Determination of axion abundance using lattice QCD

Norikazu Yamada
(KEK Theory Center/SOKENDAI)

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Collaboration with

Julien Frison, Ryuichiro Kitano,

Hideo Matsufuru, Shingo Mori

Goal

QCD θ-vacuum(spontaneous CPV),

role of instantons [poster by Mori],

…

axion abundance

Strong CP problem

Symmetries of the SM do allow the θ term,

$$\mathcal{L}_\theta = \frac{i\theta}{32\pi^2}\epsilon^{\mu\nu\sigma\rho}\mathrm{Tr}(G_{\mu\nu}G_{\sigma\rho}) = \frac{i\theta}{32\pi^2}G\tilde{G}$$

$G_{\mu\nu}$: gluon field strength

$\boldsymbol{\theta = \theta_{\mathbf{QCD}} + \theta_{\mathbf{Yukawa}}}$

NEDM exp ➡ $\boldsymbol{\theta \lesssim 10^{-10}}$ **!**

➡ **Why is $\boldsymbol{\theta}$ so small?**

Two possible solutions

$\boxed{\checkmark\, m_{\mathrm{u}} = 0}$

θ-term $\Rightarrow$ unphysical

$\boxed{\checkmark\,\text{Peccei-Quinn mechanism}}$

θ-term dynamically vanishes

$\boxed{m_u = 0?}$

In 1992, "quark masses" appears for the first time.

d

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Mass $m = 5$ to 15 MeV   Charge $= -\frac{1}{3}\,e$ $I_z = -\frac{1}{2}$

$m_d/m_s = 0.04$ to 0.06

The d-, u-, and s-quark masses are estimates of so-called "current-quark masses," with ratios m_u/m_d and m_d/m_s extracted from pion and kaon masses using chiral symmetry. The estimates of d and u masses are not without controversy and remain under active investigation. Within the literature there are even suggestions that the u quark could be essentially massless. The s-quark mass is estimated from SU(3) splitting in hadron masses.

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u

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Mass $m = 2$ to 8 MeV   Charge $= \frac{2}{3}\,e$ $I_z = +\frac{1}{2}$

$m_u/m_d = 0.25$ to 0.70

See the comment for the d quark above.

$m_u = 0?$

Latest PDG (2016)

$\boxed{\text{Light Quarks } (u, d, s)}$

OMITTED FROM SUMMARY TABLE

u-QUARK MASS

The *u*-, *d*-, and *s*-quark masses are estimates of so-called "current-quark masses," in a mass- independent subtraction scheme such as $\overline{\text{MS}}$. The ratios m_u/m_d and m_s/m_d are extracted from pion and kaon masses using chiral symmetry. The estimates of *d* and *u* masses are not without controversy and remain under active investigation. Within the literature there are even suggestions that the *u* quark could be essentially massless. The *s*-quark mass is estimated from SU(3) splittings in hadron masses.

We have normalized the $\overline{\text{MS}}$ masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35. The values of "Our Evaluation" were determined in part via Figures 1 and 2.

$\overline{\text{MS}}$ *MASS* (MeV)	*DOCUMENT ID*		*TECN*
2.2 $^{+0.6}_{-0.4}$ OUR EVALUATION	See the ideogram below.		
$2.27 \pm 0.06 \pm 0.06$	[1] FODOR	16	LATT
2.36 ± 0.24	[2] CARRASCO	14	LATT
$2.57 \pm 0.26 \pm 0.07$	[3] AOKI	12	LATT
$2.15 \pm 0.03 \pm 0.10$	[4] DURR	11	LATT
1.9 ± 0.2	[5] BAZAVOV	10	LATT
$2.24 \pm 0.10 \pm 0.34$	[6] BLUM	10	LATT

