Quark-hadron continuity under rotation: vortex continuity or boojum?

熱場の量子論とその応用
２０１８年８月２８日(火)～３０日(木)

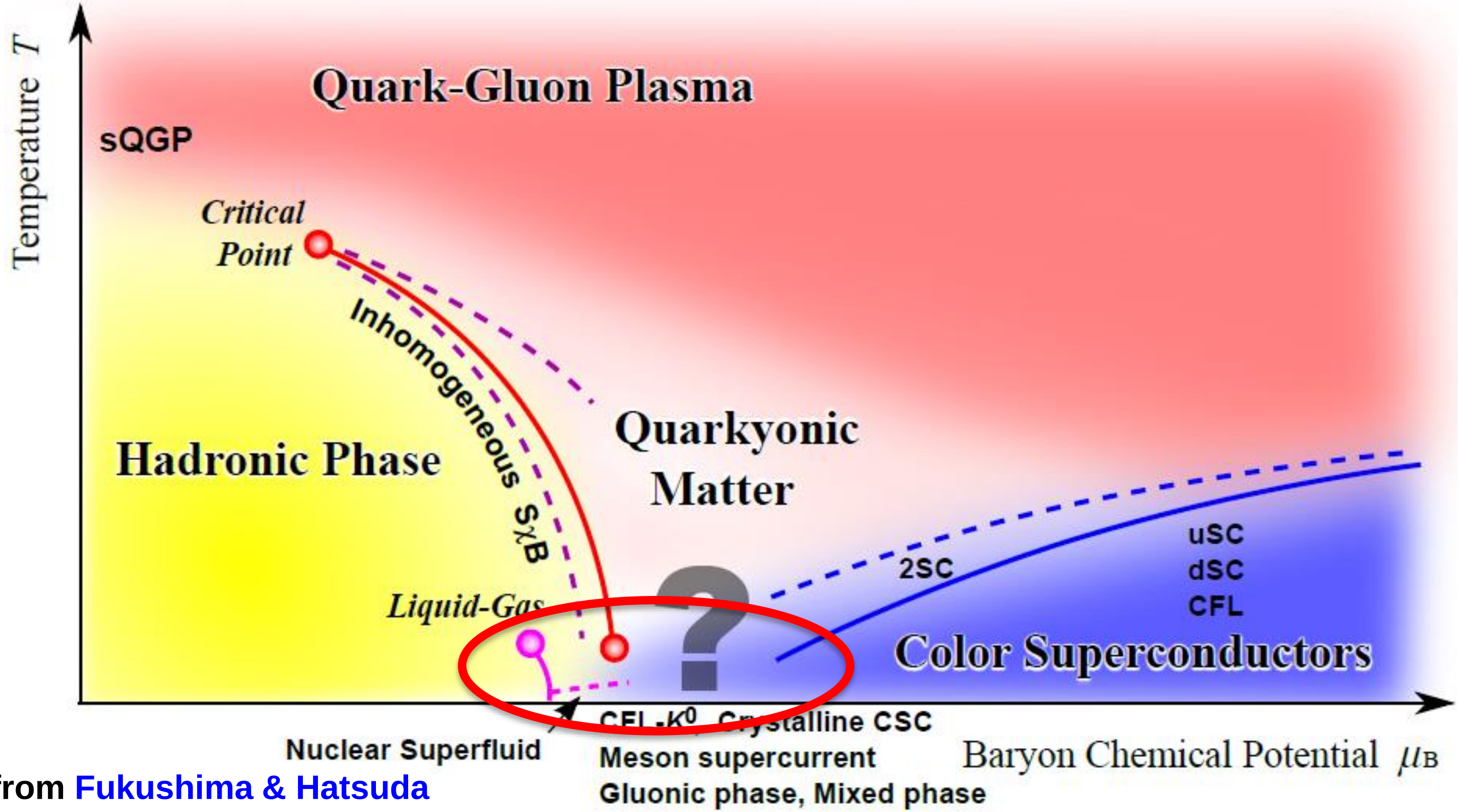
2018/8/30@ RIKEN

Muneto Nitta(新田宗土)
Chandrasekhar Chatterjee, Shigehiro Yasui(安井 繁宏)
Keio U.(慶應義塾大学)

Topological Materials Science

Keio University
1858
CALAMVS
GLADIO
FORTIOR

Topic in this talk



from Fukushima & Hatsuda
Rept.Prog.Phys. 74 (2011) 014001

Quantum Chromo Dynamics (QCD)

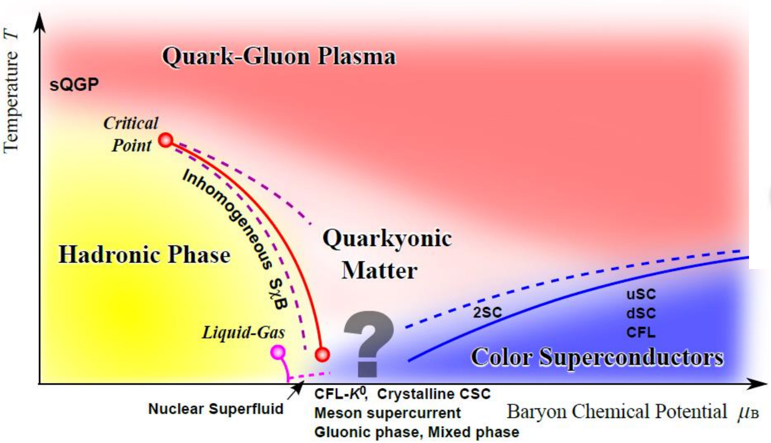
Quark matter

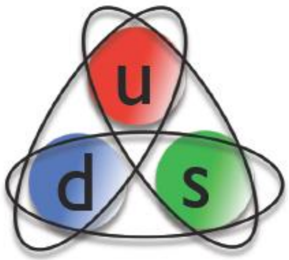
quarks

$$q^i_\alpha \quad i = \text{u,d,s flavor(global) SU(3)} \quad \alpha = \text{r,g,b color(gauge) SU(3)}$$

Color-flavor locked (CFL) phase

"Color superconductor"





@ high density

Bailin-Love('79),
Iwasaki-Iwado('95)
Alford-Rajagopal-Wilczek('98)

$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q^j_\beta q^k_\gamma \sim \mathbf{1}_{\alpha i}$$

3x3 matrix

from Fukushima & Hatsuda
Rept.Prog.Phys. 74 (2011) 014001

Color superconductivity as well as *superfluidity*

Continuity of Quark and Hadron Matter
Thomas Schäfer and Frank Wilczek
Phys. Rev. Lett. **82**, 3956 – Published 17 May 1999

(continuity or crossover)

No phase transition between hadron and CFL phases

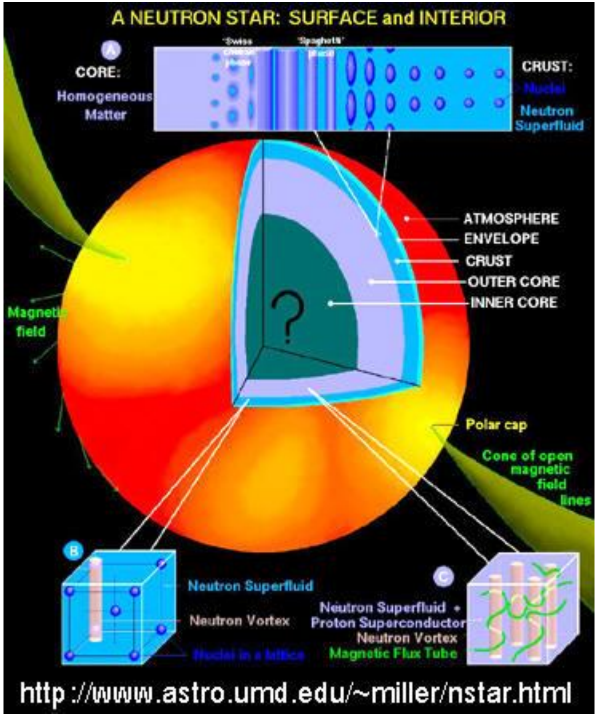
Matching of symmetries and excitations (*Nambu-Goldstone modes etc*)

