共鳴相互作用する1次元量子系の普遍性

関野裕太
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基研研究会「熱場の量子論とその応用」
Monday, September 2nd, 2019
@Yukawa Institute for Theoretical Physics, Kyoto University

Collaborators

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Tokyo Tech

Prof. Shina Tan
Peking Univ./Georgia Tech

Y. Sekino, S. Tan, & Y. Nishida, Phys. Rev. A **97**, 013621 (2018)

Y. Sekino, & Y. Nishida, in progress

Outline of this talk

1. Introduction:

   Universality near resonance & properties in 1D

2. Universal relations for 1D bosons & fermions near 2-body resonances

   Y. Sekino, S. Tan, & Y. Nishida, Phys. Rev. A **97**, 013621 (2018)

3. Summary

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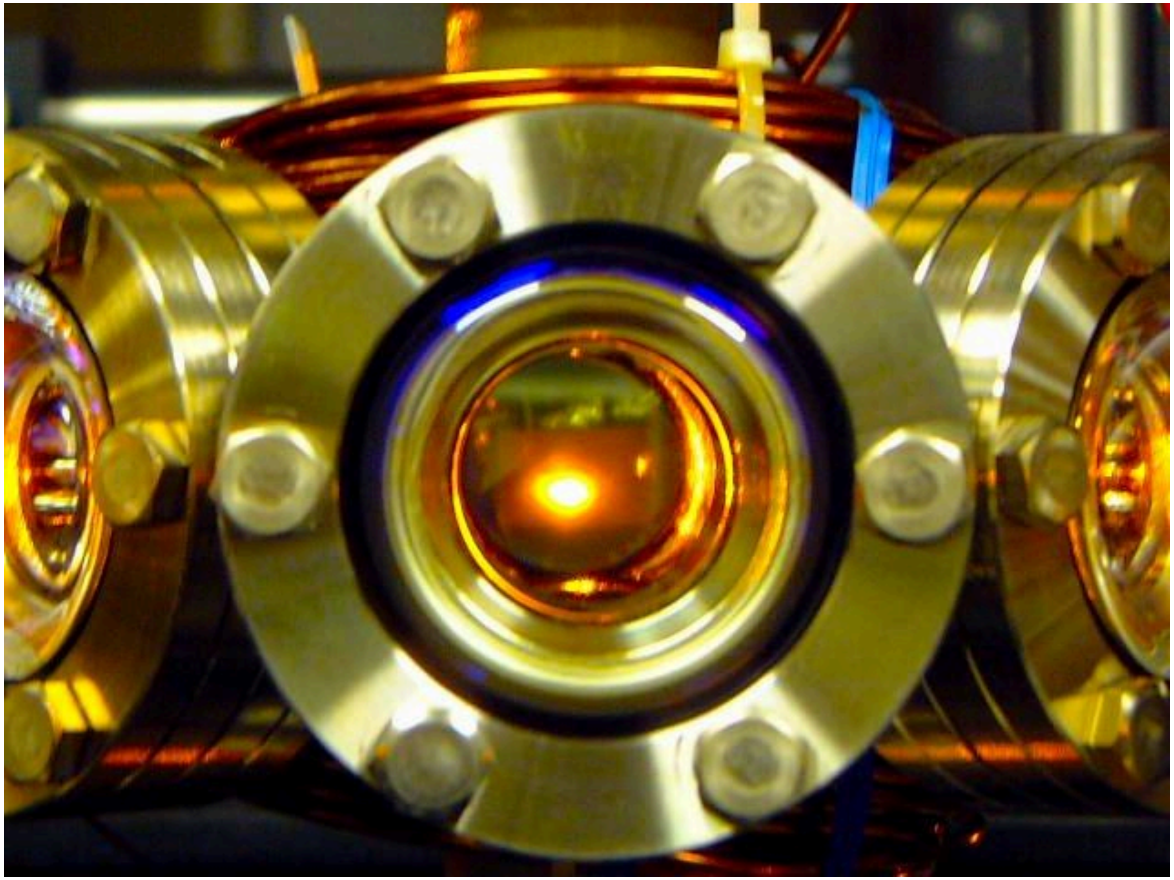
3. Summary

Ultracold atoms

Coldest systems in the universe !!

