

# Quantum criticality of magnetic catalysis in (2+1)-dimensions

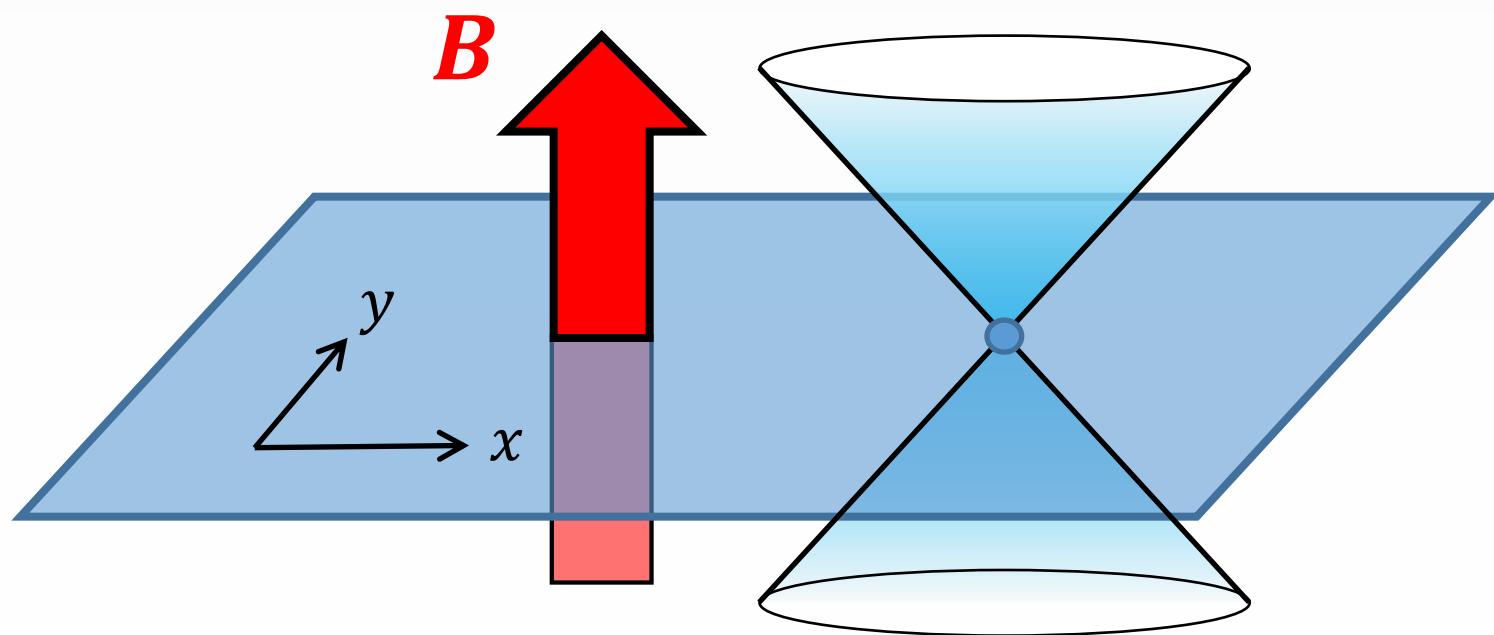
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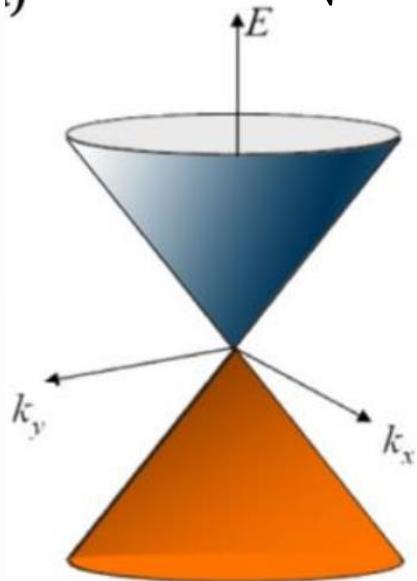
**Ref: YT, arXiv: 2005.01990**  
(to be published in Phys. Rev. Research)

# Uniform magnetic field $B$ on Dirac system

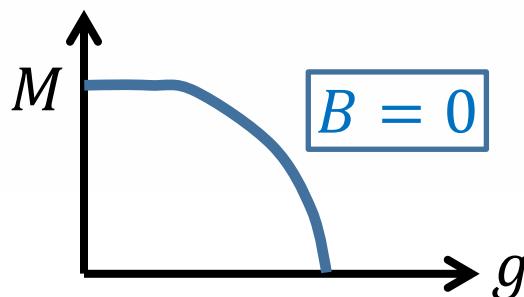


# Magnetic catalysis

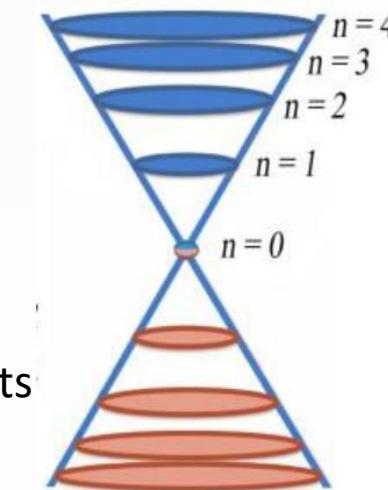
$$\epsilon(k_x, k_y) = \pm v \sqrt{k_x^2 + k_y^2}$$



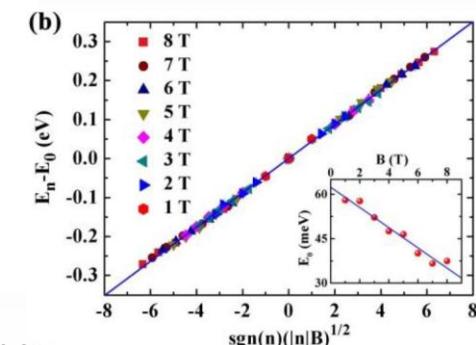
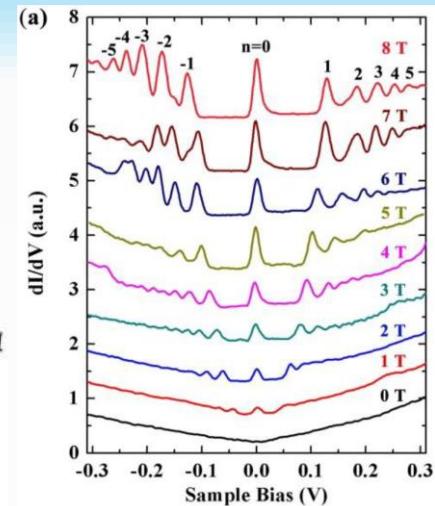
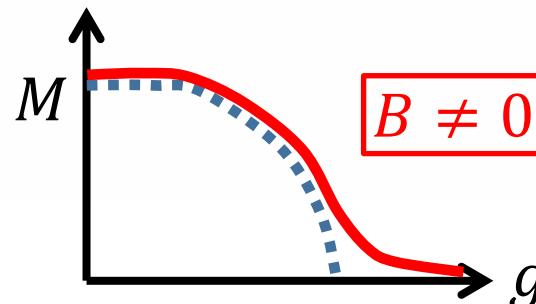
$(2+1)$  dimensions



