Partial deconfinement for some boson matrix models

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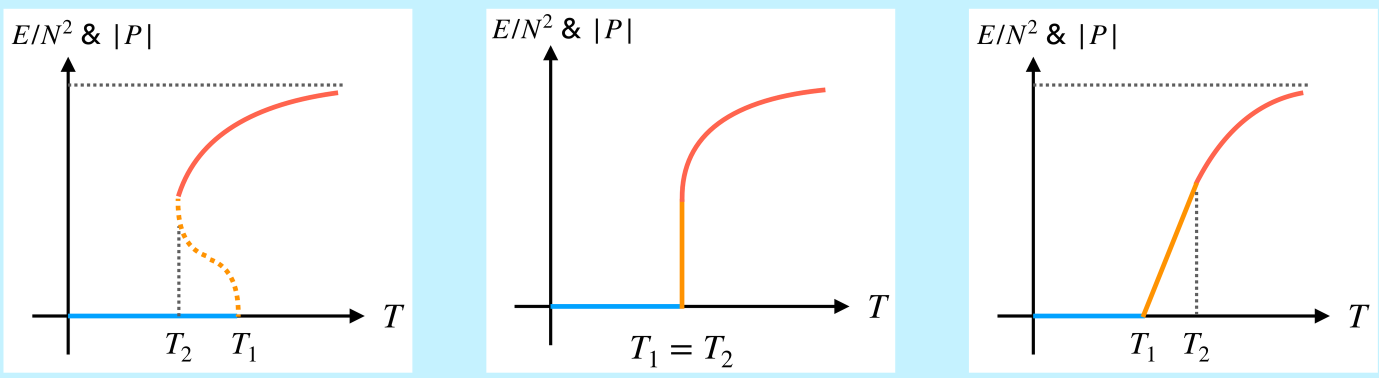
Review of partial deconfinement

Partial Deconfinement

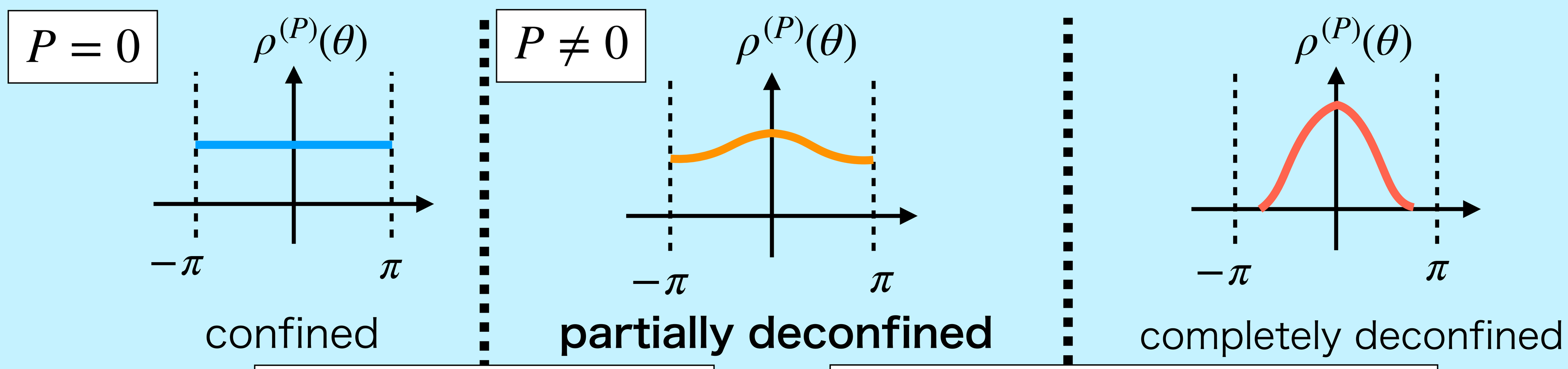
Around the critical temperature, the physical degrees of freedom those behave as in confined/deconfined phases are **coexisting in internal space (color space).**

Possibilities of phase structure in (large N) gauge theories.
[Aharony, Marsano, Minwalla, Papadodimas & Raamsdonk, (2003)/Schnitzer, (2004)]



Note that due to the large N limit as thermodynamic limit and nonlocal interaction, the situation for thermodynamic properties changes.

