熱場の量子論とその応用
TQFT 2019
YITP, Kyoto University

多体量子系の時間相関からの量子カオスの特徴づけ

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Danshita, Hanada, and MT, PTEP 2017, 083I01 (arXiv:**1606.02454**)

Cotler, Gur-Ari, Hanada, Polchinski, Saad, Shenker, Stanford, Streicher, and MT, JHEP 1705, 118 (2017) (arXiv:1611.04650)

Hanada, Shimada, and MT, Phys. Rev. E **97**, 022224 (2018) (arXiv:1702.06935)

García-García, Loureiro, Romero-Bermudez, and MT, PRL **120**, 241603 (2018) (arXiv:**1707.02197**)

García-García and MT, Phys. Rev. B **99**, 054202 (2019) (arXiv:**1801.03204**)

Gharibyan, Hanada, Shenker, and MT, JHEP 1807, 124 (2018) (arXiv:1803.08050)

Gharibyan, Hanada, Swingle, and MT, JHEP 1904, 082 (2019) (arXiv:**1809.01671**), submitted (arXiv:**1902.11086**)

Lau, Ma, Murugan, and MT, Phys. Lett. B **795**, 230 (10 August 2019) (arXiv:1812.04770)

How to characterize quantum chaos?

$\hat{H}$ = const.

$$i\frac{d}{dt}|\psi\rangle = \hat{H}|\psi\rangle \qquad |\psi(t)\rangle = \hat{\mathrm{T}}\exp\left[-i\int_0^t \hat{H}(t')dt\right]|\psi(t=0)\rangle = \exp\left(-i\hat{H}t\right)|\psi(t=0)\rangle$$

Linear dynamics

Unitary time evolution

- Energy level statistics

Correlation between levels, as in random matrices

Short range: Normalized level separation distribution, gap ratio, …
Longer range: Number variance, spectral form factor, …

cf. Bohigas-Giannoni-Schmit conjecture

- Out-of-time correlator

Classically,

$$\{x_i(t), p_j(0)\}_{\mathrm{PB}}^{\;2} = \left(\frac{\partial x_i(t)}{\partial x_j(0)}\right)^2 \to e^{2\lambda_{\mathrm{L}}t} \text{ for large } t$$

Quantum version:

OTOC: $C_T(t) = \left\langle \left| \left[\hat{W}(t), \hat{V}(t=0)\right] \right|^2 \right\rangle$

$$= \langle \hat{W}^\dagger(t)\hat{V}^\dagger(0)\hat{W}(t)\hat{V}(0)\rangle + \cdots$$

Numerically

➔ Actual energy eigenvalues needed

➔ Hard to see exponential time dependence

Our proposal (1902.11086): Singular value statistics of two-point correlators

$$G_{ab}^{(\phi)}(t) = \langle\phi|\hat{\chi}_a(t)\hat{\chi}_b(0)|\phi\rangle$$

The Sachdev-Ye-Kitaev model

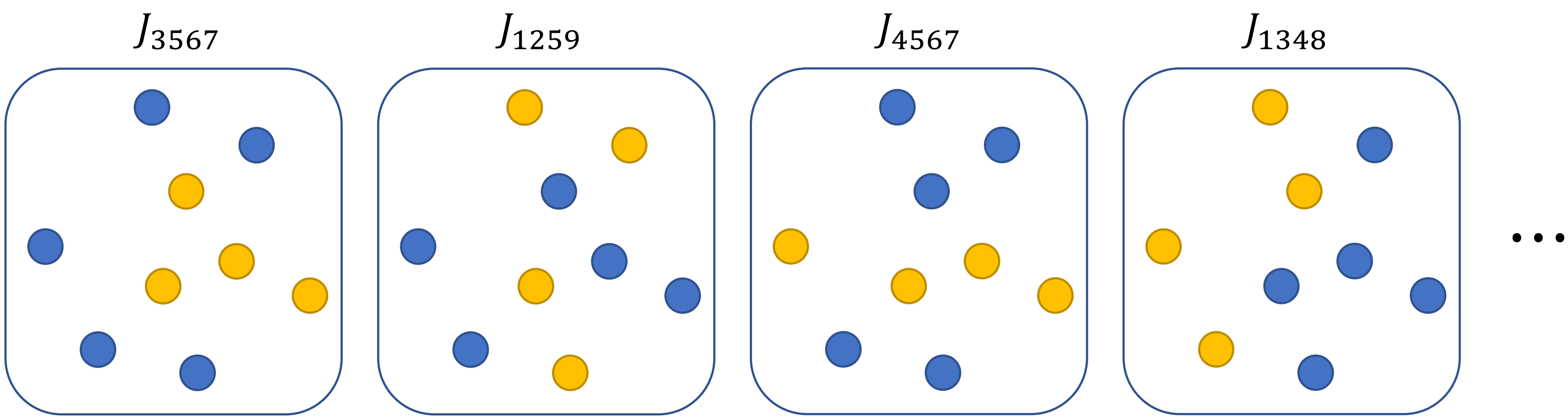
$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1\leq a<b<c<d\leq N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$$

cf. Sachdev-Ye model (1993)

[A. Kitaev, talks at KITP (2015)]

$\hat{\chi}_{a=1,2,\ldots,N}$: N Majorana fermions ($\{\hat{\chi}_a, \hat{\chi}_b\} = \delta_{ab}$)

J_{abcd} : independent Gaussian random couplings ($\langle {J_{abcd}}^2 \rangle = J^2 = 1$)



Two versions of the SYK model and large-N solvability

N Majorana- or Dirac- fermions randomly coupled to each other

[Majorana version]

$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1 \le a < b < c < d \le N} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d$$

[A. Kitaev: talks at KITP (Feb 12, Apr 7 and May 27, 2015)]

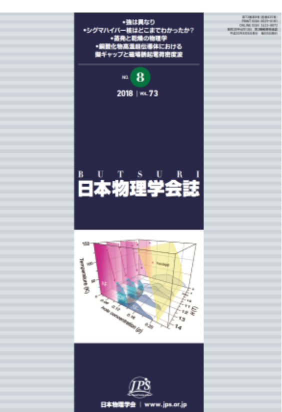
[Dirac version]

$$\hat{H} = \frac{1}{(2N)^{3/2}} \sum_{ij;kl} J_{ij;kl} \hat{c}_i{}^{\dagger} \hat{c}_j{}^{\dagger} \hat{c}_k \hat{c}_l$$

[Kitaev's talk][S. Sachdev: PRX **5**, 041025 (2015)]

"Two-body random ensemble" since 1970s

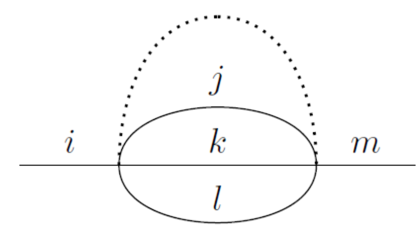
<u>Both solvable in the large-*N* limit</u>



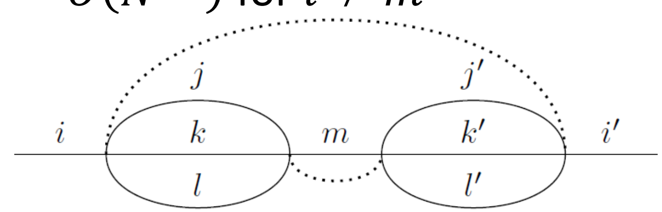
"Melon diagrams" dominate;
Reparametrization symmetry emerges

See [I. Danshita, M. Tezuka, and M. Hanada: Butsuri **73**(8), 569 (2018)] including our proposal for experimental realization [PTEP 2017]

$O(N^0)$



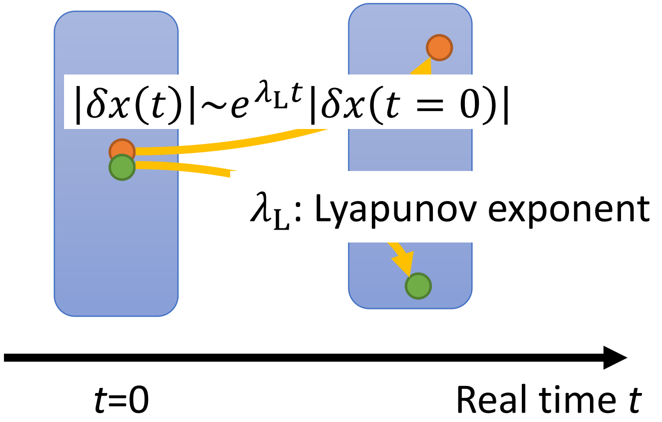
$O(N^{-2})$ for $i \neq m$



Definition of Lyapunov exponent using out-of-time-order correlators

$$F(t) = \langle \hat{W}^\dagger(t)\hat{V}^\dagger(0)\hat{W}(t)\hat{V}(0)\rangle \qquad W(t) = \mathrm{e}^{iHt} W \mathrm{e}^{-iHt}$$