Enhanced correlation effects

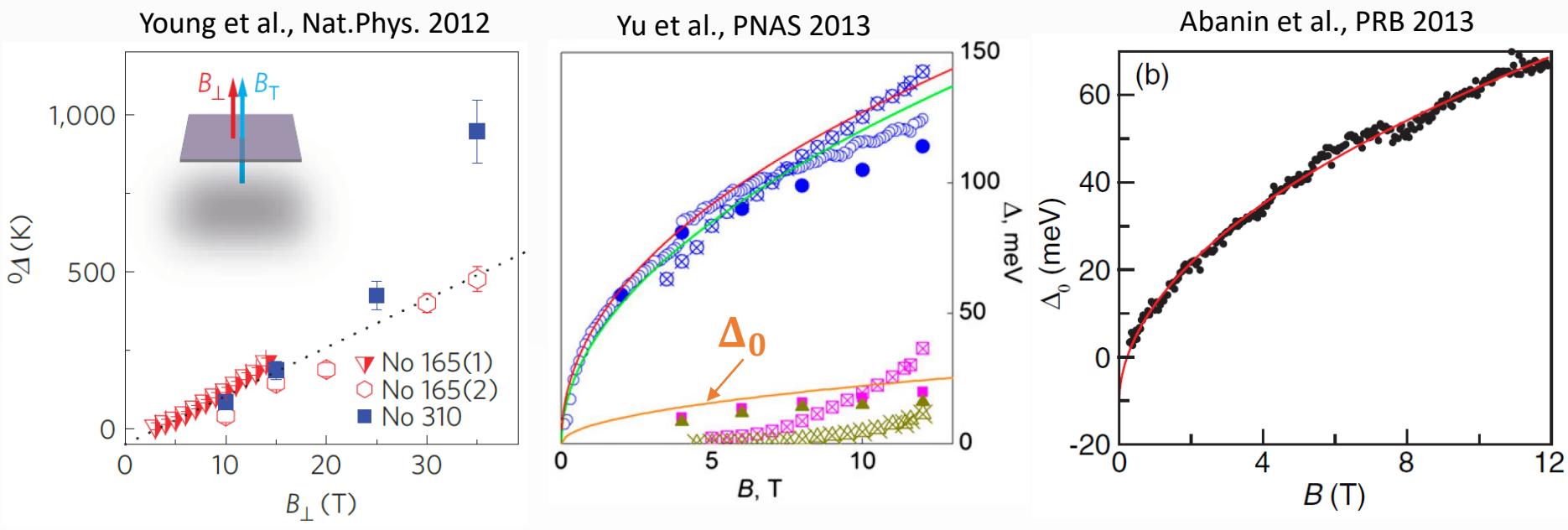


$(0+1)$  dimensions



Yin et al., PRB 2015

# Magnetic catalysis in graphene

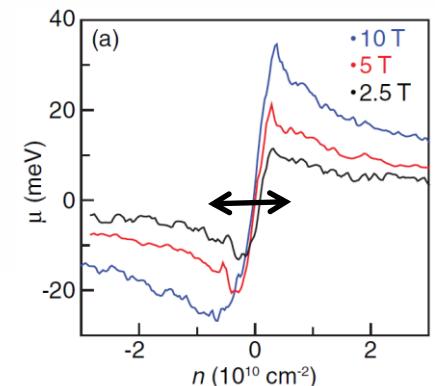


$$\Delta_0 \sim B$$

$$\Delta_0 \sim B^{>0.5}$$

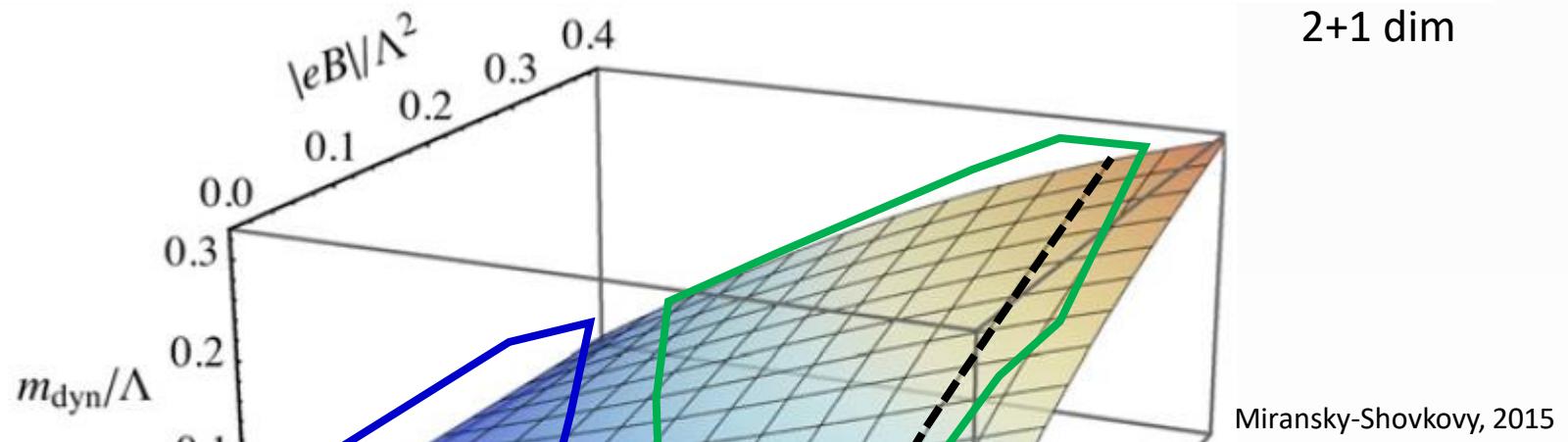
$$\Delta_0 \sim B^{\sim 0.5}$$

- Magnetic field induced gap
- $B$ -dependence changes for different experiments



# Magnetic catalysis in large $N$ limit

$$\mathcal{L} = \frac{1}{2} [\bar{\Psi}, i\gamma^\mu D_\mu \Psi] - \bar{\Psi} (\sigma + \gamma^3 \tau + i\gamma^5 \pi) \Psi - \frac{1}{2G} (\sigma^2 + \pi^2 + \tau^2)$$



