New dynamic critical phenomena of nonrelativistic photons in QCD

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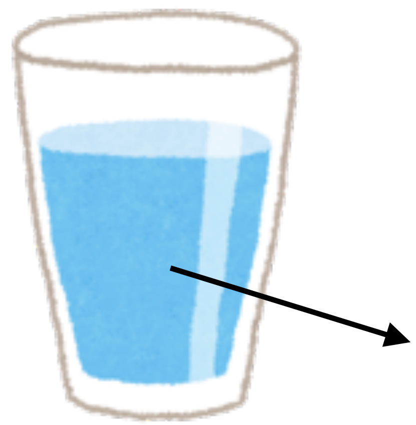
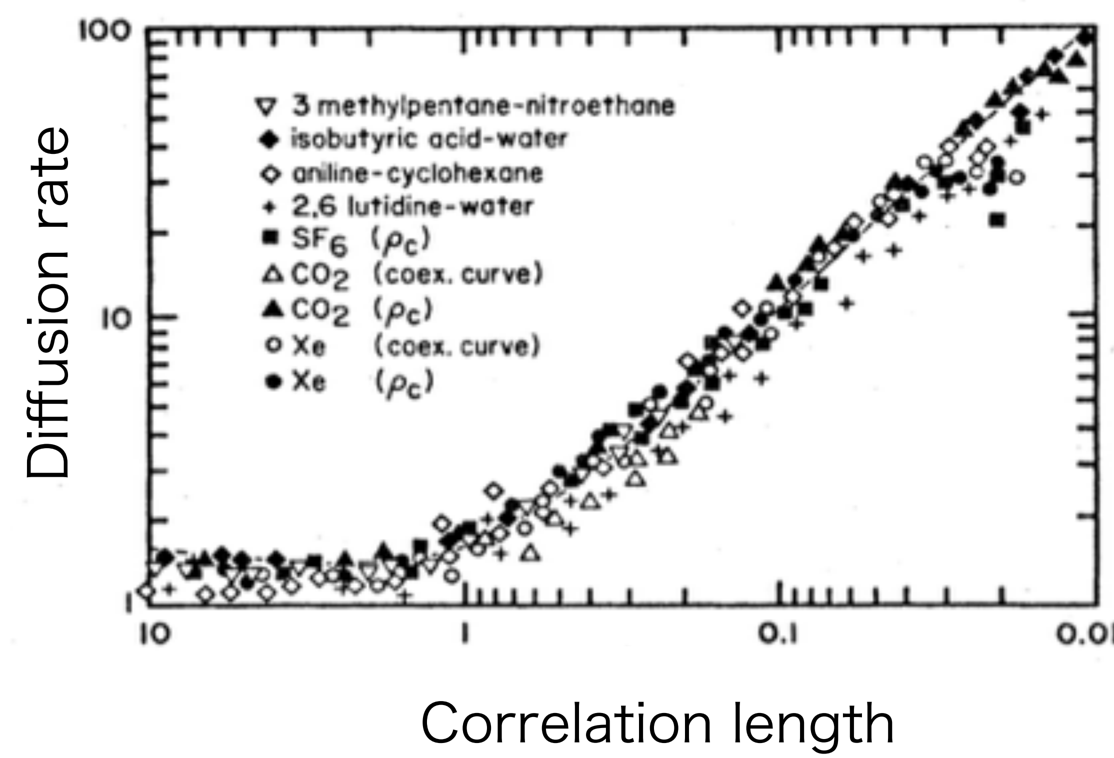
熱場の量子論 at YITP on September 4th, 2019

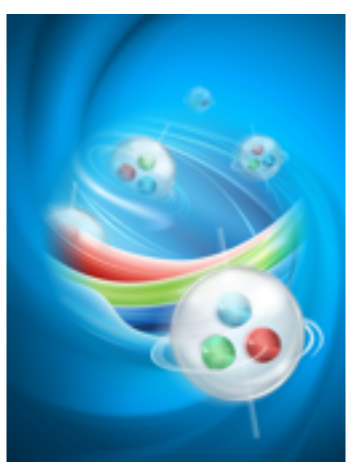
in a collaboration with Naoki Yamamoto (Keio University)

Critical phenomena





Swinney and Henry (1973)

