ユニタリー・フェルミ気体の一粒子スペクトル関数に対する和則の構築

基研研究会「熱場の量子論とその応用」
27.08.2013
Philipp Gubler (RIKEN, Nishina Center)

Collaborators:
Y. Nishida (Tokyo Tech), N. Yamamoto (University of Maryland, YITP),
T. Hatsuda (RIKEN, Nishina Center)

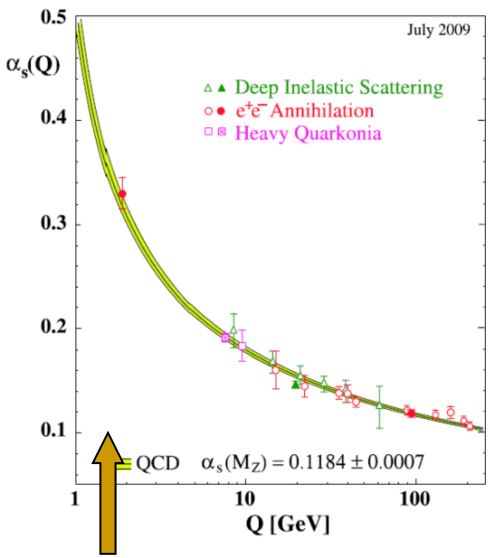
Contents

- Introduction
  - Similarities between QCD and the Unitary Fermi Gas

    $\rightarrow$ The same methods can be used ?!
- The method:
  - The Operator Product Expansion
  - Formulation of sum rules
  - MEM analysis
- First results
- Summary + Conclusions

Introduction

QCD

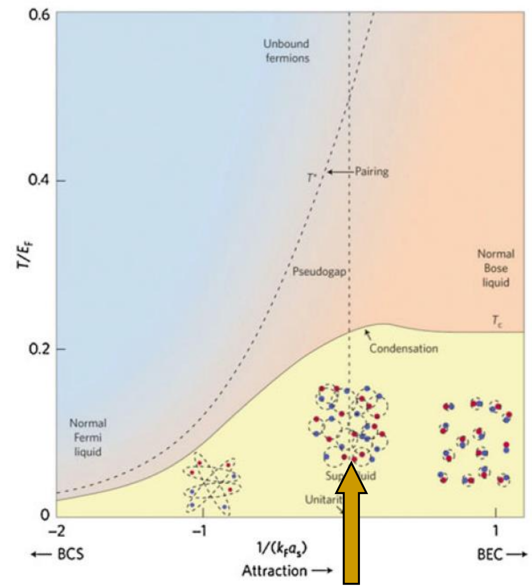


Strongly coupled at low energy
→ naïve perturbation theory does not work!

The properties of QCD matter can be characterized by a few parameters:

$$\langle \bar{q}q \rangle,\ \langle G^a_{\mu\nu} G^{a\mu\nu} \rangle, \ldots$$

The Unitary Fermi Gas



$k_F a$ is infinitely large
→ naïve perturbation theory does not work!

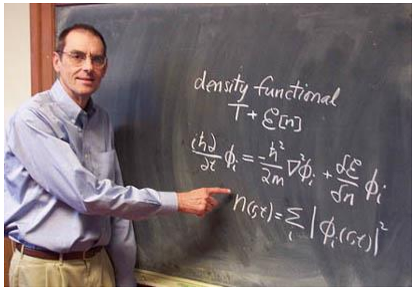
The bulk features of the unitary fermi gas can be characterized by a few parameters:

$$\xi,\ C$$

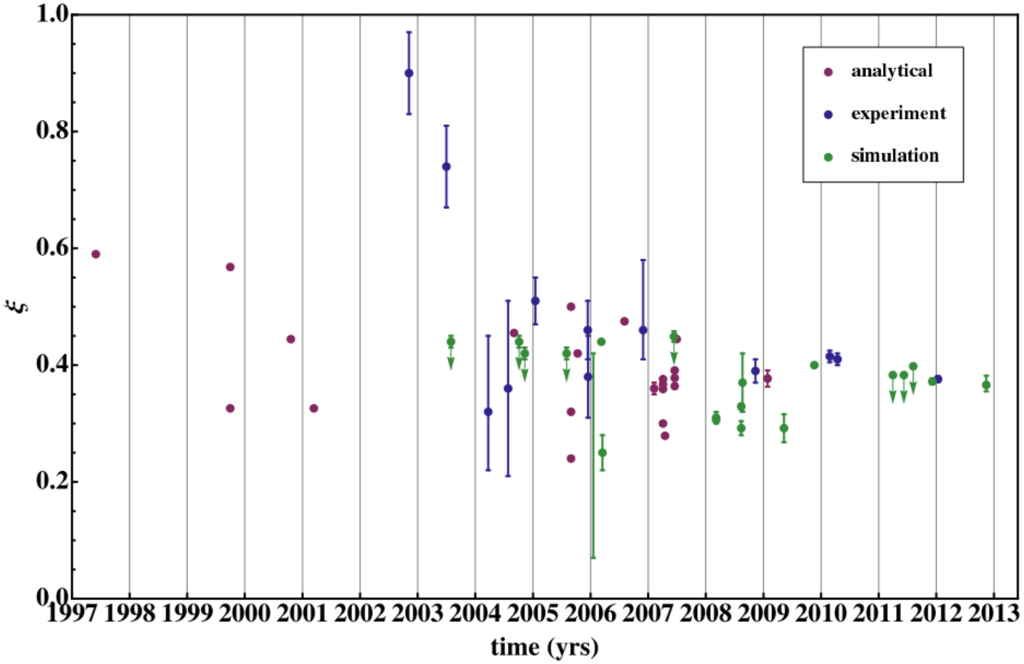
Parameters charactarizing the unitary fermi gas (1)

The Bertsch parameter ξ

$$E = \xi E_{free} = \xi \times \frac{3}{5}\frac{k_F^2}{2m}$$



M.G. Endres, D.B. Kaplan, J.-W. Lee and A.N. Nicholson, Phys. Rev. A **87**, 023615 (2013).



~ 0.37

Parameters charactarizing the unitary fermi gas (2)

The “Contact” C

$$n_\sigma(\boldsymbol{k}) \longrightarrow \frac{C}{k^4}$$

interaction energy

$$T + U = \sum_\sigma \int \frac{d^3k}{(2\pi)^3} \frac{\hbar^2 k^2}{2m} \left(n_\sigma(\boldsymbol{k}) - \frac{C}{k^4} \right) + \frac{\hbar^2}{4\pi m a} C$$

kinetic energy

$$\left(\frac{dE}{da^{-1}} \right)_S = -\frac{\hbar^2}{4\pi m} C$$

$$\left\langle n_1 \left(\boldsymbol{R} + \frac{1}{2}\boldsymbol{r} \right) n_2 \left(\boldsymbol{R} - \frac{1}{2}\boldsymbol{r} \right) \right\rangle \longrightarrow \frac{1}{16\pi^2} \left(\frac{1}{r^2} - \frac{2}{ar} \right) \mathcal{C}(\boldsymbol{R})$$

$$C = \int d^3 R\, \mathcal{C}(\boldsymbol{R})$$

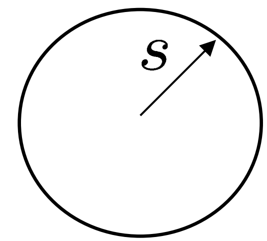
Tan relations

S. Tan, Ann. Phys. **323**, 2952 (2008); **323**, 2971 (2008); **323**, 2987 (2008).

What is the “Contact”?

$$\langle n_1(\boldsymbol{R}+\boldsymbol{r}_1)\; n_2(\boldsymbol{R}+\boldsymbol{r}_2)\rangle \longrightarrow \frac{1}{16\pi^2|\boldsymbol{r}_1-\boldsymbol{r}_2|^2}\mathcal{C}(\boldsymbol{R})$$

$$N_{\text{pair}}(\boldsymbol{R}, s) \longrightarrow \frac{s^4}{4}\mathcal{C}(\boldsymbol{R})$$



$N_1(R,s)N_2(R,s)$: Number of pairs

In field theoretical language:

Zero-Range model:

$$\mathcal{T} = \sum_\sigma \frac{1}{2m}\nabla\psi_\sigma^\dagger \cdot \nabla\psi_\sigma^{(\Lambda)}(\boldsymbol{R})$$

$$\mathcal{U} = \frac{g_0(\Lambda)}{m}\psi_1^\dagger\psi_2^\dagger\psi_2\psi_1^{(\Lambda)}(\boldsymbol{R})$$

$$\mathcal{H} = \mathcal{T} + \mathcal{U}$$

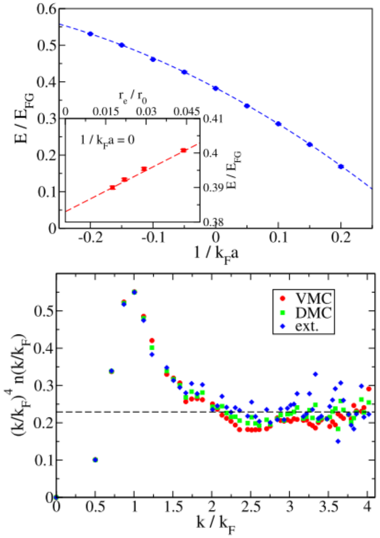
$$\Phi(\boldsymbol{R}) = g_0(\Lambda)\psi_2\psi_1^{(\Lambda)}(\boldsymbol{R})$$

$$C = \int d^3R\,\langle\Phi^\dagger\Phi(\boldsymbol{R})\rangle$$

Local four-fermion operator

E. Braaten and L. Platter, Phys. Rev. Lett. **100**, 205301 (2008).

What is the value of the “Contact”?



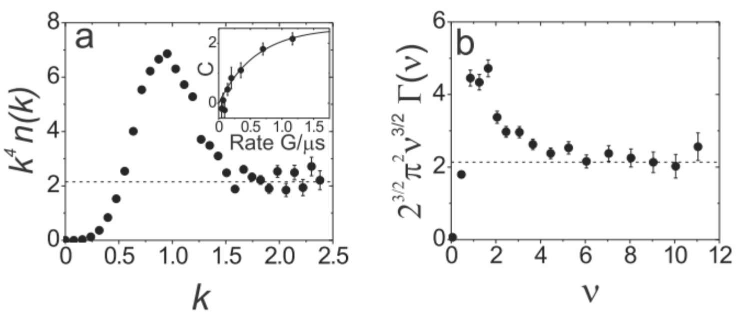
S. Gandolfi, K.E. Schmidt and J. Carlson,
Phys. Rev. A **83**, 041601 (2011).

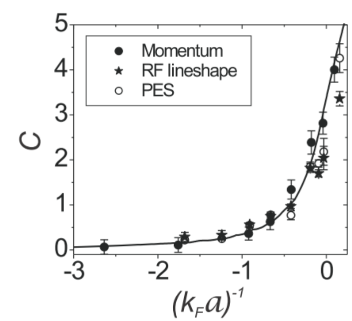
Using Quantum Monte-Carlo simulation:

$$C = \zeta \times \frac{k_F^4}{3\pi^2}$$

3.40(1)

J.T. Stewart, J.P. Gaebler, T.E. Drake, D.S. Jin,
Phys. Rev. Lett. **104**, 235301 (2010).





A new development: Use of the operator product expansion (OPE)

General OPE:
$$O_A\left(\boldsymbol{R}+\frac{1}{2}\boldsymbol{r}\right)\ O_B\left(\boldsymbol{R}-\frac{1}{2}\boldsymbol{r}\right)=\sum_C f_{A,B}^C(\boldsymbol{r})\ O_C(\boldsymbol{R})$$

$$f_{A,B}^C(\boldsymbol{r})\sim r^{d_C-d_A-d_B}$$

works well for small r!

Applied to the momentum distribution $n_\sigma(k)$:

$$n_\sigma(\boldsymbol{k})=\int \mathrm{d}^3R\int \mathrm{d}^3r e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\left\langle\psi_\sigma^\dagger\left(\boldsymbol{R}-\frac{1}{2}\boldsymbol{r}\right)\ \psi_\sigma\left(\boldsymbol{R}+\frac{1}{2}\boldsymbol{r}\right)\right\rangle$$

