Open quantum physics: recent topics in non-Hermitian systems and beyond

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Collaborators:
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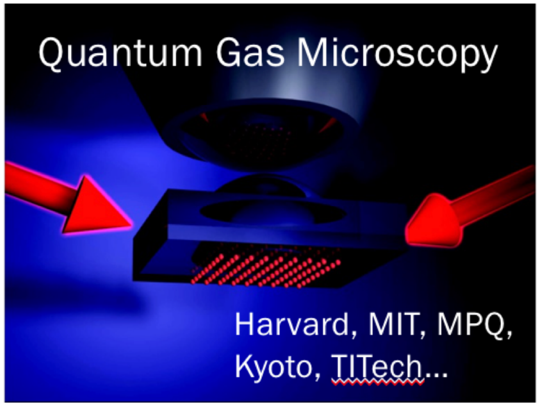
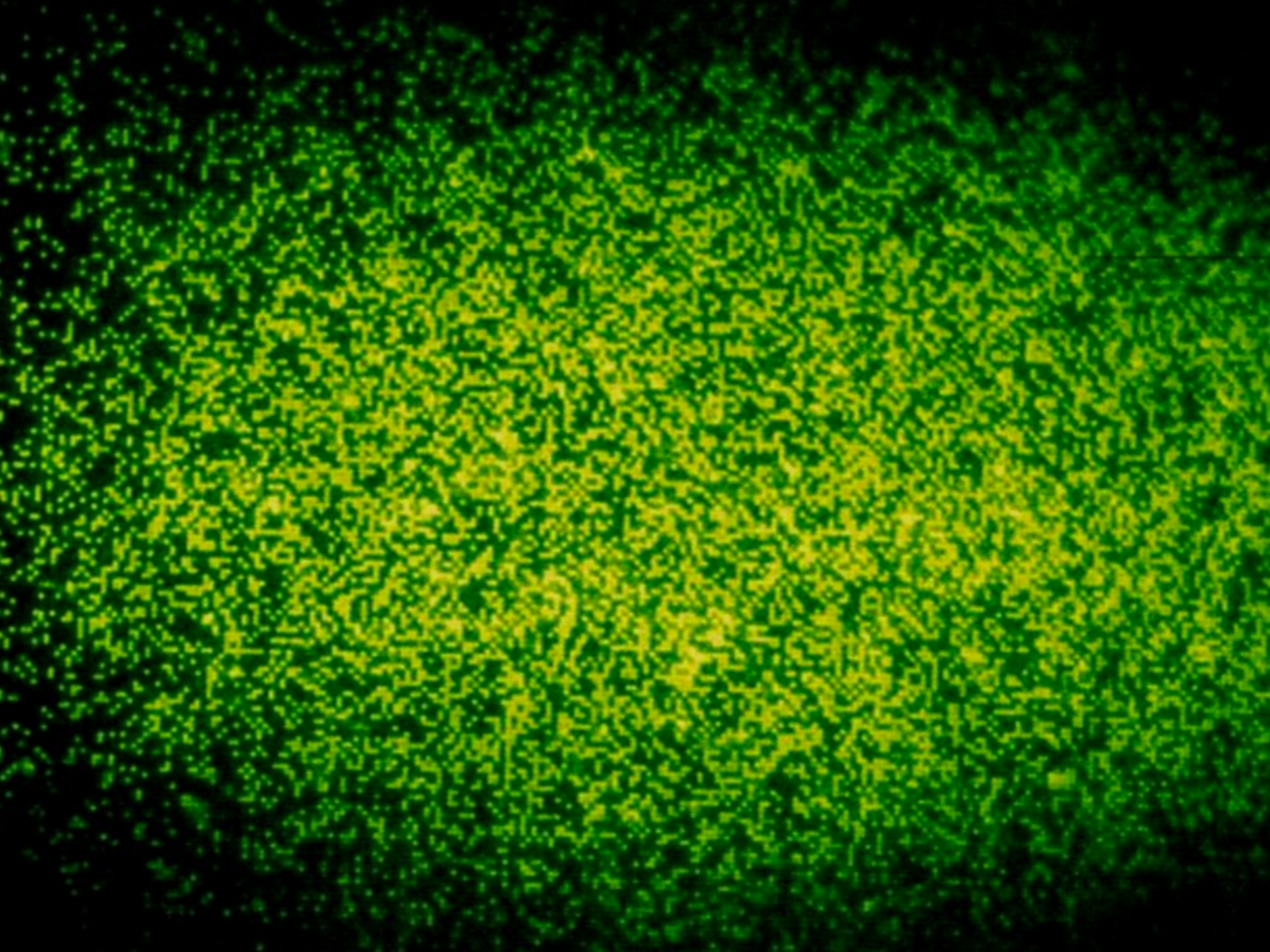
Eugene Demler (Harvard), Richard Shmidt (Harvard→MPQ), Leticia Tarruell (ICFO),
Ignacio Cirac (MPQ), Tao Shi (MPQ→CAS), Mari Carmen Banuls (MPQ)

Microscopic observation of Macroscopic Quantum systems

Revolutionary techniques to **observe** and **manipulate** many-body systems:

- Observing many-body systems at the single-atom level.





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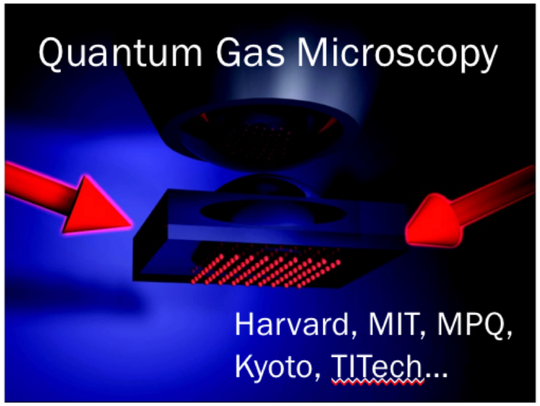
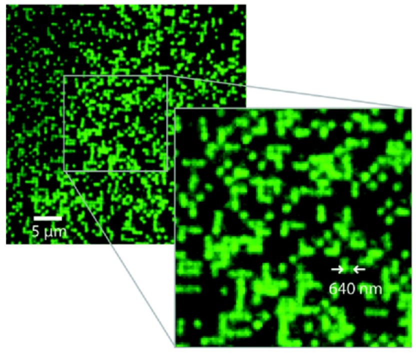


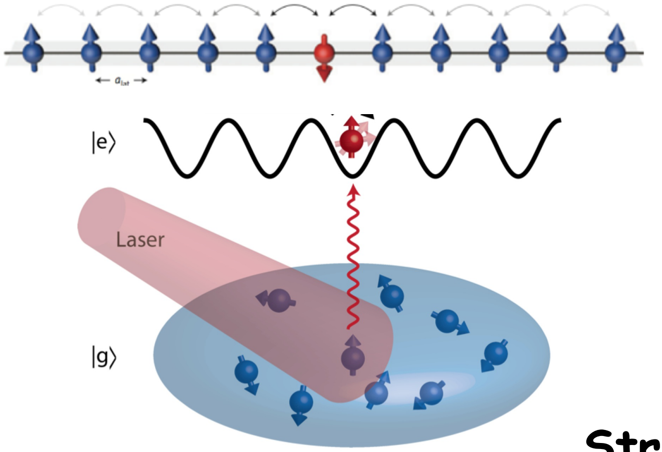
single-shot
imaging



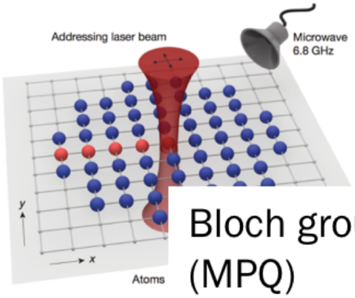
Measurement backaction on many-body systems?

- Manipulating **single** quantum particles in many-body systems.

spin
impurity



Single-site addressing



Bloch group
(MPQ)

Strongly correlated open systems?

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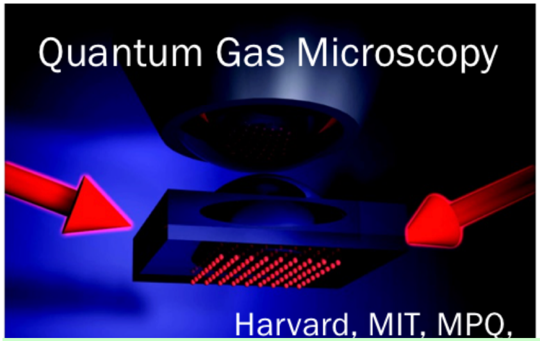

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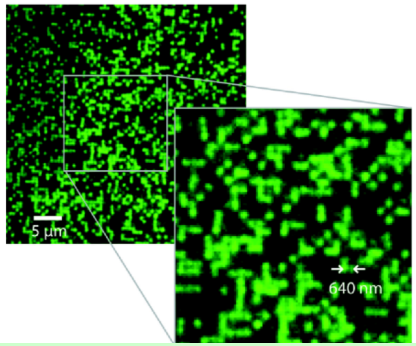
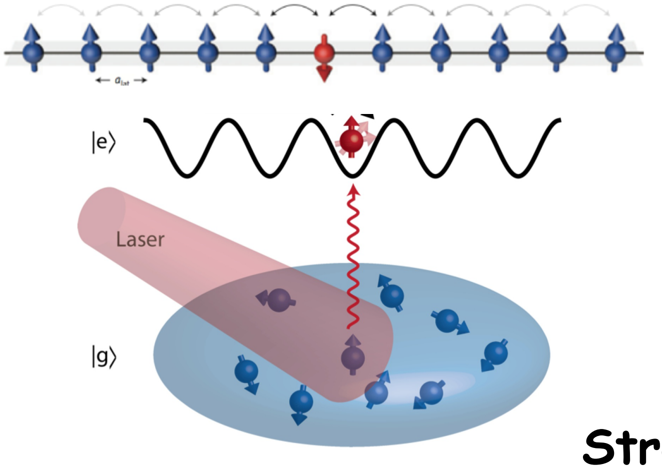


New frontier pioneered with microscopic observation and manipulation capabilities of many-body systems?

s?

spin impurity



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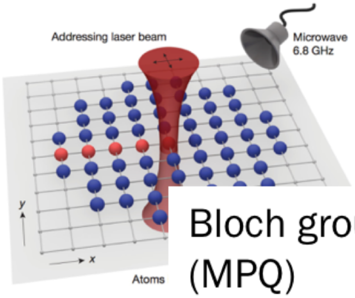

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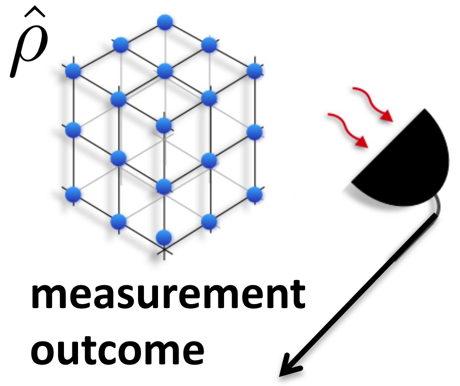
Strongly correlated open systems?

Continuous Monitoring of Quantum Systems: Quantum Trajectory



Non-unitary dynamics due to the measurement backaction.

If one does not know measurement outcomes,

→ Lindblad master equation (**dissipative dynamics**)

$$\frac{d\hat{\rho}}{dt} = \boxed{-i[\hat{H},\hat{\rho}]} - \sum_a \left(\boxed{\frac{1}{2}\hat{L}_a^\dagger \hat{L}_a \hat{\rho} + \frac{1}{2}\hat{\rho}\hat{L}_a^\dagger \hat{L}_a} - \boxed{\hat{L}_a \hat{\rho} \hat{L}_a^\dagger} \right)$$

$\hat{L}_a$: jump operator

Continuous observation

If one can **access** the information about measurement outcomes,

→ **Quantum trajectory dynamics** (keeping the **purity** of the state)

$$d|\psi\rangle = \underbrace{-i\hat{H}|\psi\rangle dt}_{\text{unitary evolution}} \underbrace{- \sum_a \frac{1}{2}\overset{\text{(norm preservation)}}{\left(\hat{L}_a^\dagger \hat{L}_a - \langle \hat{L}_a^\dagger \hat{L}_a \rangle\right)} |\psi\rangle dt}_{\text{non-unitary evolution without quantum jumps}} + \underbrace{\sum_a \left(\frac{\hat{L}_a|\psi\rangle}{\sqrt{\langle \hat{L}_a^\dagger \hat{L}_a \rangle}} - |\psi\rangle \right) dN_a}_{\text{stochastic quantum jumps}}$$

*Taking ensemble average *E* over *dN*$_a$, $\hat{\rho} = E[|\psi\rangle\langle\psi|]$ obeys Lindblad master eq.

Dalibard, Castin, Mølmer; Carmichael (1993)

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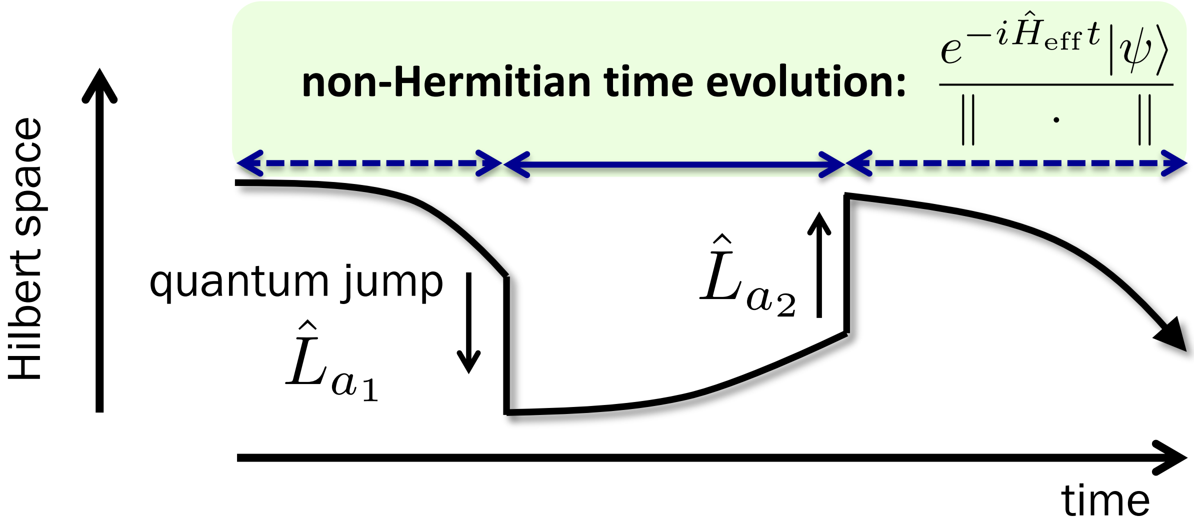

(effective non-Hermitian Hamiltonian)

$$\hat{H}_{\text{eff}} = \hat{H} - \frac{i}{2}\sum_a \hat{L}_a^\dagger \hat{L}_a$$

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Dalibard, Castin, Mølmer; Carmichael (1993)

Continuous Observation: the ability to access quantum jumps

What does differentiate **continuous observation** from dissipation?

The ability to access the information about quantum jumps.

examples of continuously monitored dynamics:

- complete time record of all quantum jumps

$$\hat{\rho}_{\mathrm{traj}}(t) = |\psi_{\mathrm{traj}}(t)\rangle\langle\psi_{\mathrm{traj}}(t)|$$

- coarse-grained information: the number of quantum jumps

post-measurement state

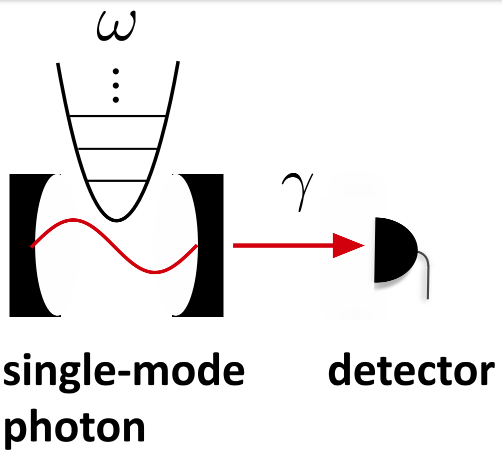
$$\hat{\rho}_{\mathrm{post}}(t) = \sum_{i\in\mathcal{D}} |\psi_{\mathrm{traj},i}(t)\rangle\langle\psi_{\mathrm{traj},i}(t)|$$

$\mathcal{D}$: subspace of quantum trajectories
determined by a number of jumps

- no-count process: **non-Hermitian** dynamics

Such continuously observed dynamics is different from the dissipative Lindblad dynamics (where the system is easily driven into, e.g., an infinite temperature state or the vaccum).

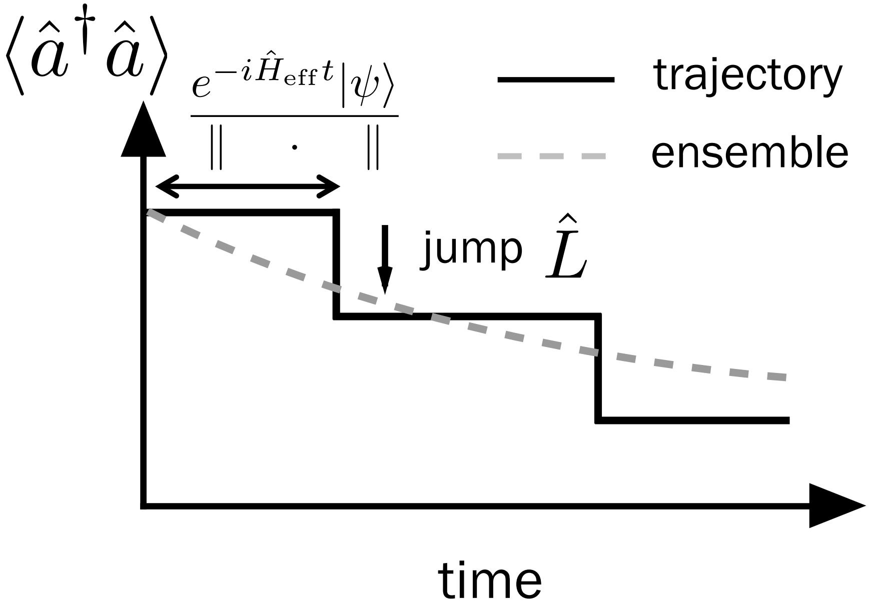
Simple example: continuous observation of cavity photons



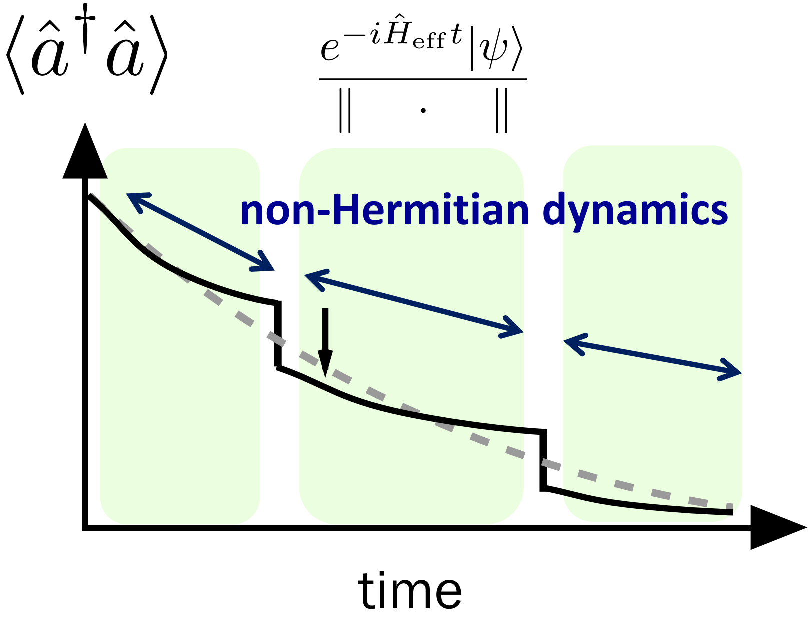
Effective Hamiltonian : $\hat{H}_{\mathrm{eff}} = \left(\omega - \frac{i\gamma}{2}\right)\hat{a}^\dagger\hat{a}$

jump operator : $\hat{L} = \sqrt{\gamma}\hat{a}$

example 1: Fock state

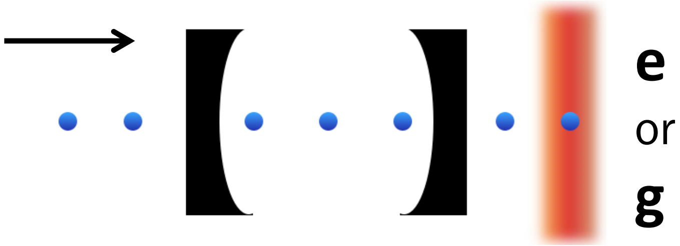


example 2: Squeezed state

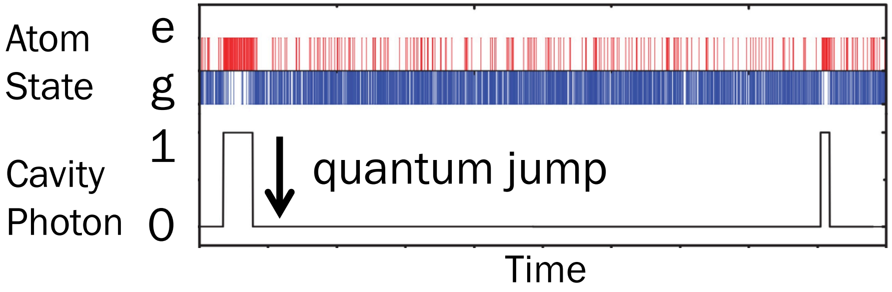


Experiments: continuous observation of small quantum systems

Microwave cavity photon



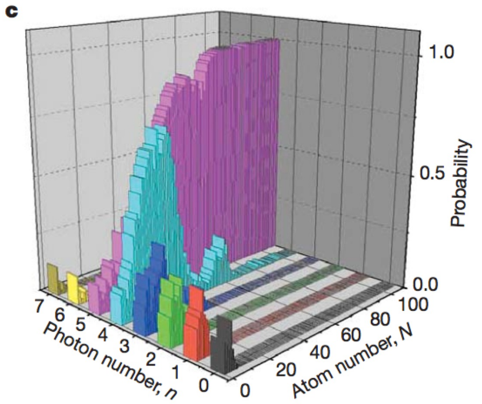
observation of quantum jumps



Gleyzes et al., Nature 446, 297 (2007)