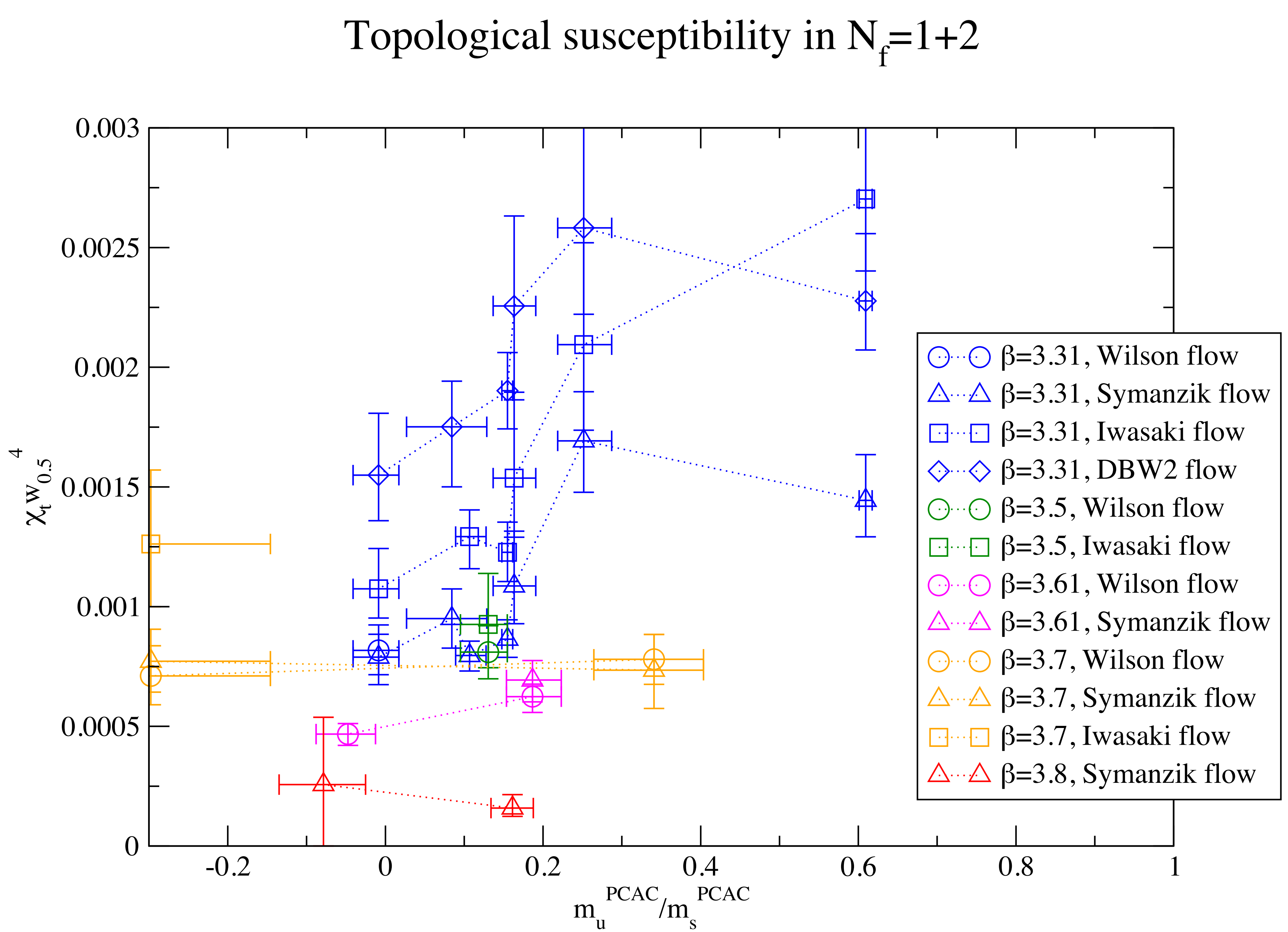
$m_{\mathrm{u}} = 0?$

$N_f = 1+2$

$m_1 \ll m_2 = m_3$

χ_t : topological susceptibility

$\chi_t \propto m_1$



J. Frison, et al., in preparation

PQ mechanism [Peccei and Quinn (77)]

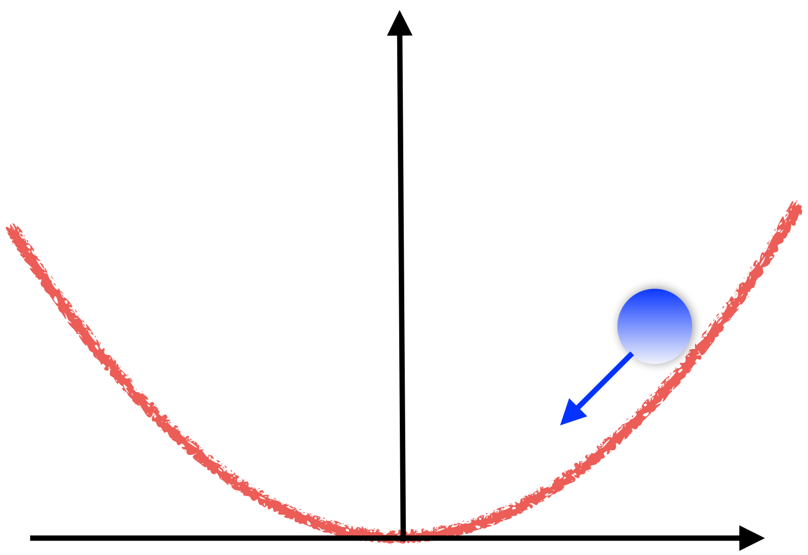
Introduce a complex scalar field $\varphi(x)=|\varphi(x)|e^{ia(x)/f_a}$

$$V(\varphi) = \lambda\left(|\varphi|^2 - \frac{f_a^2}{2}\right)^2 \xrightarrow{\text{SSB}} \langle\varphi\rangle e^{ia/f_a}$$