T = 1 μK~ 10 nK

Very pure & dilute



High controllability of ultracold atoms

- **Statistics & internal d.o.f** by atomic species and isotopes

Bosons (^{7}Li, ^{23}Na, ^{39}K, ^{41}K,⋯) Fermions (^{6}Li, ^{40}K, ^{173}Yb, ⋯)

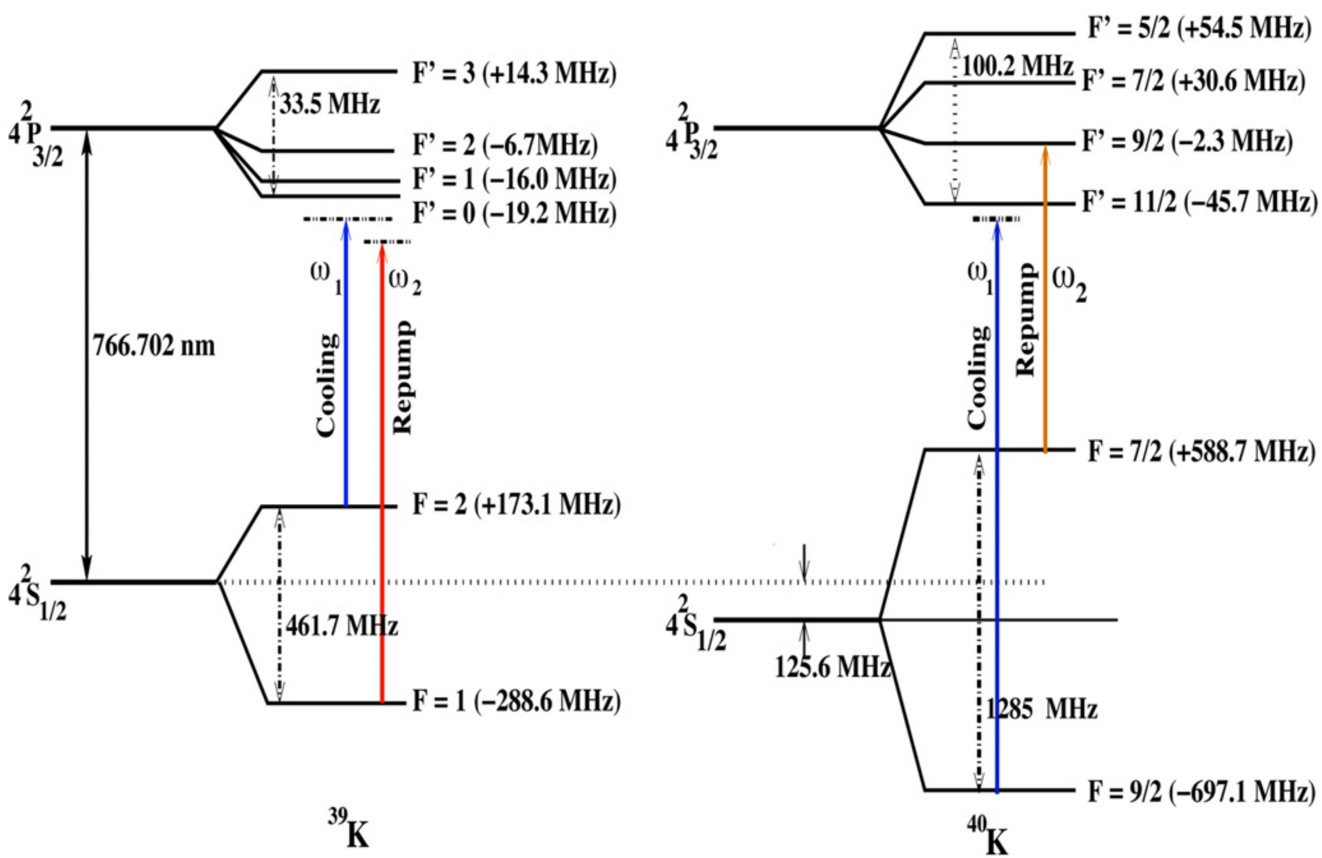


Figure 1. Hyperfine energy level diagram of D_2 transition in ^{39}K and ^{40}K.

