熱QEDにおける準粒子とultrasoftモード
（含む中川寿夫先生の追悼）

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[基研研究会「熱場の量子論とその応用」, 2013.8.26-28]

1984年頃～　有限温度の場の理論を始める

有限温度の場の理論の計算処方に関する研究

With 牲川（大阪市大）、芦田（大阪市大院生・一部）

- QCDの結合定数の温度・密度依存性

Non-Abelian gauge couplings at finite temperature in the general covariant gauge

Non-Abelian gauge couplings at finite temperature and density

Real-time non-Abelian gauge coupling at finite temperature: Temperature and vertex configuration dependences

- 有限温度の場の理論における反応比率の計算処方

Diagrammatics for finite-temperature reaction rates

Diagrammatic algorithm for evaluating finite-temperature reaction rates

1992年～

<u>有限温度における場の理論の相構造・相転移機構および準粒子等の性質</u>

＝基本方程式から、解析的処方で分析をする＝

方法1

繰り込み群方程式を用いて改良された有効ポテンシャルによる解析

RG improvement of the effective potential at finite temperature

Effective potential at finite temperature —RG improvement vs. high temperature expansion—

Phase structure of the massive scalar ϕ^4 model at finite temperature —Resummation procedure *a la* RG improvement—

方法２

実時間形式での硬熱ループ再加算されたDS方程式による解析

With 笛木（奈女大・院生・途中まで）・吉田（奈良大）

N-point vertex functions, Ward-Takahashi identities and Dyson-Schwinger equations in thermal QCD/QED in the real time hard-thermal-loop approximation

Chiral phase transitions in QED at finite temperature —Dyson-Schwinger equation analysis in the real time hard-thermal-loop approximation

Analysis of the phase structure of thermal QED/QCD through the HTL improved ladder Dyson-Schwinger equation —On the gauge dependence of the solution—

Vanishing thermal mass in the strongly coupled QCD/QED medium, PRD 85 (2012) 031902(R)

Structure of a thermal quasifermion in the QCD/QED medium, PRD 86 (2012) 096007

Hard-Thermal-Loop Resummed Dyson-Schwinger Equations(Real Time Formalism)

$$-i\Sigma_R(P) = -\frac{e^2}{2}\int \frac{d^4K}{(2\pi)^4} \times \left[\Gamma^{\mu}_{RAA} S_R(K) \Gamma^{\nu}_{RAA} G_{C,\mu\nu}(P-K) + \Gamma^{\mu}_{RAA} S_C(K)\, \Gamma^{\nu}_{AAR} G_{R,\mu\nu}(P-K) \right]$$

$$G_R^{\mu\nu}(K) = \frac{1}{\Pi_T^R - K^2 - i\varepsilon k_0} A^{\mu\nu} + \frac{1}{\Pi_L^R - K^2 - i\varepsilon k_0} B^{\mu\nu} - \frac{\xi}{K^2 + i\varepsilon k_0} D^{\mu\nu}$$

$$A^{\mu\nu} = g^{\mu\nu} - B^{\mu\nu} - D^{\mu\nu}, B^{\mu\nu} = -\frac{\tilde{K}^{\mu}\tilde{K}^{\nu}}{K^2}, D^{\mu\nu} = \frac{K^{\mu}K^{\nu}}{K^2}, \tilde{K} = (k, -k_0\vec{k})$$

$\Gamma^{\mu} = \gamma^{\mu}$ ← Ladder 近似

$$\Sigma_R(P) = (1 - A(P))\, p_i \gamma^i - B(P)\gamma_0 + C(P)$$

Themal Quasiparticleの振る舞い

Symmetric Phase（ $C=0$ ）、Landau gauge

$$S_R = \frac{1}{2}\left[\frac{1}{D_+}\left(\gamma^0+\frac{p_i\gamma^i}{p}\right)+\frac{1}{D_-}\left(\gamma^0-\frac{p_i\gamma^i}{p}\right)\right]$$

Spectral Function

$$\rho_\pm(p_0,p) = -\frac{1}{\pi}\mathrm{Im}\frac{1}{D_\pm(p_0,p)} = -\frac{1}{\pi}\mathrm{Im}\frac{1}{p_0+B(p_0,p)\mp pA(p_0,p)}$$