In terms of distribution of the Polyakov line phases, $P = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$



→ In partially-deconfined phase,
[Hanada, Ishiki & HW, (2018) / Hanada, Jevicki, Peng & Wintergerst, (2019)]

$$\rho^{(P)}(\theta) = \left(1 - \frac{M}{N}\right)\cdot \rho^{(P)}_{\rm con}(\theta) + \frac{M}{N}\cdot \rho^{(P)}_{\rm dec}(\theta)$$

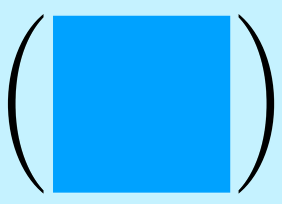
$\rho^{(P)}_{\rm dec}(\theta)$ is defined at GWW-point

$$P = \frac{M}{2N}, \quad E = E_{\rm GWW}(M), \quad S = S_{\rm GWW}(M)$$

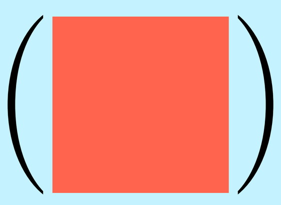
The contribution comes only from deconfined sector

• Intuitive picture

e.g) large N theories whose gauge group is $\mathrm{U}(N)$ with adjoint matters

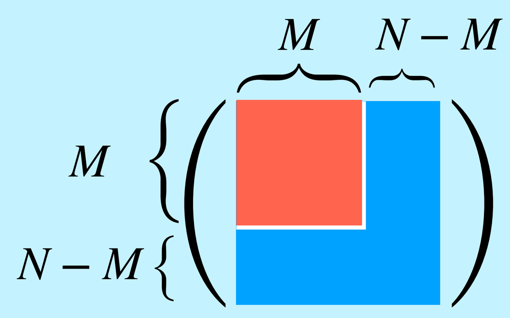


$E(T \in T_{\rm con}) \sim O(N^0)$ Confined



$E(T \in T_{\rm dec}) \sim O(N^2)$ Deconfined

What happens if $O(N^0) \lesssim E \lesssim O(N^2)$?



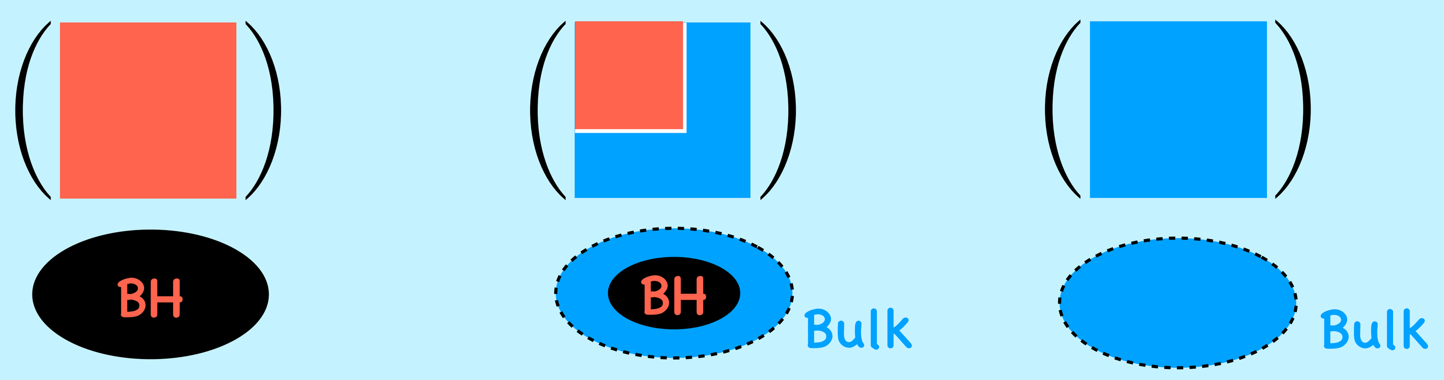
$$E = E(M) \sim \epsilon N^2, \quad S = S(M),$$

$M \sim \sqrt{\epsilon} N$: order parameter of partial deconfinement

& its gauge-equivalents

· Originally **motivated from gauge/gravity duality** [Hanada & Maltz, (2016)]

· counterpart of black hole with negative specific heat



c.f.) Confinement at large N ↔ Bose-Einstein-Condensate
[Hanada, Shimada & Wintergerst, (2019)]

Permutation symmetry among N identical bosons is **gauge symmetry**!

