ゲージ･重力対応による非平衡相転移と対称性の自発的破れ

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Outline

- Introduction
- Non-equilibrium steady state : conducting system
- Non-equilibrium phase transition
- Critical behavior
- Summary

Gauge/Gravity duality

Gravity theory on $AdS_5 \times S^5$ spacetime ⟷ $\mathcal{N}=4$ SYM theory on (3+1) spacetime

Duality

- Adding Flavor : D3-D7 Model

Probe D7-brane is embedded in $AdS_5 \times S^5$ spacetime. This corresponds to $\mathcal{N}=2$ fundamental hyper-multiplet.

$$AdS_5 \times S^5 : \; ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2} + d\Omega_5^2$$

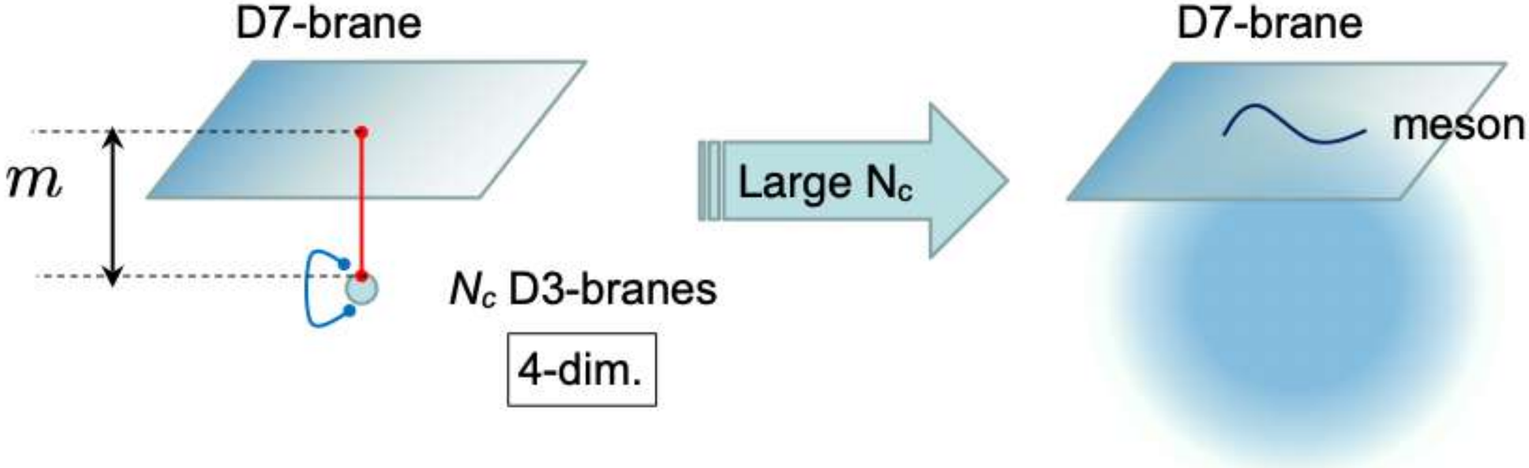
DBI action : $S_{D7} = -T_{D7} \int d^8\xi \sqrt{-\det(g_{ab} + (2\pi\alpha')F_{ab})}$

A. Karch, E. Katz, (2002)

D3-D7 at zero temperature

10-dim. flat spacetime

10-dim. $AdS_5 \times S^5$



N=4 super Yang-Mills + N=2 quark multiplet

A. Karch, E. Katz (2002).

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						
D7	X	X	X	X	X	X	X	X		
	AdS_5					S^3			S^2	

$$d\Omega_5^2 = d\theta^2 + \sin^2\theta d\Omega_3 + \cos^2\theta d\phi^2$$

Embedding function

$$\theta(z) = mz + \theta_2 z^3 + \cdots$$

m : quark mass

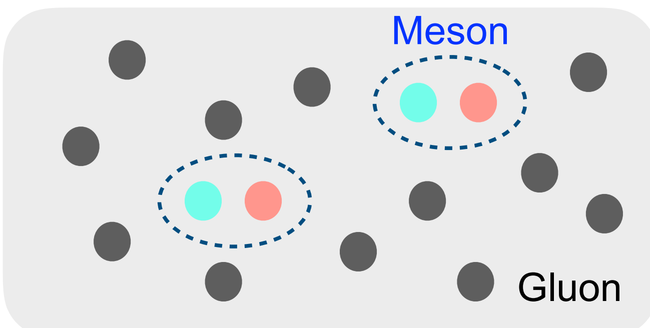
string between D3 and D7-brane ⇔ "quark"
fluctuations of D7-brane ⇔ "meson"

D3-D7 at finite temperature

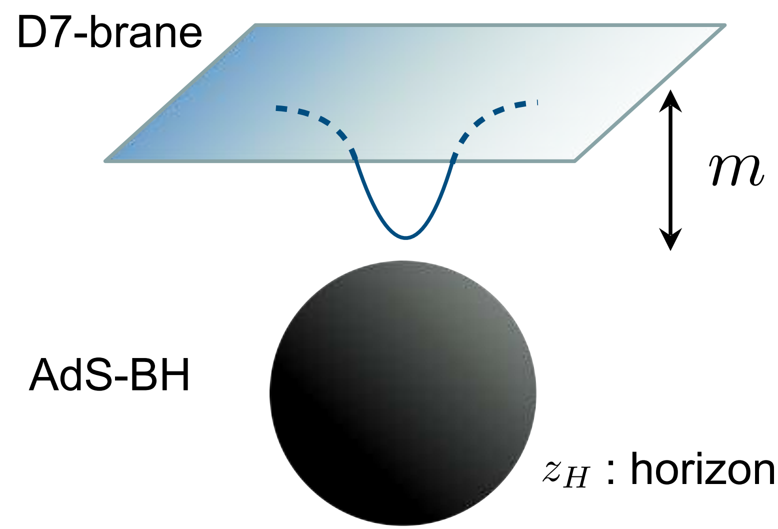
AdS-BH : $ds^2 = -\frac{1}{z^2}\frac{(1-z^4/z_H^4)^2}{1+z^4/z_H^4}dt^2 + \frac{1+z^4/z_H^4}{z^2}d\vec{x}^2 + \frac{dz^2}{z^2}$

DBI Action : $S_{D7} = -T_{D7}\int d^8\xi\sqrt{-\det(g_{ab}+(2\pi\alpha')F_{ab})}$

(3+1)-dim. *N*=4 SYM + *N*=2 quark multiplet ⟷ Duality ⟷ AdS Black hole spacetime