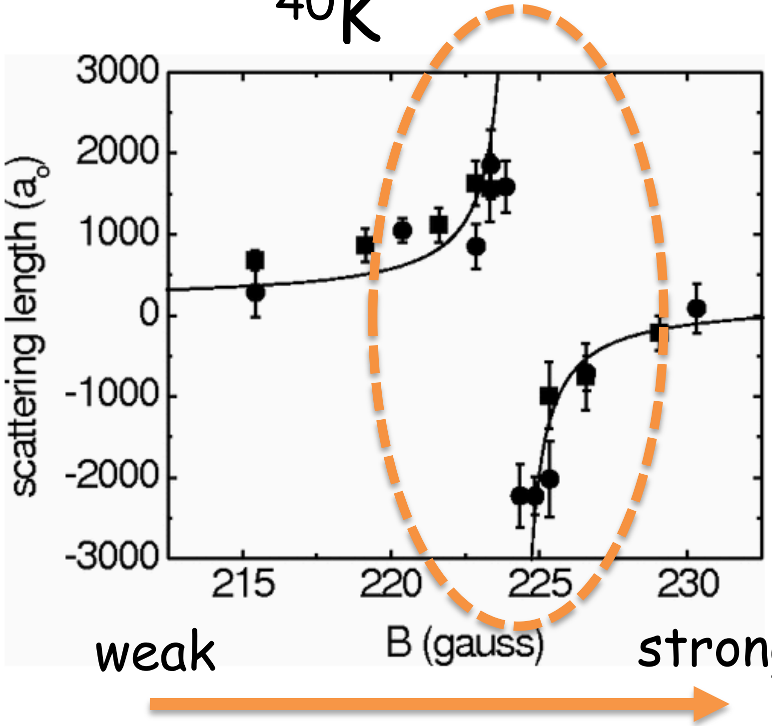
Gokhroo, et.al., (2011)

High controllability of ultracold atoms

- **Statistics & internal d.o.f** by atomic species and isotopes
- **Control of parameters**
- **Spatial geometry**

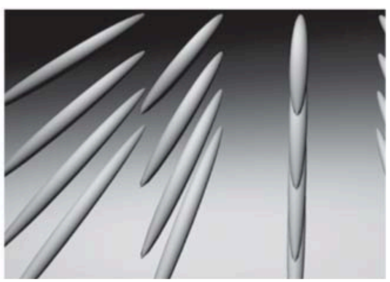
Feshbach resonance → Interaction

^{40}K

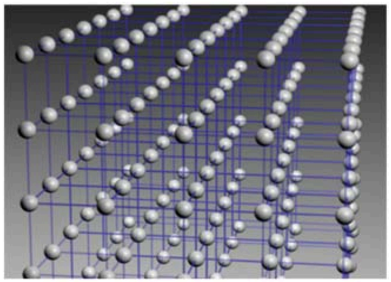


Regol and Jin, PRL (2003)

Array of 1D gases



Cubic lattice

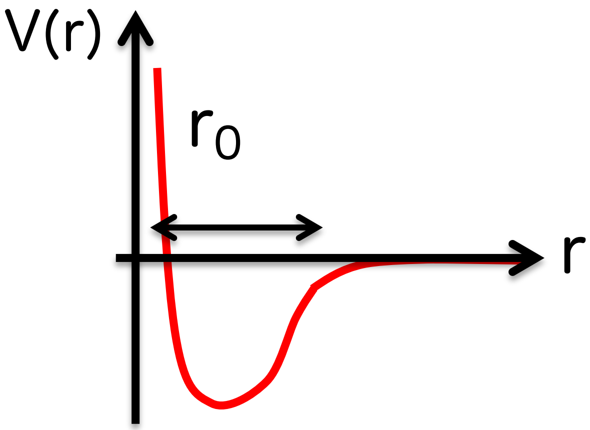


Bloch Nat. Phys. (2005)

Highly controllable systems

Ideal ground to study **universal physics near resonance**

Universalities near resonances



Resonant regime

$r_0 \ll \lambda_T, l_{mean}, |a|$ $\quad (\lambda_T \sim T^{-1/2}, l_{mean} \sim n^{-1/d})$

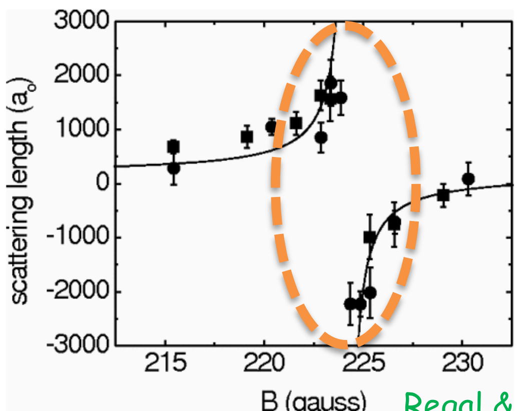
$r_0 \to 0$

Independent of details of V(r)

Universal !!

