2020/8/24 @ 熱場 online

Improvement of the SFtX method based on the gradient flow in the study of finite temperature $N_f = 2+1$ QCD

K. Kanaya*, A. Baba*, S. Ejiri[1], M. Kitazawa[2], H. Suzuki[3], Y. Taniguchi*, T. Umeda[4]

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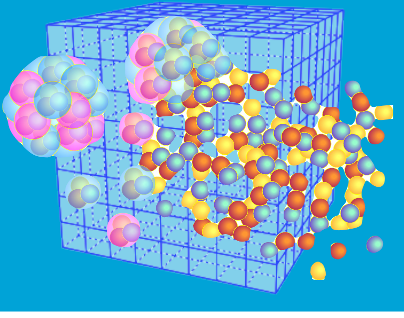


We study thermodynamic properties of lattice N_f = 2+1 QCD with improved Wilson quarks, adopting the SFtX method (small flow-time expansion method) based on the gradient flow. We study the renormalization scale and 2-loop matching coefficients in the SFtX method in N_f = 2+1 QCD. We find that the μ_0 renormalization scale of Harlander et al. improves the applicability and reliability of the SFtX method. On the other hand, no apparent improvements are seen with the 2-loop matching coefficients on this lattice, while contamination of $O(1/N_t^2)$ discretization errors is suggested at $N_t \leq 10$ in the equation of motion which was assumed in the calculation of Harlander et al.

1. SFtX method based on GF

Gradient flow: modification of fields in terms of a fictitious time t.

$$\dot{B}_\mu = D_\nu G_{\nu\mu}, \qquad B_\mu\big|_{t=0} = A_\mu$$

Narayanan-Neuberger(2006), Lüscher(2009–)

Flowed field $B\mu$ ≈ smeared $A\mu$ over a physical range $\sqrt{(8t)}$.

Operators of flowed fields have no UV divergences nor short-distance singularities at t > 0.

Lüscher-Weisz (2011)

SFtX method: a general method to correctly calculate any renormalized observables on the lattice

H. Suzuki, PTEP 2013, 083B03 (2013) [E:2015, 079201]

We are applying the SFtX method to 2+1 flavor QCD with improved Wilson quarks. The SFtX method removes difficulties associated with the chiral violation.

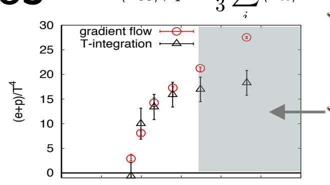
2. QCD with heavy u,d and physical s

Taniguchi et al., PRD96, 014509 (2017); D95, 054502 (2017), D99, 059904(E) (2019)

As the first systematic application of the method to QCD with dynamical quarks, we first studied the case that u and d quarks are heavier than their physical masses.

- ☑ N_F=2+1 QCD, Iwasaki gauge + NP-O(a)-improved clover
- ☑ **fine lattice $a \approx 0.07$fm** (β = 2.05), **slightly heavy ud** ($m_{PS}/m_V \approx 0.63$), **≈phys. s.**
- ☑ CP-PACS+JLQCD $T=0$ config. (28^3x56)
- ☑ WHOT-QCD $T>0$ fixed-scale config.set (32^3xNt, Nt = 4, 6, …, 16)

➤ **EoS** $\epsilon = -\langle T_{00}\rangle$, $p = \frac{1}{3}\sum_i \langle T_{ii}\rangle$

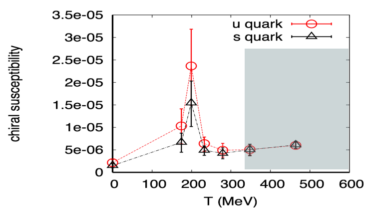


☆ EoS by SFtX agrees well with conventional integration method at $T \lesssim 300$ MeV ($Nt \geq 10$).

☆ Disagreement at $T \gtrsim 350$ MeV attributed to a-indep. lattice artifacts of $O((aT)^2 = 1/Nt^2)$ at $Nt \lesssim 8$.

➤ **Chiral susceptibility** (disconnected)

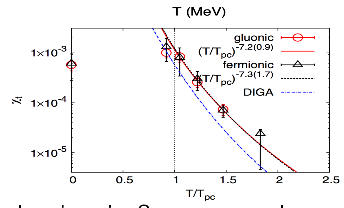
☆ Clear peak at $T_{pc} \approx 190$MeV as expected from Polyakov loop, in spite of the chiral violation due to the Wilson quark action.