Three-flavor quarks with degenerate mass

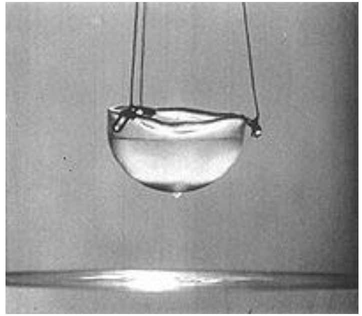
Phases	Hadron phase	Color–flavor-locked phase
	Confinement	Higgs
Quarks	Confined	Condensed
Monopoles	*Condensed?*	*Confined*
Coupling constant	Strong	Weak
Order parameters	Chiral condensate $\langle \bar{q} q \rangle$	Diquark condensate $\langle q q \rangle$
Symmetry	$SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times U(1)_{\mathrm{B}}$ $\rightarrow SU(3)_{\mathrm{L+R}}$	$SU(3)_{\mathrm{C}} \times SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times U(1)_{\mathrm{B}}$ $\rightarrow SU(3)_{\mathrm{C+L+R}}$
Fermions	**8** baryons	**8** + **1** quarks
Vectors	**8** + **1** vector mesons	**8** gluons
NG modes	**8** pions $(\bar{q} q)$ H boson	**8** + **1** pions $(\bar{q}\bar{q} q q)$ H boson

Neutron Stars

Core **Nuclear matter**

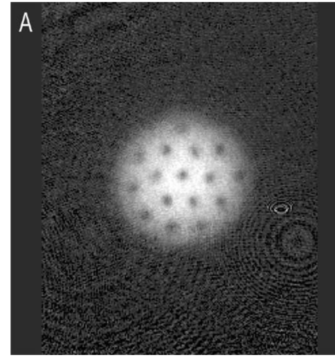


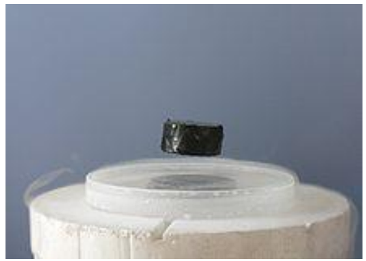
Neutron superfluid



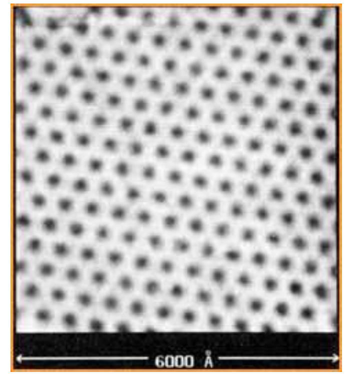
Rotation

Superfluid vortices





Magnetic field

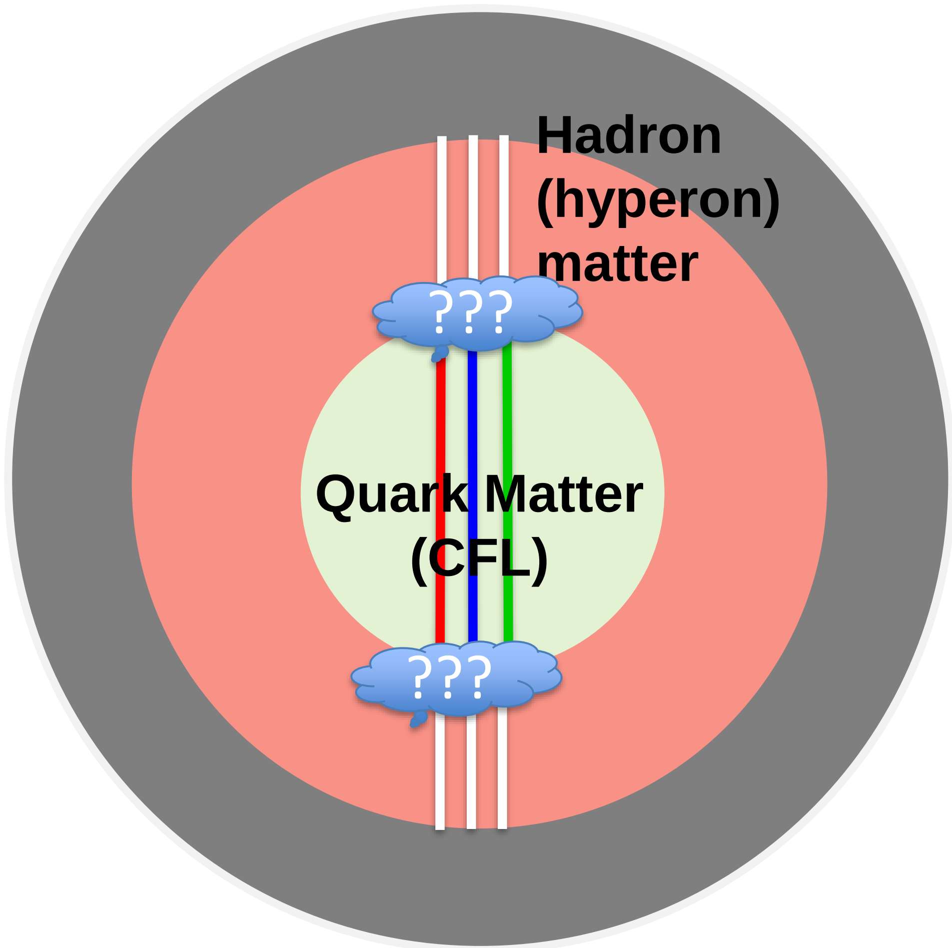


Proton super-conductor

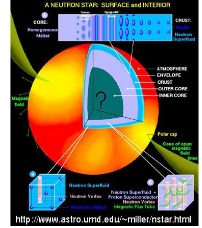
Baym&Pines ('60s)
Anderson&Itoh ('75)

vortices (Flux tubes)

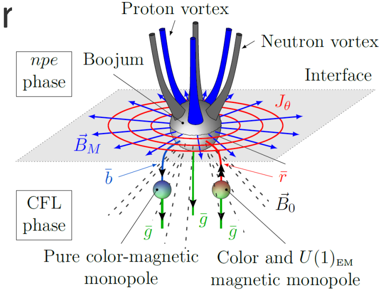
How do vortices connect?



