原子気体のボース凝縮系における最近の進展:人工ゲージ場の効果を中心として

基研研究会「熱場の量子論とその応用」
2013年8月26日(月)～28日(水)
於 京都大学 基礎物理学研究所 湯川記念館
Panasonic 国際交流ホール

笠松　健一

近畿大学　理工学部　理学科

原子気体のボース凝縮系における
最近の進展:
人工ゲージ場の効果を中心として

1, 原子気体のボースアインシュタイン凝縮（BEC）

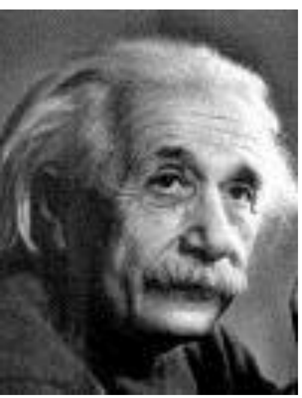
２, 超流動・量子流体力学の展開

３, 中性原子に対する磁場（Synthetic gauge field）

４, スピン軌道相互作用をもつBEC

ボース-アインシュタイン凝縮 (Bose-Einstein condensation :BEC)

S. Bose



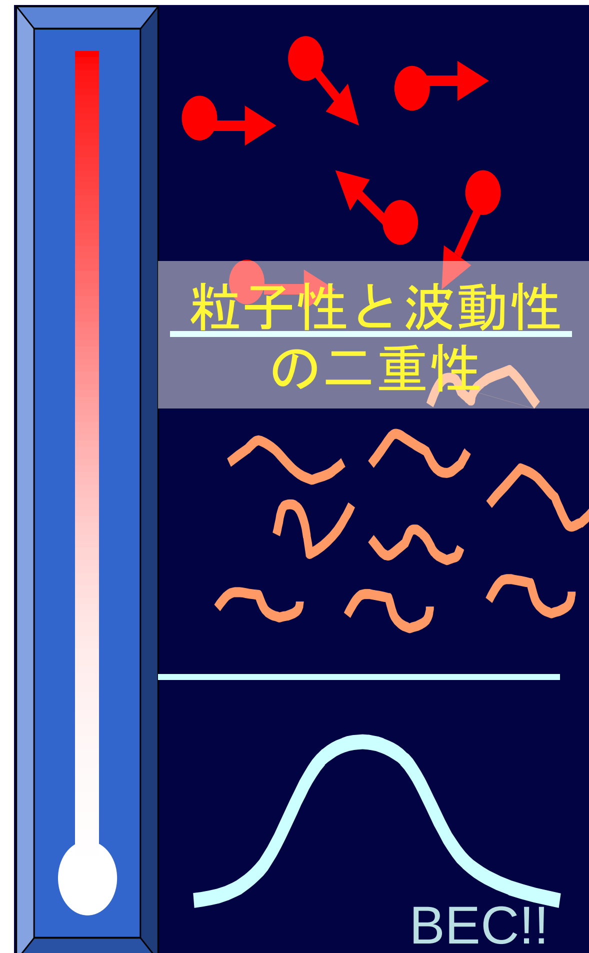
A. Einstein

ボース統計に従う粒子の集団（気体）において、ある温度以下で突然、全粒子数に匹敵する大量の粒子が、最低エネルギー状態に落ち込む現象。

温度が下がると物質波の波長が伸びる。さらに低温では波の位相がそろい始めて、粒子全体が巨大な波として振るまう。

ミクロな量子現象をマクロなスケールに出現させる。

超流動（ヘリウム原子のBEC）
超伝導（電子の対のBEC）
などの劇的な現象を生み出す源

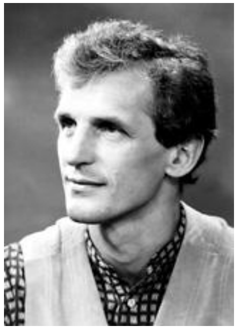


ボース-アインシュタイン凝縮
(Bose-Einstein condensation :BEC)

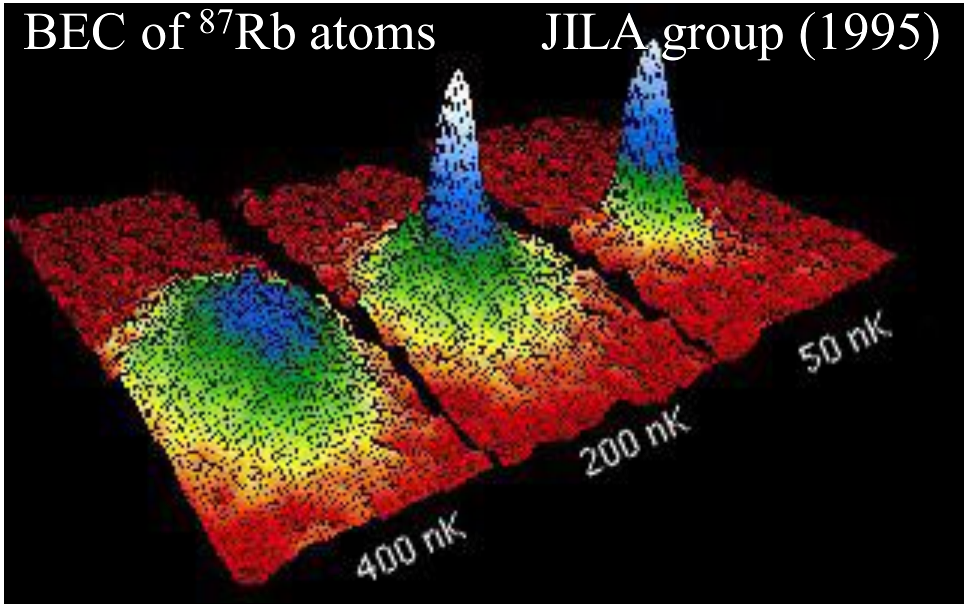
1995年・・・冷却原子気体のBECが実現

E. Cornell

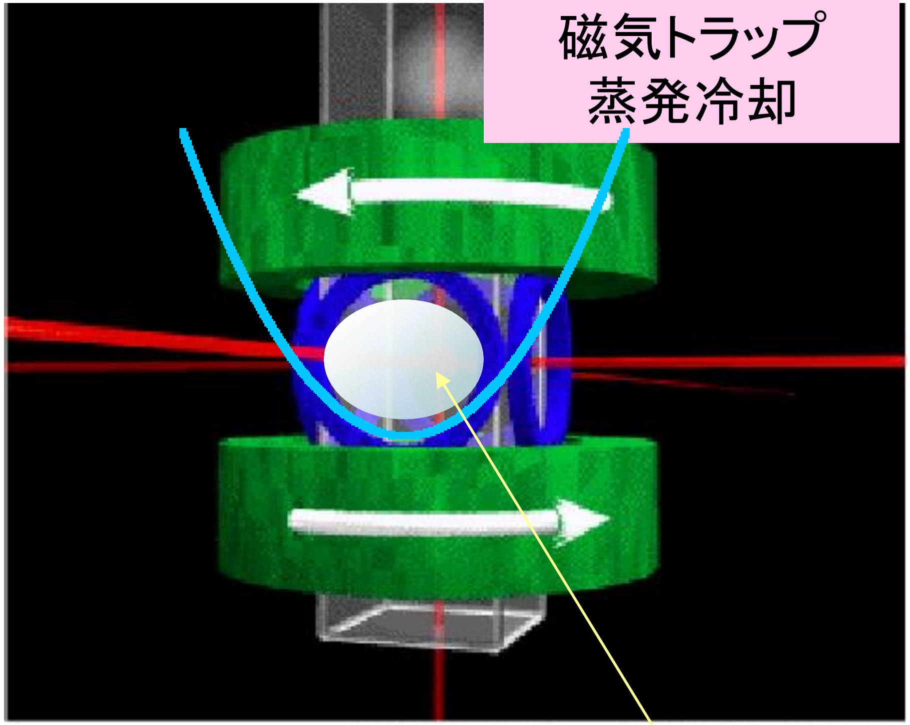
C. Wieman



W. Ketterle



レーザー冷却
磁気トラップ
蒸発冷却



温度: nK（10^{-9} K）
原子数: 10^{4-7} 個
密度: 10^{13-15} /cm^3
サイズ: 10-100 μm

冷却原子BECの特徴

- 巨視的コヒーレンスの発現 ⇔ 巨視的波動関数（秩序変数）
- 希薄気体 (small gas parameter) ・・・・ Gross-Pitaevskii 方程式

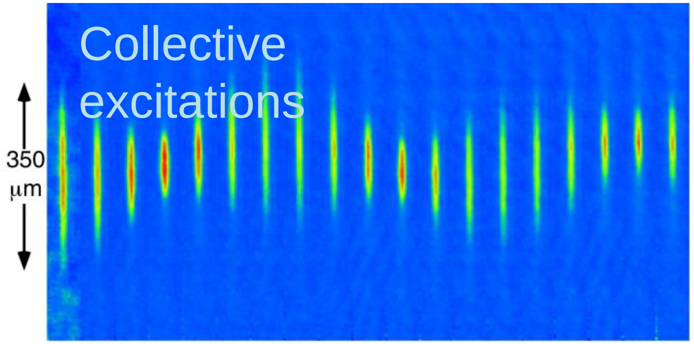
$$i\hbar\frac{\partial\Psi}{\partial t}=\left[-\frac{\hbar^2\nabla^2}{2m}+V+\frac{4\pi\hbar^2 a}{m}|\Psi|^2\right]\Psi$$

Ψ : condensate wave function
V : Trapping potential
a : s-wave scattering length

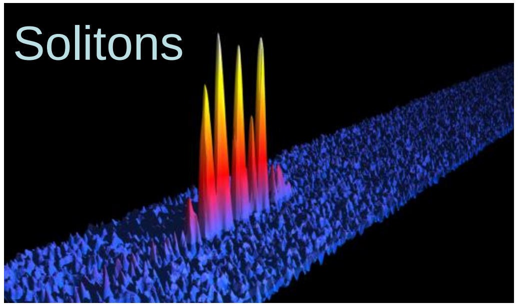
- 比較的大きな回復長（秩序変数の空間変化）
- 閉じ込めポテンシャルによる有限系
- 多成分凝縮体
- 原子間相互作用の操作（フェシュバッハ共鳴）
- 外場による凝縮体操作・・・・トラップの異方性，光格子，低次元系
  光スプーン，不純物ポテンシャル
  人工ゲージ場，．．．

BECにおける超流動性

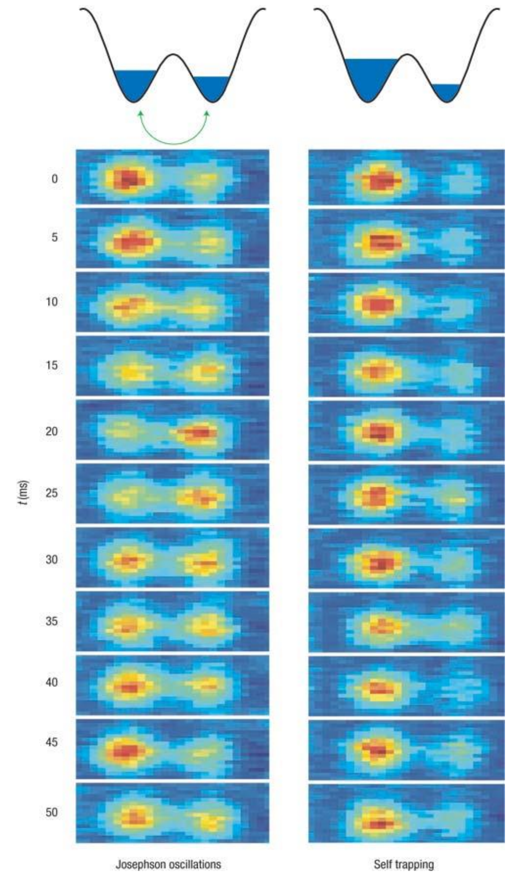
- 巨視的コヒーレンスの発現 ⇔ 巨視的波動関数（秩序変数）

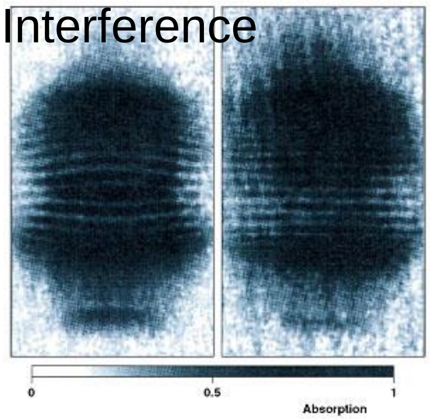


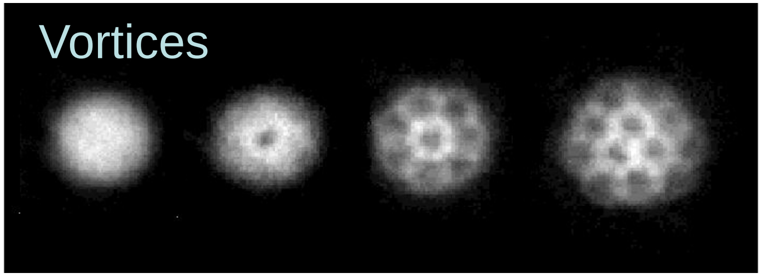
5 milliseconds per frame



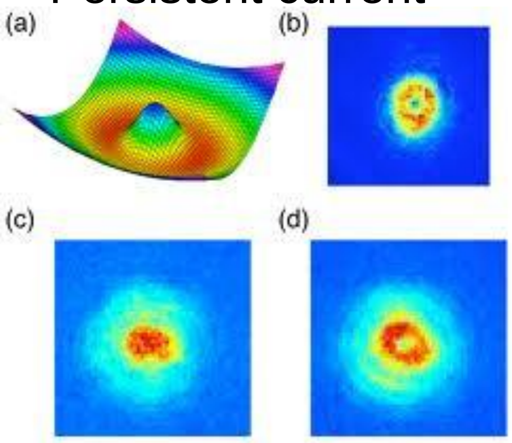
Josephson effect

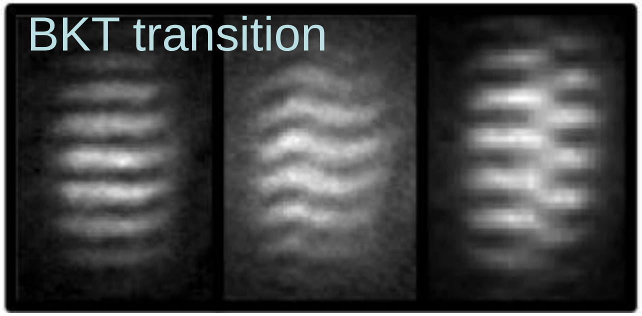






Persistent current





BECにおける量子流体力学

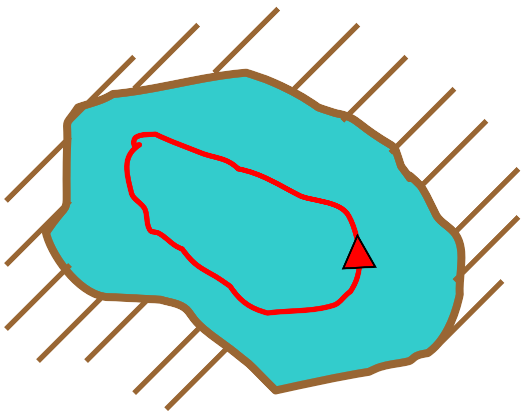
超流動体で起こる流体現象、特に「量子渦」によって支配される流体力学

循環の量子化

巨視的波動関数 $\Psi = |\Psi| \exp(i\theta)$

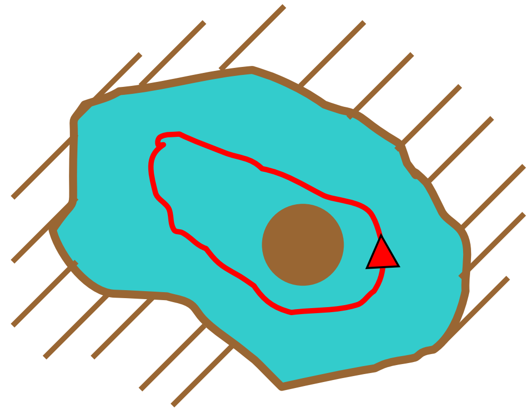
超流動速度場 $\boldsymbol{v}_s = \dfrac{\hbar}{m} \nabla \theta$

単連結領域

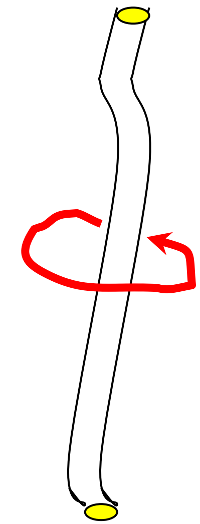


$\Gamma = \oint_C \mathbf{v}_s \cdot d\mathbf{l} = 0, \ \ \mathrm{rot}\,\mathbf{v}_s = 0$

多重連結領域



$\Gamma = \oint_C \mathbf{v}_s \cdot d\mathbf{l} = \dfrac{\hbar}{m} \oint_C \nabla \theta \cdot d\mathbf{l} = \dfrac{h}{m} n$



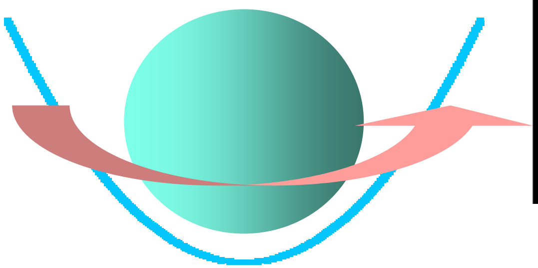
(n: 整数)

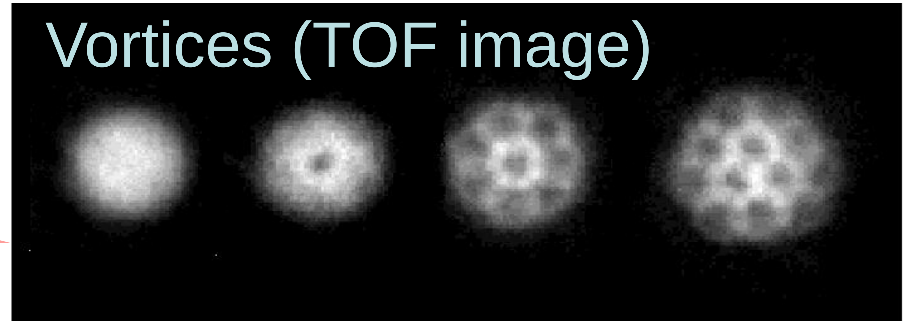
超流体が排除された渦芯と量子化された循環を持つ渦

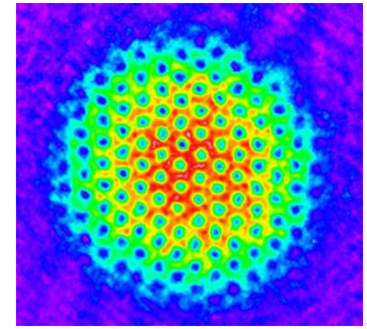
量子渦(Quantized vortex)

冷却原子気体BECにおける量子渦

Rotating BEC

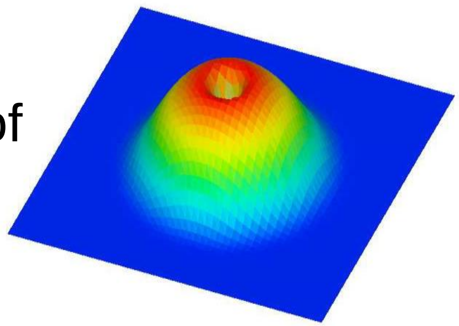


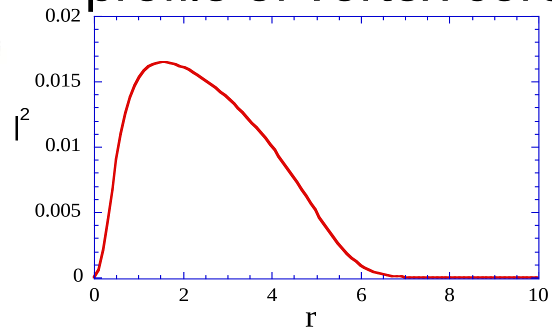


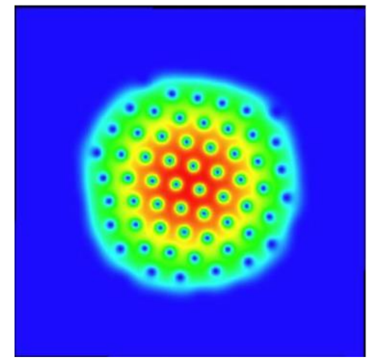


→ 高速回転

Solutions of GP eq.