Serge Haroche
2012 Nobel Prize

observation of wavefunction collapse



Guerlin et al., Nature 448, 889 (2007)

Superconducting qubit

Vijay et al., PRL 106, 110502 (2011)

Quantum dots

Vamivakas et al., Nature 467, 297 (2010)

Experiments: continuous observation of small quantum systems

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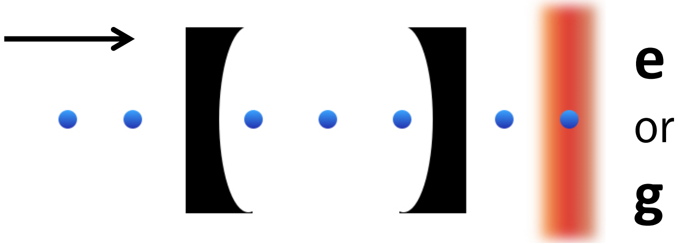

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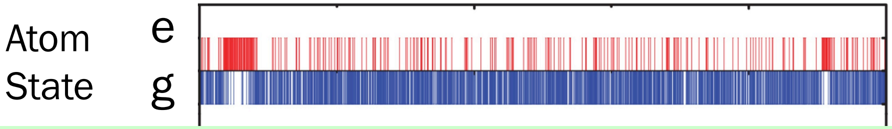

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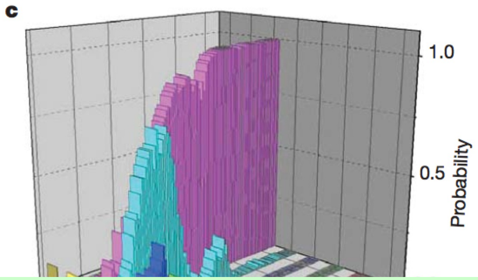


Restricted to quantum systems
with small degrees of freedom

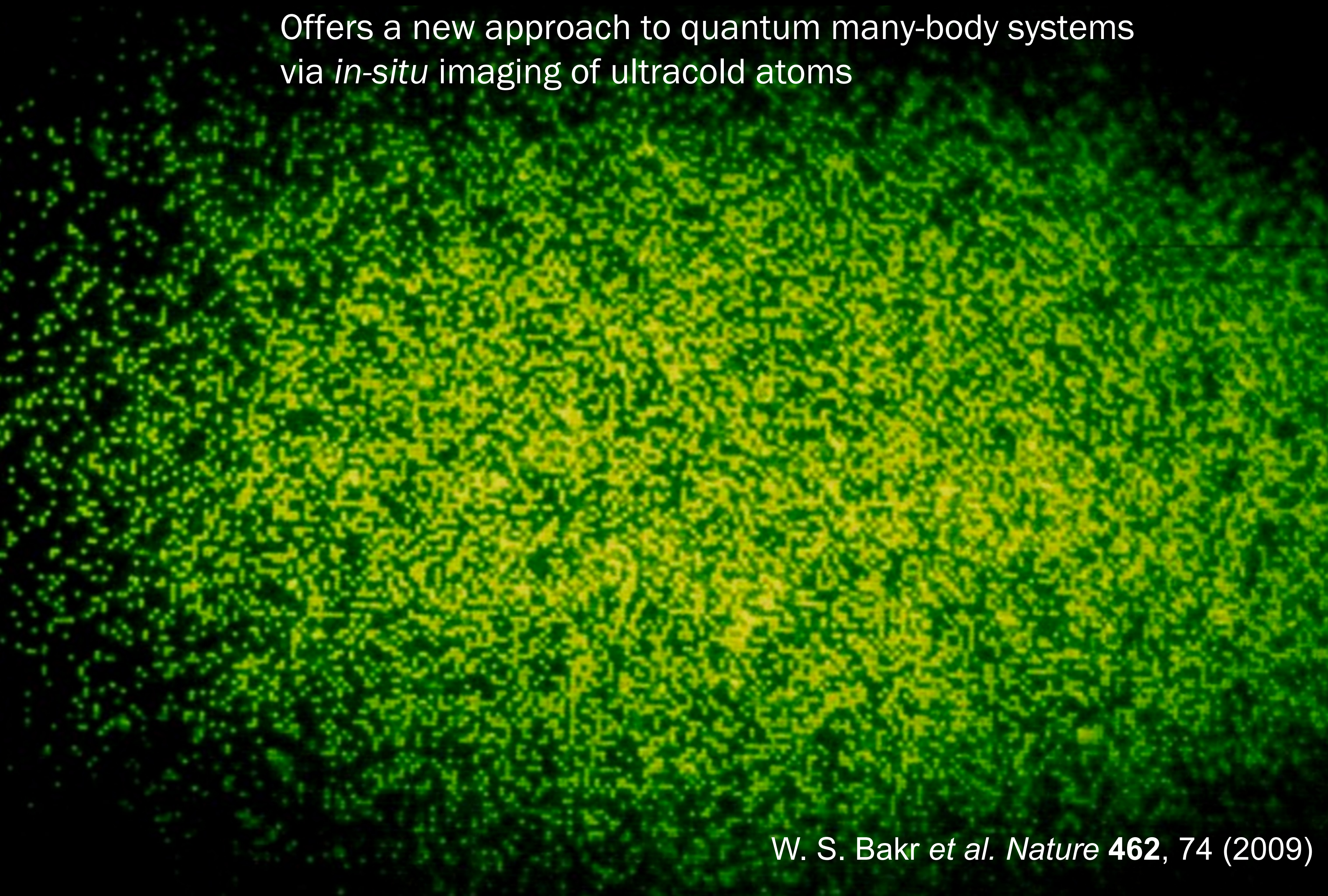
Superconducting qubit

Vijay et al., PRL 106, 110502 (2011)

Quantum dots

Vamivakas et al., Nature 467, 297 (2010)

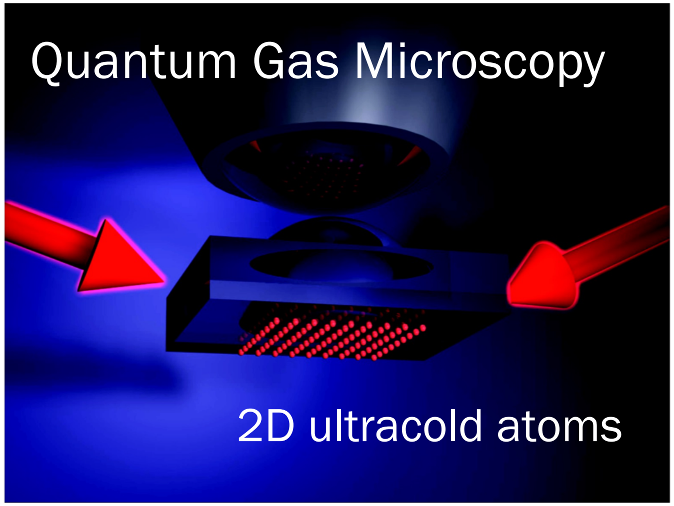
Quantum gas microscopy



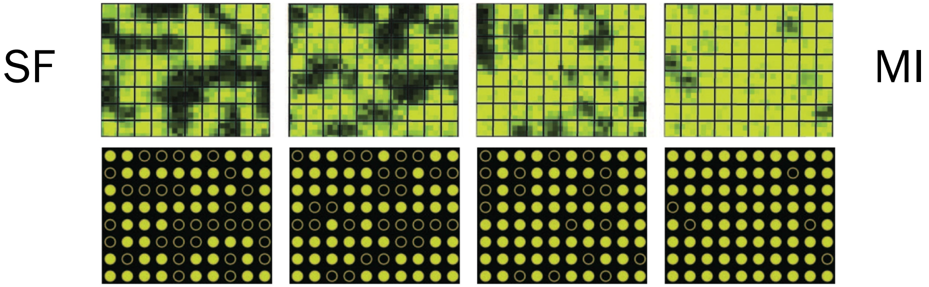
Offers a new approach to quantum many-body systems
via *in-situ* imaging of ultracold atoms

W. S. Bakr *et al. Nature* **462**, 74 (2009)

Developments in Quantum Gas Microscopy



Superfluid-to-Mott insulator transition has been observed at the **single-particle** level.

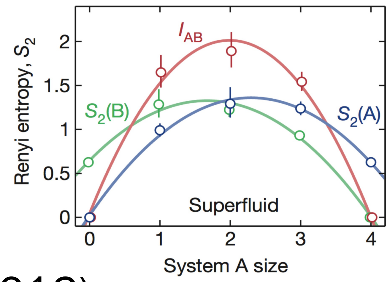


W. S. Bakr et al., Science 329, 547 (2010).

Recent breakthroughs:

Measurement of entanglement entropy:

Islam *et al. Nature* 528 77 (2015).



Observation of antiferromagnetic fermionic correlations:

Cheuk *et al*., Parsons *et al*., Boll *et al*., *Science* 353 1253-1260 (2016), Mazurenko *et al*., *Nature* 545 462 (2017).

Observation of many-body localization:

Schreiber *et al. Science* 349 842 (2015),
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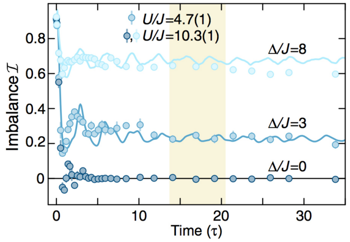

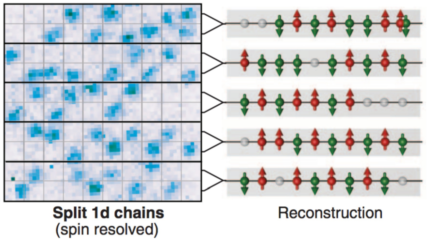

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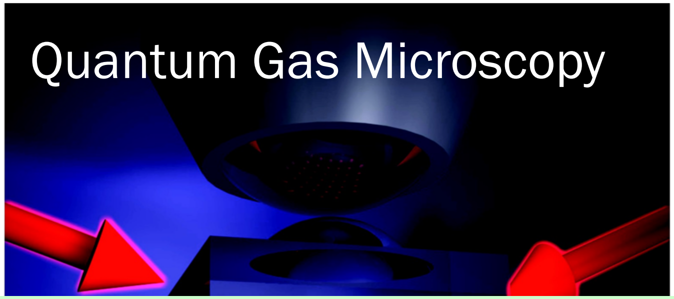

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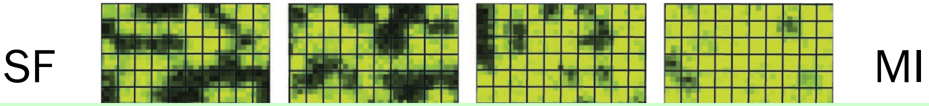
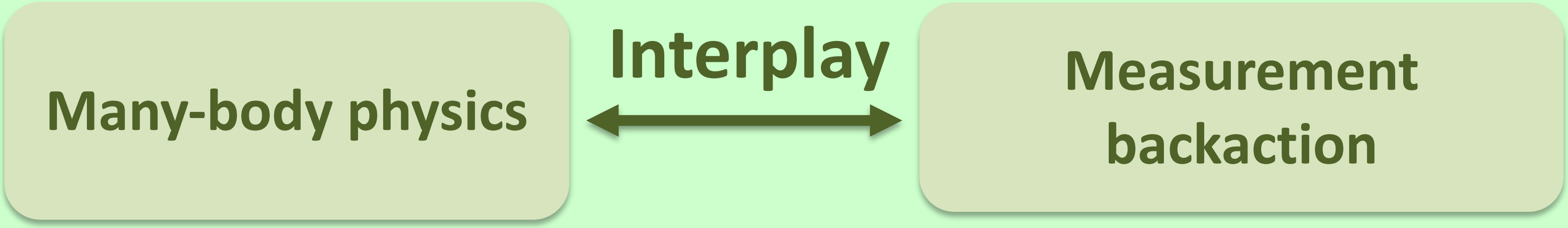


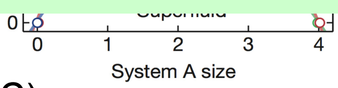
New direction: Continuous monitoring of many-body systems by QGM.



Many-body physics

Interplay

Measurement backaction



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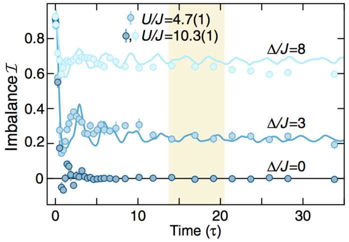
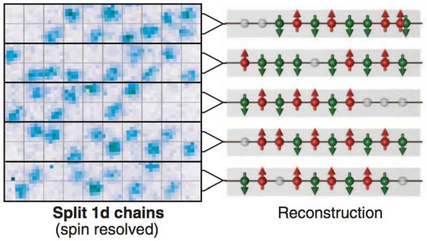
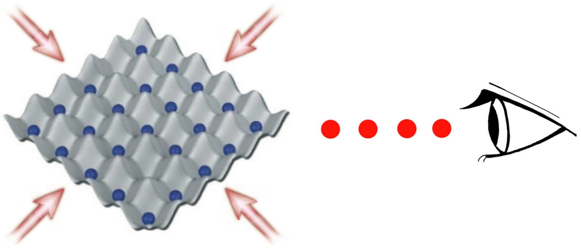






1. Quantum many-body systems under continuous observation

Wavefunction collapse and super-resolved imaging:

YA and M. Ueda, PRL 115, 095301 (2015).

YA and M. Ueda, Opt. Lett. **41**, 72-75 (2016).

Quantum critical phenomena:

YA, S. Furukawa and M. Ueda, Nat. Commun. 8, 15791 (2017).

YA, S. Furukawa and M. Ueda, PRA 94, 053615 (2016).

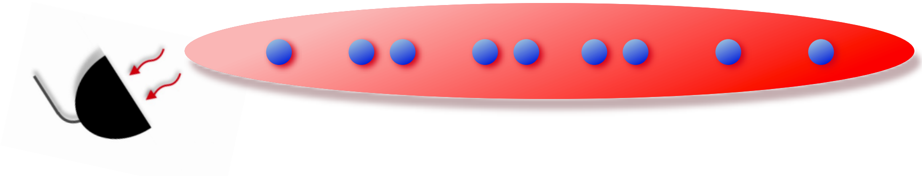
Nonequilibrium dynamics:

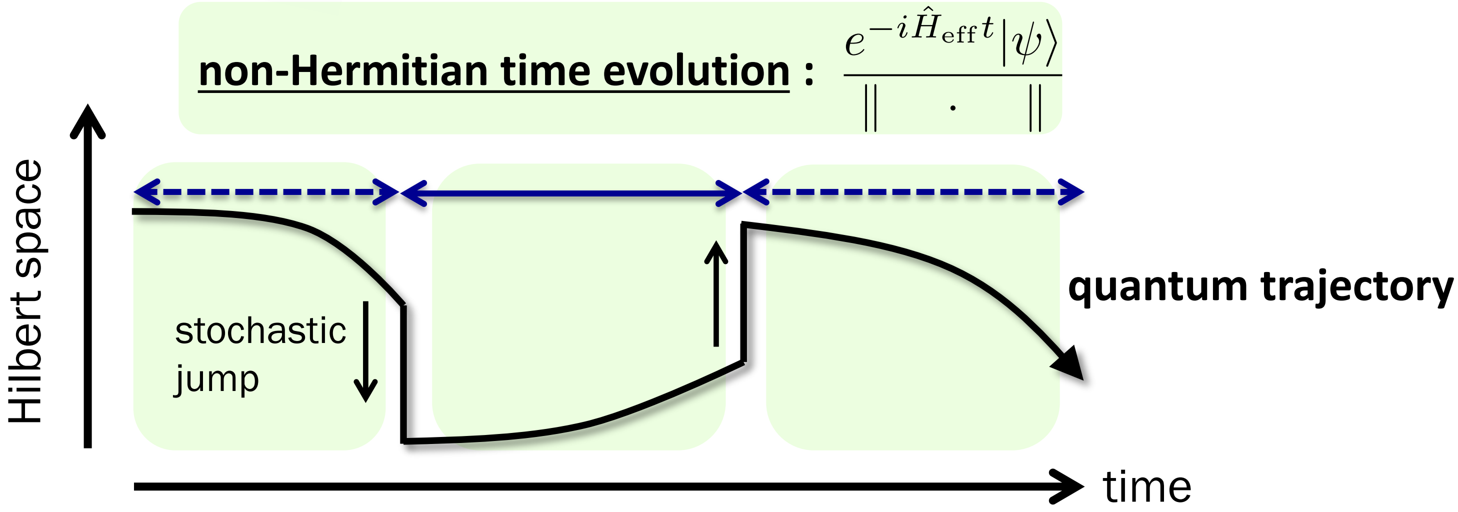
YA and M. Ueda, PRL 120, 185301 (2018).

YA and M. Ueda, PRA 95, 022124 (2017).

Quantum critical phenomena under continuous observation

Imagine that we **continuously monitor** a strongly correlated many-body system.



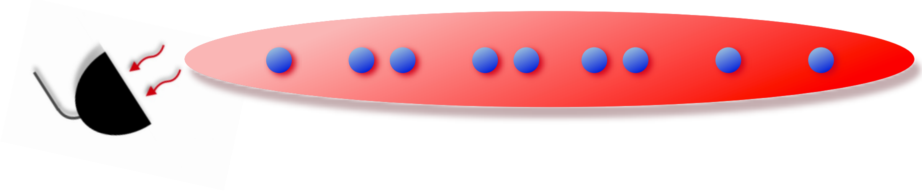


non-Hermitian time evolution : $\dfrac{e^{-i\hat{H}_{\rm eff}t}|\psi\rangle}{\|\quad\cdot\quad\|}$

effective Hamiltonian: $\hat{H}_{\rm eff} = \hat{H} \underbrace{- \frac{i}{2}\sum_a \hat{L}_a^\dagger \hat{L}_a}_{\text{measurement backaction}}$

Quantum critical phenomena under continuous observation

Q. Does such measurement backaction change universality class of quantum critical phenomena?



Idea:

(sine-Gordon model) (PT-symmetry)

Quantum Phase Transition + Spectral Singularity → *New* universality class unique to open systems?

↑

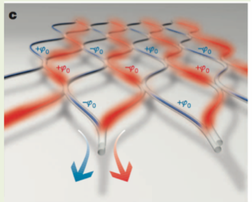
Singularity due to **non-diagonalizability**
= ***unique*** to non-Hermitian systems

Experimental studies of non-Hermitian systems

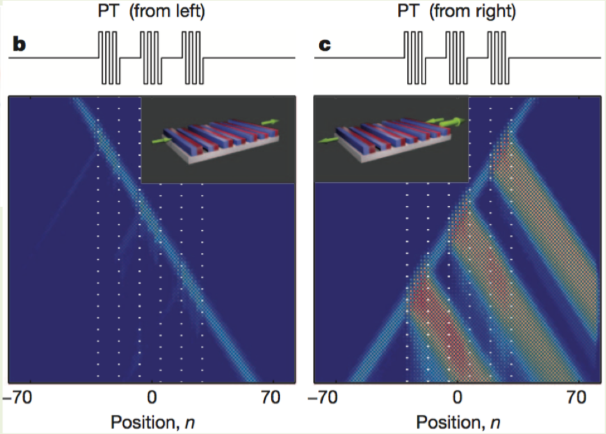
Non-hermitian systems exhibit unique features at Exceptional Point (i.e., Spectral singularity) and have attracted considerable attentions. Nat. Phys. 14, 11 (2018); Nat. Photon. 11, 741 (2017).