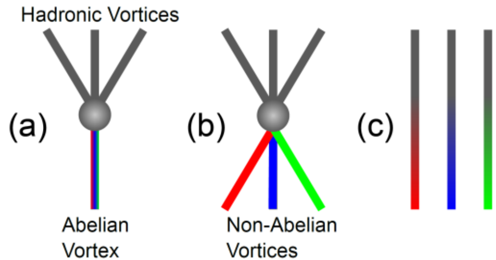
Continuity of Quark and Hadron Matter
Thomas Schäfer and Frank Wilczek
Phys. Rev. Lett. **82**, 3956 – Published 17 May 1999



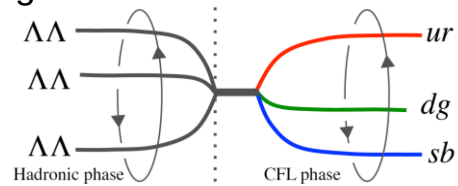
Colorful boojums at the interface of a color superconductor
Mattia Cipriani, Walter Vinci, and Muneto Nitta
Phys. Rev. D **86**, 121704(R) – Published 21 December 2012



Continuity of vortices from the hadronic to the color-flavor locked phase in dense matter
Mark G. Alford, Gordon Baym, Kenji Fukushima, Tetsuo Hatsuda, Motoi Tachibana
e-Print: arXiv:1803.05115 [hep-ph]



Quark-hadron continuity under rotation: vortex continuity or boojum?
Chandrasekhar Chatterjee, Muneto Nitta, Shigehiro Yasui
e-Print: arXiv:1806.09291 [hep-ph]



Anyonic particle-vortex statistics and the nature of dense quark matter
Aleksey Cherman, Srimoyee Sen, Laurence G. Yaffe
e-Print: arXiv:1808.04827 [hep-th]

What is Boojum?

Boojum trees in Arizona

Boojum

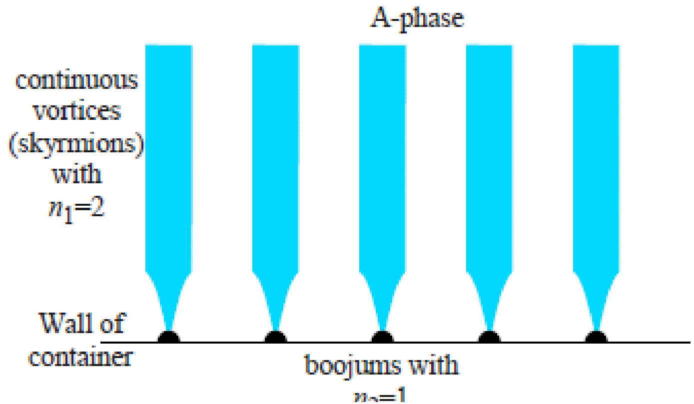
A particularly dangerous kind of Snark

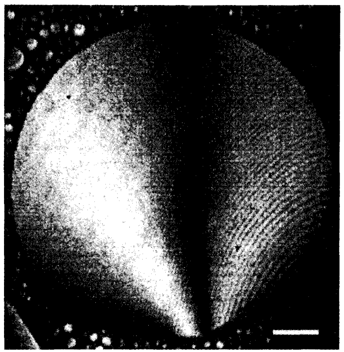
"The Hunting Of Snark"
Lewis Carroll

What is Boojum?

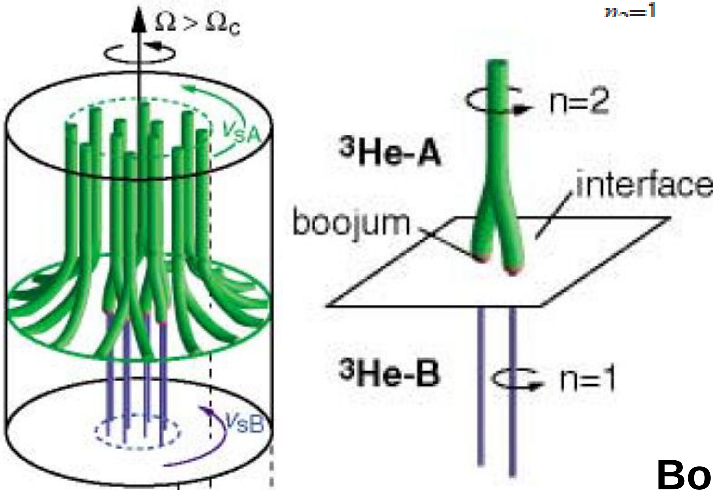
In physics, named by
D.Mermin (1976)

Boojums on the container wall in superfluid ^{3}He-A

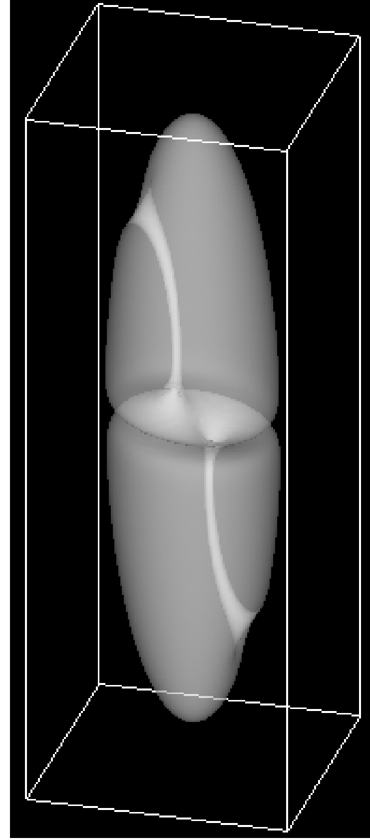




Boojum in liquid crystal

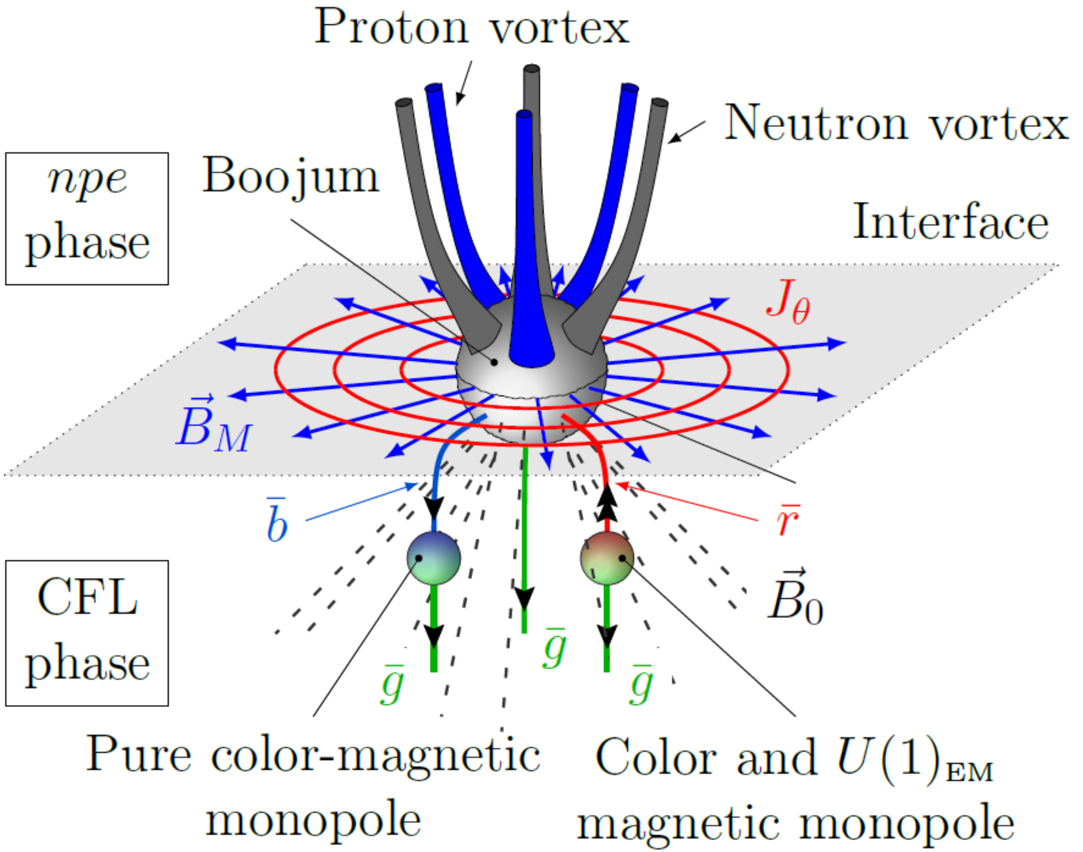


Boojums in ^{3}He A-B phase boundary



Boojum in two component BECs
Kasamatsu-Takeuchi-MN-Tsubota, JHEP1011:068,2010 [arXiv:1002.4265]

