熱場の量子論とその応用 2018

光格子冷却原子系における Dirac fermion量子シミュレータの構築法と そのトポロジカル物性

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Y. K, I. Ichinose, Y. Takahashi, Sci. Rep. 8, 10699 (2018)

Summary

How to construct 1D topological insulating model
→ 1D generalized Wilson-Dirac model

Generalize matrix:
Implement four Peierls phases

Experimental proposal

I. Two parallel tilted optical lattice → Energy offset term

II. Laser assisted hopping scheme → Dirac gamma matrices

Symmetry aspect

Only requirement for Piers phase

$$\theta_a = \theta_b \pm \pi.$$

This condition preserves Chiral symmetry.

→ 1D Wilson-Dirac type simulator in topological BDI class.

Application

- Simulator of 1D topological insulator
- (1+1)-D lattice Gross-Neveu model

Outline

1. Introduction

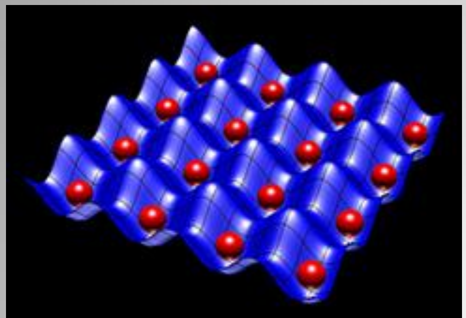
2. 1D generalized Wilson-Dirac model

3. General construction scheme

4. Symmetry aspects and non-trivial topological phase

5. Other related topics

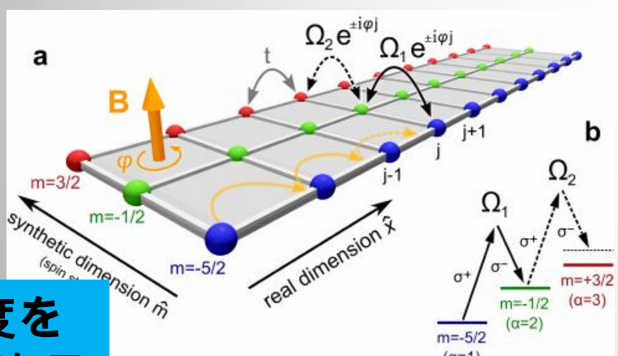
Ultracold atomic gas in an optical lattice & quantum simulator



- Parameter controllability
- Synthetic dimension, artificial gauge field
- High-resolution measurement scheme

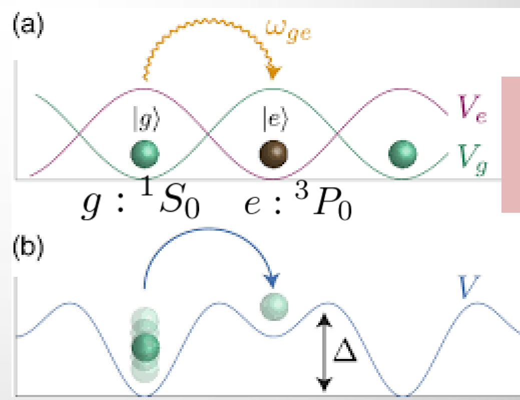
= Ideal platform of quantum simulator for various theoretical models

Recent experimental progress



内部自由度を用いた人工次元＆人工磁場

M. Mancini, et al., Science **349**, 6255 (2015).



長寿命励起状態を用いた人工磁場

cf. ^{173}Yb

F. Gerbier and J. Dalibard, New J. Phys. **12**, 033007(2010).

実験では光格子冷却原子系（フェルミ、ボソン）で原子の内部自由度を巧みに用いた人工ゲージ場等が実装できる段階に技術が進歩してきている

Ultracold atomic gas in an optical lattice & quantum simulator

Quantum Simulation:

- 1D topological insulating models.
- Simulate relativistic physics

Question：

How to realize 1D lattice Dirac fermion in cold-atomic gas in an optical lattice ?