Review papers

・ K. K., and M. Tsubota, Prog. Low Temp. Phys. **16**, 351 (2009), ed. by W. P. Halperin and M. Tsubota (Elsevier).

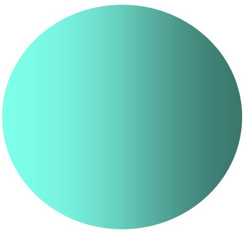
・ A. L. Fetter, Rev. Mod. Phys. **81**, 647 (2009).

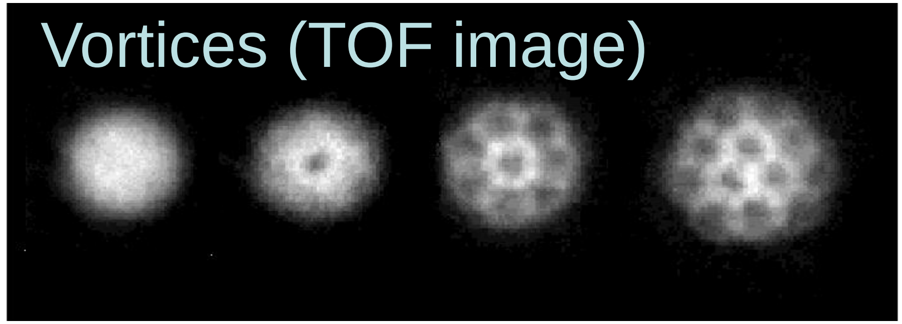
・ M. Tsubota, K. K, M. Kobayashi, Chap.3 in *Novel Superfluids*, ed. by K. H. Bennemann and J. B. Ketterson,

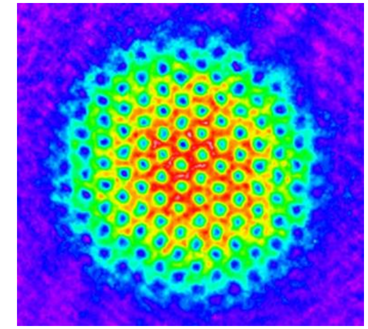
・ M. Tsubota, M. Kobayashi, H. Takeuchi, Phys. Rep., **522**, 191-238 (2013)

冷却原子気体BECにおける量子渦

Rotating BEC







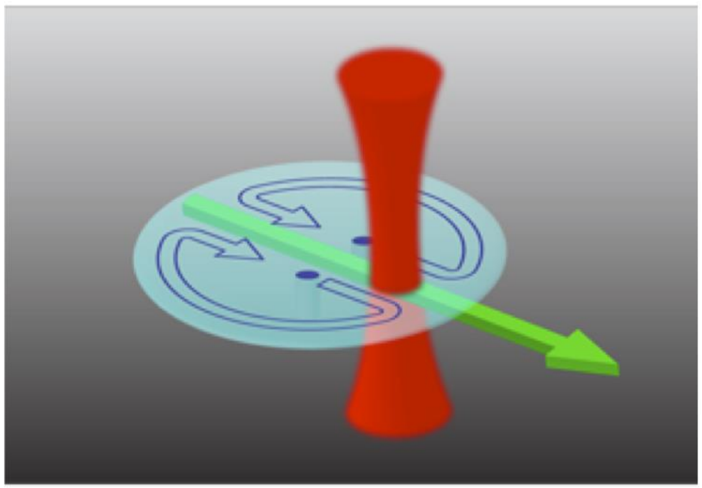
高速回転

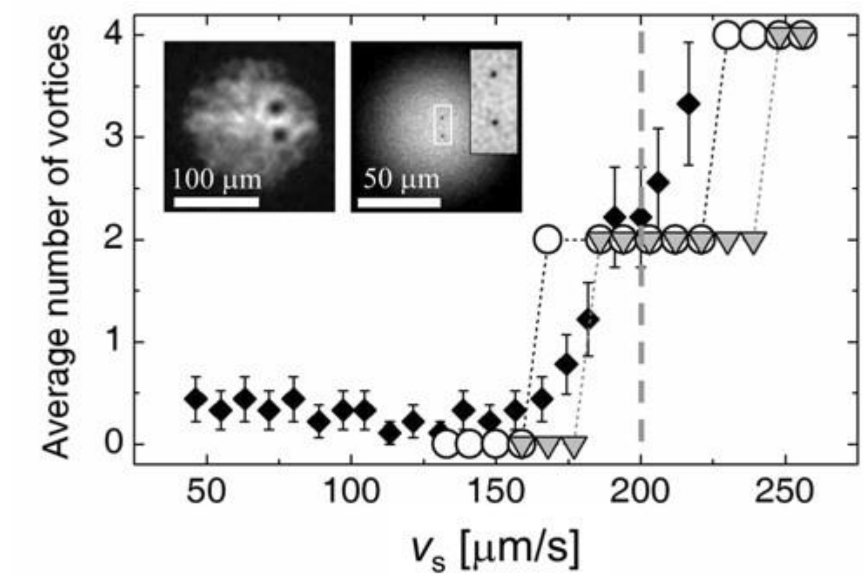
Time of flight (TOF)

量子渦の制御と観測

Neely, et al., PRL **104**, 160401 (2010)

Vortex dipole (渦-半渦ペア) の生成とダイナミクスの観測

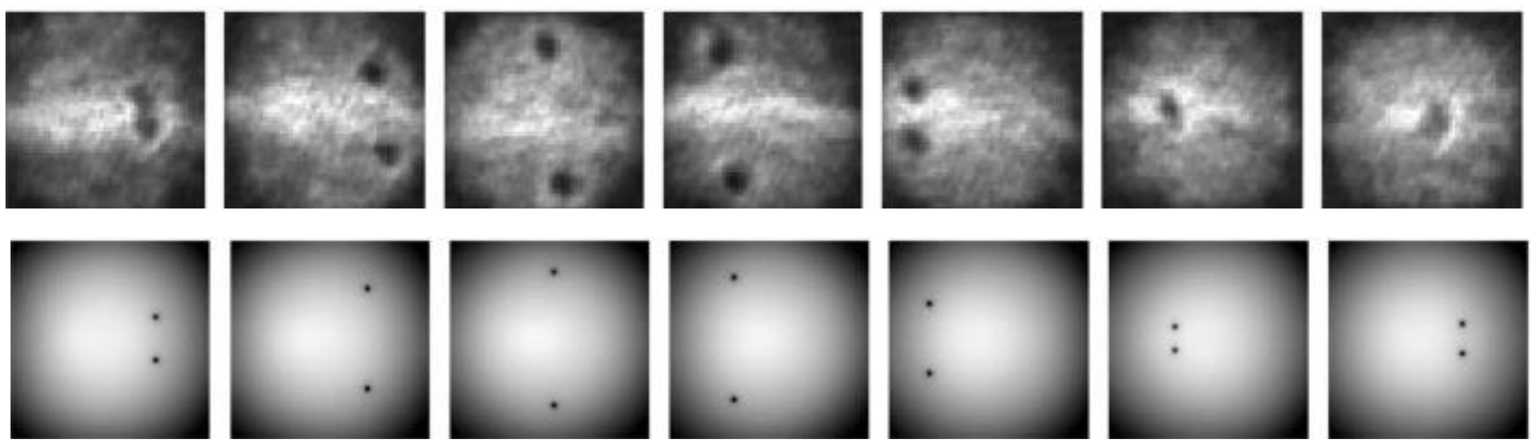




実験と理論の定量的比較

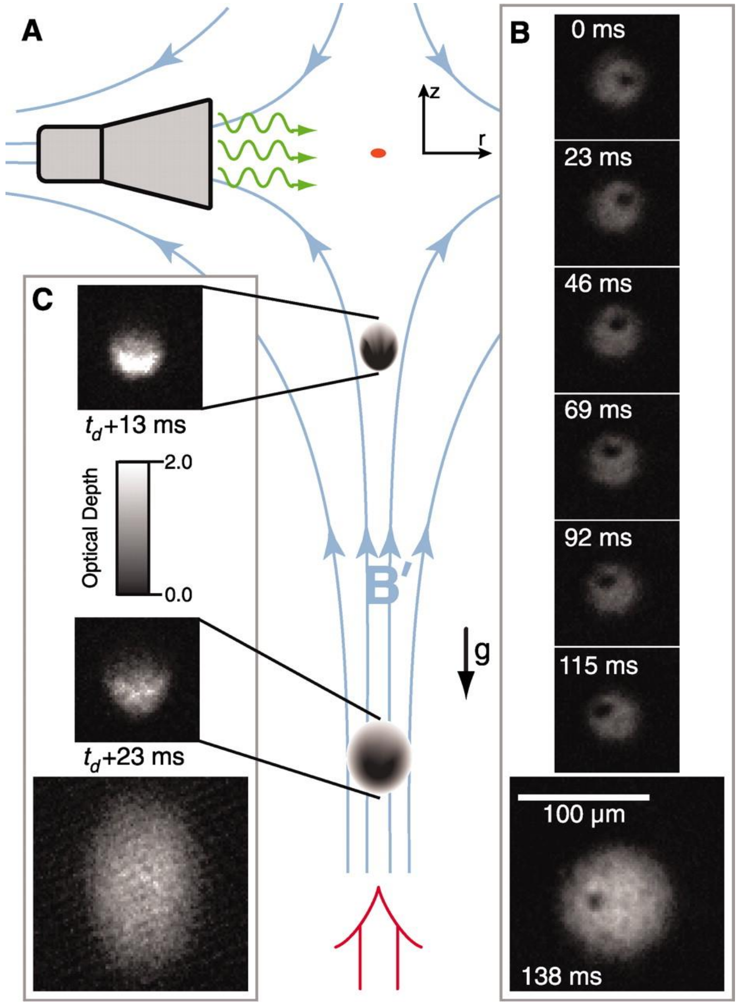
実験

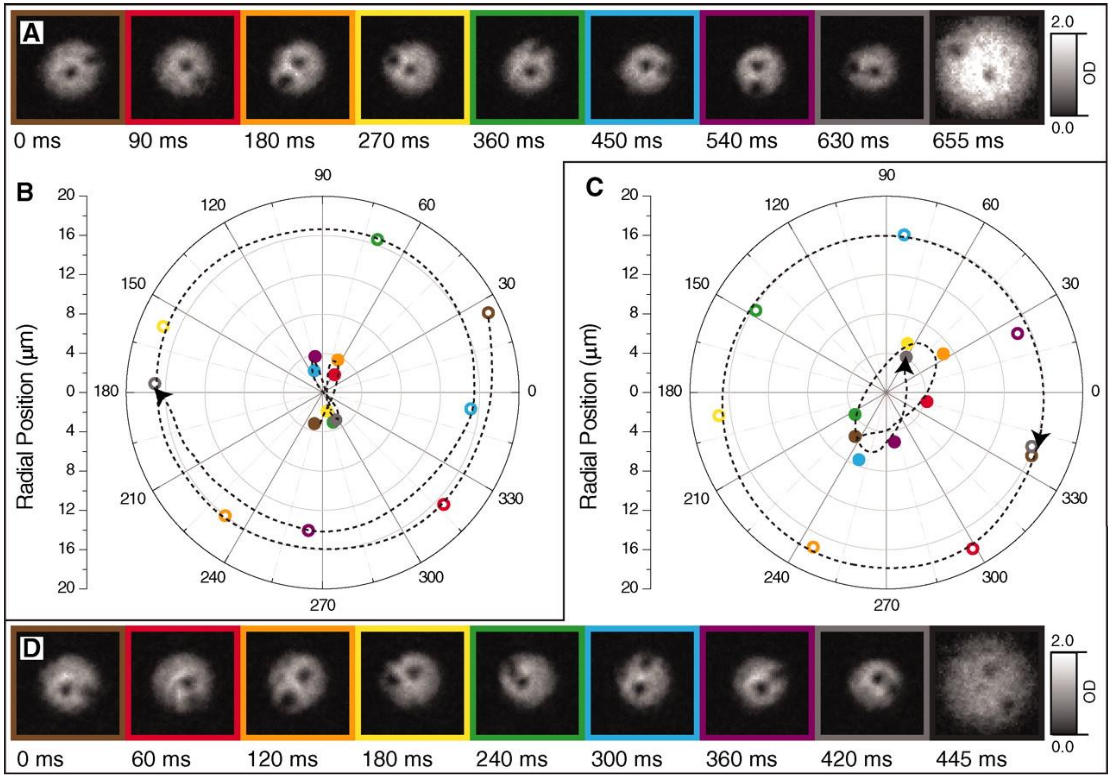
理論



量子渦の制御と観測

D.V. Freilich, et al., Science **329**, 1182 (2010)





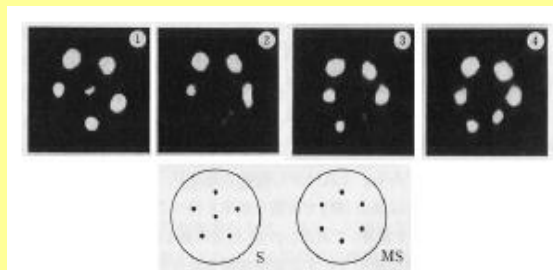
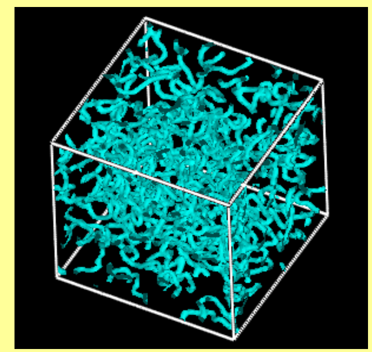
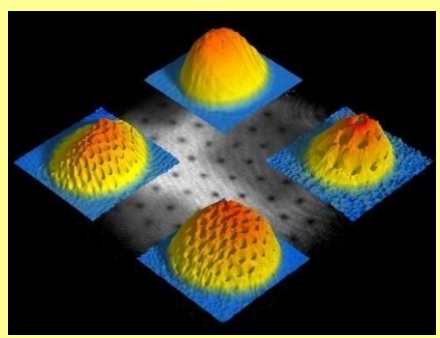
Real-Time Dynamics of a Single Vortex Line and Vortex Dipoles

