Twisted Schwinger effect and geometric effects in quantum tunneling

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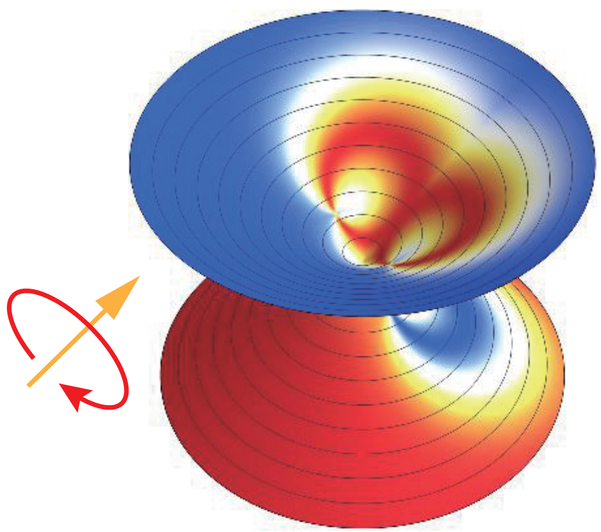
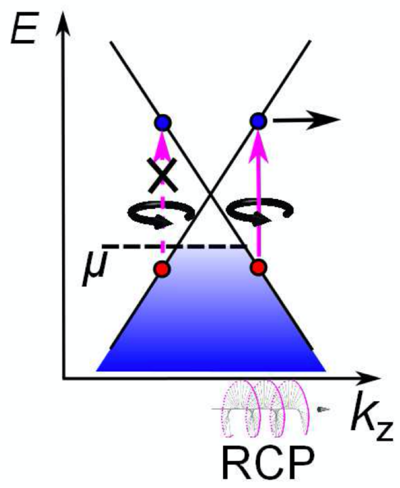


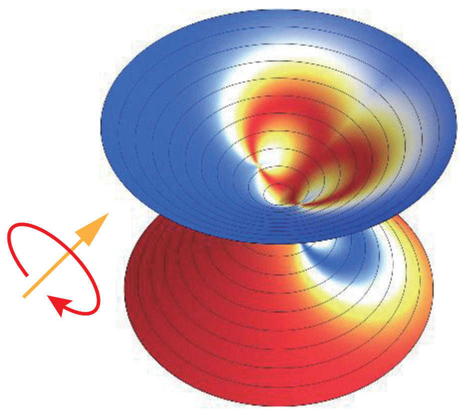
Take home message



(non)-linear optics

1. Energy-momentum conservation
2. Optical selection rule

e.g. Q. Ma et al. Nat. Phys. '17



non-perturbative optics

$$H(t)$$

1. ~~Energy-momentum conservation~~
2. Nonadiabatic Geometric effect

Berry 1990

Takayoshi, Wu, Oka 2020

Geometric effects in near “adiabatic” time evolution

Solution of the td-Schrodinger equation at slow parameter change

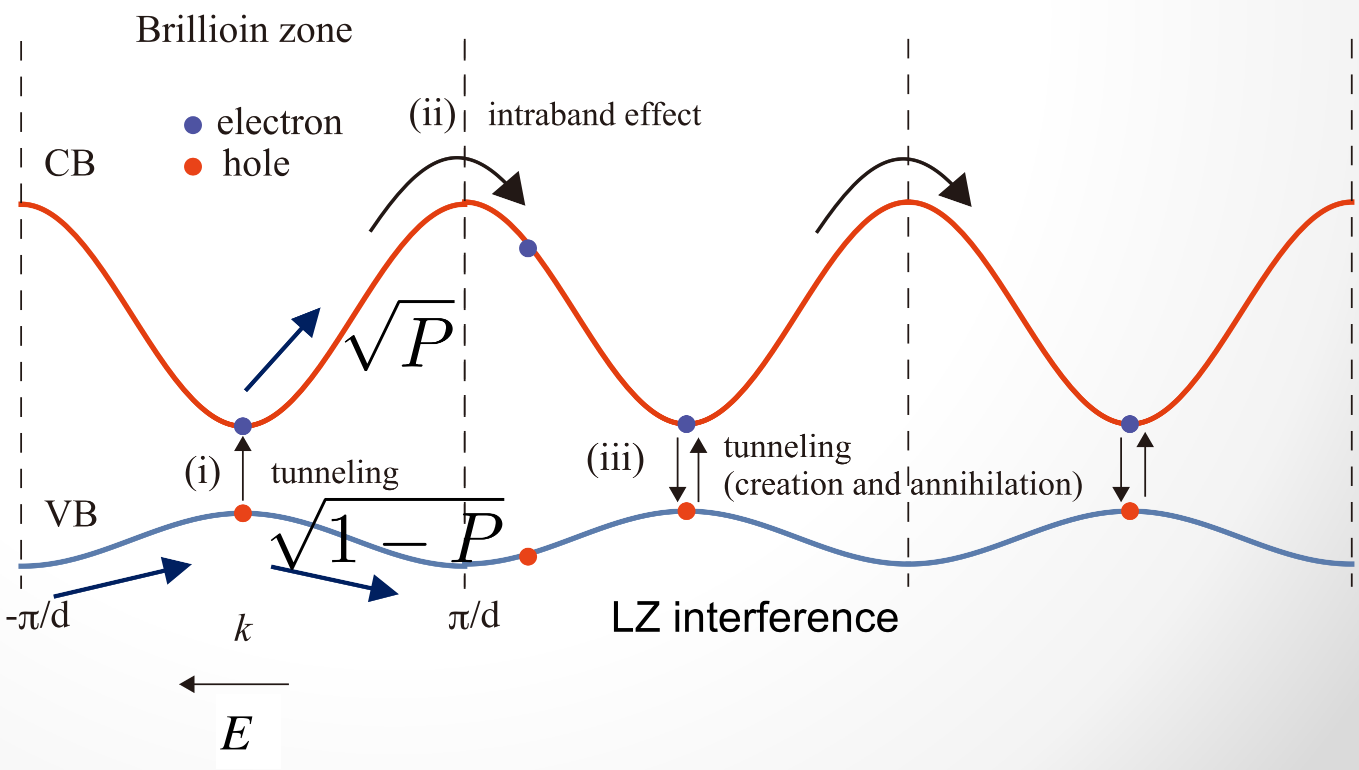
single tunneling approximation

$$|\psi(t)\rangle \simeq \sqrt{1-P}|\psi_n(\boldsymbol{\lambda}(t))\rangle e^{-i\int_0^t dt\varepsilon_n(\boldsymbol{\lambda}(t))} e^{i\tilde{\gamma}_n} + \mathrm{O}(\sqrt{P})$$

Example:
Insulator in E-field

$$\hat{\mathcal{H}}(\boldsymbol{k}+\boldsymbol{A}(t))$$

A=-Et



Geometric effects in near “adiabatic” time evolution

single tunneling approximation

$$|\psi(t)\rangle \simeq \sqrt{1-\boxed{P}}\,\psi_n(\boldsymbol{\lambda}(t))\rangle e^{-i\int_0^t dt\varepsilon_n(\boldsymbol{\lambda}(t))} e^{i\tilde{\gamma}_n} + \mathrm{O}(\sqrt{P})$$

2 1

1. (nonadiabatic) Geometric phase

Berry 1984
Aharonov-Anandan 1988

$$\tilde{\gamma} = \int_{\lambda_0}^{\lambda_t} d\lambda \langle \psi_n | i\partial_\lambda | \psi_n \rangle + \dots$$