Dispersion Relation

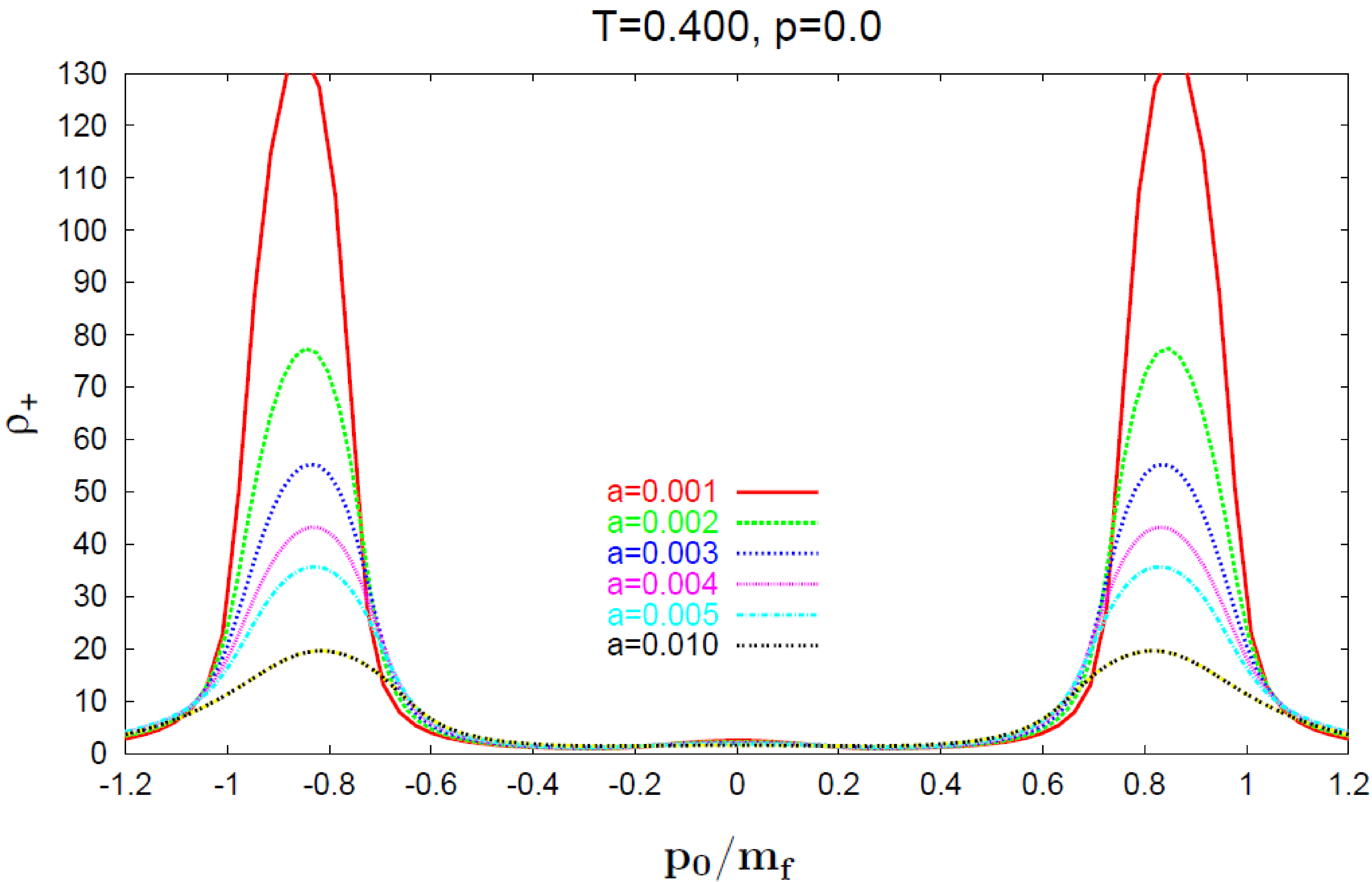
$$\mathrm{Re}[D_+(p_0=\omega,p)]=0$$

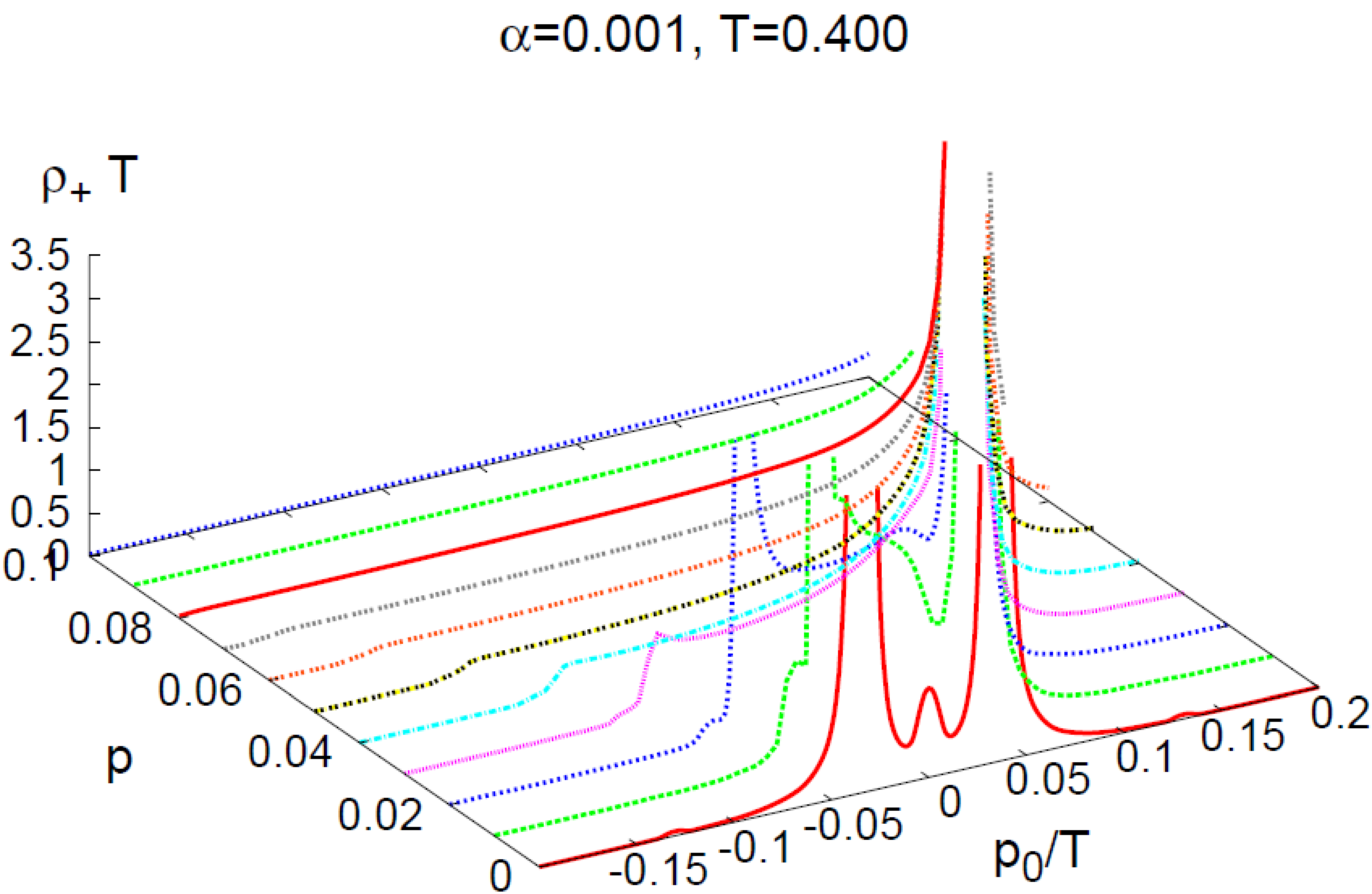
Decay Constant

$$\gamma(p)=\frac{1}{2}\mathrm{Im}\left[D_+(p_0=\omega,p)\right]\left[\frac{\partial}{\partial p_0}\mathrm{Re}\left[D_+(p_0,p)\right]\Big|_{p_0=\omega}\right]^{-1}$$

Spectral Function

$$\alpha = \frac{e^2}{4\pi}$$





α=0.001, T=0.400
ρ+ T
3.5
3
2.5
2
1.5
1
0.5
0
0.1
0.08
0.06
0.04
0.02
0
p
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
p0/T

Thermal Mass

Leading

$$m_f^2 = \frac{1}{8}e^2T^2$$

補正後

$$\left(\frac{m_f^*}{m_f}\right)^2 = 1 - \frac{4e}{\pi}\left[-\frac{e}{2\pi} + \sqrt{\frac{e^2}{4\pi^2} + \frac{1}{3}}\,\right]$$

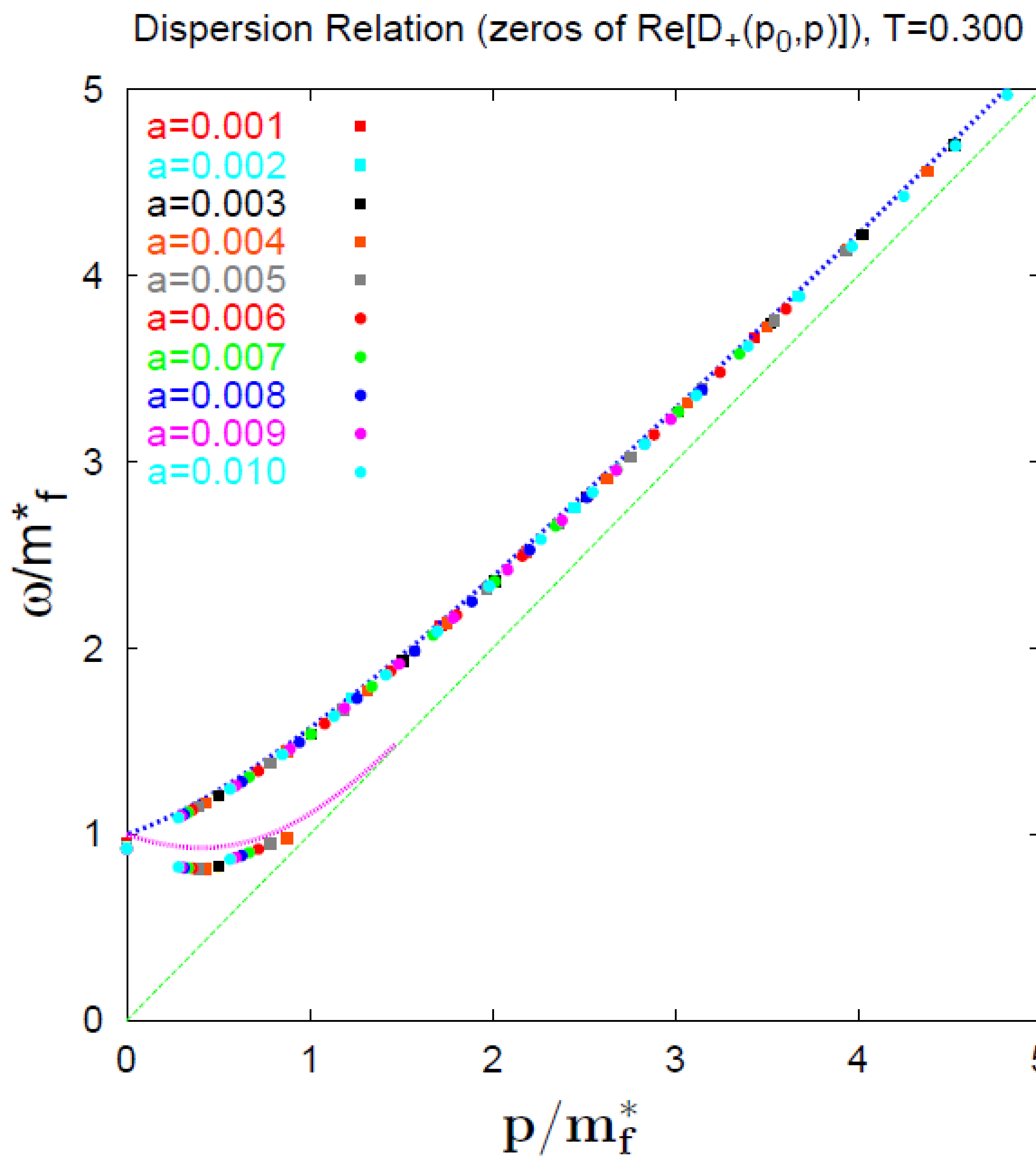
$$(m_D^*)^2 = m_D^2 - \frac{1}{\pi}e^2Tm_D^*$$