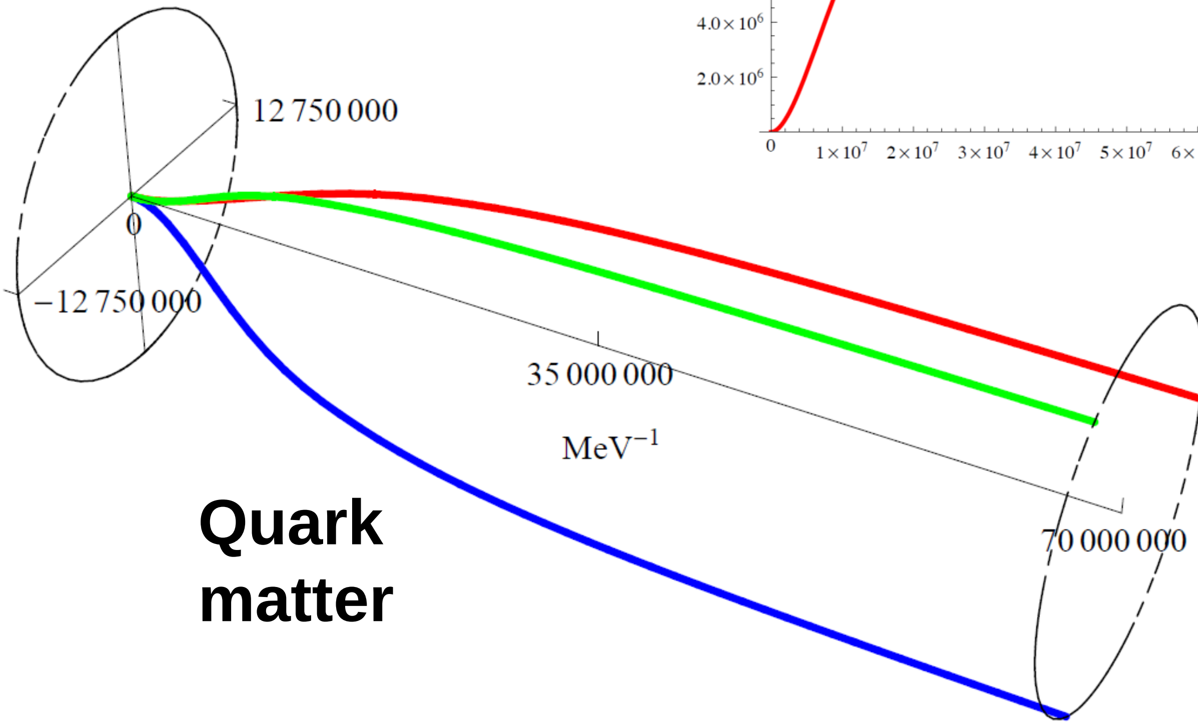
Colorful boojum at interface of quark matter



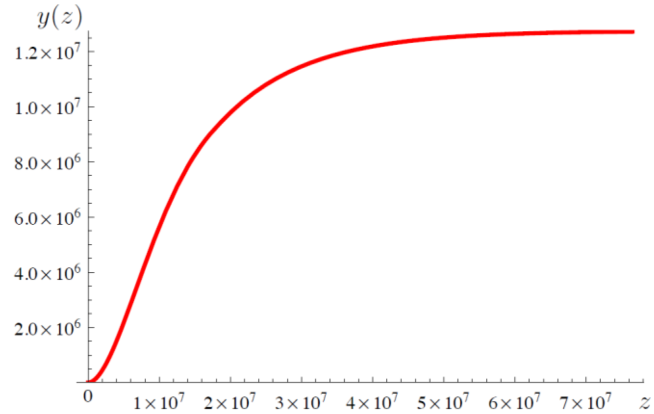
Cipriani,Vinci & MN, Phys.Rev.D86:121704,2012 [arXiv:1208.5704]

Neutron boojum
= Colorful boojum

Nuclear matter

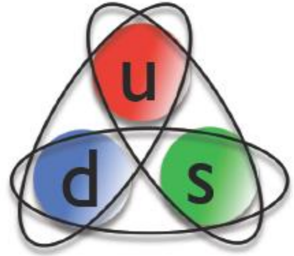


Quark matter



Cipriani,Vinci & MN, Phys.Rev.D86:121704,2012 [arXiv:1208.5704]

Color superconductor



$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_j^{\beta} q_k^{\gamma}$$

$$\alpha = 1,2,3 \ (\mathrm{r,g,b}) \quad i = 1,2,3 \ (\mathrm{u,d,s})$$

$$\Phi_{\alpha i} = \begin{pmatrix} d_{[g}s_{b]} & s_{[g}u_{b]} & u_{[g}d_{b]} \\ d_{[b}s_{r]} & s_{[b}u_{r]} & u_{[b}d_{r]} \\ d_{[r}s_{g]} & s_{[r}u_{g]} & u_{[r}d_{g]} \end{pmatrix} \begin{matrix} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{matrix}$$

$$\bar{u} = ds \quad \bar{d} = sb \quad \bar{s} = ud$$

$$G = U(1)_{\mathrm{B}} \times SU(3)_{\mathrm{C}} \times SU(3)_{\mathrm{F}}$$

$$\Phi_{\alpha i} \to e^{i\alpha} g_{\mathrm{color}} \Phi_{\alpha i} g_{\mathrm{flavor}}$$

Landau-Ginzburg model from QCD

Iida&Baym('01)
Giannakis&Ren('02)
Iida,Matsuura,
Tachibana&Hatsuda('04)

$$\mathcal{L} = \mathrm{Tr}(K_0 \mathcal{D}_0 \Phi^\dagger \mathcal{D}_0 \Phi - K_3 \mathcal{D}_i \Phi^\dagger \mathcal{D}_i \Phi),$$
$$+ \frac{\varepsilon}{2} F_{0i}^2 - \frac{1}{4\lambda} F_{ij}^2 - V,$$

$$V_{\mathrm{GL}} = \mathrm{Tr}\left[\Phi^\dagger \left\{ \left(\alpha + \frac{2\epsilon}{3} \right) \mathbf{1}_3 + \epsilon X_3 \right\} \Phi \right]$$
$$+ \beta_1 [\mathrm{Tr}(\Phi^\dagger \Phi)]^2 + \beta_2 \mathrm{Tr}[(\Phi^\dagger \Phi)^2],$$

GL parameters

$$\alpha = 4N(\mu)\log\frac{T}{T_c}, \quad \beta_{1,2} = \frac{7\zeta(3)}{8(\pi T_c)^2} N(\mu) \equiv \beta,$$

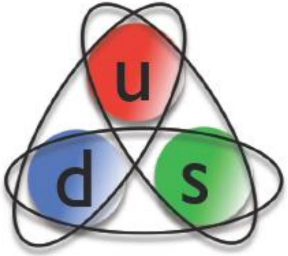
$$K_3 = \frac{1}{3} K_0 = \frac{7\zeta(3)}{12(\pi T_c)^2} N(\mu), \quad \epsilon = N(\mu)\frac{m_s^2}{\mu^2}\log\frac{\mu}{T_c},$$

density of state

$$N(\mu) = \frac{\mu^2}{2\pi^2}$$

For a while we consider high density limit, where strange quark mass can be neglected.
We also ignore E&M interaction. Later we take into account these.