S. Middelkamp, et al., PRA **84**, 011605(R) (2011)
R. Navarro, et al., PRL **110**, 225301 (2013)

原子気体BECにおける量子乱流

量子渦の制御と観測技術の確立

→　「量子乱流（Quantum turbulence）」状態の解明

	Vortex array	Vortex tangle
Superfluid He		
Atomic BEC		

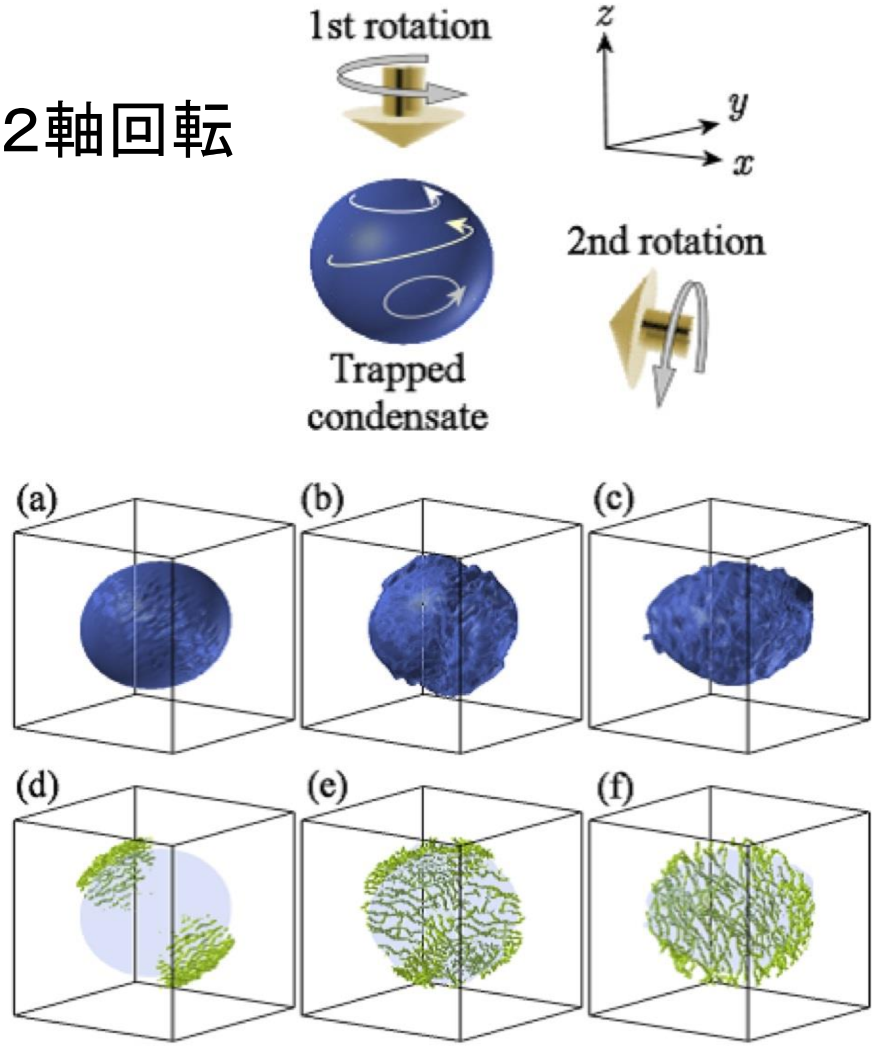
原子気体BECにおける量子乱流

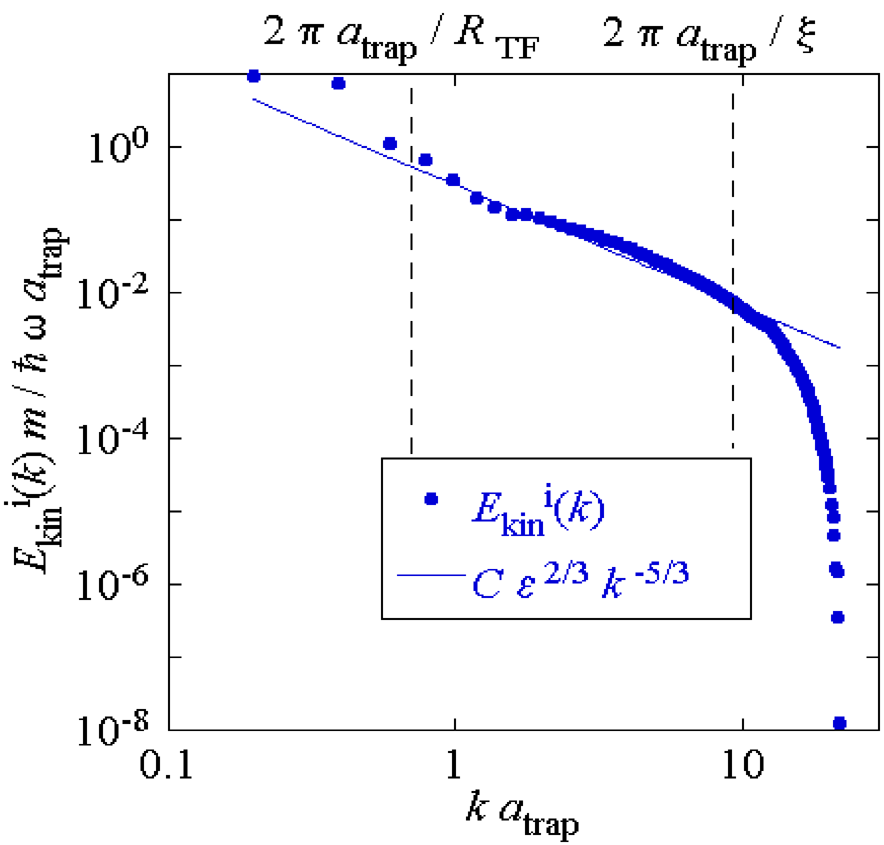
(1)流れを系に加えることができない。

(2)有限系である。十分な慣性領域がとれるか？

M. Kobayashi and M. Tsubota,
PRA **76**, 045603 (2007)

２軸回転





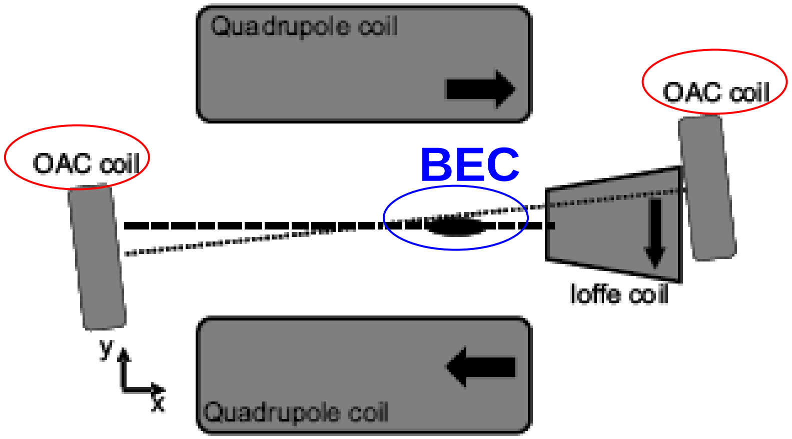
Kolmogorov law $E(k) \propto k^{-5/3}$

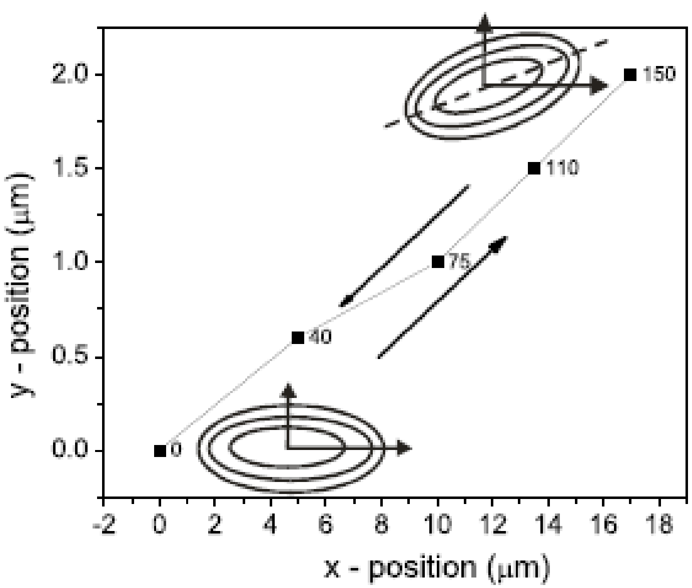
原子気体BECにおける量子乱流

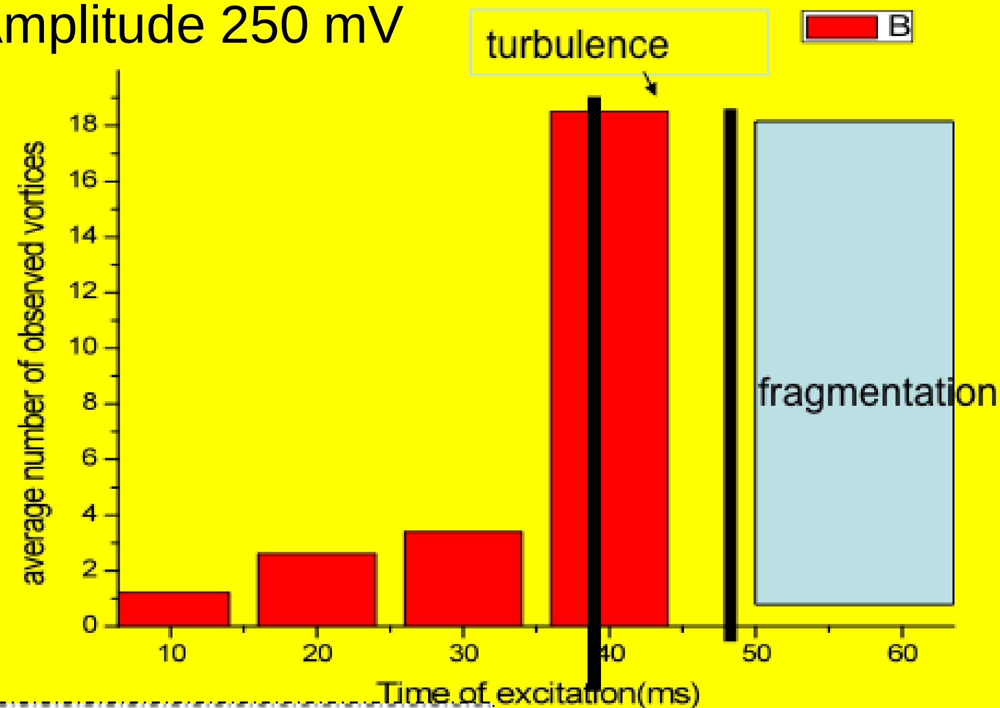
E. A. L. Henn, et al., PRA **79**, 043618 (2009)

Excitation frequency : 200Hz

Amplitude 250 mV



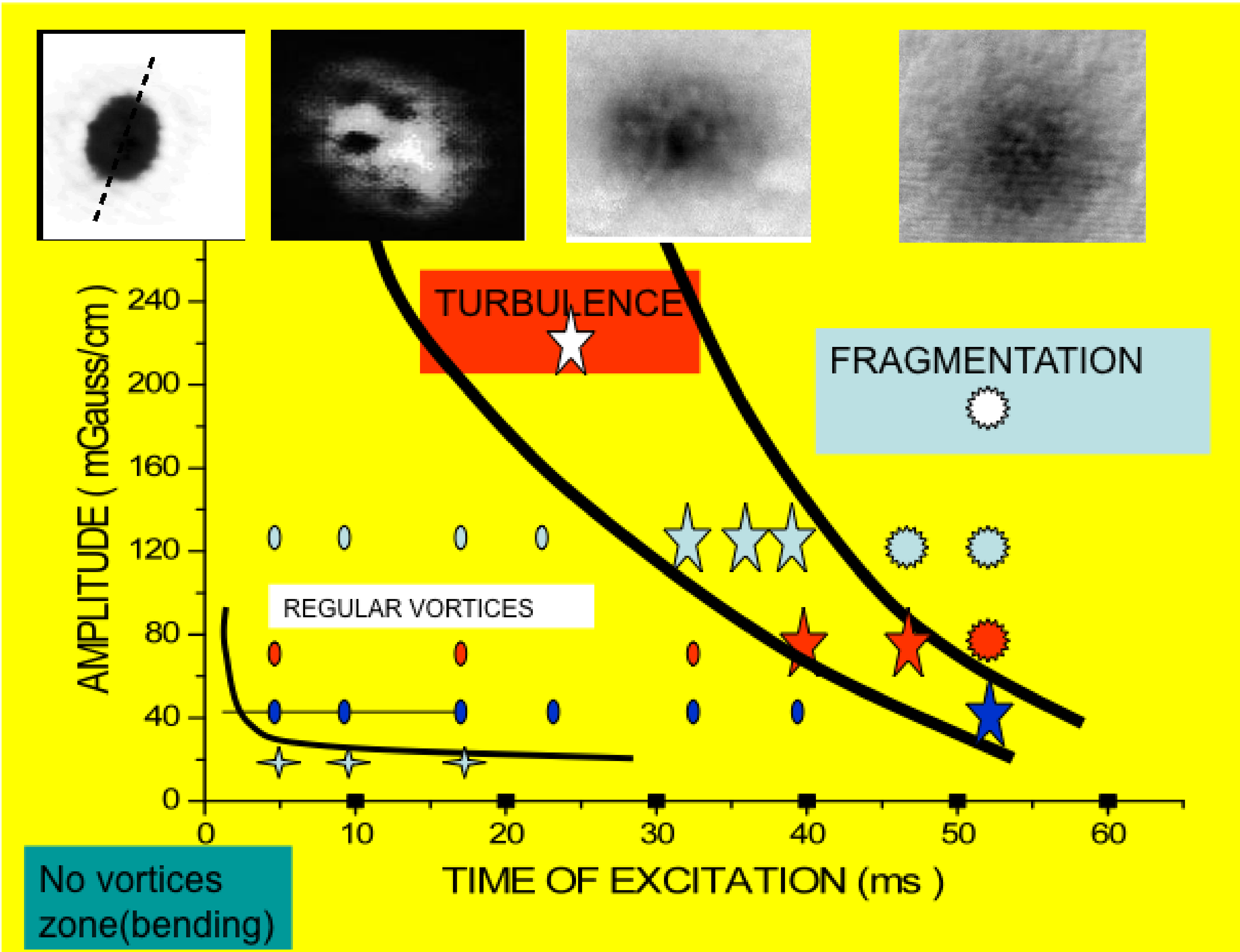




E. A. L. Henn, et al.,
PRL **103**, 045301 (2009)

原子気体BECにおける量子乱流

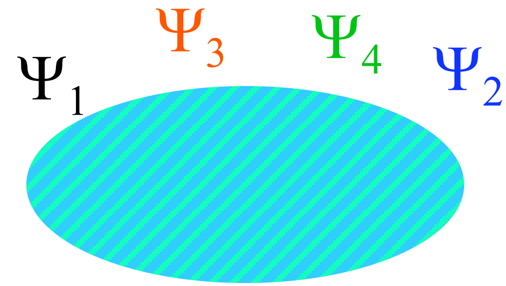
E. A. L. Henn, et al., PRL **103**, 045301 (2009)



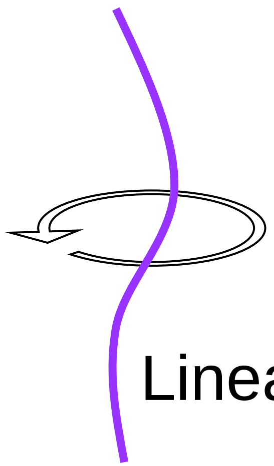
多成分Bose-Einstein凝縮体

多成分Bose Einstein 凝縮体

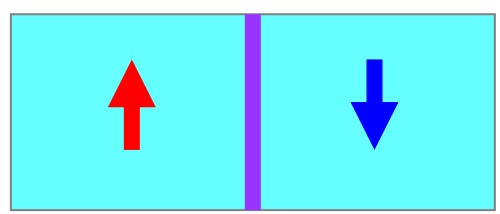
→ 多成分秩序変数



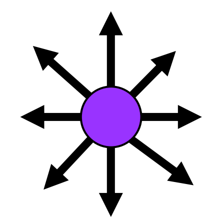
渦，ソリトン，ドメインウォール
モノポール，ブージャム



Planer defect



Point defect



多成分超流動系における多彩なダイナミクス

多成分Bose-Einstein凝縮

スピノールBEC

Kawaguchi and Ueda, Phys. Rep. **520**, 253 (2012)
Stamper-Kurn and Ueda, RMP **85**, 1191 (2013)

アルカリ原子の持つスピン自由度 (F=I+J) を利用する

D. S. Hall et al., PRL **81**, 1539 (1998)	^{87}Rb $\vert F=1, m_F=-1\rangle$, $\vert 2, 1\rangle$
J. Stenger et al., Nature **396**, 345 (1998)	^{23}Na $\vert 1, -1\rangle$, $\vert 1, 0\rangle$, $\vert 1, 1\rangle$
M.D. Barrett et al., PRL **87**, 010404 (2001)	^{87}Rb $\vert 1, -1\rangle$, $\vert 1, 0\rangle$, $\vert 1, 1\rangle$
H. Schmaljohann, et al., PRL **92**, 040402 (2004) M.S. Cheng, et al., PRL **92**, 140403 (2004) T. Kuwamoto, et al., PRA **69**, 063604 (2004)	^{87}Rb $\vert 2, -2\rangle$, $\vert 2, -1\rangle$, $\vert 2, 0\rangle$, $\vert 2, 1\rangle$, $\vert 2, 2\rangle$

BEC mixture

異なる種類の原子・同位体の原子を利用する

G.Modugno et al., PRL 89, 190404 (2002)	^{41}K-^{87}Rb
D. J. McCarron, et al., PRA 84, 011603 (2011)	^{133}Cs-^{87}Rb
S. B. Papp, et al., PRL **101**, 040402 (2008)	^{85}Rb-^{87}Rb
Y. Takasu et al., PRL 91, 040404 (2003)	$^{168, 170, 172, 174, 176}$Yb

多成分Bose-Einstein凝縮

スピノールBEC

Kawaguchi and Ueda, Phys. Rep. **520**, 253 (2012)
Stamper-Kurn and Ueda, RMP **85**, 1191 (2013)

アルカリ原子の持つスピン自由度 (F=I+J) を利用する

D. S. Hall et al., PRL **81**, 1539 (1998)	^{87}Rb $\|F=1, m_F=-1>$, $\|2, 1>$
J. Stenger et al., Nature **396**, 345 (1998)	^{23}Na $\|1, -1>$, $\|1, 0>$, $\|1, 1>$
M.D. Barrett et al., PRL **87**, 010404 (2001)	^{87}Rb $\|1, -1>$, $\|1, 0>$, $\|1, 1>$
H. Schmaljohann, et al., PRL **92**, 040402 (2004) M.S. Cheng, et al., PRL **92**, 140403 (2004) T. Kuwamoto, et al., PRA **69**, 063604 (2004)	^{87}Rb $\|2, -2>$, $\|2, -1>$, $\|2, 0>$, $\|2, 1>$, $\|2, 2>$

BEC mixture

異なる種類の原子・同位体の原子を利用する

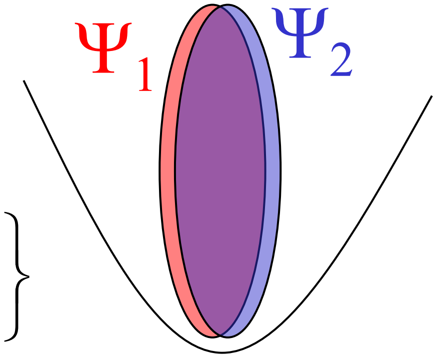
G.Modugno et al., PRL 89, 190404 (2002)	^{41}K-^{87}Rb
D. J. McCarron, et al., PRA 84, 011603 (2011)	^{133}Cs-^{87}Rb
S. B. Papp, et al., PRL **101**, 040402 (2008)	^{85}Rb-^{87}Rb
Y. Takasu et al., PRL 91, 040404 (2003)	$^{168, 170, 172, 174, 176}$Yb

2成分BEC系の定式化

Gross-Pitaevskii (GP) energy functional

$$E[\Psi_1,\Psi_2]=\int d^3r\left\{\sum_{j=1,2}\left[\frac{\hbar^2}{2m_j}|\nabla\Psi_j|^2+V_j|\Psi_j|^2\right]\right.$$

$$\left.+\frac{g_{11}}{2}|\Psi_1|^4+\frac{g_{22}}{2}|\Psi_2|^4+g_{12}|\Psi_1|^2|\Psi_2|^2\right\}$$



Trapping potential

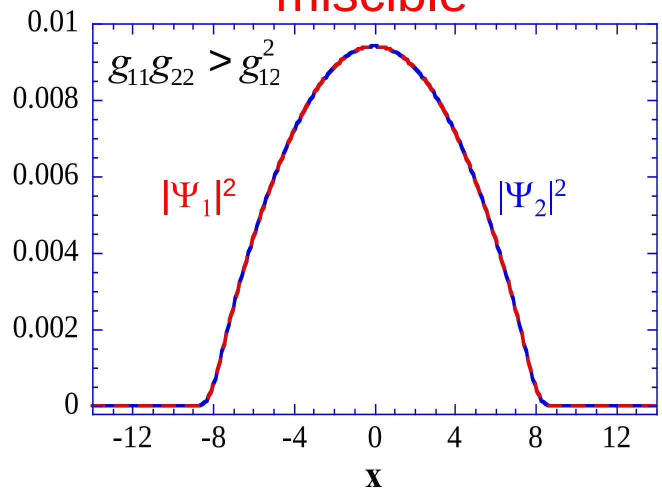
Trapping potential $\quad V_j=\frac{1}{2}m_j\omega^2\left(r^2+\lambda^2z^2\right)$

Atomic interaction $\quad g_{jk}=\frac{2\pi\hbar^2a_{jk}}{m_{jk}}$ $\quad a_{ij}$: s-wave scattering length

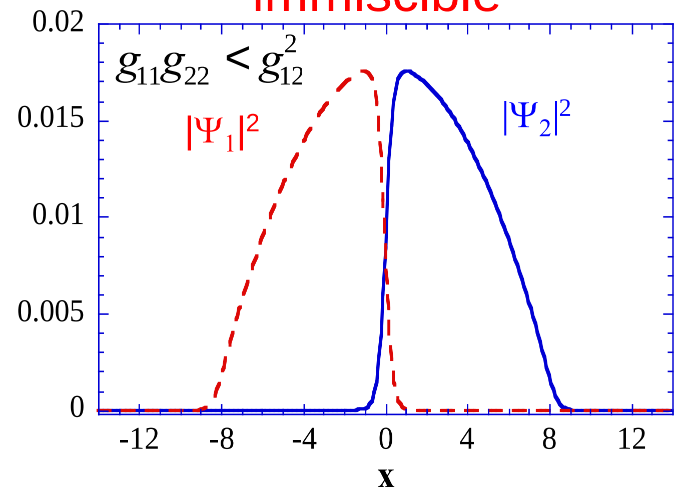
$$m_{jk}^{-1}=m_j^{-1}+m_k^{-1}$$

Ground state

miscible



immiscible

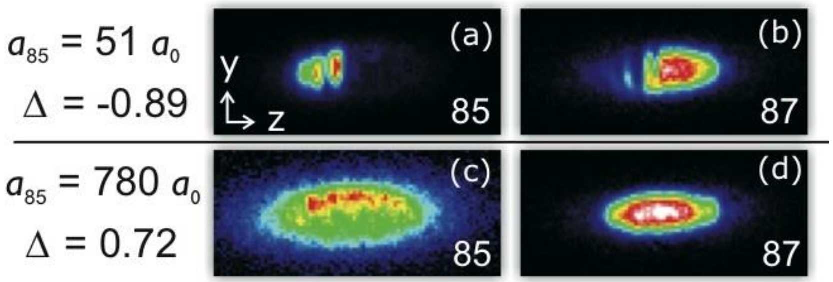


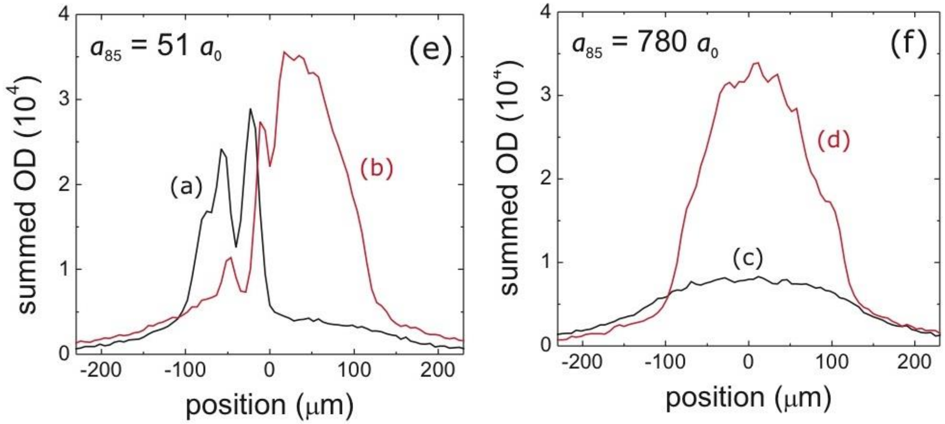
2成分BEC系の実験

^{85}Rb - ^{87}Rb

$$\Delta = \frac{a_{85}a_{87}}{a_{85-87}^2} - 1$$

Papp, et al., PRL **101**, 040402 (2008)