$$m_{\text{dyn}} \equiv \bar{\sigma} \simeq \frac{gg_c|eB|}{2(g_c - g)\Lambda}$$

$$m_{\text{dyn}} = \bar{\sigma} \simeq 0.446\sqrt{|eB|}$$

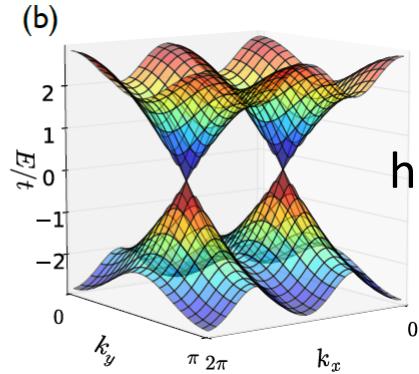
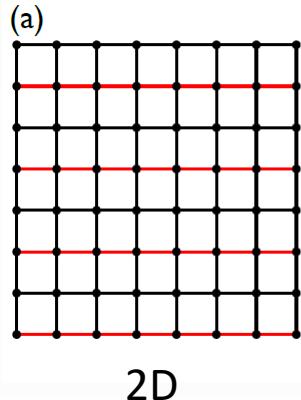
$$m_{\text{dyn}} = \bar{\sigma} \simeq m_{\text{dyn}}^{(0)} \left( 1 + \frac{(eB)^2}{12 \left( m_{\text{dyn}}^{(0)} \right)^4} \right)$$

# Question

What is the true criticality of magnetic catalysis beyond large N?

$$M(g = g_c) \sim B^{x??}$$

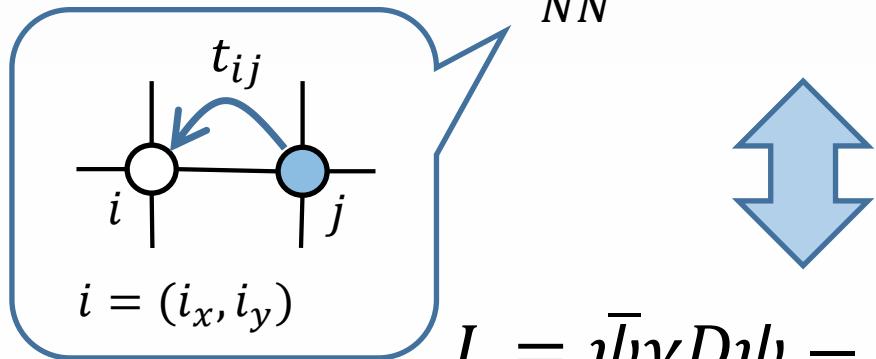
# spinless fermions on $\pi$ -flux square lattice ((2+1)D staggered fermion)



half filling  $\rightarrow$  2 Dirac cones (total 4 components)

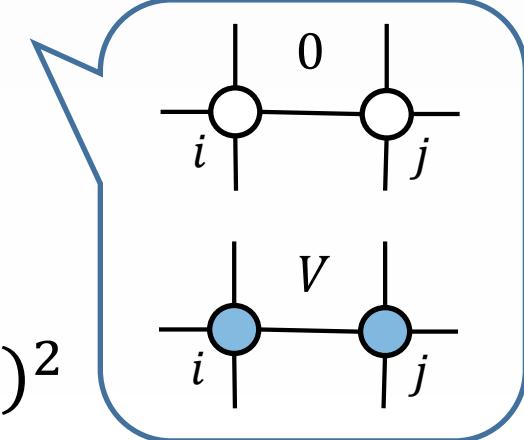
QMC : Wang et al. NJP 2014, Li et al. NJP 2015, etc...

$$H = \sum_{NN} t_{ij} c_i^+ c_j + \sum_{NN} V n_i n_j$$

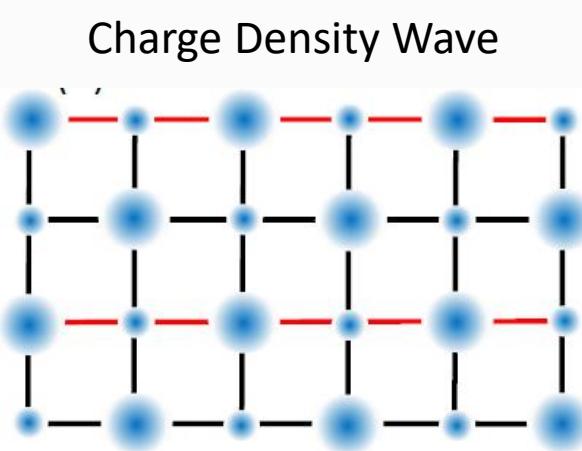


$$L = \bar{\psi} \gamma D \psi - g(\bar{\psi} \psi)^2$$

$$n_i = c_i^+ c_i$$

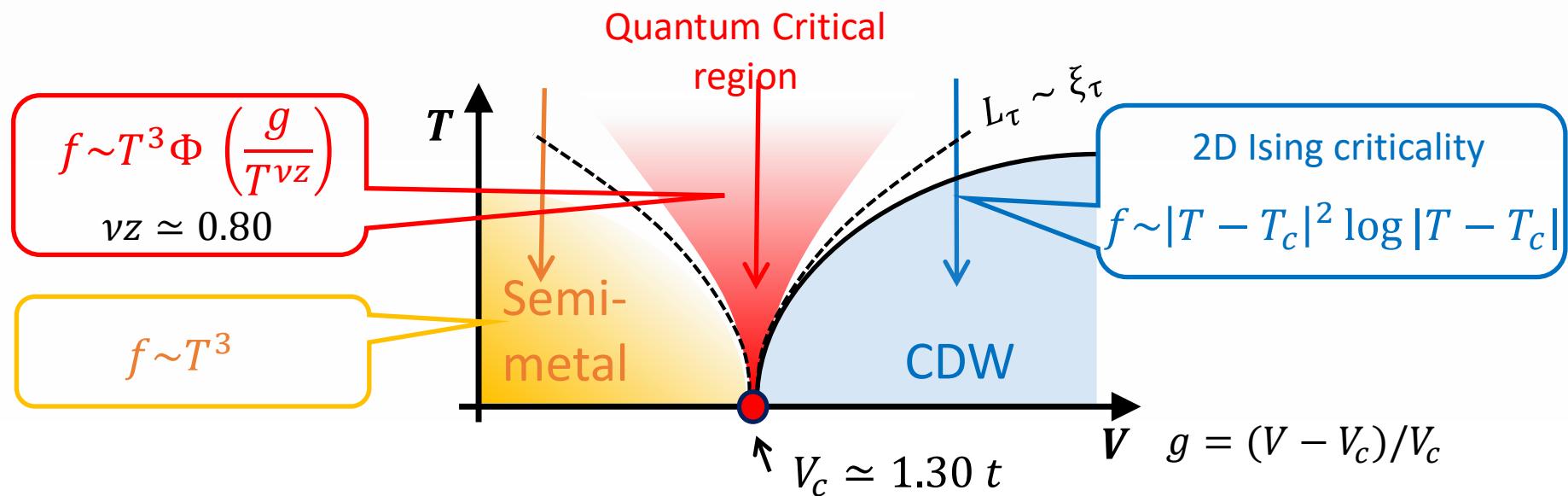
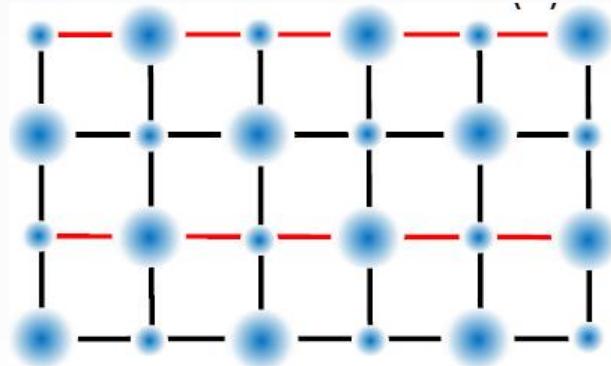