➤ **Topological susceptibility**

☆ Gluonic and fermionic definitions agree, even at $a \neq 0$ and with Wilson-type quarks!

☆ Power-low behavior consistent with DIGA.



Consistency among different methods suggests our lattice is fine enough.

The SFtX method works well.

3. Improvement of the SFtX method

Taniguchi et al., PRD102, 014510 (2020)

In the application of the SFtX method to QCD at physical u, d, s quark masses (physical point QCD) and other more complicated observables, we found that we need to improve the method.

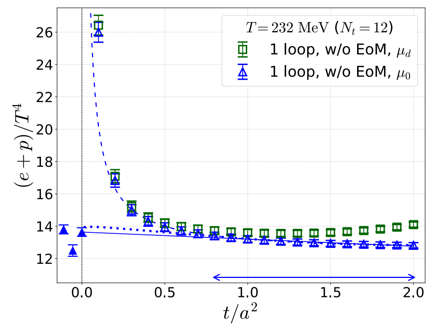
Besides generating finite-temperature configurations at the physical point, we carried out **tests of the renormalization scale and 2-loop matching coefficients in the SFtX method, using the heavy u, d configurations**. We found that a proper choice of the renormalization scale extends the stability and applicability of the SFtX method much.

3.1 Renormalization scale

The renormalization scale of the matching coefficients in the SFtX method has a freedom to choose its value μ within a range. Besides the conventional $\mu_d = 1/\sqrt{(8t)}$, we test a new scale $\mu_0 = 1/\sqrt{(2e^{\gamma}t)}$, which may extend signal to larger t.

We find that the μ_0-scale

- is consistent with the μ_d-scale,
- improves the signal at large t,
- extends the stability and applicability of the SFtX method.

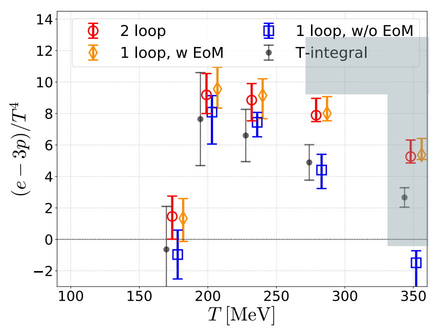


3.2 Two-loop matching coefficients

We have been using 1-loop matching coefficients of Makino and Suzuki (2014). Recently, Harlander et al. calculated 2-loop coeff.'s for EMT, using equation of motion (EoM) for quarks (2018). Use of 2-loop coeff.'s may improve data at small intermediate t.

In our test, we find that the 2-loop

- is consistent with 1-loop besides the effects of EoM,
- shows no apparent improvement on our fine lattice,
- suggests EoM gets $O(1/N_t^2)$ artifacts at $N_t \leq 10$.



4. QCD with physical u, d, s quarks

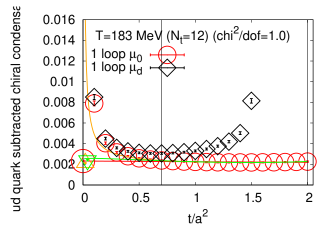
Kanaya et al., PoS Lattice2019, 088 (2020)

Extending the study in 2+1 flavor QCD with heavy u,d quarks, we are generating config.'s at the physical point on a slightly coarser lattice:

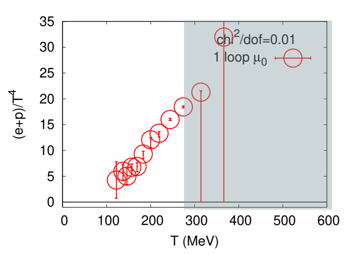
- ☑ N_F=2+1 QCD, Iwasaki gauge + NP-O(a)-improved clover
- ☑ **semi-fine lattice $a \approx 0.09$fm** (β = 1.9), **physical u,d,s**
- ☑ PACS-CS $T=0$ config. (32^3x64), fine-tuned to phys.pt.
- ☑ WHOT-QCD $T>0$ fixed-scale configs. (32^3xNt, Nt = 4, 5, …, 18, $T \approx$122-549MeV)

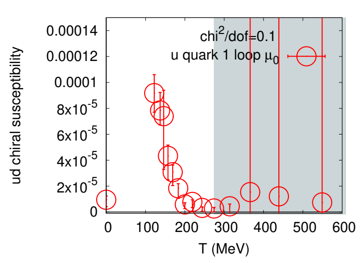
on-going

We find that the μ_0-scale improves well.