Color superconductor



$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_j^{\beta} q_k^{\gamma}$$

$$\alpha = 1,2,3\ (\mathrm{r,g,b}) \quad i = 1,2,3\ (\mathrm{u,d,s})$$

Ground state

$$\Phi_{\alpha i} = \begin{array}{cl} \begin{pmatrix} \Delta & 0 & 0 \\ 0 & \Delta & 0 \\ 0 & 0 & \Delta \end{pmatrix} & \begin{array}{l} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{array} \\ \bar{u} = ds \quad \bar{d} = sb \quad \bar{s} = ud & \end{array}$$

color-flavor locked (CFL)

$$G = U(1)_{\mathrm{B}} \times SU(3)_{\mathrm{C}} \times SU(3)_{\mathrm{F}}$$

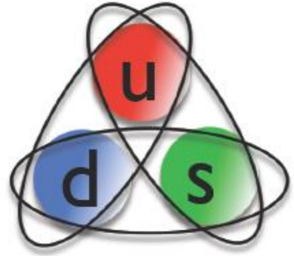
$$\to H = SU(3)_{\mathrm{C+F}}$$

$$g_{\mathrm{color}} = g_{\mathrm{flavor}}^{-1}$$

$\cancel{U(1)_{\mathrm{B}}}$ superfluidity

$\cancel{SU(3)_{\mathrm{C}}}$ color superconductivity

Color superconductor



$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_j^{\beta} q_k^{\gamma}$$

Integer quantized superfluid vortex

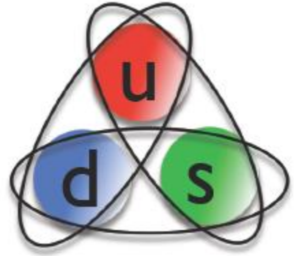
$$\alpha = 1,2,3\ (\mathrm{r,g,b}) \quad i = 1,2,3\ (\mathrm{u,d,s})$$

$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix} \begin{array}{l} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{array}$$

$$\bar{u} = ds \qquad \bar{d} = sb \qquad \bar{s} = ud$$

Iida & Baym, Forbes & Zhitnitsky('02)

Color superconductor



$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_j^{\beta} q_k^{\gamma}$$

$$\alpha = 1,2,3\ (\mathrm{r}, \mathrm{g}, \mathrm{b}) \quad i = 1,2,3\ (\mathrm{u}, \mathrm{d}, \mathrm{s})$$

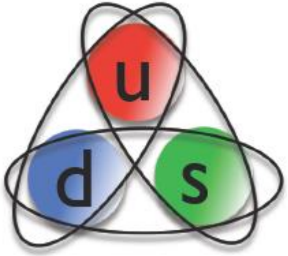
1/3 quantized vortex

$$\Phi_{\alpha i} = \begin{array}{cl} \begin{pmatrix} \Delta_1(r) e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix} & \begin{array}{l} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{array} \end{array}$$

$$\bar{u} = ds \quad \bar{d} = sb \quad \bar{s} = ud$$

Balachandran, Digal & Matsuura (BDM) ('05)
Nakano, MN & Matsuura ('07), Eto & MN ('09)

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$= \boxed{\exp\left(\frac{i\theta}{3}\right)} \underbrace{\exp\frac{i\theta}{3}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}}_{\text{SU(3) color}} \begin{pmatrix} \Delta_1(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

1/3 quantized

SU(3) color

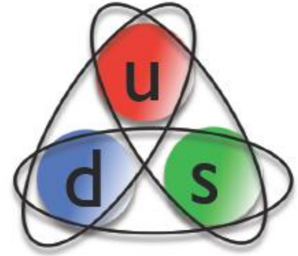
$$F_{12} \sim \oint r\, d\theta A_\theta$$

→ **color flux tube**

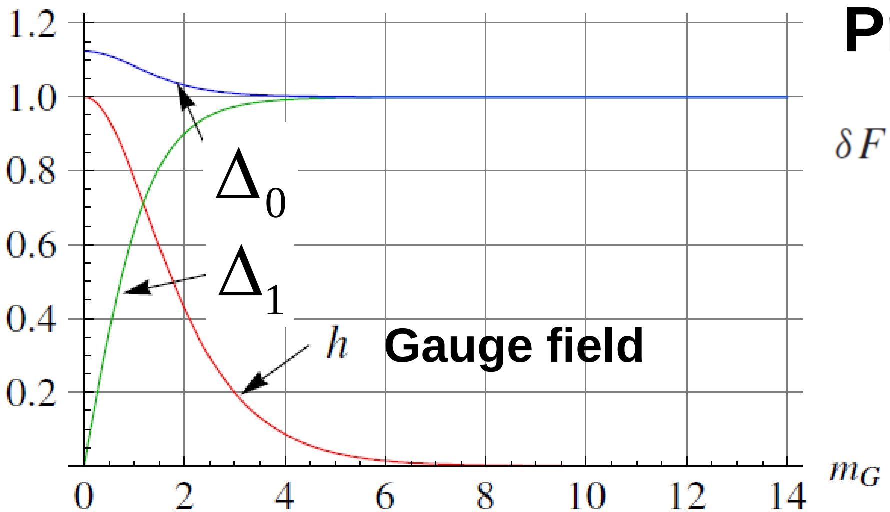
Superfluid vortex

Non-Abelian vortex

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$



Profiles

$F \equiv \Delta_1 + 2\Delta_0,$ **trace**

$G \equiv \Delta_1 - \Delta_0,$ **traceless**

$$\delta F = q_1 \sqrt{\frac{\pi}{2m_1 r}}\, e^{-m_1 r} - \frac{1}{3m_1^2 r^2} + O\left((m_1 r)^{-4}\right)$$

Long tail of a superfluid vortex

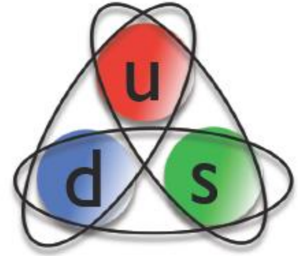
$$\delta G = q_8 \sqrt{\frac{\pi}{2m_8 r}}\, e^{-m_8 r}$$

$$\delta h = -\frac{2}{3}\frac{m_g^2}{m_g^2 - m_8^2}\delta G$$

confined color flux

Eto & MN ('09)

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$= \boxed{\exp\left(\frac{i\theta}{3}\right)} \exp\frac{i\theta}{3}\begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