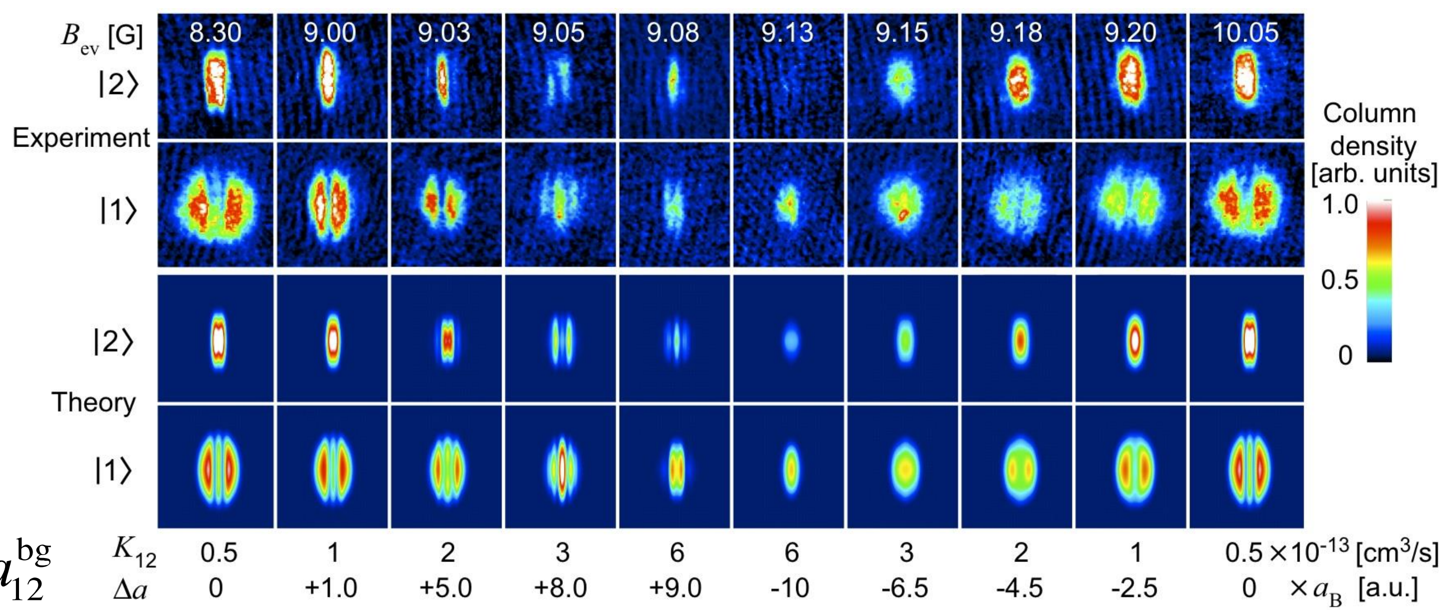


Tojo, et al., PRA **82**, 033609 (2010)

^{87}Rb

| F=1, m$_F$=1>

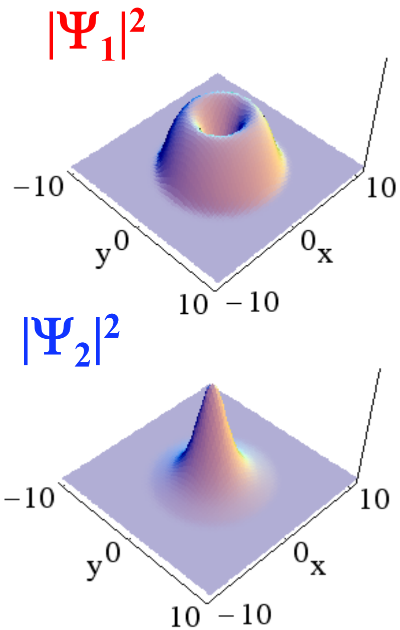
| F=2, m$_F$=−1>

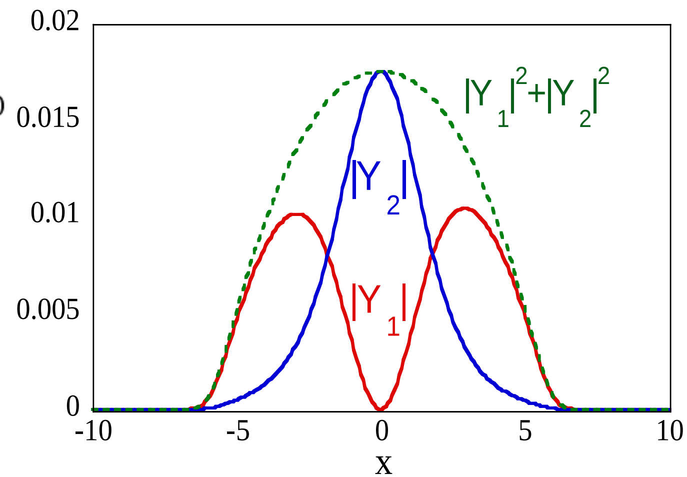


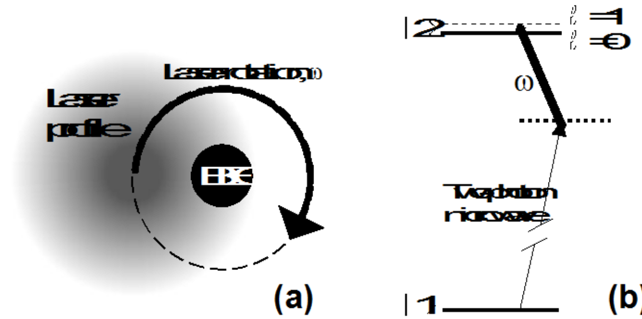
$$\Delta a = a_{12}(B) - a_{12}^{\mathrm{bg}}$$

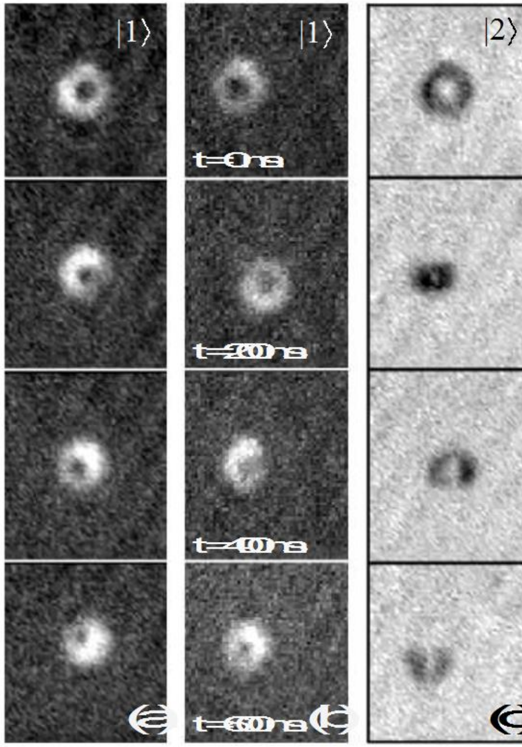
2成分BECにおける量子渦

Matthews et al., PRL **83**, 2498 (1999)



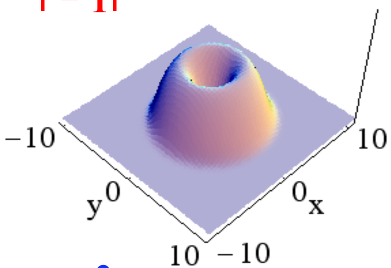




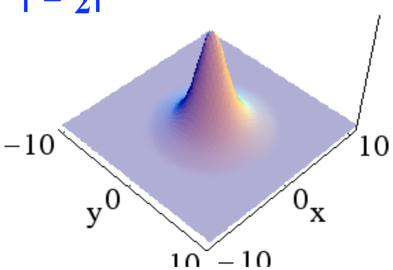


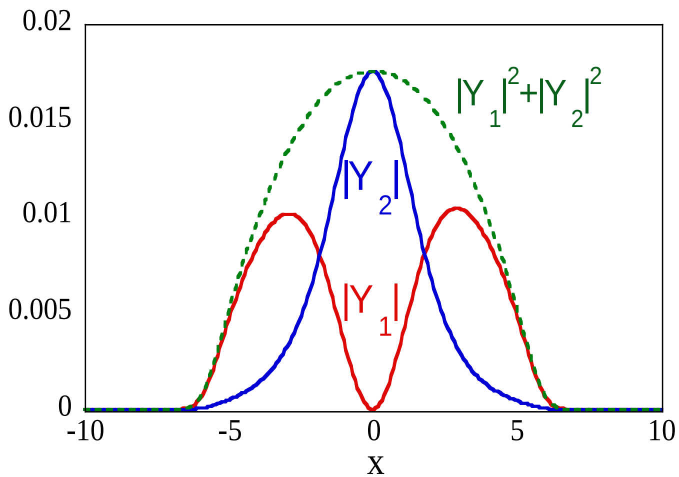
2成分BECにおける量子渦

$|\Psi_1|^2$



$|\Psi_2|^2$



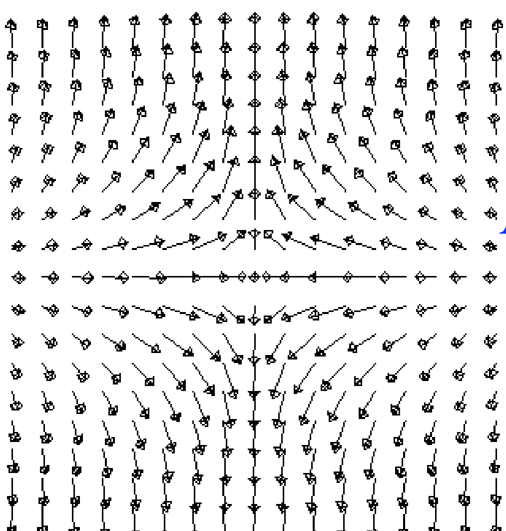


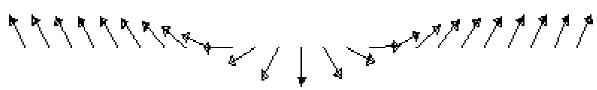
$$\boldsymbol{\Psi} = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \sqrt{n_1} e^{i\theta_1} \\ \sqrt{n_2} e^{i\theta_2} \end{pmatrix}$$

$$= \sqrt{n_{\mathrm{T}}}\, e^{i\frac{\theta_{\mathrm{T}}}{2}} \begin{pmatrix} \cos(\vartheta/2) e^{-i\frac{\varphi}{2}} \\ \sin(\vartheta/2) e^{i\frac{\varphi}{2}} \end{pmatrix}$$

pseudo-spin:

$$\boldsymbol{S} = \chi^{\dagger} \boldsymbol{\sigma} \chi = \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} = \begin{pmatrix} \sin\vartheta \cos\varphi \\ \sin\vartheta \sin\varphi \\ \cos\vartheta \end{pmatrix}$$

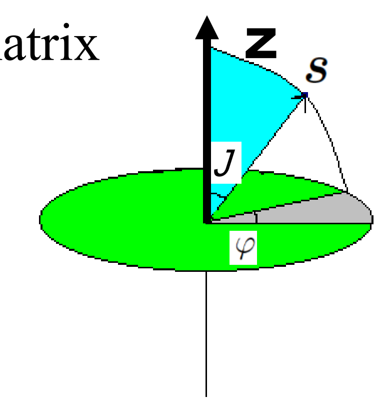




2D skyrmion

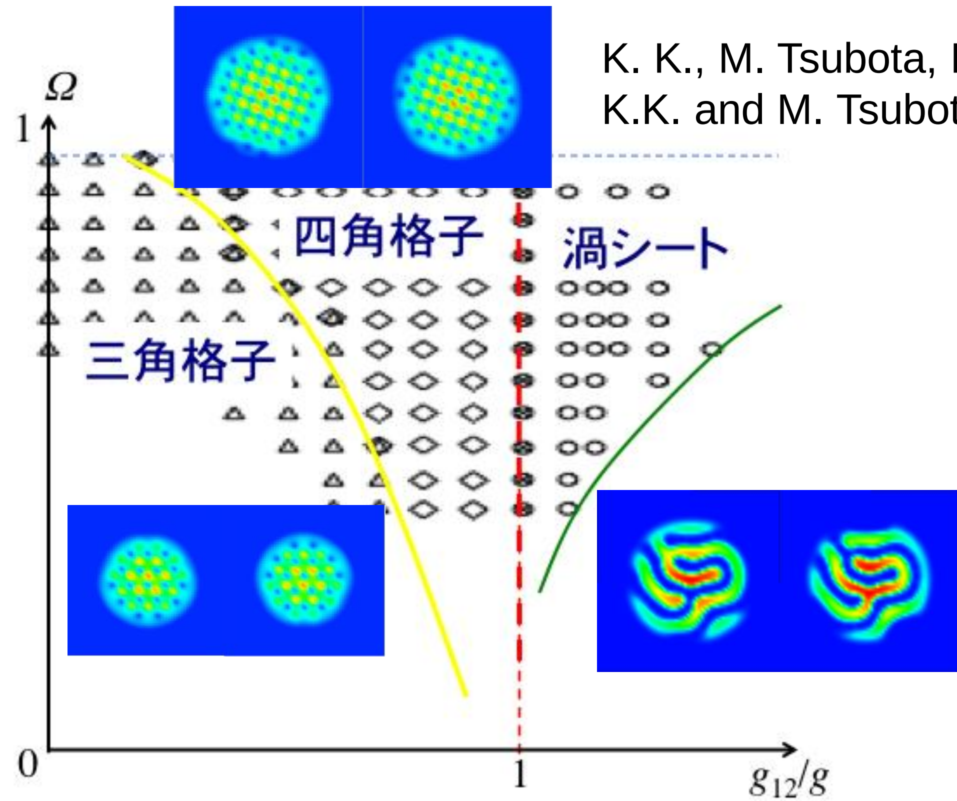
Anderson-Toulouse vortex

σ : Pauli matrix

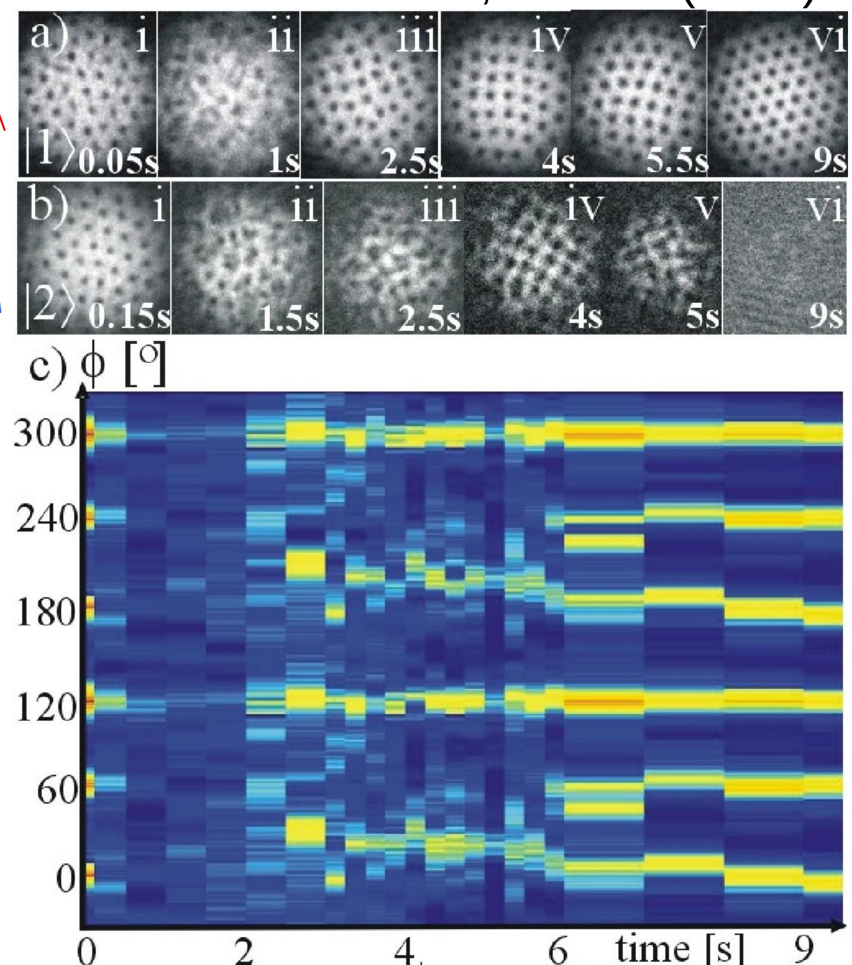


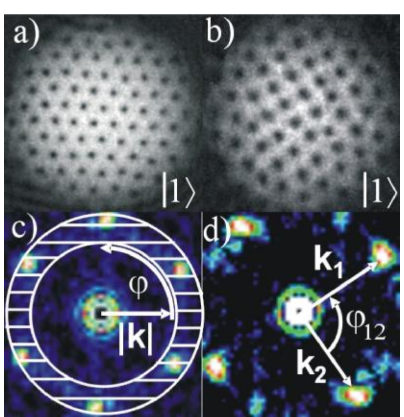
2成分BECにおける量子渦

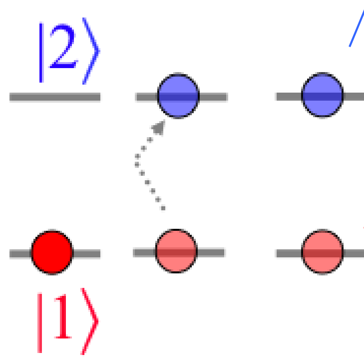
K. K., M. Tsubota, M. Ueda, PRL **91,** 150406 (2003)
K.K. and M. Tsubota, PRA **79**, 023606 (2009)



V. Schweikhard, et al.,
PRL **93**, 210403 (2004)

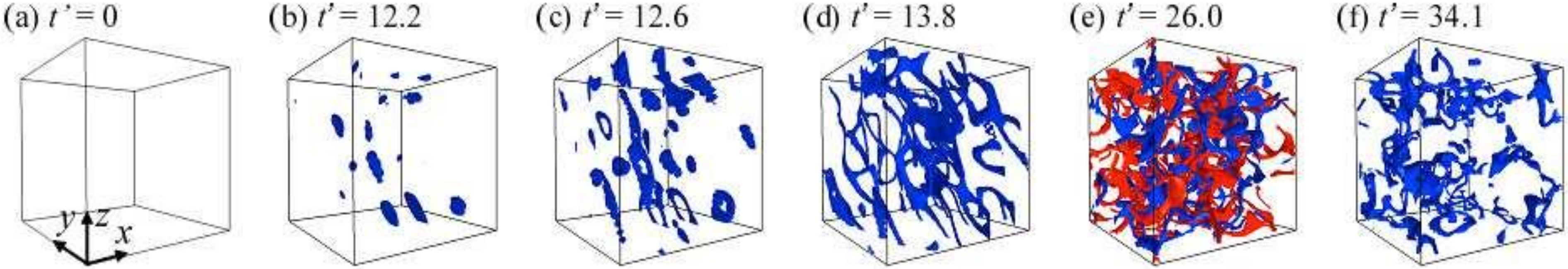






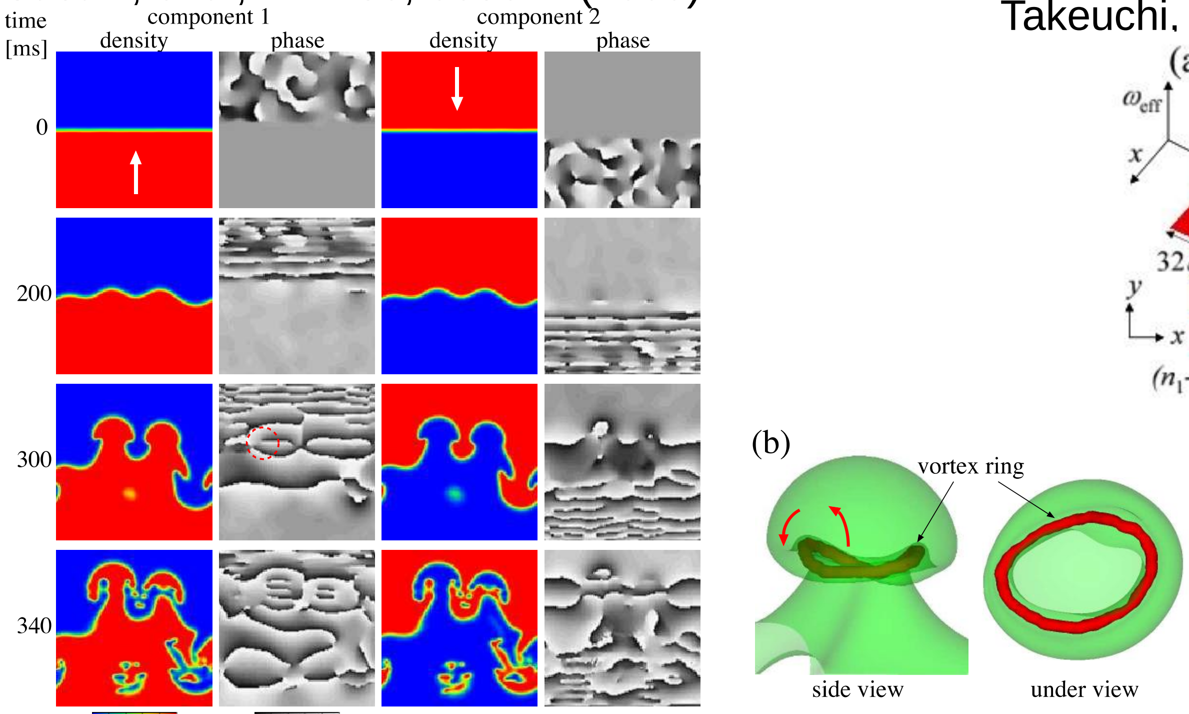
2成分BECにおける量子流体現象

２成分対向超流動 Ishino, Takeuchi, Tsubota., PRL **105**, 205301 (2010)