Applications: polarization, quantum Hall effect, TI,
geometric effect in HHG, …

2. Nonadiabatic Geometric effect

Berry 1990

Takayoshi, Wu, Oka 2020

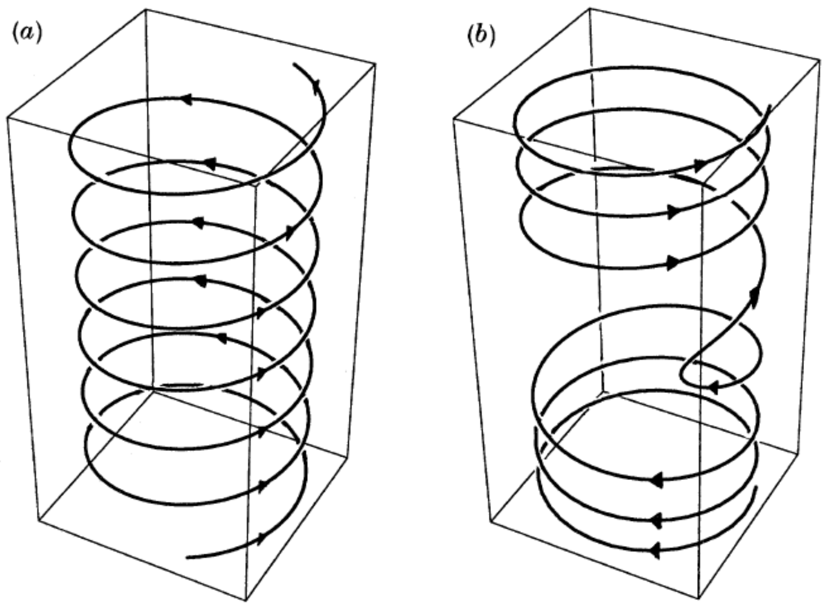
also Kitamura Nagaosa Morimoto 2019

Geometric amplitude factors in adiabatic quantum transitions

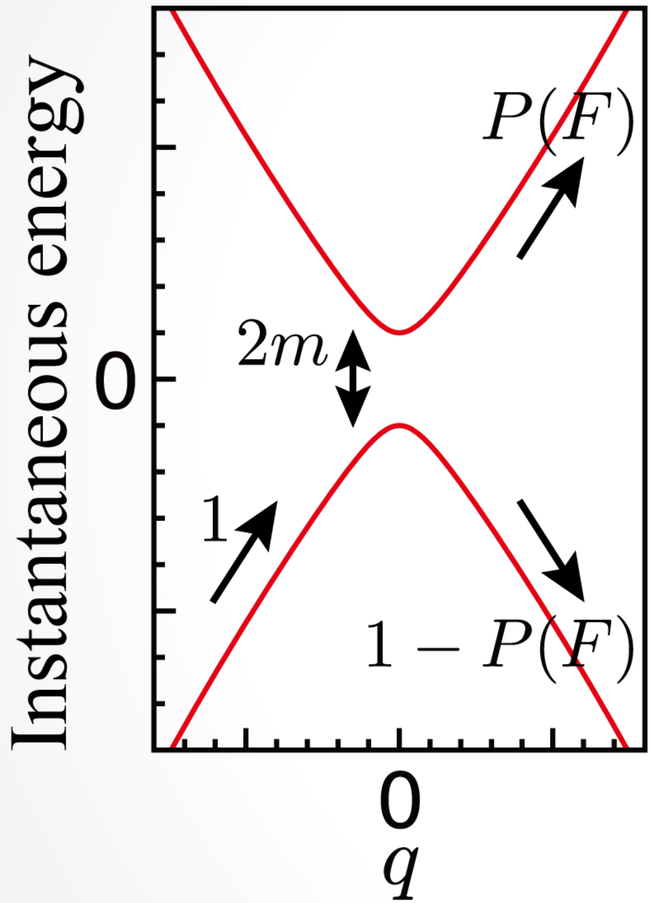
By M. V. Berry
H. H. Wills Physics Laboratory, Tyndall Avenue, Bristol BS8 1TL, U.K.

Proc. Roy. Soc. London 430, 405 (1990)

$$\boldsymbol{H}(\tau) = (\Delta \cos \phi(\tau), \Delta \sin \phi(\tau), A\tau)$$



Landau-Zener model



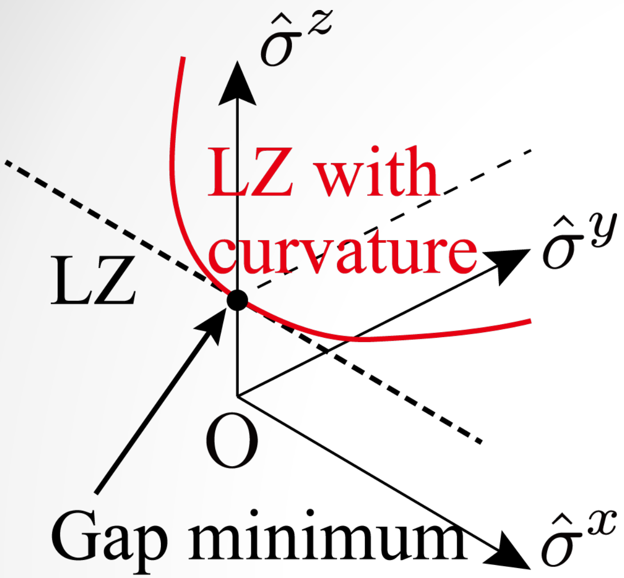
$$\hat{\mathcal{H}}(q) = m\hat{\sigma}^z + vq\hat{\sigma}^x$$

$$q = -Ft$$

Landau-Zener formula

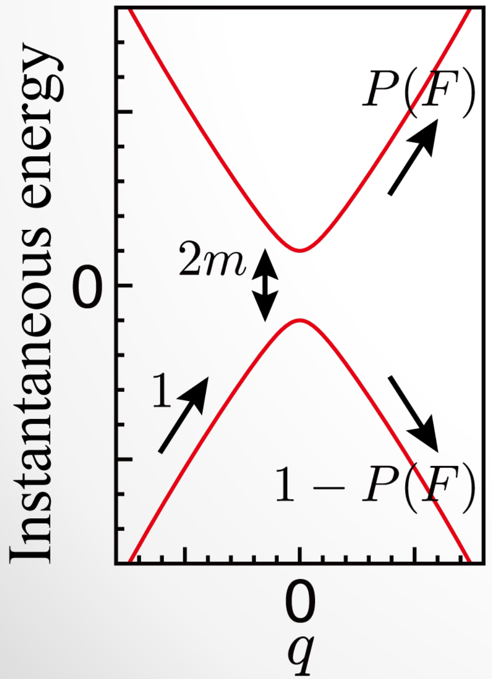
$$P(F) = \exp\left[-\pi \frac{m^2}{|vF|} \right]$$

Twisted Landau-Zener model



$$\hat{\mathcal{H}}(q) = m\hat{\sigma}^z + vq\hat{\sigma}^x \boxed{+ \frac{1}{2}\kappa_{\parallel} v^2 q^2 \hat{\sigma}^y}$$
$$q = -Ft$$

curvature



Tunneling formula

$$P(F) = \exp\left[-\pi\frac{(m + \boxed{\kappa_{\parallel} vF/4})^2}{|vF|}\right]$$

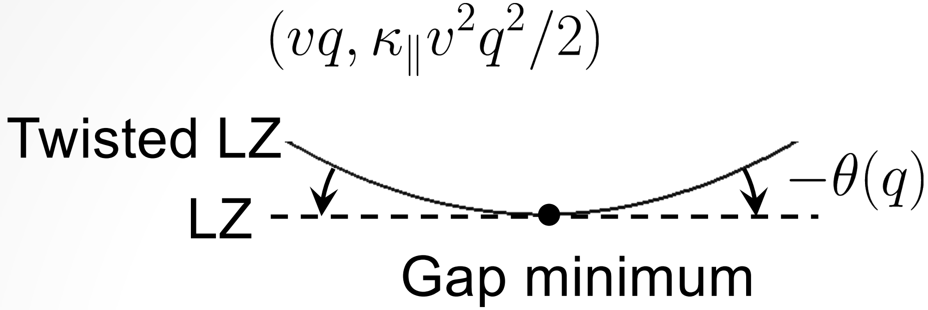
Nonadiabatic geometric effect

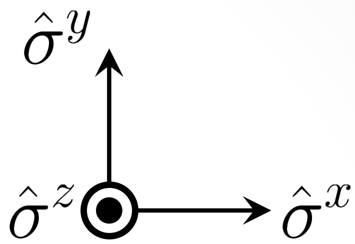
Berry 1990 (first order in κ)