$$(m_f^*)^2 = m_f^2 - \frac{1}{2\pi}e^2Tm_D^*$$

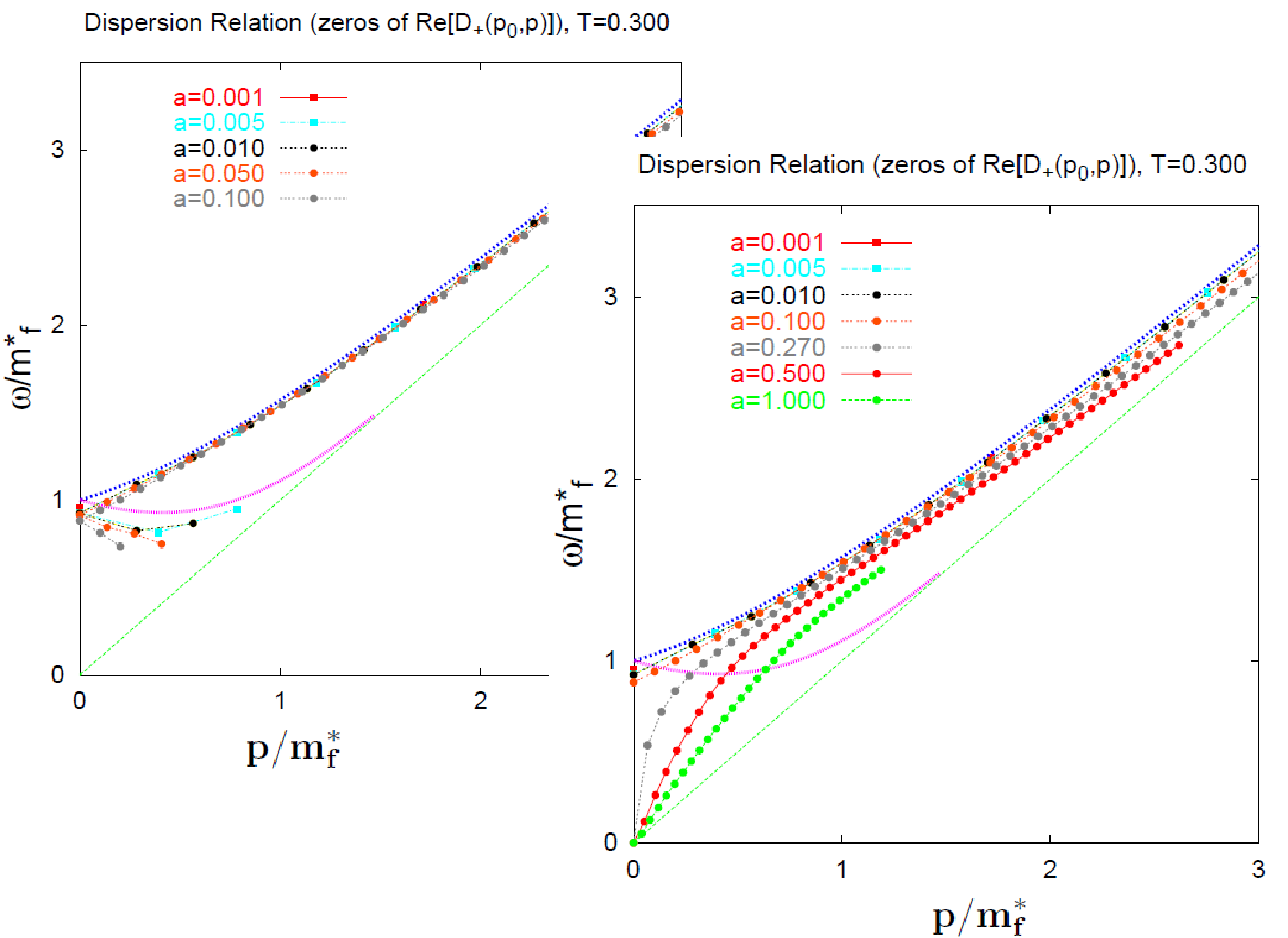
$$\mathrm{Re}[D_+(p_0 = \omega, p = 0)] = 0$$