Optics

periodic gain-loss

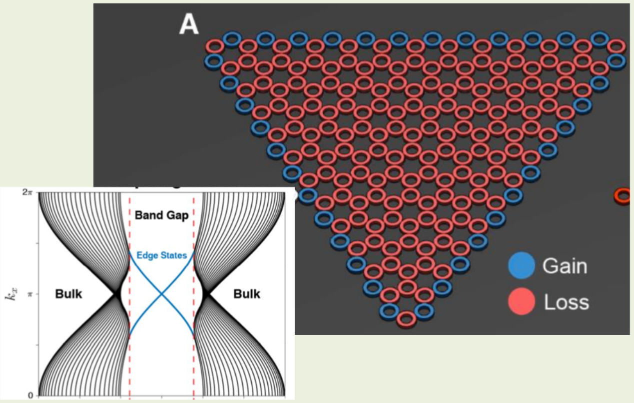


unidirectionality



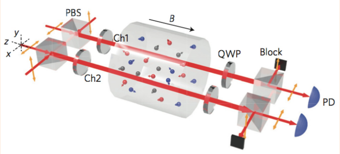
Regensberger et al. Nature 488 167 (2012)

“Lasing” edge mode

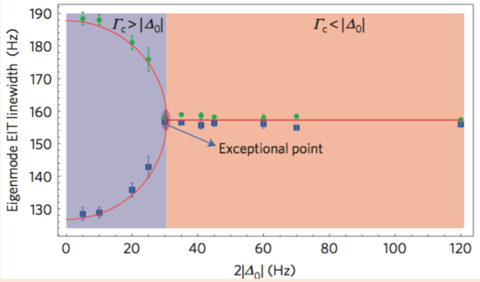


Harari et al. Science 10.1126/science.aar4003 (2018)

Thermal atomic gas



bifurcation in collective spin mode

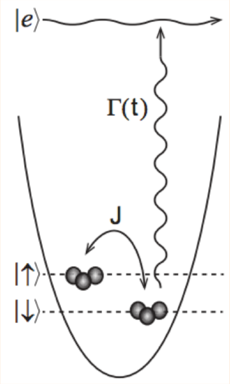


Peng et al. Nature Phys. 12, 1139 (2016)

*Open quantum systems:

ultracold atoms
J. Li et al., arXiv:1608.05061

quantum walks
L. Xiao et al., Nat. Phys. (2017)



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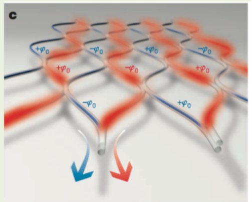

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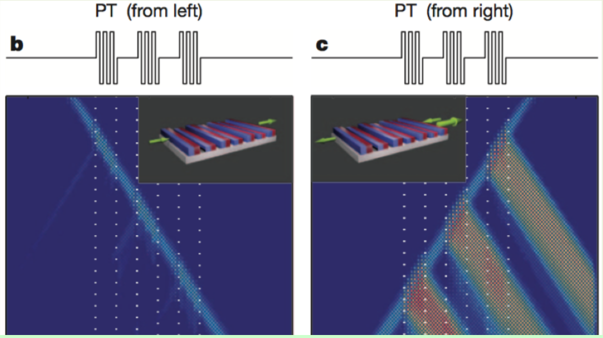

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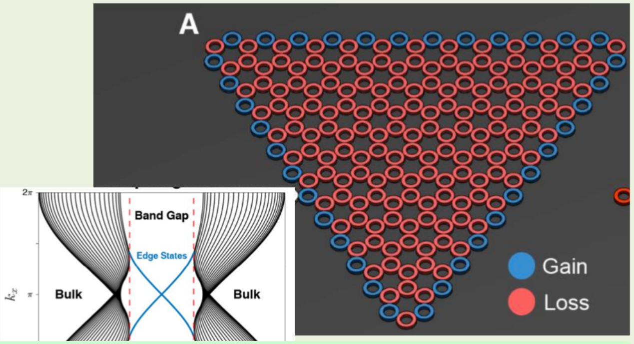
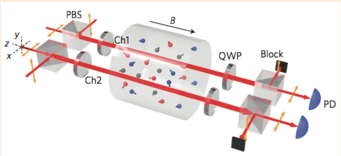


One-body (mean-field) phenomena have been well explored.

We extend the idea of PT-symmetry to many-body systems.

Thermal atomic gas



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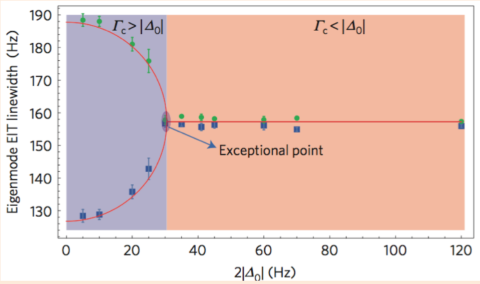

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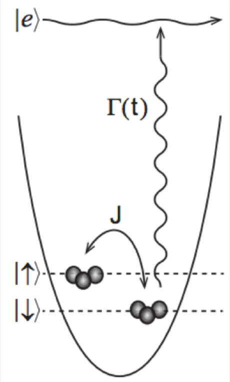
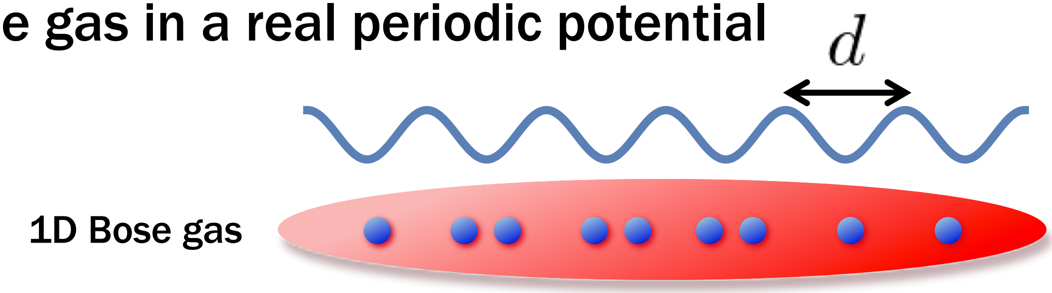


Many-body paradigm: BKT transition and sine-Gordon model

1D Bose gas in a real periodic potential



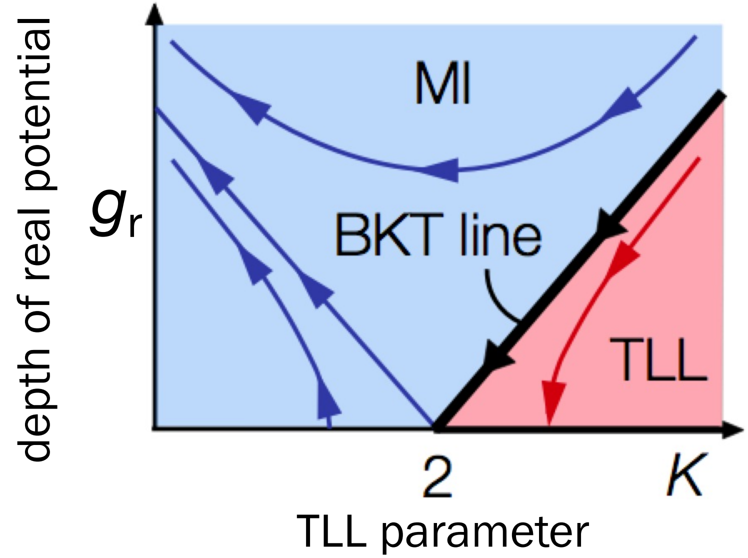
shallow periodic potential
(far-detuned light)

$$U(x) = U_{\rm r} \cos\left(\frac{2\pi x}{d}\right)$$

Low-energy Hamiltonian (sine-Gordon model):

$$\hat{H} = \int dx \left\{ \frac{\hbar v}{2\pi} \left[K(\partial_x \hat{\theta})^2 + \frac{1}{K}(\partial_x \hat{\phi})^2 \right] + V(\hat{\phi}) \right\}, \quad V(\hat{\phi}) = \frac{g_{\rm r}}{\pi} \cos(2\hat{\phi})$$

RG flows & Phase diagram:



gapped phase
(Mott insulator (MI))

↕ Berezinskii-Kosterlitz-Thouless transition

gapless quantum critical phase
(Tomonaga-Luttinger liquid (TLL))

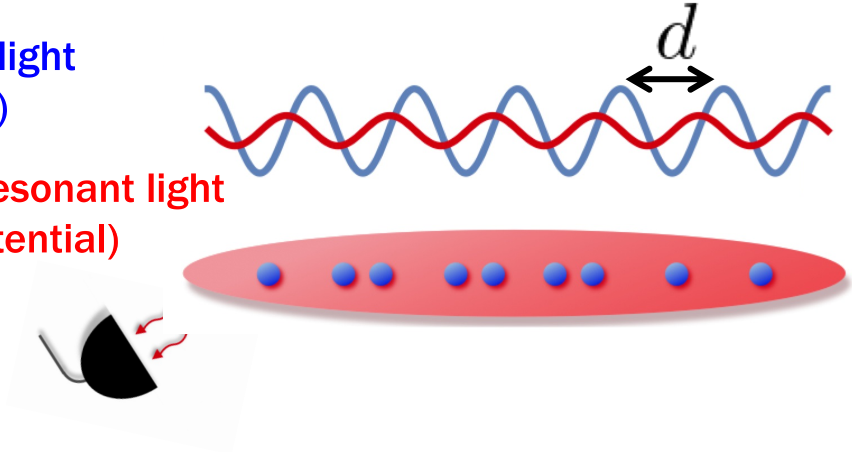
2016 Nobel Prize

Under continuous observation: Generalized sine-Gordon model

1D Bose gas subject to spatially modulated one-body loss

a far-detuned light
(real potential)

a weak near-resonant light
(imaginary potential)

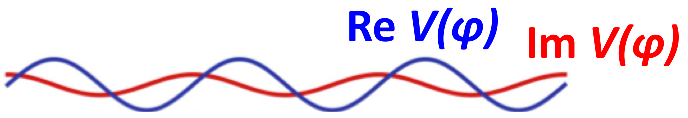


$$U(x) = \underline{U_\mathrm{r}\cos\left(\frac{2\pi x}{d}\right)}\; \underline{-iU_\mathrm{i}\sin\left(\frac{2\pi x}{d}\right)} -iU_\mathrm{i}$$

periodic gain-loss
structure

Low-energy Hamiltonian (Generalized sine-Gordon model):

$$\hat{H} = \int dx \left\{ \frac{\hbar v}{2\pi}\left[K(\partial_x\hat{\theta})^2 + \frac{1}{K}(\partial_x\hat{\phi})^2\right] + V(\hat{\phi})\right\}$$



$$V(\hat{\phi}) = \frac{g_\mathrm{r}}{\pi}\cos(2\hat{\phi}) \boxed{-\frac{ig_\mathrm{i}}{\pi}\sin(2\hat{\phi})}$$

g_r : depth of **real** potential

g_i : depth of **imaginary** potential

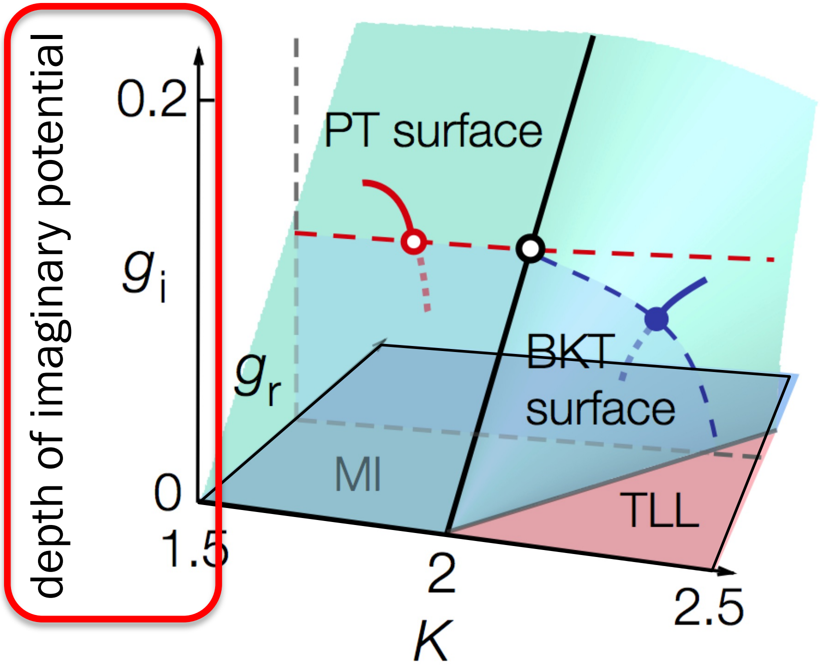
$$\hat{\mathcal{P}}\hat{\phi}\hat{\mathcal{P}} = -\hat{\phi} \quad \hat{\mathcal{T}}i\hat{\mathcal{T}} = -i \quad \rightarrow \quad [\hat{H}, \hat{\mathcal{P}}\hat{\mathcal{T}}] = 0$$

PT symmetry

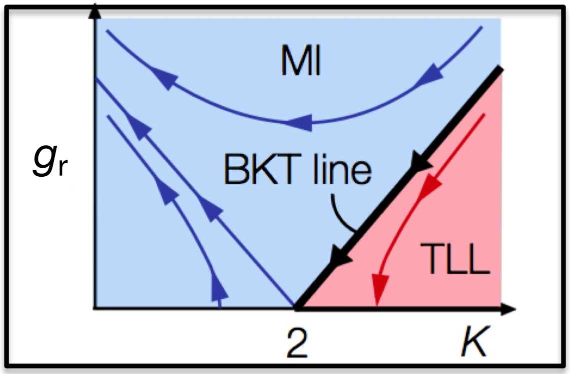
Bender & Boettcher PRL (1998)

Phase diagram of PT-symmetric sine-Gordon model

PT-symmetric sine-Gordon

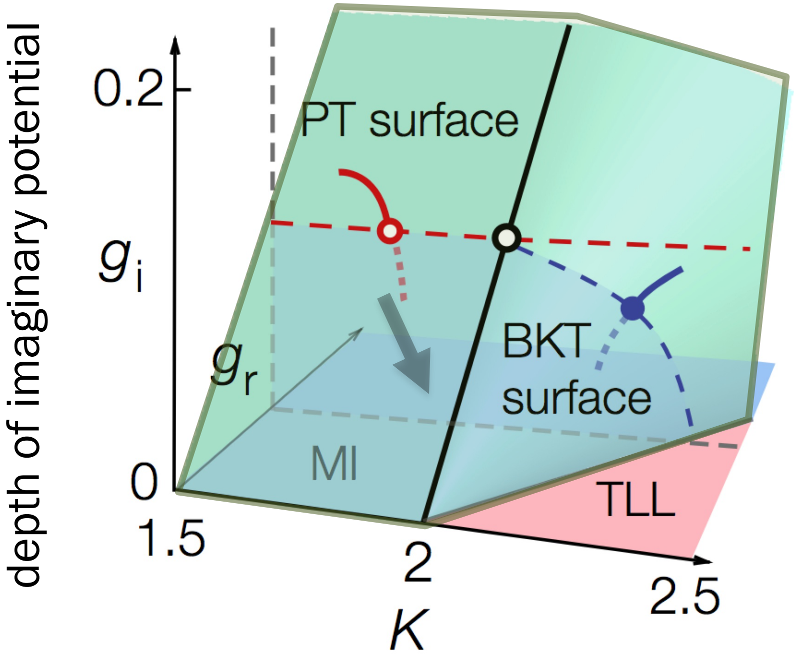


*c.f.) conventional sine-Gordon



Phase diagram of PT-symmetric sine-Gordon model

PT-symmetric sine-Gordon



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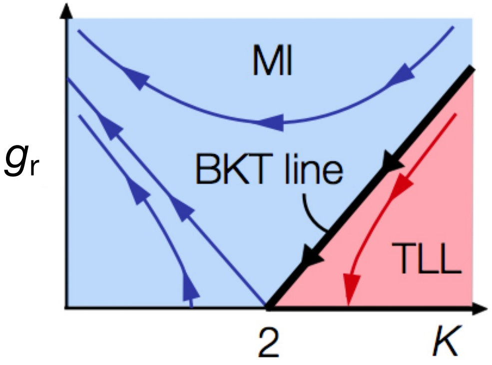
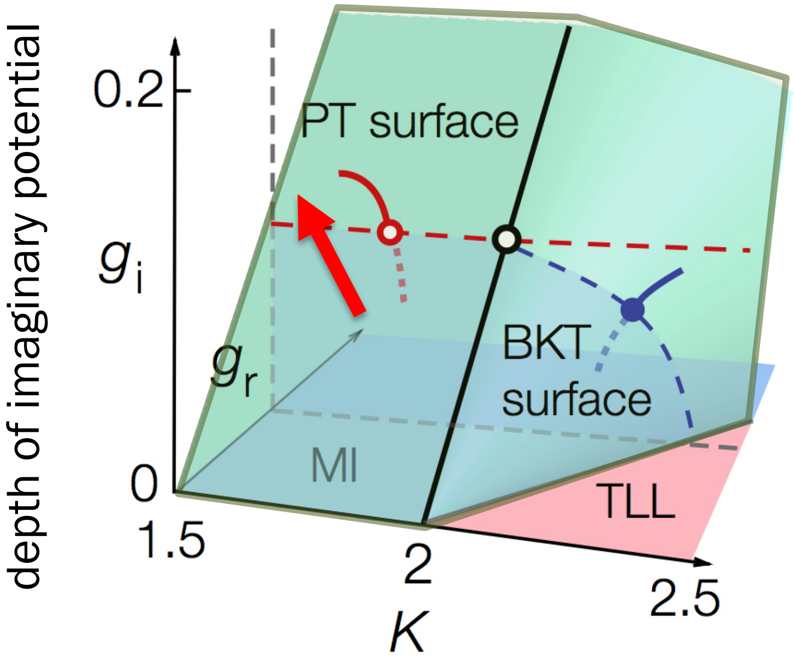


2D phase boundary:

- MI phase (below) and TLL phase (above)

Phase diagram of PT-symmetric sine-Gordon model

PT-symmetric sine-Gordon



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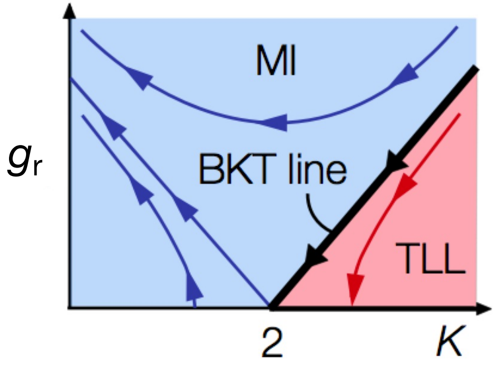

2D phase boundary:

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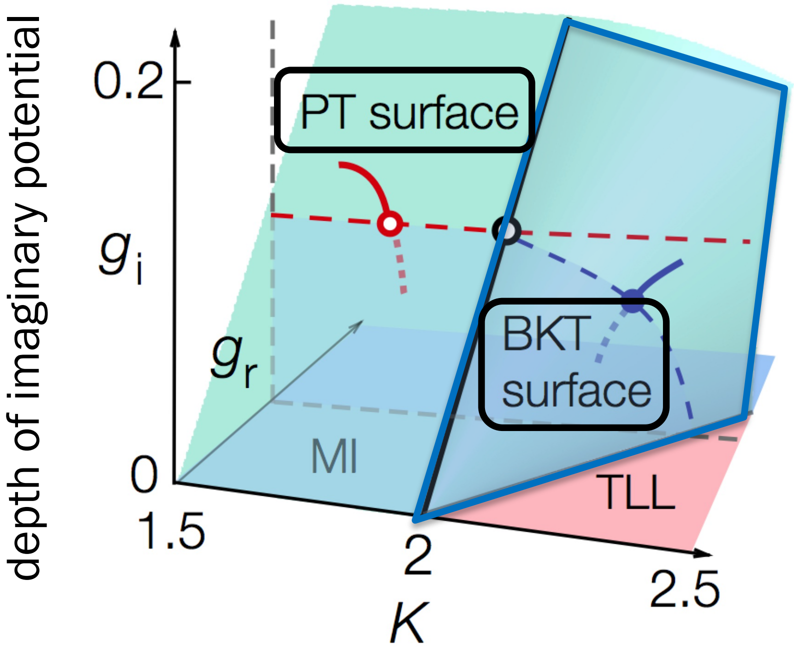
Phase diagram of PT-symmetric sine-Gordon model

PT-symmetric sine-Gordon



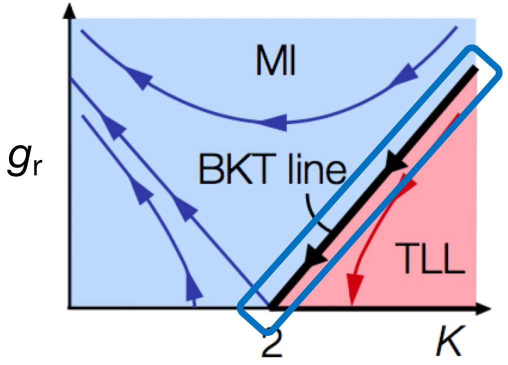
2D phase boundary:

- MI phase (below) and TLL phase (above)

Two phase transitions:

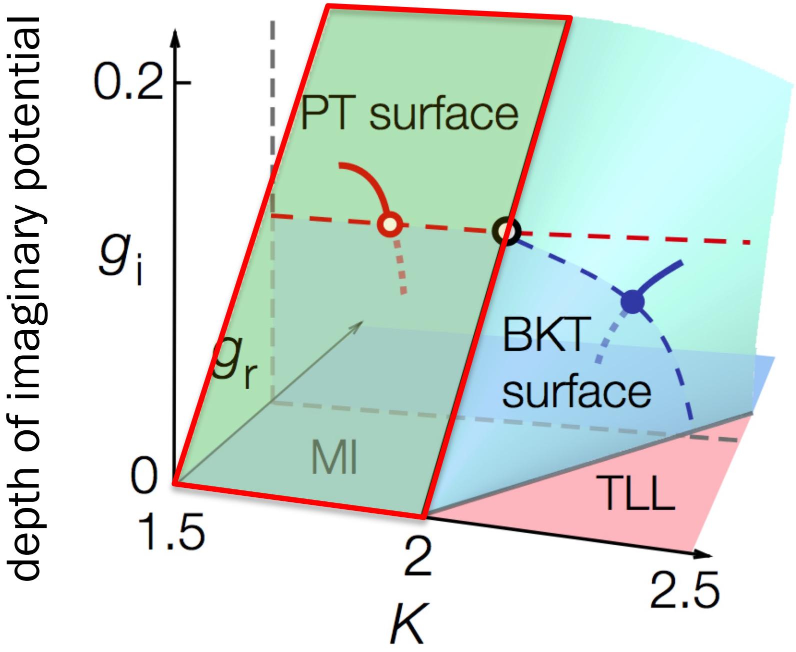
- BKT transition (K>2)
- **PT transition** (K<2)

*c.f.) conventional sine-Gordon

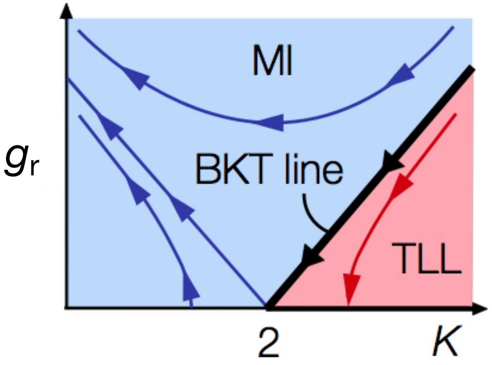


Phase diagram of PT-symmetric sine-Gordon model

PT-symmetric sine-Gordon



*c.f.) conventional sine-Gordon



2D phase boundary:

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Two phase transitions:

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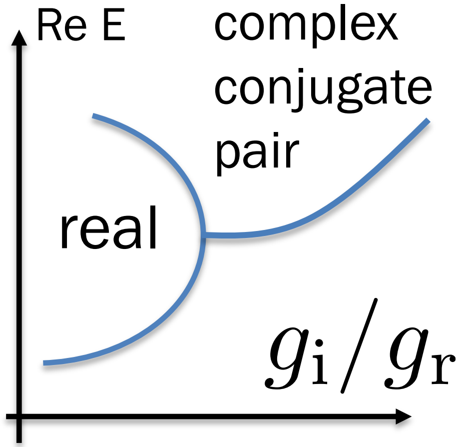
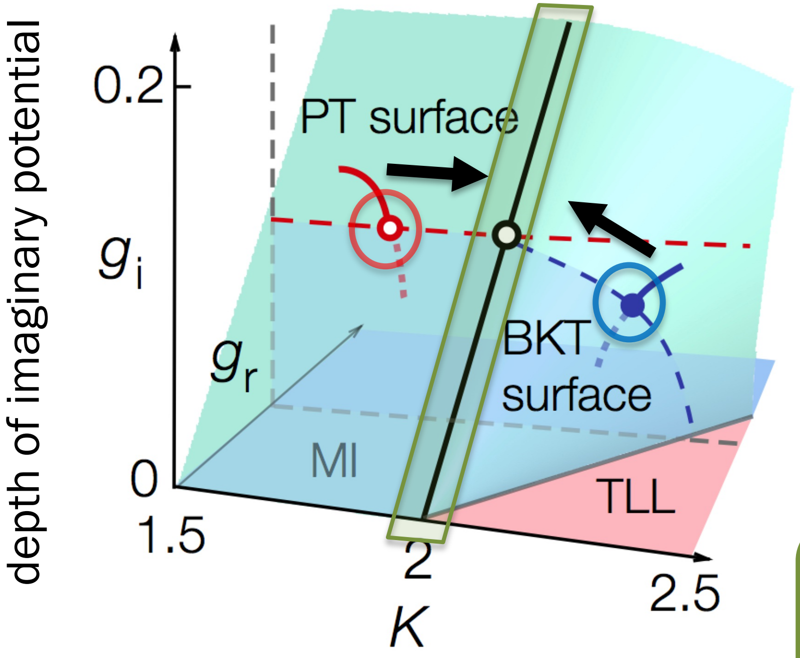
Measurement-induced QPT

Spectral Singularity



Phase diagram of PT-symmetric sine-Gordon model

PT-symmetric sine-Gordon



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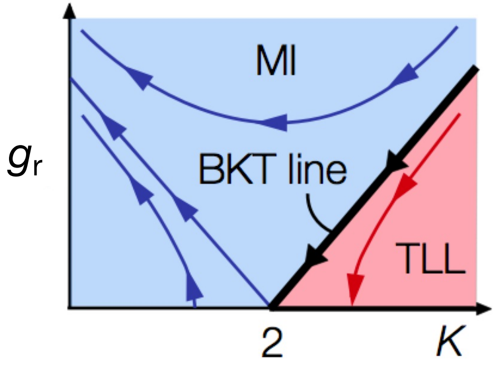


2D phase boundary:

- MI phase (below) and TLL phase (above)

Two phase transitions:

- BKT transition (K>2)
- **PT transition** (K<2)

↖ Measurement-induced QPT
Spectral Singularity

Merging line:

- Each point = RG **fixed** point.
- **Scale invariance** at K=2, g_s=g_c

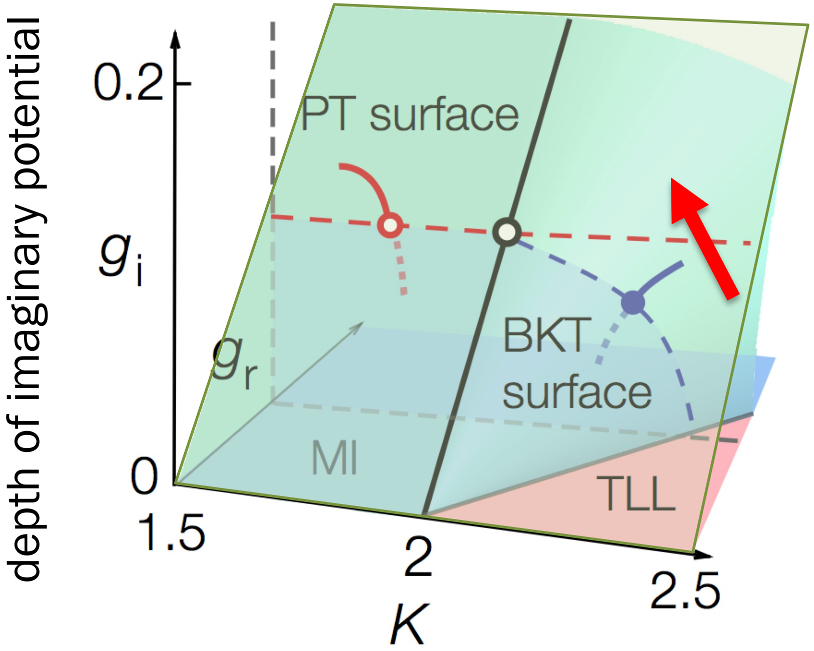
Spectral singularity
+
quantum critical point
→ a nontrivial RG fixed point.

New universality class.

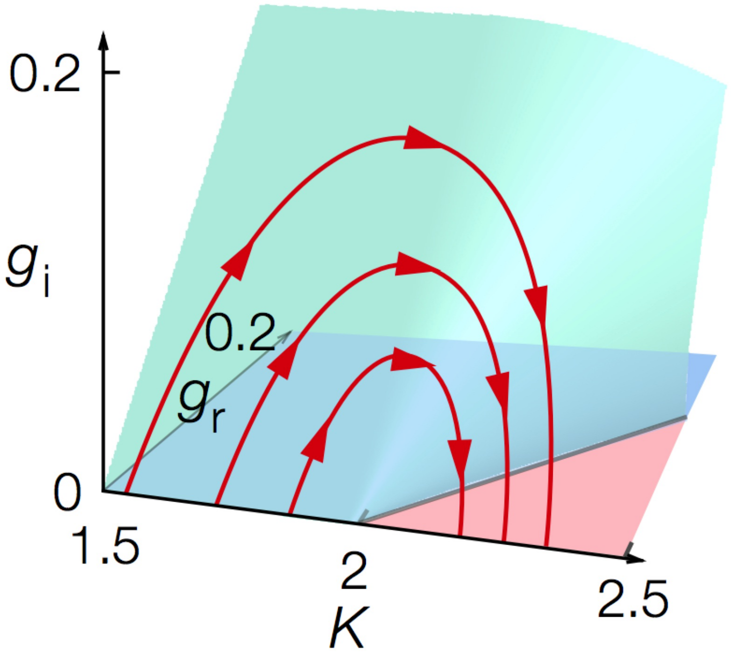
cf. non-unitary CFT [Ikhlef et al., PRL 116, 130601 (2016)].

New type of RG flows: Violation of c-theorem

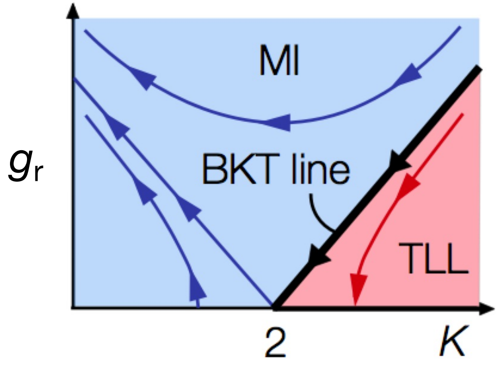
PT-symmetric sine-Gordon



PT broken (g_i>g_r)



*c.f.) conventional sine-Gordon



- **Semicircular** RG flows appear:

  → anomalous **increase** of TLL parameter K

  = **varying** critical exponent:

$$\langle \hat{\Psi}^{\dagger}(r)\hat{\Psi}(0)\rangle \propto (1/r)^{\frac{1}{2K}}$$

- **Violation of c-theorem**

  (Zamorodchikov, 1984) $\frac{dc}{dl} < 0$

c : central charge l : RG scale

YA, S. Furukawa and M. Ueda, Nat. Commun. 8, 15791 (2017).

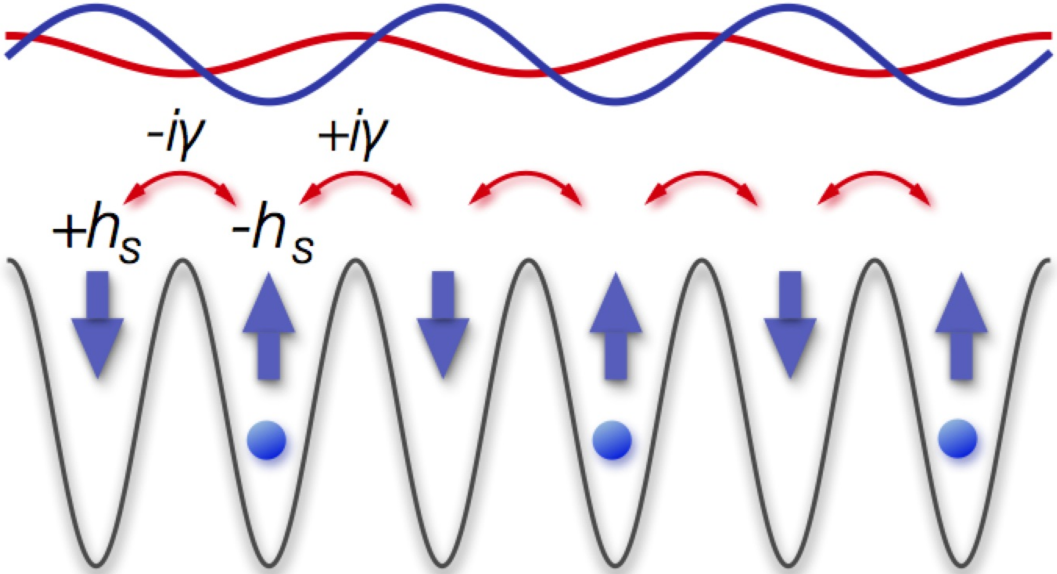
Numerical test of RG phase diagram (exact diagonalization)

PT-symmetric spin-chain model:

$$\hat{H}_{\mathrm{L}}=\sum_{m=1}^{N}\left[-\left(J+(-1)^{m}i\gamma\right)\left(\hat{S}^{x}_{m}\hat{S}^{x}_{m+1}+\hat{S}^{y}_{m}\hat{S}^{y}_{m+1}\right)+\Delta\hat{S}^{z}_{m}\hat{S}^{z}_{m+1}+(-1)^{m}h_{s}\hat{S}^{z}_{m}\right]$$

Correspondence to the effective field theory: $(-\Delta, h_s, \gamma) \Leftrightarrow (K, g_c, g_s)$

Physical implementation in ultracold atoms:



PT symmetric shallow potential

deep optical lattice (**irrelevant** to critical behavior)

hard-core bosons

↑ : presence

↓ : absence

Numerical test of RG phase diagram (exact diagonalization)

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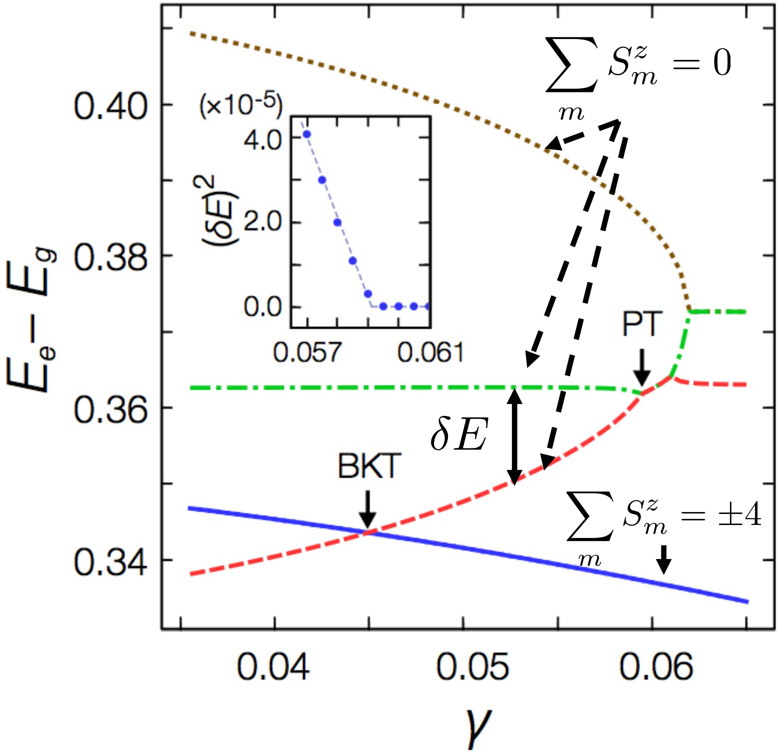
$$\hat{H}_{\mathrm{L}} = \sum_{m=1}^{N} \left[-\left(J + (-1)^m i\gamma\right) \left(\hat{S}^x_m \hat{S}^x_{m+1} + \hat{S}^y_m \hat{S}^y_{m+1} \right) + \Delta \hat{S}^z_m \hat{S}^z_{m+1} + (-1)^m h_s \hat{S}^z_m \right]$$

Correspondence to the effective field theory: $(-\Delta, h_s, \gamma) \Leftrightarrow (K, g_c, g_s)$

Typical low-energy excitation spectrum



BKT transition point:
crossing of appropriate energy levels
(level spectroscopy)

K. Nomura, J. Phys. A 28, 5451 (1995)

PT transition point:
merging of two energy levels
with square-root scaling
(characteristic of exceptional point)

T. Kato, *Perturbation theory for linear operators* (1966)

YA, S. Furukawa and M. Ueda, Nat. Commun. 8, 15791 (2017).

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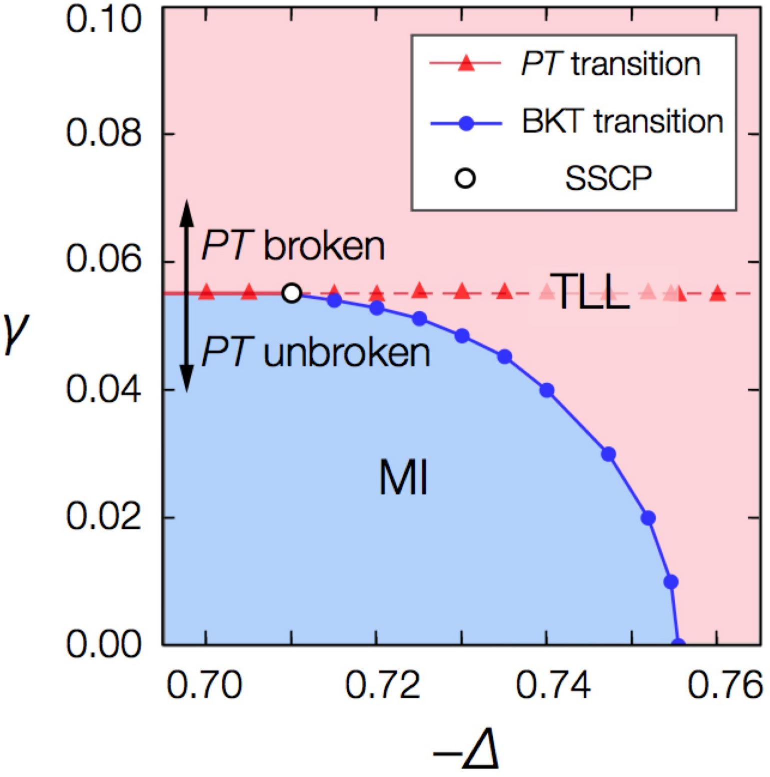
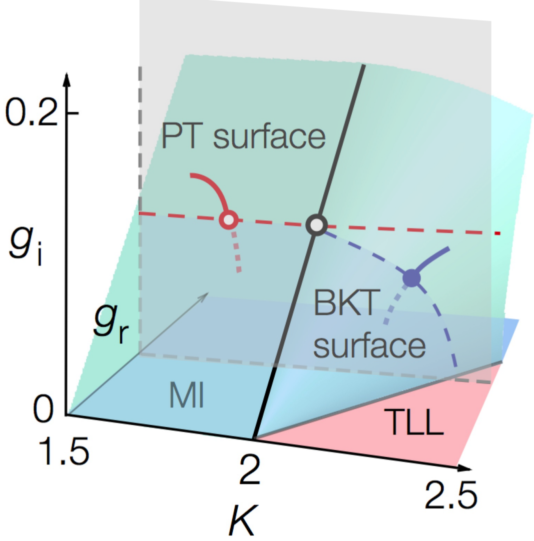
Correspondence to the effective field theory: $(-\Delta, h_s, \gamma) \Leftrightarrow (K, g_c, g_s)$

Phase diagram (fixed h_s)



fixed g_c



Numerical **supports** for RG phase diagram.

YA, S. Furukawa and M. Ueda, Nat. Commun. 8, 15791 (2017).

Numerical test of anomalous RG flows (iTEBD)

PT-symmetric spin-chain model:

$$\hat{H}_{\mathrm{L}} = \sum_{m=1}^{N} \left[-\left(J + (-1)^m i\gamma\right) \left(\hat{S}^x_m \hat{S}^x_{m+1} + \hat{S}^y_m \hat{S}^y_{m+1} \right) + \Delta \hat{S}^z_m \hat{S}^z_{m+1} + (-1)^m h_s \hat{S}^z_m \right]$$

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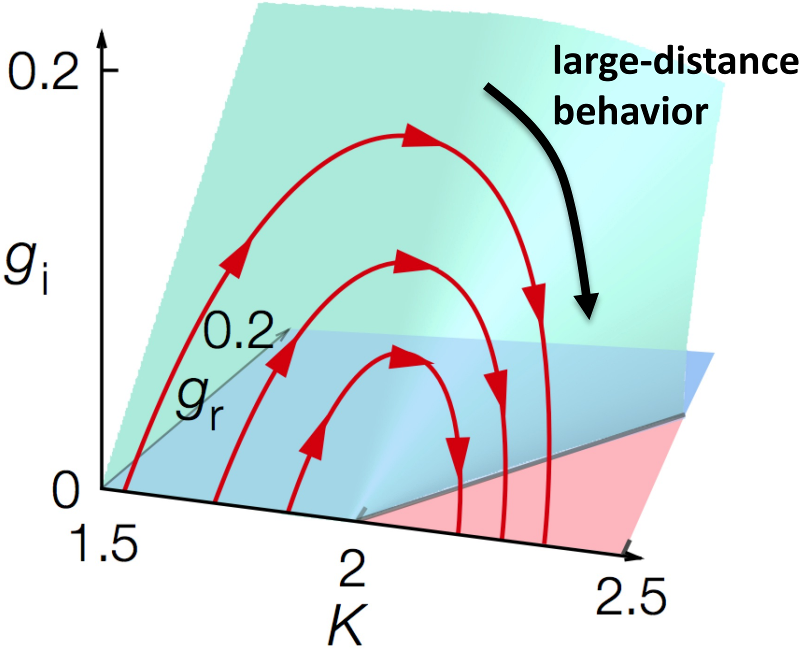
PT broken (g_i>g_r)



Semicircular RG flows:

- **Varying** critical exponent in a **larger** distance
- **Violation of c-theorem**

Method:

infinite time-evolving block decimation (**iTEBD**) algorithm (G. Vidal, PRL 98, 070201 (2007))

imaginary-time evolution → ground state

$$\frac{\exp(-\hat{H}\tau)|\Psi_0\rangle}{\|\exp(-\hat{H}\tau)|\Psi_0\rangle\|}$$

YA, S. Furukawa and M. Ueda, Nat. Commun. 8, 15791 (2017).