Our preliminary results at the physical point, using 1-loop matching coeff.'s with the μ_0-scale, suggest:





☆ e+p ≈ KS (cont. lim.), e-3p ≈ 3 * KS but with large errors.

☆ u,d show a sharper chiral trans. than s.

☆ T_{pc}^{phys} < 157 MeV

☆ $T \approx$122-157 MeV may be critical

(cf.) T_{pc}^{phys} = 156.5 ± 1.5 MeV

Bazavov et al. PLB795, 15 (2019), HISQ

Need more statistics at low T's.
Need continuum extrapolation too.

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WHOT-QCD Collaboration:

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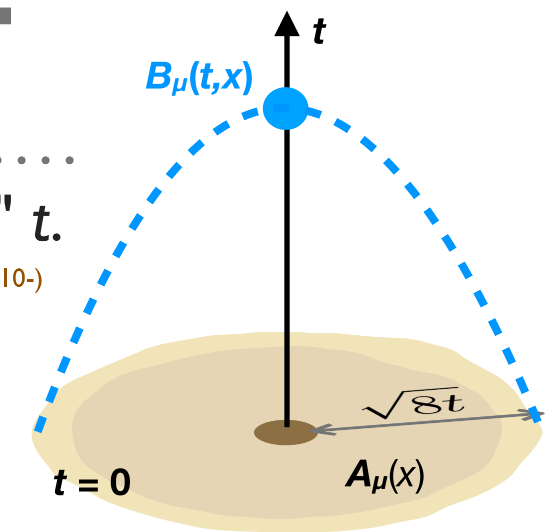
Tomonaga Center for the History of the Universe

SFtX method based on GF



Gradient Flow: a gauge-inv. smearing of fields in terms of a fictitious "time" t.

Narayanan-Neuberger (2006), Lüscher (2010-)

Operators of flowed fields are

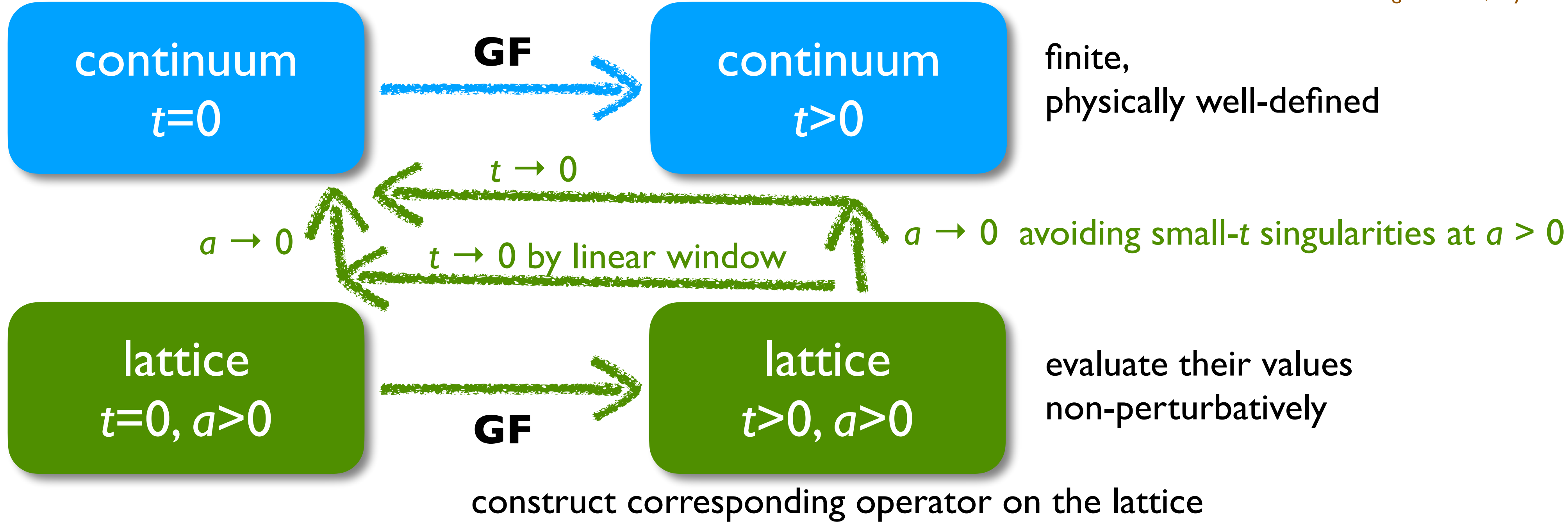
free from UV divergences nor short-distance singularities.