Dirac fermion

- 1次元トポロジカル絶縁体
- 格子上のDirac Fermion

物性物理学、格子ゲージ理論の理論モデルに広く現れるフェルミ粒子系トイモデルを冷却原子系上で深く理解する

・実験に先立ち理論から考え得る目標を提示する
・技術的困難をクリアーにする
・その系でシミュレートされうる物理現象を予測する

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2. 1D generalized Wilson-Dirac model
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1D generalized Wilson-Dirac model

$\Psi_j = {}^t(a_j, b_j)$: spinor

$$H^{(g)}_{\mathrm{GWDM}} = \sum_j \Psi_j^\dagger \Gamma_z(\Delta)\Psi_j - \sum_j \left[\Psi_{j+1}^\dagger \Gamma_z^h(\theta_a,\theta_b)\Psi_j + \mathrm{h.c.}\right] + \sum_j \left[\Psi_{j+1}^\dagger \Gamma_x(\theta^+,\theta^-)\Psi_j + \Psi_{j+1}\Gamma_x^*(\theta^+,\theta^-)\Psi_j^\dagger\right],$$

Gamma matrix in 1D system

$$\gamma_0 \to \sigma_z$$
$$\gamma_1 \to \sigma_x$$

Generalized gamma matrix

$$\Gamma_z(\Delta) = \begin{bmatrix} \Delta & 0 \\ 0 & -\Delta \end{bmatrix} = \Delta\sigma_z,$$

$$\Gamma_z^h(\theta_a,\theta_b) = \begin{bmatrix} |J_a|e^{i\theta_a} & 0 \\ 0 & |J_b|e^{i\theta_b} \end{bmatrix},$$

$$\Gamma_x(\theta^+,\theta^-) = \begin{bmatrix} 0 & |J_{ab}^-|e^{i\theta^-} \\ |J_{ab}^+|e^{-i\theta^+} & 0 \end{bmatrix},$$

Parameters

$|J_a|,\ |J_b|,\ |J_{ab}^+|,\ |J_{ab}^-|$ Hopping amplitudes

$\theta_a,\ \theta_b,\ \theta^+,\ \theta^-$ Peierls phases

Δ Energy-offset

1D generalized Wilson-Dirac model: Ordinary form

The model is reduced to (1+1)-D ordinary Wilson-Dirac model = Standard model of 1D topological insulator

Set parameter

$$|J_a| = |J_b| = t',\ |J^+_{ab}| = |J^-_{ab}| = t$$
$$\theta_a = 0,\ \theta_b = \pi,\ \theta^+ = -\theta^- = -\pi/2$$

$$\bar{\Phi}_j = \Psi^\dagger_j \gamma_0$$

$$H_{GWDM} = -\Delta \sum_j \bar{\Phi}_j \Phi_j - t \sum_j \left[\bar{\Phi}_{j+1} \Phi_j + \bar{\Phi}_j \Phi_{j+1} \right] + t' \sum_j \left[\bar{\Phi}_{j+1} \gamma_1 \Phi_j - \bar{\Phi}_j \gamma_1 \Phi_{j+1} \right],$$

$t = t',\ -2 < \Delta/t < 2$ → **Non-trivial topological phase** $W_N = 1$

$t = t',\ \Delta/t < -2,\ \Delta/t > 2$ → **Trivial insulating phase** $W_N = 0$

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1D generalized Wilson-Dirac model: Fermion atom picture

$$H^{(g)}_{\mathrm{GWDM}} = H_{\mathrm{spinOL}} + H_{\mathrm{ahop}} + H_{\mathrm{bhop}} + H^{+}_{ab} + H^{-}_{ab},$$

$$H_{spinOL} = \sum_j \Delta(a^\dagger_j a_j - b^\dagger_j b_j)$$

Energy-offset term

$$H_{ahop} = -\sum_j |J_a| e^{i\theta_a} a^\dagger_j a_{j+1} + \mathrm{h.c.}$$

$$H_{bhop} = -\sum_j |J_b| e^{i\theta_b} b^\dagger_j b_{j+1} + \mathrm{h.c.}$$

$$H^{+}_{ab} = -\sum_j |J^{+}_{ab}| e^{i\theta^{+}} a^\dagger_j b_{j+1} + \mathrm{h.c.}$$

exchange

$$H^{-}_{ab} = -\sum_j |J^{-}_{ab}| e^{i\theta^{-}} a^\dagger_j b_{j-1} + \mathrm{h.c.}$$