1/3 quantized

SU(3) color

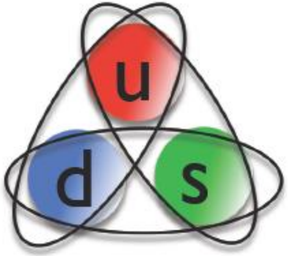
$$F_{12} \sim \oint r d\theta A_\theta$$

→ **color flux tube**

Superfluid vortex

Non-Abelian vortex

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r) e^{i\theta} \end{pmatrix}$$

$$= \boxed{\exp\left(\frac{i\theta}{3}\right)} \exp\frac{i\theta}{3} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r) \end{pmatrix}$$

1/3 quantized

SU(3) color

$$F_{12} \sim \oint r\, d\theta A_\theta$$

→ color flux tube

Superfluid vortex

Non-Abelian vortex

1/3 quantized vortex

$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r) e^{i\theta} \end{pmatrix}$$

$$= \boxed{\exp\left(\frac{i\theta}{3}\right)} \underline{\exp \frac{i\theta}{3} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}} \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r) \end{pmatrix}$$

1/3 quantized

SU(3) color

$$F_{12} \sim \oint r d\theta A_\theta \rightarrow$$

color flux tube

Comment

Non-Abelian vortices were discovered earlier in the context of Supersymmetry and String theory ('03)

Superfluid vortex

Non-Abelian vortex

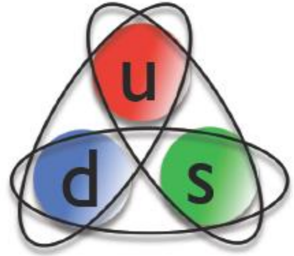
Non-Abelian vortices

Color Fluxes

$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$



Abelian vortex

No flux

$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$

Which are energetically favored?

Non-Abelian vortices

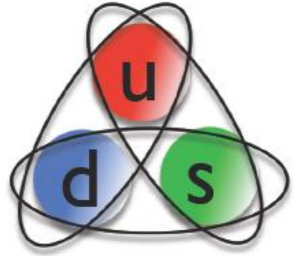
Color Fluxes

$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$

Split



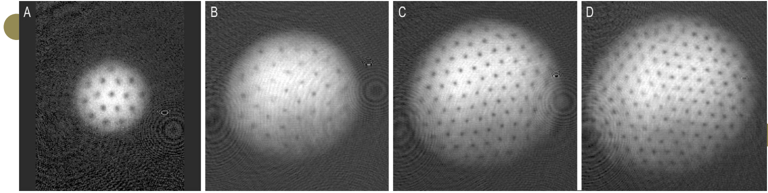
Abelian vortex

No flux

$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$

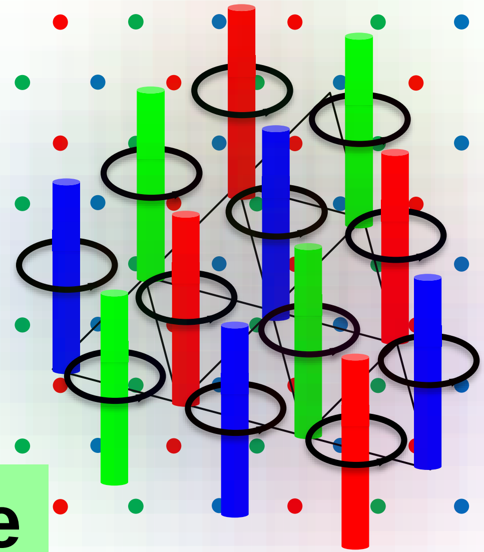
$$E[\ ] = E[\ ] = E[\ ] = \frac{1}{9}E[\ ]$$

Nakano, MN & Matsuura (‘07), simulation by Alford et.al (‘16)



Abrikosov vortex lattice

Colorful vortex lattice



$$\Phi_{\alpha i} = \left(\begin{array}{c|cc} \Delta_1(r)e^{i\theta} & 0 & 0 \\ \hline 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{array}\right)$$

$$H = SU(3)_{\mathrm{C+F}}$$
$$\downarrow$$
$$K = [SU(2)\times U(1)]_{\mathrm{C+F}}$$

@ vortex core

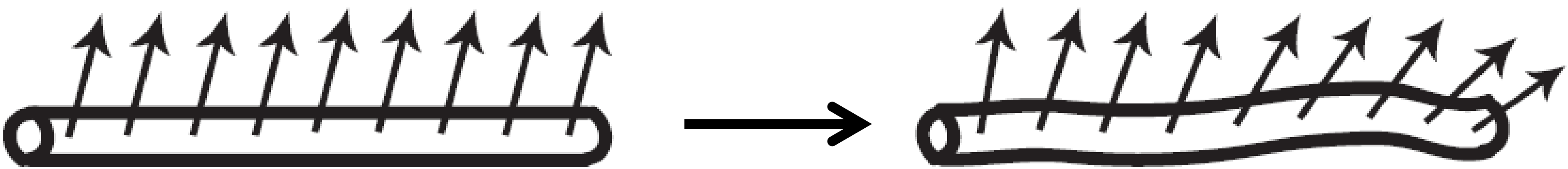
Nambu-Goldstone modes localized around the vortex

$$\mathbf{C}\times\frac{H}{K} = \mathbf{C}\times\frac{SU(3)_{\mathrm{C+F}}}{SU(2)\times U(1)} = \mathbf{C}\times\mathbf{C}P^2$$

Continuous family of solutions exists

Eto,Nakano&MN('09)

= Gapless modes propagating along the vortex line



"ground state" **1+1 dim effective theory** **fluctuations**

Effective theory of orientational modes

Eto, Nakano & MN('09)

$$\frac{H}{K} = \frac{SU(3)_{\mathrm{C+F}}}{SU(2)\times U(1)} = \mathbf{C}P^2$$

1+1 dim. world-sheet theory

$$\phi = \left(\phi^1, \phi^2, \phi^3\right)$$

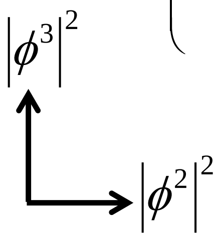
homogenous coordinates

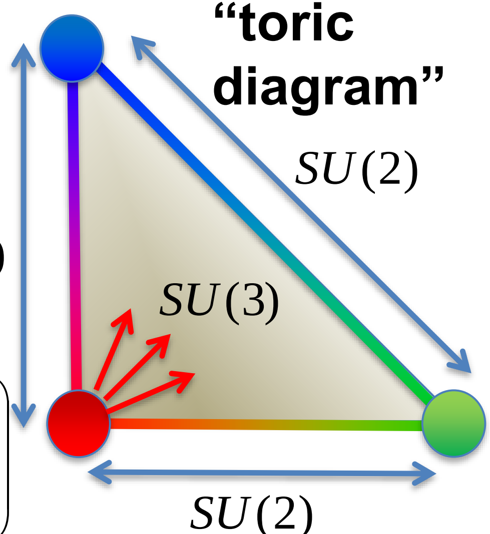
$$\mathcal{L}_{\mathbb{C}P^2} = C \sum_{\alpha=0,3} K_\alpha [\partial^\alpha \phi^\dagger \partial_\alpha \phi + (\phi^\dagger \partial^\alpha \phi)(\phi^\dagger \partial_\alpha \phi)],$$

$$\phi^\dagger \phi = 1$$

"toric diagram"