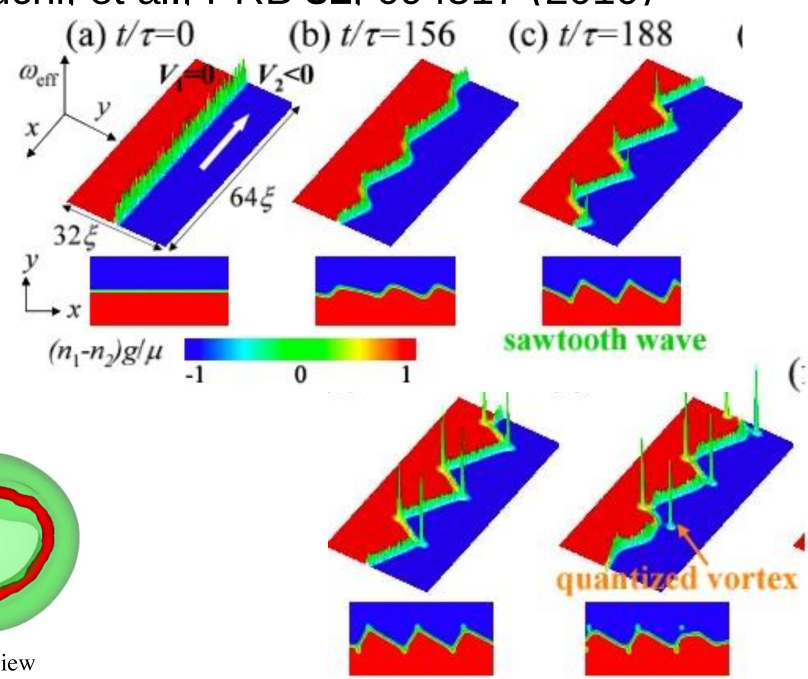
レイリーテイラー不安定性

Sasaki, et al, PRA **80**, 063611 (2009)



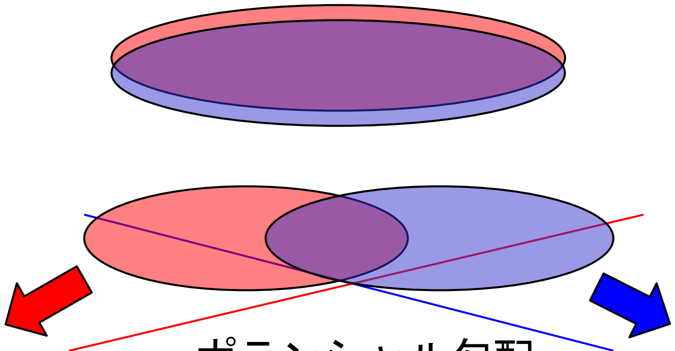
ケルビンヘルムホルツ不安定性

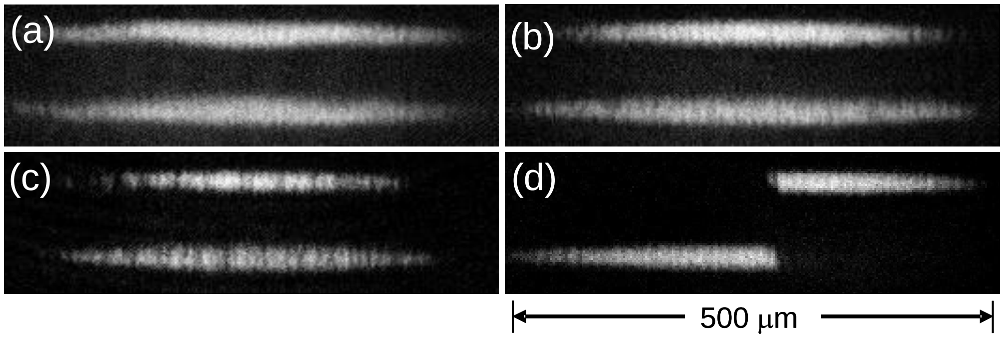
Takeuchi, et al., PRB **81**, 094517 (2010)

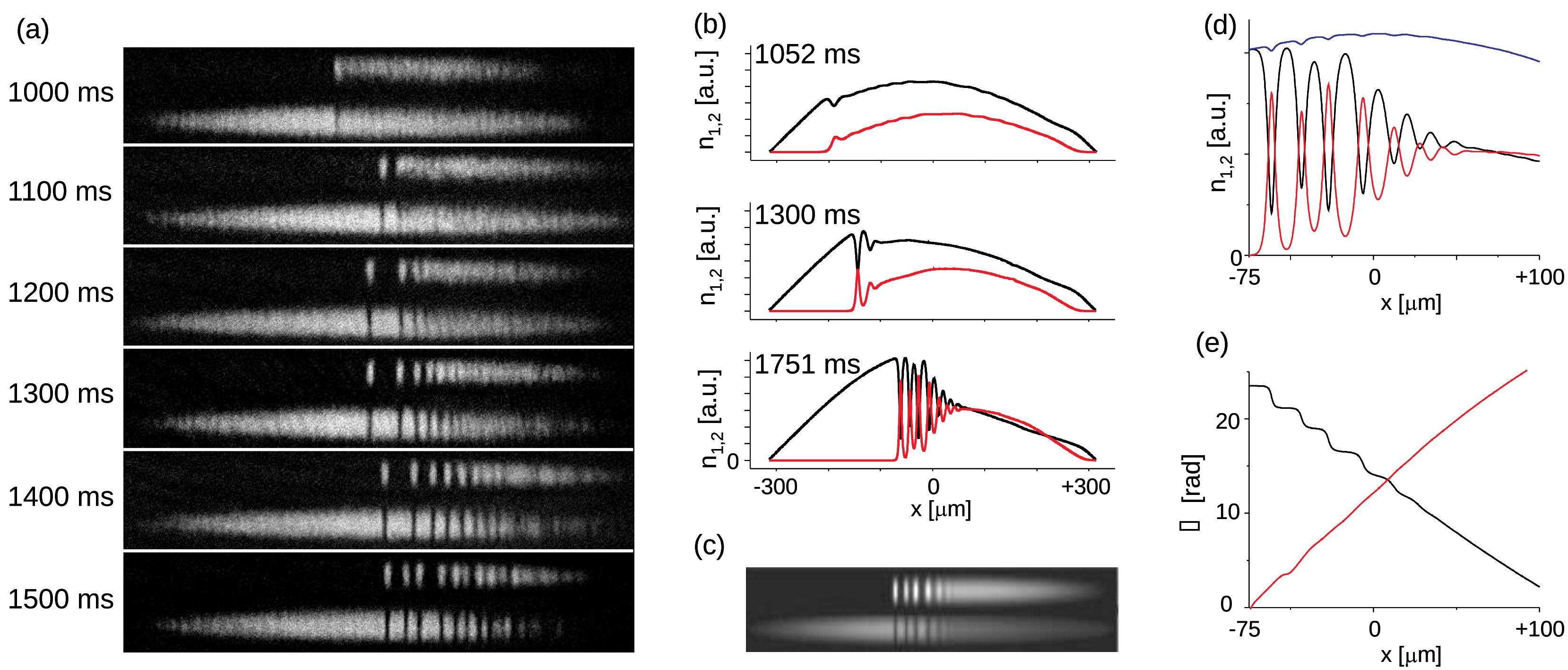


2成分BECにおける量子流体現象

２成分対向超流動 Hamner, et al., PRL **106,** 065302 (2011)

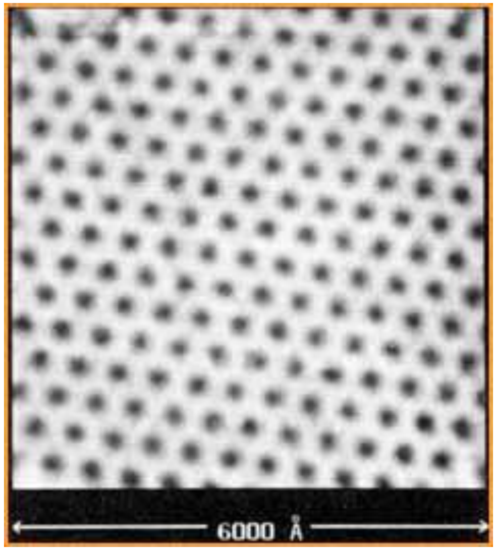




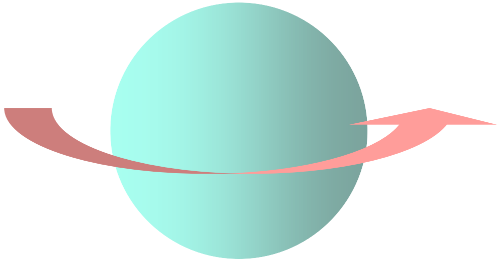


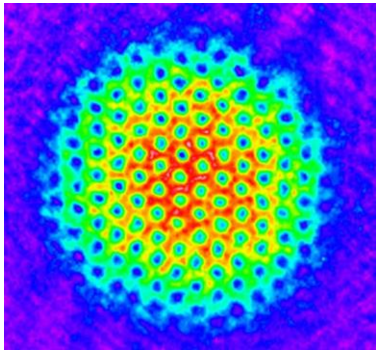
冷却原子系における「人工ゲージ場」

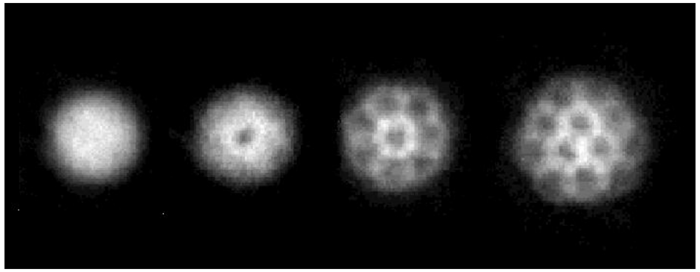
Superconductivity under a magnetic field



Rotating BEC







In a frame of reference rotating with $\boldsymbol{\Omega} = \Omega\hat{z}$

$$\frac{p^2}{2m} - \boldsymbol{\Omega}\cdot\boldsymbol{L} = \boxed{\frac{(\boldsymbol{p}-m\boldsymbol{\Omega}\times\boldsymbol{r})^2}{2m}} - \frac{1}{2}m\Omega^2 r^2$$

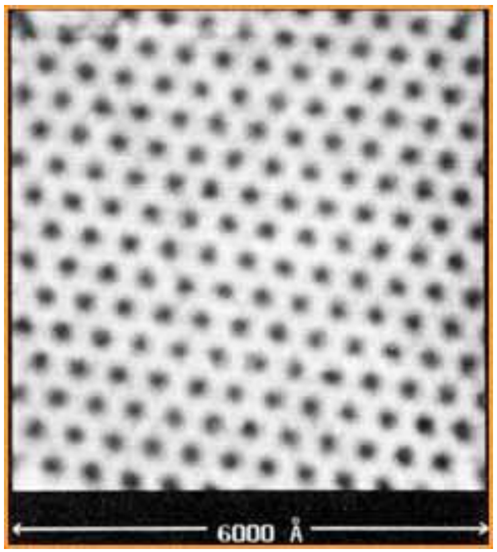
analogue of the Hamiltonian of a electron in a magnetic field

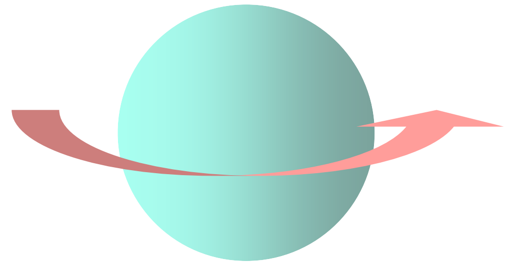
$$\frac{(-i\hbar\nabla - e\mathbf{A}/c)^2}{2m}$$

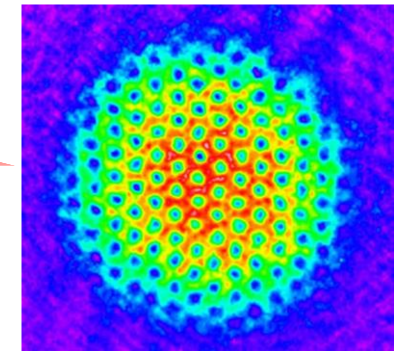
冷却原子系における「人工ゲージ場」

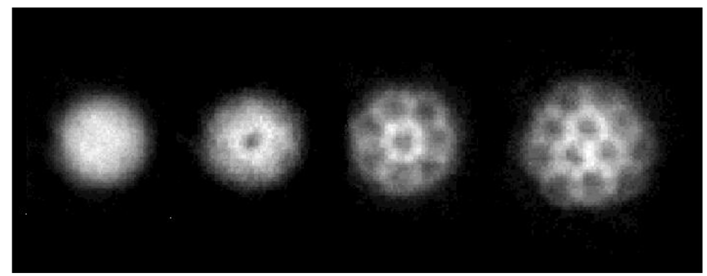
Superconductivity under a magnetic field

Rotating BEC









In a frame of reference rotating with $\boldsymbol{\Omega} = \Omega\hat{z}$

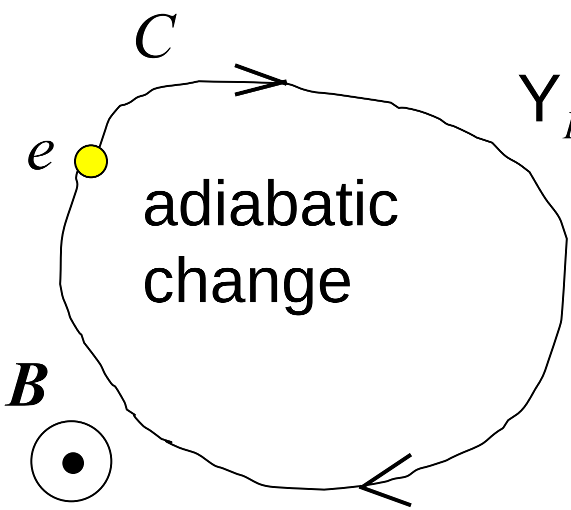
$$\frac{p^2}{2m} - \boldsymbol{\Omega}\cdot\boldsymbol{L} = \frac{(\boldsymbol{p} - m\boldsymbol{\Omega}\times\boldsymbol{r})^2}{2m} - \frac{1}{2}m\Omega^2 r^2$$

centrifugal potential weakens
the confining potential of a BEC

中性原子に対する「磁場」

< 幾何学的位相 >

$$H(\boldsymbol{R}(t))\Psi_R = E_R \Psi_R$$



$$\Psi_{R(t_f)}(t_f) = \exp\left\{ -\frac{i}{\hbar}\int_{t_i}^{t_f} dt' E[\boldsymbol{R}(t')] + i\gamma(C) \right\} \Psi_{R(t_i)}(t_i)$$

Berry phase

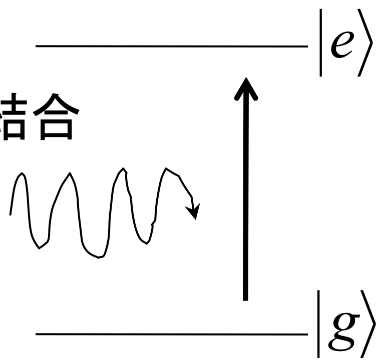
$$\boxed{\gamma(C) = i\oint_C \left\langle \Psi_{R(t)} \middle| \nabla_{R(t)} \Psi_{R(t)} \right\rangle \cdot d\boldsymbol{R}(t)}$$

e.g. Electrons in a magnetic field

Aharanov-Bohm phase $\gamma(C) = \dfrac{eBS}{\hbar} = \dfrac{2\pi\Phi}{\Phi_0}$

中性原子に対する「磁場」

<Toy model>　二つの内部状態をもつ原子を考える



ラビ結合

$$H=\left(\frac{p^2}{2m}+V\right)\hat{1}+U \qquad U=\frac{\hbar\Omega}{2}\begin{pmatrix}\cos\theta & e^{-i\phi}\sin\theta\\ e^{i\phi}\sin\theta & -\cos\theta\end{pmatrix}$$

$V(\boldsymbol{r})$: 外部ポテンシャル (内部状態とは独立)

$\Omega(\boldsymbol{r})$: ラビ振動数

$\theta(\boldsymbol{r})$: mixing angle

$\phi(\boldsymbol{r})$: phase angle

これらは空間依存

Local eigenstate of $U(\boldsymbol{r})$（dressed state）

$$|\chi_1\rangle=\begin{pmatrix}\cos(\theta/2)\\ e^{i\phi}\sin(\theta/2)\end{pmatrix} \qquad |\chi_2\rangle=\begin{pmatrix}e^{i\phi}\sin(\theta/2)\\ \cos(\theta/2)\end{pmatrix}$$

（直交基底）

状態ベクトル　$|\Psi(\boldsymbol{r},t)\rangle=\sum_{j=1,2}\psi_j(\boldsymbol{r},t)|\chi_j(\boldsymbol{r})\rangle$

中性原子に対する「磁場」

$$|\chi_1\rangle = \begin{pmatrix} \cos(\theta/2) \\ e^{i\phi}\sin(\theta/2) \end{pmatrix} \qquad |\chi_2\rangle = \begin{pmatrix} e^{i\phi}\sin(\theta/2) \\ \cos(\theta/2) \end{pmatrix} \qquad |\Psi(\boldsymbol{r},t)\rangle = \sum_{j=1,2} \psi_j(\boldsymbol{r},t)\,|\chi_j(\boldsymbol{r})\rangle$$

The atomic state is $|\chi_1\rangle$ at $t=0$, following the dressed state adiabatically（ψ_2 is negligible）.

$$i\hbar\frac{\partial}{\partial t}|\Psi(\boldsymbol{r},t)\rangle = H|\Psi(\boldsymbol{r},t)\rangle$$

$$i\hbar\frac{\partial\psi_1}{\partial t} = \left[\frac{(\boldsymbol{p}-\boldsymbol{A})^2}{2m} + V + \frac{\hbar\Omega}{2} + W\right]\psi_1$$

geometric potential

$$\boldsymbol{A}(\boldsymbol{r}) = i\hbar\langle\chi_1|\nabla\chi_1\rangle = \frac{\hbar}{2}(\cos\theta - 1)\nabla\phi$$

$$W(\boldsymbol{r}) = \frac{\hbar^2}{2m}\left|\langle\chi_2|\nabla\chi_1\rangle\right|^2 = \frac{\hbar^2}{8m}\left[(\nabla\theta)^2 + \sin^2\theta(\nabla\phi)^2\right]$$

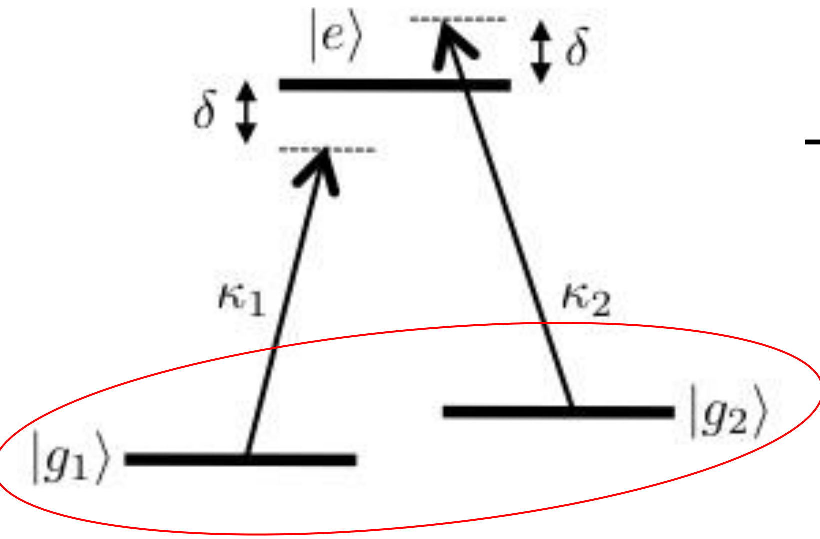
only depedent on the spatial derivative of θ and ϕ

中性原子に対する「磁場」

The lifetime of excited state $|e\rangle$ is generally very short.