exchange

Hopping terms
with **Peierls phase**
Origin: Dirac-gamma matrix

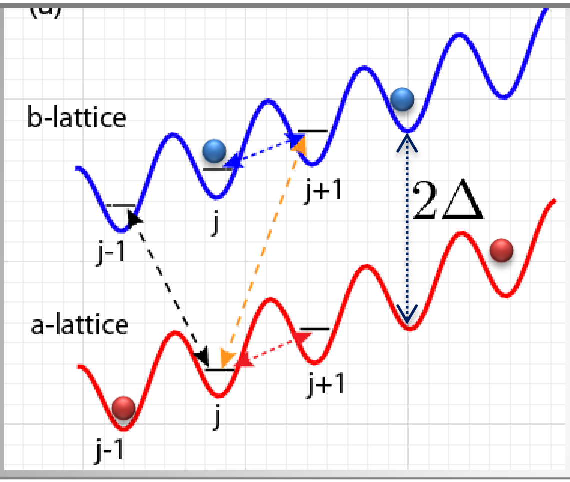
How to implement Dirac-gamma matrices in optical lattice system ?

General construction scheme

"Combine two experimental techniques"

*O.Mandel, et.al, PRL **91**, 010407 (2003).

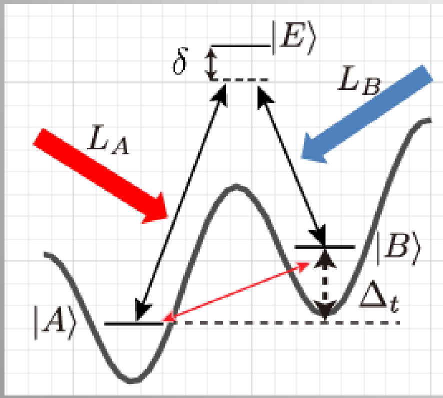
I. Two parallel tilted optical lattice → **Energy offset term**

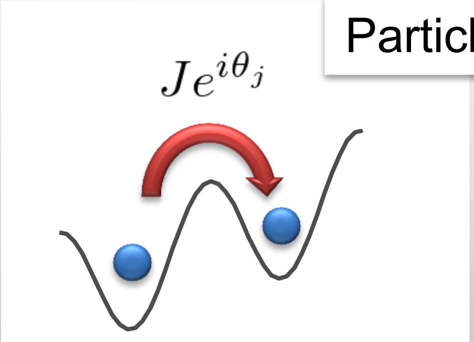


- Spin dependent lattice (vector light shift) + Zeeman splitting
- Lattice tilt: Gravity, Electric field gradient