# CDW phase transition @ $B = 0$



$Z_2$  (sublattice)  
symmetry breaking

$$M = \frac{1}{L_x L_y} \sum_i (-1)^{|i|} n_i$$



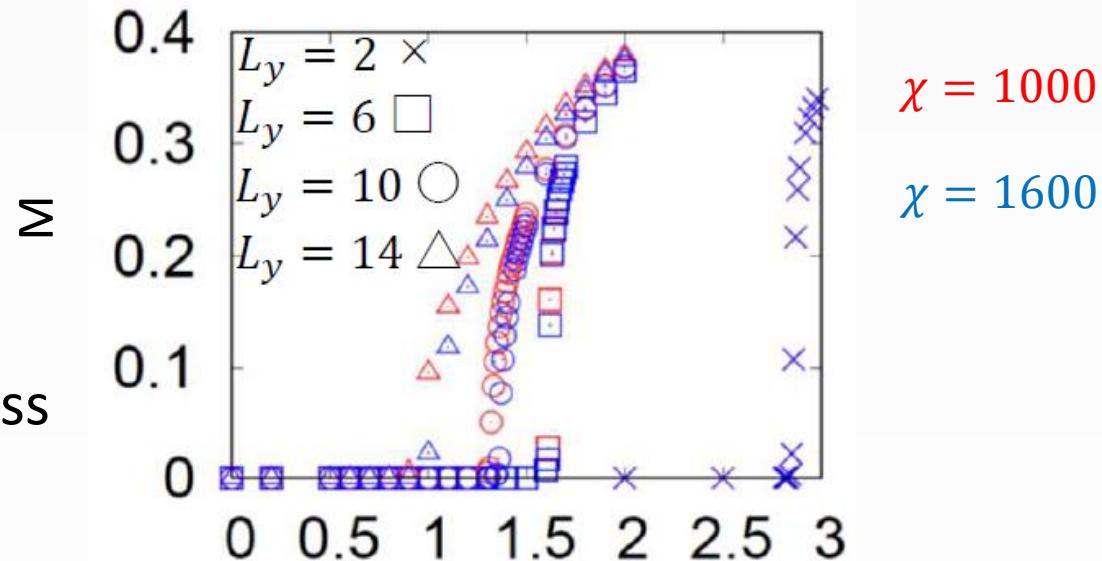
# iDMRG result for $B = 0$

YT, PRB 2019

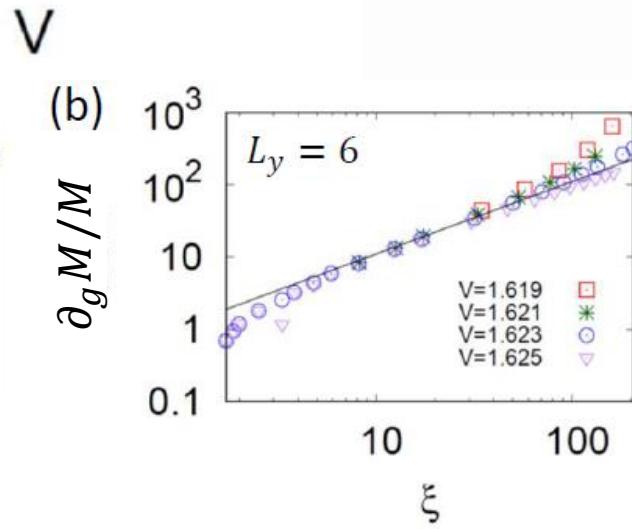
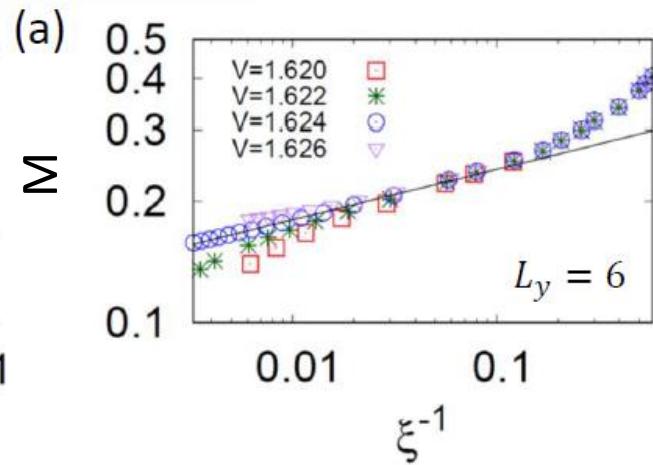
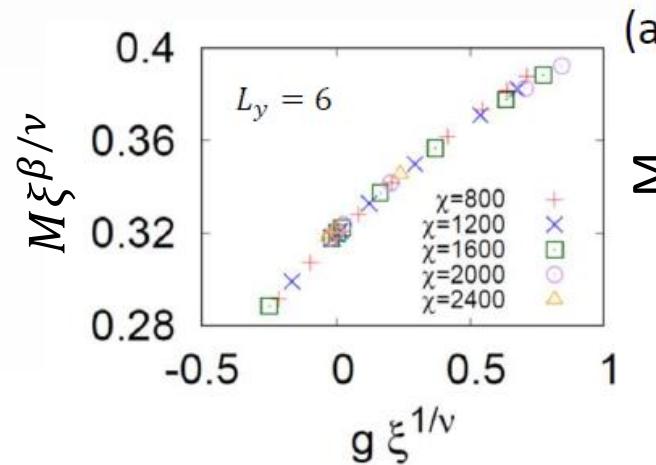
$T = 0$