Polyakov loop ↔ Off-Diagonal Long Range Order (ODLRO)

Analysis of matrix models

1. gauged Gaussian matrix model

[Hanada, Jevicki, Peng & Wintergerst (2019)]

$$S = N\sum_{I=1}^{D}\int_0^\beta dt\,\mathrm{Tr}\left\{\frac{1}{2}(D_t X_I)^2 + \frac{1}{2}X_I^2\right\} \quad X_I : N\times N \text{ Hermitian mat.} \quad D_t X_I = \partial_t X_I - i[A_t, X_I]$$

At critical temperature $T_c = 1/\log D$

$$\langle E(T_c)\rangle = \left\langle \frac{N}{\beta}\int dt\,\mathrm{Tr}\,X_I^2\Big|_{T=T_c}\right\rangle = \frac{D}{2}(N^2 - M^2) + \left(\frac{D}{2} + \frac{1}{4}\right)M^2$$

$$= DN^2\langle x^2\rangle = D(N^2 - M^2)\langle x^2\rangle_{\rm con} + DM^2\langle x^2\rangle_{\rm dec}$$

with $x \equiv \{\sqrt{N}X_{I,ii}, \sqrt{2N}\mathrm{Re}X_{I,ij}, \sqrt{2N}\mathrm{Im}X_{I,ij}\}$ $(i > j)$

$$\rightarrow \langle x^2\rangle_{\rm con} = \int dx\, x^2 \rho^{(X)}_{\rm con}(x) = \frac{1}{2}, \quad \langle x^2\rangle_{\rm dec} = \int dx\, x^2\rho^{(X)}_{\rm dec}(x) = \frac{1}{D}\left(\frac{D}{2} + \frac{1}{4}\right)$$

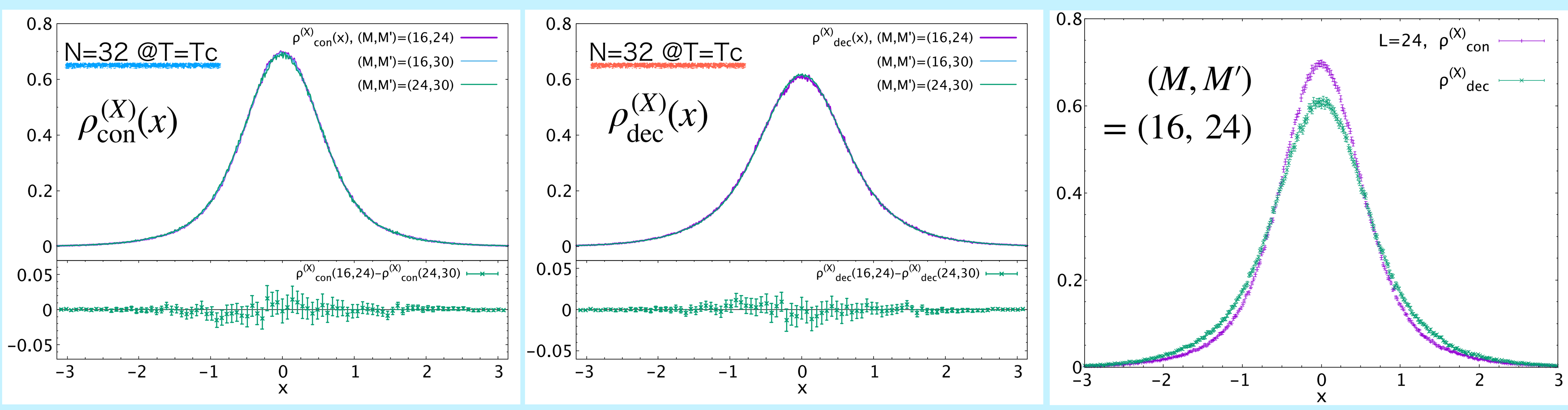
We expect

$$\rho^{(X)}(x; M) = \left(1 - \left(\frac{M}{N}\right)^2\right)\rho^{(X)}_{\rm con}(x) + \left(\frac{M}{N}\right)^2 \rho^{(X)}_{\rm dec}(x) \quad \cdots \quad (\star)$$

Estimation of $\rho^{(X)}_{\rm con}(x)$, $\rho^{(X)}_{\rm dec}(x)$ numerically; (for $D = 2$ case)

Since they should be uniquely determined.

By solving simultaneous eq. (★) with different M and M' (as $M = 2NP$)



· $\rho^{(X)}_{\rm con}(x)$, $\rho^{(X)}_{\rm dec}(x)$ are independent on M or M' (up to finite-size effect.)

· The values of their variances are 1/2 and 5/8 ($D = 2$).

· They do not coincide the Gaussian distribution with above variances.
At low-temp.(i.e. confined phase), the distribution approaches to it.