$$H_{\text{spinOL}} = \sum_{j=1} \Delta(a_j^\dagger a_j - b_j^\dagger b_j)$$

II. Laser assisted hopping scheme → **Dirac gamma matrices**



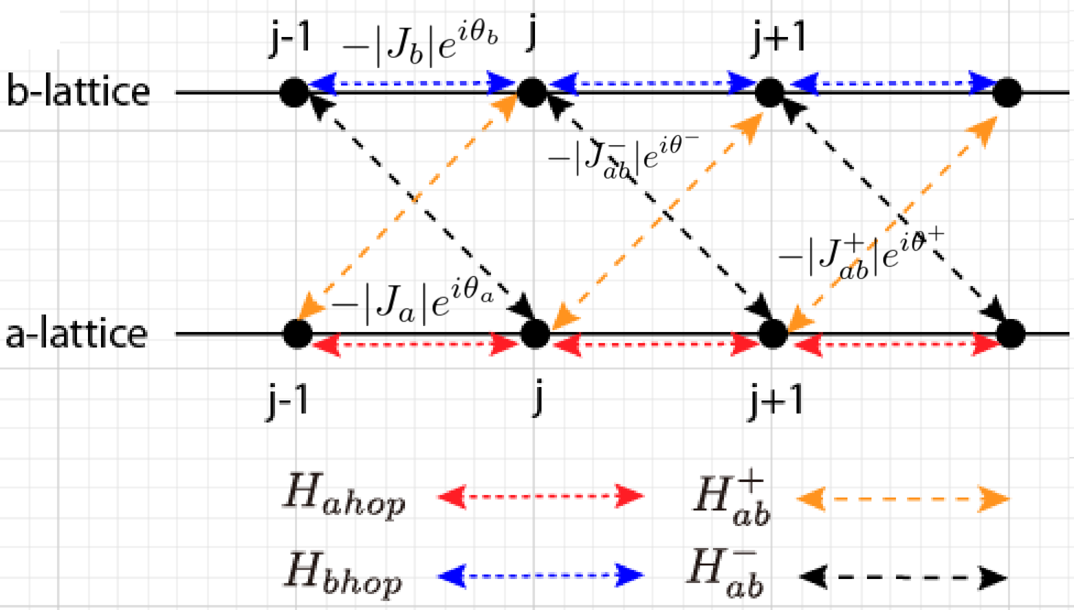


Particle hops with **Peierls** phase

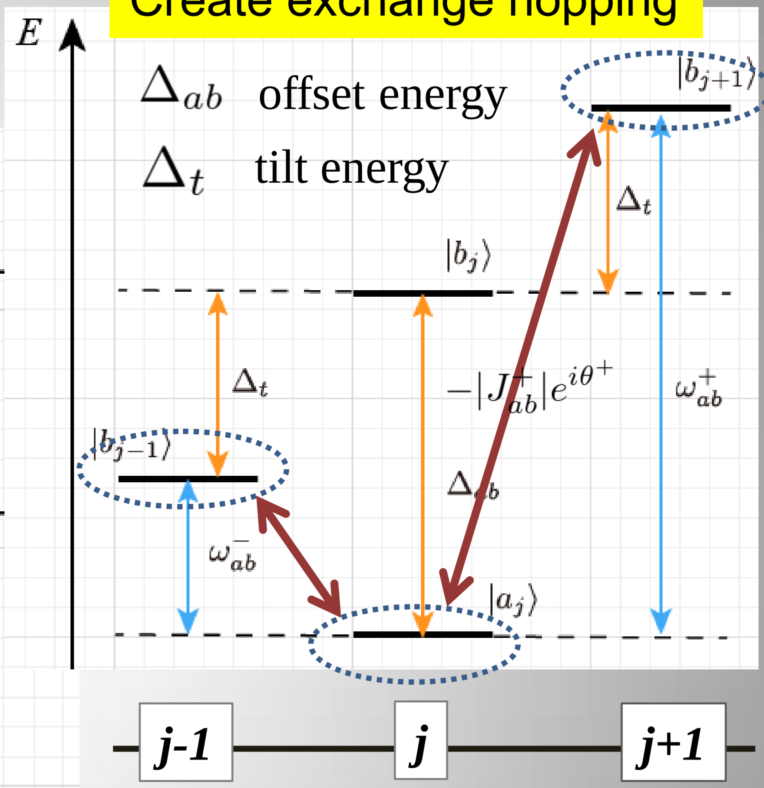
Apply four kinds of laser-assisted hopping

Implement "four kinds of laser-assisted hopping"

- Lattice tilt
- Choice suitable Energy levels
- Use both polarized lasers π or $\sigma^{\pm}$



Create exchange hopping



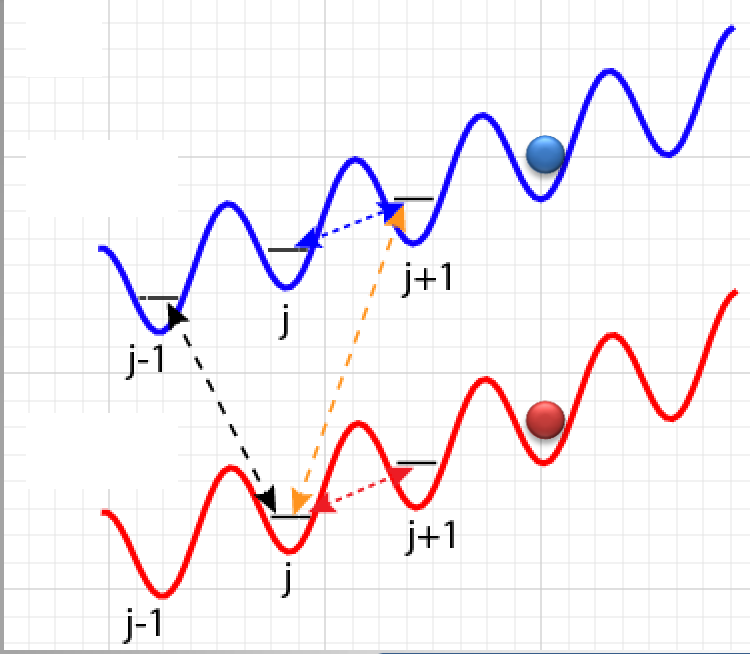
$$H_{ahop} = -\sum_j |J_a|e^{i\theta_a} a_j^\dagger a_{j+1} + \text{h.c.}$$