$$M = \frac{1}{L_x L_y} \sum_i (-1)^{|i|} n_i$$

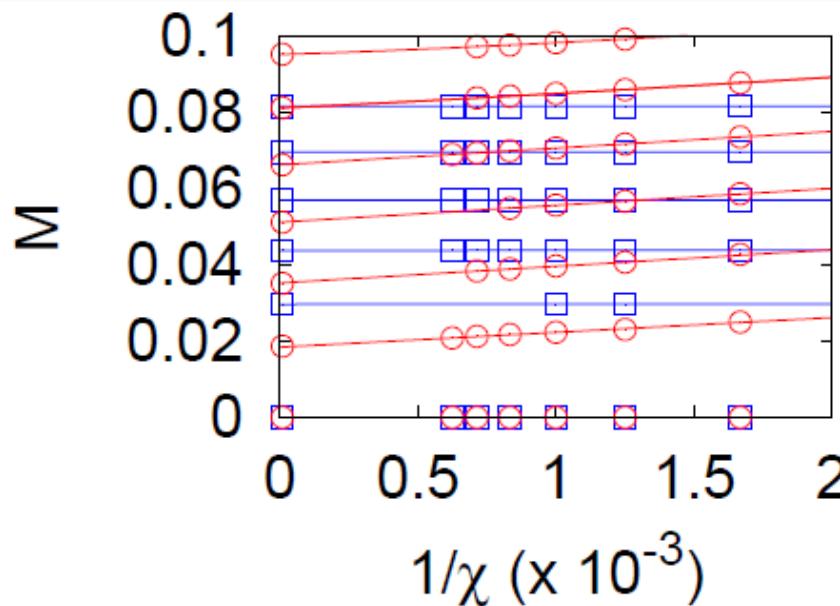
Criticality is simply  
(1+1)D Ising universality class



$\xi$  = correlation length  $\leftarrow$  finite  $\chi$



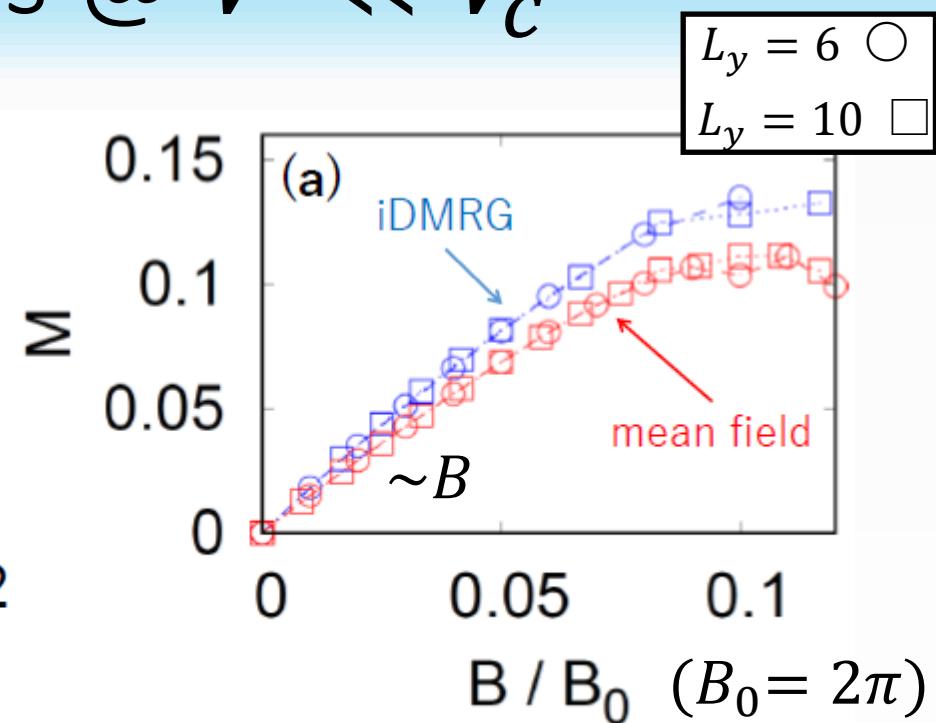
# Magnetic catalysis @ $V \ll V_c$



Extrapolation to  $\chi \rightarrow \infty$  works well

$$M \sim B$$

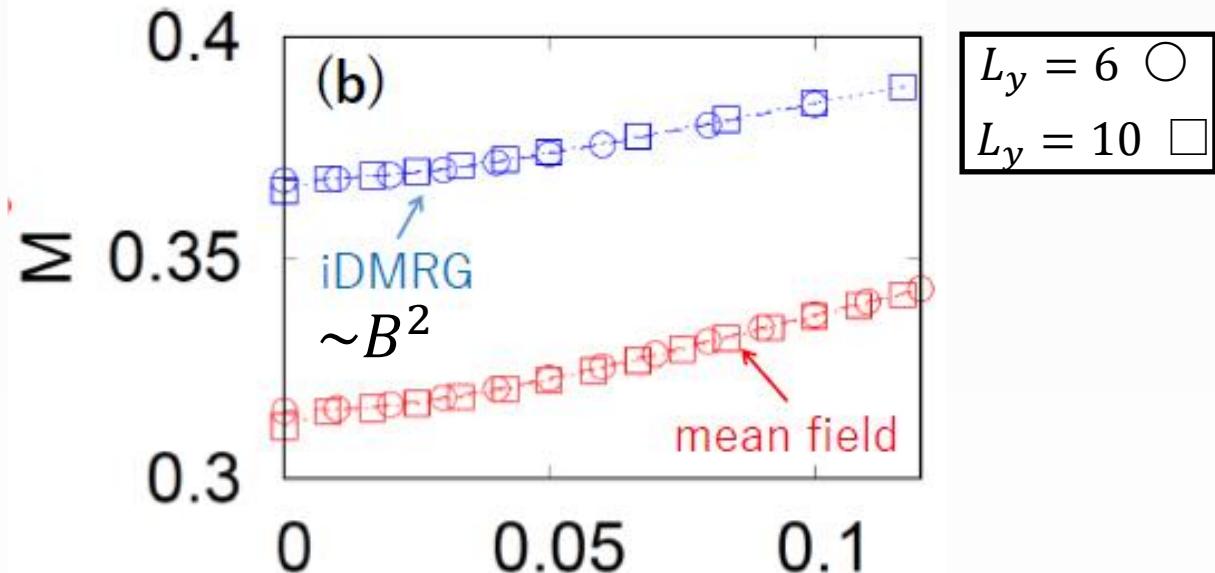
- negligible size dependence for  $L_y = 6$  and  $L_y = 10$
- enhancement of  $M$  by quantum fluctuation