Small Flow-*time* eXpansion (SF*t*X) method

a general method to correctly calculate any renormalized observables non-perturbatively on the lattice

H. Suzuki, PTEP 2013, 083B03 (2013)
Y. Taniguchi et al., Phys.Rev. D 96, 014509 (2017)

physical obs. we want



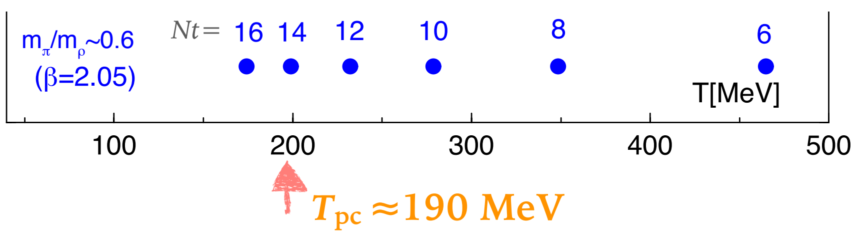
Because we can construct a lattice operator directly from the continuum operator,
this method is applicable also to observables whose base symmetry is broken on the lattice
(Poincaré inv., **chiral sym.**, etc.)

⇨ energy-momentum tensor

⇨ **QCD with Wilson-type quarks,** to cope with problems due to chiral violation.

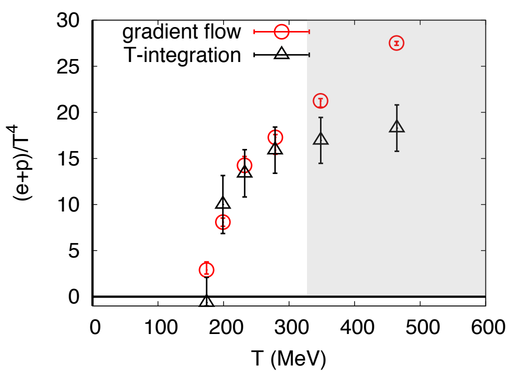
Our **first test** of SFtX with dynamical quarks

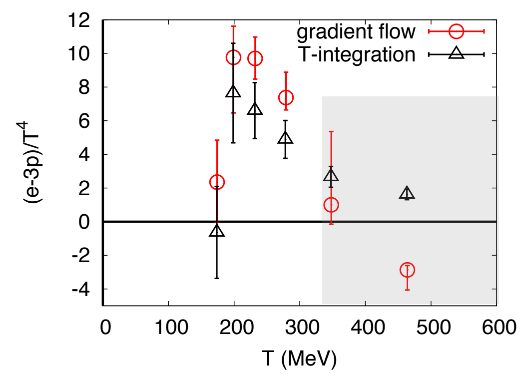
Taniguchi et al., Phys. Rev. D 96, 014509 (2017); Phys. Rev. D 95, 054502 (2017)



- 2+1 flavor QCD with non-perturbatively improved Wilson quarks.
- Slightly heavy ud quarks ($m_\pi/m_\rho \approx 0.63$) with ≈physical s quark ($m_{\eta ss}/m_\phi \approx 0.74$).
- Fine lattice ($a \approx 0.07$fm with improved action).

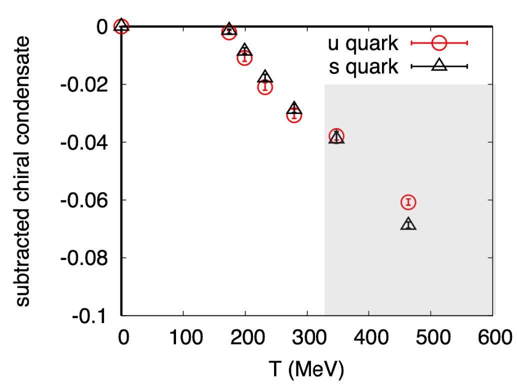
EoS

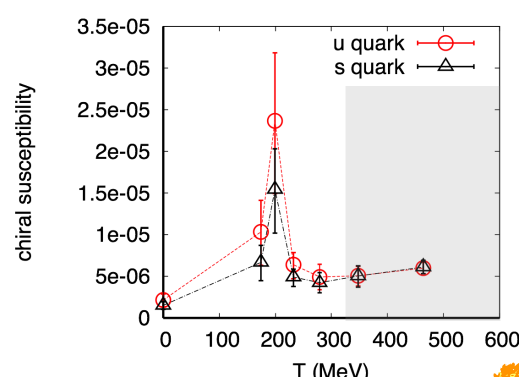




- ☑ Reproduce the results of conventional integral method.
- ☐ Disagreement at $T \geq 350$ MeV due to $O((aT)^2 = 1/Nt^2)$ lattice artifact at $Nt \lesssim 8$.