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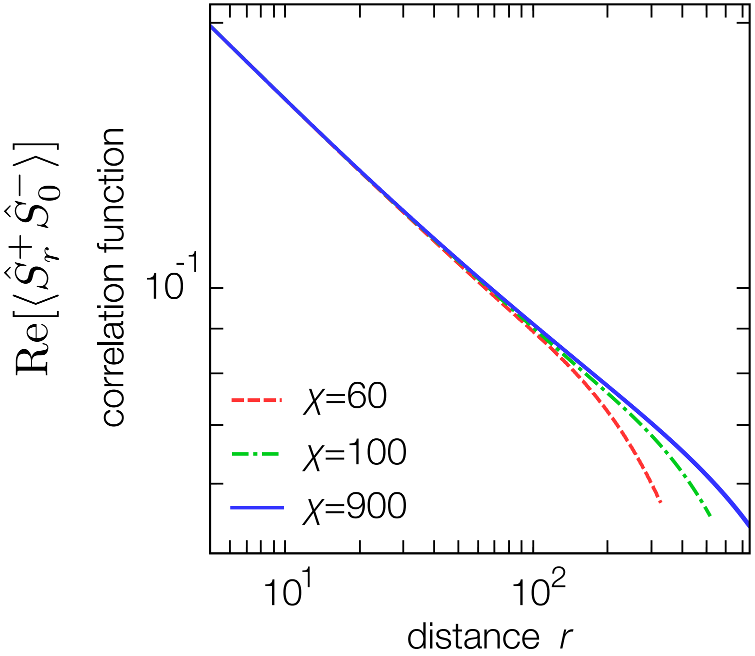
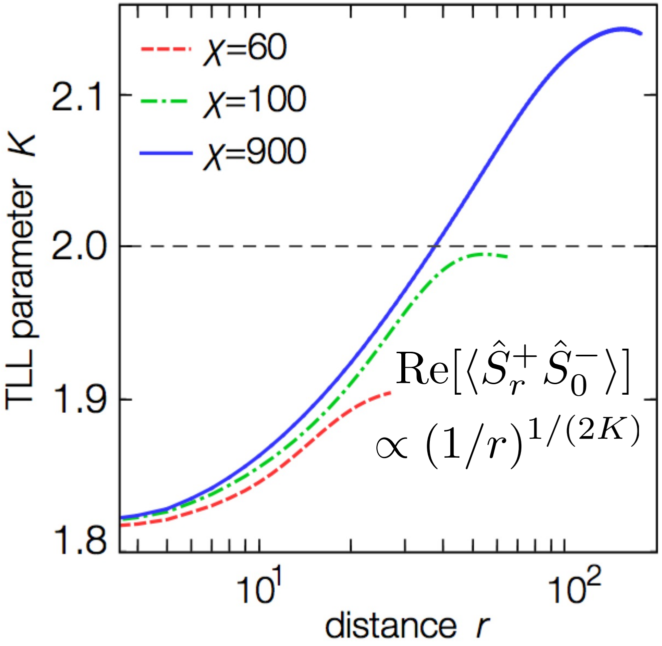
Critical decay in PT broken phase



Varying TLL parameter *K*



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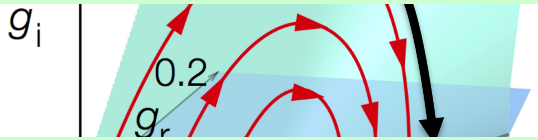
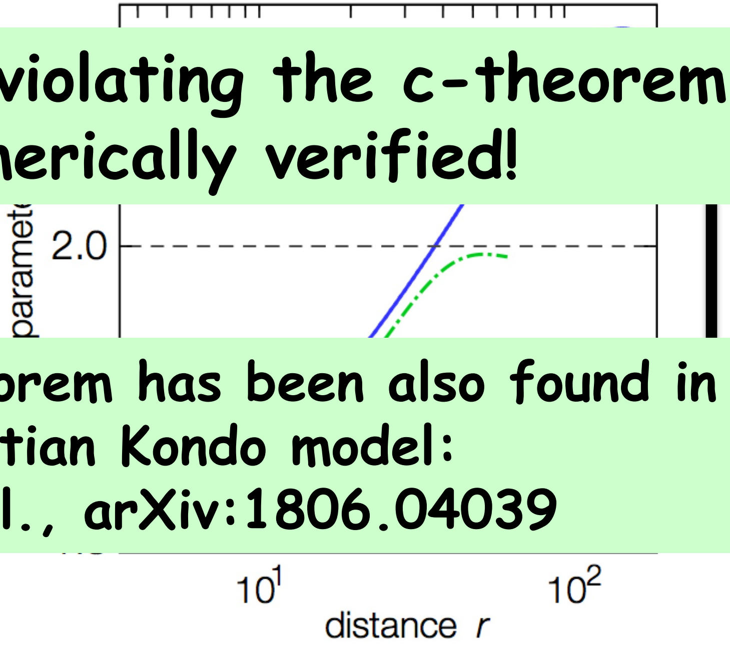
Correspondence to the effective field theory: $(-\Delta, h_s, \gamma) \Leftrightarrow (K, g_c, g_s)$

RG analysis

Varying TLL parameter *K*

Anomalous RG flows violating the c-theorem have been numerically verified!

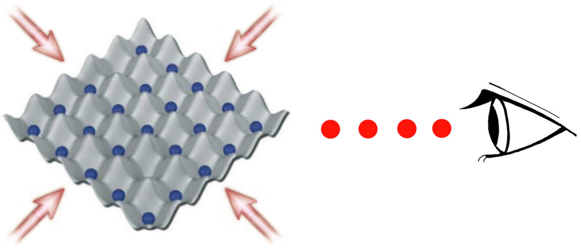




*** Violation of the g-theorem has been also found in the non-Hermitian Kondo model: M. Nakagawa et al., arXiv:1806.04039**

Semicircular RG flows
= **Enhancement** of TLL parameter
in a **larger** distance

YA, S. Furukawa and M. Ueda, Nat. Commun. 8, 15791 (2017).



1. Quantum many-body systems under continuous observation

Wavefunction collapse and super-resolved imaging:

YA and M. Ueda, PRL 115, 095301 (2015).

YA and M. Ueda, Opt. Lett. **41**, 72-75 (2016).

Quantum critical phenomena:

YA, S. Furukawa and M. Ueda, Nat. Commun. 8, 15791 (2017).

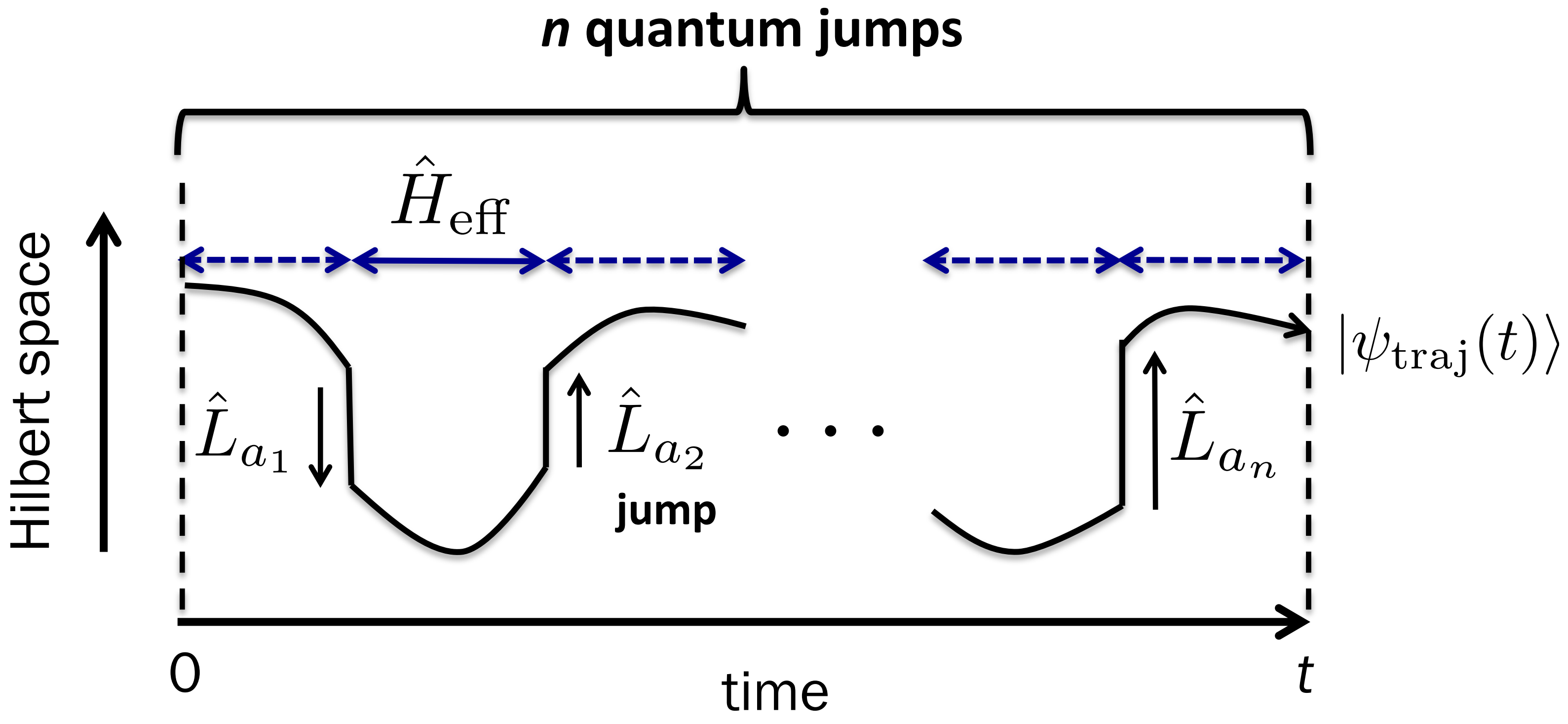
YA, S. Furukawa and M. Ueda, PRA 94, 053615 (2016).

Nonequilibrium dynamics (short summary):

YA and M. Ueda, PRL 120, 185301 (2018).

YA and M. Ueda, PRA 95, 022124 (2017).

Full-counting “dynamics” under continuous observation



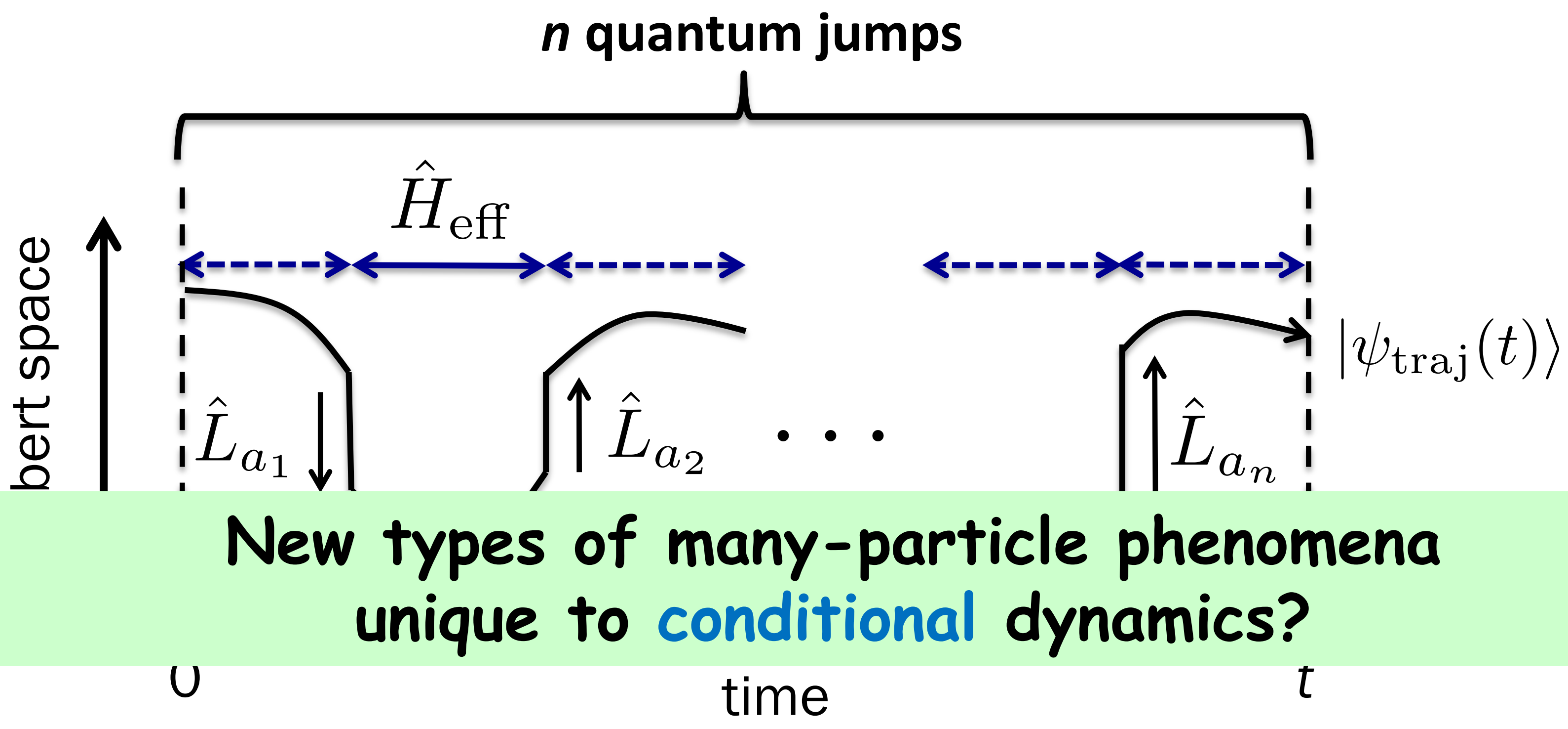
Post-measurement density matrix conditioned on the number of jumps *n* :

$$\hat{\rho}^{(n)}_{\text{post}}(t) \propto \sum_{i \in \mathcal{D}_n} |\psi_{\text{traj},i}(t)\rangle\langle\psi_{\text{traj},i}(t)|$$

$\mathcal{D}_n$: subspace of quantum trajectories
having *n* jumps

***different** from the unconditional dynamics (= Lindblad dynamics)

Full-counting "dynamics" under continuous observation



New types of many-particle phenomena unique to conditional dynamics?

Post-measurement density matrix conditioned on the number of jumps *n* :

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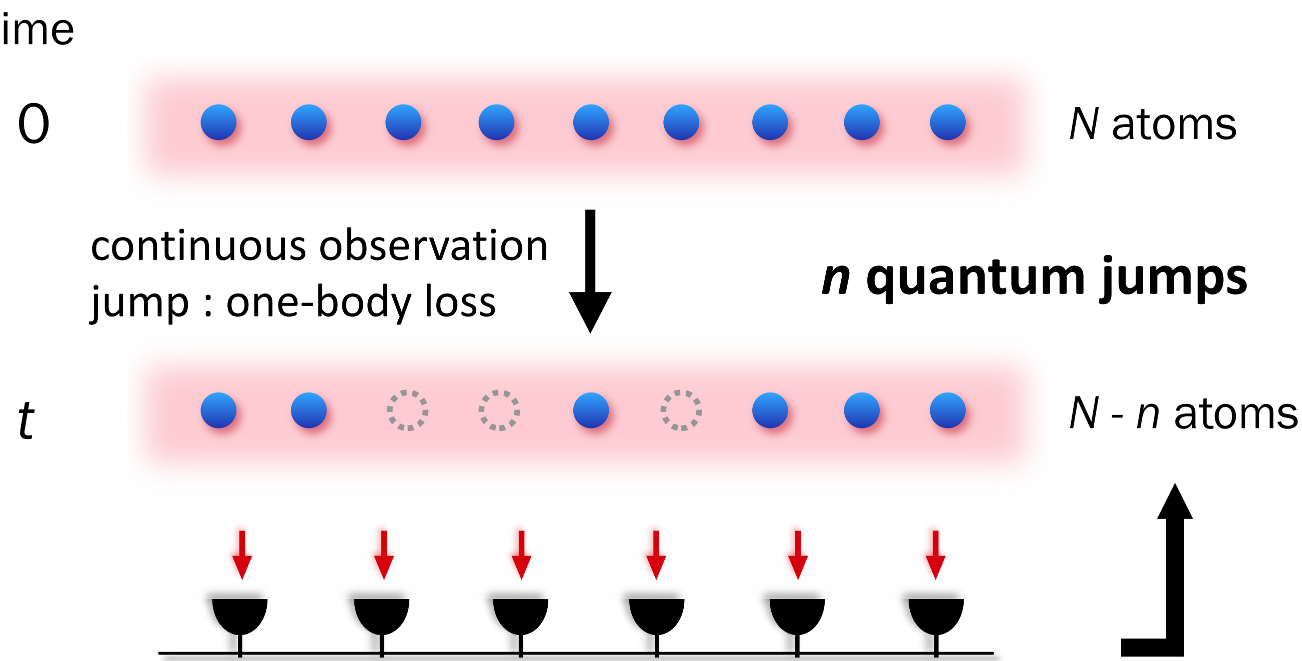
$\mathcal{D}_n$: subspace of quantum trajectories having *n* jumps

***different** from the unconditional dynamics (= Lindblad dynamics)

Propagation beyond the Lieb-Robinson bound (summary)

Setup



time

0

N atoms

continuous observation
jump : one-body loss

***n* quantum jumps**

t

N - *n* atoms

Measuring total atom number by QGM

Conditional dynamics

Ensemble average over trajectories including *n* jumps during [0,*t*]

$$\hat{\rho}^{(n)}(t)$$

YA and M. Ueda, PRL 120, 185301 (2018).

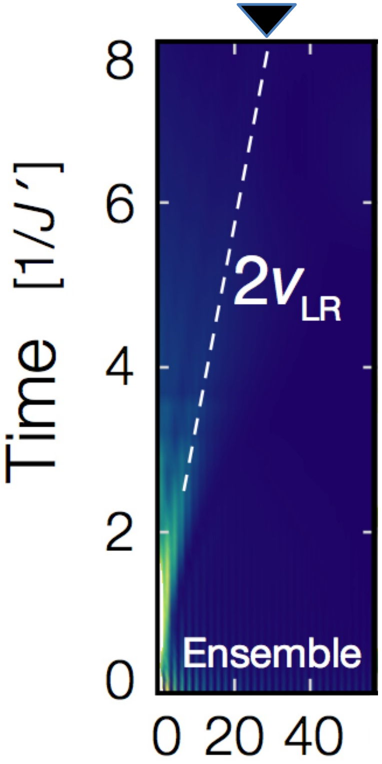
Propagation beyond the Lieb-Robinson bound (summary)

Exact calculations of quench dynamics for solvable model

Lindblad dynamics: $C(l,t) = \mathrm{Tr}[\hat{\rho}(t)\hat{c}_l^\dagger \hat{c}_0]$ $\hat{\rho}(t) = \sum_n P_n(t)\hat{\rho}^{(n)}(t)$

Lieb-Robinson (LR) bound for Lindblad dynamics

Poulin PRL 104, 190401 2010



YA and M. Ueda, PRL 120, 185301 (2018).

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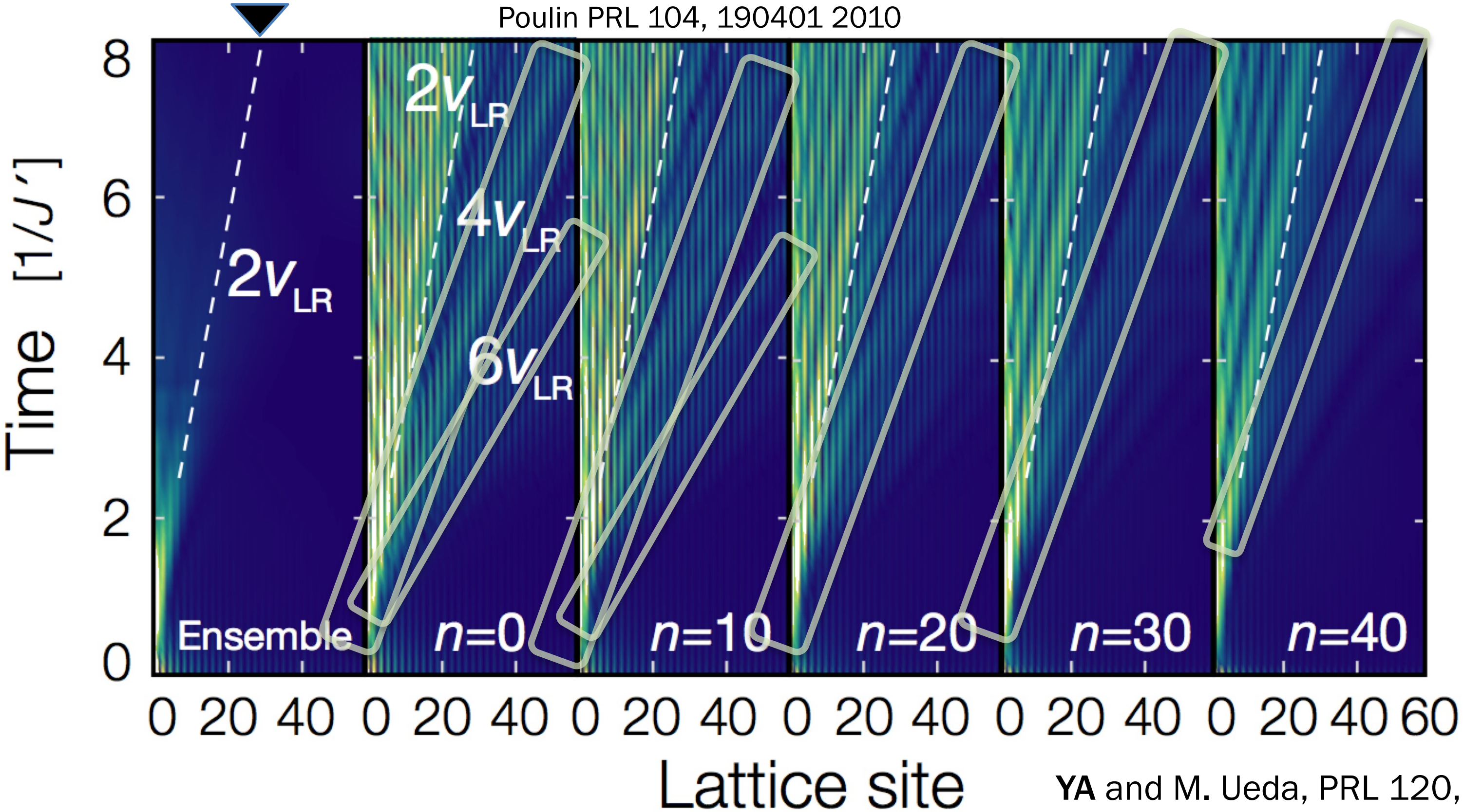
conditional dynamics: $C^{(n)}(l,t)=\mathrm{Tr}[\hat{\rho}^{(n)}(t)\hat{c}_l^\dagger\hat{c}_0]$

Lieb-Robinson (LR) bound for Lindblad dynamics

Poulin PRL 104, 190401 2010



YA and M. Ueda, PRL 120, 185301 (2018).