Takayoshi, Wu, Oka 2020

(with general form)

Derivation by unitary transformation





"Untwist" $\hat{U}(q) = \exp\left[-i\frac{\theta(q)}{2}\hat{\sigma}^z\right]$ $\quad q = -Ft$

$\hat{\mathcal{H}}_{\mathrm{LZ}}(q) = \hat{U}^\dagger(q)\hat{\mathcal{H}}_{\mathrm{TLZ}}(q)\hat{U}(q) - \hat{U}^\dagger(q)(i\partial_t)\hat{U}(q)$ + higher order

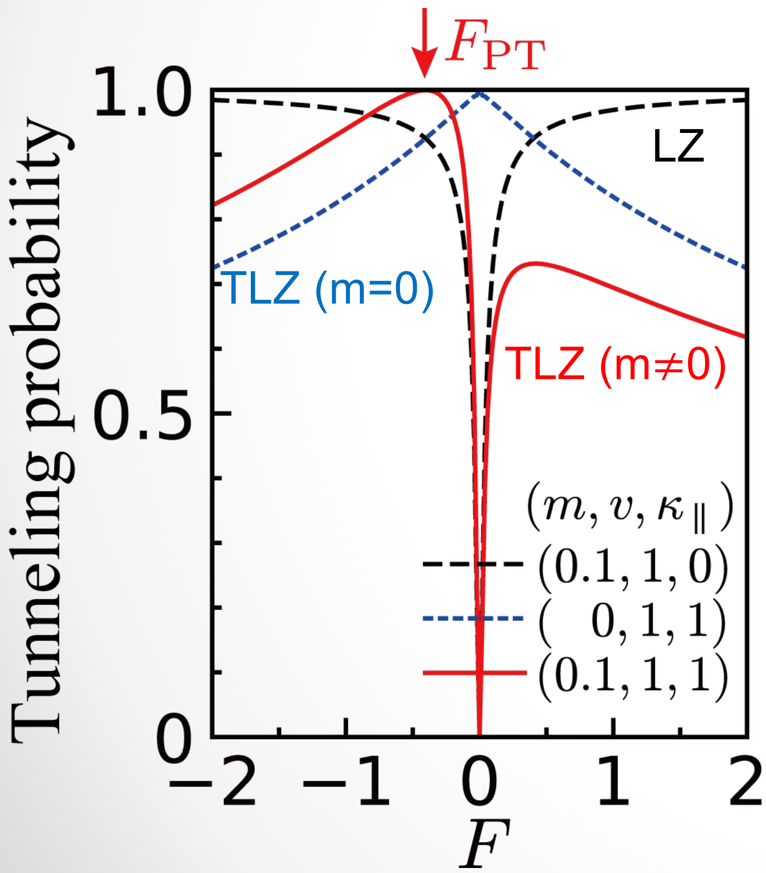
LZ model with an effective gap

$$\hat{\mathcal{H}}_{\mathrm{LZ}} = m_{\mathrm{eff}}\hat{\sigma}^z + vq\hat{\sigma}^x$$
$$m_{\mathrm{eff}} = m + \kappa_{\parallel} vF/4$$

Twisted Landau-Zener model

$$P(F) = \exp\left[-\pi \frac{(m+\kappa_{\parallel} vF/4)^2}{|vF|} \right]$$

Effective gap depends on the speed F



Rectification:

$$P(|F|)/P(-|F|) = \exp(-\pi m \kappa_{\parallel}) \neq 1$$

for $m\kappa_{\parallel} \neq 0$

Perfect tunneling:

$m_{\mathrm{eff}} = 0$ at $F_{\mathrm{PT}} = -4m/(\kappa_{\parallel} v)$

Counter diabaticity:

For large $|F|$, $P(F) = \exp(-\pi \kappa_{\parallel}^2 |vF|/16)$

Takayoshi, Wu, Oka 2020

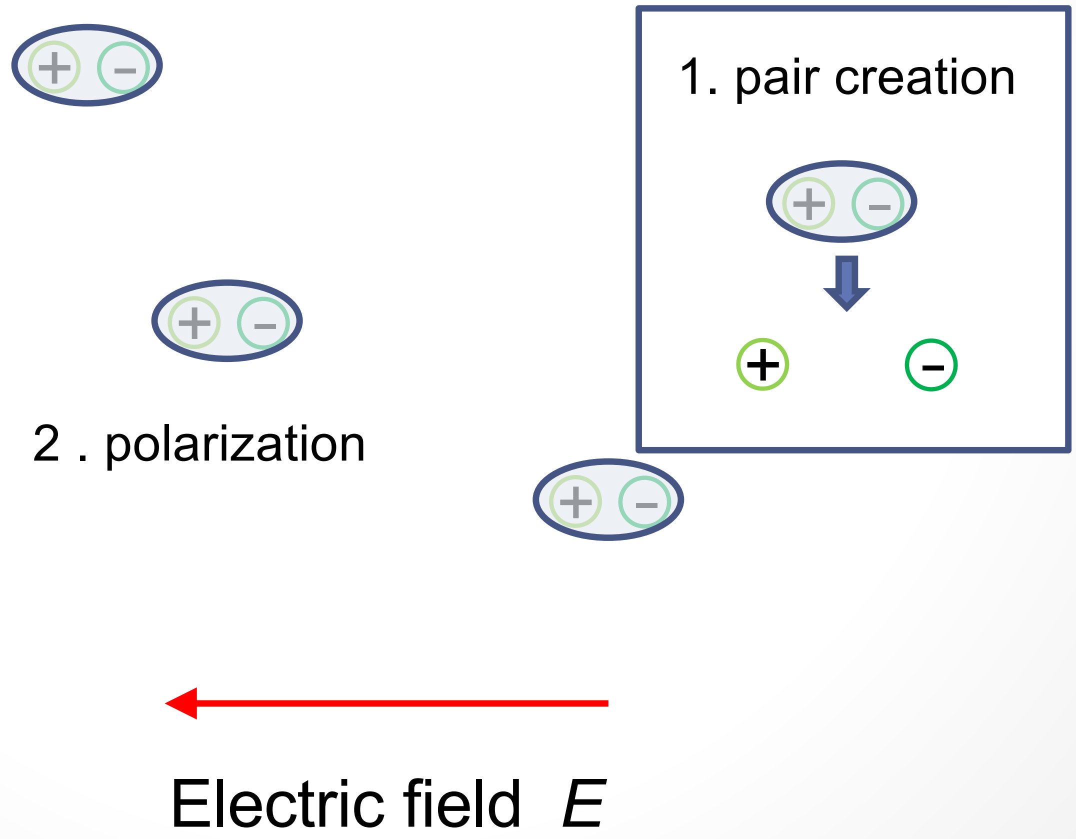
Schwinger effect

In strong E-field

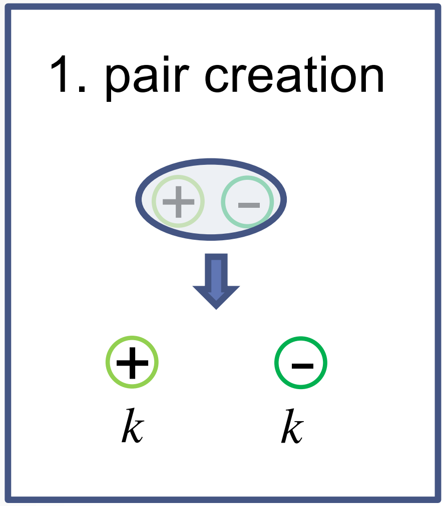
vacuum →
1. pair creation
2. induced polarization