Classical:
Infinitesimally different initial states



$$\{x(t), p(0)\}_{\mathrm{PB}}{}^2 = \left(\frac{\partial x(t)}{\partial x(0)}\right)^2 \to e^{2\lambda_{\mathrm{L}} t}$$

Consider operators V and W,

$$\begin{aligned} C(t) &= \langle |[W(t), V(t=0)]|^2 \rangle \\ &= 2\big(1 - \mathrm{Re}\, F(t)\big) \end{aligned}$$

quantifies strength of quantum scrambling

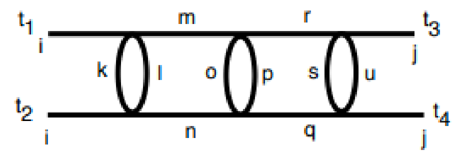
"Black holes are fastest quantum scramblers"

[P. Hayden and J. Preskill 2007] [Y. Sekino and L. Susskind 2008] [Shenker and Stanford 2014]

Chaos bound $\lambda_{\mathrm{L}} = 2\pi k_{\mathrm{B}} T/\hbar$

[J. Maldacena, S. H. Shenker, and D. Stanford, JHEP08(2016)106]

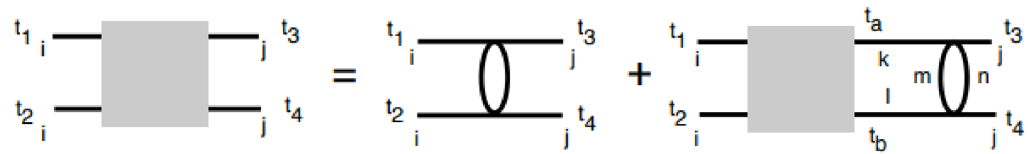
Out-of-time-ordered correlators (OTOCs)



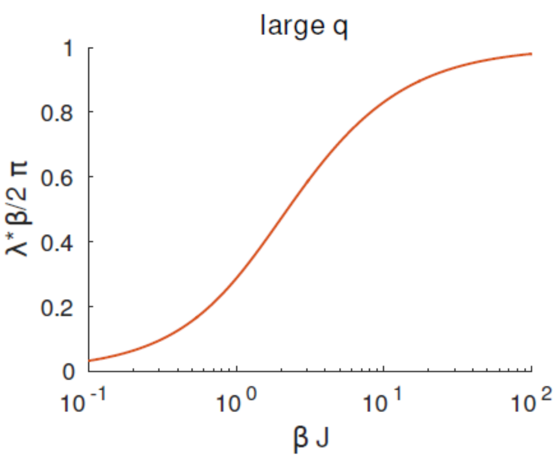
$\langle \hat{\chi}_i(t_1)\hat{\chi}_i(t_2)\hat{\chi}_j(t_3)\hat{\chi}_j(t_4)\rangle$

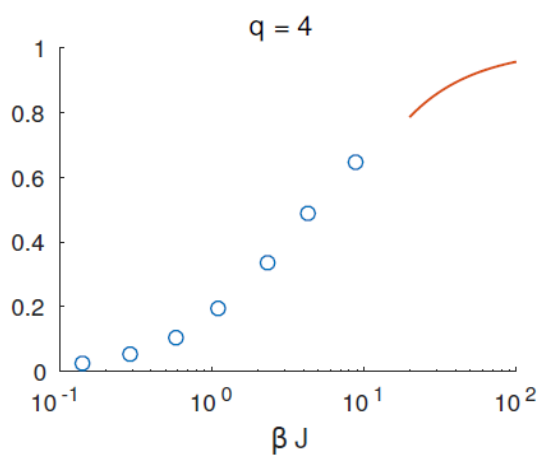
(a)

Regularized OTOC can be calculated for large-*N* SYK model, satisfies the chaos bound $\lambda_\mathrm{L} = 2\pi k_\mathrm{B} T/\hbar$ at low *T* limit



$$\Gamma(t_1,t_2,t_3,t_4) = \Gamma_0(t_1,t_2,t_3,t_4) + \int dt_a dt_b\, \Gamma(t_1,t_2,t_a,t_b) K(t_a,t_b,t_3,t_4)$$





SYK$_q$: *q*-fermion interactions

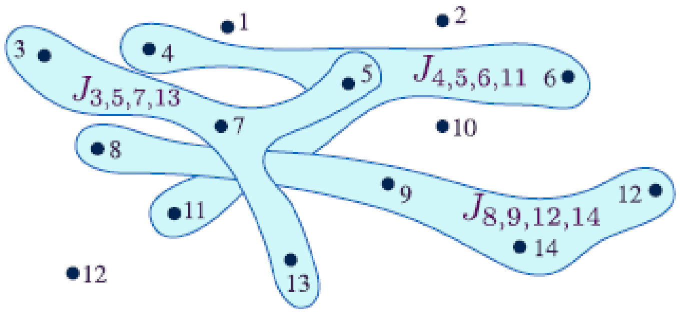
[Kitaev's talks]

[J. Polchinski and V. Rosenhaus, JHEP 1604 (2016) 001]

[J. Maldacena and D. Stanford, Phys. Rev. D **94**, 106002 (2016)]

Holographic connection to gravity

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

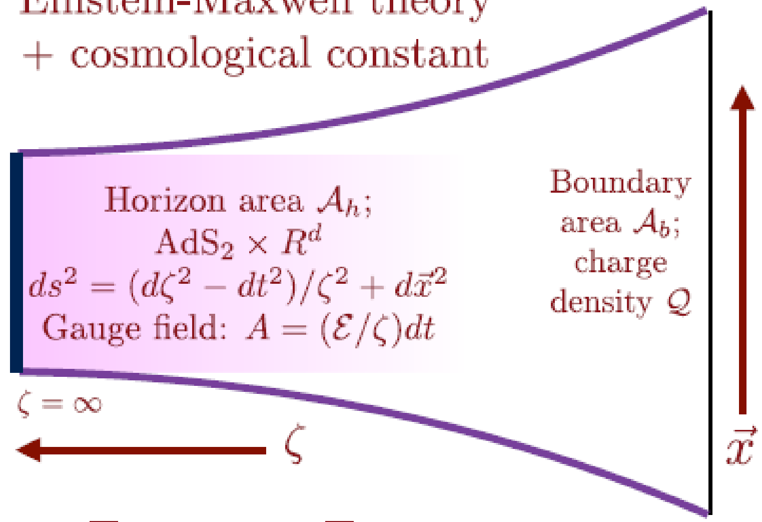
$$-\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \ \tau > 0 \\ e^{-2\pi\mathcal{E}} \, |\tau|^{-1/2} & , \ \tau < 0. \end{cases}$$

Known "equation of state" determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density $\mathcal{S}$ obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

Einstein-Maxwell theory + cosmological constant



$$\mathcal{L} = \overline{\psi} \Gamma^\alpha D_\alpha \psi + m \overline{\psi} \psi$$

$$-\left\langle \psi(\tau) \overline{\psi}(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \ \tau > 0 \\ e^{-2\pi\mathcal{E}} \, |\tau|^{-1/2} & , \ \tau < 0. \end{cases}$$

"Equation of state" relating $\mathcal{E}$ and $\mathcal{Q}$ depends upon the geometry of spacetime far from the AdS_2

Black hole thermodynamics (classical general relativity) yields

$$\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

[S. Sachdev, Phys. Rev. X **5**, 041025 (2015)]

NMR experiment for the SYK model

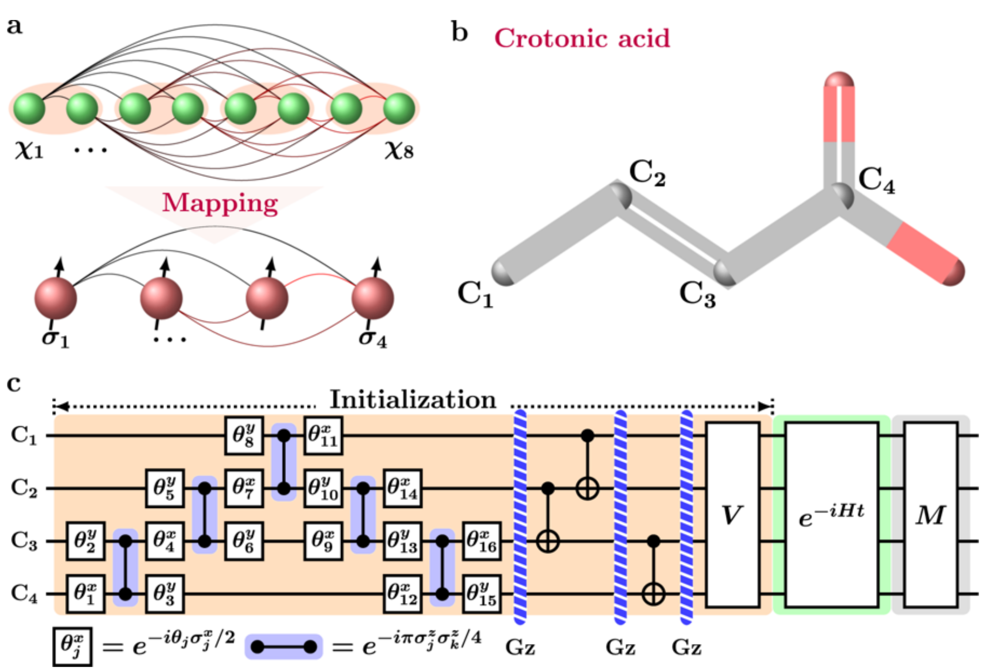
“Quantum simulation of the non-fermi-liquid state of Sachdev-Ye-Kitaev model” Zhihuang Luo, Yi-Zhuang You, Jun Li, Chao-Ming Jian, Dawei Lu, Cenke Xu, Bei Zeng and Raymond Laflamme, npj Quantum Information **5**, 53 (2019)