Heat bath $T = \frac{\sqrt{2}}{\pi z_H}$



Quark mass : m

Chiral condensate : $\langle q\bar{q}\rangle = -2\theta_2 + \frac{1}{3}m^3$

D7-brane embedding function

$\theta(z) = mz + \theta_2 z^3 + \cdots$

Conducting system in D3-D7

Gauge field $A_x(t,z) = -Et + a_x(z),$: external electric field

$A_y(x) = Bx$: external magnetic field

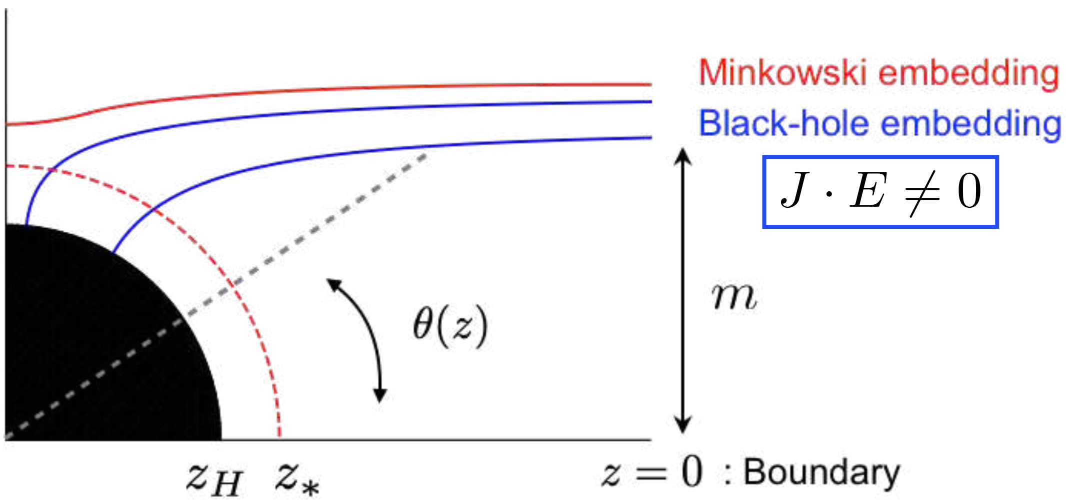
➡ Current density : $J^2 = (2\pi\alpha')^2 |g_{tt}| g_{xx}^2 \cos^6\theta(z_*)$

effective horizon : $z_*^4 = \left(2F(E,B) - \sqrt{(2F(E,B)-1)^2 - 1} - 1\right) z_H^4$

Solve EOM for D7-brane embedding function numerically

⬇

The quark mass is obtained from the asymptotic form.



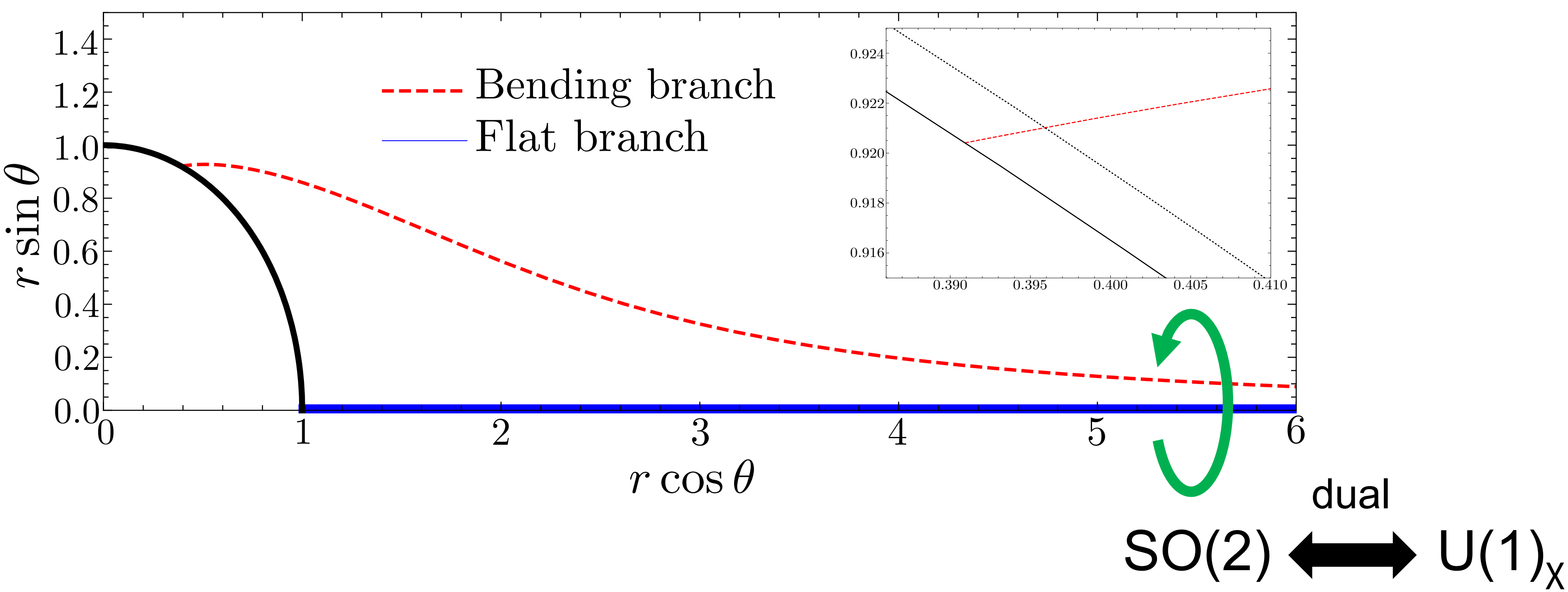
In massless case, are there possible solutions of D7-brane configuration?

- trivial one : $\theta(z) = 0$ (always solution)

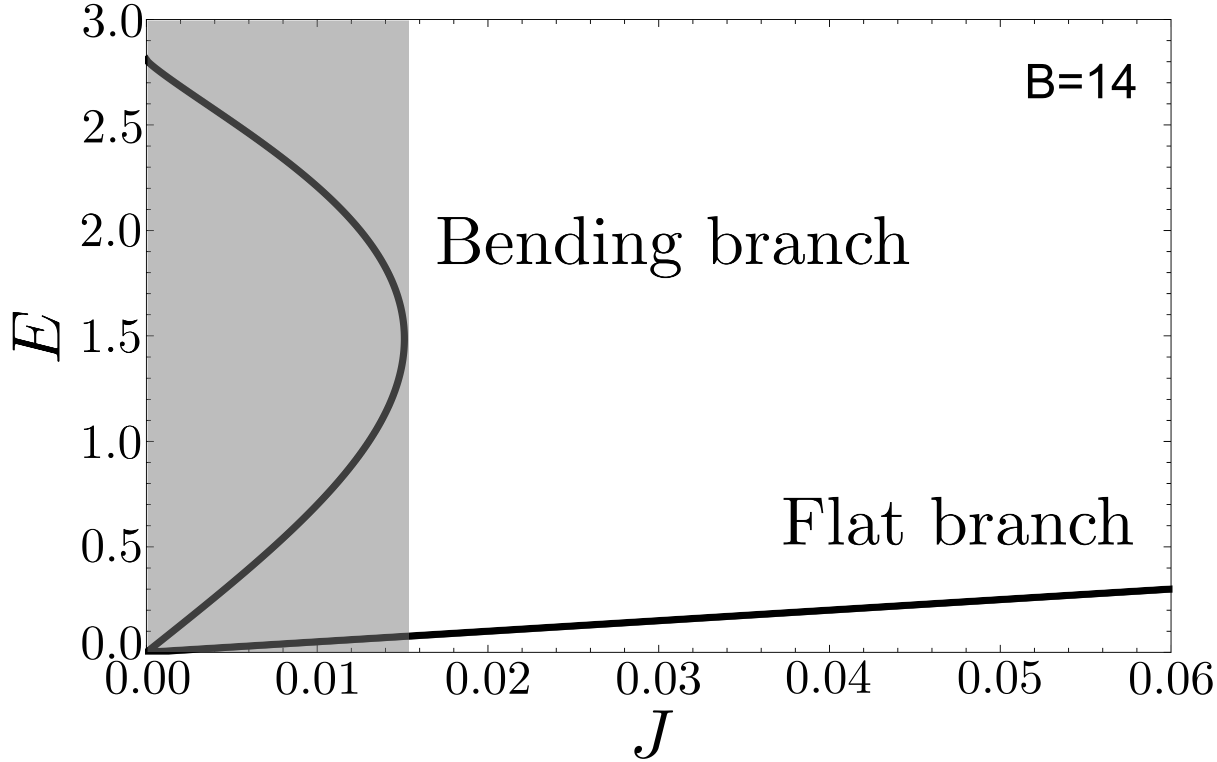
  SO(2) rotational symmetry is preserved. Flat branch