Dirac 1928

Sauter 1931
Heisenberg-Euler 1936
Schwinger 1951



Schwinger effect



Creation probability (k=0)

$$p = \exp\left(-\pi \frac{E_S}{E}\right)$$

Schwinger limit

$$E_S = \frac{m^2 c^3}{e\hbar} = 1.3 \times 10^{18} \mathrm{V/m}$$

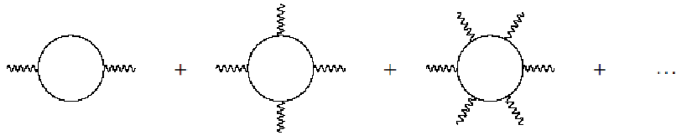
Schwinger's work

Heisenberg-Euler Effective Lagrangian

$$\mathcal{L}(A_{ext}) = -i \ln \int D[\psi, \bar{\psi}] e^{i \int d^D x L(A_{ext})}$$

$$= -i \ln \mathrm{Det}\left[i\partial\!\!\!/ + ieA\!\!\!/ - m \right]$$

Schwinger 1951

$=$ 

generating function of nonlinear responses

Vacuum decay rate (sum over tunneling channels)

$$2\,\mathrm{Im}\mathfrak{L} = \frac{1}{4\pi} \sum_{n=1}^{\infty} s_n^{-2} \exp(-m^2 s_n)$$

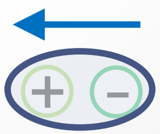
$$= \frac{\alpha^2}{\pi^2} \mathcal{E}^2 \sum_{n=1}^{\infty} n^{-2} \exp\left(\frac{-n\pi m^2}{e\mathcal{E}} \right). \qquad (6.41)$$

pair creation

$\oplus$ $\ominus$

Vacuum Polarization

$$X(E) = -\frac{\partial}{\partial E} \mathrm{Re}\mathcal{L}$$



Loschmidt echo and the Heisenberg-Euler Effective Lagrangian

$$\Xi(t) = \langle \Psi_0(\boldsymbol{\lambda}(t)) | \hat{T} e^{-i \int_0^t H(\boldsymbol{\lambda}(s)) ds} | \Psi_0(\boldsymbol{\lambda}(0)) \rangle e^{i \int_0^t E_0(A(s)) ds}$$

$$\Xi(t) \sim e^{itV\mathcal{L}(E)}$$

many-body ground state

For dc *E*-field $\mathcal{L}(E) = -\lim_{t\to\infty} \frac{i}{tV} \ln \langle 0 | \hat{T} e^{-i \int_0^\tau E\hat{X}(s) ds} | 0 \rangle$

$\hat{X}$: position operator

Oka, Aoki 2005

$$\Xi_{\boldsymbol{k}}(t) = \langle \psi_0(\boldsymbol{\lambda}(t)) | \psi(t) \rangle \simeq \sqrt{1 - P_{\boldsymbol{k}}} e^{i\tilde{\gamma}_{\boldsymbol{k}}}$$

Induced polarization = nonadiabatic geometric phase

$$\mathrm{Re}\mathcal{L}(E) = -E \int_{\mathrm{BZ}} \frac{d\boldsymbol{k}}{(2\pi)^d} \frac{\tilde{\gamma}(\boldsymbol{k})}{2\pi} \qquad X(E) = -\frac{\partial}{\partial E} \mathrm{Re}\mathcal{L}$$

Schwinger effect = Landau-Zener tunneling

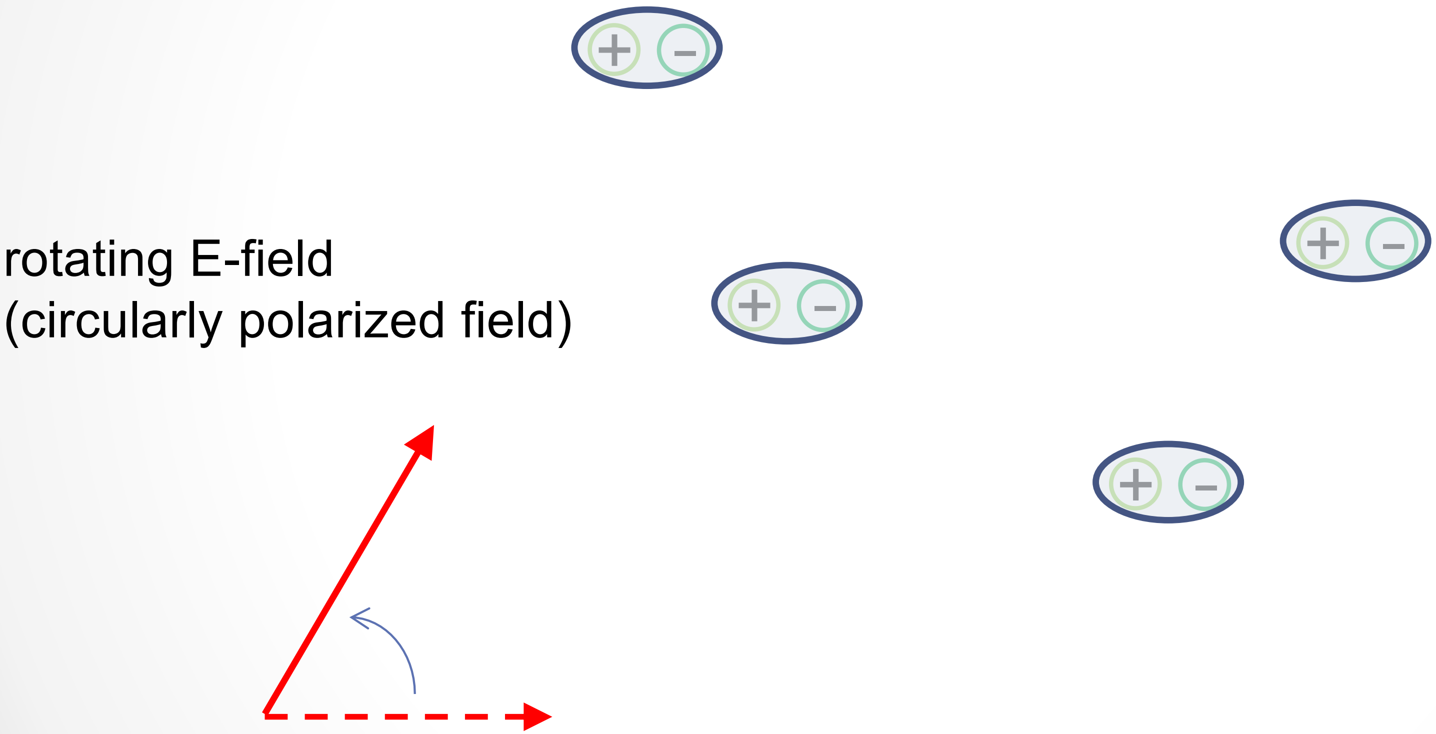
$$\mathrm{Im}\mathcal{L}(E) = -E \int_{\mathrm{BZ}} \frac{d\boldsymbol{k}}{(2\pi)^d} \frac{1}{4\pi} \ln[1 - P(\boldsymbol{k})]$$

Consistent with Schwinger 1951, agrees with the Berry phase theory of polarization Resta, King-Smith Vanderbilt at $E \to 0$.