Dispersion relation

$$\mathrm{Re}[D_+(p_0 = \omega, p)] = 0$$

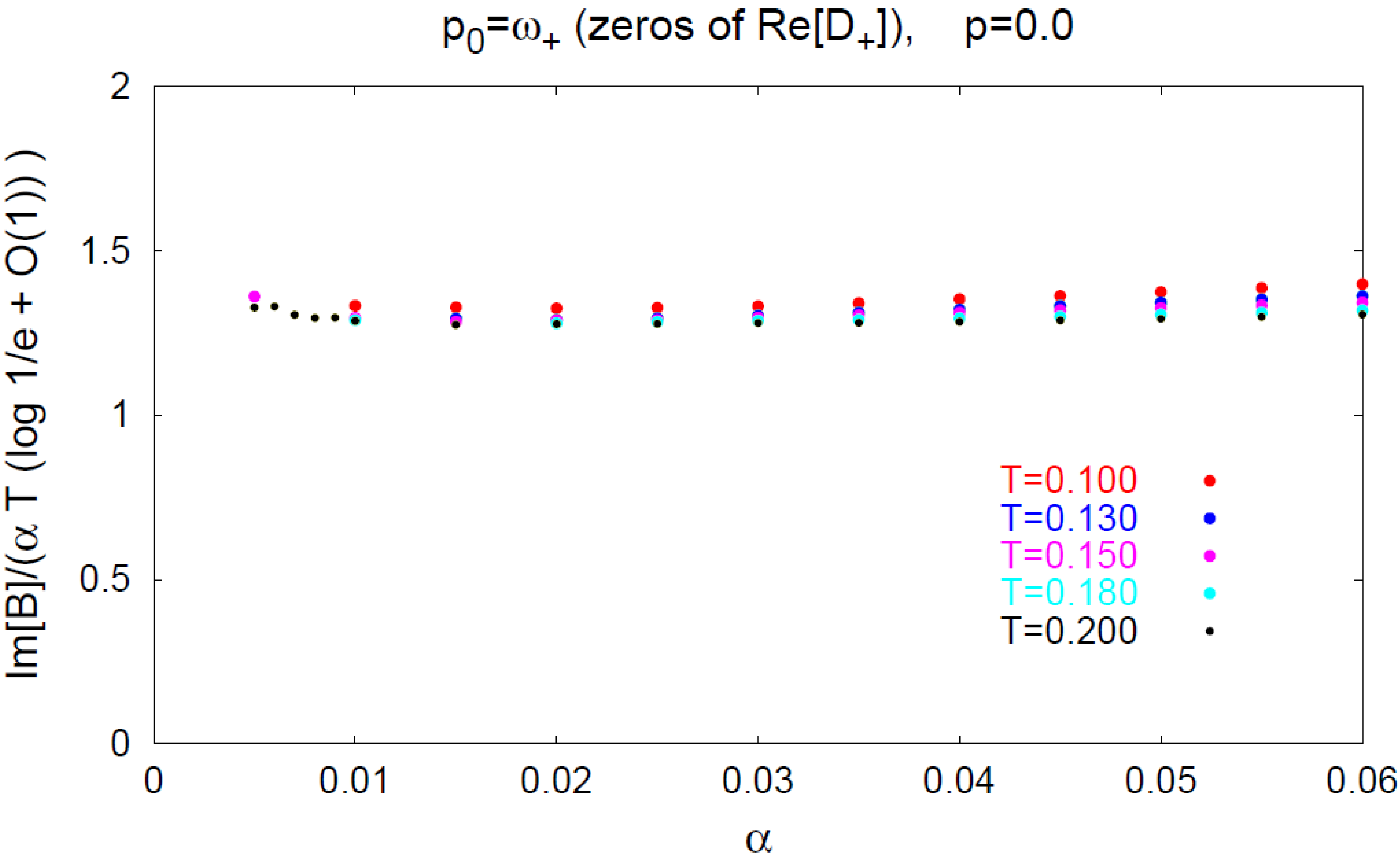


small coupling のとき
HTLの振る舞いを再現
している

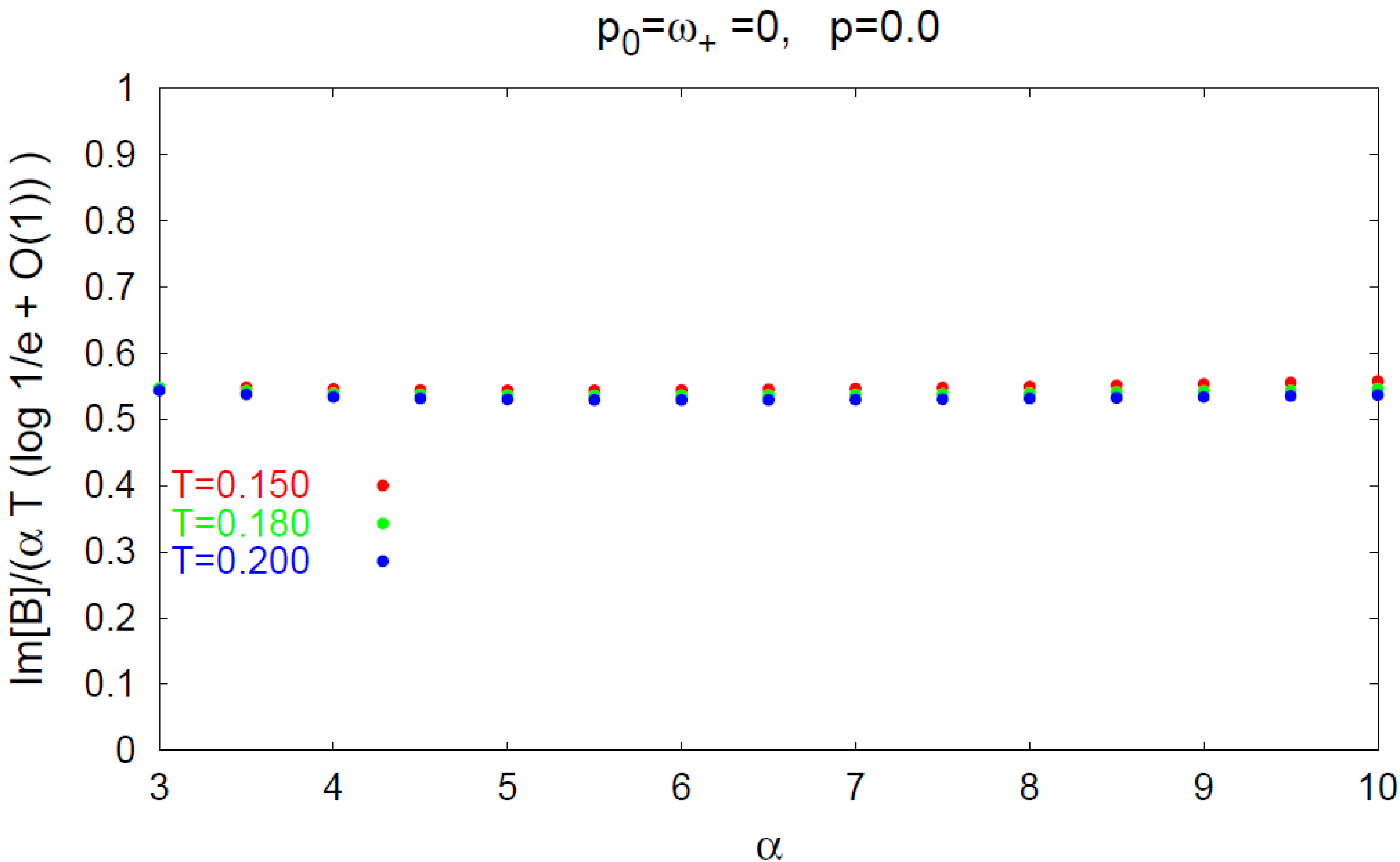


Dispersion Relation (zeros of Re[D+(p0,p)]), T=0.300
a=0.001
a=0.005
a=0.010
a=0.050
a=0.100
ω/m*f
p/m*f
Dispersion Relation (zeros of Re[D+(p0,p)]), T=0.300
a=0.001
a=0.005
a=0.010
a=0.100
a=0.270
a=0.500
a=1.000
ω/m*f
p/m*f