- non-trivial one : $\theta(z) \neq 0$ (in electric field and magnetic field)

  SO(2) rotational symmetry is broken. Bending branch



J-E characteristics and transitions



- We choose the current density as a control parameter.
- There is a multi-valued region at given J.
- Which solution is the most stable?

➡ We assume that the most stable solution has the lowest "thermodynamic potential".

Thermodynamic potential

$$\tilde{F}_{D7} = \lim_{\varepsilon \to 0} \left[\int_{\varepsilon}^{z_*} dz \tilde{\mathcal{H}}_{D7} - L_{\text{count}}(\varepsilon) \right]$$

z_* : effective horizon

ε : cutoff near boundary

Hamiltonian density : $\tilde{\mathcal{H}}_{D7} = \dot{A}_x \frac{\partial \mathcal{L}_{D7}}{\partial \dot{A}_x} + A'_x \frac{\partial \mathcal{L}_{D7}}{\partial A'_x} - \mathcal{L}_{D7}$

$$= g_{xx} \sqrt{|g_{tt}| g_{zz}} \sqrt{\frac{J^2 - |g_{tt}| g_{xx}^2 \cos^6 \theta}{g_{xx} E^2 - |g_{tt}| g_{xx}^2 - |g_{tt}| B^2}}$$

Counterterm : $L_{\text{count}}(\varepsilon) = \frac{1}{4\varepsilon^4} - \frac{m^2}{2\varepsilon^2} + \frac{5}{12} m^4 - \frac{E^2 + B^2}{2} \log \Lambda\varepsilon$

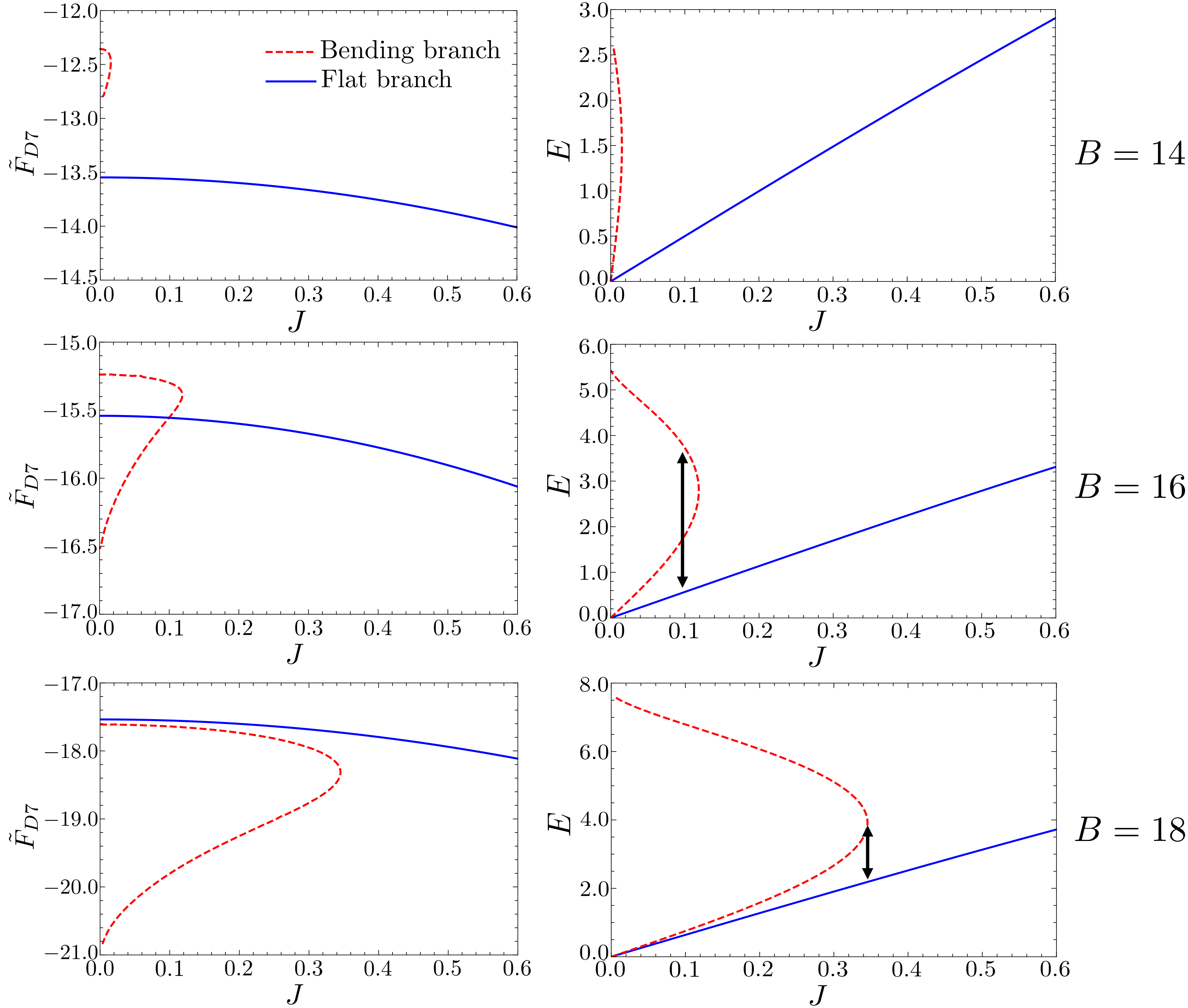
massless

Dimension [1/ε]

In our study, we have chosen Λ as one of the possible choices so that $\partial^2 \mathcal{L}_{D7} / \partial E^2 = 0$ or $\partial^2 \mathcal{L}_{D7} / \partial B^2 = 0$ for vacuum $(T = 0,\ E = 0,\ B = 0)$.

zero polarizability

zero susceptibility



Bending branch
Flat branch
$\tilde{F}_{D7}$
E
J
$B = 14$
$B = 16$
$B = 18$

Order parameter : chiral condensate

We want a parameter which characterizes this phase transition.

→ Chiral condensate

Chiral condensate roughly corresponds to the non-flatness of the D7-brane configuration.