$k\to\infty$

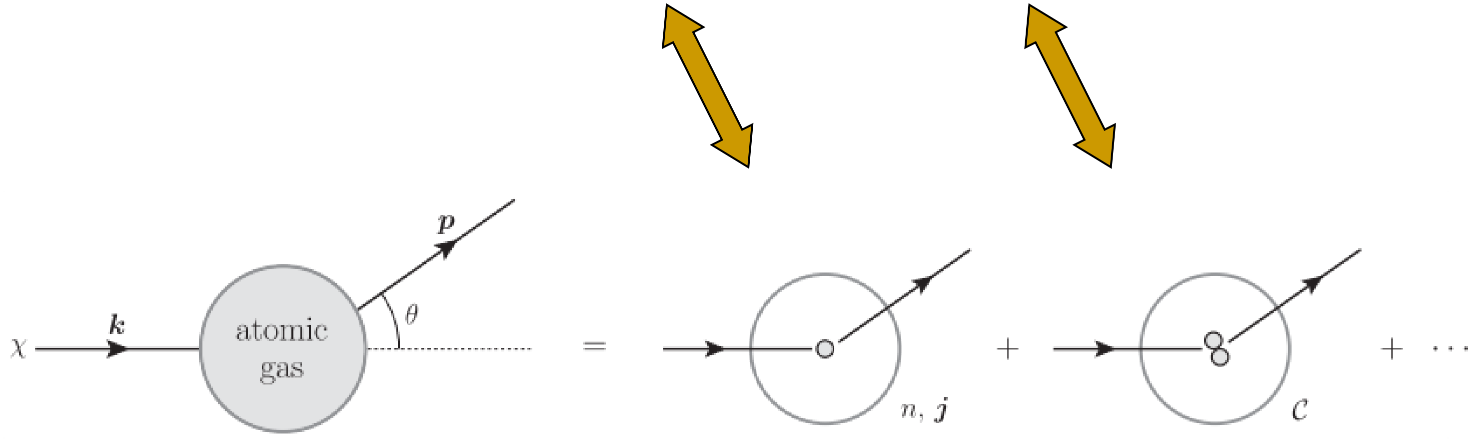
$$\frac{1}{k^4}\underbrace{\int \mathrm{d}^3R\langle\Phi^\dagger\Phi(\boldsymbol{R})\rangle}_{C}$$

E. Braaten and L. Platter, Phys. Rev. Lett. **100**, 205301 (2008).

Another result derived using the OPE (1)

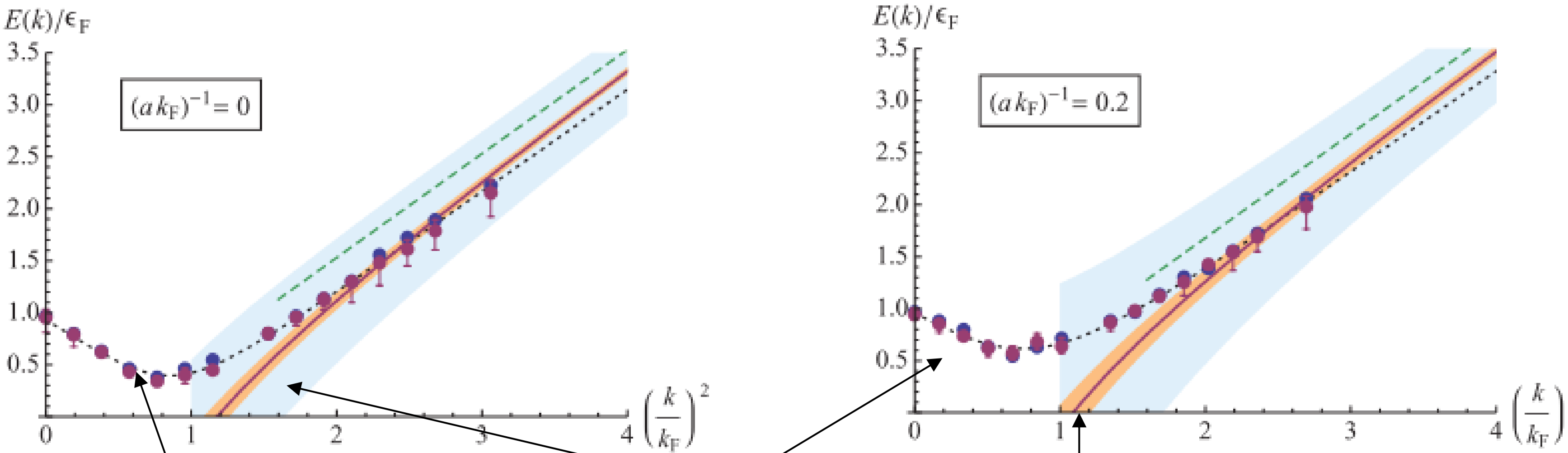
$$E_\uparrow(\boldsymbol{k}) = \left[1 + 32\pi\frac{n_\downarrow}{a_f|\boldsymbol{k}|^4} - 7.54\frac{\mathcal{C}_f}{|\boldsymbol{k}|^4} + O(\boldsymbol{k}^{-6})\right]\frac{\boldsymbol{k}^2}{2m}$$