Also possible to see the coexistence in a different manner.(discussed in paper)

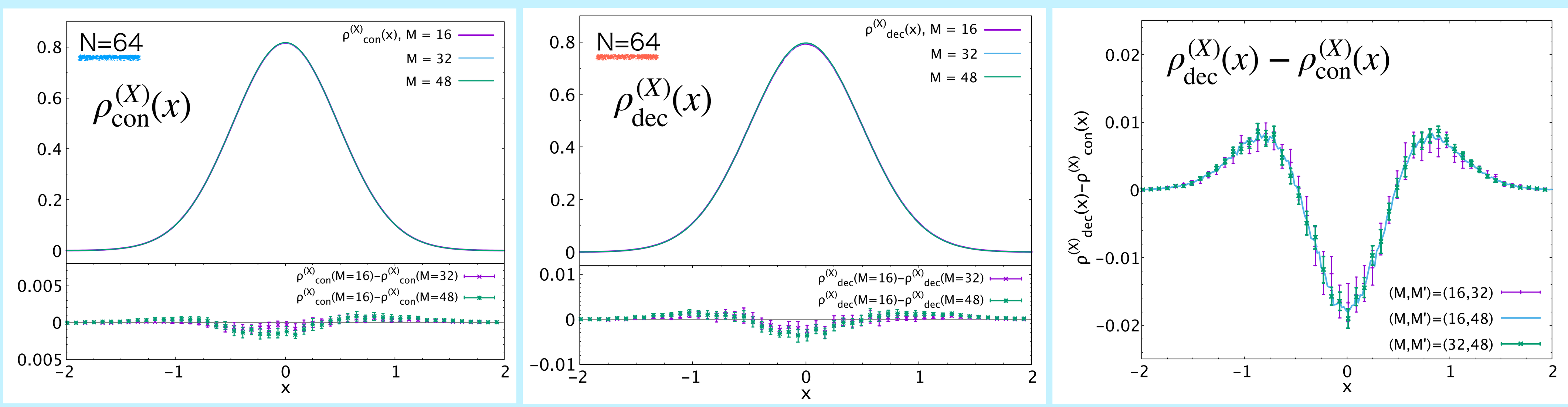
2. Yang-Mills matrix model (bosonic-BFSS)

c.f.) [Bergner, Bodendorfer, Hanada, Rinaldi, Schafer & Vranas, (2019)]

$$S = N\sum_{I=1}^{D}\int_0^\beta dt\,\mathrm{Tr}\left\{\frac{1}{2}(D_t X_I)^2 - \frac{1}{4}[X_I, X_J]^2\right\}$$

Showing the hysteresis in the narrow region

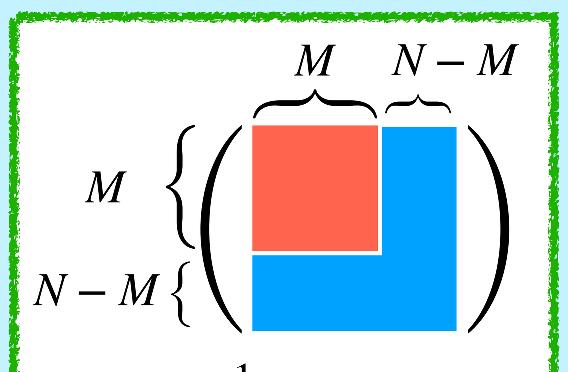
→ We apply the same method as Gaussian MM



Introducing a term to try to constrain configs.

$$\Delta S = \begin{cases} \frac{\gamma}{2}\left(|P_M| - \frac{1+\delta}{2}\right)^2 & \left(|P_M| > \frac{1+\delta}{2}\right) \\ \frac{\gamma}{2}\left(|P_M| - \frac{1-\delta}{2}\right)^2 & \left(|P_M| < \frac{1-\delta}{2}\right) \\ \frac{\gamma}{2}\left(|P_{N-M}| - \delta\right)^2 & \left(|P_{N-M}| > \delta\right) \end{cases}$$

$$P_M = \frac{1}{M}\sum_{j=1}^{M} e^{i\theta_j} \quad P_{N-M} = \frac{1}{N-M}\sum_{j=M+1}^{N} e^{i\theta_j}$$



$|P_M| \approx \frac{1}{2}$, $|P_{N-M}| \approx 0$ can be realized

Note : small nontrivial M dependence may be found.

It is the difference from Gaussian MM (due to interaction?)

· Energy contribution only comes from deconfined sector;

$$E_{\rm con} \equiv \left\langle -\frac{3N}{4\beta}\int_0^\beta dt\,\mathrm{Tr}\,[X_{{\rm con}I}, X_{{\rm con}J}]^2\right\rangle, \quad E_{\rm dec} \equiv E - E_{\rm con}$$

$E^{(0)}_{\rm con}$ is computed by the configs. with $|P| \approx 0$ and $|P_M| \approx 0$