Through the coupling to quarks

$$\Rightarrow \quad \theta \to \theta + \frac{a(x)}{f_a}$$

$\Rightarrow$ Minimizing axion potential



$\Downarrow$

$$\boxed{\theta + \frac{a}{f_a} = 0 \quad \text{(dynamically selected)}}$$

$\Rightarrow$ Strong CP problem gone.

Axion mass

Axion mass:

$$m_a^2(T) = \chi_t(T) / 2 f_a^2$$

χ_t : topological susceptibility

$$\chi_t = -\frac{1}{V}\frac{1}{Z}\frac{d^2 Z}{d\theta^2}\bigg|_{\theta=0} = \frac{\langle Q^2 \rangle}{V}$$

Q: topological charge

$$Q = \frac{1}{32\pi^2}\int d^4x\, G\tilde{G}$$

At T=0, $\chi_t \approx [80\ \mathrm{MeV}]^4$

$$\Rightarrow \quad m_a \approx 6 \times 10^{-6}\,\mathrm{eV}\left(\frac{10^{12}\mathrm{GeV}}{f_a}\right)$$

axion abundance

Evolution of coherent component:

$$\boxed{\ddot{a} + 3H(T)\dot{a} + m_a(T)^2 a = 0}$$

$$3H(T)^2 M_{\rm pl}^2 \propto T^4$$

$$m_a^2(T) = \chi_t(T) / 2 f_a^2$$

Initially, $3H(T) \gg m_a(T) \Rightarrow \dot{a} = 0$

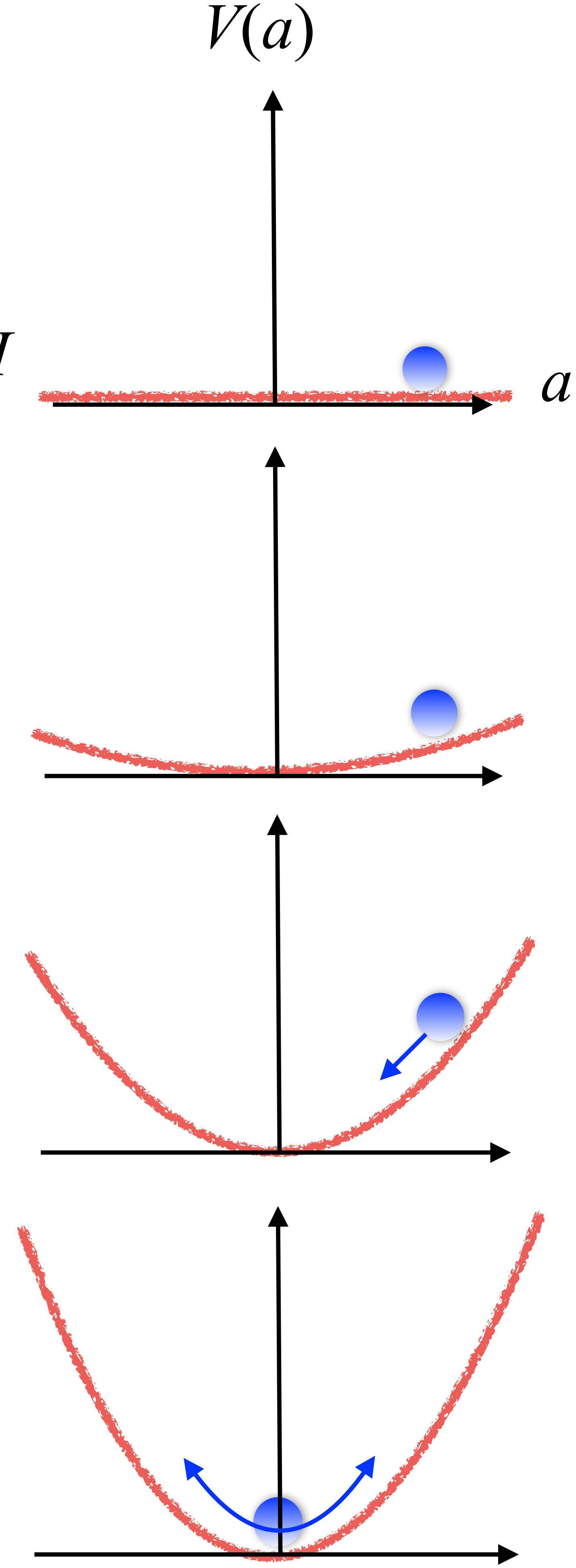
When $3H(T^*) \sim m_a(T^*) \Rightarrow a$ starts to oscillate.