$$\Gamma_\uparrow(\boldsymbol{k}) = \left[32\pi\left(1 - \frac{4}{a_f^2|\boldsymbol{k}|^2}\right)\frac{n_\downarrow}{|\boldsymbol{k}|^3} + 44.2\frac{\mathcal{C}_f}{a_f|\boldsymbol{k}|^5} + O(\boldsymbol{k}^{-6})\right]\frac{\boldsymbol{k}^2}{2m}$$



Y. Nishida, Phys. Rev. A **85**, 053643 (2012).

Another result derived using the OPE (2)



OPE results

Quantum Monte Carlo simulation

P. Magierski, G. Wlazłowski and A. Bulgac, Phys. Rev. Lett. **107**, 145304 (2011).

Y. Nishida, Phys. Rev. A **85**, 053643 (2012).

Novel idea

Use the OPE to formulate sum rules and analyze them with MEM.

Sum rules have been formulated already in earlier works:

W.D. Goldberger and I.Z. Rothstein, Phys. Rev. A **85**, 013613 (2012).

$$i\mathcal{G}_\uparrow(k) \equiv \int dy e^{iky}\left\langle T\left[\psi_\uparrow\left(x+\frac{y}{2}\right)\psi_\uparrow^\dagger\left(x-\frac{y}{2}\right)\right]\right\rangle$$

$$\mathcal{G}_\uparrow(k) = \frac{1}{k_0 - \epsilon_{\boldsymbol{k}} - \Sigma_\uparrow(k) + i0^+}$$

$$y = \frac{k}{k_{\mathrm{F}}}$$

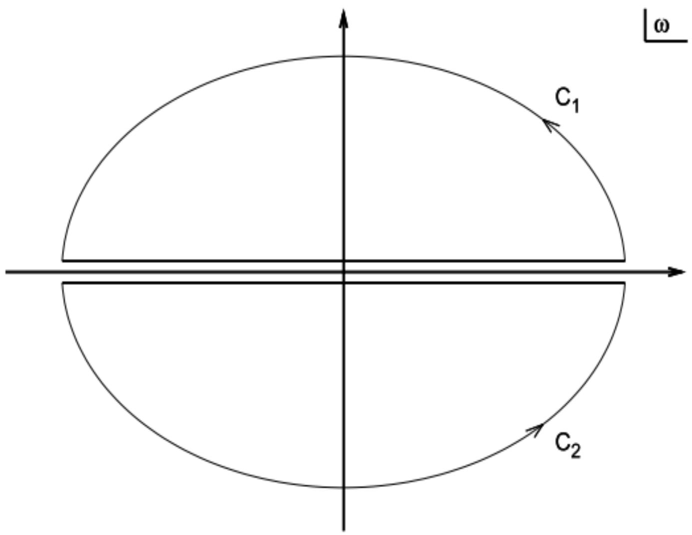
$$\Sigma_\uparrow(\omega + i0^+, y) =$$

$$-\frac{4\sqrt{2}}{3\pi}\frac{1}{\sqrt{y^2/2 - \omega - i0^+}}$$

$$+\frac{4}{3\pi^2}\zeta\left[\frac{1}{\omega + y^2 + i0^+} - \frac{\sqrt{3}}{2\pi}\frac{1}{\omega - y^2/2 + i0^+} - \frac{1}{\pi}\frac{3\omega - y^2}{y(y^2 - 2\omega - i0^+)^{3/2}}\log\left(\frac{1+\sqrt{3}\sqrt{1-2\omega/y^2 - i0^+}}{-1+\sqrt{3}\sqrt{1-2\omega/y^2 - i0^+}}\right) + \frac{1}{y^2}L\left(\frac{\omega}{y^2}\right)\right]$$

$$-\frac{\sqrt{2}}{5\pi}\xi\frac{y^2 - \omega}{(\sqrt{y^2/2 - \omega - i0^+})^5}$$