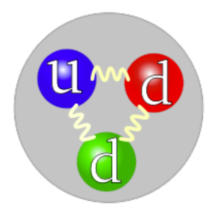
E.g. S-wave resonance in 3D

✓ Ultracold atoms



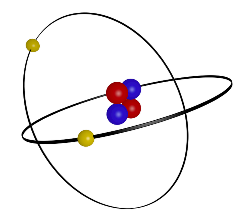
Regal & Jin, PRL (2003)

✓ Neutrons



$|a|/r_0 \approx 18$

✓ ^{4}He atoms



$|a|/r_0 \approx 20$

→ Unitary Fermi gas, Efimov effect, …

1D systems near resonances

Bosons near an **s-wave** resonance

3D gases

Fermions near a **p-wave** resonance

Confined to 1D

Olshanii, PRL (1998)

Granger & Blume, PRL (2004).

1D bosons near an **even-wave** resonance

1D fermions near an **odd-wave** resonance

Bose-**Fermi** correspondence

$$V_B(x) = -\frac{2}{m a_B}\delta(x)$$

$$V_F(x) = -\frac{2a_F}{m}\delta'(x)\tilde{D}_x$$

$$(\hbar = 1)$$

Lieb & Liniger, Phys. Rev. (1963).

Girardeau & Olshanii, PRA (2004).

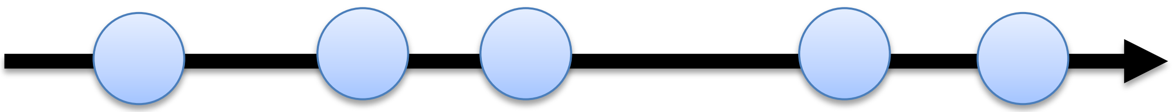
Regularized differential : $\tilde{D}_x = \lim_{x \to +0}\left(\frac{\partial}{\partial x}\bullet\right)$

Bose-Fermi correspondence (1)

Hard-core bosons | **Free fermions**

$$V_B(x) = -\frac{2}{m a_B}\delta(x) \qquad \Longleftrightarrow \qquad V_F(x) = 0$$

$$\to +\infty$$



Bose-Fermi mapping

$$\Psi_B(x_1, \cdots, x_N) = A\Psi_F(x_1, \cdots, x_N) \quad A = \prod_{i<j} \mathrm{sgn}(x_{ij})$$