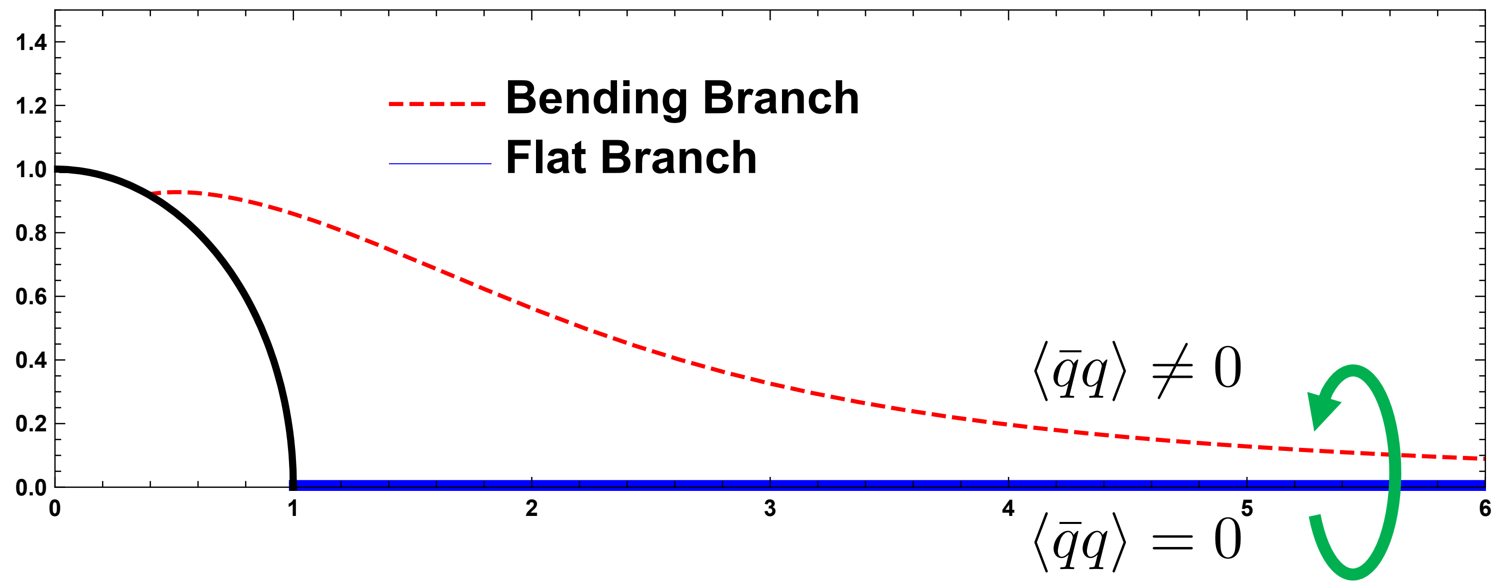
D7-brane embedding function :

$$\theta(z) = \cancel{m}z + \theta_2 z^3 + \cdots$$

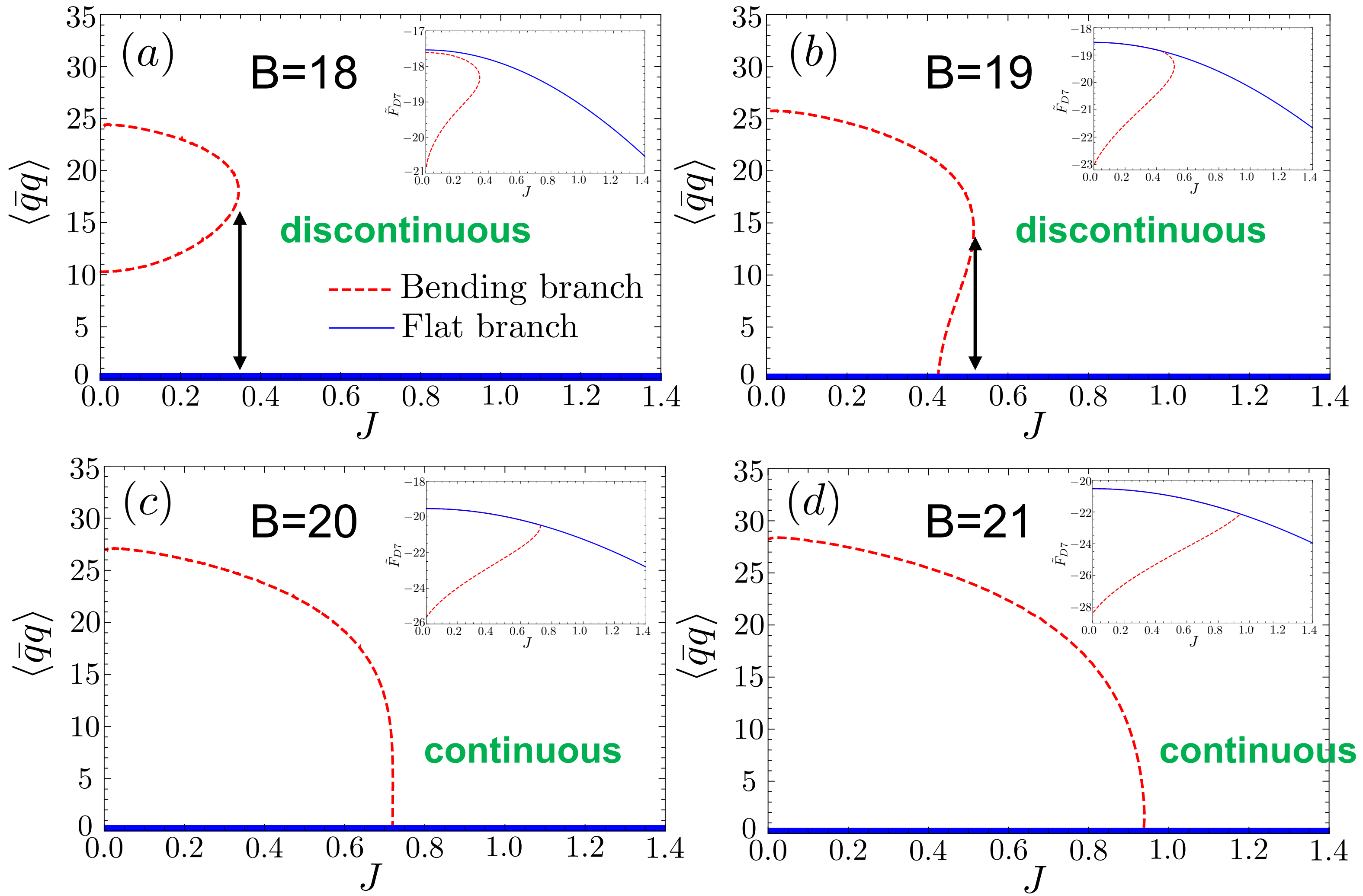
Chiral condensate :