$$n_a(T_0) \approx n_a(T^*)(T_0/T^*)^3$$

$$\sim m_a(T^*) f_a^2 \theta_{a,\rm i}^2 (T_0/T^*)^3$$

$(\theta_{a,\rm i} = \theta + a_{\rm ini} / f_a)$

Need to know T dependence of $\chi_t(T)$



T≫T*
$m_a \sim 0 \ll 3H$

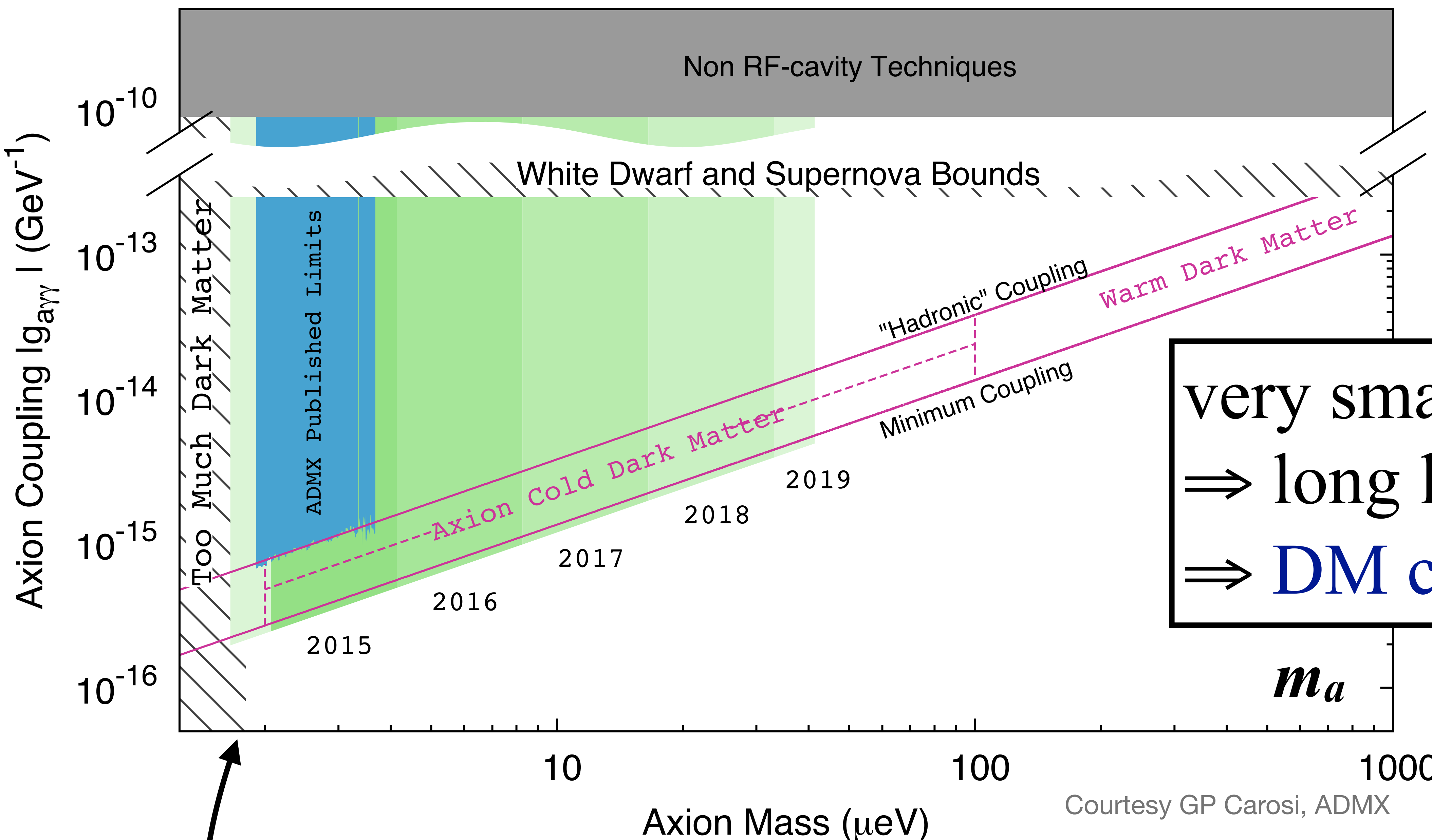
T>T*
$m_a \neq 0 < 3H$

T≈T*
$m_a^* \approx 3H$

T=T₀
$m_{a0} > 3H_0$

Constraints on axion mass

k_{model} / f_a



m_a

very small $1/f_a$

$\Rightarrow$ long life time

$\Rightarrow$ DM candidate