$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$





$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

Linear dispersion: type I NG mode

Hadron matter with *degenerate quark mass*

⇒ Hyperon matter

Baryon: hyperon $\Lambda = uds$, $\Sigma = uus$, $\Xi = uss$

$$\Delta_{\Lambda\Lambda} = \langle \Lambda\Lambda \rangle \neq 0$$

$$8 \times 8 = 1 + 8 + 8 + 10 + 10^* + 27$$

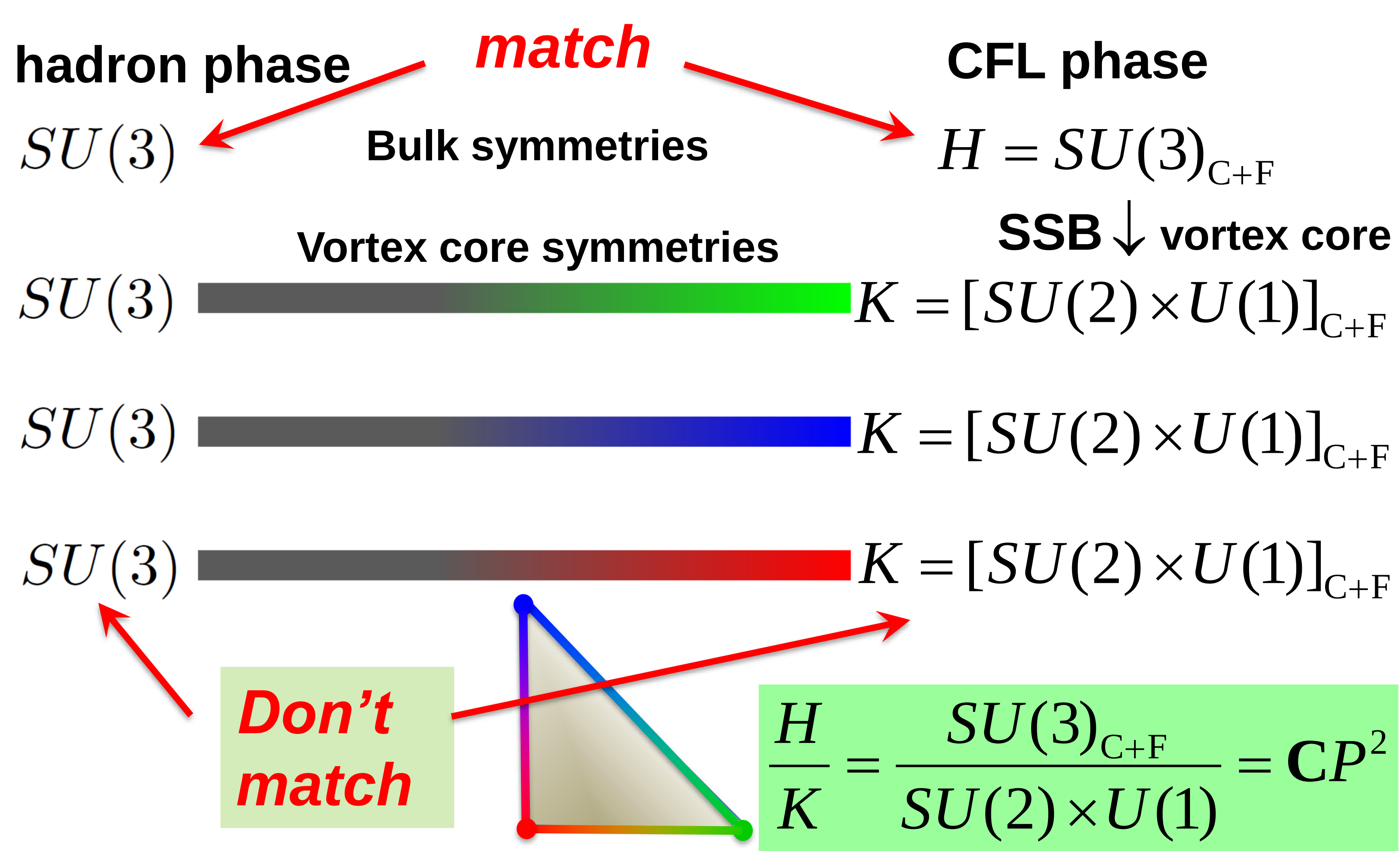
$$-\sqrt{1/8}\,\Lambda\Lambda + \sqrt{3/8}\,\Sigma\Sigma + \sqrt{4/8}\,N\Xi$$

$U(1)_B$ spontaneous symmetry breaking

⇒ superfluidity

⇒ vortices under rotation

How these ΛΛ vortices connect to non-Abelian vortices in CFL phase?



match
hadron phase
CFL phase
$SU(3)$
Bulk symmetries
$H = SU(3)_{\mathrm{C+F}}$
SSB $\downarrow$ vortex core
Vortex core symmetries
$SU(3)$
$K = [SU(2) \times U(1)]_{\mathrm{C+F}}$
$SU(3)$
$K = [SU(2) \times U(1)]_{\mathrm{C+F}}$
$SU(3)$
$K = [SU(2) \times U(1)]_{\mathrm{C+F}}$
Don't match
$\frac{H}{K} = \frac{SU(3)_{\mathrm{C+F}}}{SU(2) \times U(1)} = \mathbf{C}P^2$

hadron phase ***match*** **CFL phase**

$SU(3)$ **Bulk symmetries** $H = SU(3)_{\mathrm{C+F}}$

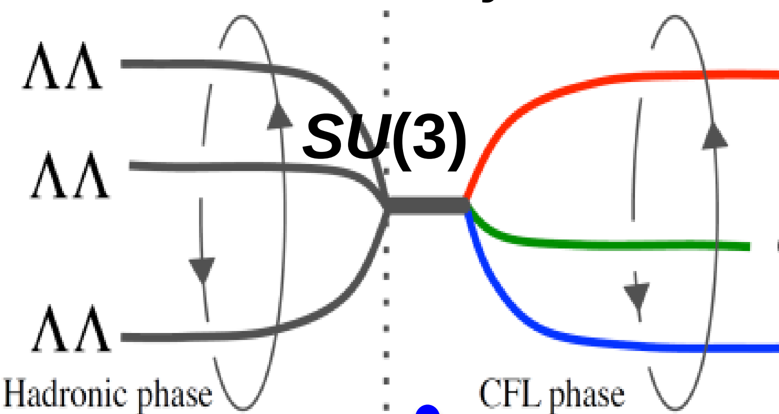
SSB $\downarrow$ **vortex core**

Vortex core symmetries

$SU(3)$

$SU(3)$

$SU(3)$

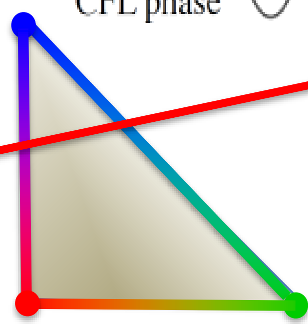


$_{ur}K = [SU(2)\times U(1)]_{\mathrm{C+F}}$

$_{dg}K = [SU(2)\times U(1)]_{\mathrm{C+F}}$

$_{sb}K = [SU(2)\times U(1)]_{\mathrm{C+F}}$

Don't have to match



$$\frac{H}{K} = \frac{SU(3)_{\mathrm{C+F}}}{SU(2)\times U(1)} = \mathbf{C}P^2$$

Bogoliubov-de Gennes equation in hadron phase

$$\begin{pmatrix} -\frac{\vec{\nabla}^2}{2m_B} - \mu_B & e^{i\theta}|\Delta_{\Lambda\Lambda}| \\ e^{-i\theta}|\Delta_{\Lambda\Lambda}| & \frac{\vec{\nabla}^2}{2m_B} + \mu_B \end{pmatrix} \begin{pmatrix} u_B \\ v_B \end{pmatrix} = \mathcal{E} \begin{pmatrix} u_B \\ v_B \end{pmatrix}$$

$$\theta \to \theta + \alpha, \qquad (u, v) \to (e^{i\alpha/2}u, e^{-i\alpha/2}v)$$