Use of (quasi)degenerate ground states $\{|g_j\rangle, j=1,\dots,N\}$ is preferred.

「Λ–level scheme」



$\{|g_1\rangle,|e\rangle,|g_2\rangle\}$ basis $U=\dfrac{\hbar}{2}\begin{pmatrix} 2\delta & \kappa_1^* & 0 \\ \kappa_1 & 0 & \kappa_2 \\ 0 & \kappa_2^* & 2\delta \end{pmatrix}$

Three eigenstates $\begin{cases} |D\rangle \\ |\pm\rangle = \left(|B\rangle \pm |e\rangle\right)/\sqrt{2} \end{cases}$

Dark state $|D\rangle = \left(\kappa_2|g_1\rangle - \kappa_1|g_2\rangle\right)/\kappa$

Bright state $|B\rangle = \left(\kappa_1^*|g_1\rangle + \kappa_2^*|g_2\rangle\right)/\kappa$

$\kappa = \sqrt{|\kappa_1|^2 + |\kappa_2|^2}$

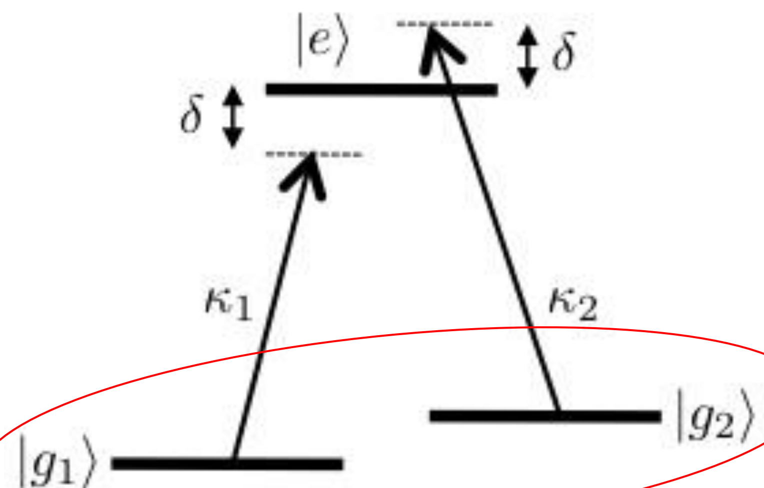
中性原子に対する「磁場」

The lifetime of excited state $|e\rangle$ is generally very short.

Use of (quasi)degenerate ground states $\{|g_j\rangle, j=1,\dots,N\}$ is preferred.

「Λ–level scheme」

$\{|g_1\rangle, |e\rangle, |g_2\rangle\}$ basis $U = \frac{\hbar}{2}\begin{pmatrix} 2\delta & \kappa_1^* & 0 \\ \kappa_1 & 0 & \kappa_2 \\ 0 & \kappa_2^* & 2\delta \end{pmatrix}$



Three eigenstates $\begin{cases} |D\rangle \\ |\pm\rangle = \left(|B\rangle \pm |e\rangle\right)/\sqrt{2} \end{cases}$

$$|\Psi(\mathbf{r})\rangle = \sum_{X=D,\pm} \psi_X(\mathbf{r})|X(\mathbf{r})\rangle \approx \psi_D(\mathbf{r})|D(\mathbf{r})\rangle$$

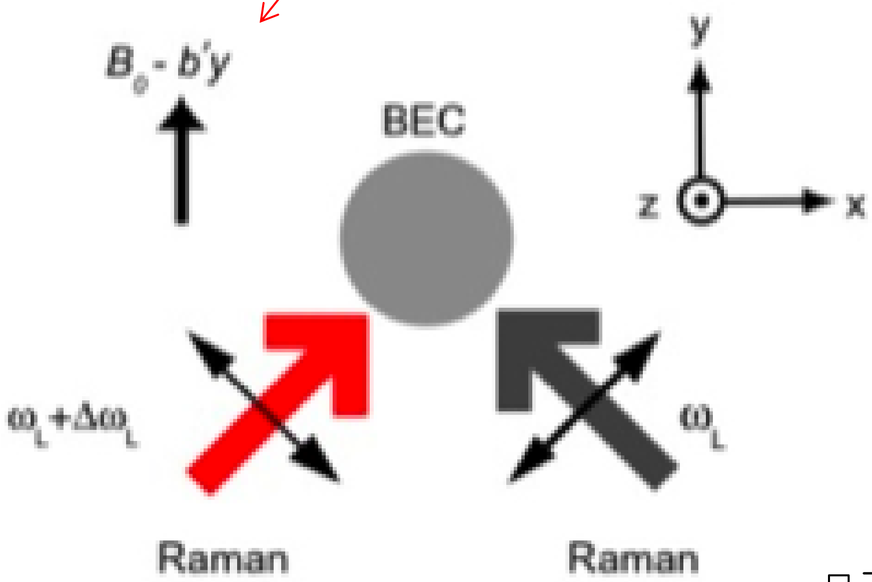
$$i\hbar\frac{\partial \psi_D}{\partial t} = \left[\frac{(\mathbf{p}-\mathbf{A})^2}{2m} + V + W\right]\psi_D$$

$$\mathbf{A}(\mathbf{r}) = i\hbar\langle D|\nabla D\rangle$$

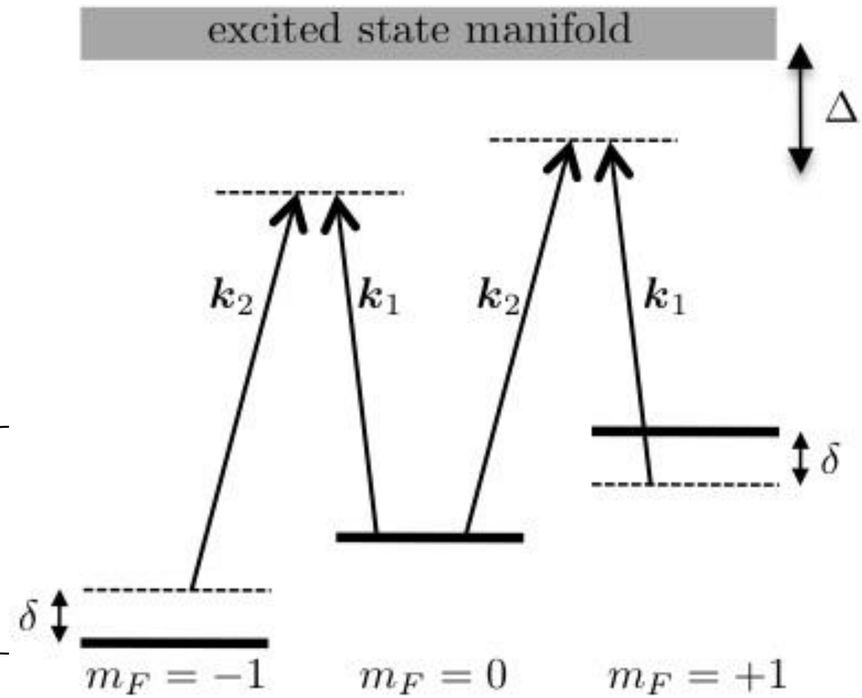
中性原子に対する「磁場」

Lin, *et al*. Phys. Rev. Lett. **102**, 130401 (2009)
Nature **462**, 628 (2009)

real magnetic field



^{87}Rb atom F=1 hyperfine level



The Zeeman split due to the real magnetic field

中性原子に対する「磁場」

Lin, *et al.* Phys. Rev. Lett. **102**, 130401 (2009)
Nature **462**, 628 (2009)

$\{|m_F=-1\rangle, |m_F=0\rangle, |m_F=+1\rangle\}$ basis

$$U=\frac{\hbar}{2}\begin{pmatrix} -2\delta & \kappa & 0 \\ \kappa^* & 0 & \kappa \\ 0 & \kappa^* & 2\delta \end{pmatrix}$$

$$\kappa=\kappa^{(0)}\exp\left[i(\mathbf{k}_1-\mathbf{k}_2)\cdot\mathbf{r}\right]=\kappa^{(0)}\exp(ik_d x)$$

eigenstate with the lowest eigenvalue

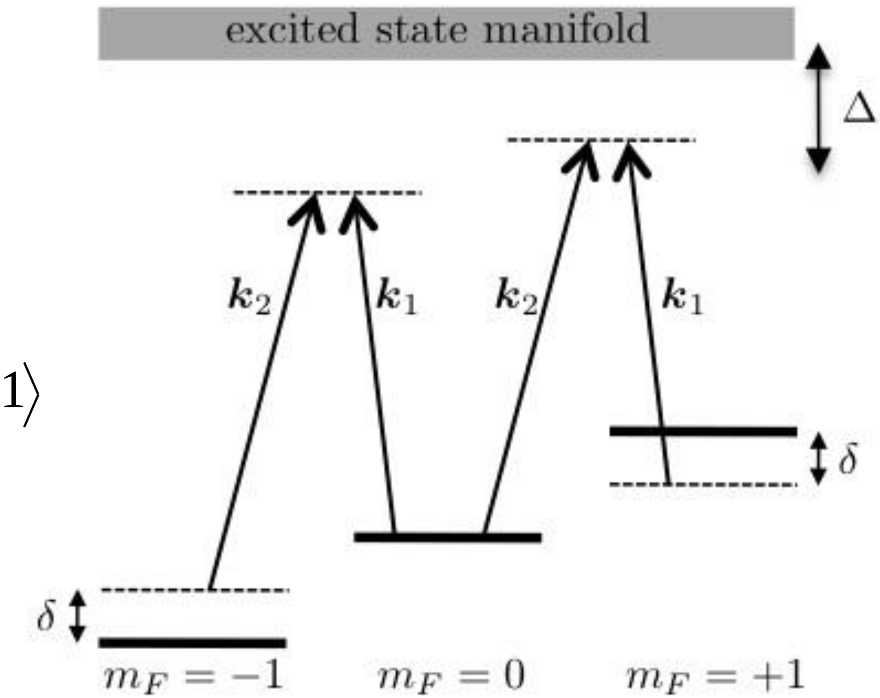
$$|\chi\rangle=e^{i\phi}\cos^2\frac{\theta}{2}|-1\rangle-\frac{\sin\theta}{\sqrt{2}}|0\rangle+e^{-i\phi}\sin^2\frac{\theta}{2}|+1\rangle$$

$$\boldsymbol{A}(\boldsymbol{r})=i\hbar\langle\chi|\nabla\chi\rangle=-\hbar k_{\mathrm{d}}\boldsymbol{e}_x\cos\theta$$

$$\tan\theta=\frac{\kappa^{(0)}}{\sqrt{2}\delta}$$

$$\boxed{\boldsymbol{A}(\boldsymbol{r})\propto\delta\boldsymbol{e}_x\propto\delta'(y-y_0)\,\boldsymbol{e}_x}$$

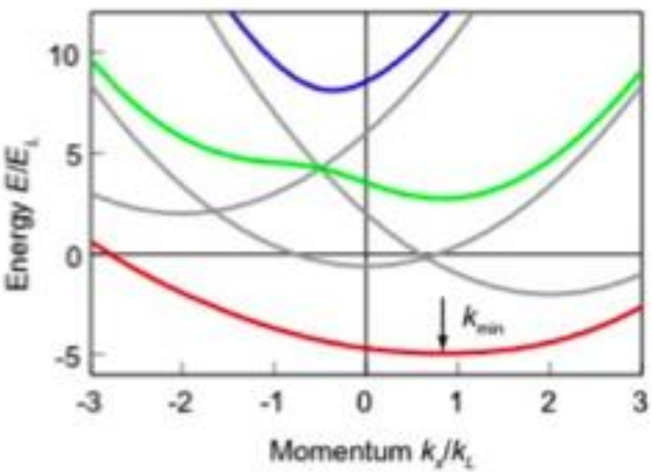
^{87}Rb atom F=1 hyperfine level

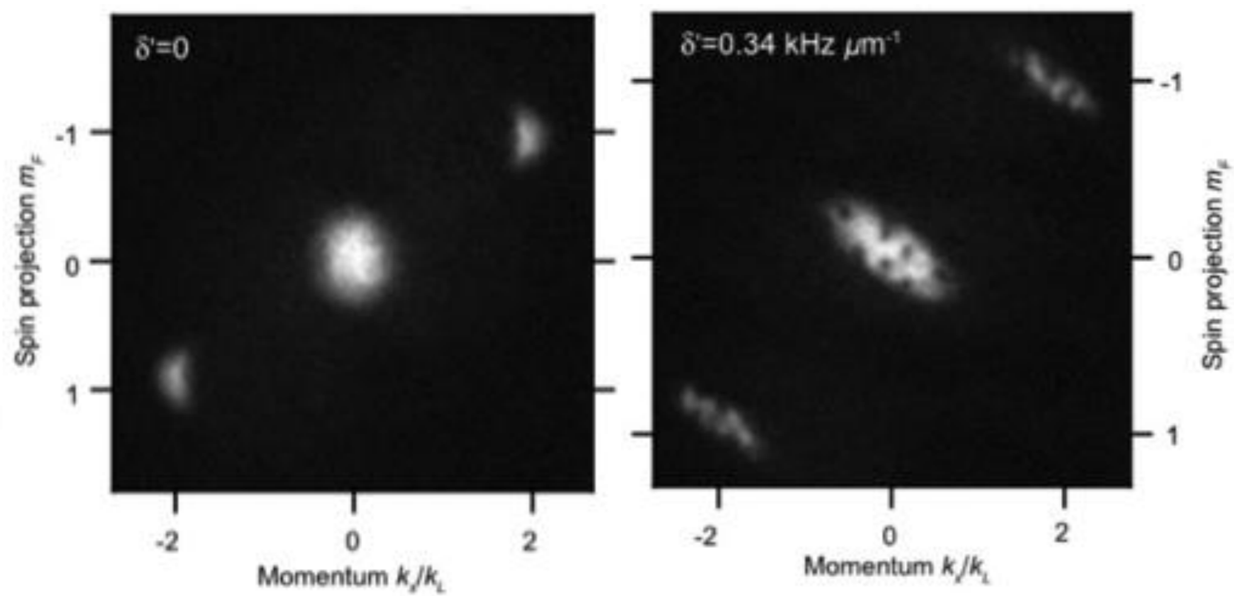


Uniform magnetic field along the z-axis

中性原子に対する「磁場」

Lin, *et al.* Phys. Rev. Lett. **102**, 130401 (2009)
Nature **462**, 628 (2009)





$$H_x \approx \frac{\hbar^2}{2m^*}\left(k_x - k_{\min}\right)^2$$

$$= \frac{\hbar^2}{2m^*}\left(k_x - \frac{q^* A^*}{\hbar}\right)^2$$

$$\boxed{\boldsymbol{A}(\boldsymbol{r}) \propto \delta \boldsymbol{e}_x \propto \delta(y - y_0)\,\boldsymbol{e}_x}$$

Uniform magnetic field along the z-axis

中性原子に対する「磁場」

光格子中の原子系における Hofstadter-Harper　ハミルトニアンの実現

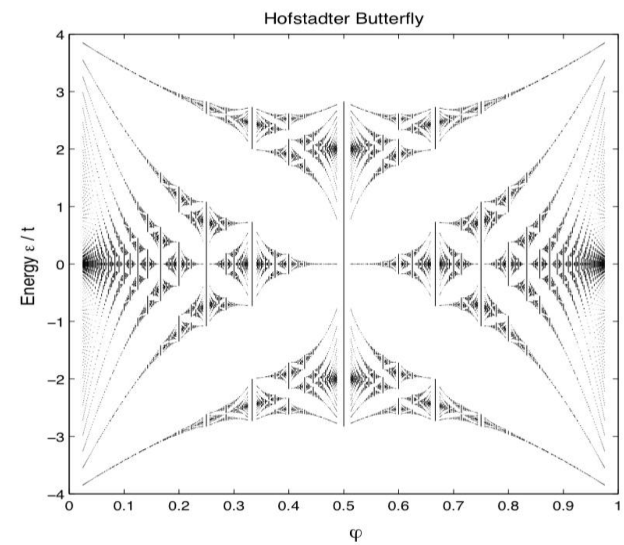
Peierls phase

$$\hat{H} = -\sum_{\langle j,j'\rangle} \frac{t_{j,j'}}{2}\left(\hat{a}_j^\dagger \hat{a}_{j'} e^{-iA_{j,j'}} + \mathrm{h.c.}\right) + \sum_j E_j \hat{N}_j + \sum_j \frac{U_j}{2} \hat{N}_j\left(\hat{N}_j - 1\right)$$

D. Jaksch and P. Zoller, New J. Phys. **5**, 56 (2003)

E. Mueller, et al., PRA **70**, 041603 (2004)

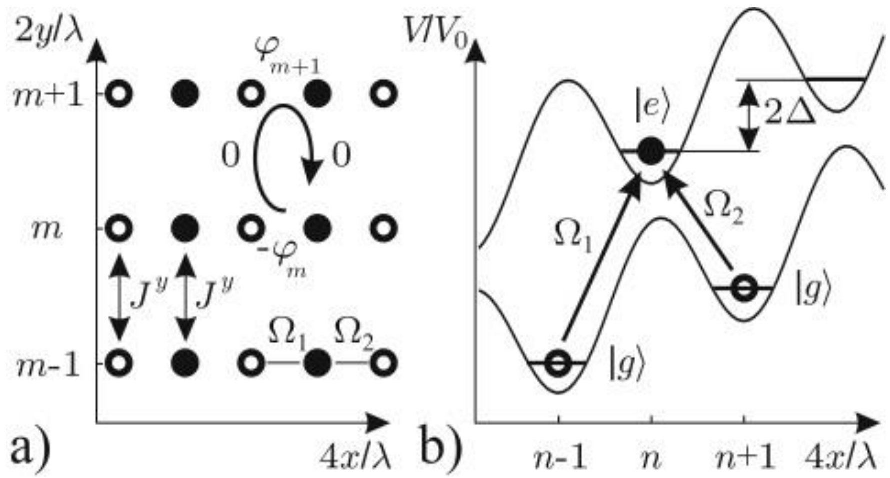
A.S. Sorensen et al., PRL **94**, 086803 (2005).



