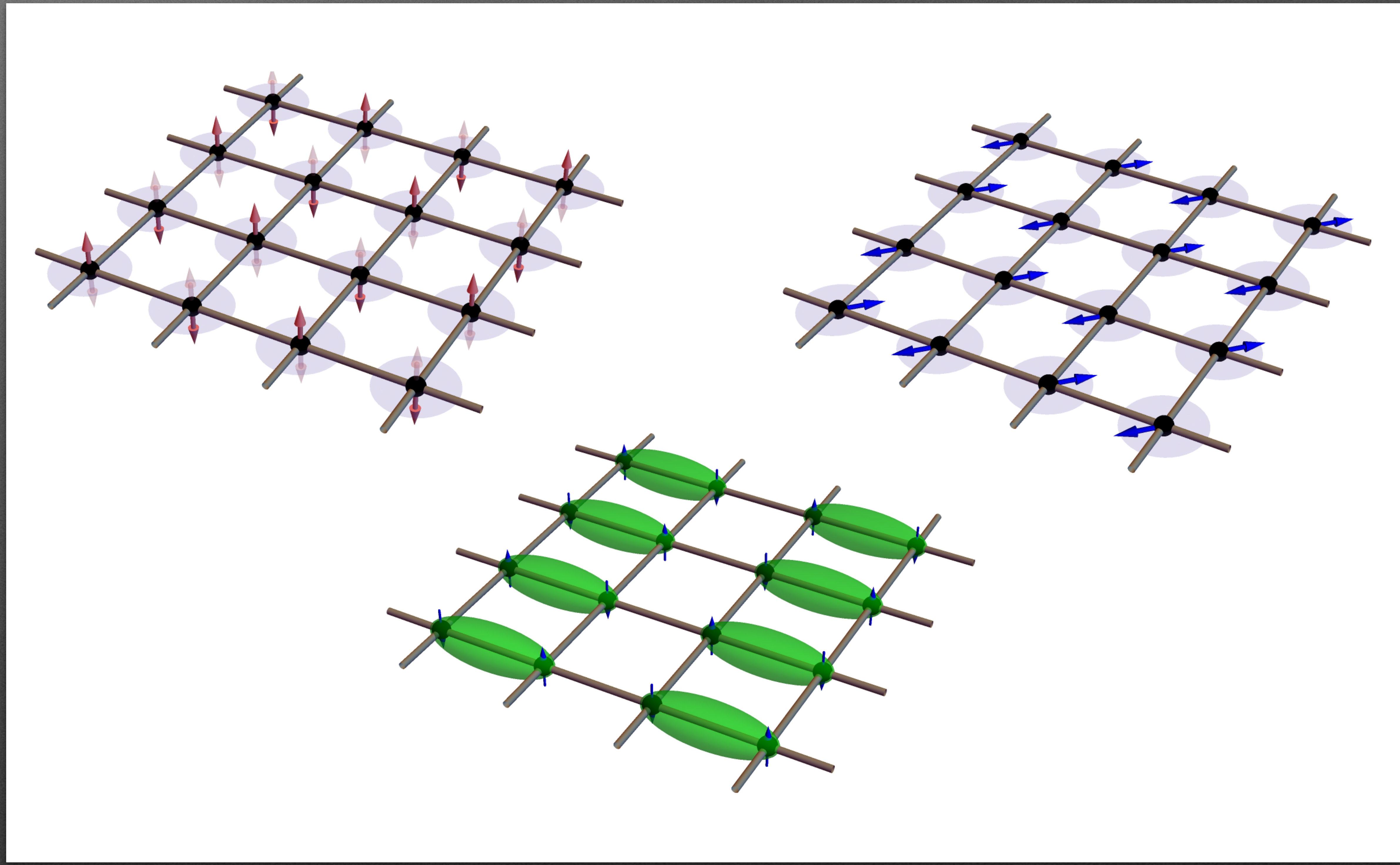

Novel QCP at finite T?