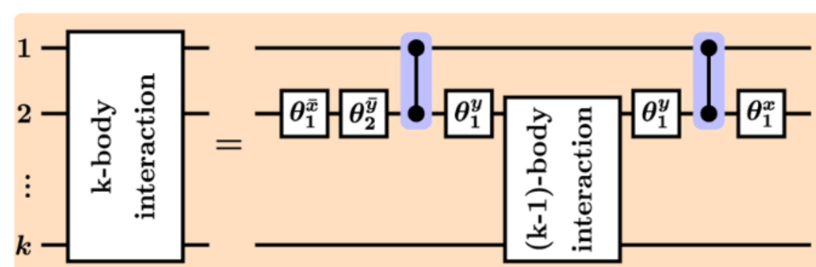
$$H = \frac{J_{ijkl}}{4!}\chi_i\chi_j\chi_k\chi_l + \frac{\mu}{4}C_{ij}C_{kl}\chi_i\chi_j\chi_k\chi_l$$

$$\chi_{2i-1} = \frac{1}{\sqrt{2}}\sigma_x^1\sigma_x^2\cdots\sigma_x^{i-1}\sigma_z^i,\ \chi_{2i} = \frac{1}{\sqrt{2}}\sigma_x^1\sigma_x^2\cdots\sigma_x^{i-1}\sigma_y^i.$$

$$H = \sum_{s=1}^{70} H_s = \sum_{s=1}^{70} a_{ijkl}^s\sigma_{\alpha_i}^1\sigma_{\alpha_j}^2\sigma_{\alpha_k}^3\sigma_{\alpha_l}^4$$

$$e^{-iH\tau} = \left(\prod_{s=1}^{70} e^{-iH_s\tau/n}\right)^n + \sum_{s<s'}\frac{[H_s, H_{s'}]\tau^2}{2n} + O(|a|^3\tau^3/n^2),$$





$SYK_4 + SYK_2$: Large-N calculation for OTOC

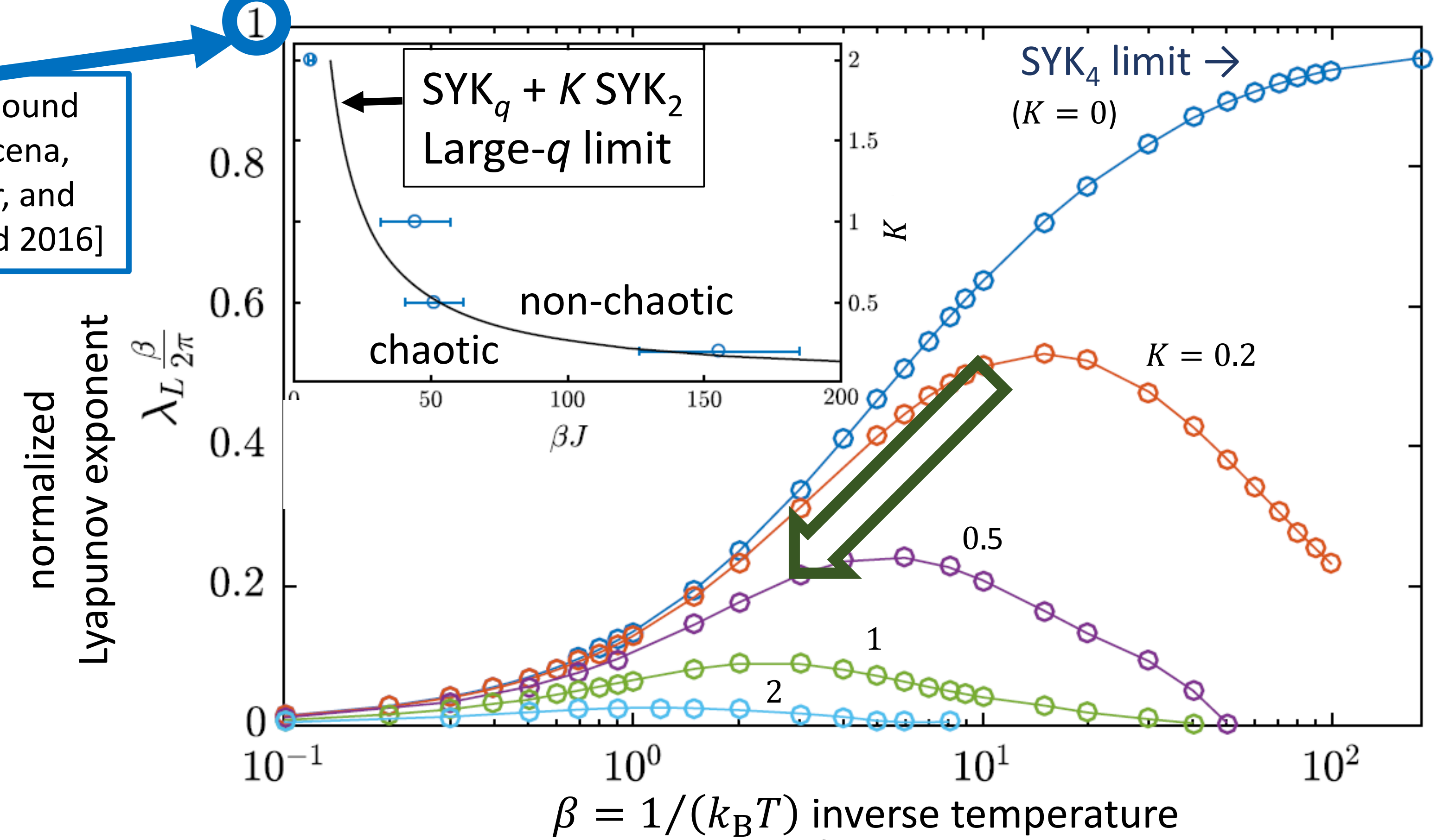
1707.02197

$$\hat{H} = \sum_{1\le a<b<c<d}^{N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d + i\sum_{1\le a<b}^{N} K_{ab}\hat{\chi}_a\hat{\chi}_b$$

SYK_4 SYK_2

K_{ab}: standard deviation $\frac{K}{\sqrt{N}}$

Chaos bound [Maldacena, Shenker, and Stanford 2016]



A. M. Garcia-Garcia, B. Loureiro, A. Romero-Bermudez, and MT, PRL **120**, 241603 (2018)

Deviation from the chaos bound as SYK_2 component is introduced

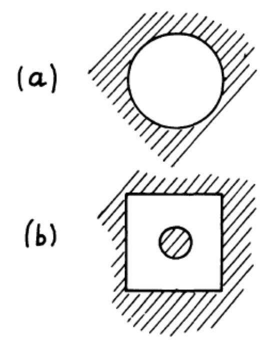
Contents

✓The Sachdev-Ye-Kitaev model
  ✓Large-*N* solvability: conformal symmetry and maximal chaos
  ✓Deformation and suppression of maximal chaos 1707.02197
- Characterization of chaos in random systems 1702.06935
  - Quantum Lyapunov spectrum 1809.01671
  - Singular value statistics of two-point correlators 1902.11086

Other related works: 1801.03204, 1812.04770

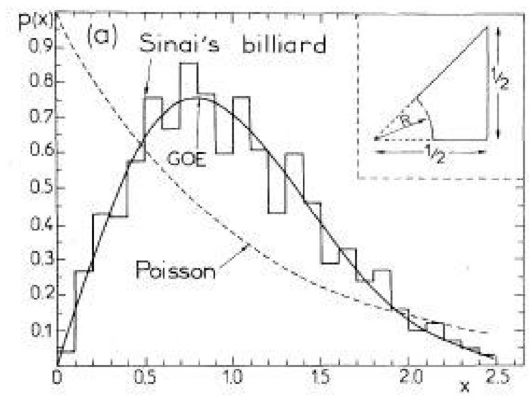
The Bohigas-Giannoni-Schmit conjecture

Assume quantum mechanical systems with a classical limit



circular: integrable

Sinai billiard: chaotic



“Spectral statistics of chaotic systems can be described as a random matrix”

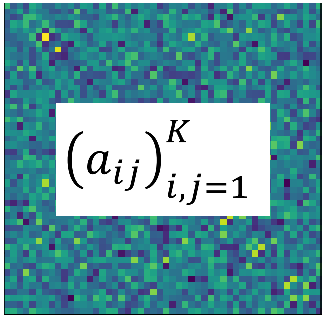
O. Bohigas, M. J. Giannoni, and C. Schmit, Phys. Rev. Lett. 52, 1 (1984);
J. de Phys. Lett. 45, 1015 (1984).