Schwinger effect by circularly polarized field

"Twisted Schwinger effect"

Takayoshi, Wu, Oka 2020

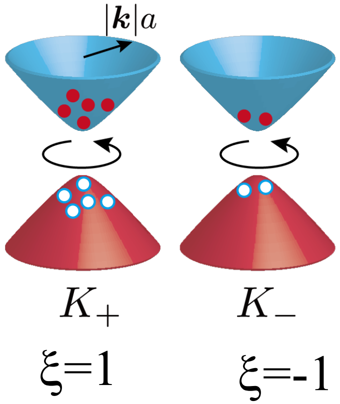


$$E = E_0\,(\cos \Omega t,\ \sin \Omega t,\ 0)$$

Application to Dirac systems in rotating E-field

1. 2D gapped Dirac system

$$\hat{\mathcal{H}} = v[\xi(k_x + eA_x)\hat{\sigma}^x + (k_y + eA_y)\hat{\sigma}^y] + m\hat{\sigma}^z$$

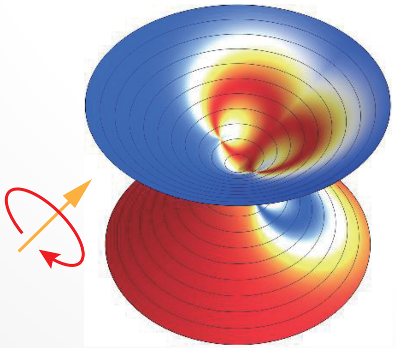


Circularly polarized laser

$$A(-\sin(\Omega t), \cos(\Omega t), 0)$$

2. 3D gapless Dirac system

$$\hat{\mathcal{H}}_{3\mathrm{D}} = v \sum_{j=x,y,z} \hat{\gamma}^0 \hat{\gamma}^j (q_j + eA_j) \qquad \hat{\gamma}^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \ \hat{\gamma}^j = \begin{pmatrix} 0 & \hat{\sigma}^j \\ -\hat{\sigma}^j & 0 \end{pmatrix}$$



2D gapped Dirac system

$$\hat{\mathcal{H}} = v[\xi(k_x + eA_x)\hat{\sigma}^x + (k_y + eA_y)\hat{\sigma}^y] + m\hat{\sigma}^z$$

Circularly polarized laser

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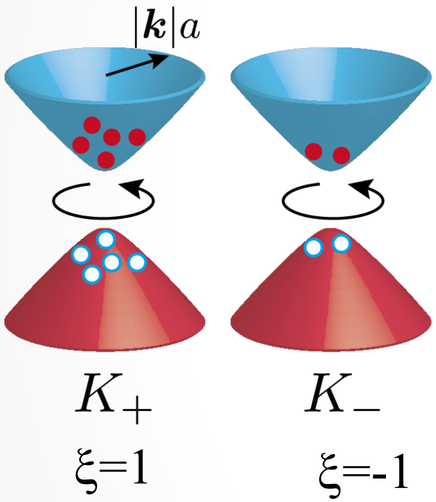


"*Nonperturbative* Valley-selective circular dichroism"

$$\mathcal{P}_\xi(\boldsymbol{k}) = \exp\left(-\pi\frac{m_{\text{eff}}^2}{veE}\right)$$

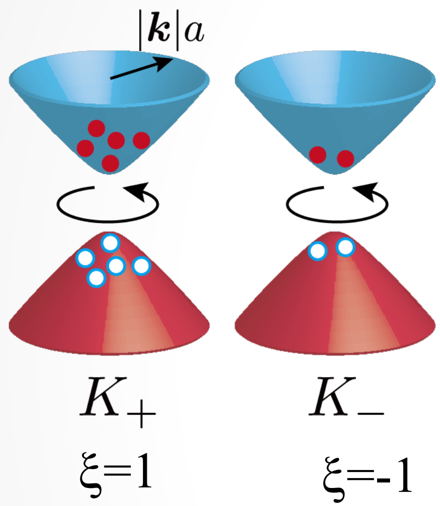
$$m_{\text{eff}} = M - \xi\frac{\Omega m}{4M}$$

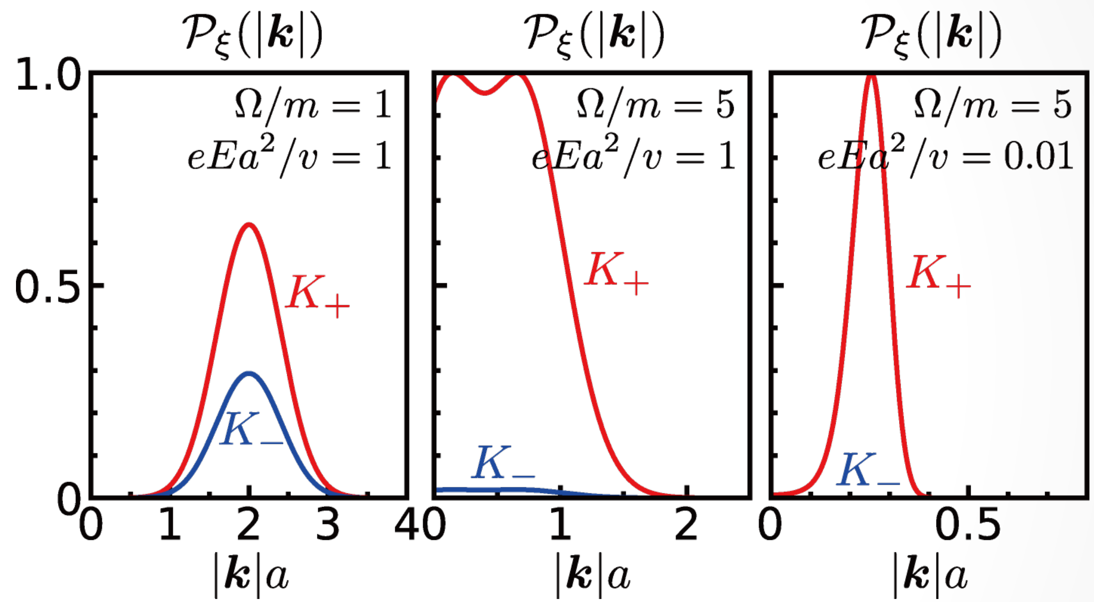
$$M = \sqrt{v^2\left(|\boldsymbol{k}| - \frac{eE}{|\Omega|}\right)^2 + m^2}$$

Takayoshi, Wu, Oka 2020

2D gapped Dirac system

$$\hat{\mathcal{H}} = v[\xi(k_x + eA_x)\hat{\sigma}^x + (k_y + eA_y)\hat{\sigma}^y] + m\hat{\sigma}^z$$