Λ hyperon acquires a phase (1/2) × 2π = π around a vortex

Λ=uds

At quark level…

$$(q)_{\alpha i} = \begin{pmatrix} u_r & d_r & s_r \\ u_g & d_g & s_g \\ u_b & d_b & s_b \end{pmatrix}$$

Aharanov-Bohm(-like) phase or holonomy

$$Q_{\Lambda\Lambda} = \frac{1}{6} \begin{pmatrix} +1 & +1 & +1 \\ +1 & +1 & +1 \\ +1 & +1 & +1 \end{pmatrix}$$

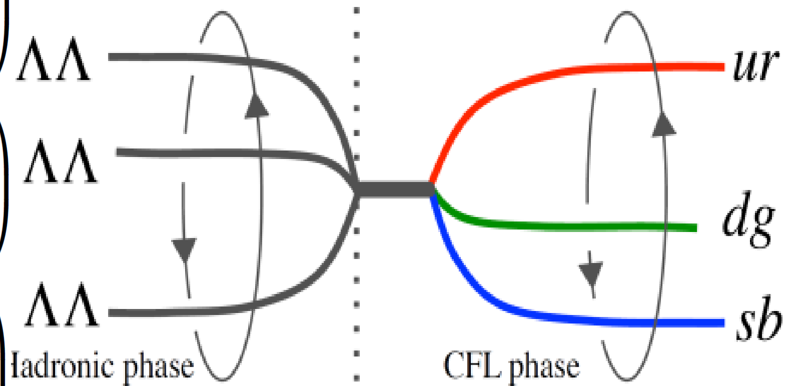
Aharanov-Bohm(-like) phase matching

$$(q)_{\alpha i} = \begin{pmatrix} u_r & d_r & s_r \\ u_g & d_g & s_g \\ u_b & d_b & s_b \end{pmatrix}$$

$$Q_{\Lambda\Lambda} = \frac{1}{6}\begin{pmatrix} +1 & +1 & +1 \\ +1 & +1 & +1 \\ +1 & +1 & +1 \end{pmatrix}$$

$$Q_{\Lambda\Lambda} = \frac{1}{6}\begin{pmatrix} +1 & +1 & +1 \\ +1 & +1 & +1 \\ +1 & +1 & +1 \end{pmatrix}$$

$$Q_{\Lambda\Lambda} = \frac{1}{6}\begin{pmatrix} +1 & +1 & +1 \\ +1 & +1 & +1 \\ +1 & +1 & +1 \end{pmatrix}$$



$$Q_{ur} = \frac{1}{2}\begin{pmatrix} -1 & 0 & 0 \\ 0 & +1 & +1 \\ 0 & +1 & +1 \end{pmatrix}$$

$$Q_{dg} = \frac{1}{2}\begin{pmatrix} +1 & 0 & +1 \\ 0 & -1 & 0 \\ +1 & 0 & +1 \end{pmatrix}$$

$$Q_{sb} = \frac{1}{2}\begin{pmatrix} +1 & +1 & 0 \\ +1 & +1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$Q_{\Lambda\Lambda} \neq Q_{ur},\, Q_{dg},\, Q_{sb}$$

$$Q_{ur+dg+sb}(= Q_{ur} + Q_{dg} + Q_{sb}) = 3Q_{\Lambda\Lambda}$$

Collaborators

Hadron, **HEP**, **Cond-mat**

Eiji Nakano（Kouchi）, **Taeko Matsuura**（Hokkaido）,
Minoru Eto（Yamagata）, **Naoki Yamamoto**（Keio）,
Shigehiro Yasui(Titech) , **Kazunori Itakura**（KEK),
Yuji Hirono（BNL）, **Takuya Kanazawa**(RIKEN),
Takanori Fujiwara(Ibaragi), **Takahiro Fukui**(Ibaragi),
Walter Vinci（USC）, **Mattia Cipriani**（Pisa）,
Michikazu Kobayashi（Kyoto）, **Chandrasekhar Chatterjee** (Keio)

[1] with Nakano,Matsuura, Phys.Rev.D78:045002,2008 [arXiv:0708.4096]
[2] with Eto, Phys.Rev.D80:125007,2009 [arXiv:0907.1278]
[3] with Eto,Nakano, Phys.Rev.D80:125011,2009 [arXiv:0908.4470]
[4] with Eto,Yamamoto, Phys.Rev.Lett.104:161601,2010 [arXiv:0912.1352]
[5] with Yasui,Itakura, Phys.Rev.D81:105003,2010 [arXiv:1001.3730]
[6] with Hirono,Kanazawa, Phys.Rev.D83:085018,2011 [arXiv:1012.6042]
[7] with Eto,Yamamoto, Phys.Rev.D83:085005,2011 [arXiv:1101.2574]
[8] with Fujiwara,Fukui,Yasui, Phys.Rev.D84:076002,2011 [arXiv:1105.2115]
[9] with Hirono, Phys.Rev.Lett.109:062501,2012 [arXiv:1203.5059]
[10] with Vinci,Cipriani, Phys.Rev.D86:085018,2012 [arXiv:1206.3535]
[11] with Cipriani,Vinci, Phys.Rev.D86:121704,2012 [arXiv:1208.5704]
[12] with Kobayashi, PTEP:021B01,2014 [arXiv:1307.6632]
[13] with Eto,Hirono,Yasui, PTEP 2014 (2014) 012D01, invited review [arXiv:1308.1535]
[14] with Kobayashi,Nakano, JHEP 1406,130:2014 [arXiv:1311.2399]
[15] with Chatterjee, Phys.Rev. D93: 065050, 2016 [arXiv:1512.06603]
[16] with Chatterjee, Cipriani, Phys. Rev. D 93, 065046 (2016) [arXiv:1602.01677]
[17] with Chatterjee, Phys.Rev. D95 (2017) 085013 [arXiv:1612.09419]

Non-Abelian Statistics of Majorana fermions

[18] with Yasui,Itakura, Phys.Rev.B83:134518,2011 [arXiv:1010.3331]
[19] with Yasui,Itakura, Nucl.Phys.B859:261-268,2012 [arXiv:1109.2755]
[20] with Hirono,Yasui,Itakura,Phys.Rev.B86:014508,2012 [arXiv:1203.0173]
[21] with Yasui,Hirono,Itakura,Phys.Rev.E87:052142,2013 [arXiv:1204.1164]

Chiral symmetry breaking

[22] with Shiiki Phys.Lett.B658:143-147,2008 [arXiv:0708.4091]
[23] with Nakano,Matsuura, Phys.Lett.B672:61-64,2009 [arXiv:0708.4092]
[24] with Eto,Nakano, Nucl.Phys.B821:129-150,2009 [arXiv:0903.1528]
[25] with Eto,Hirono, PTEP 2014 (2014) 033B01 [arXiv:1309.4559]