Justifications:

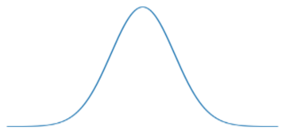
Non-linear sigma-model (Andreev 1993, Altland 2015)
Gutzwiller trace formula in terms of periodic orbits (Berry 1985, Gutzwiller 1990, Sieber, Richter, Braun, Muller, Heusler, …)

Also more examples including systems without clear classical version

Gaussian random matrices



$a_{ij} = a_{ji}^*$



Gaussian distribution

Density $\propto e^{-\frac{\beta K}{4}\mathrm{Tr}H^2} = \exp\left(-\frac{\beta K}{4}\sum_{i,j}^{K}|a_{ij}|^2\right)$

Real (β=1): Gaussian Orthogonal Ensemble (GOE)

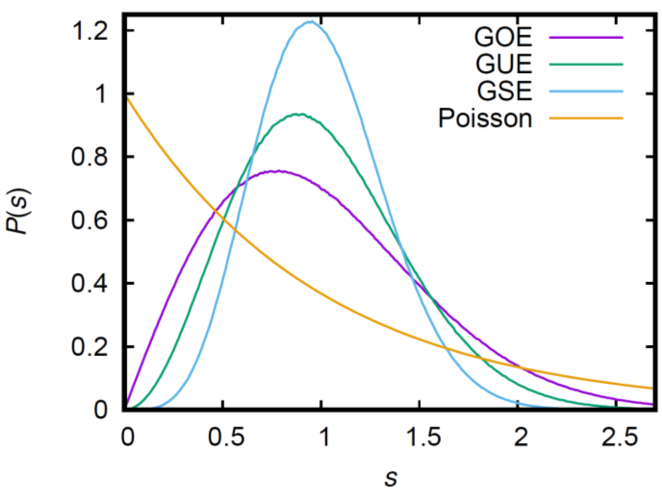
Complex (β=2): G. Unitary E. (GUE)

Quaternion (β=4): G. Symplectic E. (GSE)

Joint distribution function for eigenvalues $\{e_j\}$

Level repulsion

$$p(e_1, e_2, \ldots, e_K) \propto \prod_{1\le i<j\le K} |e_i - e_j|^{\beta} \prod_{i=1}^{K} e^{-\beta K e_i^2/4}$$



- $P(s)$: Distribution of normalized level separation

$$s_j = \frac{e_{j+1} - e_j}{\Delta(\bar{e})}$$

GOE/GUE/GSE: $P(s) \propto s^{\beta}$ at small s, has e^{-s^2} tail

Uncorrelated (Poisson): $P(s) = e^{-s}$

- $\langle r \rangle$: Average of neighboring gap ratio

$$r = \frac{\min(e_{i+1} - e_i\,,\ \ e_{i+2} - e_{i+1})}{\max(e_{i+1} - e_i\,,\ \ e_{i+2} - e_{i+1})}$$

	Uncorrelated	GOE	GUE	GSE
$\langle r \rangle$	2log 2 – 1 = 0.38629...	0.5307(1)	0.5996(1)	0.6744(1)

[Y. Y. Atas *et al.* PRL 2013]

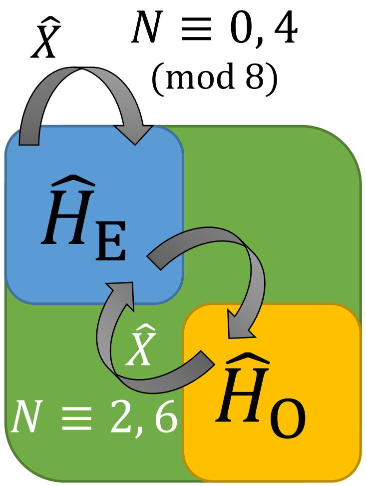
N mod 8 classification of Majorana SYK$_{q=4}$

$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1\le a<b<c<d\le N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$$

SPT phase classification for class BDI:
$\mathbb{Z} \rightarrow \mathbb{Z}_8$ due to interaction
[L. Fidkowski and A. Kitaev, PRB 2010, PRB 2011]

Introduce $N/2$ complex fermions $\hat{c}_j = \frac{(\hat{\chi}_{2j-1}+\mathrm{i}\hat{\chi}_{2j})}{\sqrt{2}}$

$\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$ respects the complex fermion parity
Even ($\hat{H}_\mathrm{E}$) and odd ($\hat{H}_\mathrm{O}$) sectors: $L = 2^{N/2-1}$ dimensions



N mod 8	0	2	4	6
η	-1	+1	+1	-1
$\hat{X}^2$	**+1**	+1	**-1**	-1
$\hat{X}$ maps H_E to	H_E	H_O	H_E	H_O
Class	**AI**	**A+A**	**AII**	**A+A**
Gaussian ensemble	**GOE**	**GUE**	**GSE**	**GUE**

$$\hat{X} = \hat{K}\prod_{j=1}^{N/2}(\hat{c}_j^\dagger + \hat{c}_j)$$

$$\hat{X}\hat{c}_j\hat{X} = \eta\hat{c}_j^\dagger;\ [\hat{X},\hat{H}] = 0$$

[You, Ludwig, and Xu, PRB 2017]

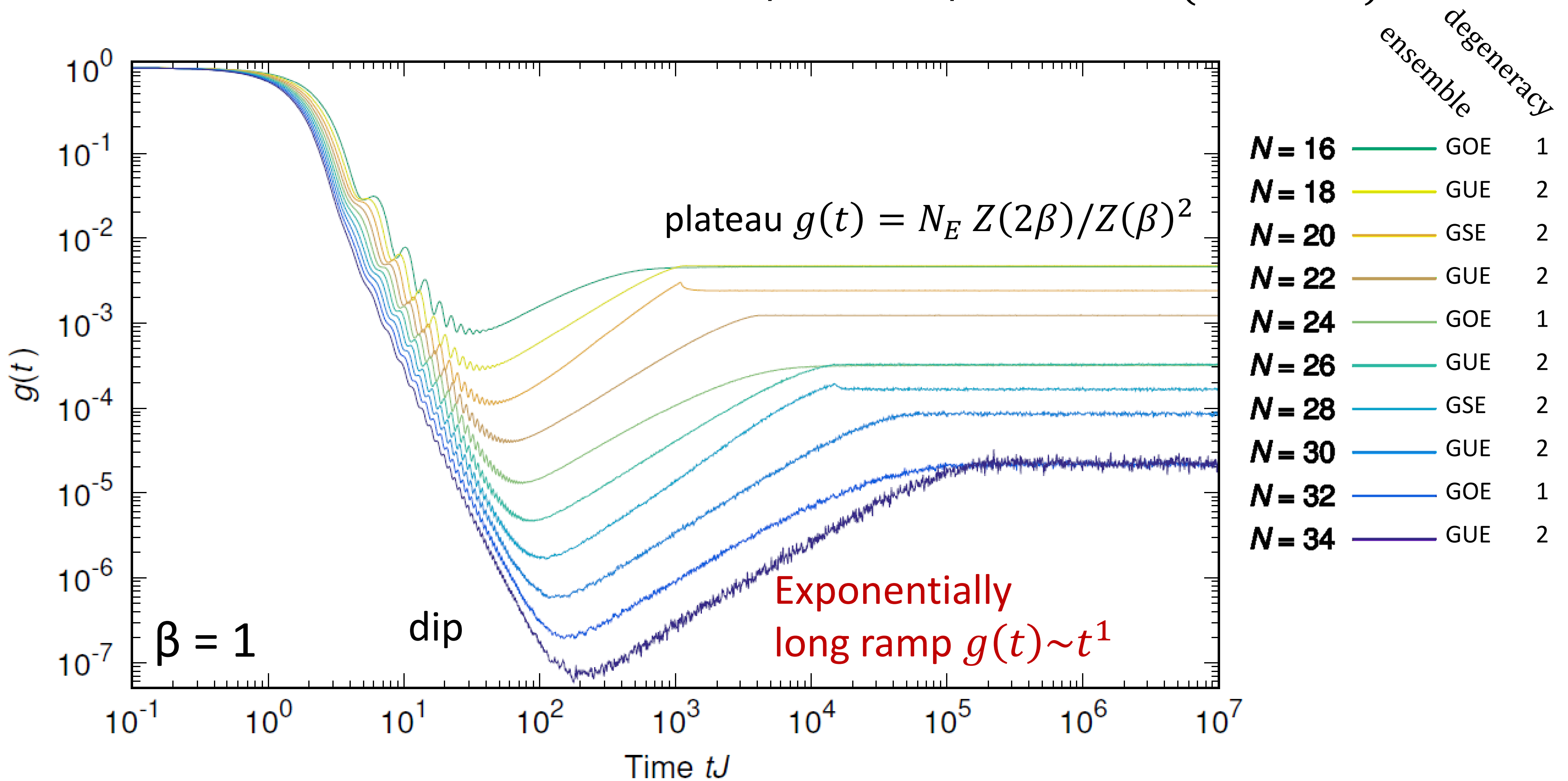
[Fadi Sun and Jinwu Ye, 1905.07694]
for SYK$_q$, supersymmetric SYK