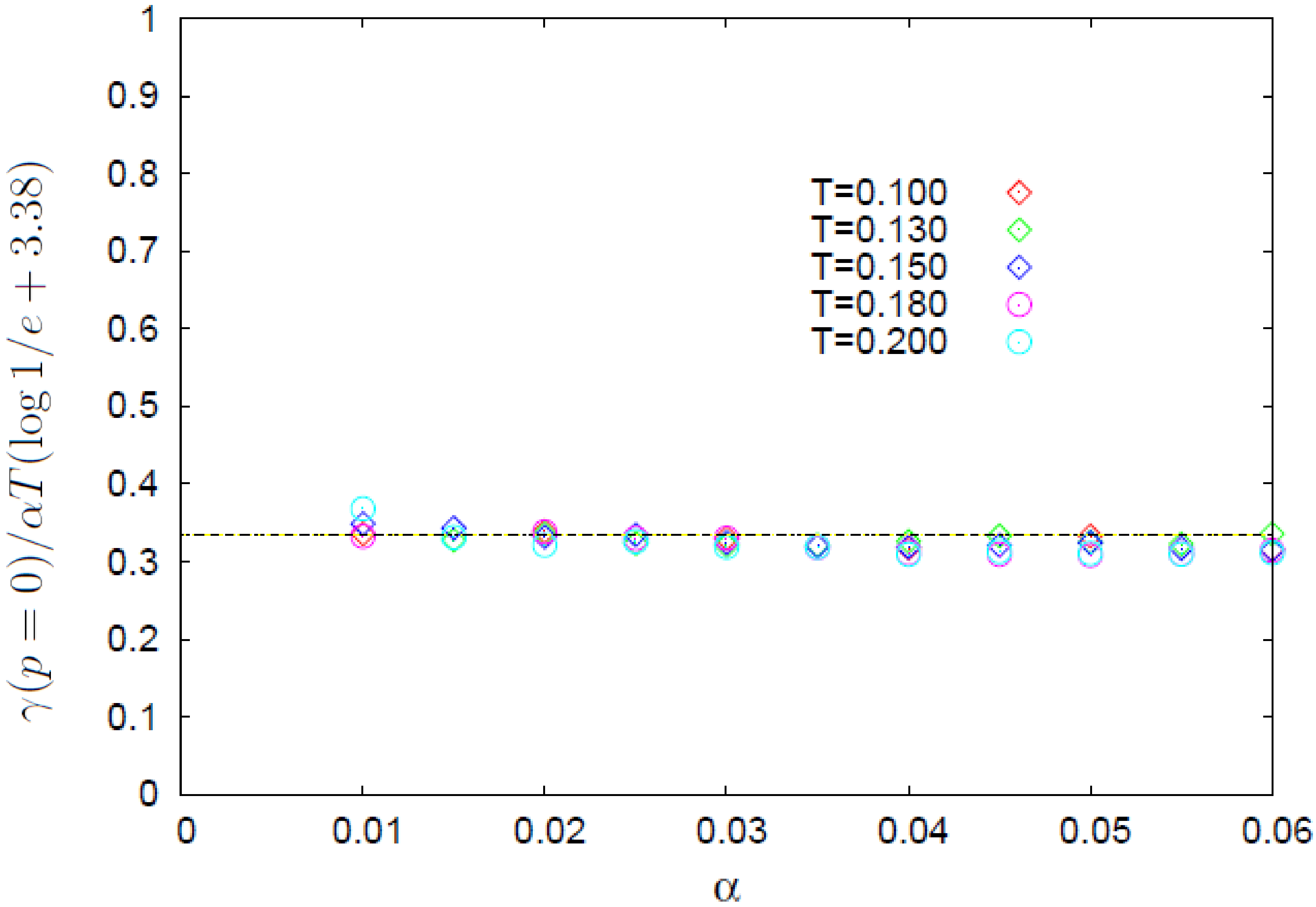
Small Coupling での Decay Constant



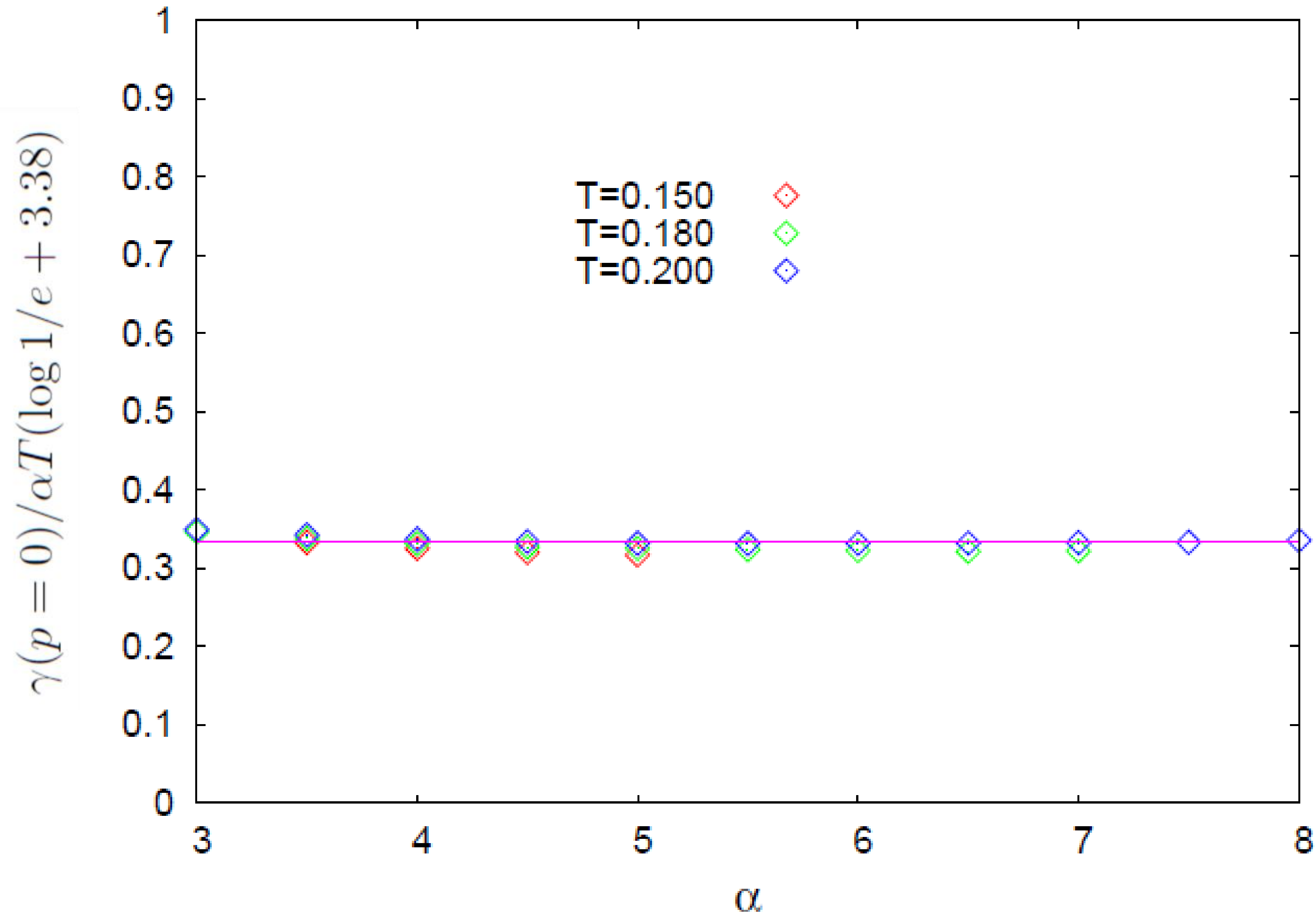
Large Coupling での Decay constant



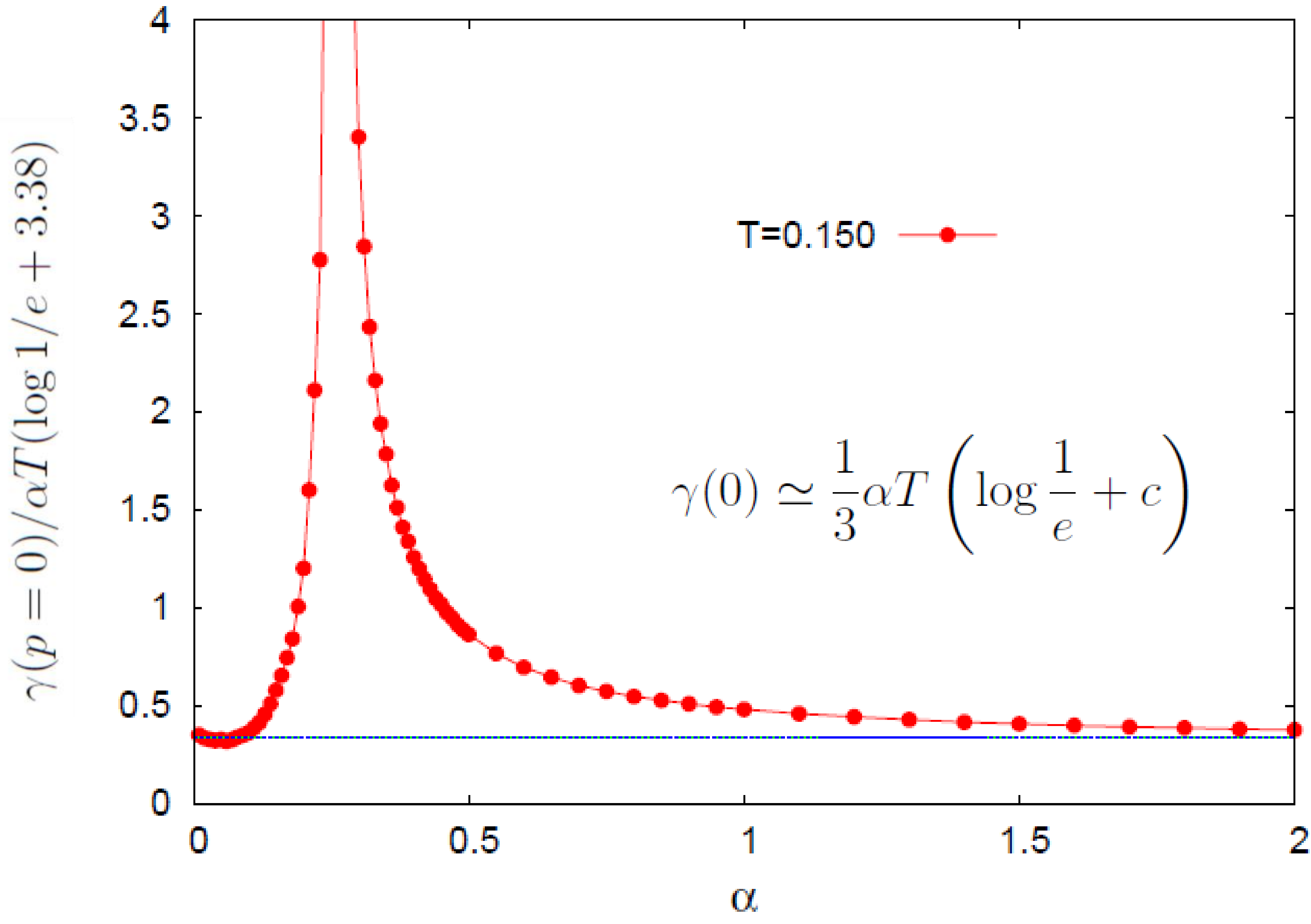
Small α とは 係数およびO(1)の値が異なる