$$g = \frac{0.5 - V_c}{V_c} \simeq -0.62$$

$$g_{MF} = \frac{0.3 - V_{c,MF}}{V_{c,MF}} \simeq -0.62$$

# Magnetic catalysis @ $V \gg V_c$



$$M - M_0 \sim B^2$$

$$B / B_0$$

$$g = \frac{2.0 - V_c}{V_c} \simeq 0.54$$

$$M(g = 0.54, B) > M_{MF}(0.54, B)$$

$$g_{MF} = \frac{1.2 - V_{c,MF}}{V_{c,MF}} \simeq 0.54$$

$M(g, B = 0) \sim g^\beta$

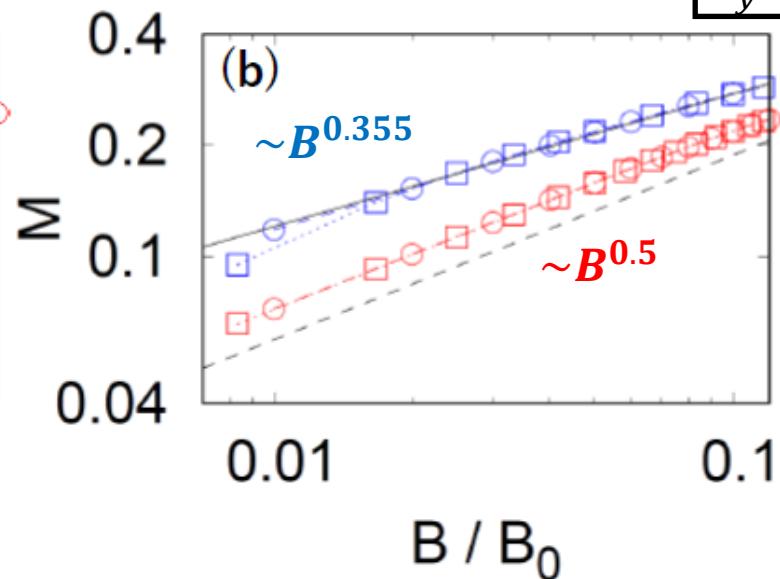
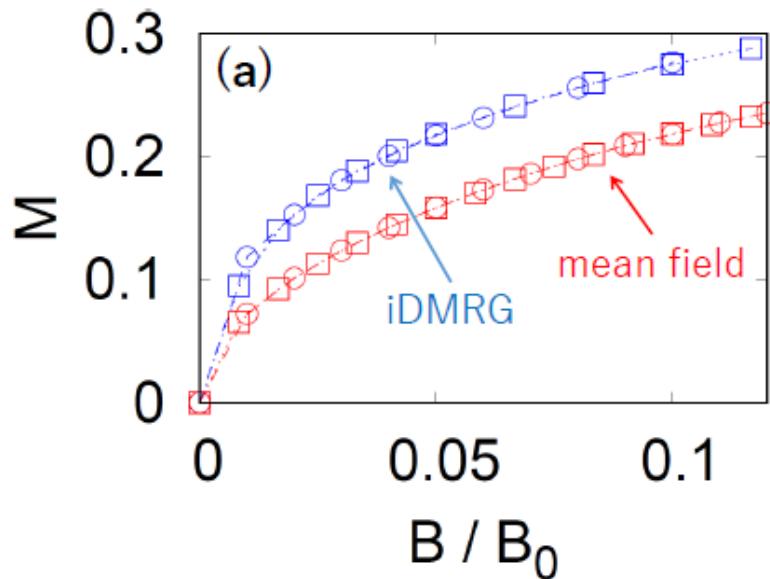
$\beta \simeq 0.5 \sim 0.6$

$M(g, B = 0) \sim (g_{MF})^{\beta_{MF}}$

$\beta_{MF} = 1$

# Magnetic catalysis @ $V = V_c$

$L_y = 6$  ○  
 $L_y = 10$  □



Fitting for  $B$ -fields where  $M(L_y = 6) \simeq M(L_y = 10)$

→  $M \sim B^{0.355(6)}$

\*mean field (large  $N$ ):  $M \sim \sqrt{B}$

# Scaling ansatz

$\varepsilon_{sing}$  = singular part of the ground state energy density      ( $\varepsilon = \varepsilon_0 + \varepsilon_{sing}$ )

$$\varepsilon_{sing}(g, h, l_B^{-1}) = b^{-D} \varepsilon_{sing}(b^{y_g} g, b^{y_h} h, bl_B^{-1})$$

normalized  
interaction

conjugate  
field

$$H \rightarrow H - hM$$

$b > 0$  : scale factor

$y_g, y_h$  : scaling dimensions

$$D = d + z = 2 + 1 = 3$$

$z = 1$ : dynamical exponent

Conventional finite size scaling:

$$\varepsilon_{sing}(L^{-1}) = b^{-D} \varepsilon_{sing}(bL^{-1}) \quad L = (\text{isotropic}) \text{ system size}$$

$$M(g = 0, l_B^{-1}) \sim (l_B^{-1})^{\beta/\nu} \sim B^{\beta/2\nu} \quad \frac{\beta}{2\nu} = 0.355(6)$$

\*exact condition for the hypothesis to hold is not known → future study

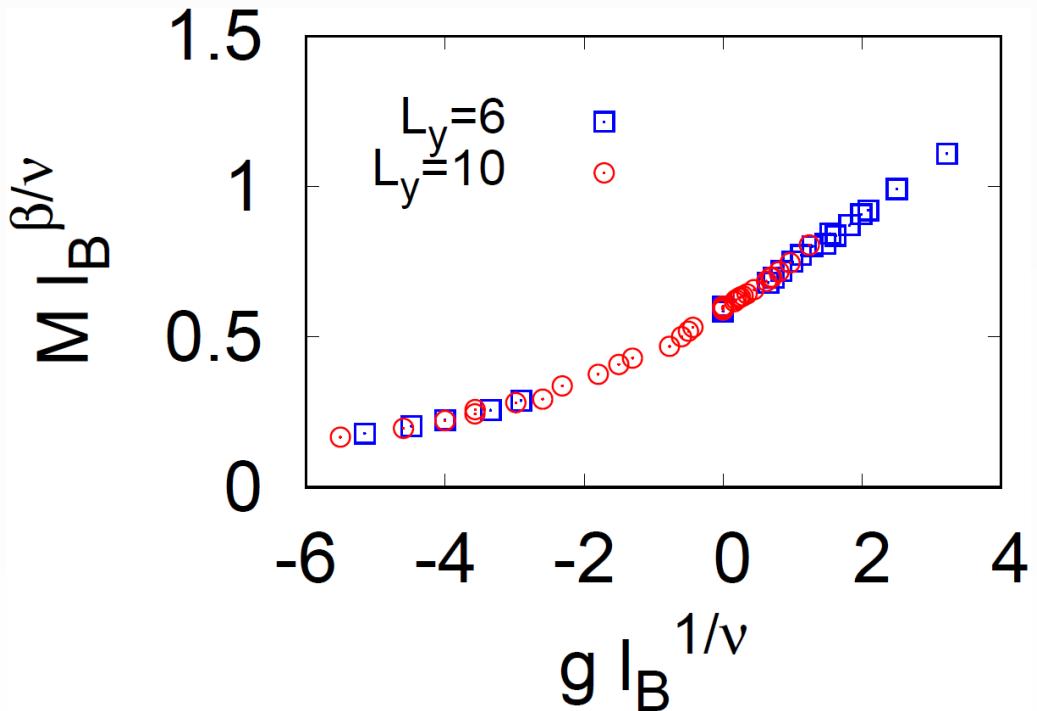
# Data collapse

$$M(g, l_B^{-1}) = l_B^{-\beta/\nu} \Phi(gl_B^{1/\nu})$$

Previous studies @  $B = 0$

Hesselmann-Wassel, PRB 2016

Huffman-Chandrasekharan, PRD 2017



Method	$\nu$	$\eta$
$4 - \epsilon$ , 1st order [9]	0.709	0.577
$4 - \epsilon$ , 2st order [9]	0.797	0.531
FRG (linear cutoff) [10 and 11]	0.927	0.525
FRG (exp. cutoff) [10]	0.962	0.554
FRG [13]	0.929	0.602
$1/N$ expansion [11]	0.738	0.635
CT-INT (GS) [18]	0.80(3)	0.302(7)
MQMC (GS) [19]	0.77(3)	0.45(2)
LCT-INT (GS) [20]	0.80(3)	0.302(7)
CT-INT (finite $T$ ), here	0.74(4)	0.275(25)
Fermion Bag QMC (GS)	0.88(2)	0.54(6)