SYK: sparse matrix, but energy spectral statistics strongly resemble that of the corresponding (dense) Gaussian ensemble [Cotler, …, MT, JHEP 2017]

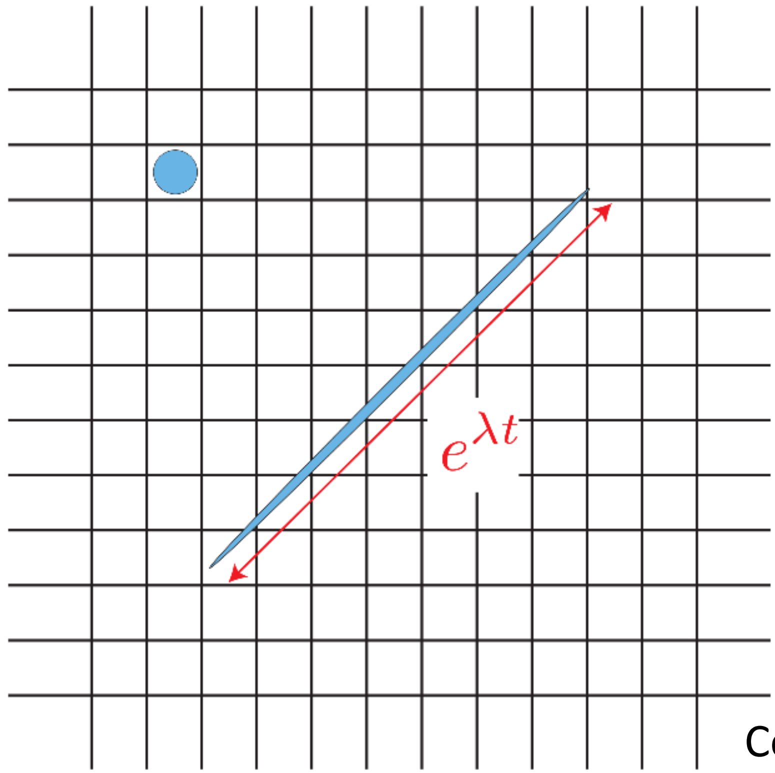
The spectral form factor $g(\beta,t) = \frac{\langle |Z(\beta,t)|^2 \rangle_J}{\langle Z(\beta) \rangle_J^{\ 2}}$

1611.04650

$$Z(\beta,t) = Z(\beta + \mathrm{i}t) = \mathrm{Tr}\left(\mathrm{e}^{-\beta\hat{H} - \mathrm{i}\hat{H}t}\right)$$



Lyapunov growth of phase space



Coarse-grained

- Just one direction?
- If more than one, what are relations between λ?

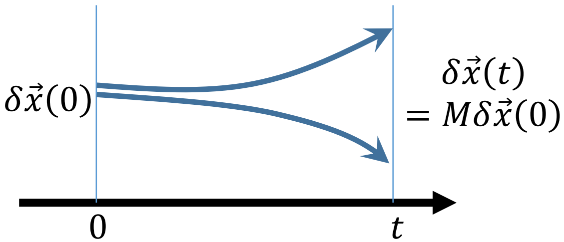
1809.01671

Quantum Lyapunov spectrum

Finite-time **classical Lyapunov spectrum**: obeys RMT statistics for chaos

[Hanada, Shimada, and MT: PRE **97**, 022224 (2018)]

Singular values of $M_{ij} = \left(\frac{\partial x_i(t)}{\partial x_j(0)}\right)$ **at finite *t***: $\{s_k(t)\} = \{e^{\lambda_k t}\}$



$L = \{x_i(t), p_j(0)\}_{\mathrm{PB}}^{\ 2} = \left(\frac{\partial x_i(t)}{\partial x_j(0)}\right)^2 \to e^{2\lambda_{\mathrm{L}} t}$ for large t

OTOC: $C_T(t) = \left\langle \left|\left[\widehat{W}(t), \widehat{V}(t=0)\right]\right|^2 \right\rangle = \langle \widehat{W}^\dagger(t)\widehat{V}^\dagger(0)\widehat{W}(t)\widehat{V}(0)\rangle + \cdots$

Quantum Lyapunov spectrum: Define $\widehat{M}_{ab}(t)$ as (anti)commutator of $\widehat{O}_a(t)$ and $\widehat{O}_b(0)$

$$\widehat{L}_{ab}(t) = \left[\widehat{M}(t)^\dagger \widehat{M}(t)\right]_{ab} = \sum_{j=1}^{N} \widehat{M}_{ja}(t)^\dagger \widehat{M}_{jb}(t)$$

For $N \times N$ matrix $\langle \phi | \widehat{L}_{ab}(t) | \phi \rangle$, obtain singular values $\{s_k(t)\}_{k=1}^{N}$.

The Lyapunov spectrum is defined as $\left\{\lambda_k(t) = \frac{\log s_k(t)}{2t}\right\}$.

Quantum Lyapunov spectrum for SYK model + modification

$$\hat{H} = \sum_{1\le a<b<c<d}^{N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d + i\sum_{1\le a<b}^{N} K_{ab}\hat{\chi}_a\hat{\chi}_b$$

J_{abcd}: s. d. $= \frac{\sqrt{6}}{N^{3/2}}$

K_{ab}: s. d. $= \frac{K}{\sqrt{N}}$

- Define $\hat{L}_{ab}(t) = \sum_{j=1}^{N} \widehat{M}_{ja}(t)\widehat{M}_{jb}(t)$ for time-dependent anticommutator $\widehat{M}_{ab}(t) = \{\hat{\chi}_a(t), \hat{\chi}_b(0)\}$.

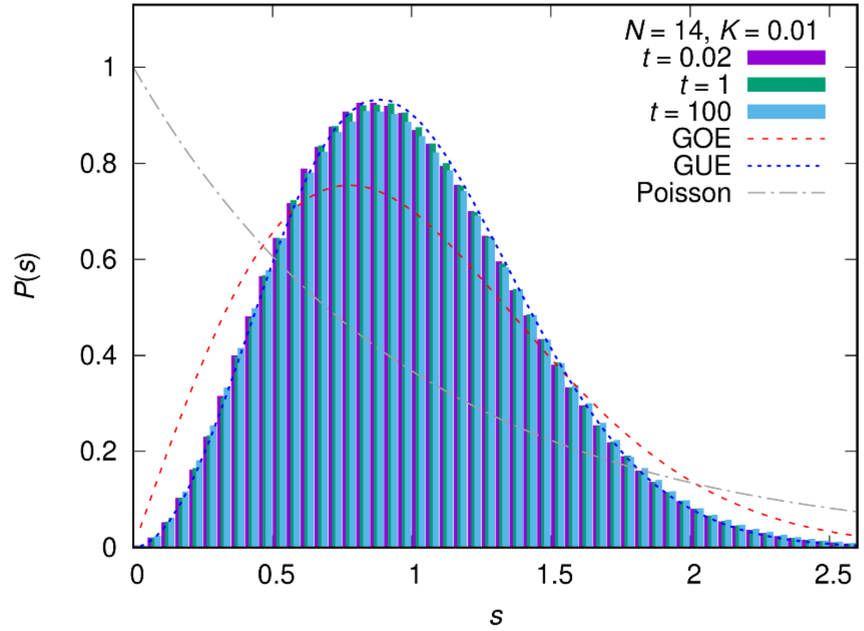
- Obtain the singular values $\{a_k(t)\}_{k=1}^{K}$ of $\langle\phi|\hat{L}_{ab}(t)|\phi\rangle$