$$\langle \bar{q}q \rangle = -2\theta_2 + \cancel{\frac{1}{3}m^3}$$

massless



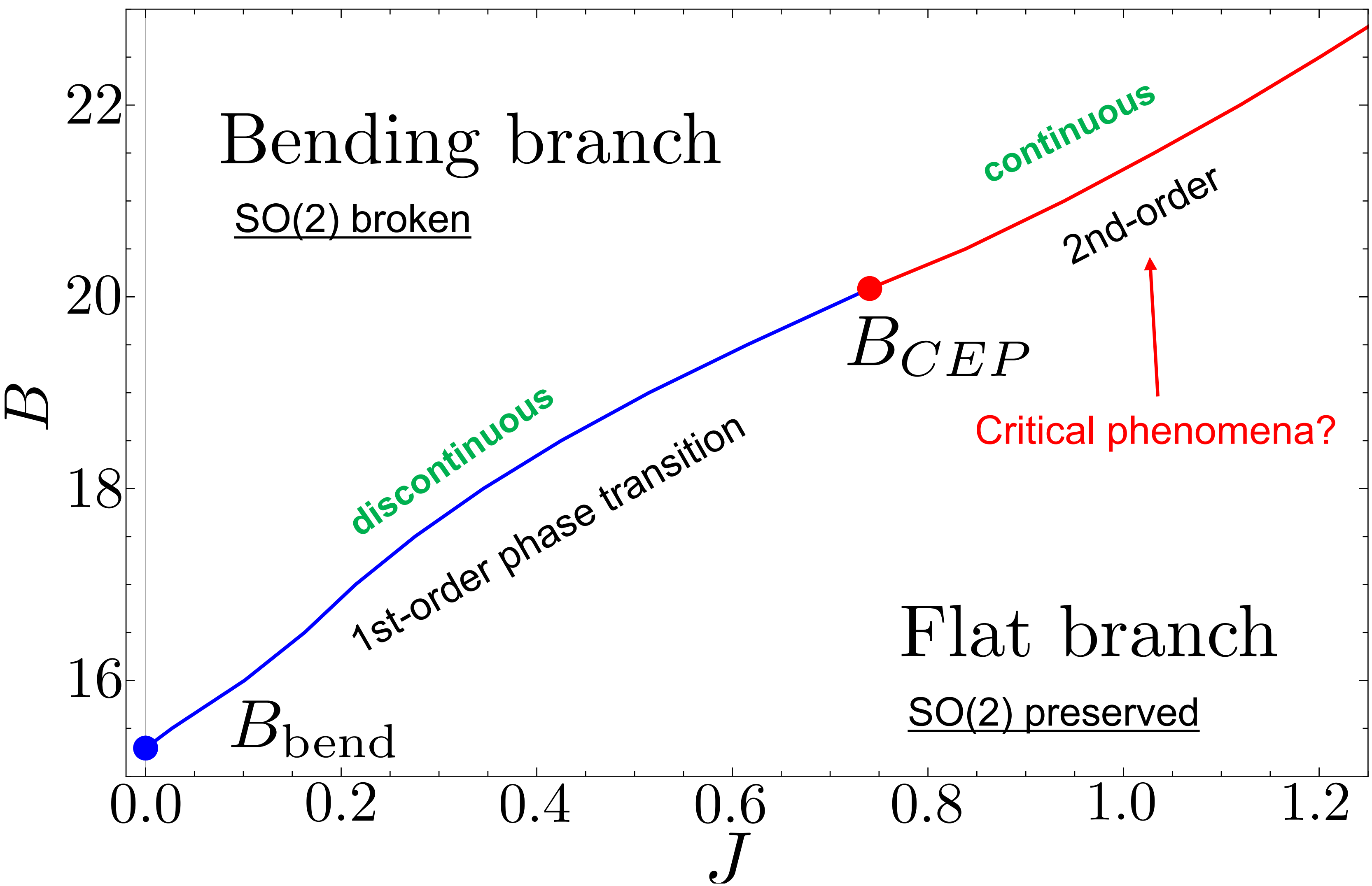
J vs Chiral condensate



When B becomes larger, the chiral condensate in the bending branch is changed from multi-valued to single valued.

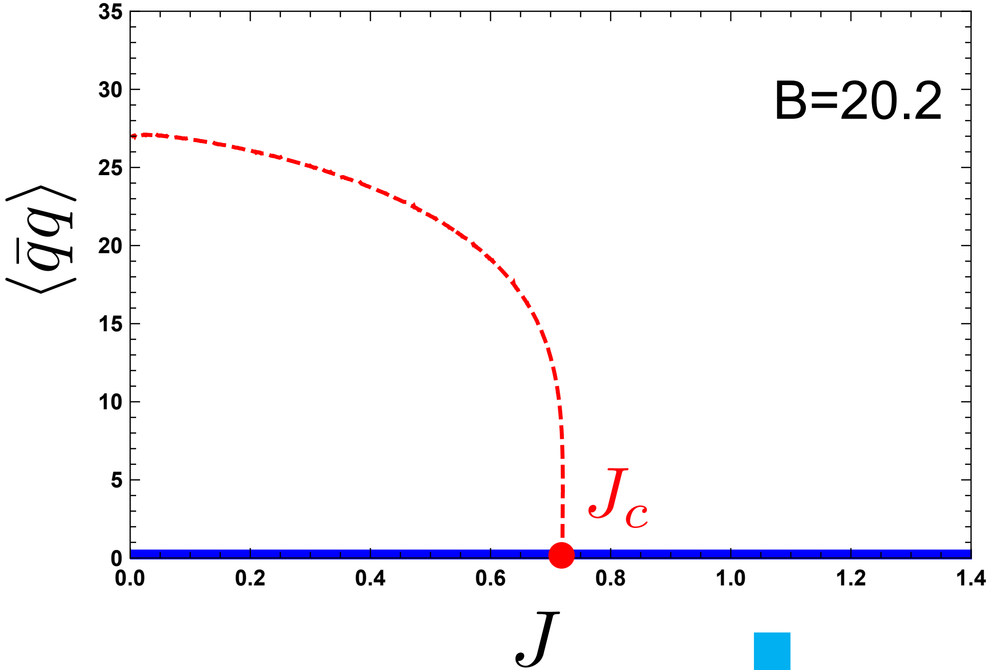
The discontinuous jump of the chiral condensate is changed to the continuous transition.

Phase diagram

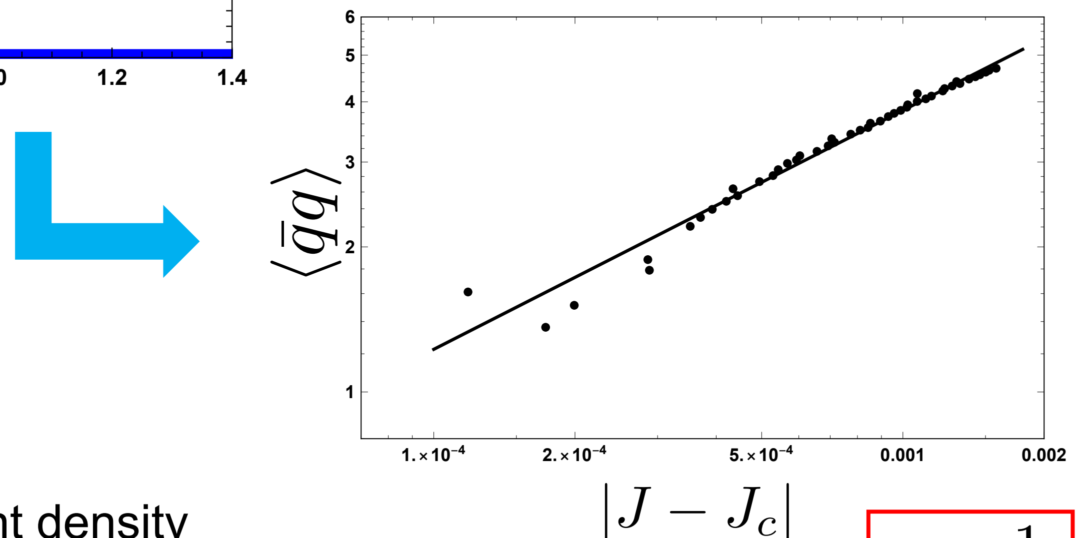


If we control the current density at given B, the symmetry is spontaneously broken.