$$E_B = E_F = E$$

Girardeau, J. Math. Phys. (1960),

Correspondences of

$$Z_B = Z_F = Z = \sum_E e^{-E/k_B T}$$

Thermodynamics

$$|\Psi_B|^2 = |\Psi_F|^2 = |\Psi|^2$$

Density correlations in cord. space

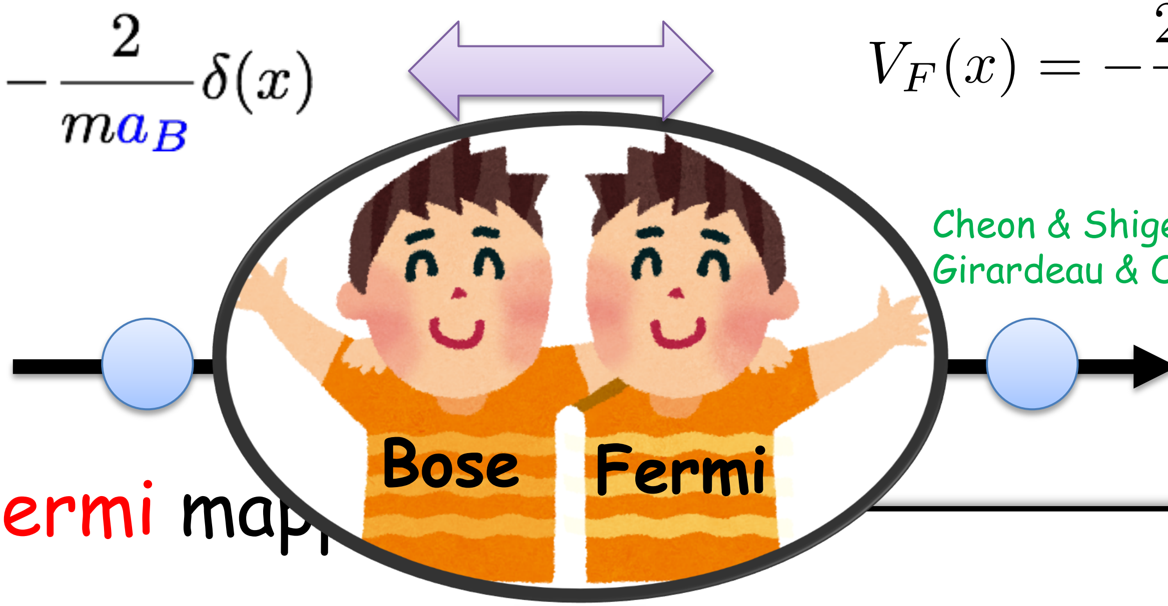
Bose-Fermi correspondence (2)

Bosons $a_B = a_F = a$ **Fermions**

$$V_B(x) = -\frac{2}{m a_B}\delta(x)$$

$$V_F(x) = -\frac{2a_F}{m}\delta'(x)\tilde{D}_x$$

Cheon & Shigehara, PRL (1999);
Girardeau & Olshanii, PRA (2004).



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$$|\Psi_B|^2 = |\Psi_F|^2$$

Density correlations in cord. space

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3. Summary

Purpose of our study

1D bosons near an **even-wave** resonance ⟺ **1D fermions** near an **odd-wave** resonance

How does similarity and difference appear **in universal properties** b/w bosons & fermions ??

$$r_0 \ll \lambda_T,\ n^{-1},\ |a|$$

Universal relations

- ✓ Exact constraints for any (a, T, n)
- ✓ Characterized by quantities called "contact(s)"

Tan, Ann. Phys. (2008).

Universal relations for bosons

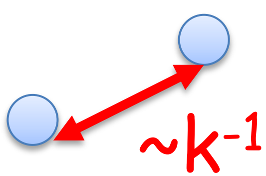
✓ Tail of momentum distribution (MD):

Olshanii & Dunjko, PRL (2003).

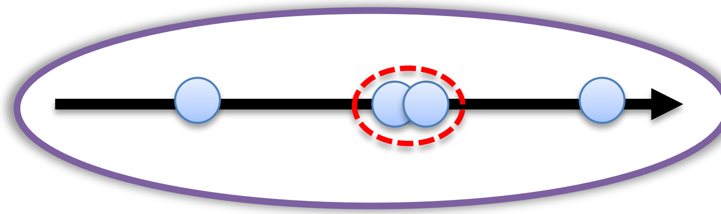
$$\rho_B(k) = \left\langle \tilde{\psi}_k^\dagger \tilde{\psi}_k \right\rangle = \frac{4C_2}{a^2 k^4} + O(1/k^5)$$

$r_0 \ll k^{-1} \ll \lambda_T, n^{-1}, |a|$

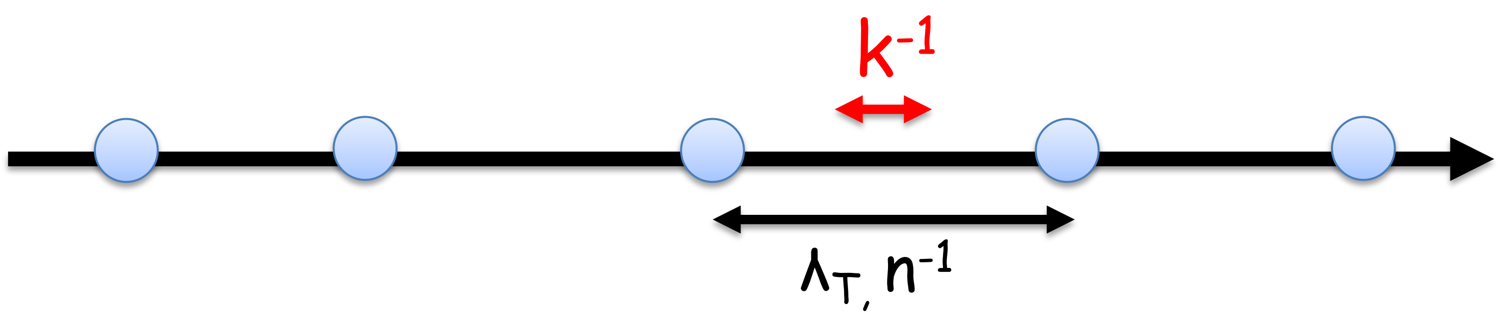
Contribution from
2 particles at short range