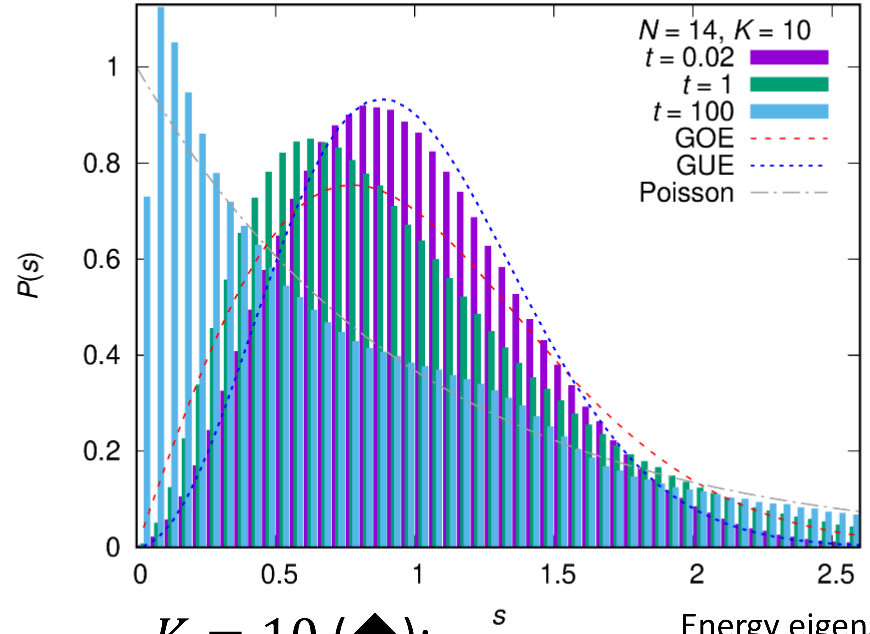
- Quantum Lyapunov spectrum: $\left\{\lambda_k(t) = \frac{\log a_k(t)}{2t}\right\}_{k=1,2,\dots,K}$ (also dependent on state ϕ)

Other possibilities: see Rozenbaum, Ganeshan, and Galitski: PRB **100**, 035112 (2019), Hallam, Morley, and Green: 1806.05204

Spectral statistics of quantum Lyapunov spectrum: SYK

1809.01671

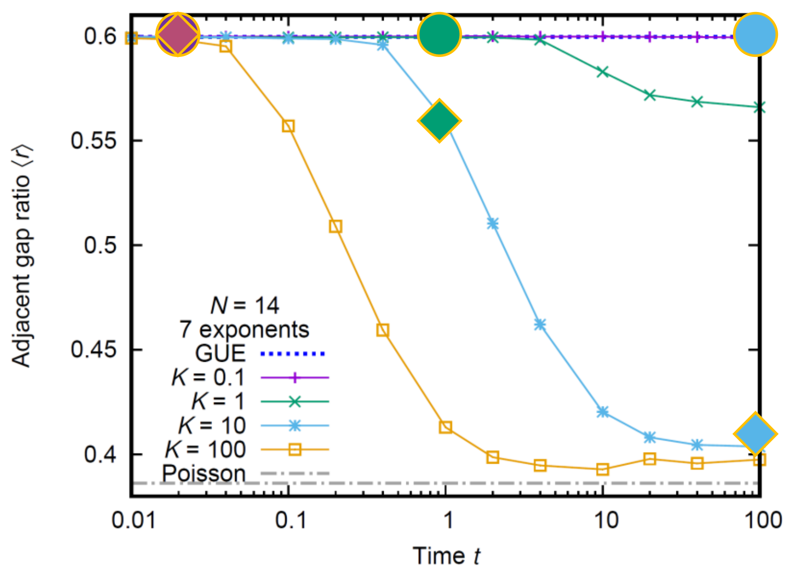




$K = 0.01$ (●): Remains GUE for long time

$K = 10$ (◆): Approaches Poisson

Energy eigenstates
$N/2$ larger exponents



$\langle r \rangle$: average of $\frac{\min(s_i, s_{i+1})}{\max(s_i, s_{i+1})}$

(fixed-i unfolding: unfold each gap $g_i = \lambda_{i+1} - \lambda_i$ using its average $\langle g_i \rangle_J$, $s_i = g_i / \langle g_i \rangle_J$)

QLS: The case of the random field XXZ model

$$\hat{H} = \sum_{i}^{N} \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_{i+1} + \sum_{i}^{N} h_i \widehat{S_i^z}$$ h_i: uniform distribution $[-W, W]$

Many-body localization (MBL) transition at $W = W_c \sim 3.5$

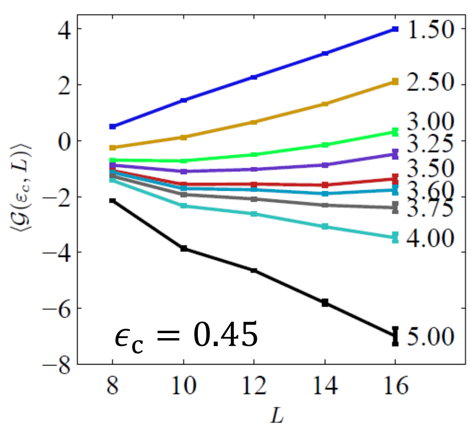
(though recently disputed; e.g. $W_c \geq 5$ proposed in E. V. H. Doggen et al., [1807.05051] using large systems with time-dependent variational principle & machine learning)

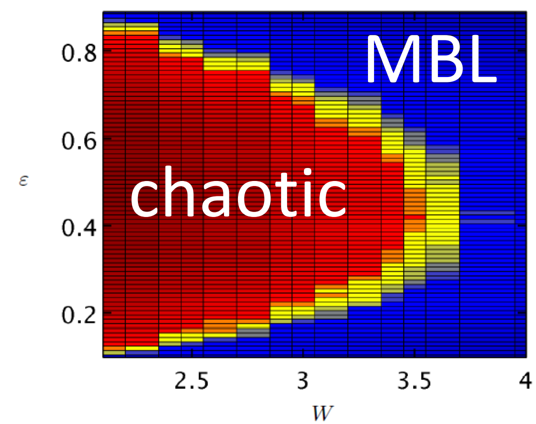
e.g. M. Serbyn, Z. Papic, and D. A. Abanin, Phys. Rev. X **5**, 041047 (2015) (arXiv:1507.01635)

Matrix element of local perturbation

$$\mathcal{G}(\varepsilon, L) = \ln \frac{|V_{n,n+1}|}{E'_{n+1} - E'_n}$$

Energy separation of neighboring energy eigenstates





cf. MBL in short-range SYK [García-García and MT, Phys. Rev. B **99**, 054202 (2019)]; Localization of fermions on quasiperiodic lattice with attractive on-site interaction [Phys. Rev. A **82**, 043613 (2010)]