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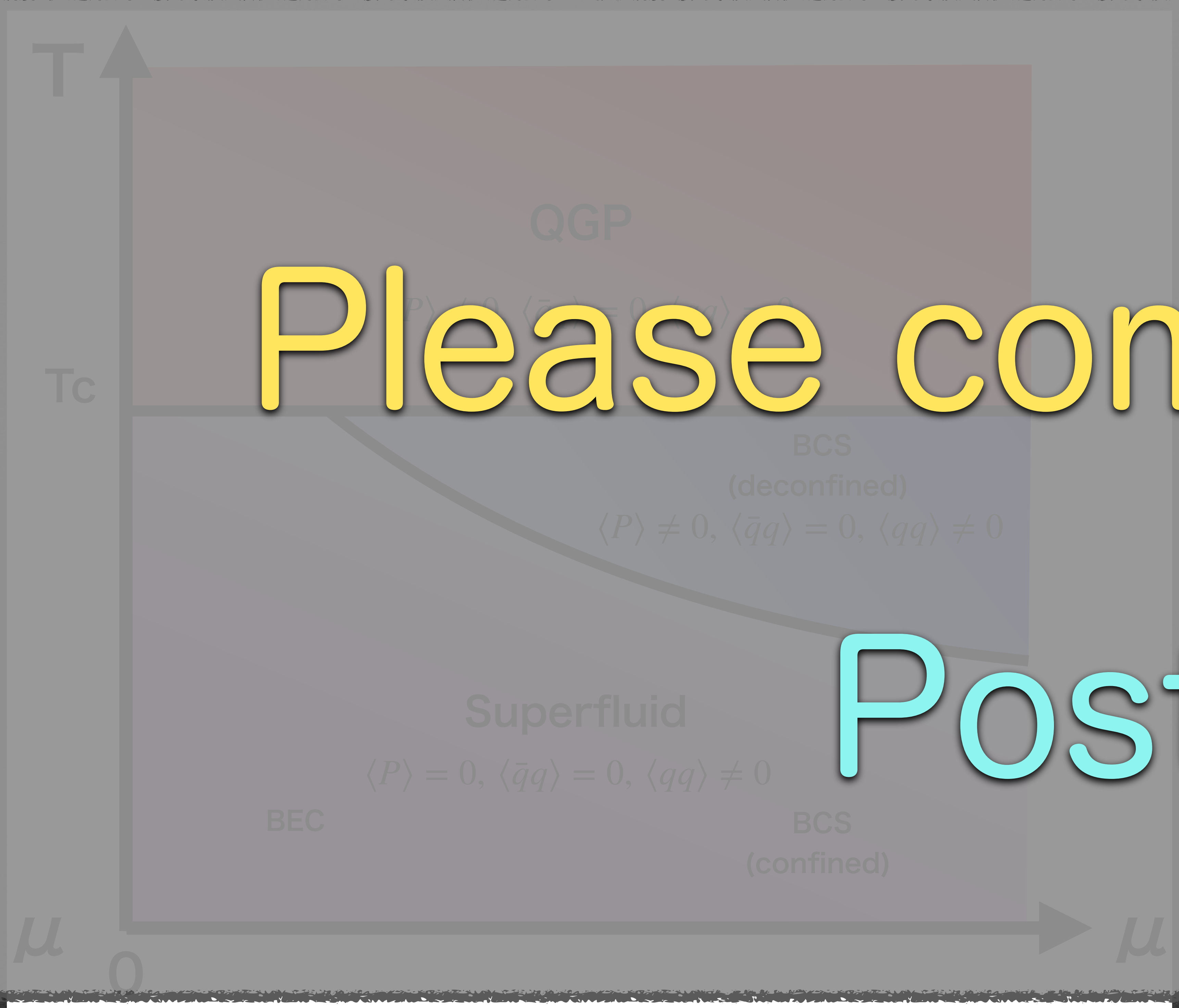

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Please come to my poster!

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