Construct the sum rules from analiticity (as in QCD)



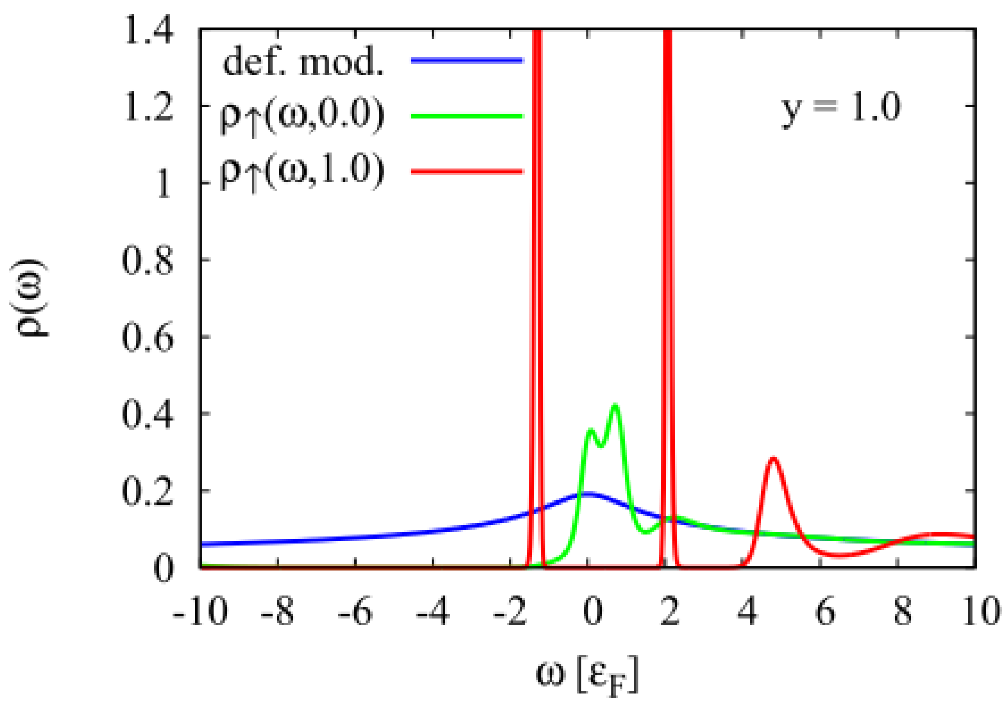
$$
\begin{aligned}
&\int_{-\infty}^{\infty} d\omega \mathscr{K}(\omega)\mathrm{Im}\Sigma_{\uparrow}(\omega+i0^{+},y) \\
&= \frac{8\sqrt{2}}{3\pi}\int_{y^2/2}^{\infty} d\omega \sqrt{\omega - y^2/2}\,\mathscr{K}'(\omega) \\
&+ \frac{4}{3\pi}\zeta\left[\sqrt{3}\mathscr{K}\left(\frac{y^2}{3}\right) - \mathscr{K}(-y^2)\right] \\
&+ \frac{4}{3\pi^2 y}\zeta\int_{y^2/3}^{y^2/2} d\omega \sqrt{y^2 - 2\omega}\left[6\mathscr{K}'(\omega) - (y^2 - 3\omega)\mathscr{K}''(\omega)\right] \\
&+ \frac{4}{3\pi^2 y^2}\zeta\int_{y^2/3}^{y^2} d\omega \mathscr{K}(\omega)\mathrm{Im}\left[L\left(\frac{\omega}{y^2}\right)\right] \\
&- \frac{8\sqrt{2}y^2}{15\pi}\xi\int_{y^2/2}^{\infty} d\omega \sqrt{\omega - y^2/2}\left[3\mathscr{K}''(\omega) + (\omega - y^2)\mathscr{K}'''(\omega)\right]
\end{aligned}
$$