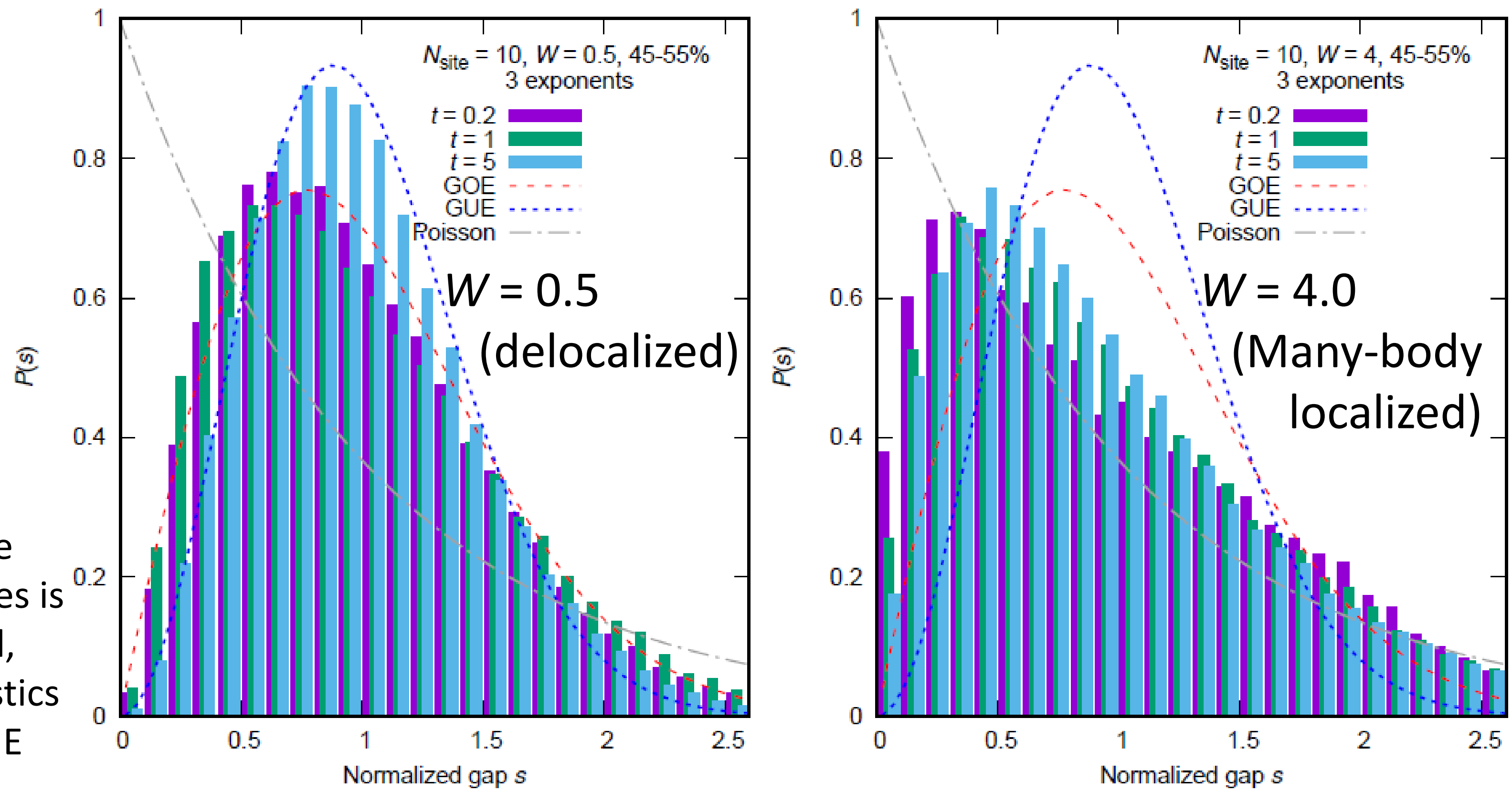
1809.01671

Spectral statistics of QLS for random field XXZ

$$\hat{H} = \sum_i^N \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_{i+1} + \sum_i^N h_i \widehat{S_i^z}$$

h_i: uniform distribution $[-W, W]$

$$\hat{M}_{ab}(t) = [\widehat{S_a^+}(t), \widehat{S_b^-}(0)]$$



- Exponential growth of the singular values is not observed, but the statistics approach GUE

Quantum Lyapunov spectrum distinguishes chaotic and non-chaotic phases

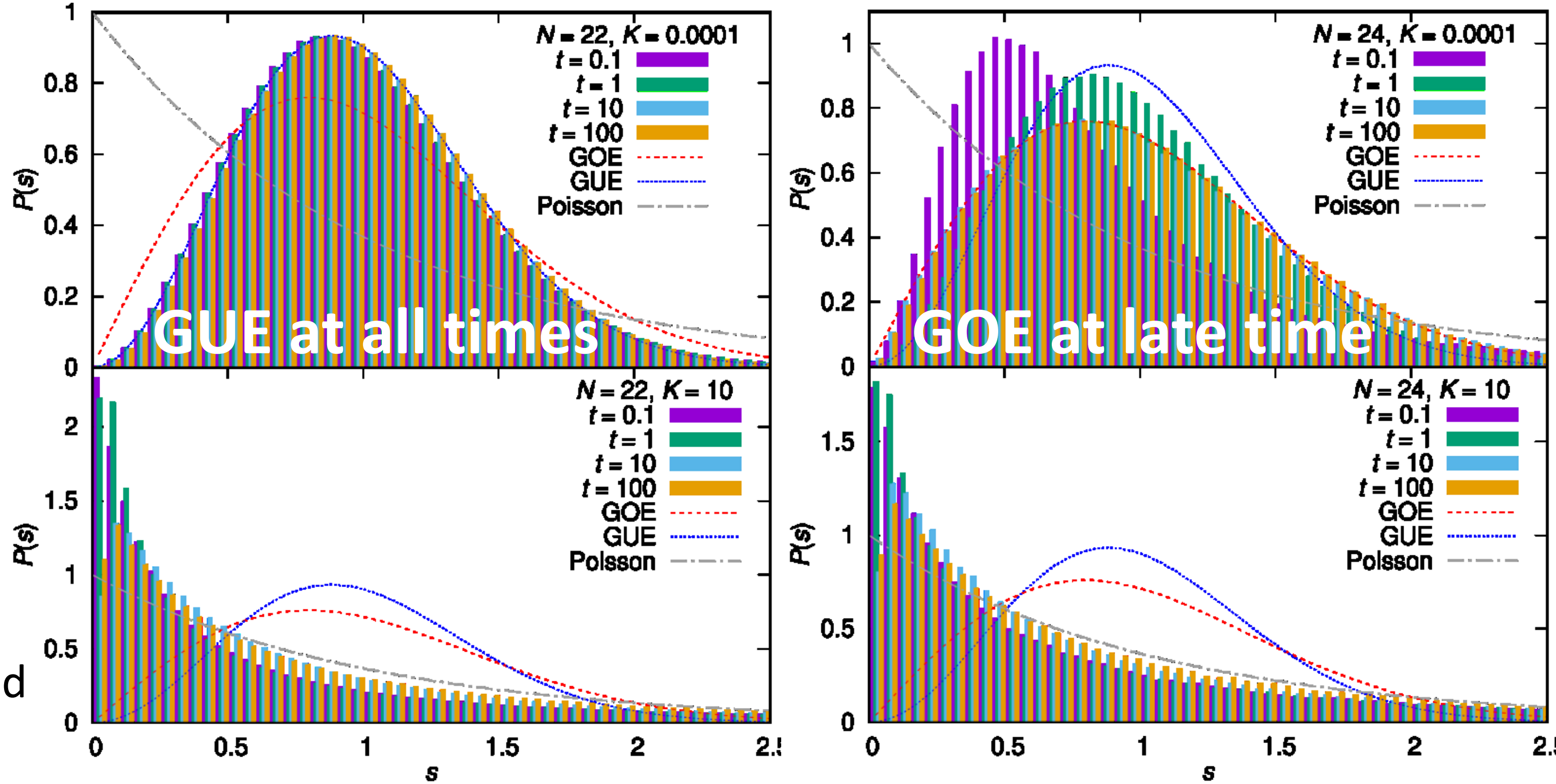
Singular value statistics of **two-point time correlators**

$$G_{ab}^{(\phi)}(t) = \langle \phi | \hat{\chi}_a(t) \hat{\chi}_b(0) | \phi \rangle \text{ as a matrix}$$

$$\lambda_j(t) = \log\left[\text{singular values of}\left(G_{ab}^{(\phi)}(t)\right)\right]$$

SYK$_4$

Strongly perturbed



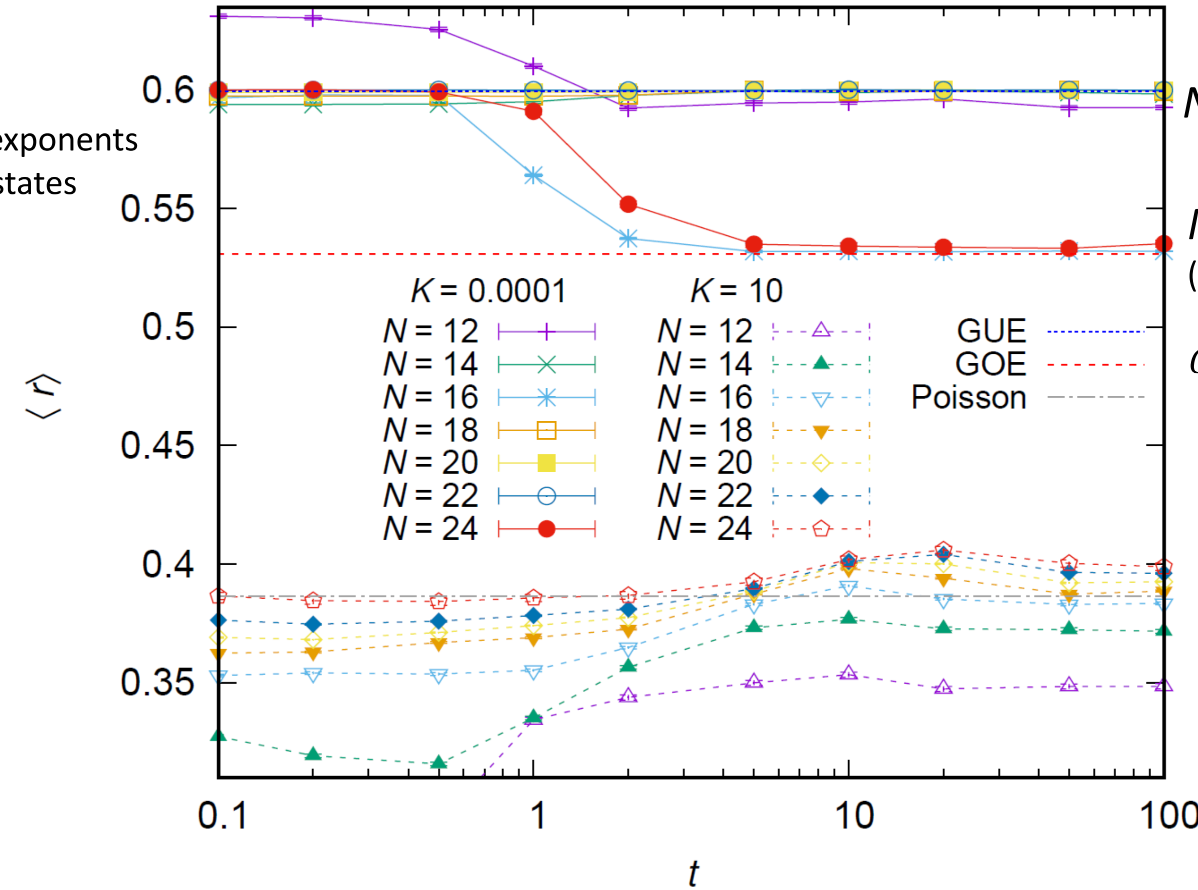
1902.11086

$\langle r \rangle$: average of the adjacent gap ratio $\frac{\min(\lambda_{i+1}-\lambda_i\,,\lambda_{i+2}-\lambda_{i+1})}{\max(\lambda_{i+1}-\lambda_i\,,\lambda_{i+2}-\lambda_{i+1})}$