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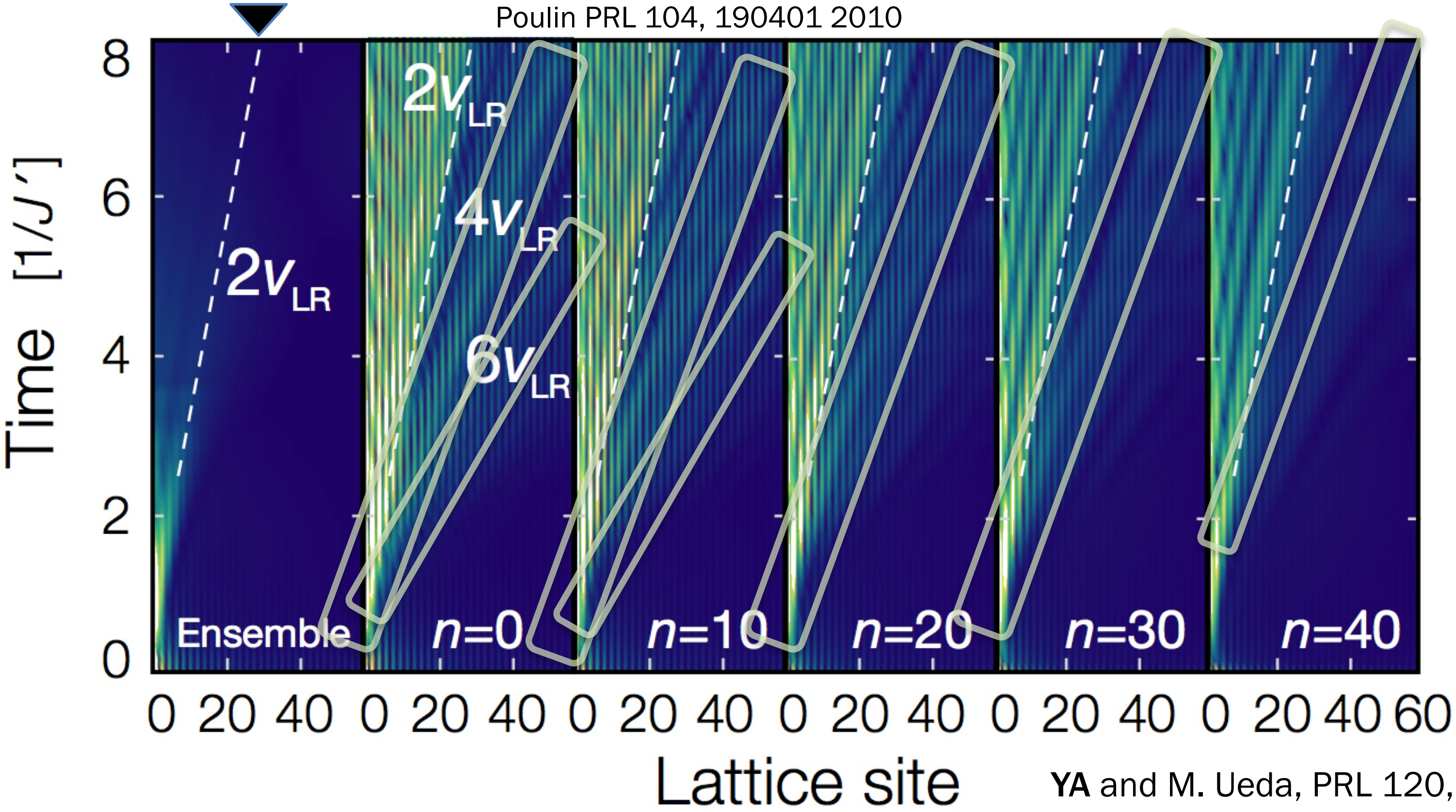
Lindblad dynamics: $C(l,t) = \mathrm{Tr}[\hat{\rho}(t)\hat{c}_l^\dagger \hat{c}_0]$ $\hat{\rho}(t) = \sum P_n(t)\hat{\rho}^{(n)}(t)$

Propagation beyond the LR bound can (probabilistically) appear as a result of the global postselection.

Lieb-Robinson (LR) bound for Lindblad dynamics

Poulin PRL 104, 190401 2010



YA and M. Ueda, PRL 120, 185301 (2018).

Classical non-Hermitian systems: when continuous observation reduces to one-body dissipative dynamics

Continuous observation (non-Hermitian): $\hat{H}_{\mathrm{eff}} = \hat{H} - \frac{i}{2}\sum_a \hat{L}_a^\dagger \hat{L}_a$

Markovian dissipative dynamics: $\frac{d\hat{\rho}}{dt} = -i[\hat{H}, \hat{\rho}] - \sum_a \left(\frac{1}{2}\hat{L}_a^\dagger \hat{L}_a \hat{\rho} + \frac{1}{2}\hat{\rho}\hat{L}_a^\dagger \hat{L}_a - \hat{L}_a \hat{\rho} \hat{L}_a^\dagger \right)$

Sometimes, continuous observation and dissipation can be **equivalent** and reduce to **classical** (**one-body**) dynamics:

Case I: Loss process in the **single-particle sector**

$\hat{L}_a$ annihilates the particle → Jump term is trivial $\hat{L}_a \hat{\rho} \hat{L}_a^\dagger \propto |\mathrm{vac}\rangle\langle\mathrm{vac}|$

Case II: Loss process with a **coherent state**

(e.g., classical waves of photons and phonons, or BEC)

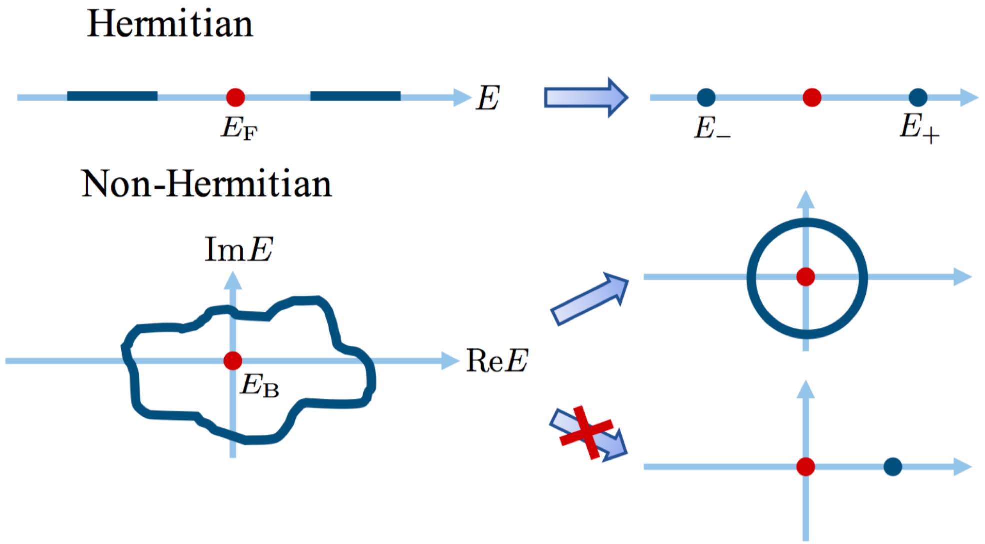
$\hat{L}_a$ is an eigenstate of $\hat{\rho}$ → Jump term just modifies the normalization.

(Essentially) classical phenomena, yet still interesting and directly relevant to experiments in classical optics & mechanics.

Topological aspects of non-Hermitian systems?

Topological aspects of non-Hermitian systems (short summary)

Definition of allowed continuous deformations



Band gap not closed

= *E*E_F (●) not touched.

⇨ E_B (●) not touched.

Two non-Hermitian Hamiltonians are equivalent $H(\boldsymbol{k}) \sim H'(\boldsymbol{k})$

iff ∃ $H_\lambda(\boldsymbol{k})$ ($\lambda \in [0,1]$) s.t.

(i) $H_0(\boldsymbol{k}) = H(\boldsymbol{k})$, $H_1(\boldsymbol{k}) = H'(\boldsymbol{k})$ and

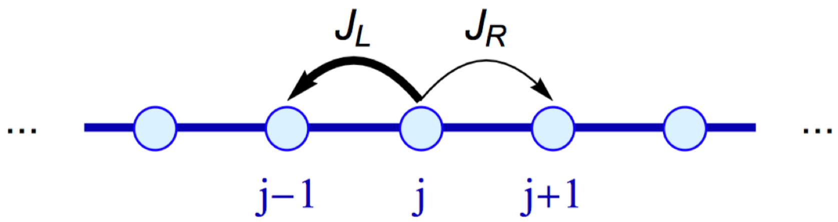
(ii) $\det[H_\lambda(\boldsymbol{k}) - E_\mathrm{B}] \neq 0 \ \forall \lambda \in [0,1]$ and $\boldsymbol{k} \in \mathrm{B.Z.}$

- Consistent with the definition in Hermitian systems.
- Parameter fine tuning not required → “Generic” nonunitary operation allowed.

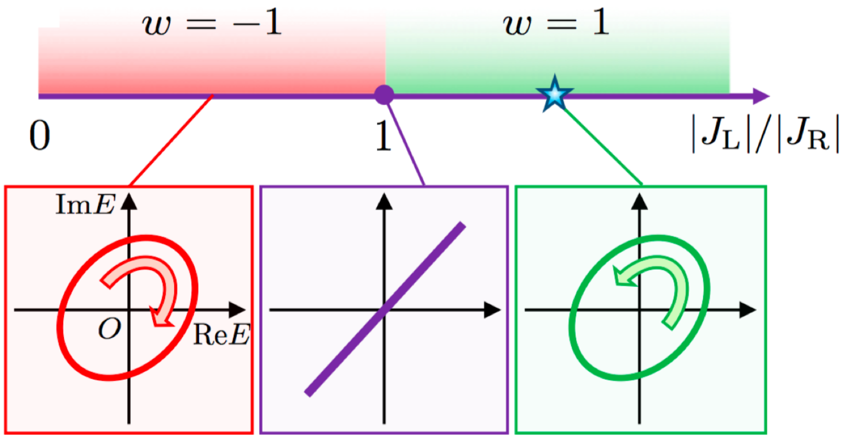
Gong, **YA**, Kawabata, Takasan, Higashikawa and Ueda, arXiv:1802.07964 [to appear in PRX]

Topological aspects of non-Hermitian systems (short summary)

Minimal model:
Asymmetric hopping of a single particle on 1D ring.



[c.f. Hatano and Nelson, PRL 77, 570 (1996)]



$$w = \int_{-\pi}^{\pi} \frac{dk}{2\pi i} \mathrm{Tr}[H^{-1}(k)\partial_k H(k)]$$

$$= \int_{-\pi}^{\pi} \frac{dk}{2\pi i} \frac{E'(k)}{E(k)}$$

***Robust against on-site disorder**
***General classification possible**
(in parallel with Floquet systems
c.f. Roy & Harper, PRB 96, 155118 (2017))

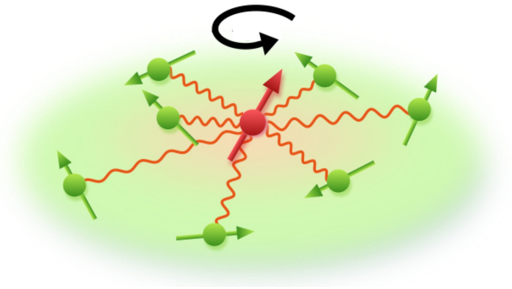
Gong, **YA**, Kawabata, Takasan, Higashikawa and Ueda, arXiv:1802.07964 [to appear in PRX]

Title:

Open quantum physics: non-Hermitian systems and beyond

So far, we focused on open quantum systems (weakly) coupled to a Markovian environment.

What happens if **strong system-environment entanglement** is present? (Theoretical analysis **beyond** the Born-Markov approximation needed).



2. Strongly correlated open systems: Nonequilibrium quantum impurities

YA, T. Shi, M. C. Bañuls, J. I. Cirac and E. Demler, PRL 121, 026805 (2018).

YA, T. Shi, M. C. Bañuls, J. I. Cirac and E. Demler, PRB 98, 024103 (2018).

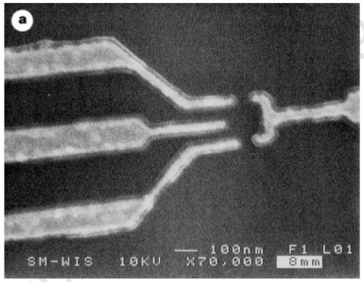
M. Kanász-Nagy, **YA,** T. Shi, C. P. Moca, T. N. Ikeda, S. Foelling, J. I. Cirac, G. Zarand and E. Demler, PRB 97, 155156 (2018).

YA, R. Schmidt, L. Tarruell and E. Demler, PRB 97, 060302(R) (2018).

Quantum impurity: a paradigm in many-body physics

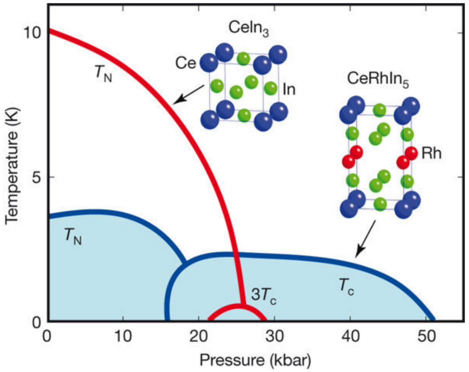
- **Relevance to a variety of physical systems:**

quantum dots



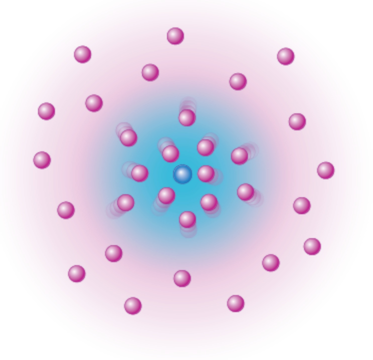
Goldhaber-Gordon et al.,
Nature 391, 156 (1998).

heavy electron materials



Monthoux et al.,
Nature 450, 1177 (2007).

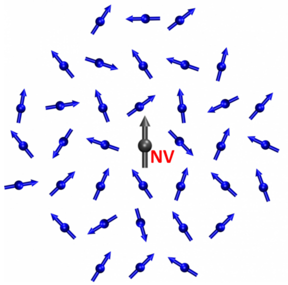
ultracold atoms



Physics 9, 86 (2016).

- **Prototypical open system**

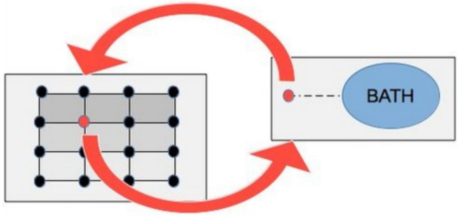
decoherence



Shin et al., PRB 88, 161412 (2013).

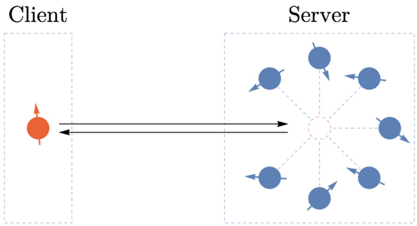
- **Numerical method**

DMFT solver



From Web page of LMU
theoretical nanophysics.

- **Universal quantum computation**

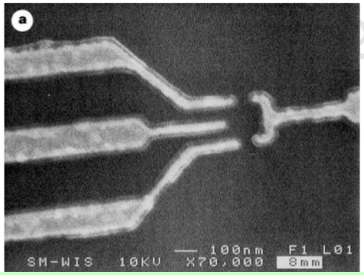


M. C. Tran & J. M. Taylor,
arXiv:1801.04006

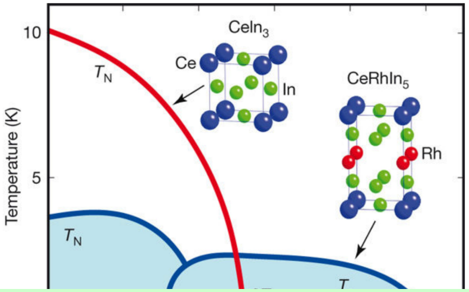
Quantum impurity: a paradigm in many-body physics

- Relevance to a variety of physical systems:

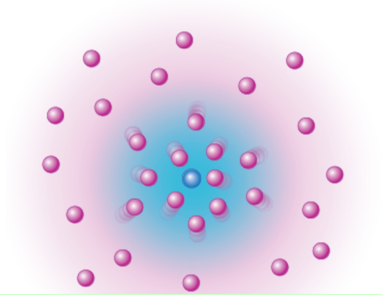
quantum dots



heavy electron materials



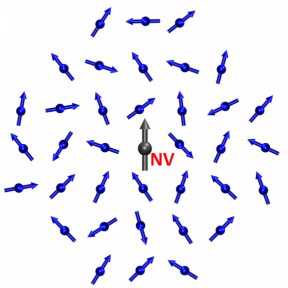
ultracold atoms



New theoretical approach to solve generic quantum spin impurity problems (including Kondo or central spin models)?

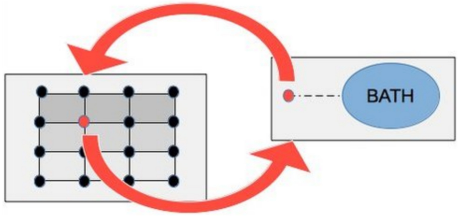
- Prototypical open system
- Numerical method
- Universal quantum computation

decoherence

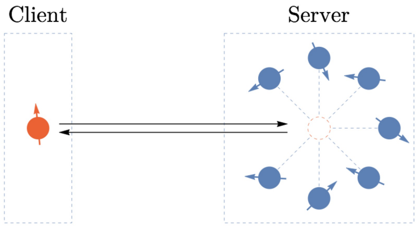


Shin et al., PRB 88, 161412 (2013).

DMFT solver



From Web page of LMU theoretical nanophysics.



M. C. Tran & J. M. Taylor, arXiv:1801.04006

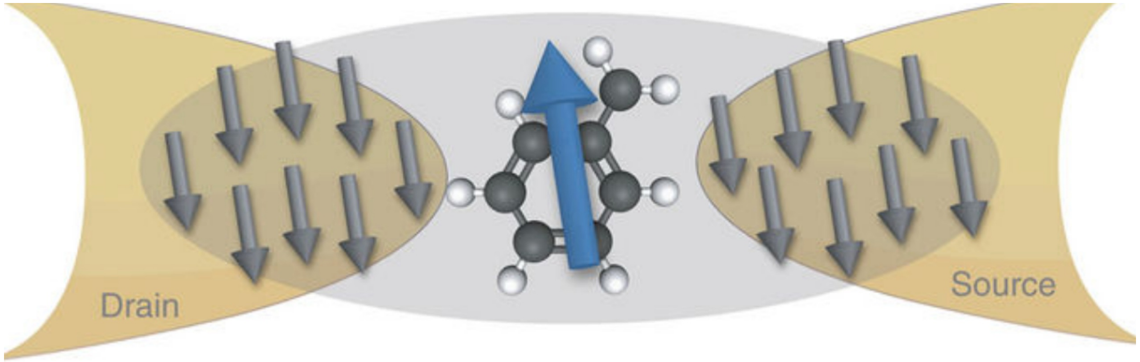
New "disentangling" canonical transformation

Essential feature of spin-impurity systems:

strong entanglement between impurity and bath

(*this strong correlation invalidates the Markov approximation).

Kondo singlet



impurity
spin 1/2

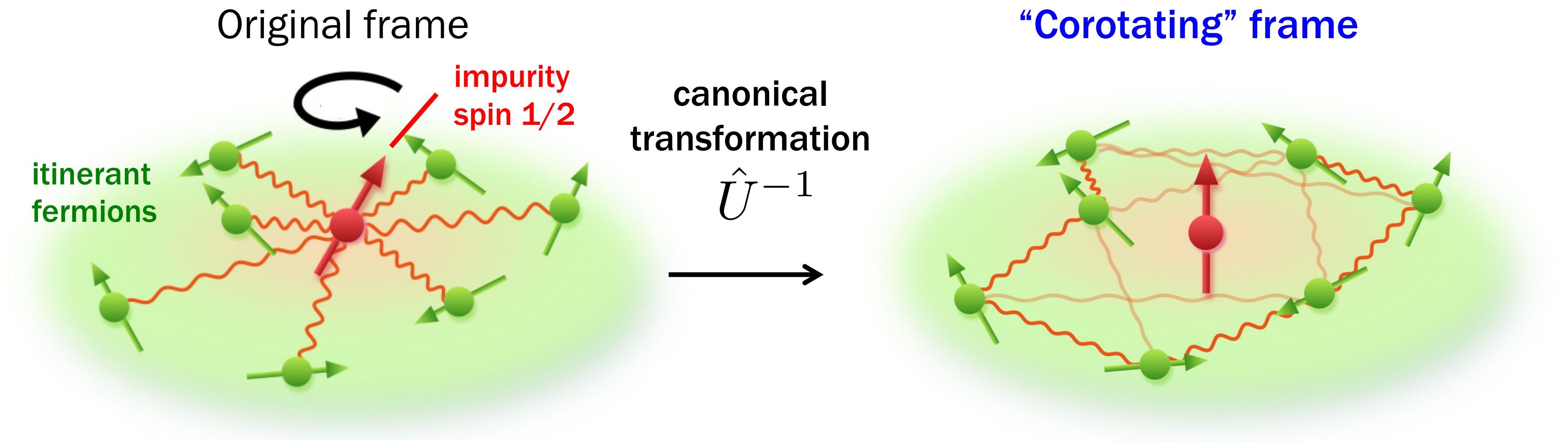
itinerant fermions

Nat. Commun. 8, 15210 (2017)

$$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_{\mathrm{imp}}|\Psi_{\downarrow}\rangle - |\downarrow\rangle_{\mathrm{imp}}|\Psi_{\uparrow}\rangle\right)$$

New "disentangling" canonical transformation

Construct a "disentangling" canonical transformation *U*



$$|\Psi_{\text{tot}}\rangle$$

Strong entanglement

$$\hat{U}^{-1}|\Psi_{\text{tot}}\rangle = |\Psi_{\text{imp}}\rangle|\Psi_{\text{bath}}\rangle$$

Impurity spin is **decoupled** from bath!

Efficient variational states: $|\Psi_{\text{var}}\rangle = \hat{U}|\Psi_{\text{imp}}\rangle|\Psi_{\text{bath}}\rangle$ ↑

Gaussian states

YA, T. Shi, M. C. Bañuls, J. I. Cirac and E. Demler, PRL 121, 026805 (2018).

YA, T. Shi, M. C. Bañuls, J. I. Cirac and E. Demler, PRB 98, 024103 (2018).