$$H_{ab}^+ = -\sum_j |J_{ab}^+|e^{i\theta^+} a_j^\dagger b_{j+1} + \text{h.c.}$$

$$H_{bhop} = -\sum_j |J_b|e^{i\theta_b} b_j^\dagger b_{j+1} + \text{h.c.}$$

$$H_{ab}^- = -\sum_j |J_{ab}^-|e^{i\theta^-} a_j^\dagger b_{j-1} + \text{h.c.}$$

Feasible theoretical proposal: Concrete example $^{171}\mathrm{Yb}$



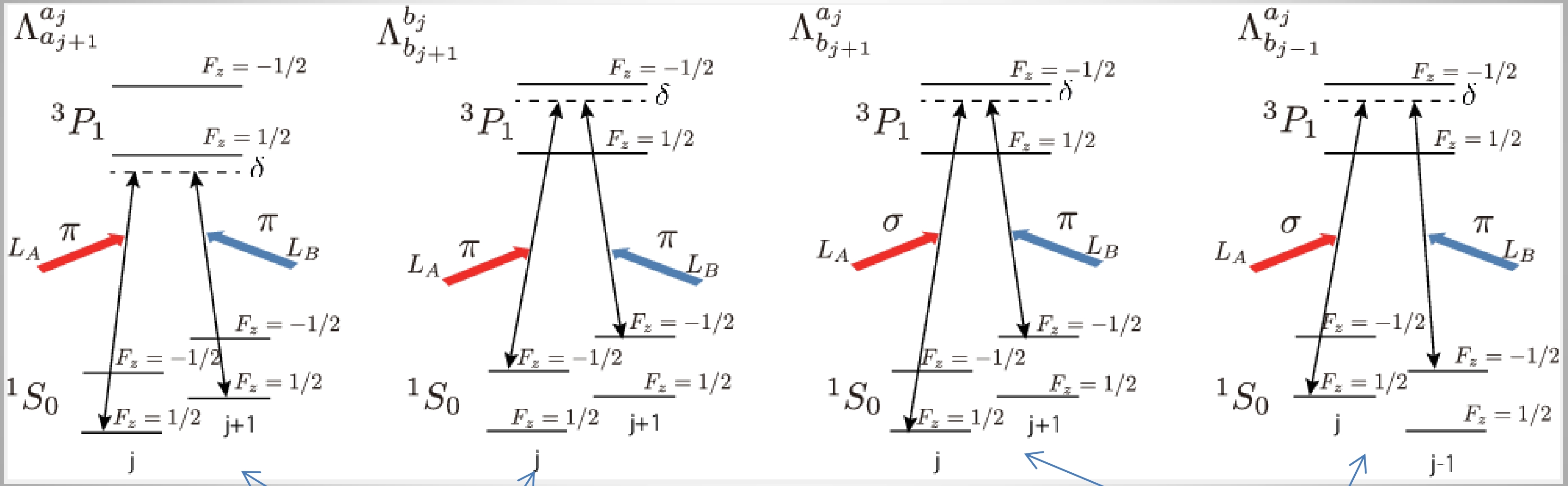
- Making use of polalization of lasers in the laser-assisted hopping scheme π or $\sigma^{\pm}$
- Large energy splitting in $^3P_1$ excited state manifold.
- Select suitable polalizability for $^3P_1$ excited states.

Pseudo-spin picture			Experimental candidate
			$^{171}\mathrm{Yb}$
			Two hyperfine state in $^1S_0$
Up spin	$\vert a\rangle$	a_j	$\vert {}^1S_0,\ F_z = 1/2\rangle$
Down spin	$\vert b\rangle$	b_j	$\vert {}^1S_0,\ F_z = -1/2\rangle$

Schematic figure of four kinds of laser-assisted hopping

TABLE I. Four kind of laser-assisted hopping by using the hyperfine structure of ^{171}Yb