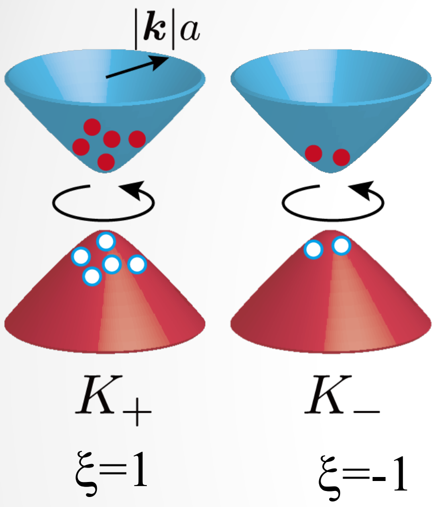


$\xi\Omega > 0$: Optically allowed

$\xi\Omega < 0$: Optically forbidden

Note: Selection rule is replaced by nonadiabatic geometric effects

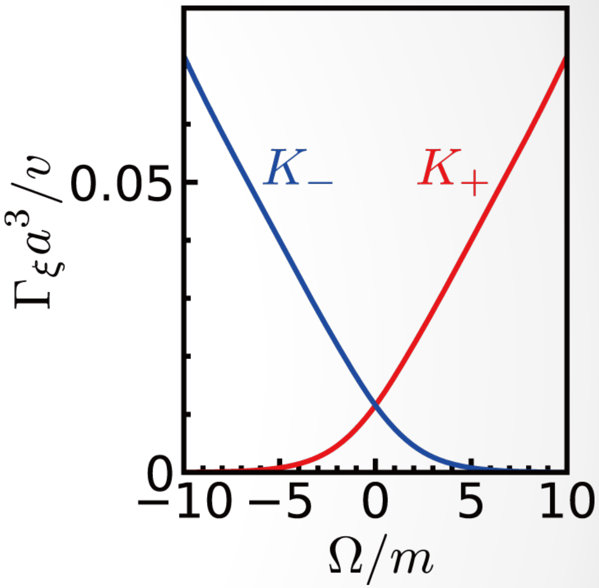
2D gapped Dirac system



Total production rate

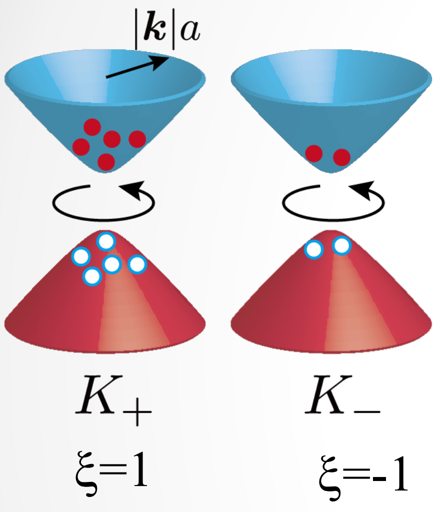
$$\Gamma_\xi \equiv \frac{|\Omega|}{(2\pi)^3}\int d\boldsymbol{k}\mathcal{P}_\xi(\boldsymbol{k})$$

$$\gamma = \frac{\mathcal{P}_+(\boldsymbol{k})}{\mathcal{P}_-(\boldsymbol{k})} = \exp\left(\frac{\pi\xi\Omega m}{veE}\right)$$



measurable with chiral dichroism

2D gapped Dirac system



3D gapped Dirac in dc-E field

$$2\,\mathrm{Im}\,\mathfrak{L} = \frac{1}{4\pi}\sum_{n=1}^{\infty} s_n^{-2}\exp(-m^2 s_n)$$

$$= \frac{\alpha^2}{\pi^2}\mathcal{E}^2\sum_{n=1}^{\infty} n^{-2}\exp\left(\frac{-n\boxed{\pi m^2}}{e\mathcal{E}}\right). \qquad (6.41)$$

Schwinger 1951

2D gapped, rotating E

$$\Gamma_\xi \simeq \frac{eE}{(2\pi)^2}\sqrt{\frac{eE}{v}}\exp\left(-\pi\frac{E_{S,\xi}}{E}\right)$$

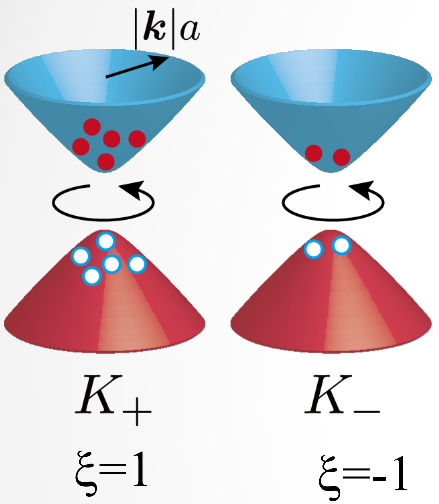
Schwinger limit

$$E_{S,\xi} \equiv (\overline{m}_{\mathrm{eff},\xi})^2/(ve) = \boxed{(m-\xi\Omega/4)^2}/(ve)$$

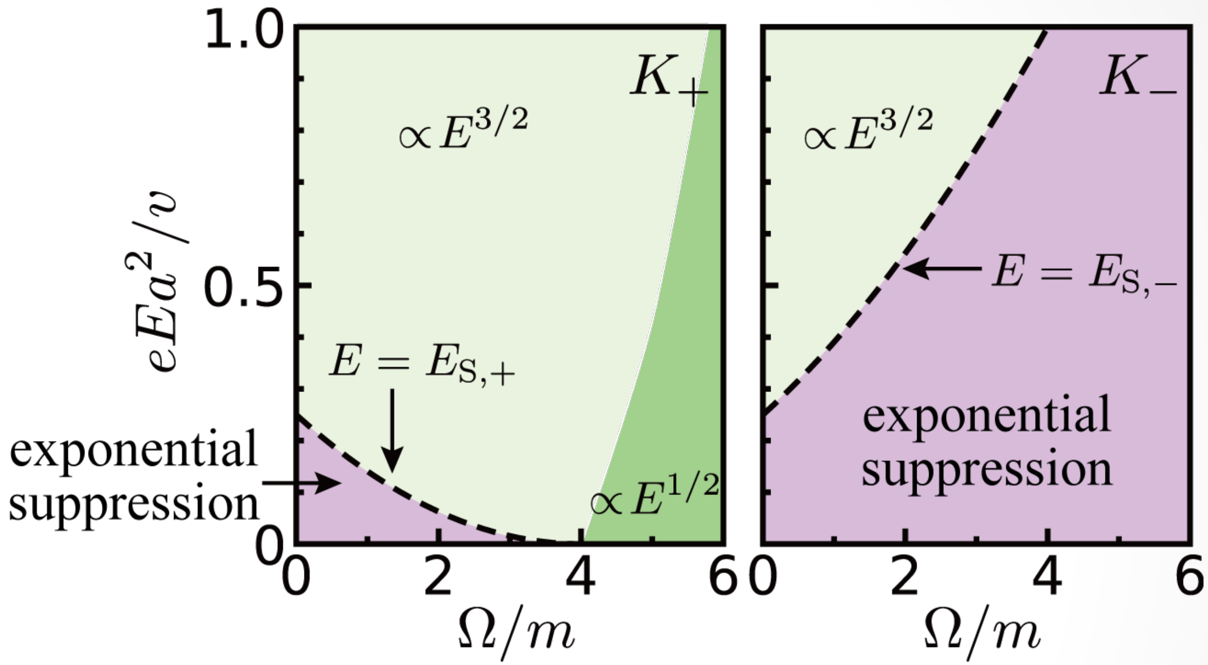
Vanish at $\Omega=4m$ (for $\xi=1$)

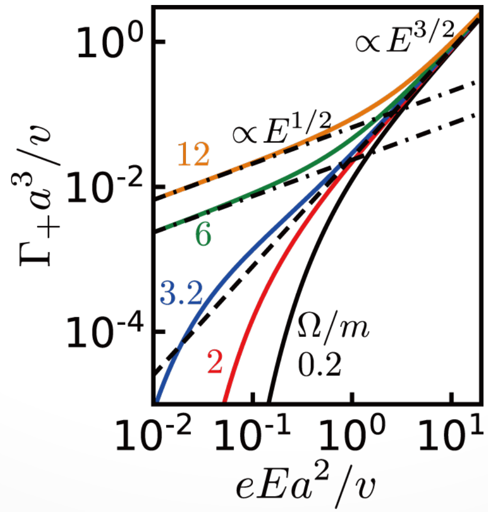
What does this mean?