Critical exponent



We define and calculate the critical exponent near the critical value of the current density.



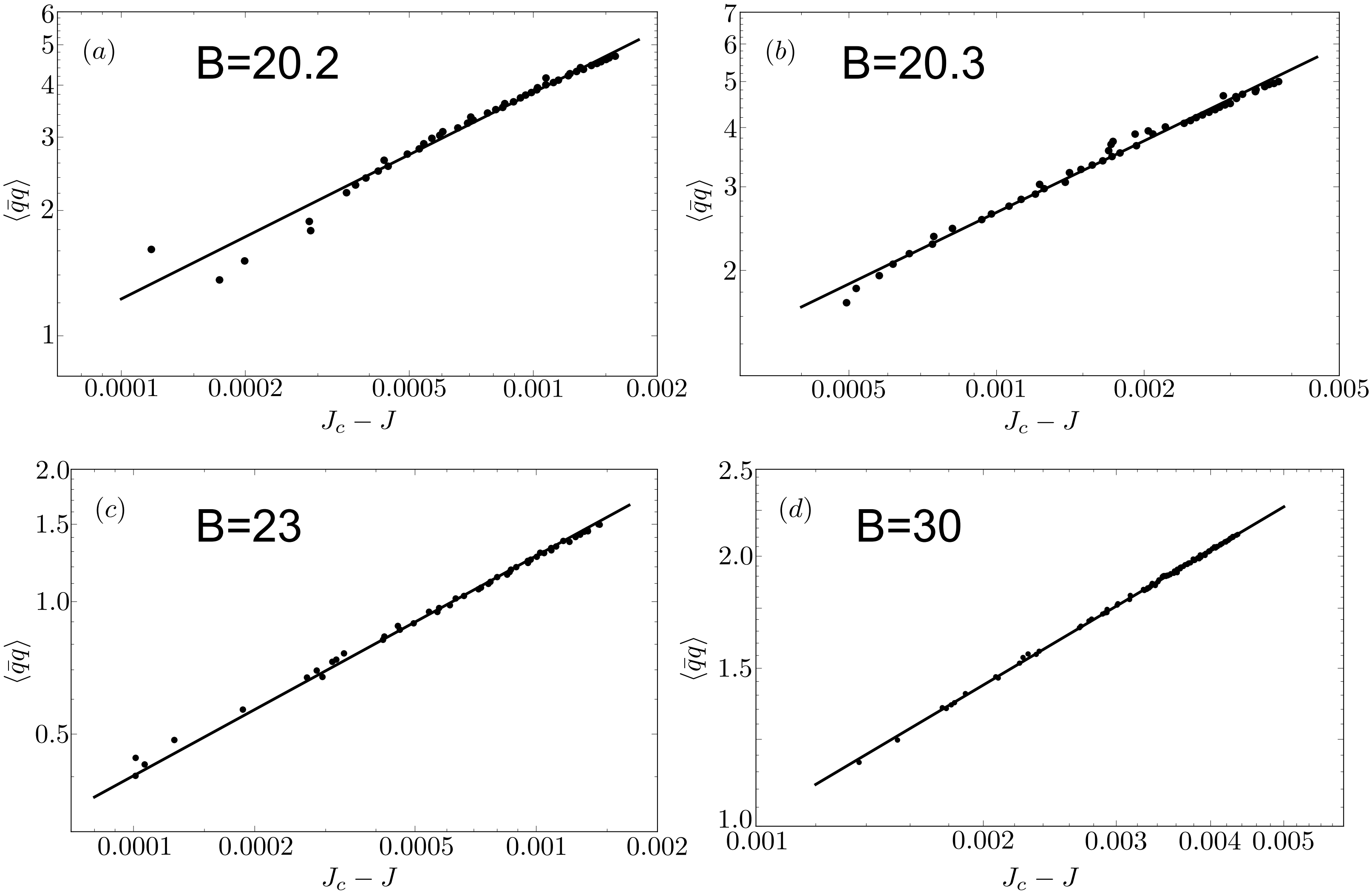
$$\langle \bar{q}q \rangle \propto |J - J_c|^{\kappa}$$

κ : critical exponent

J_c : critical value of the current density

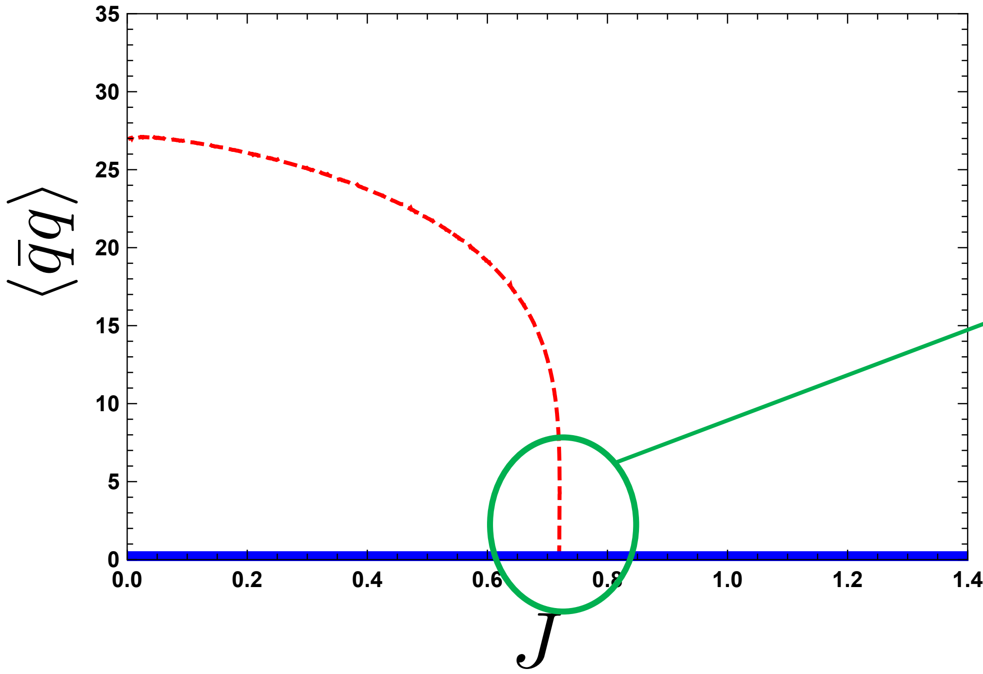
$$\kappa = \frac{1}{2}$$

Critical exponent



$\kappa = 1/2$ is always satisfied along the 2nd-order phase transition line.