New “disentangling” canonical transformation

1. Notice **parity** symmetry in a Hamiltonian of a generic spin-impurity system:

$$\hat{H} = \sum_{nl\alpha} h_{nl}\hat{\Psi}^\dagger_{n\alpha}\hat{\Psi}_{l\alpha} + \frac{1}{4}\sum_{\gamma=x,y,z}\hat{\sigma}^\gamma_{\mathrm{imp}}\cdot\sum_{n\alpha\beta} g^\gamma_n\hat{\Psi}^\dagger_{n\alpha}\sigma^\gamma_{\alpha\beta}\hat{\Psi}_{n\beta} - \frac{h_{\mathrm{i}}}{2}\hat{\sigma}^z_{\mathrm{imp}}$$

invariance under π rotation of impurity and bath spins around *z* axis:

$$\hat{\sigma}^{x,y} \to \hat{\mathbb{P}}^{-1}\hat{\sigma}^{x,y}\hat{\mathbb{P}} = -\hat{\sigma}^{x,y} \longrightarrow \boxed{[\hat{H},\hat{\mathbb{P}}] = 0}$$

parity operator:

$$\hat{\mathbb{P}} = e^{i\pi\hat{\sigma}^z_{\mathrm{imp}}/2}\underbrace{e^{i\pi(\sum_n \hat{\sigma}^z_n + \hat{N})/2}}_{\text{bath parity: } \hat{\mathbb{P}}_{\mathrm{bath}}}$$

spin operator of bath mode *n*: $\hat{\sigma}^z_n$

2. Find the unitary transformation *U* mapping the parity to the impurity spin-1/2:

$$\hat{U}^\dagger\hat{\mathbb{P}}\hat{U} = \hat{\sigma}^x_{\mathrm{imp}}$$

⟶ impurity is **decoupled** !

$$\boxed{[\hat{\tilde{H}},\hat{\sigma}^x_{\mathrm{imp}}] = 0}$$

Transformed Hamiltonian

$$\hat{U} = \frac{1}{\sqrt{2}}\left(1 + i\hat{\sigma}^y_{\mathrm{imp}}\hat{\mathbb{P}}_{\mathrm{bath}}\right)$$

$$\hat{\tilde{H}} = \hat{U}^\dagger\hat{H}\hat{U}$$

$$= \sum_{lm\alpha} h_{lm}\hat{\Psi}^\dagger_{l\alpha}\hat{\Psi}_{m\alpha} + \frac{1}{4}\sum_{l}\left[g^x_l\sigma^x_{\mathrm{imp}}\hat{\sigma}^x_l + \underline{\hat{\mathbb{P}}_{\mathrm{bath}}\left(-ig^y_l\hat{\sigma}^y_l + g^z_l\sigma^x_{\mathrm{imp}}\hat{\sigma}^z_l\right)}\right]$$

effective bath-bath interaction mediated by the impurity

$\sigma^x_{\mathrm{imp}} = \pm 1$ is c-number.

New “disentangling” canonical transformation

1. Notice **parity** symmetry in a Hamiltonian of a generic spin-impurity system:

$$\hat{H} = \sum_{nl\alpha} h_{nl} \hat{\Psi}^\dagger_{n\alpha} \hat{\Psi}_{l\alpha} + \frac{1}{4} \sum_{\gamma=x,y,z} \hat{\sigma}^\gamma_{\text{imp}} \cdot \sum_{n\alpha\beta} g^\gamma_n \hat{\Psi}^\dagger_{n\alpha} \sigma^\gamma_{\alpha\beta} \hat{\Psi}_{n\beta} - \frac{h_{\text{i}}}{2} \hat{\sigma}^z_{\text{imp}}$$

invariance under π rotation of impurity and bath spins around *z* axis:

$$\hat{\sigma}^{x,y} \to \hat{\mathbb{P}}^{-1} \hat{\sigma}^{x,y} \hat{\mathbb{P}} = -\hat{\sigma}^{x,y} \longrightarrow \boxed{[\hat{H}, \hat{\mathbb{P}}] = 0}$$

Time-dependent variational principle

↓

Efficient variational imaginary- and real-time evolutions applicable to in- and out-of-equilibrium problems!

$U^\dagger \mathbb{P} U = \hat{\sigma}^x_{\text{imp}}$ ⟶ impurity is **decoupled** ! $\boxed{[\tilde{H}, \hat{\sigma}^x_{\text{imp}}] = 0}$

Transformed Hamiltonian

$$\hat{\tilde{H}} = \hat{U}^\dagger \hat{H} \hat{U} \longleftarrow \hat{U} = \frac{1}{\sqrt{2}} \left(1 + i \hat{\sigma}^y_{\text{imp}} \hat{\mathbb{P}}_{\text{bath}}\right)$$

effective bath-bath interaction mediated by the impurity

$$= \sum_{lm\alpha} h_{lm} \hat{\Psi}^\dagger_{l\alpha} \hat{\Psi}_{m\alpha} + \frac{1}{4} \sum_l \left[g^x_l \sigma^x_{\text{imp}} \hat{\sigma}^x_l + \underline{\hat{\mathbb{P}}_{\text{bath}} \left(-i g^y_l \hat{\sigma}^y_l + g^z_l \sigma^x_{\text{imp}} \hat{\sigma}^z_l \right)} \right]$$

$\uparrow$

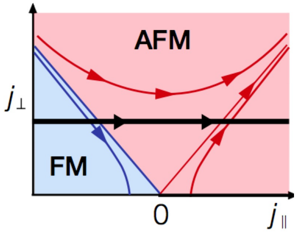
$\sigma^x_{\text{imp}} = \pm 1$ is c-number.

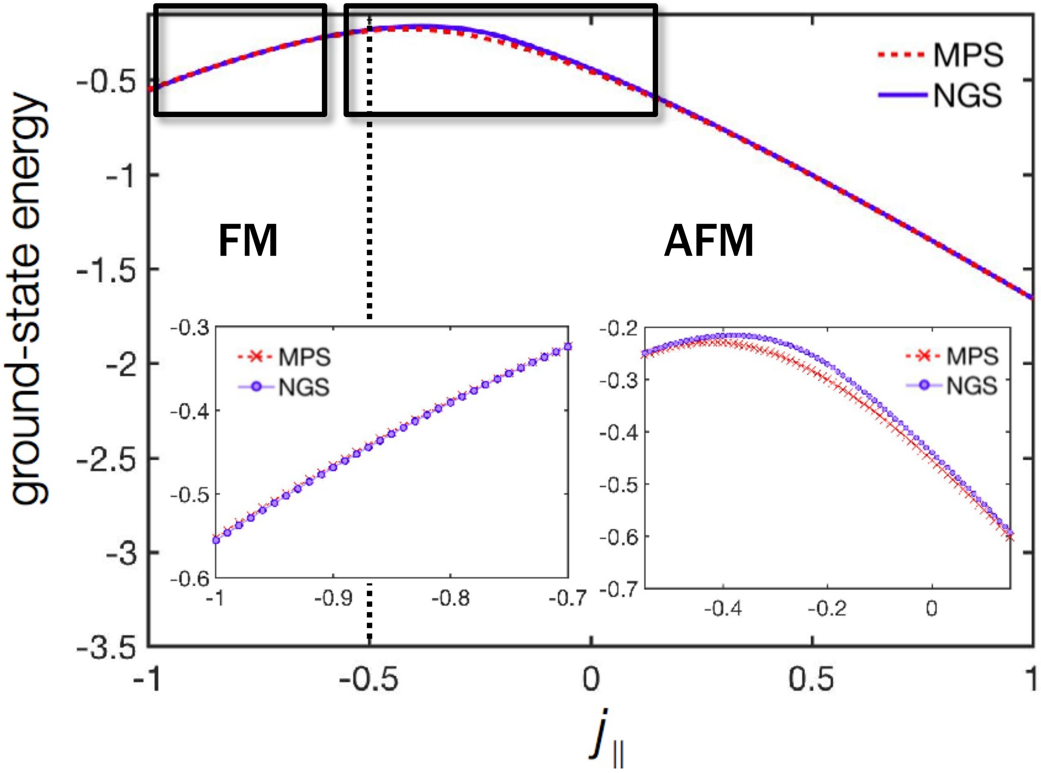
Benchmark tests with MPS results in and out of equilibrium

Ground-state energies (L=200)

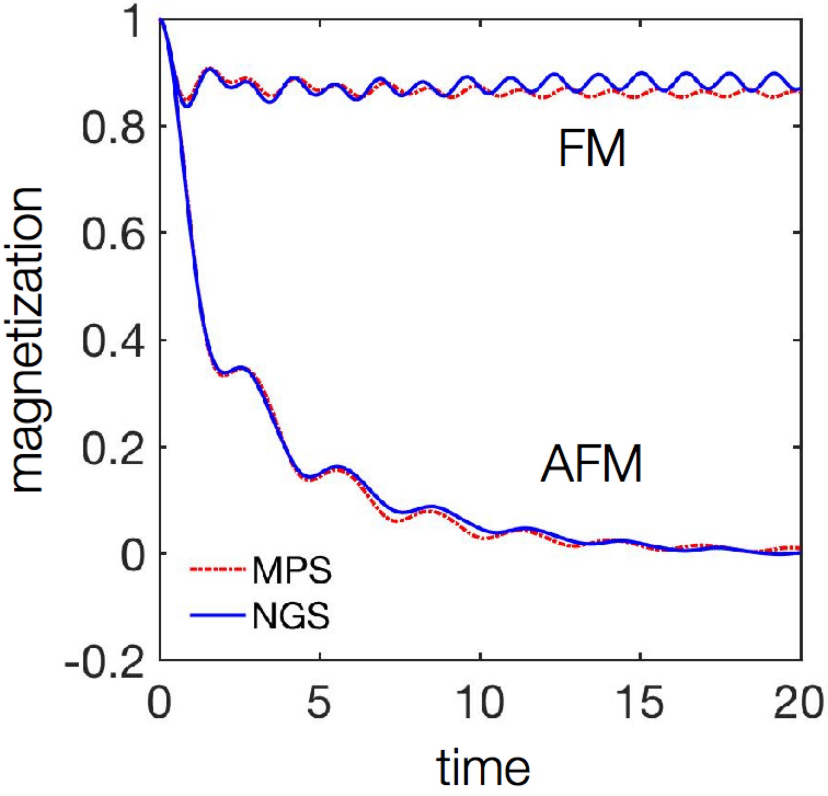
red: MPS (D=280)

blue: our variational states





Quench dynamics of the impurity



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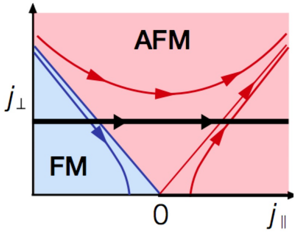
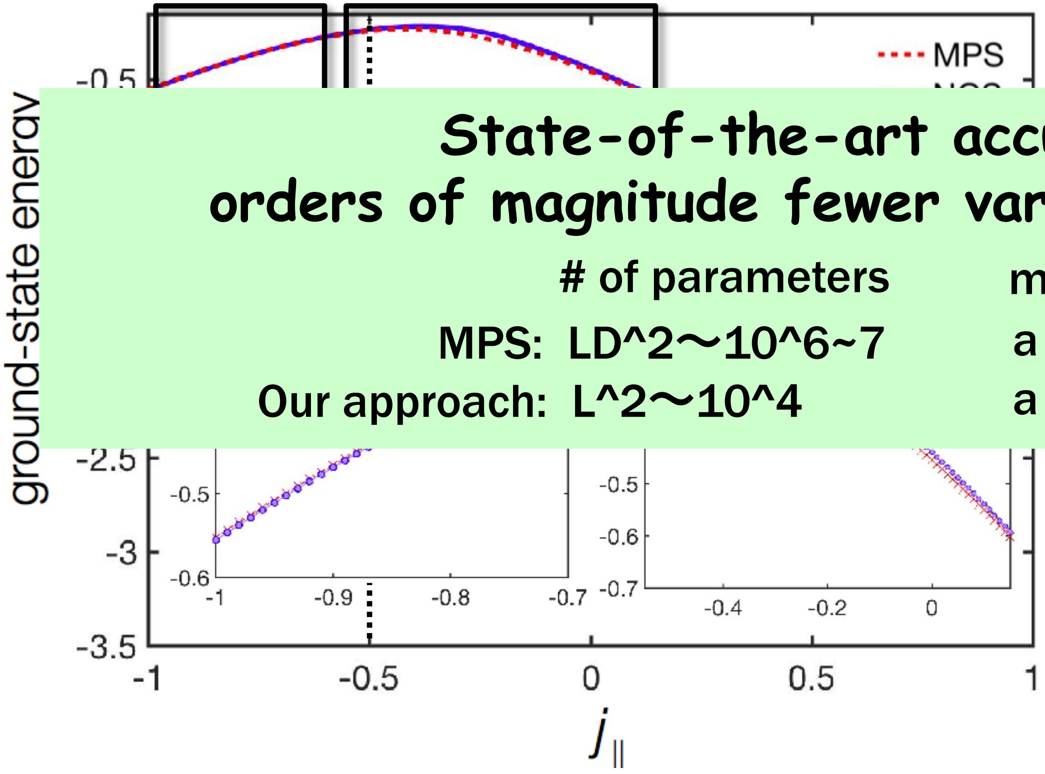
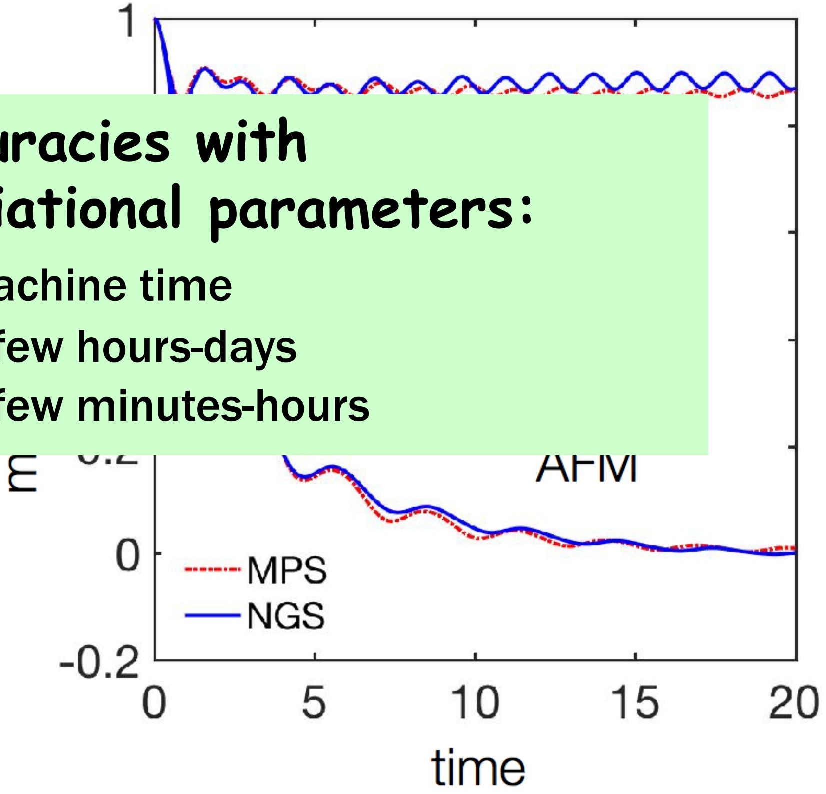


Quench dynamics of the impurity





State-of-the-art accuracies with orders of magnitude fewer variational parameters:

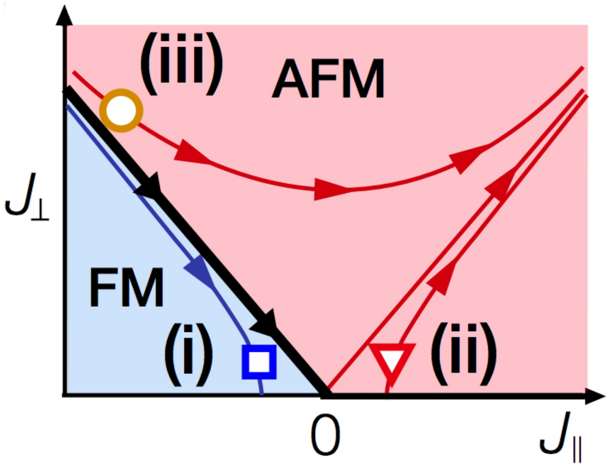
	# of parameters	machine time
MPS:	LD^2〜10^6~7	a few hours-days
Our approach:	L^2〜10^4	a few minutes-hours

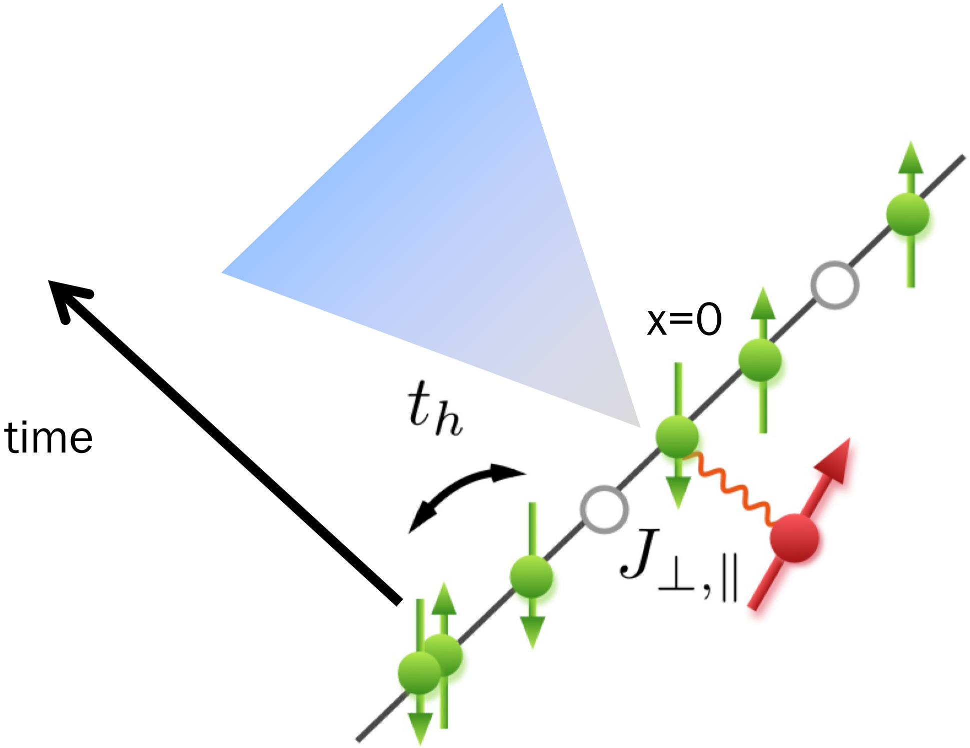
Spatiotemporal dynamics of correlations after quench

initial state: $|\Psi_0\rangle = |\uparrow\rangle_{\mathrm{imp}}|\mathrm{Fermi\ sea}\rangle$

Quench the impurity-bath interaction at site *x*=0.

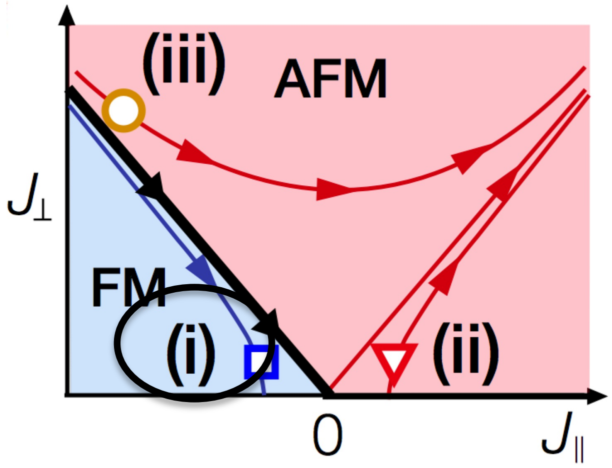
Real-time formation and spread of the impurity-bath correlation?