chiral cond. / chiral susceptibility

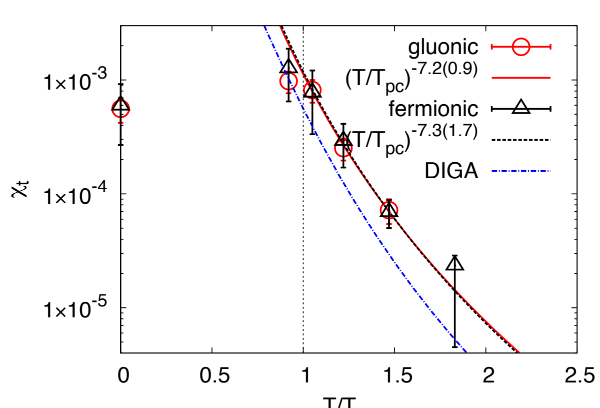




- ☑ Crossover around $T_{pc} \approx 190$ MeV.
- ☑ Peak higher with decreasing m_q.

The SFtX method enables us to directly measure observables, even with chiral violation due to Wilson-type quarks!

topological susceptibility



- ☑ Gluonic and fermionic definitions agree well.
- ☑ Power low consistent with a prediction of Dilute Instanton Gas model.

Improvement of the SFtX method

Taniguchi et al., Phys. Rev. D 102, 014510 (2020)

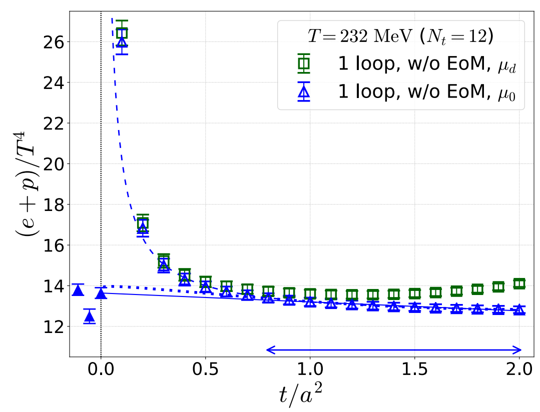
We need to improve the method for application on a coarser lattice and/or more complicated operators.

Here, we test improvements in the **matching coefficients**, which matches flowed operators at $t > 0$ to the target observable at $t = 0$ using perturbatively known properties and thus makes the $t \to 0$ extrapolation smoother.

Choice of the renormalization scale

Conventional choice: $\mu_d(t) \equiv \dfrac{1}{\sqrt{8t}}$

A new proposal: $\mu_0(t) \equiv \dfrac{1}{\sqrt{2e^{\gamma_E} t}}$

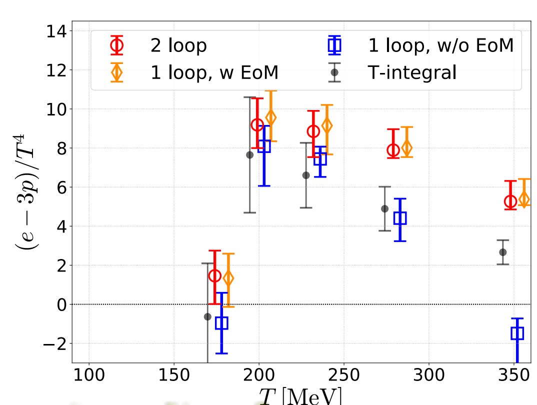


We find that the μ_0-scale
- is consistent with the μ_d-scale
- improves the signal at large t,
- **extends the stability and applicability of the SFtX method.**

2-loop matching coefficients

2-loop matching coefficients are now available for EMT. Harlander-Kluth-Lange, EPJC 78:944 (2018)

Equation of Motion (EoM) for quarks used in their calculation to reduce # of mixing operators.



We find that the 2-loop results
- are consistent with 1-loop besides the effects of EoM,
- show no apparent improvement on our fine lattice,
- suggest: EoM gets $O(1/N_t^2)$ artifacts at $N_t \lesssim 10$.

Details as well as application to the physical point QCD are discussed in the poster.