Probability of
"2-body contact"



Few-body d.o.f → short-range scale



Universal relations for bosons

✓ Tail of momentum distribution (MD):

Olshanii & Dunjko, PRL (2003).

$$\rho_B(k) = \left\langle \tilde{\psi}_k^\dagger \tilde{\psi}_k \right\rangle = \frac{4C_2}{a^2 k^4} + O(1/k^5)$$

$r_0 << k^{-1} << \lambda_T, n^{-1}, |a|$

✓ Adiabatic relation:

$$\left(\frac{dE}{da^{-1}}\right)_S = -\frac{C_2}{m}$$

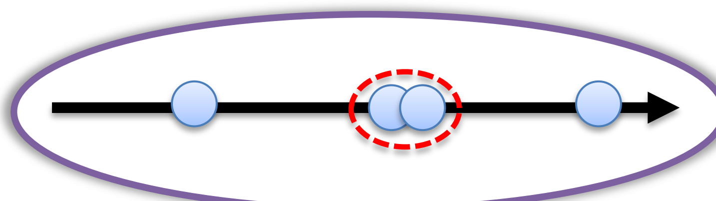
Lieb & Liniger, Phys. Rev. (1963).

✓ Energy relation (ER):

$$E = \int \frac{dk}{2\pi} \frac{k^2}{2m} \rho_B(k) - \frac{C_2}{ma}$$

Valiente, Europhys. Lett. (2012).

2-body contact C_2



Comparison b/w bosons & fermions

✓ Tail of MD ($r_0 \ll k^{-1} \ll \lambda_T, n^{-1}, |a|$):

$$\rho_B(k) = \frac{4C_2}{a^2 k^4} + O(1/k^5) \quad \text{(boson)}$$

$$\rho_F(k) = \frac{4C_2}{k^2} + O(1/k^3) \quad \text{(fermion)}$$

Cui, PRA (2016)

✓ Adiabatic relation: $\left(\frac{dE}{da^{-1}}\right)_S = -\frac{C_2}{m}$ (boson & fermion)

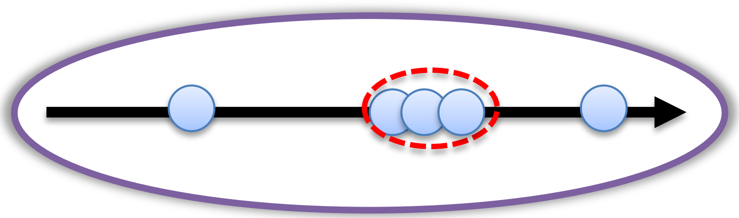
✓ ER:

$$E = \int \frac{dk}{2\pi} \frac{k^2}{2m} \rho_B(k) - \frac{C_2}{ma} \quad \text{(boson)}$$

$$E = \int \frac{dk}{2\pi} \frac{k^2}{2m} \left(\rho_F(k) - \frac{4C_2}{k^2}\right) + \frac{C_2}{ma} + \boxed{\frac{2C_3}{m}} \quad \text{(fermion)}$$

YS, Tan, & Nishida PRA (2018)

3-body contact C_3



RG analysis in QFT !!

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Summary

1D **bosons** near an **even-wave** resonance ⟷ 1D **fermions** near an **odd-wave** resonance

Bose-**Fermi** correspondence

✓ Universal relations are
1. Exact constraint for any parameters
2. Characterized by C_2 & C_3, which are identical b/w bosons & fermions