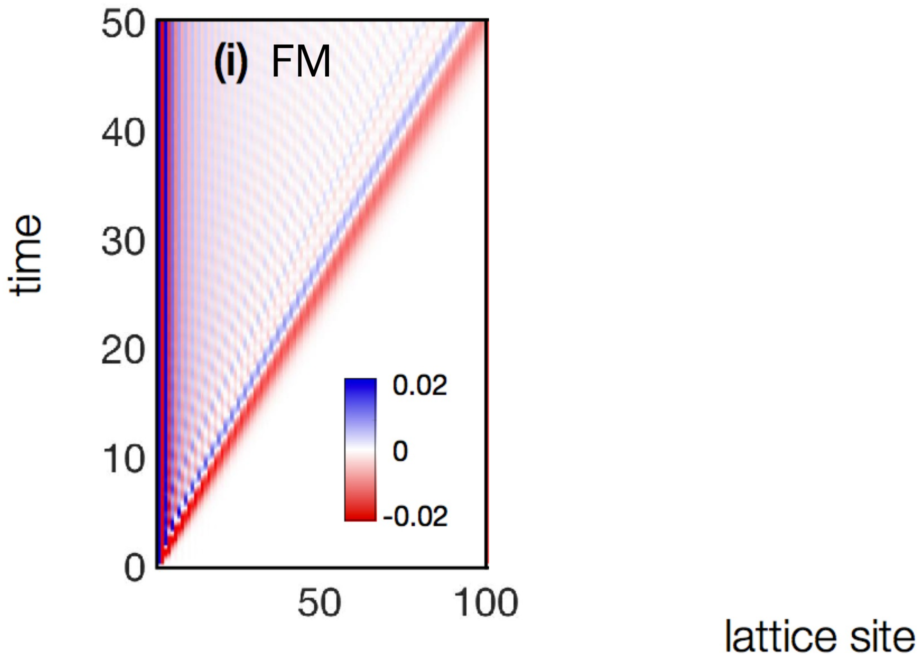


Spatiotemporal dynamics of correlations after quench

(i) FM phase: formation of triplet correlation



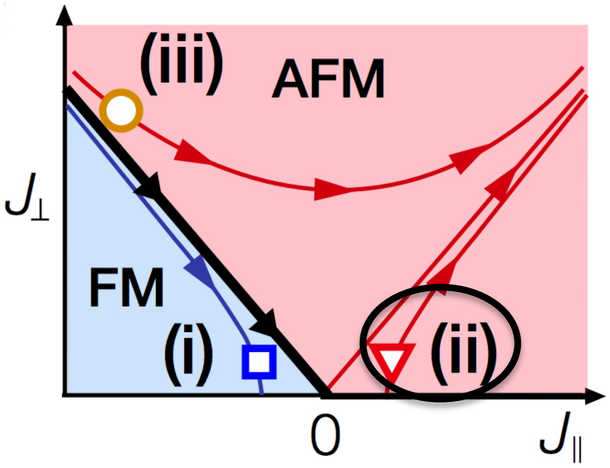
impurity-bath spin correlation $\langle\hat{\sigma}^z_{\mathrm{imp}}\hat{\sigma}^z_x\rangle$



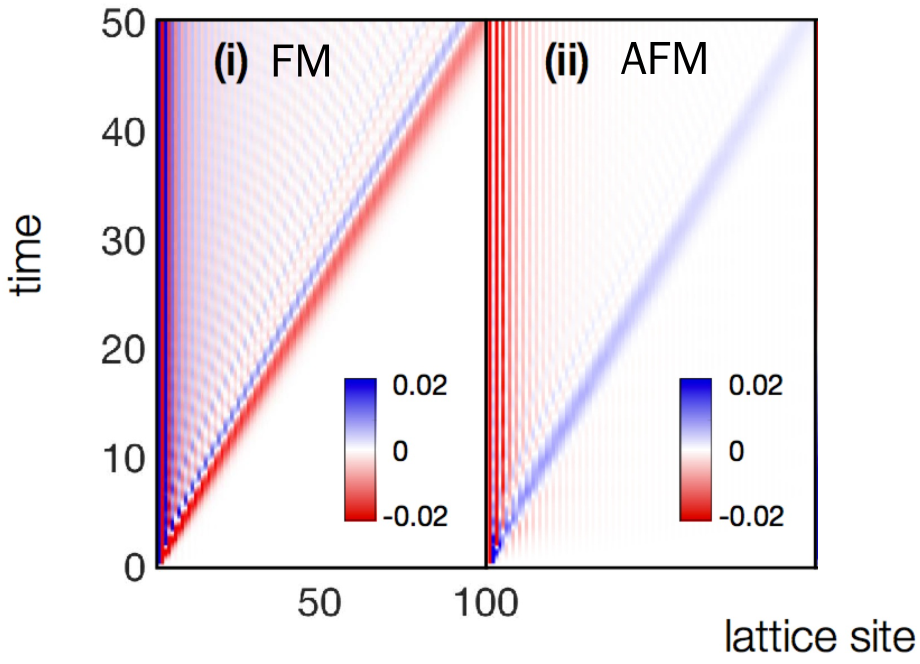
Spatiotemporal dynamics of correlations after quench

(i) **FM phase: formation of triplet correlation**

(ii) **AFM phase: formation of singlet correlation**



impurity-bath spin correlation $\langle \hat{\sigma}^z_{\rm imp} \hat{\sigma}^z_x \rangle$



Spatiotemporal dynamics of correlations after quench

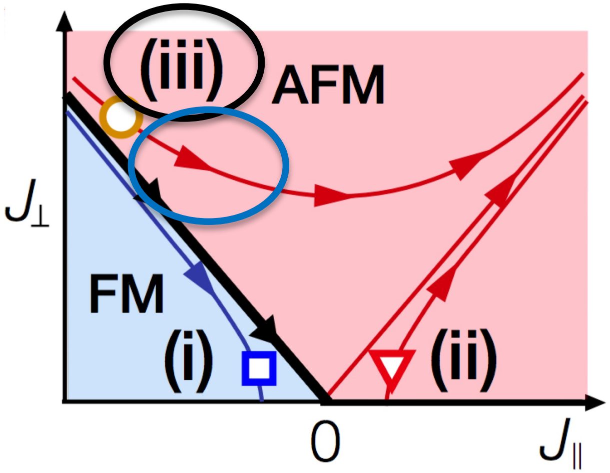
(iii) **FM→AFM crossover : mimicking RG flow in time**

short-time = high-energy physics → FM (Jz<0)

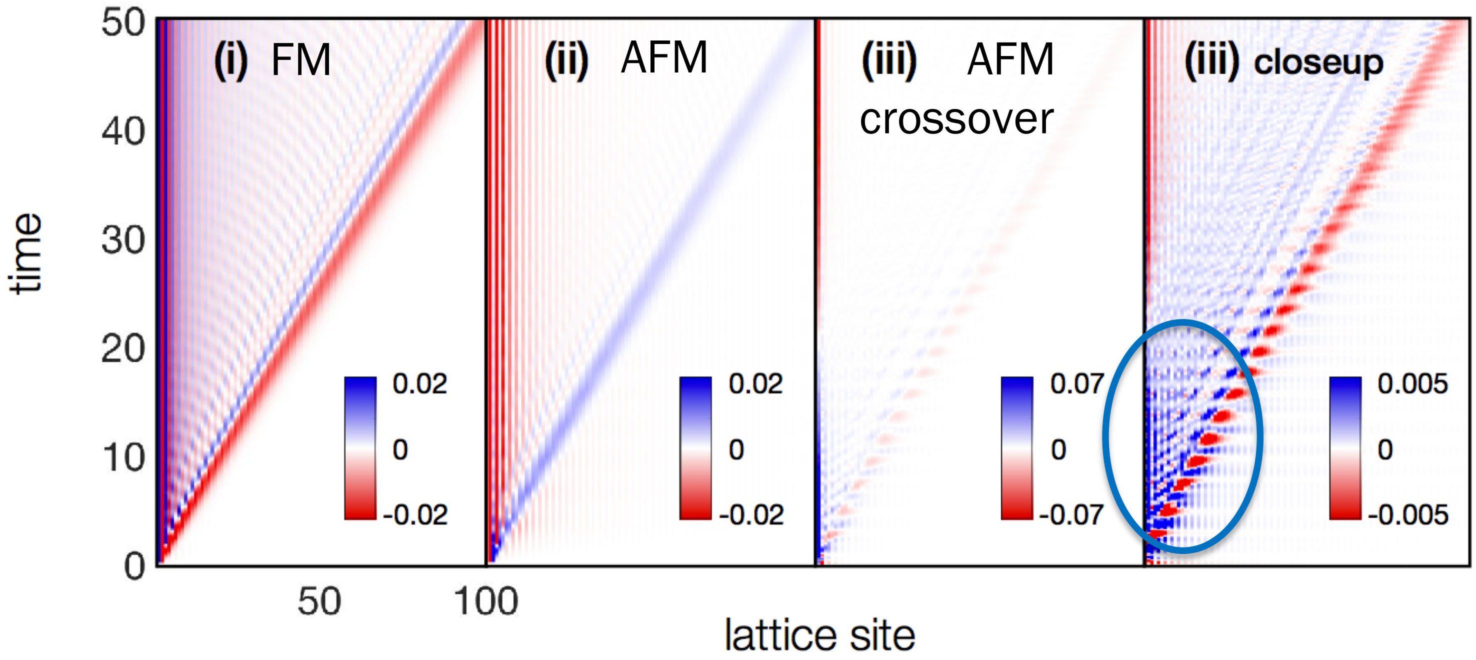
long-time = low-energy physics → AFM (J>0)

Effective temperature after quench: $T_{\mathrm{eff}} \simeq \dfrac{2}{\pi t}$

Nordlander et al., PRL 83, 808 (1999)



impurity-bath spin correlation $\langle \hat{\sigma}^z_{\mathrm{imp}} \hat{\sigma}^z_x \rangle$



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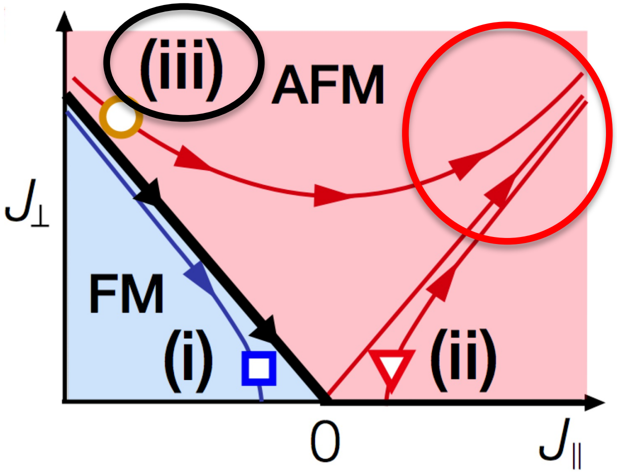
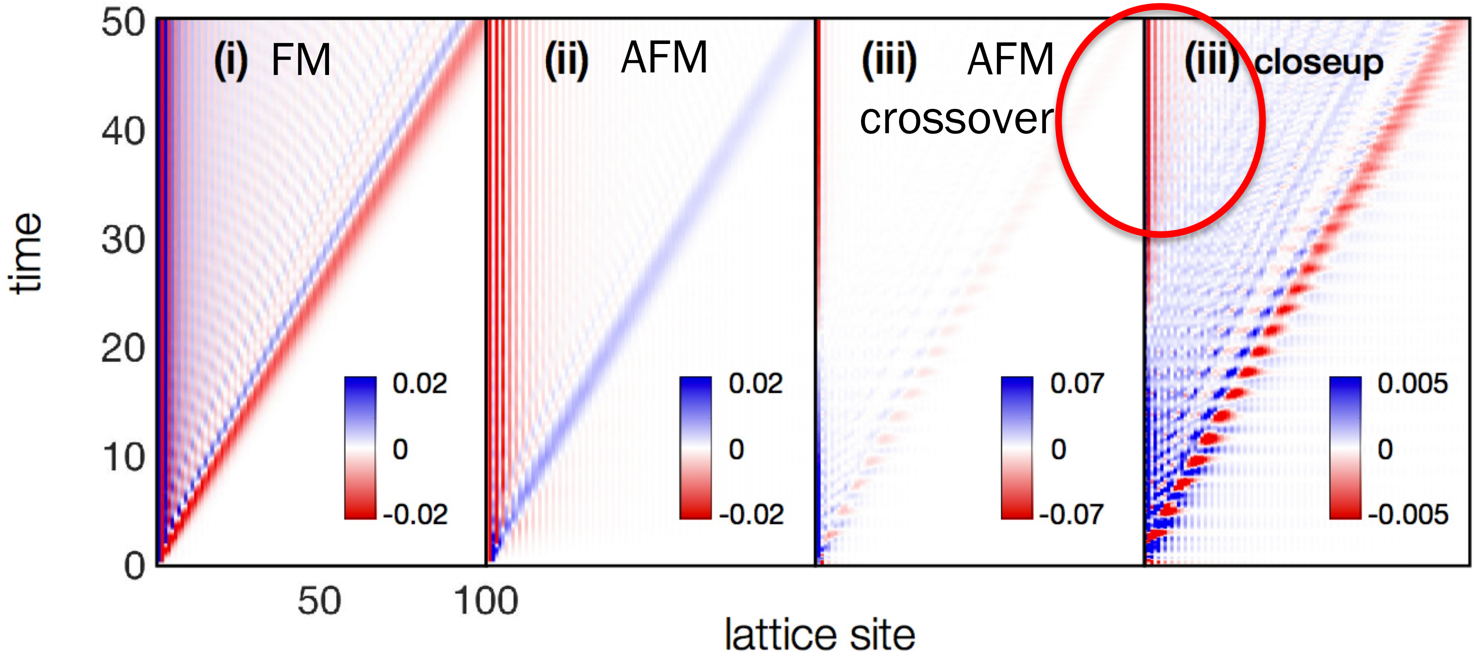


impurity-bath spin correlation $\langle \hat{\sigma}^z_{\text{imp}} \hat{\sigma}^z_x \rangle$



Spatiotemporal dynamics of correlations after quench

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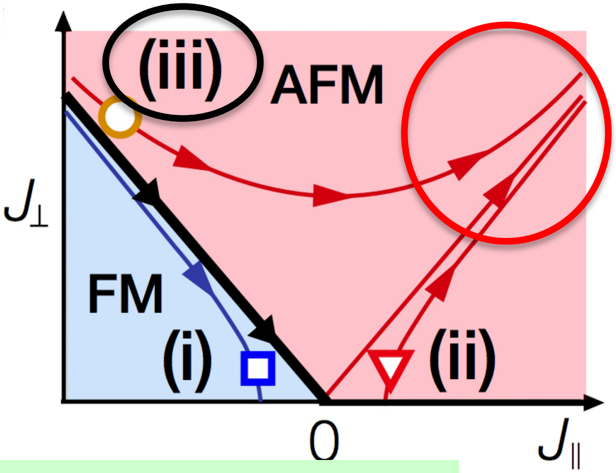
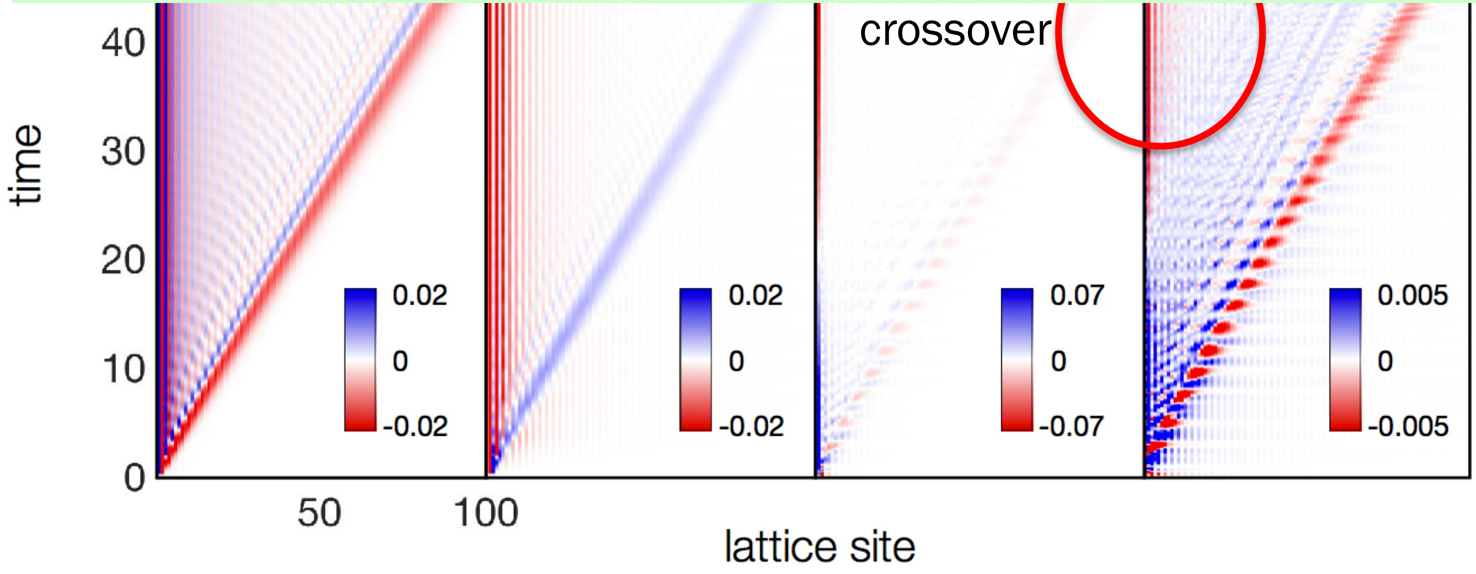
Nordlander et al., PRL 83, 808 (1999)



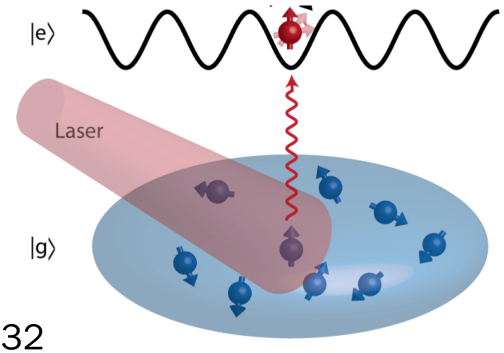
Long-time many-body dynamics in previously unexplored regime.

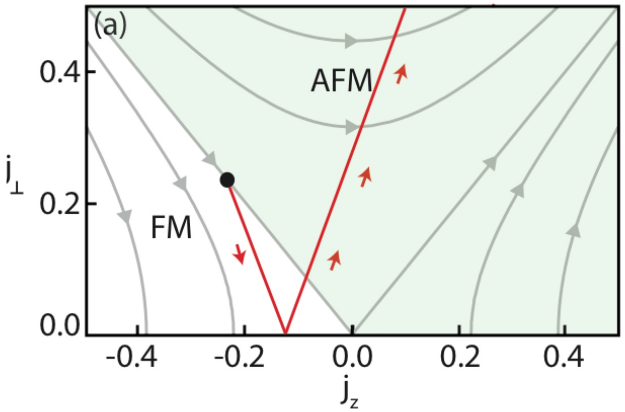


Spatiotemporal dynamics of correlations after quench

Experimental proposal with Yb atoms:

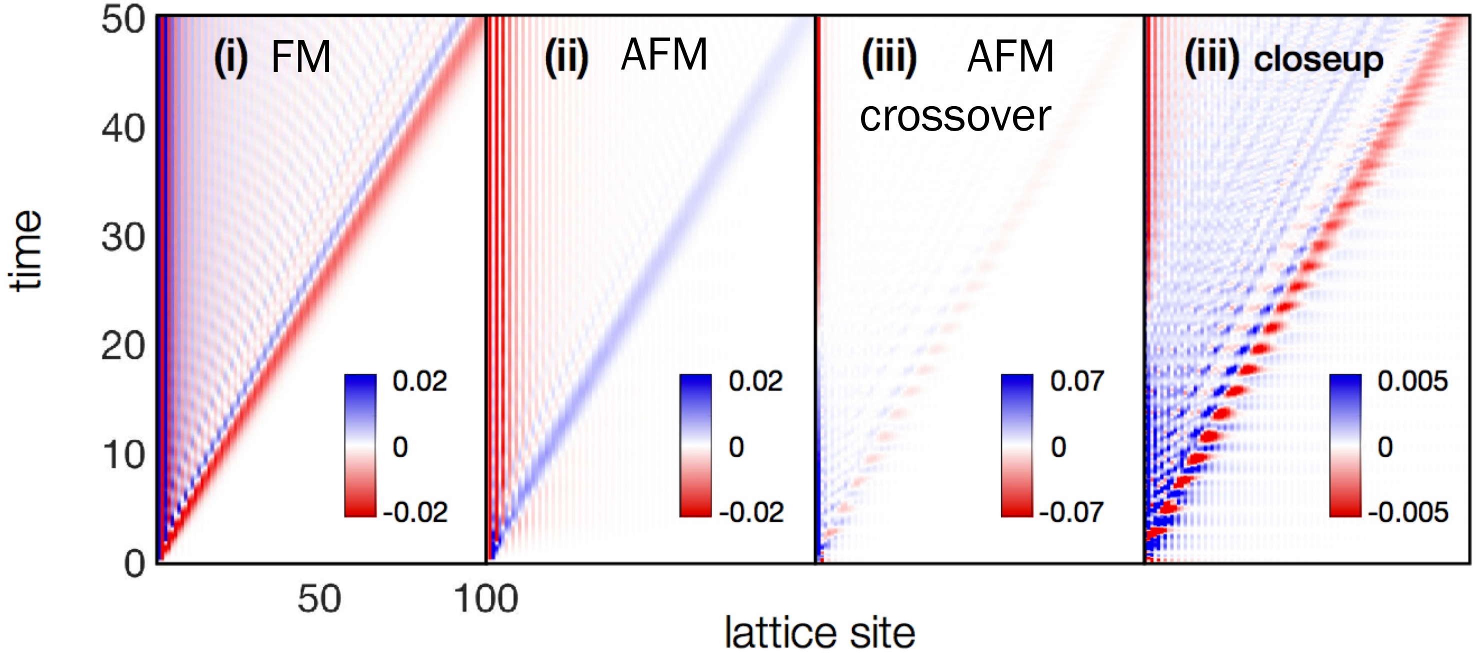
M. Kanász-Nagy, **YA**, T. Shi, C. P. Moca,
T. N. Ikeda, S. Foelling, J. I. Cirac,
G. Zarand and E. Demler, arXiv:1801.01132





✔ Spatiotemporal dynamics can be measured with QGM.

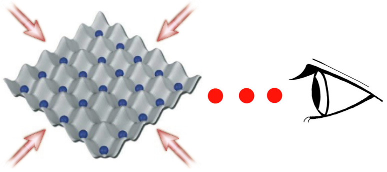
✔ Both FM and AFM regimes can be reached by Floquet engineering.



Summary: Quantum Many-Body Physics in Open Systems

I: Many-body physics under continuous observation & Non-Hermitian physics

- New types of critical phenomena, nonequilibrium dynamics and topology.

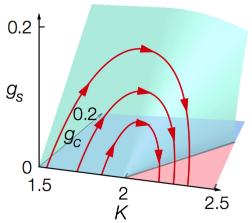


YA and M. Ueda, PRL 115, 095301 (2015).

YA, S. Furukawa and M. Ueda, Nat. Commun. 8, 15791 (2017).

YA and M. Ueda, PRL 120, 185301 (2018).

Z. Gong, **YA**, K. Kawabata, K. Takasan, S. Higashikawa
and M. Ueda, arXiv:1802.07964 [to appear in PRX].



II: Strongly correlated open quantum systems: nonequilibrium quantum impurity

Versatile and efficient theoretical approach to generic spin-impurity systems.

- Applicable to in- and out-of-equilibrium problems.

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