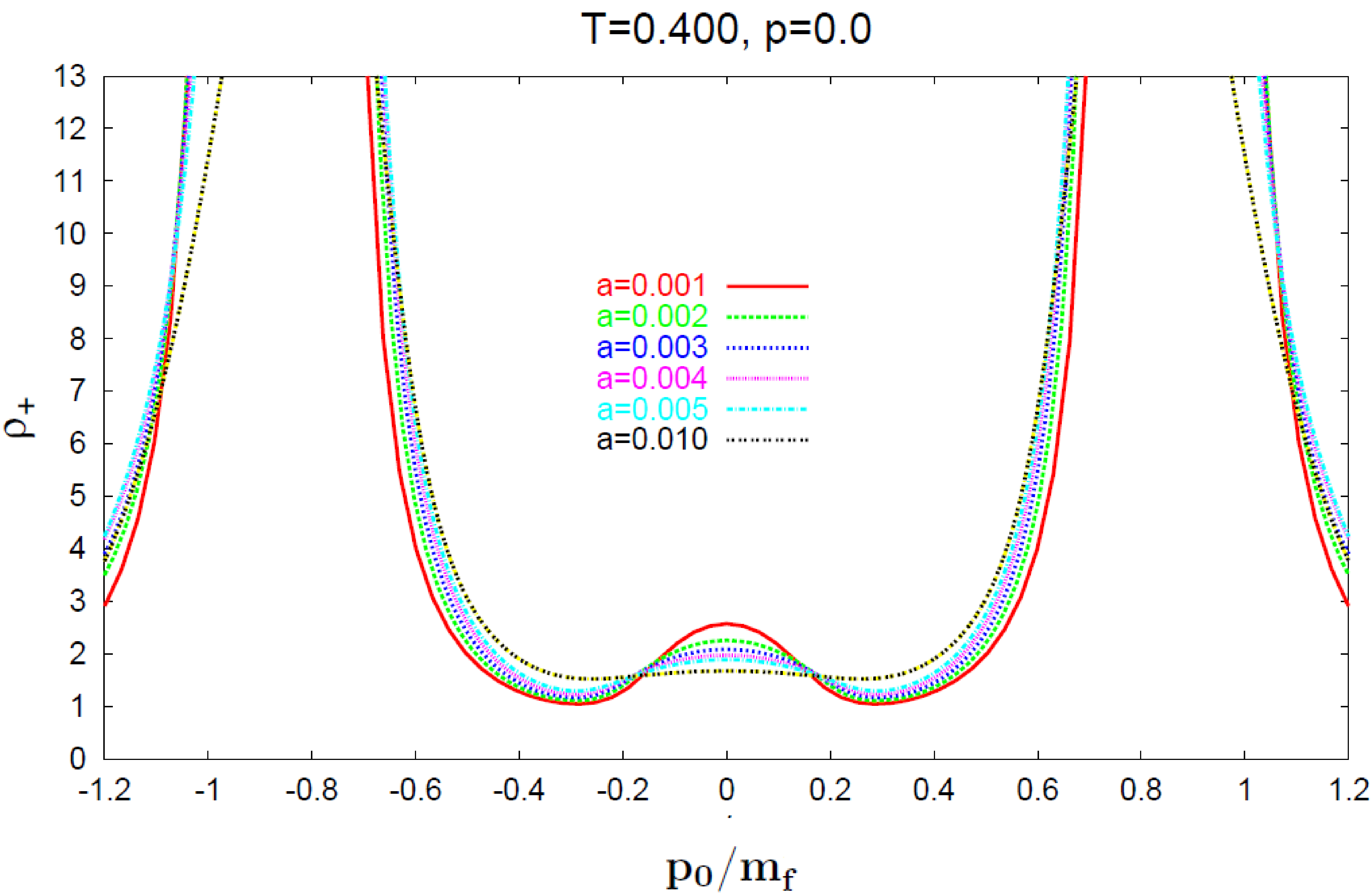
$\gamma(p=0)/\alpha T(\log 1/e + 3.38)$
α
T=0.100
T=0.130
T=0.150
T=0.180
T=0.200
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.01
0.02
0.03
0.04
0.05
0.06