2D gapped Dirac system



"phase diagram"





$$\Gamma_\xi \;\to\; \frac{eE}{(2\pi)^2}\sqrt{\frac{eE}{v}}$$ large E

$$\Gamma_\xi \simeq \frac{|\Omega|}{(4\pi)^2 v}\sqrt{|\Omega| m}\sqrt{\frac{eE}{v}}$$ small E, $\Omega > 4m$

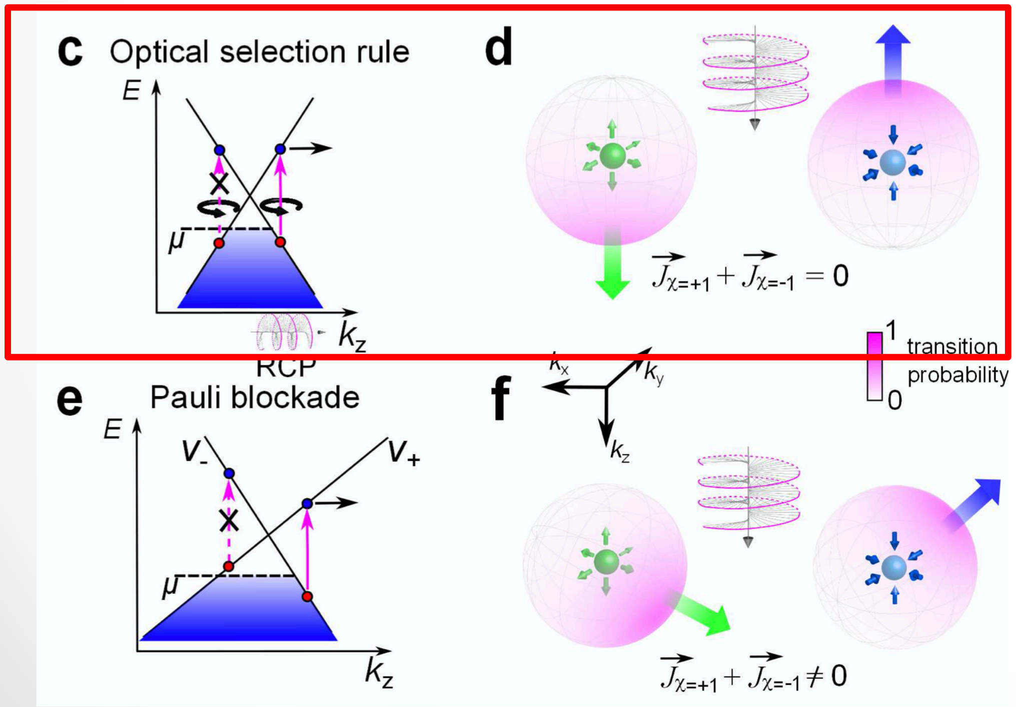
3D gapless Dirac system

$$\hat{\mathcal{H}}_{3\mathrm{D}} = v \sum_{j=x,y,z} \hat{\gamma}^0 \hat{\gamma}^j (q_j + eA_j) \qquad \hat{\gamma}^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \; \hat{\gamma}^j = \begin{pmatrix} 0 & \hat{\sigma}^j \\ -\hat{\sigma}^j & 0 \end{pmatrix}$$

= Two 3D Weyl fermions with ξ=1, -1 = two 2D Dirac ($m \leftrightarrow k_z$)

Circularly polarized laser $A(-\sin(\Omega t), \cos(\Omega t), 0)$

"Circular photo-Galvanic effect"



2nd order perturbation theory Chan et al. PRB '17

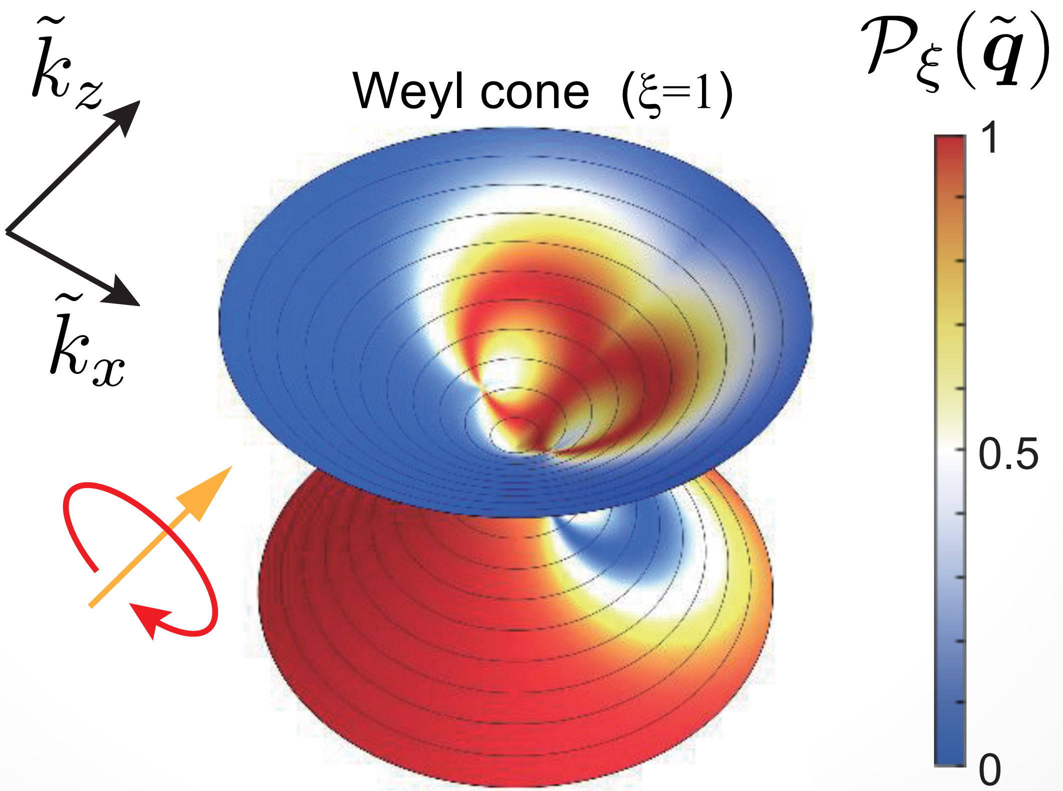
Q. Ma et al. Nat. Phys. '17

TaAs

3D gapless Dirac system

$$\hat{\mathcal{H}}_{3\mathrm{D}} = v \sum_{j=x,y,z} \hat{\gamma}^0 \hat{\gamma}^j (q_j + eA_j) \qquad \hat{\gamma}^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \ \hat{\gamma}^j = \begin{pmatrix} 0 & \hat{\sigma}^j \\ -\hat{\sigma}^j & 0 \end{pmatrix}$$