two internal state

○ $|g\rangle$

● $|e\rangle$



(Humburg) J. Struck, PRL **108**, 225304 (2012), arXiv:1304.5520

(MaxPlanck) M. Aidelsburger, PRL **107**, 225301 (2011), arXiv:1308.0321

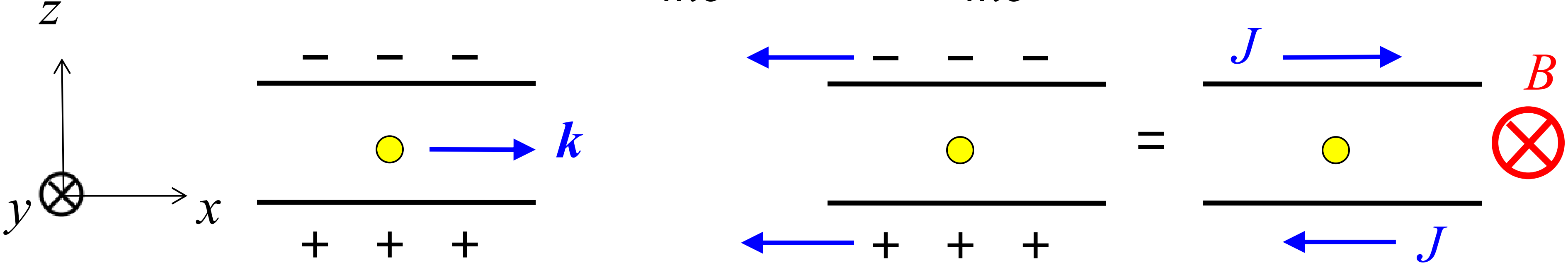
(MIT) H. Miyake, arXiv:1308.1431

スピン軌道相互作用 (SOC)

→ スピンホール効果, トポロジカル絶縁体, etc

粒子が動く座標系における磁場と粒子の磁気モーメントとの相互作用。 $= -\boldsymbol{\mu} \cdot \boldsymbol{B}_{SO}(\boldsymbol{k})$

$\boldsymbol{E} = E_0 \hat{z}$ のとき $\quad \boldsymbol{B}_{\mathrm{SO}} = \dfrac{\hbar}{mc^2}(\boldsymbol{E} \times \boldsymbol{k}) = \dfrac{\hbar}{mc^2} E_0(-k_y, k_x, 0)$



Rashba SOC $\quad -\boldsymbol{\mu} \cdot \boldsymbol{B}_{\mathrm{SO}}(\boldsymbol{k}) \propto \sigma_x k_y - \sigma_y k_x \quad (\sigma_x k_x + \sigma_y k_y)$

2次元電子系（GaAs系, etc）

Dresselhaus SOC $\quad -\boldsymbol{\mu} \cdot \boldsymbol{B}_{\mathrm{SO}}(\boldsymbol{k}) \propto -\sigma_x k_y - \sigma_y k_x \quad (-\sigma_x k_x + \sigma_y k_y)$

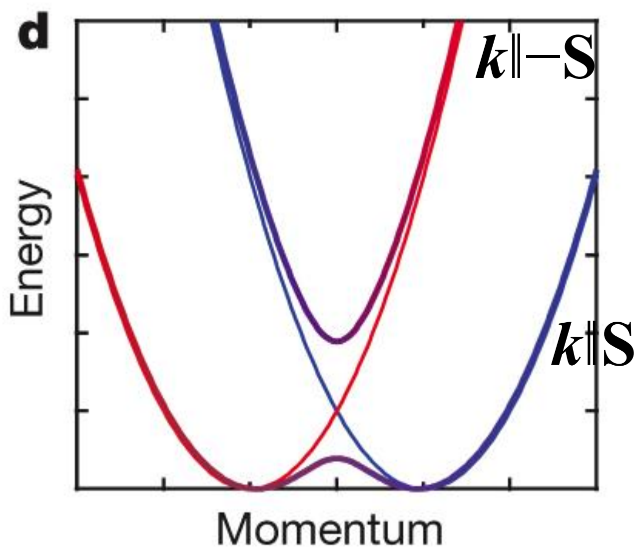
半導体の結晶構造に起因

SOCをもつボソン系

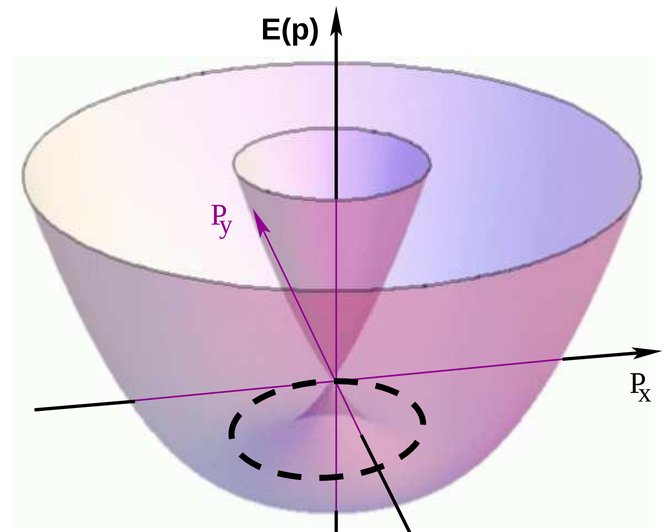
Rashba 型相互作用

$$H = \frac{\hbar^2}{2m}\left[\boldsymbol{k}^2 - 2\kappa\left(k_x\sigma_x + k_y\sigma_y\right)\right]$$

Single-particle dispersion



Minima at $k_\perp = \pm\kappa$



Degenerate ground state for different azimuthal angle

・最低エネルギー状態が縮退。 相互作用のある系での凝縮状態の構造？
縮退は原子間相互作用、トラップの非等方性、、、などにより解ける。

Stanescu, et al., PRA **78**, 023616 (2008)

冷却原子系における
スピン軌道相互作用

Lin, et al., Nature **471**, 83 (2011)

Hamiltonian

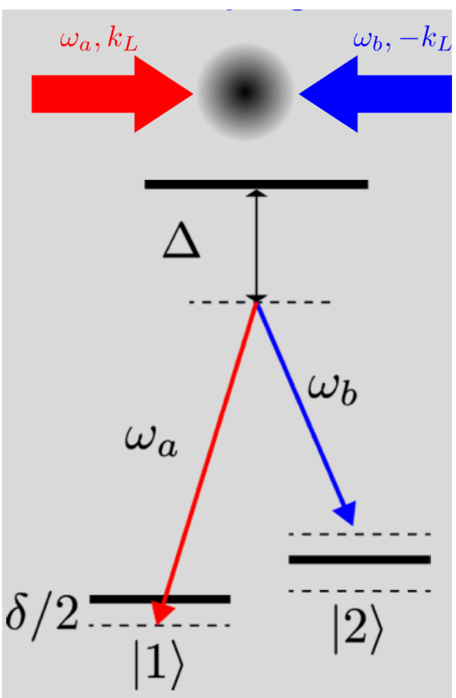
$$H_{\text{eff}}^{(2)} = -\frac{h^2\nabla^2}{2m}\hat{1} + \delta\hat{\sigma}_z + \begin{bmatrix} 0 & \tilde{\Omega}e^{-2ik_L x} \\ \tilde{\Omega}e^{+2ik_L x} & 0 \end{bmatrix}$$

Pseudospin representation

$$\left|-\right\rangle = e^{-ip/4}\left\{\cos(k_L x)\left|1\right\rangle + \sin(k_L x)\left|2\right\rangle\right\}$$

$$\left|\urcorner\right\rangle = e^{ip/4}\left\{-\sin(k_L x)\left|1\right\rangle + \cos(k_L x)\left|2\right\rangle\right\}$$

Raman coupling



Unitary transformation

$$\hat{U}(\boldsymbol{r}) = e^{-i\hat{\sigma}_z \pi/4} e^{ik_L x\hat{\sigma}_y}$$

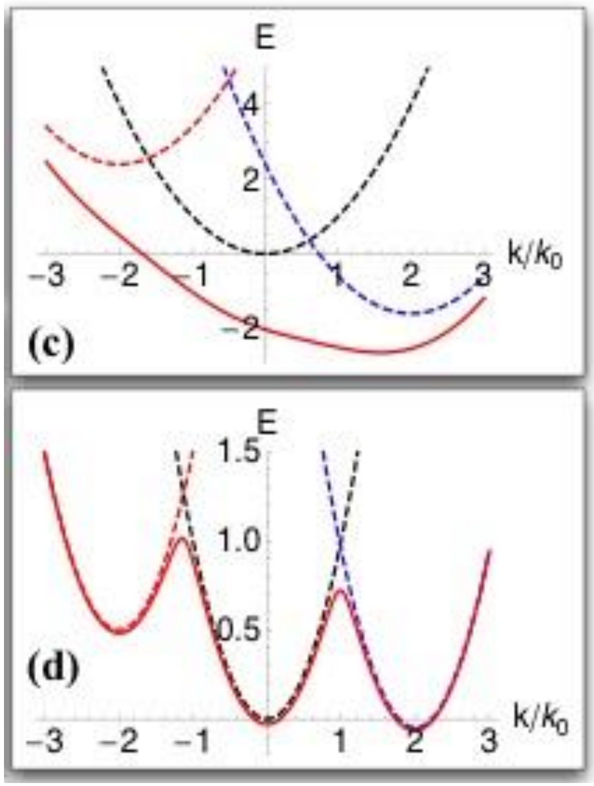
$$H'^{(2)}_{\text{eff}} = \hat{U}^{\dagger}(\boldsymbol{r}) H_{\text{eff}}^{(2)} \hat{U}(\boldsymbol{r})$$

$$= -\frac{\hbar^2}{2m}\left(i\nabla - k_L \hat{s}_x \hat{x}\right)^2 - d\hat{s}_x + \tilde{W}\hat{s}_z$$

Rashba+Dresselhaus type vector potential
(Non-Abelian gauge field)

冷却原子系における
スピン軌道相互作用

Raman coupling



Boson ^{87}Rb F=1 $\left|m_F = 1,\ 0,\ -\cancel{1}\right\rangle$

Lin et al., Nature **471**, 83 (2011)

Fermion ^{40}K F=9/2 $\left|m_F = 9/2,\ 7/2\right\rangle$

Wang et al., PRL **109**, 095301 (2012)

^{6}Li F=3/2 $\left|m_F = 3/2,\ 1/2\right\rangle$

$$H'^{(2)}_{\text{eff}} = \hat{U}^{\dagger}(\boldsymbol{r})\, H^{(2)}_{\text{eff}}\, \hat{U}(\boldsymbol{r})$$

$$= -\frac{\hbar^2}{2m}\left(i\nabla - k_L \hat{s}_x \hat{x}\right)^2 - d\hat{s}_x + \tilde{W}\hat{s}_z$$

Rashba+Dresselhaus type vector potential
(Non-Abelian gauge field)

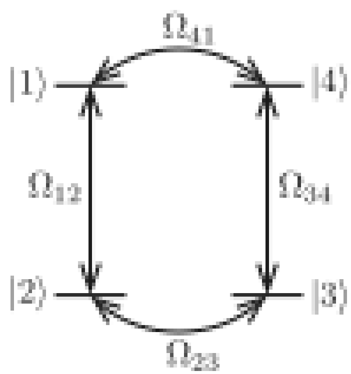
様々なスピン軌道相互作用の生成法の提案

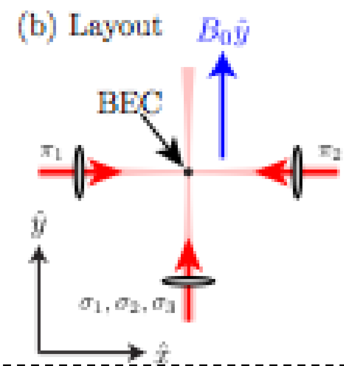
Rashba
+Dresselhause (1D) $s_x\hat{\boldsymbol{x}}$ Lin et al., Nature **471**, 83 (2011)

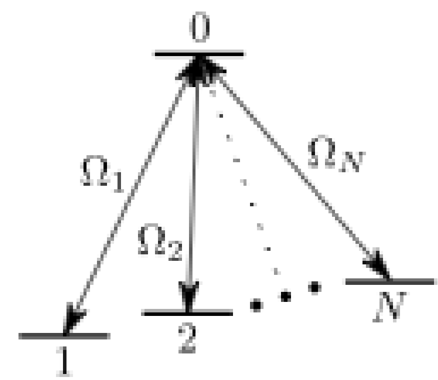
Rashba (2D) $s_x\hat{\boldsymbol{x}}+s_y\hat{\boldsymbol{y}}$

Campbell et al., PRA. **84**, 025602 (2011)

Juzeliūnas et al., PRA **81**, 053403 (2010)

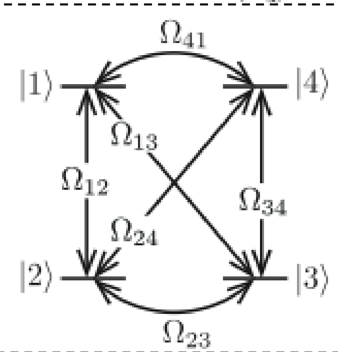


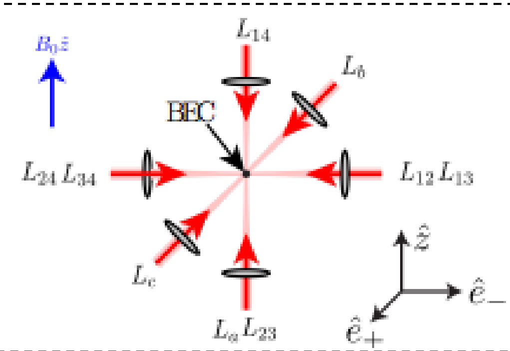




Rashba (3D) $s_x\hat{\boldsymbol{x}}+s_y\hat{\boldsymbol{y}}+s_z\hat{\boldsymbol{z}}$

Anderson et al., PRL **108,** 235301 (2012)





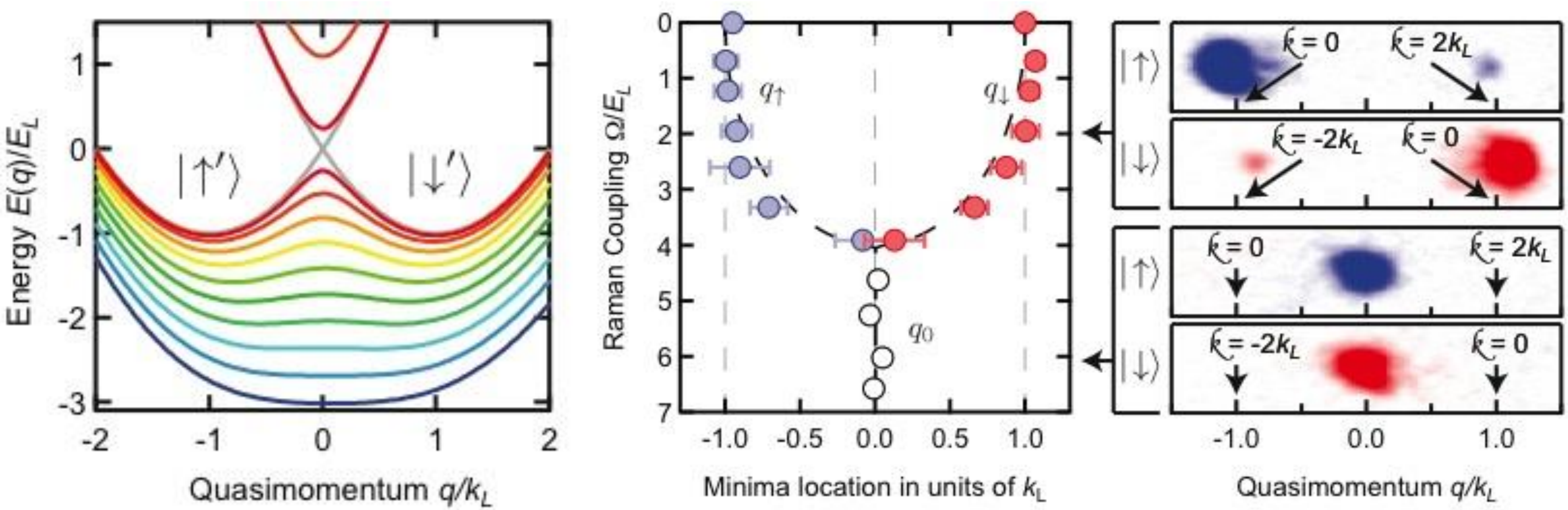
磁場勾配を用いた提案 Xu et al., RPA **87**, 063634 (2013)
Anderson et al., arXiv:1306.2606

冷却原子系におけるスピン軌道相互作用の実験的観測

Lin et al., Nature **471**, 83 (2011)

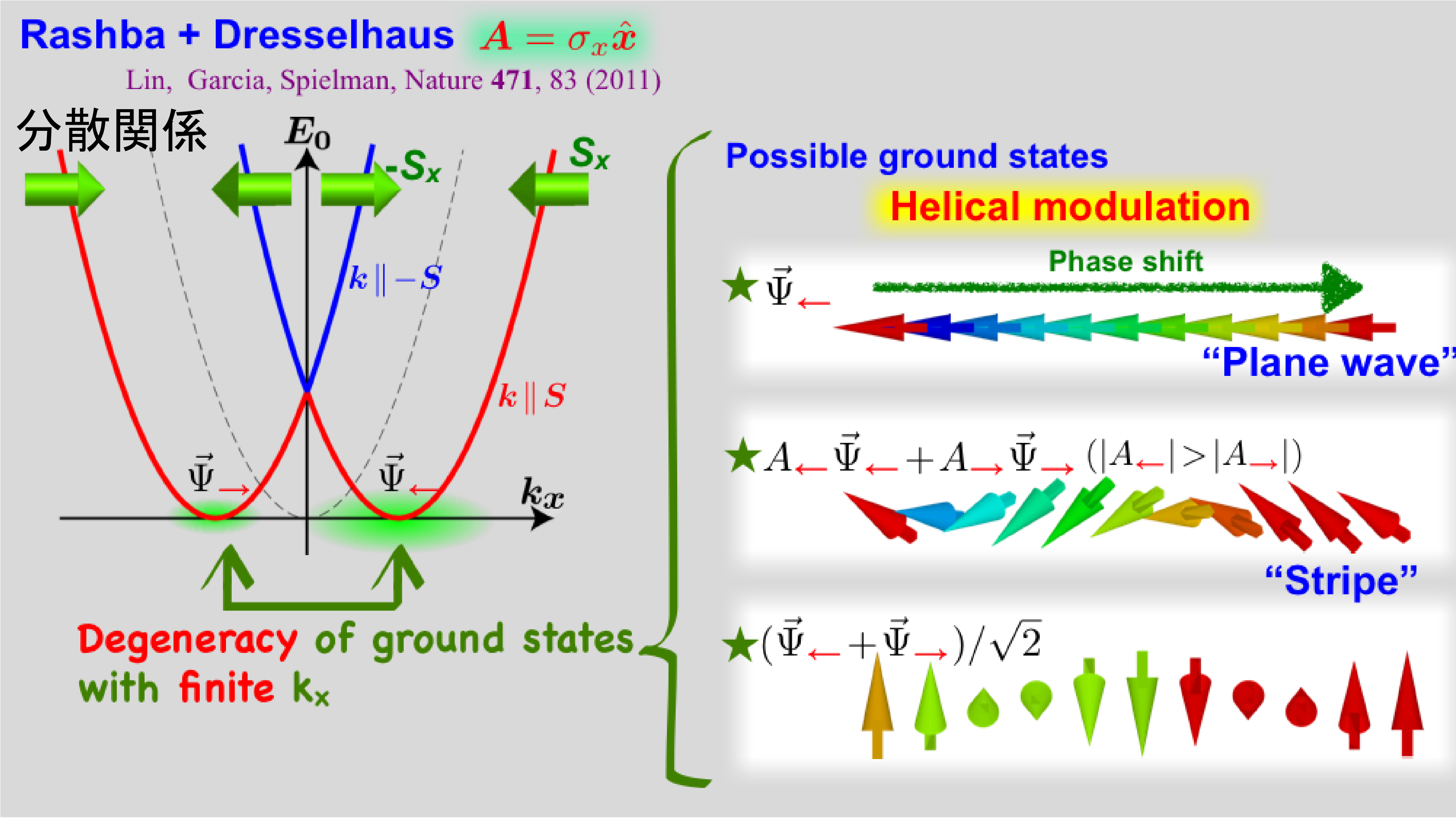
$$H'^{(2)}_{\mathrm{eff}} = -\frac{\hbar^2}{2m}\left(i\nabla - k_L \hat{s}_x \hat{x}\right)^2 - d\hat{s}_x + \tilde{W}\hat{s}_z$$