Small α と 係数およびO(1)の値は同じ

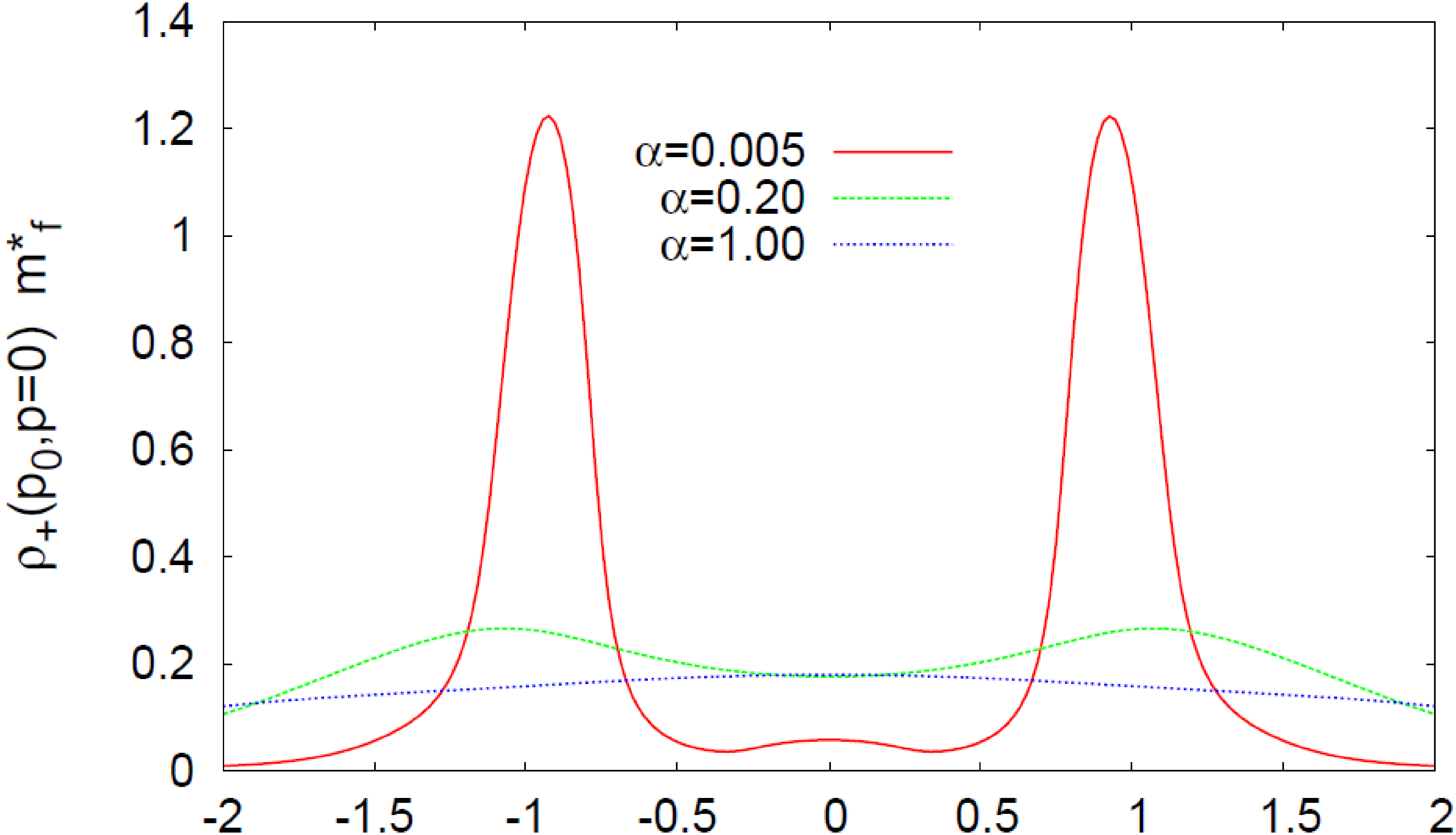


$\gamma(p=0)/\alpha T(\log 1/e + 3.38)$
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
α
T=0.150
$\gamma(0) \simeq \frac{1}{3}\alpha T\left(\log\frac{1}{e}+c\right)$