	$\Lambda^{a_j}_{a_{j+1}}$	$\Lambda^{b_j}_{b_{j+1}}$	$\Lambda^{a_j}_{b_{j+1}}$	$\Lambda^{a_j}_{b_{j-1}}$
$\vert A\rangle$	$^1S_0,\ F_z=1/2$	$^1S_0,\ F_z=-1/2$	$^1S_0,\ F_z=1/2$	$^1S_0,\ F_z=1/2$
$\vert B\rangle$	$^1S_0,\ F_z=1/2$	$^1S_0,\ F_z=-1/2$	$^1S_0,\ F_z=-1/2$	$^1S_0,\ F_z=-1/2$
$\vert E\rangle$	$^3P_1,\ F_z=1/2$	$^3P_1,\ F_z=-1/2$	$^3P_1,\ F_z=-1/2(1/2)$	$^3P_1,\ F_z=-1/2(1/2)$
(L_A, L_B)	(π,π)	(π,π)	$(\sigma(\pi)\ ,\pi(\sigma))$	$(\sigma(\pi),\pi(\sigma))$



$$H_{\rm ahop} = -\sum_j J_a a^\dagger_{j+1} a_j + \text{h.c.},$$

$$H_{\rm bhop} = -\sum_j J_b b^\dagger_{j+1} b_j + \text{h.c.},$$

$$H^+_{ab} = \sum_j J^+_{ab} a^\dagger_j b_{j+1} + \text{h.c.}$$

$$H^-_{ab} = \sum_j J^-_{ab} a^\dagger_j b_{j-1} + \text{h.c.},$$

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Symmetry classification

Symmetry				d							
AZ	Θ	Ξ	Π	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	−1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	−1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	−1	−1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	−1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	−1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

*S. Ryu, et.al.,
New J. Phys.12, 065010 (2010).

For the 1D model
Quantum simulator of topological model:
⇒ **Target: BDI class**:
- Chiral $\mathcal{S} = \mathcal{T} \cdot \mathcal{C}$
- Time reversal $\mathcal{T}$
- Charge conjugation (particle-hole) $\mathcal{C}$

Four Piers phase parameters

$-|J_a|e^{i\theta_a}$ $-|J_b|e^{i\theta_b}$ $-|J_{ab}^+|e^{i\theta^+}$ $-|J_{ab}^-|e^{i\theta^-}$

$$H^{(g)}_{\mathrm{GWDM}} = \sum_j \Psi_j^\dagger \Gamma_z(\Delta)\Psi_j - \sum_j \left[\Psi_{j+1}^\dagger \Gamma_z^h(\theta_a,\theta_b)\Psi_j + \mathrm{h.c.}\right] + \sum_j \left[\Psi_{j+1}^\dagger \Gamma_x(\theta^+,\theta^-)\Psi_j + \Psi_{j+1}\Gamma_x^*(\theta^+,\theta^-)\Psi_j^\dagger\right]$$

Change physical properties
- **symmetries of the Hamiltonian**,
- energy spectrum,
- ground state

"How to design 1D GWDM to be BDI class"

Symmetry aspect

$$|J_a| = |J_b| = |J_{ab}^{+}| = |J_{ab}^{-}| = 1$$

$$H_{\text{bulk}}(k) = \begin{bmatrix} \Delta - 2\cos(k+\theta_a) & e^{-ik+i\theta^{+}} + e^{ik+i\theta^{-}} \\ e^{-ik-i\theta^{-}} + e^{ik-i\theta^{+}} & -\Delta - 2\cos(k+\theta_b) \end{bmatrix},$$

Chiral $\mathcal{S}$

Symmetric about E=0

$$\theta_a = \theta_b \pm \pi.$$

This condition preserves Chiral symmetry.

TR and CC

$$H_{\text{bulk}}^{\text{CS}}(k) = [\Delta - 2\cos(k+\theta_a)]\sigma_z + \left[2\cos\left(k - \frac{\theta^{+}-\theta^{-}}{2}\right)\right]\tilde{\sigma}_x\left(\frac{\theta^{+}+\theta^{-}}{2}\right).$$

Pauli matrices rotated around the z-spin axis are given as

$$\tilde{\sigma}_x(\phi) = \begin{bmatrix} 0 & e^{i\phi} \\ e^{-i\phi} & 0 \end{bmatrix}$$

We find

$$U_T = \exp[i(\theta^{+} + \theta^{-})\sigma_z]$$

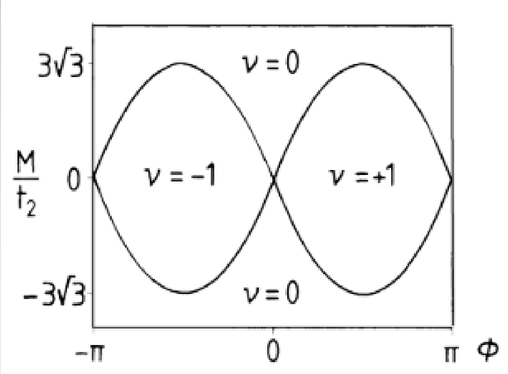
$$U_C = \sigma_y \exp[i(\theta^{+} + \theta^{-})\sigma_z]$$

We need not to implement the ordinary 1D Wilson-Dirac fermion to simulate topological properties.