This study (GS@ $B \neq 0$ )

0.80(2)

0.36(8)

$$\beta = 0.80(2), \nu = 0.54(3), V_c = 1.30(2)t$$

$$\left(\frac{\beta}{2\nu}\right)_{collapse}$$

= 0.34(2) consistent with

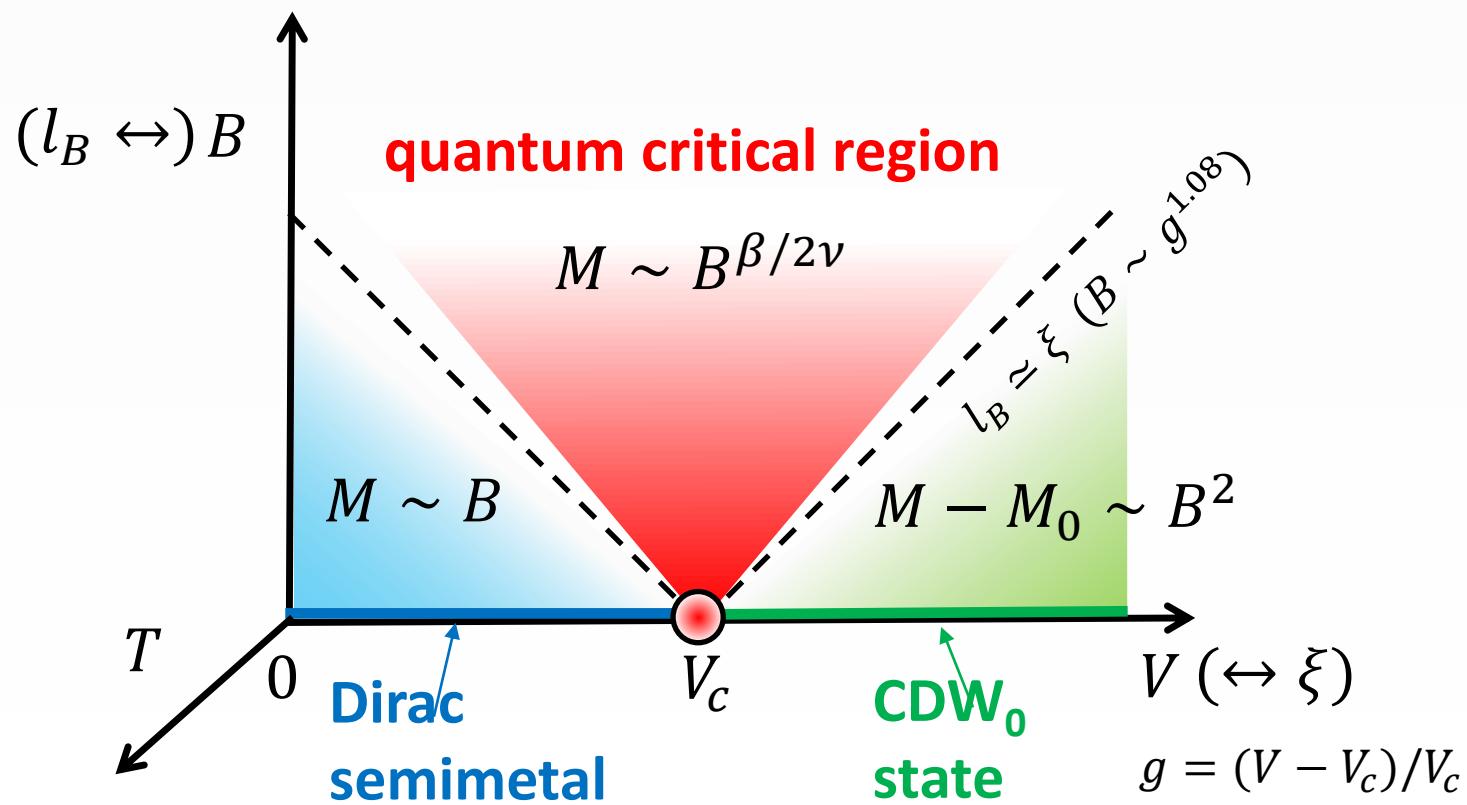
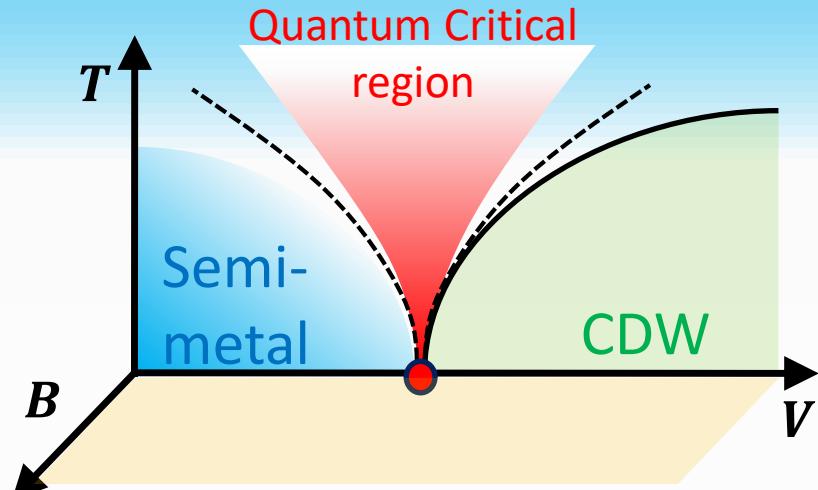
$$\left(\frac{\beta}{2\nu}\right)_{g=0} = 0.355(6)$$