Analytical approach



$$J = (2\pi\alpha') \sqrt{|g_{tt}| g_{xx}^2}\Big|_{z=z_*} \cos^3\theta(z_*)$$
$$= G(z_*) \cos^3\theta(z_*)$$

$$\theta(z_*) \ll 1$$

$$J \sim G(z_*) \left(1 - \frac{\theta(z_*)^2}{2} + \cdots\right)^3$$
$$= G(z_*) \left(1 - \frac{3}{2}\theta(z_*)^2 + \cdots\right)$$

In the limit of $\theta(z_*) \to 0\,,\ J = J_c\,.$

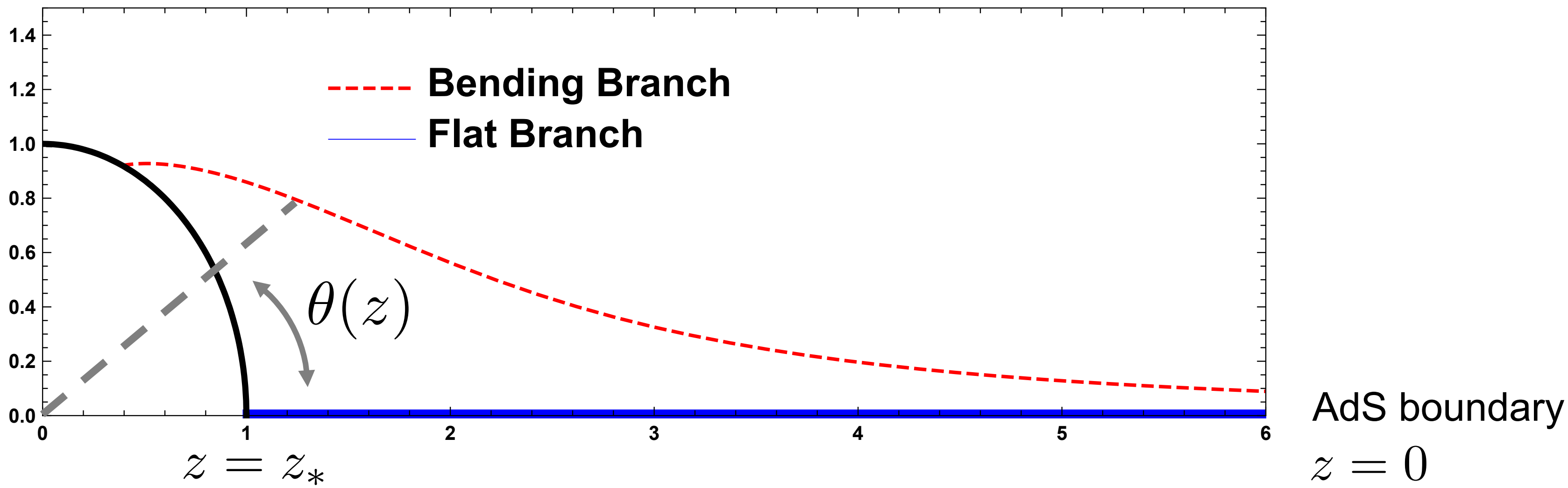
$$J = J_c + a_2\theta(z_*)^2 + a_4\theta(z_*)^4 + \cdots$$

The current density is written as the even function of $\theta(z_*)$.

Analytical approach

On the other hand, the chiral condensate is obtained from the asymptotic form of the $\theta(z)$ near the AdS boundary z=0.

$$\theta(z) = \cancel{mz} + \theta_2 z^3 + \cdots$$



If we switch the sign of $\theta(z)$, the sign of the chiral condensate must also be switched.

$$\langle \bar{q}q \rangle = b_1 \theta(z_*) + b_3 \theta(z_*)^3 + \cdots$$

Analytical approach

$$J = J_c + a_2\theta(z_*)^2 + a_4\theta(z_*)^4 + \cdots$$

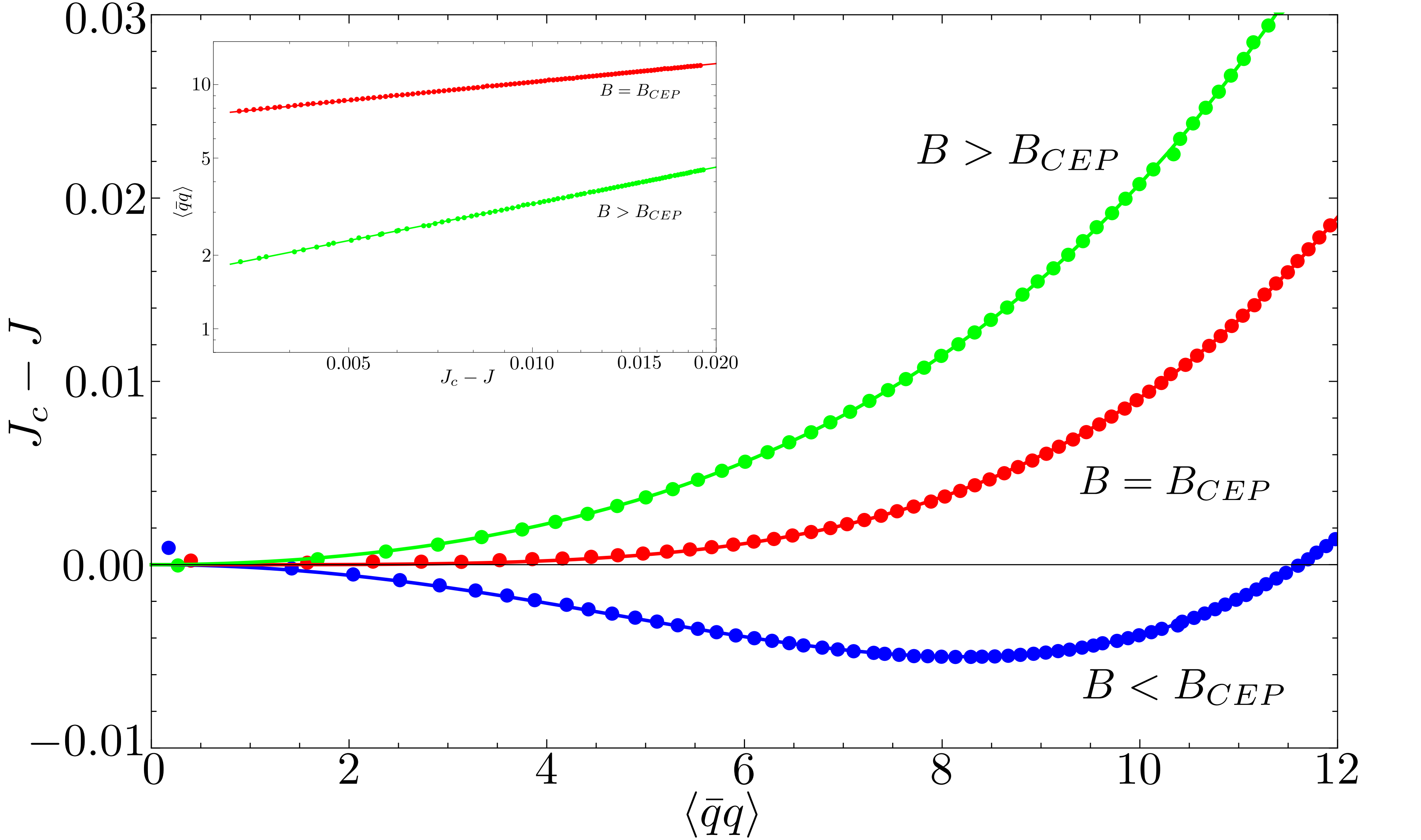
$$\langle \bar{q}q \rangle = b_1\theta(z_*) + b_3\theta(z_*)^3 + \cdots$$

$$\Rightarrow \quad \langle \bar{q}q \rangle \sim (J_c - J)^{1/2} \quad \left(\theta(z_*) \ll 1\right)$$