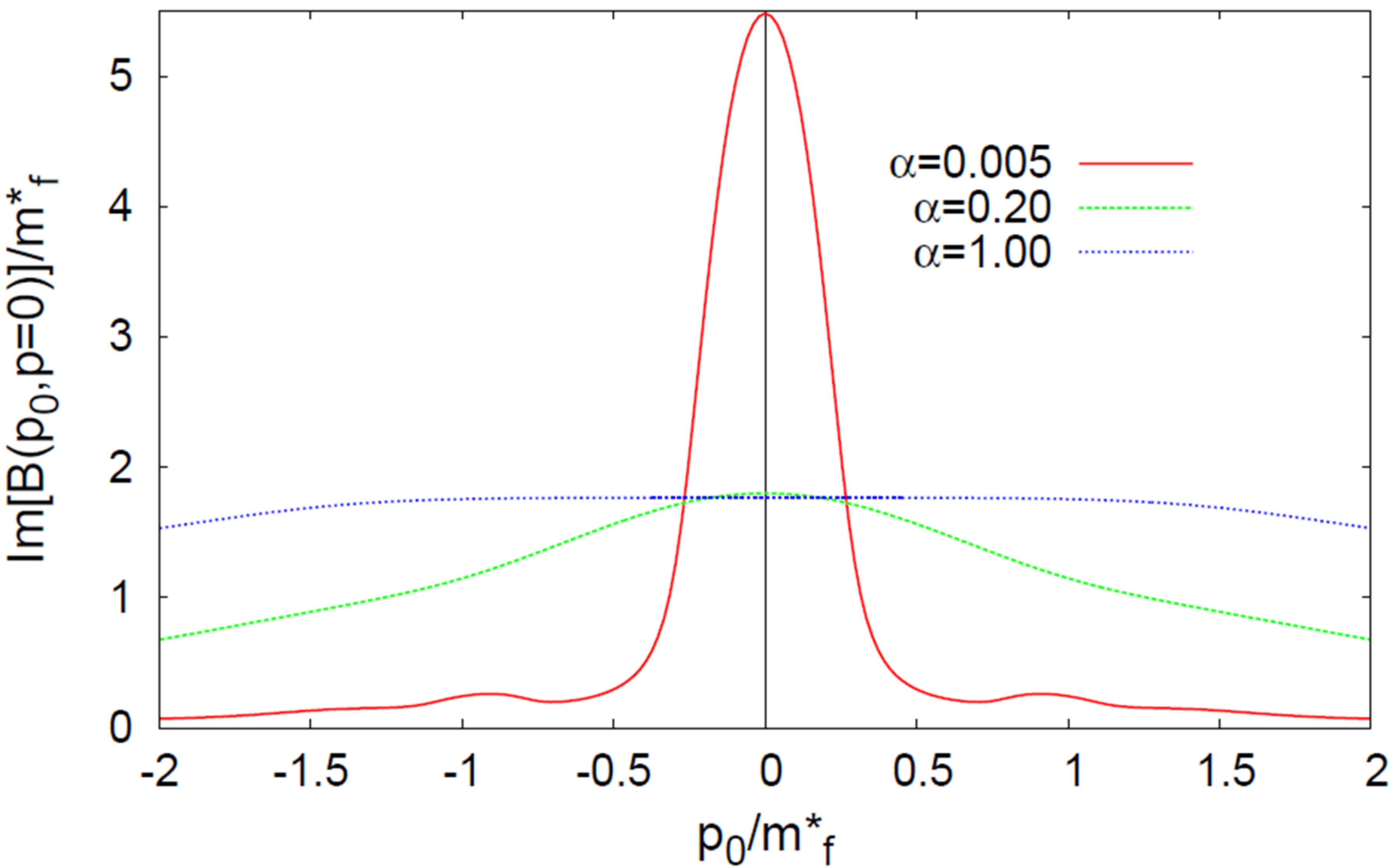


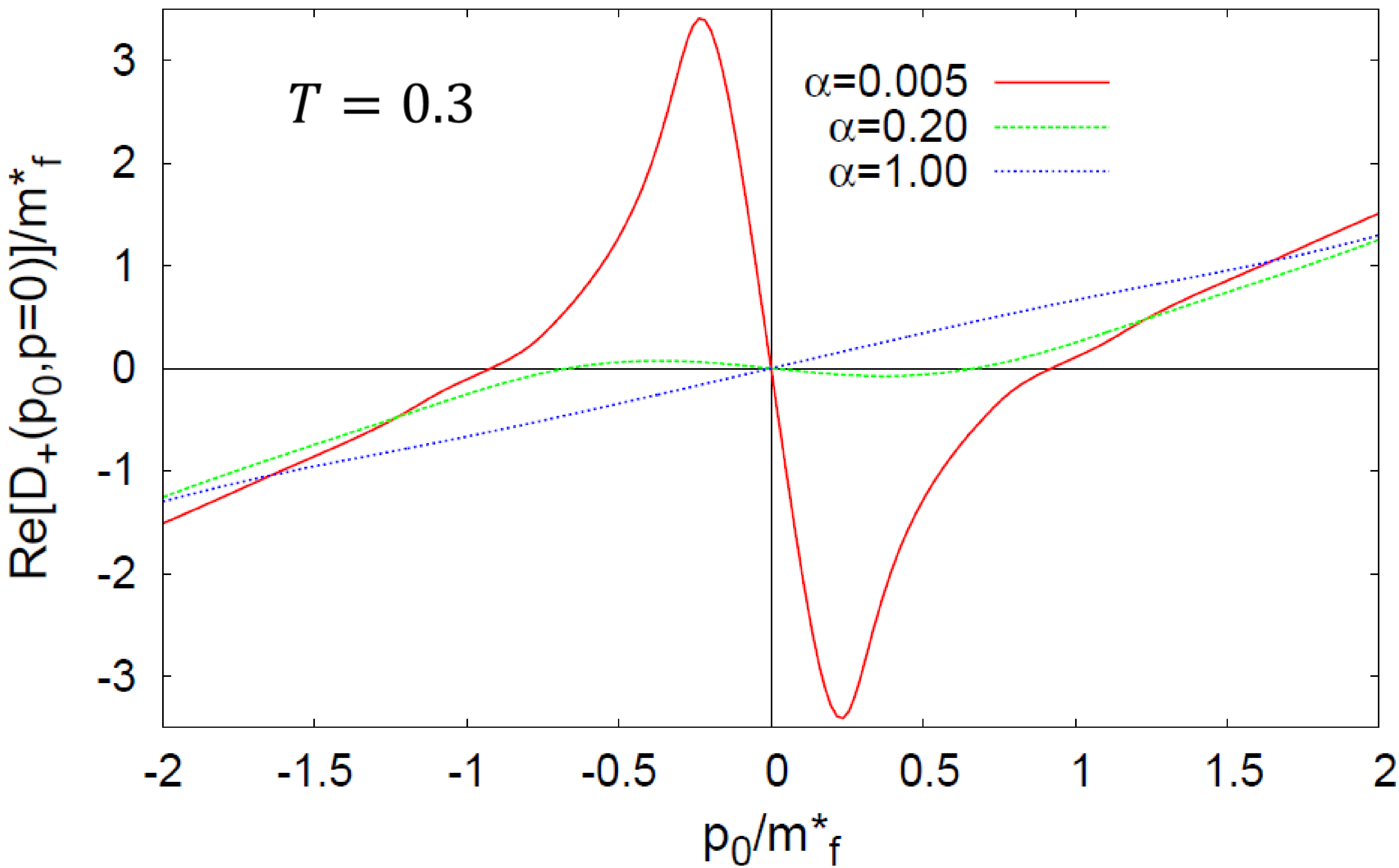
T=0.400, p=0.0
a=0.001
a=0.002
a=0.003
a=0.004
a=0.005
a=0.010
ρ_+
p_0/m_f
13
12
11
10
9
8
7
6
5
4
3
2
1
0
-1.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
1.2

$T = 0.3$



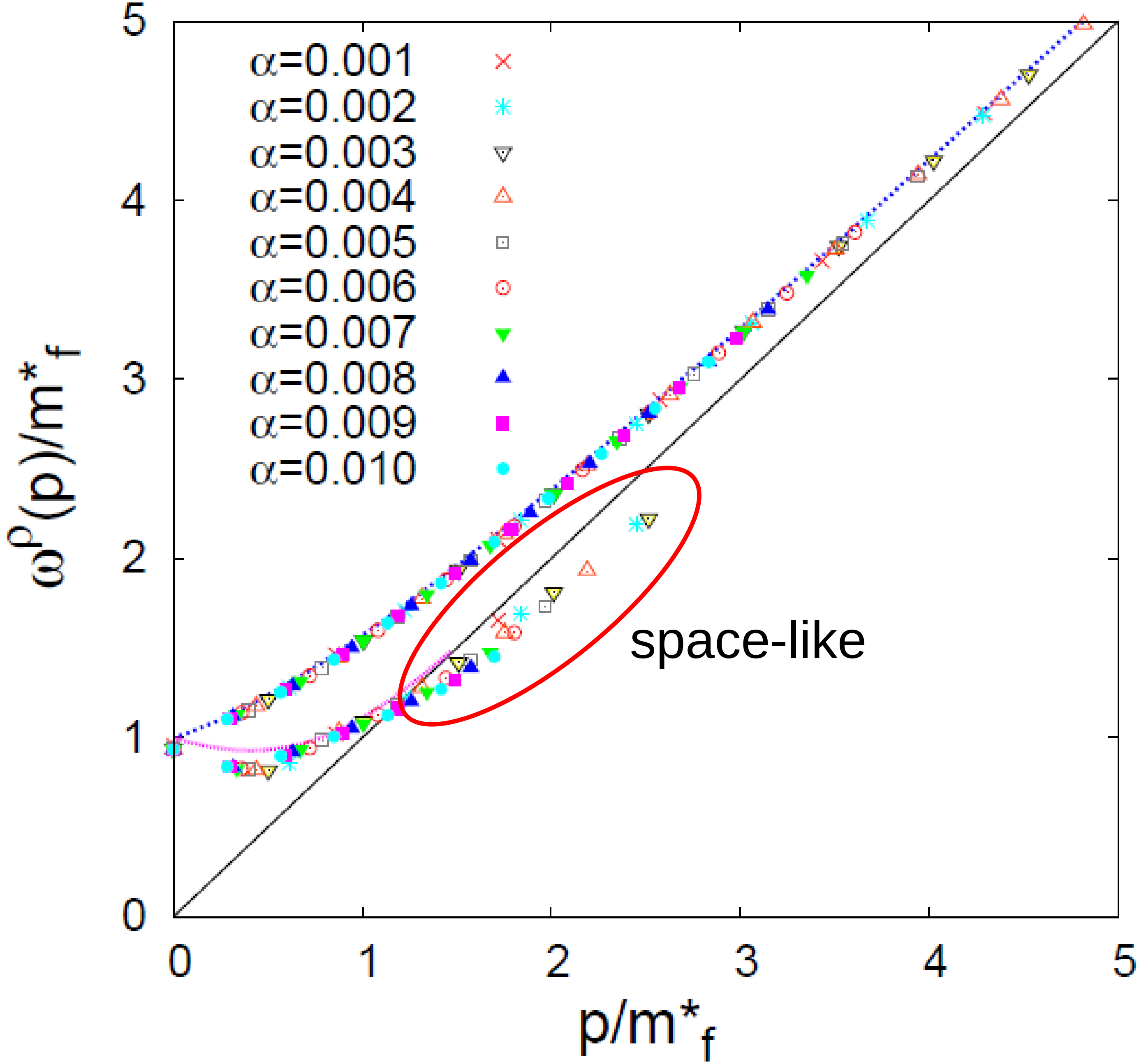
$T = 0.3$





$\mathrm{Re}[D_+(p_0, p)] = 0$ での傾きが Z^{-1}

$T = 0.3$



α=0.001
α=0.002
α=0.003
α=0.004
α=0.005
α=0.006
α=0.007
α=0.008
α=0.009
α=0.010
space-like
ω^ρ(p)/m*_f
p/m*_f
0
1
2
3
4
5

まとめ（主な結論）

- small αでは、（T≦0.2のとき）HTLの振る舞いを再現する
- large α または large Tでは
  熱的質量が小さくなり、最終的に0となる
- Decay constant は、（T=0.1～0.2の範囲で）
  small α および large α でともに

$$\gamma(0) \simeq \frac{1}{3}\alpha T \left(\log\frac{1}{e} + c\right)$$

  と同じ振る舞いを示す。他の解析の結果と
  係数も一致する
- small αで、p～0のとき ρに 3rd peak が存在する
  $\mathrm{Im}[D_+(p_0=\omega,p)]$ が大きい

$$Z_{us}^{-1} = \frac{\partial}{\partial p_0}\mathrm{Re}\left[D_+(p_0=\omega,p)\right]\Big|_{p_0=\omega} < 0$$

  ⇒ 物理的状態ではない！