# Phase diagram

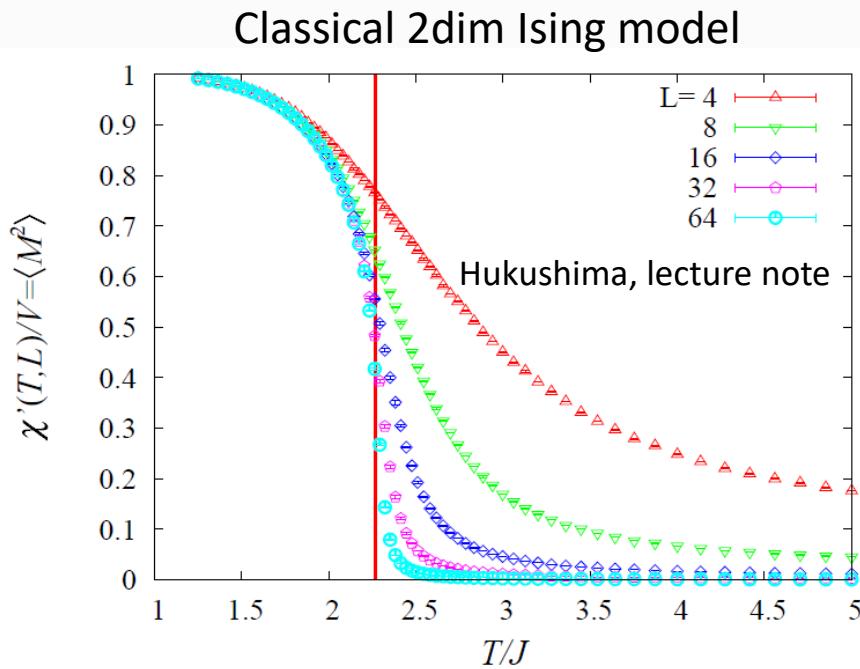
Two length scales

$\xi \sim |V - V_c|^{-\nu}$  : correlation length @  $B = 0$

$l_B = 1/\sqrt{B}$  : magnetic length

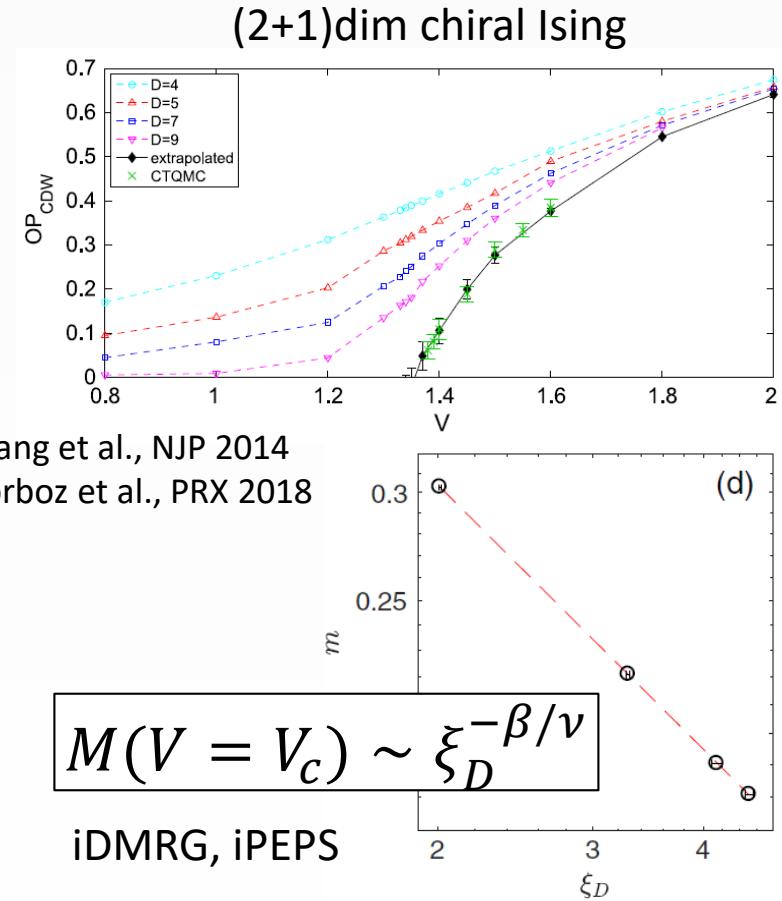


# Comparison: Overestimated $M$ for finite $L$ & $\xi_{bond}$



$$M(T = T_c) \sim L^{-\beta/\nu}$$

Monte Carlo



Magnetic catalysis = finite size effect by magnetic field

# Discussions

## - Ground state energy, diamagnetism

$$\varepsilon(g, l_B) = \varepsilon_0 + \frac{\varepsilon_{sing} \left( gl_B^{1/\nu}, 1 \right)}{l_B^3} + \dots$$

At critical point  $g = 0$  :

$$\varepsilon(0, B) = \varepsilon_0 + \alpha \cdot B^{3/2} + \dots$$

$$m_{orb} = -\frac{\partial \varepsilon(B)}{\partial B} \sim -\sqrt{B}$$

## - Finite temperature effect

$$M = l_B^{-\beta/\nu} \Psi(T/\sqrt{B})$$

*Is there “inverse magnetic catalysis”?*

← QMC would be possible

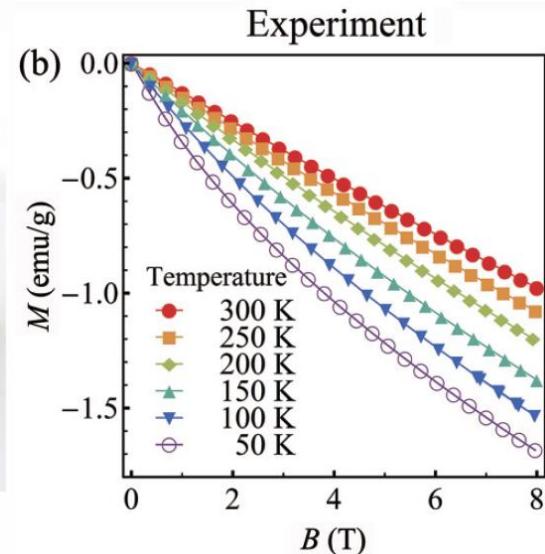
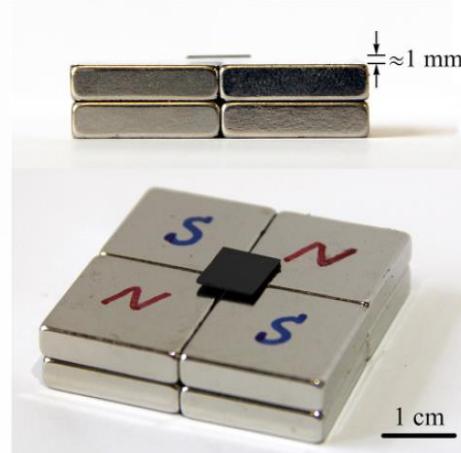
(1+1)D CFT

$$\varepsilon(L) = \varepsilon_0 - \frac{\pi c v}{6 L^2} + \dots$$

(PBC)

$c$  = central charge

$v$  = velocity of excitation



Li et al., PRB 2015

# Summary

Non-perturbative iDMRG study of magnetic catalysis

in  $(2 + 1)$ D staggered fermions

$M(V = V_c, B) \sim B^{\beta/2\nu}$  governed by  $(2+1)\text{dim } N = 4$  chiral Ising universality class