Note that in this analytical discussion, we did not impose the condition of $B > B_{CEP}$.

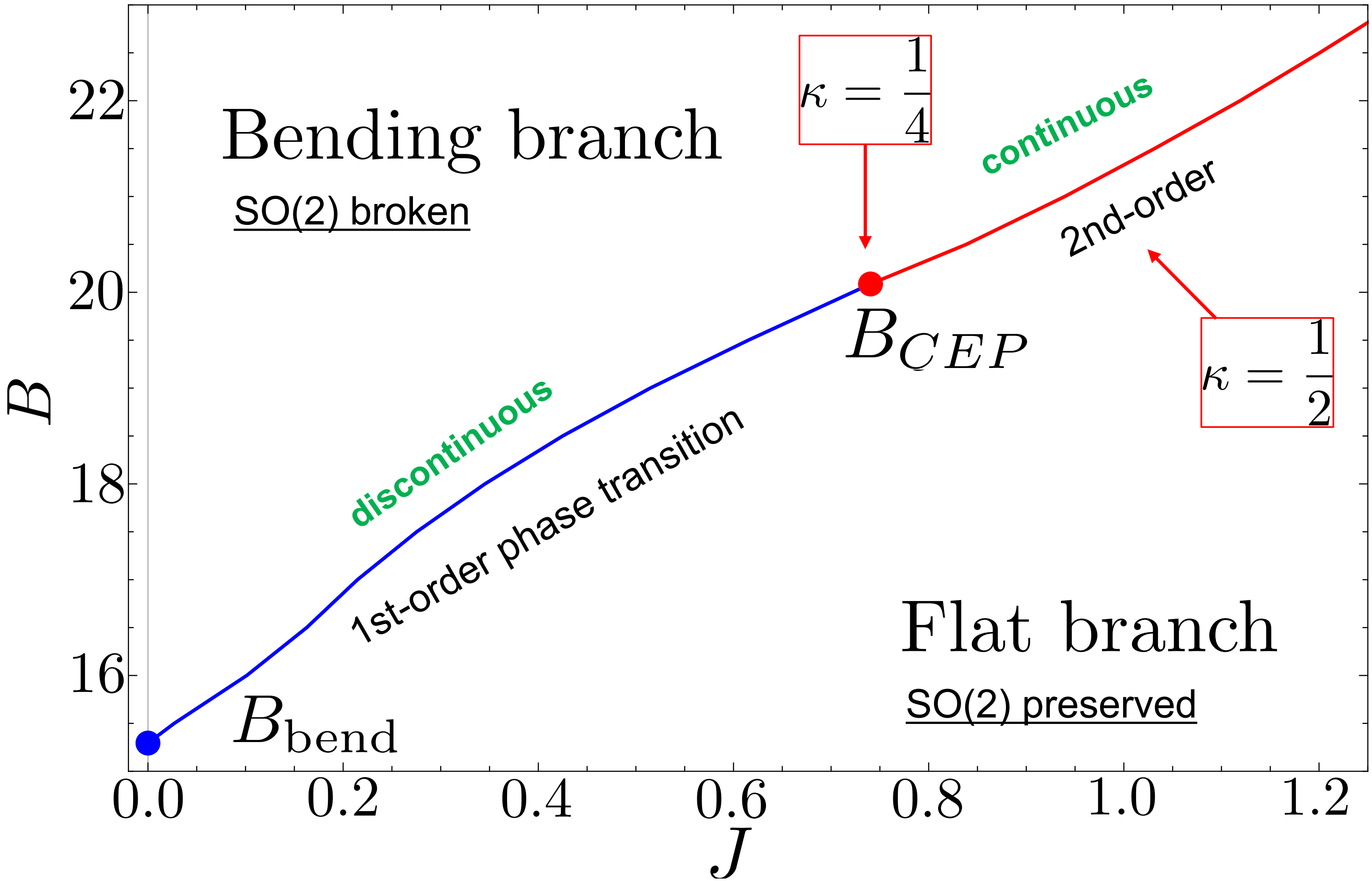
Combined with the numerical results, it appears that the relationship between the chiral condensate and the current density near $B = B_{CEP}$ has the form of

$$J \sim J_c + c_2(B - B_{CEP}) \langle \bar{q}q \rangle^2 - c_4 \langle \bar{q}q \rangle$$



$$J \sim J_c + c_2(B - B_{CEP}) \langle \bar{q}q \rangle^2 - c_4 \langle \bar{q}q \rangle$$

Phase diagram



$$\langle \bar{q}q \rangle \propto |J - J_c|^{\kappa}$$

Summary

- We study the D3-D7 conducting system in the presence of both the electric field and the magnetic field.
- The non-equilibrium phase transition (spontaneously symmetry breaking) occurs between the bending branch and the flat branch.
- Two Remarks:
  - ➢ Two branches are completely separated by the 1st-order phase transition or the 2nd-order phase transition.
  - ➢ The critical exponent κ is 1/2 at 2nd-order phase transition point in or 1/4 at $B=B_{CEP}$ in our system.

Future perspective

- Other critical exponents?
- Nambu-Goldstone mode?

Review: Landau theory and static critical exponent in equilibrium phase transitions

- Free energy near the critical point in ferromagnets

$$F = F_0 + aM^2 + bM^4 - HM$$

M : magnetization (order parameter)

H : external magnetic field

$$\Delta M \propto |T - T_c|^{\beta} \quad (T < T_c)$$

$$M \propto H^{1/\delta} \quad (T = T_c)$$

$$\chi \propto |T - T_c|^{-\gamma} \quad (T < T_c,\ T > T_c)$$

$$C_v \propto |T - T_c|^{-\alpha} \quad (T < T_c)$$

Susceptibility : $\chi = \dfrac{\partial M}{\partial H}$

Specific heat : $C_v = -T\dfrac{\partial^2 F}{\partial T^2}$

- Static critical exponents

$$\beta = \frac{1}{2},\ \ \delta = 3,\ \ \gamma = 1,\ \ \alpha = 0$$

- Critical amplitude ratio

$$\chi_{T\,>\,T_c}/\chi_{T\,<\,T_c} = 2$$

These values correspond with those in the mean-field theory.