分散関係　　　　　　TOF imaging



スピン軌道相互作用をもつBEC

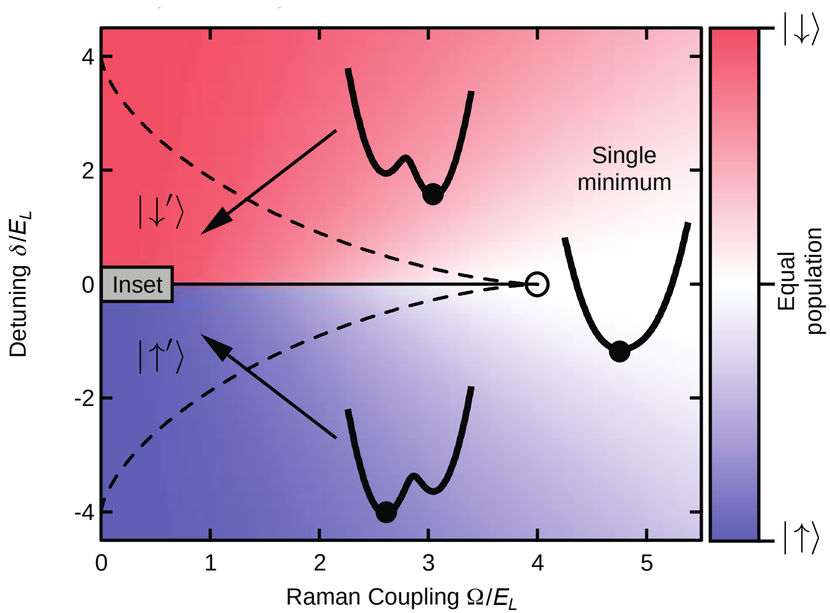
基底状態　ボース凝縮

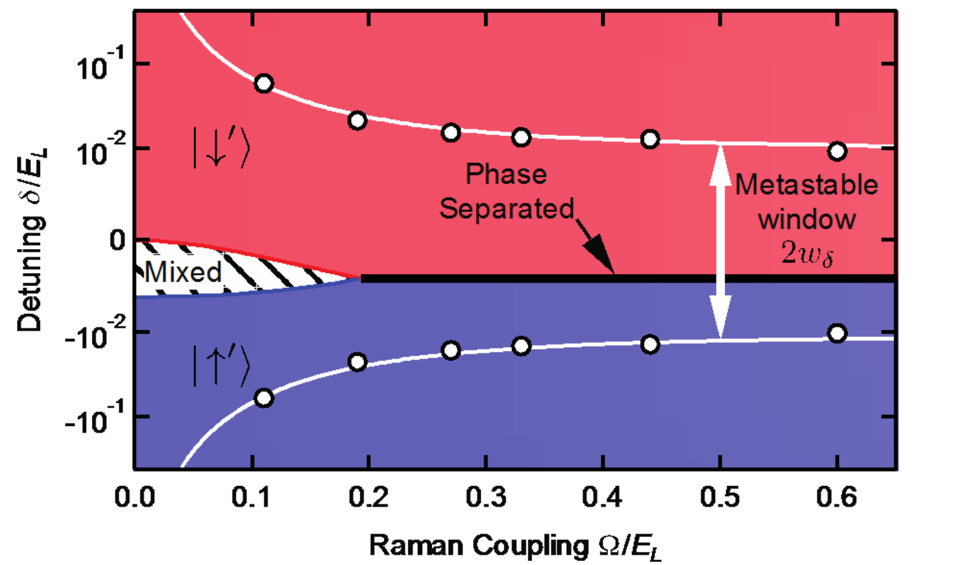


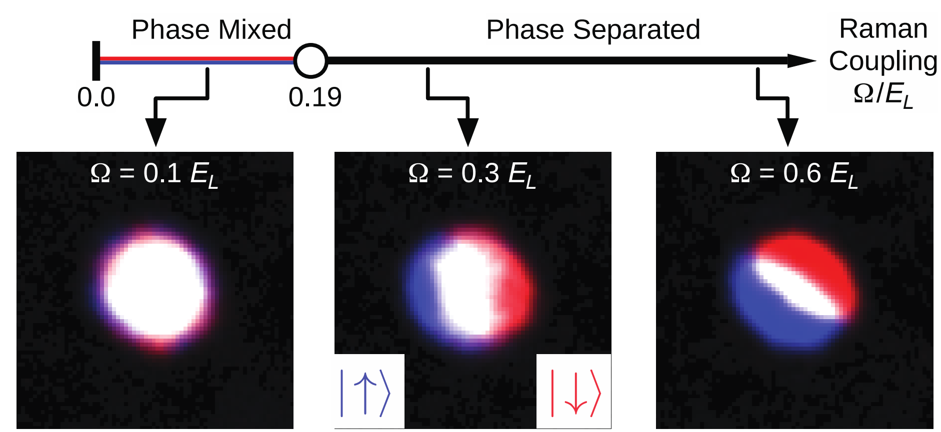
Rashba+Dresselhaus SOC BEC

Lin et al., Nature **471**, 83 (2011)

$$H = \int d\mathbf{r}\,\Psi_\mu^*\left[\frac{\hbar^2}{2m}\left(-i\nabla - k_L\hat{\sigma}_x\hat{x}\right)^2 - \delta\hat{\sigma}_x + \tilde{\Omega}\hat{\sigma}_z\right]\Psi_\nu + \frac{1}{2}\int d\mathbf{r}\, n_\mu g_{\mu\nu} n_\nu$$







$$E_{\text{int}} = \int d\mathbf{r}\left(\frac{g+g_{12}}{2}n^2 + \frac{g-g_{12}}{2}S_z^2\right)$$

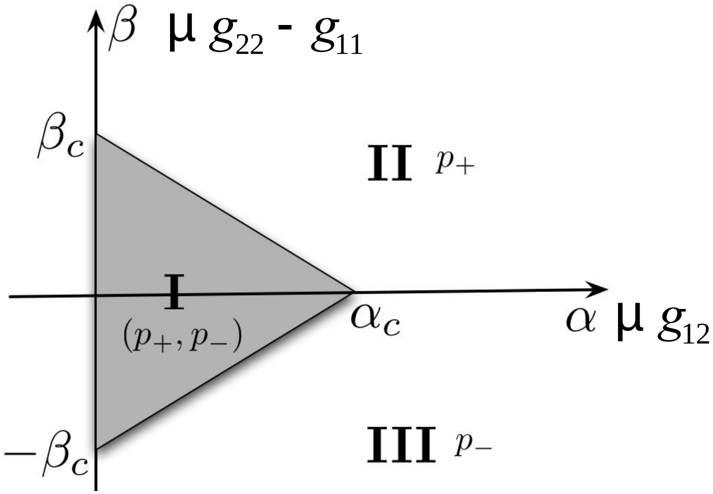
$$n = n_1 + n_2 \qquad g = g_{11} \cong g_{22}$$

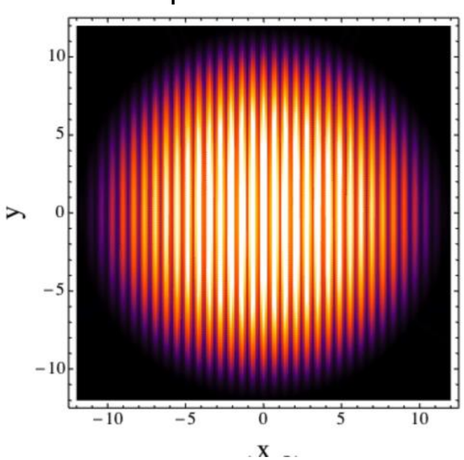
$$S_z = n_1 - n_2 \qquad g_{12} = g + \frac{\Omega_R}{2E_L}$$

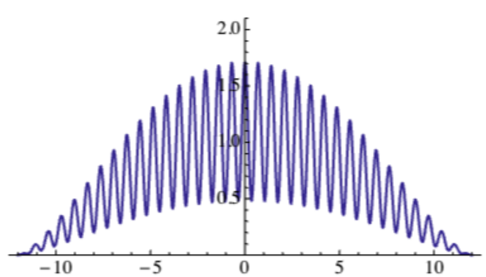
Rashba+Dresselhaus SOC BEC

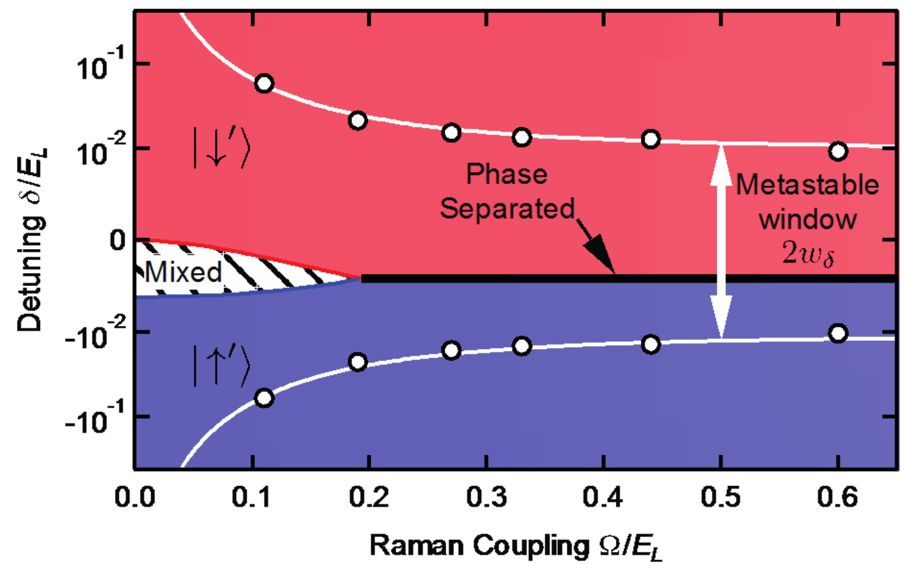
Lin et al., Nature **471**, 83 (2011)

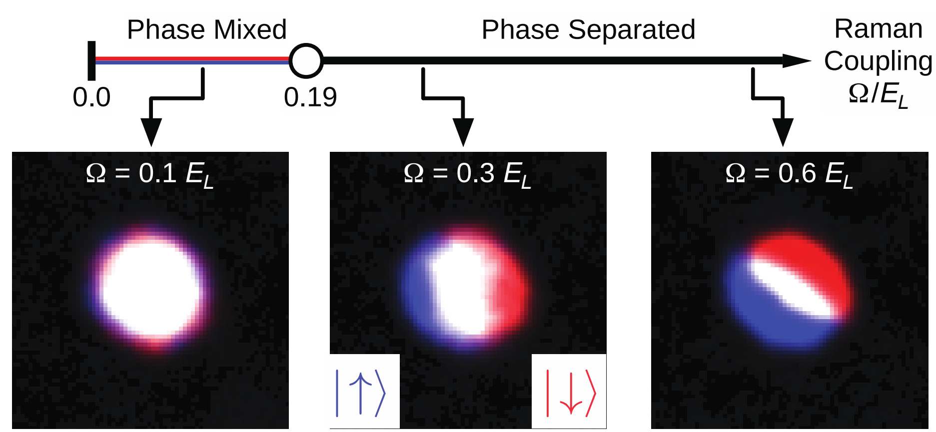
$$H = \int dr \Psi_\mu^* \left[\frac{h^2}{2m} \left(-i\nabla - k_L \hat{\sigma}_x \hat{x} \right)^2 - \delta \hat{\sigma}_x + \tilde{\Omega} \hat{\sigma}_z \right] \Psi_\nu + \frac{1}{2} \int dr\, n_\mu g_{\mu\nu} n_\nu$$











Ho, Zhang PRL **107**, 150403 (2011)

2D Rashba SOC BEC（トラップ系）

$$\boldsymbol{A} = \hbar\kappa\left(\sigma_x \hat{\boldsymbol{x}} + \sigma_y \hat{\boldsymbol{y}}\right)$$

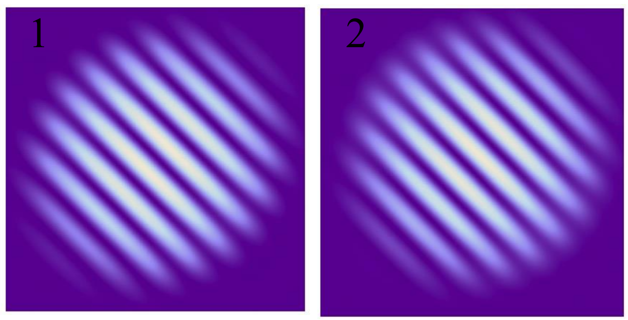
$$E_{GP} = \int d\boldsymbol{r}\left[\frac{1}{2m}\Psi^*_\mu(\boldsymbol{r})\left(-i\hbar\nabla - \boldsymbol{A}\right)^2_{\mu\nu}\Psi_\nu(\boldsymbol{r}) + Vn + \frac{c_0}{2}n^2 + \frac{c_2}{2}S_z^2\right]$$

$$c_0 = \frac{g + g_{12}}{2}$$

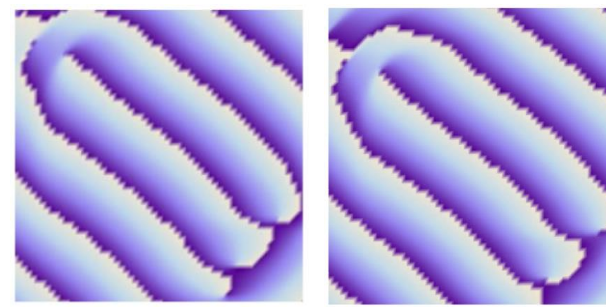
Strong interaction

Wang et al., PRL **106**, 160403 (2010)

n_1 n_2

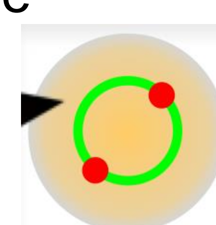


θ_1 θ_2



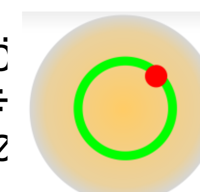
“stripe phase”

$$\psi \sim \begin{pmatrix} \cos \boldsymbol{k}\cdot\boldsymbol{r} \\ i\sin \boldsymbol{k}\cdot\boldsymbol{r} \end{pmatrix}$$



“plane wave phase”

$$\psi = \frac{1}{\sqrt{2}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \begin{pmatrix} \\ -e^{i\theta_k} \end{pmatrix}$$



$$c_2 = \frac{g - g_{12}}{2}$$

immiscible miscible

2D Rashba SOC BEC (トラップ系)

$$\boldsymbol{A} = \hbar\kappa\left(\sigma_x \hat{\boldsymbol{x}} + \sigma_y \hat{\boldsymbol{y}}\right)$$

$$E_{GP} = \int d\boldsymbol{r}\left[\frac{1}{2m}\Psi_\mu^*(\boldsymbol{r})\left(-i\hbar\nabla - \boldsymbol{A}\right)_{\mu\nu}^2 \Psi_\nu(\boldsymbol{r}) + Vn + \frac{c_0}{2}n^2 + \frac{c_2}{2}S_z^2\right]$$

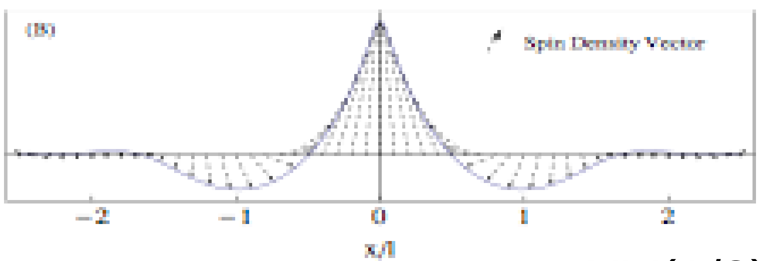
Non-interacting case

Wu, et al., Chin. Phys. Lett. **28**, 097102 (2011)

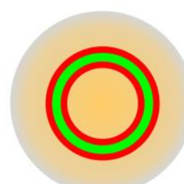
$$\phi = \begin{pmatrix} \phi_\uparrow(r) \\ \phi_\downarrow(r)e^{i\theta} \end{pmatrix}$$

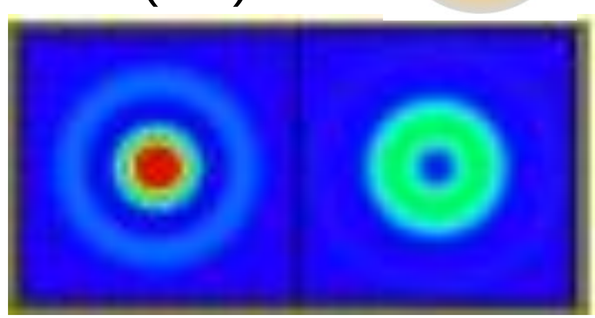
Half-quantum vortex

$$c_0 = \frac{g + g_{12}}{2}$$

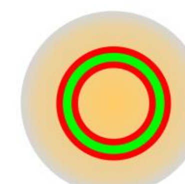


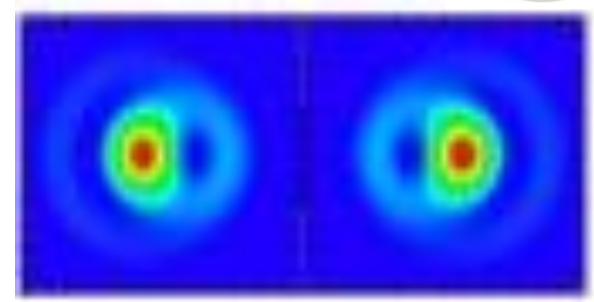
HV(1/2)





Superposition of HV(1/2)





Weak interaction

Hu, et al., PRL **108**, 010402 (2012)

$$c_2 = \frac{g - g_{12}}{2}$$

immiscible

miscible

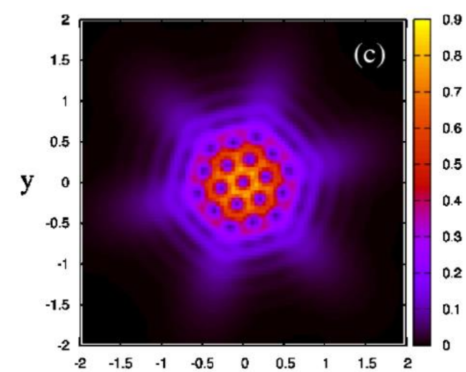
2D Rashba SOC BEC（トラップ系）

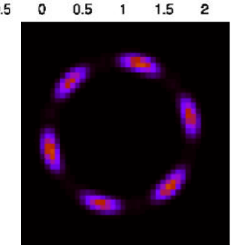
$$\boldsymbol{A} = \hbar\kappa\left(\sigma_x\hat{\boldsymbol{x}} + \sigma_y\hat{\boldsymbol{y}}\right)$$

$$E_{GP} = \int d\boldsymbol{r}\left[\frac{1}{2m}\Psi_\mu^*(\boldsymbol{r})\left(-i\hbar\nabla - \boldsymbol{A}\right)^2_{\mu\nu}\Psi_\nu(\boldsymbol{r}) + Vn + \frac{c_0}{2}n^2 + \frac{c_2}{2}S_z^2\right]$$

$$c_0 = \frac{g + g_{12}}{2}$$

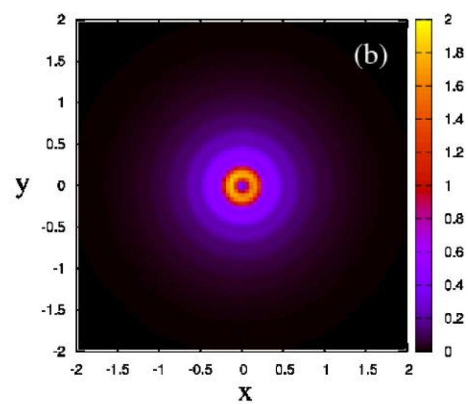
Vortex lattice





Sinha, et al., PRL **106**, 160403 (2011)

HV(3/2)



$$c_2 = \frac{g - g_{12}}{2}$$

immiscible

miscible

まとめ

1, 原子気体のボースアインシュタイン凝縮（BEC）

2, 超流動・量子流体力学の展開

3, 中性原子に対する磁場（Synthetic gauge field）

4, スピン軌道相互作用をもつBEC

ボース凝縮系における超流動，量子流体力学は新しい技術の発展により新たな展開を迎えている。

様々な分野との交流が重要