We use a Borel kernel:

$$\mathscr{K}_n(\omega, M) = (\omega - \mu)^n e^{-(\omega-\mu)^2/M^2}$$

Imaginary part is obtained from the sum rules + MEM

Results for the imaginary part of the self energy



Preliminary

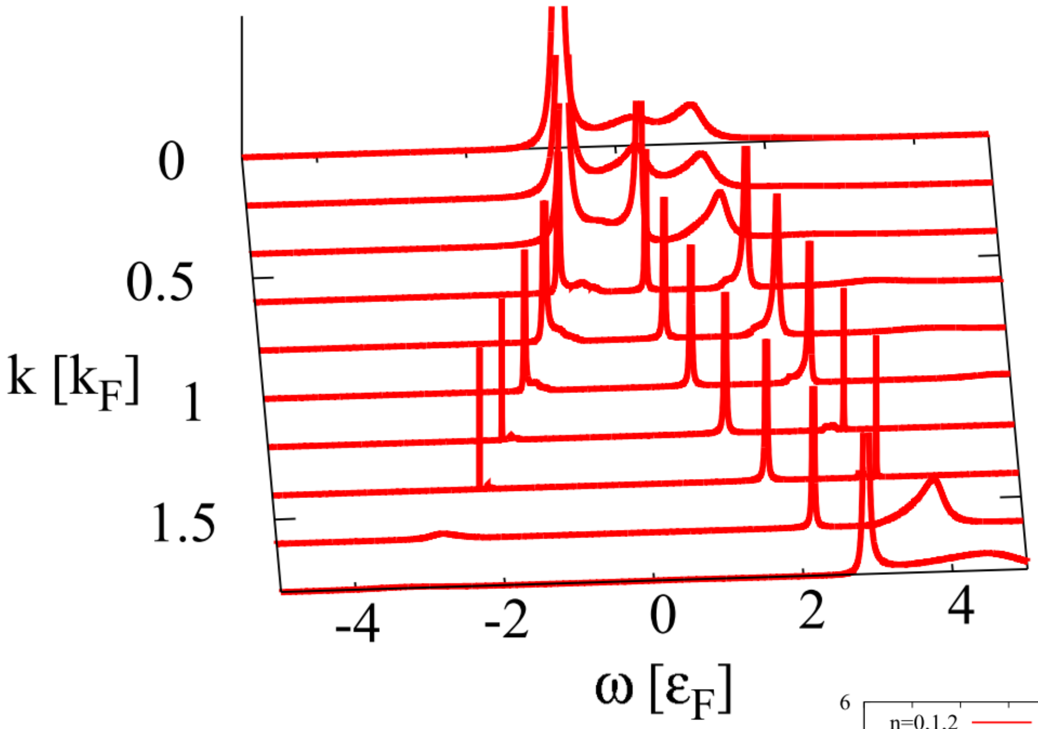
$$\mathrm{Re}\Pi(\omega) = \frac{1}{\pi} \mathrm{P} \int_{-\infty}^{\infty} d\omega' \frac{\mathrm{Im}\Pi(\omega')}{\omega' - \omega}$$

Real part is obtained by numerical integration.

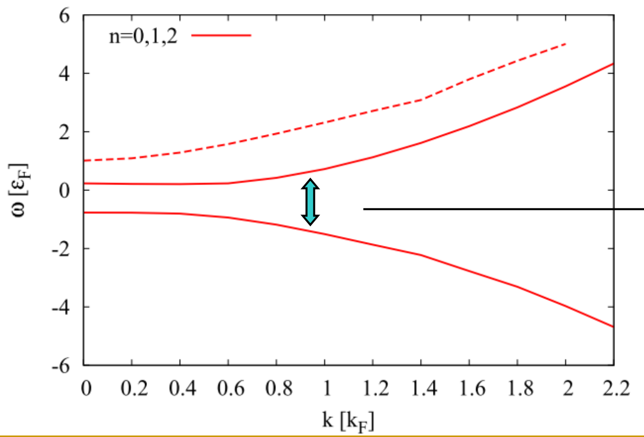
$$\mathcal{G}_{\uparrow}(k) = \frac{1}{k_0 - \epsilon_{\boldsymbol{k}} - \Sigma_{\uparrow}(k) + i0^+}$$

Taking the imaginary part of $G_{\uparrow}(k)$ leads to the single particle spectral density.

Spectral density



Preliminary



Agrees well with experiment!

pairing gap: 0.49 ε_F

Summary + Conclusions

- Unitary Fermi Gas is a strongly coupled system that can be studied experimentally
  - Test + Challenge for theory
- Operator product expansion techniques have been applied to this system recently
- We have formulated sum rules for the single particle self energy and are analyzing these by using MEM
- Using this approach, we can extract the superfluid pairing gap