Phase structure of non-trivial topological phase

Same phase structure
with the ones of
the “topological Haldane model”.

*F. D. M. Haldane, PRL **61**, 2015 (1988).



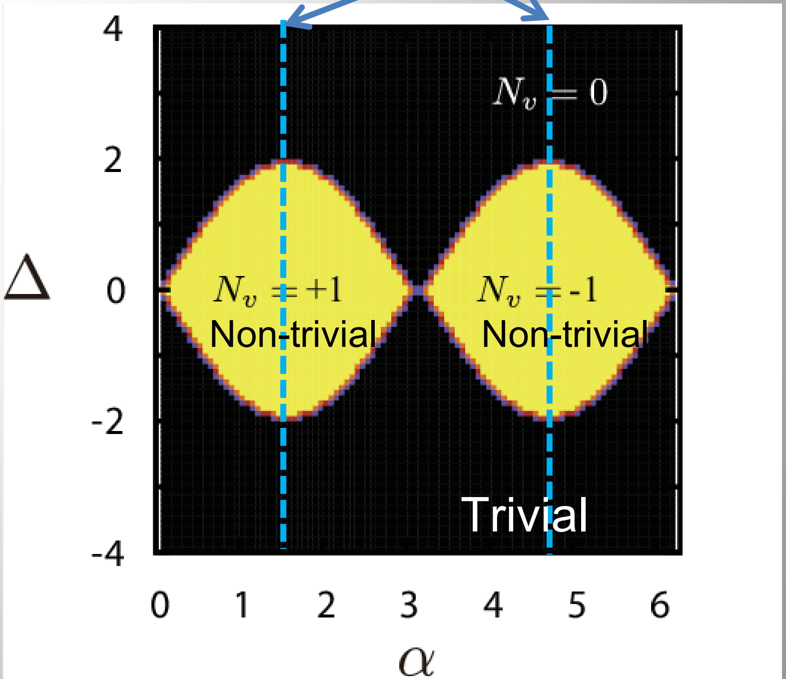
$$\theta_a = \theta_b \pm \pi. \qquad \alpha = -\theta_a - (\theta^+ - \theta^-)/2$$

$$\begin{aligned}\mathbf{d}(k) &= (d_x(k), d_z(k)) \\ &= (2\cos k, \Delta - 2\cos(k-\alpha))\end{aligned}$$

- Bulk-Topological properties can be characterized by

$$N_v = \int_{-\pi}^{\pi} \frac{dk}{2\pi} \frac{\mathbf{d}(k)}{|\mathbf{d}(k)|} \times \frac{d}{dk}\left(\frac{\mathbf{d}(k)}{|\mathbf{d}(k)|}\right).$$

Ordinary Wilson–Dirac model



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Quantum simulator of (1+1)-D lattice Gross-Neveu model

Possibility of atomic quantum simulator for high-energy physics

$$V_{int} = \sum_j V a_j^\dagger a_j b_j^\dagger b_j = \sum_j \frac{V}{2}(\Psi_j^\dagger \gamma_0 \Psi_j)^2,$$

$^{173}\mathrm{Yb}$

Finite s-wave scattering

(1+1)-D Gross-Neveu model (Wilson fermion)

*D. J. Gross and A. Neveu, PRD 10, 3235 (1974).

$$\begin{aligned} &H_{GWDM} + V_{int} \\ &= -\Delta \sum_j \bar{\Phi}_j \Phi_j - t \sum_j \left[\bar{\Phi}_{j+1} \Phi_j + \bar{\Phi}_j \Phi_{j+1} \right] \\ &+ t' \sum_j \left[\bar{\Phi}_{j+1} \gamma_1 \Phi_j - \bar{\Phi}_j \gamma_1 \Phi_{j+1} \right] + \sum_j \frac{V}{2} (\bar{\Phi}_j \Phi_j)^2 \end{aligned}$$

There exists non-trivial phase transition

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