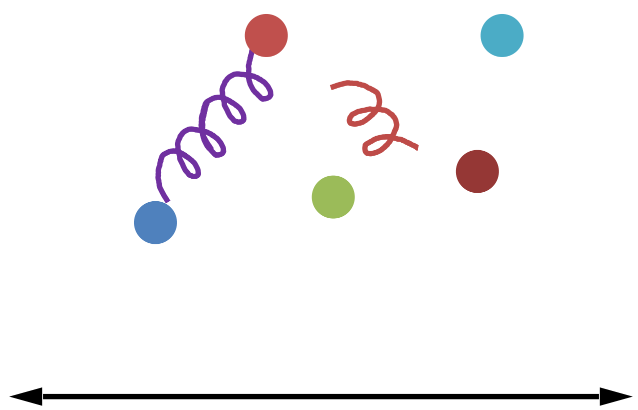


from Brookhaven National Laboratory

Dynamic universality class

P. C. Hohenberg and B. I. Halperin (1977)

Microscopic theory



$\xi \gg \Lambda^{-1}$

Dynamic universality class

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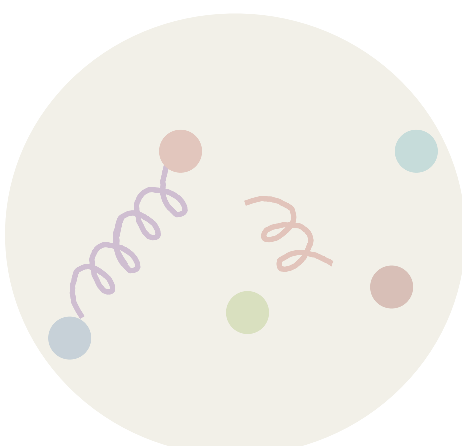


$\xi \gg \Lambda^{-1}$

Integrating out

Effective theory

Hydrodynamic variables:

- Order parameters
- Conserved charge densities
- Nambu-Goldstone modes

Same Symmetries

Dynamic universality class

P. C. Hohenberg and B. I. Halperin (1977)

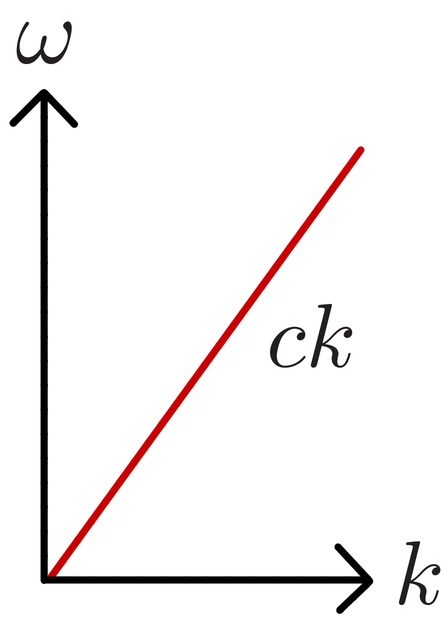
Hohenberg and Halperin: Theory of dynamic critical phenomena

TABLE I. Some dynamical models treated by renormalization-group methods.

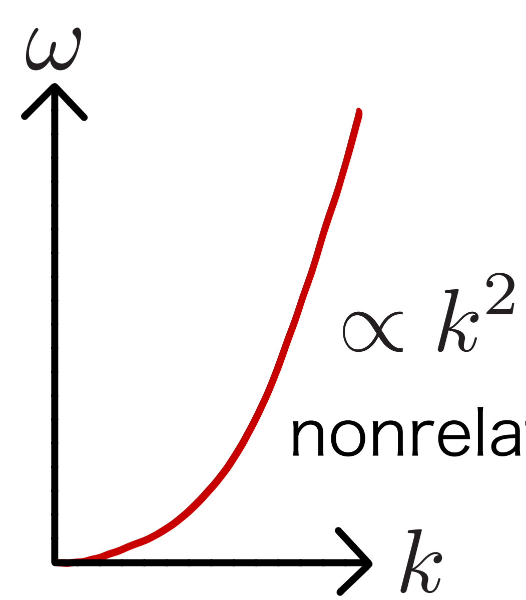
Model	Designation	System	Dimension order of parameter	Non-conserved fields	Conserved fields	Non-vanishing Poisson bracket
Relaxational	A	Kinetic Ising anisotropic magnets	n	ψ	None	None
	B	Kinetic Ising uniaxial ferromagnet	n	None	ψ	None
	C	Anisotropic magnets structural transition	n	ψ	m	None
Fluid	H	Gas–liquid binary fluid	1	None	ψ $\mathbf{j}$	$\{\psi, \mathbf{j}\}$
Symmetric planar magnet	E	Easy-plane magnet, $h_z = 0$	2	ψ	m	$\{\psi, m\}$
Asymmetric planar magnet	F	Easy-plane magnet, $h_z \neq 0$ superfluid helium	2	ψ	m	$\{\psi, m\}$
Isotropic antiferromagnet	G	Heisenberg antiferromagnet	3	ψ	$\mathbf{m}$	$\{\psi, \mathbf{m}\}$
Isotropic ferromagnet	J	Heisenberg ferromagnet	3	None	ψ	$\{\psi, \psi\}$