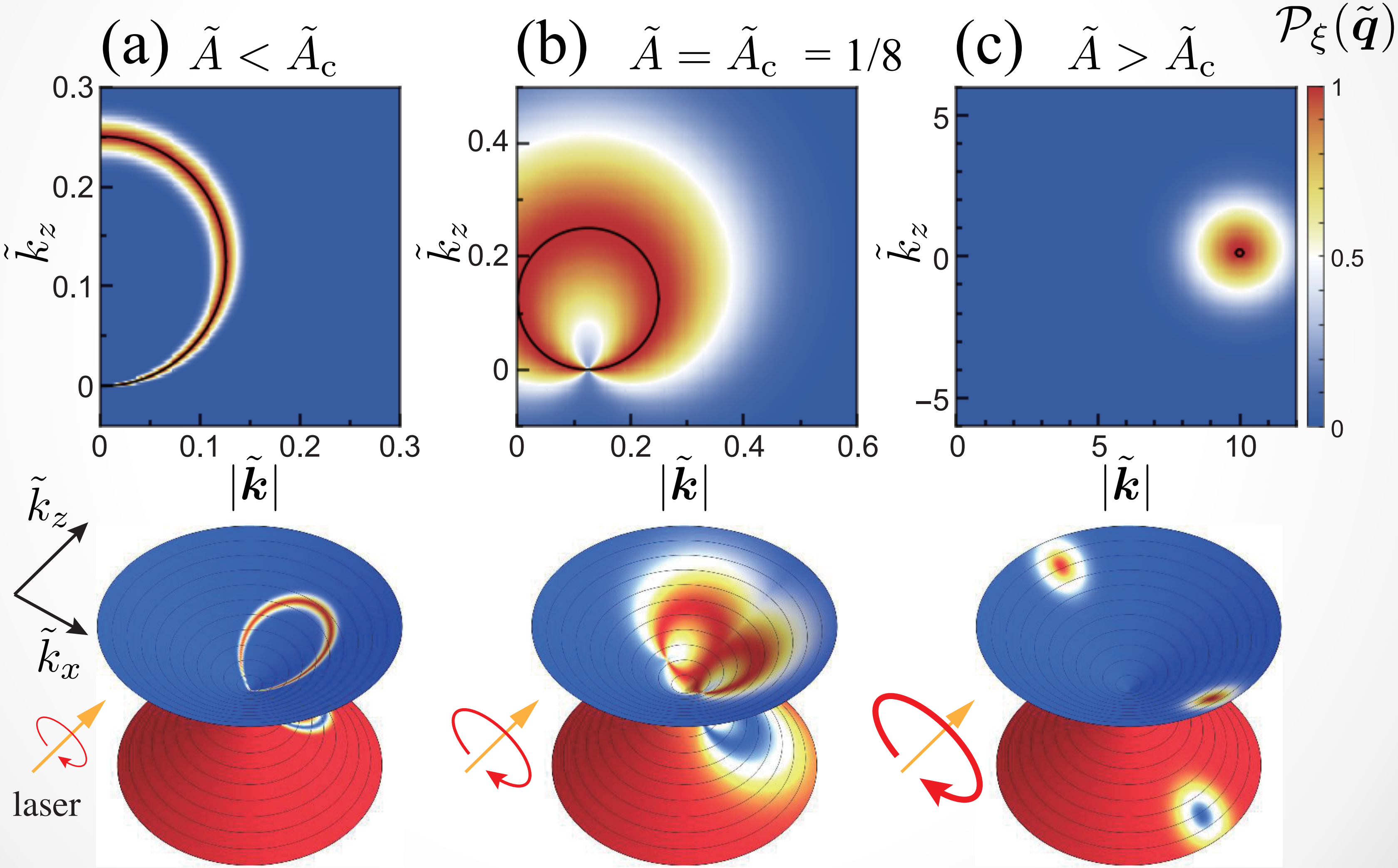
= Two 3D Weyl fermions with ξ=1, -1 = two 2D Dirac ($m \leftrightarrow k_z$)



$k_z \rightarrow -k_z$ for the Weyl cone with ξ=-1

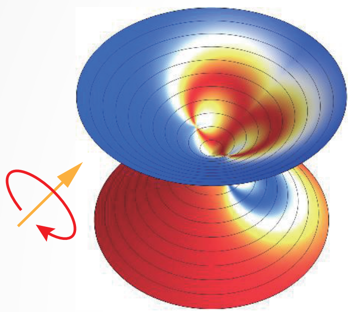
3D gapless Dirac system

scaling parameter $\tilde{A} = veA/|\Omega| = veE/\Omega^2$



3D gapless Dirac system

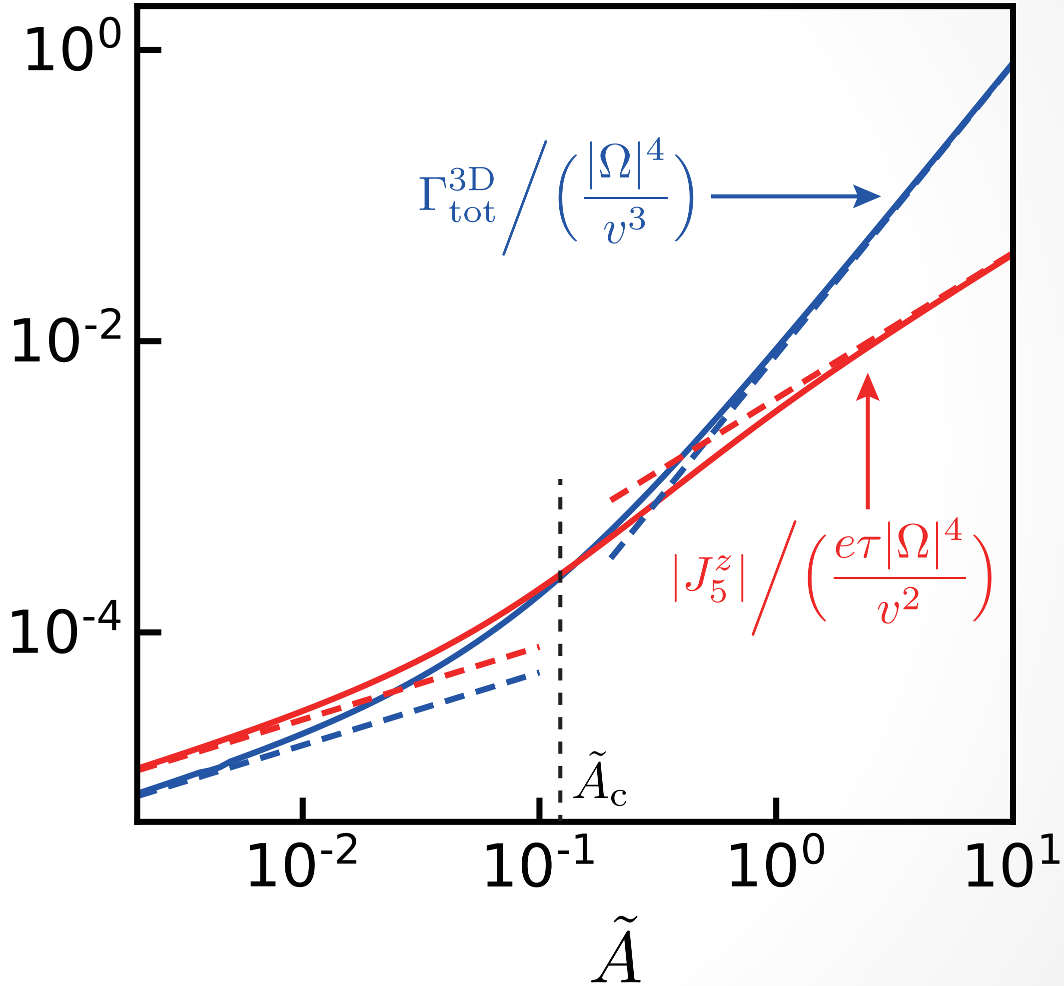
scaling parameter $\tilde{A} = veA/|\Omega| = veE/\Omega^2$



distribution shifted in k_z

→ current J_5^z

$$J_5 = J_{\xi=+1} - J_{\xi=-1}$$



Summary

Takayoshi, Wu, Oka 2020

1. Slow is not adiabatic!

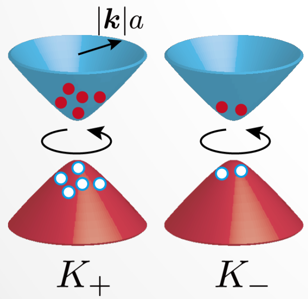
$$|\psi(t)\rangle \simeq \sqrt{1-\boxed{P}}\,|\psi_n(\boldsymbol{\lambda}(t))\rangle e^{-i\int_0^t dt\varepsilon_n(\boldsymbol{\lambda}(t))} e^{i\tilde{\gamma}_n} + \mathrm{O}(\sqrt{P})$$

$$P(F) = \exp\left[-\pi \boxed{\frac{(m+\kappa_{\parallel} vF/4)^2}{|vF|}}\right]$$

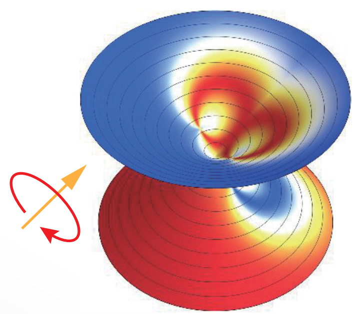
Adiabaticity = large $\boxed{}$
≠ slow parameter change F

2. Twisted Schwinger effect = Nonperturbative optical effect

"Valley-selective circular dichroism"



"Circular photo-Galvanic effect"



Guiding principle

~~1. Energy-momentum conservation~~
2. Nonadiabatic Geometric effect

Berry 1990