Uncorrelated (Poisson): $2\log 2 - 1 \approx 0.386$

Correlated: larger (GOE: 0.5307, GUE: 0.5996 etc.) [Atas *et al.*, PRL 2013]

SYK, larger $N/2$ exponents
ϕ: energy eigenstates
fixed-i unfolded



N mod 8 = 2, 4, 6: GUE

N mod 8 = 0: GOE
(the matrix is symmetric)

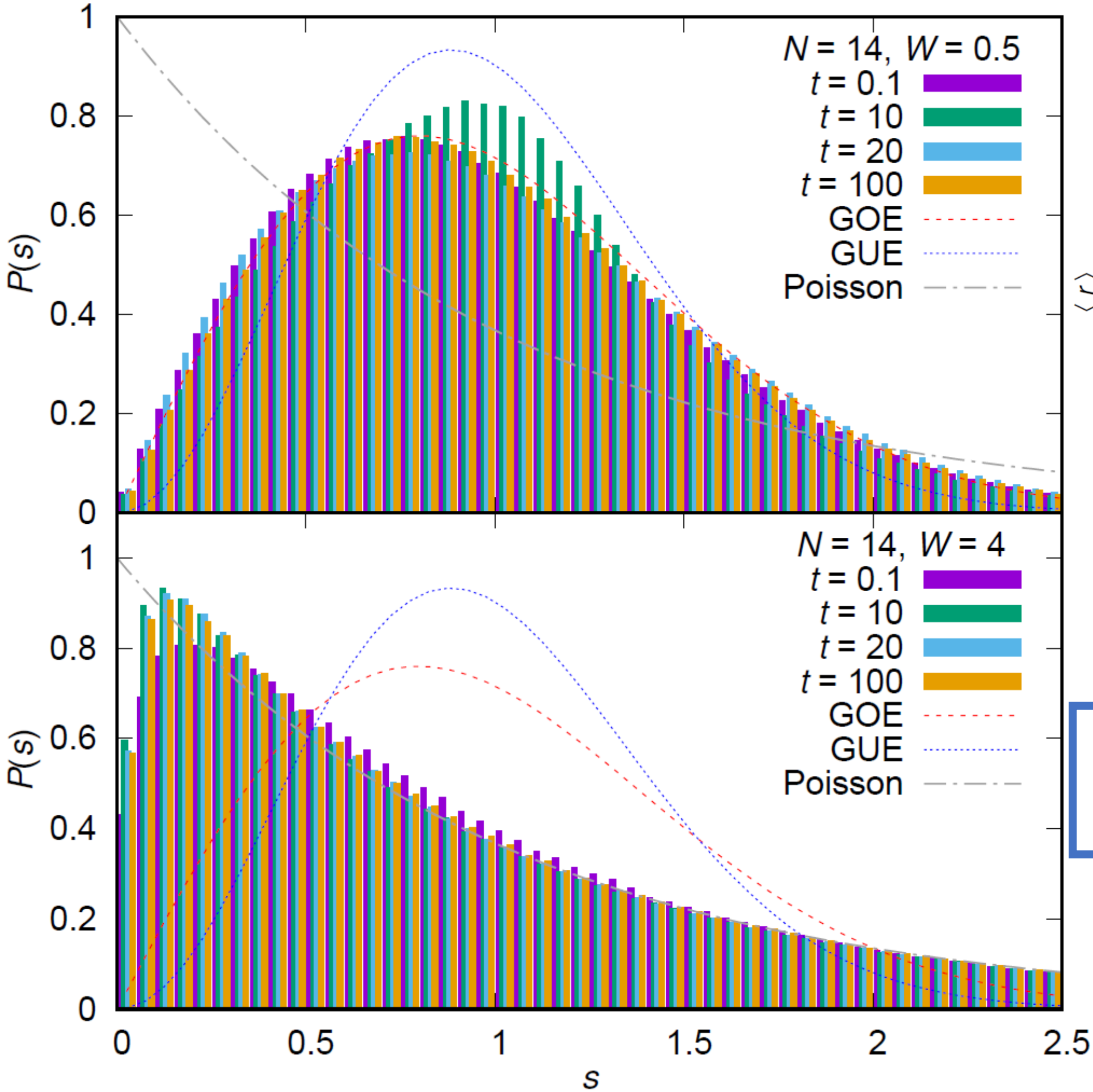
$G_{ab}^{(\phi)} = \langle\phi|\hat{\chi}_a(t)\hat{\chi}_b(0)|\phi\rangle$

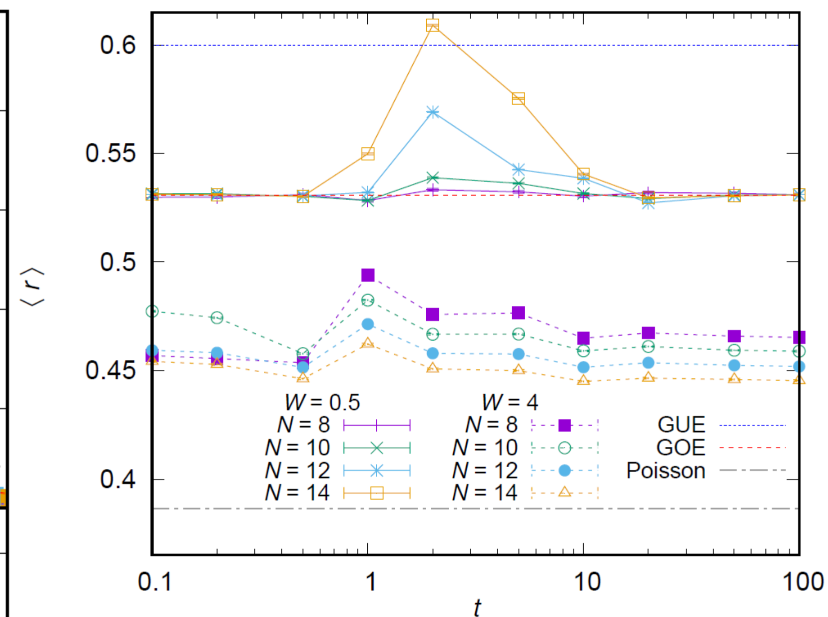
At late time, for two-point correlator singular values,
Random matrix behavior ⇔ chaotic

Two-point time correlator: XXZ model

$$\hat{H} = \sum_{i}^{N} \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_{i+1} + \sum_{i}^{N} h_i \widehat{S_i^z} \qquad G_{ij}^{(\phi)}(t) = \langle \phi | \sigma_{+,i}(t) \sigma_{-,j}(0) | \phi \rangle$$

h_i: uniform distribution $[-W, W]$





Random matrix behavior ⇔ chaotic for both early time and late time

Summary

✓The Sachdev-Ye-Kitaev model

✓Large-*N* solvability: conformal symmetry and maximal chaos

✓Deformation and suppression of maximal chaos 1707.02197

➢ $\hat{H} = \sum_{1\le a<b<c<d}^{N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d + i\sum_{1\le a<b}^{N} K_{ab}\hat{\chi}_a\hat{\chi}_b$

✓Characterization of chaos in random systems 1702.06935

✓Quantum Lyapunov spectrum 1809.01671

➢ $\langle\phi|\sum_{j=1}^{N}\hat{M}_{ja}(t)^\dagger\hat{M}_{jb}(t)\,|\phi\rangle$, $\hat{M}_{ab}(t) = \{\hat{\chi}_a(t), \hat{\chi}_b(0)\}$

✓Singular value statistics of two-point correlators 1902.11086

➢ $G_{ab}^{(\phi)}(t) = \langle\phi|\hat{\chi}_a(t)\hat{\chi}_b(0)|\phi\rangle$

Other related works: 1801.03204, 1812.04770