Gapless modes are important.

Electromagnetic waves



Maxwell equations in vacuum



nonrelativistic photon

Today's talk

Chirality and Quantum anomaly

- Chirality

right-handed: Q_{R} left-handed: Q_{L}

$$Q_5 \equiv Q_{\mathrm{R}} - Q_{\mathrm{L}} \quad \text{chirality charge}$$

$$Q \equiv Q_{\mathrm{R}} + Q_{\mathrm{L}} \quad \text{total charge}$$

- Quantum anomaly

$$\partial_t n_5 + \boldsymbol{\nabla} \cdot \boldsymbol{j}_5 = C \boldsymbol{E} \cdot \boldsymbol{B}$$

Outline

- Setup
- Static critical phenomena
- Dynamic critical phenomena
- Conclusion

Setup

- Massless two-flavor QCD at finite T and $\boldsymbol{B}_{\mathrm{ex}}$
- Chiral symmetry:

$\boldsymbol{B}_{\mathrm{ex}} = 0$		$\boldsymbol{B}_{\mathrm{ex}} \neq 0$		ground state
$\mathrm{SU(2)_L \times SU(2)_R}$	$\longrightarrow$	$\mathrm{U(1)}^{\tau^3} \times \mathrm{U(1)}_5^{\tau^3}$	$\longrightarrow$	$\mathrm{U(1)}^{\tau^3} \times \cancel{\mathrm{U(1)}_5^{\tau^3}}$
$q = \begin{pmatrix} u \\ d \end{pmatrix}$ "isospin"		$q \to \mathrm{e}^{\mathrm{i}\alpha\tau^3}\mathrm{e}^{\mathrm{i}\alpha_5\gamma_5\tau^3} q$		$\langle \bar{q} q \rangle \neq 0$

- Second-order chiral phase transition

Hydrodynamic variables

- (Complex) order parameter: Φ
- Chirality charge density: n_5
- Electric field: $\boldsymbol{E}$
- Magnetic field: $\boldsymbol{B} = \boldsymbol{B}_{\rm ex} + \delta\boldsymbol{B}$

Ginzburg-Landau Theory

$$F = \int_{\mathrm{V}} \left(\frac{r}{2}|\Phi|^2 + \frac{u}{4}|\Phi|^4 + \frac{a}{2}|\boldsymbol{\nabla}\Phi|^2 + \frac{1}{2}n_5^2 + \frac{1}{2}\boldsymbol{E}^2 + \frac{1}{2}\boldsymbol{B}^2 \right)$$

- Near second-order phase transition ⟶ small order parameters
- Long-range behavior ⟶ derivative expansion
- QCD symmetries ⟶ constraints on the expansion

chiral symmetry and CPT symmetries

Ginzburg-Landau Theory

$$F = \int_{\mathrm{V}} \left(\frac{r}{2}|\Phi|^2 + \frac{u}{4}|\Phi|^4 + \frac{a}{2}|\boldsymbol{\nabla}\Phi|^2 + \frac{1}{2}n_5^2 + \frac{1}{2}\boldsymbol{E}^2 + \frac{1}{2}\boldsymbol{B}^2 \right)$$

$$h_0 \equiv \langle|\Phi|\rangle \sim \tau^{\beta}, \quad a \sim \xi^{\eta}, \quad \xi \sim \tau^{-\nu} \quad \left(\tau \equiv \frac{T - T_{\mathrm{c}}}{T_{\mathrm{c}}}\right)$$

Wilson-Fisher fixed point (2-components)

Dynamics

- Preliminary

$$\Phi = (h + h_0)\mathrm{e}^{\mathrm{i}\pi^0/h_0}$$

Nambu-Goldstone mode

amplitude mode

(linearly decoupled)

Langevin theory 1/2

- π^0 and n_5

$$\partial_t \pi^0 = -\kappa \frac{\delta F}{\delta \pi^0} + \int_V \left[\pi^0, n_5\right] \frac{\delta F}{\delta n_5} + \xi_{\pi^0}$$

$$\partial_t n_5 = \lambda \boldsymbol{\nabla}^2 \frac{\delta F}{\delta n_5} + \int_V \left[n_5, \pi^0\right] \frac{\delta F}{\delta \pi^0} + \int_V \left[n_5, E^i\right] \frac{\delta F}{\delta E^i} + \xi_{n_5}$$

$$\left[\pi^0(t, \boldsymbol{r}), n_5(t, \boldsymbol{r}')\right] = h_0 \delta(\boldsymbol{r} - \boldsymbol{r}')\delta(t - t')$$

$$\left[n_5(t, \boldsymbol{r}), E^i(t, \boldsymbol{r}')\right] = C B_i \delta(\boldsymbol{r} - \boldsymbol{r}')\delta(t - t')$$

Anomalous commutation relation

S. Adler and D. Boulware (1969)

Langevin theory 2/2

- Maxwell equations:

$$\partial_t \boldsymbol{E} = \boldsymbol{\nabla} \times \boldsymbol{B} - \boldsymbol{j}, \quad \partial_t \boldsymbol{B} = -\boldsymbol{\nabla} \times \boldsymbol{E}$$

- Current term:

$$\boldsymbol{j} = \sigma \boldsymbol{E} + C n_5 \boldsymbol{B} + \frac{C}{h_0} \boldsymbol{\nabla} \pi^0 \times \boldsymbol{E} + \boldsymbol{\xi}$$

Chiral magnetic effect

$$\langle \xi_i(t, \boldsymbol{r}) \xi_j(t, \boldsymbol{r}') \rangle = 2\sigma_{ij} \delta(\boldsymbol{r} - \boldsymbol{r}') \delta(t - t')$$

Linear analysis

- Linearize: $\boldsymbol{B} = \boldsymbol{B}_{\mathrm{ex}} + \delta \boldsymbol{B}$ etc.
- Hydrodynamic (gapless) modes:
  - 1 diffusive mode
  - nonrelativistic photon:

$$\boxed{\omega = \pm\sqrt{\frac{v^2}{C^2 B_{\mathrm{ex}}^2} - D^2 \boldsymbol{k}_\perp^2} - \mathrm{i} D \boldsymbol{k}_\perp^2} \qquad (\boldsymbol{k}_\perp \perp \boldsymbol{B}_{\mathrm{ex}})$$

$$v^2 \equiv a h_0^2, \qquad D \equiv \frac{\kappa}{2} + \frac{\sigma v^2}{2 C^2 B_{\mathrm{ex}}^2}$$

Dynamic scaling hypotheses

- $\omega \sim \xi^{-z}$ for all terms

$$\omega = \pm\sqrt{\frac{v^2}{C^2 B_{\mathrm{ex}}^2} - D^2 \boldsymbol{k}_\perp^2} - \mathrm{i} D \boldsymbol{k}_\perp^2$$

$$D \sim \frac{v}{C B_{\mathrm{ex}}} \sim \xi^{-z+2}$$

- From statics: $v^2 \equiv a h_0^2 \sim \xi^{d-2}$
- Dynamic critical exponent:

$$\boxed{z = \frac{d}{2} + 1}$$

New dynamic universality class
beyond Hohenberg and Halperin's classification

Conclusion

- Dynamic critical phenomena of the second-order chiral phase transition under external magnetic field with dynamical electromagnetic fields
- Hydrodynamic modes: Nonrelativistic photon
- New dynamic universality class beyond conventional classification