Axion abundance ≤ DM abundance

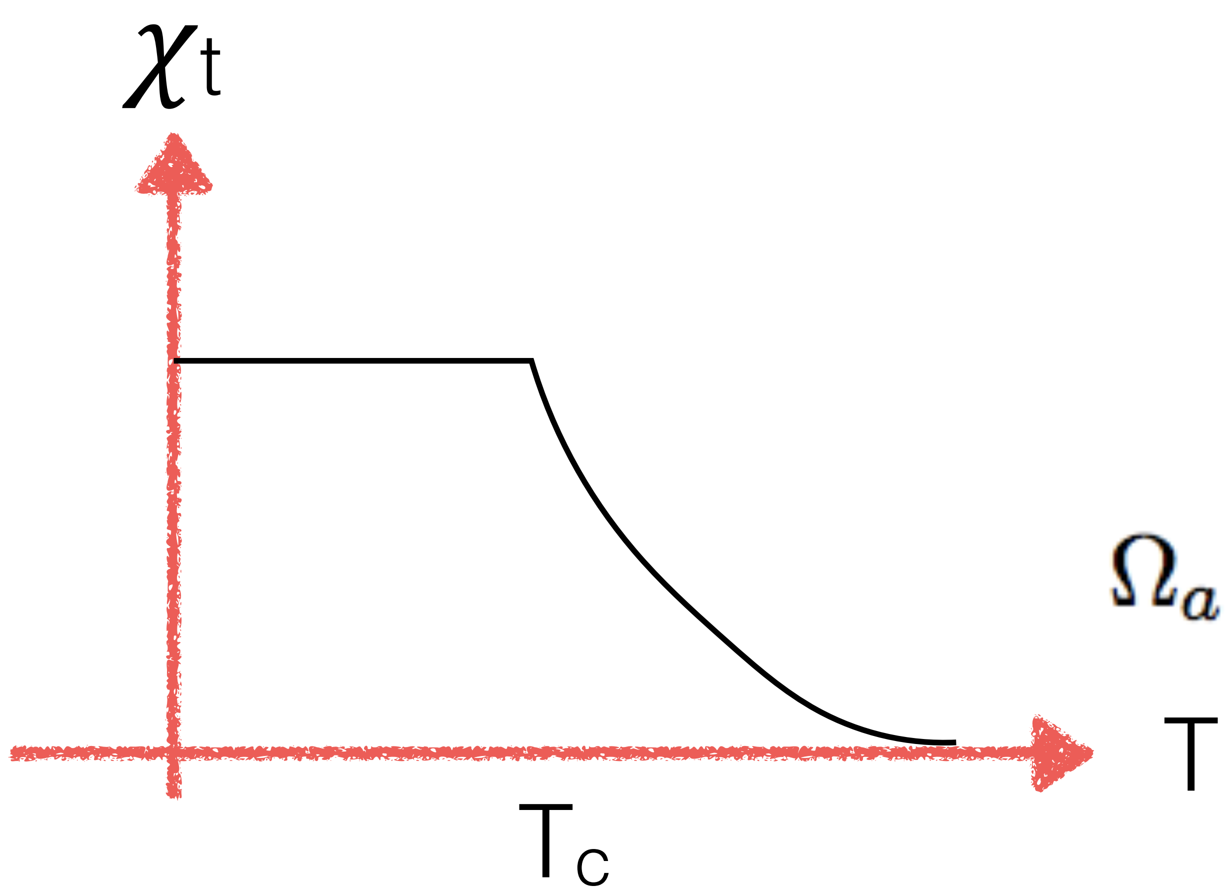
Over-closure bound depending on $\chi_t(T)$

$\chi_t(T)$ from Instanton calculus [Gross, Pisarski, Yaffe (1981)]

Estimate of axion abundance requires $\chi_t(T)$ in $T_c < T < 10\ T_c$.

Dilute Instanton Gas Approximation (DIGA) is often used.

$$\chi_t(T) = m_a^2(T) f_a^2 \propto m_u m_d m_s \Lambda_{\rm QCD}^b T^{-8}$$



Instanton action = $e^{-8\pi^2/g^2}$

b: beta function

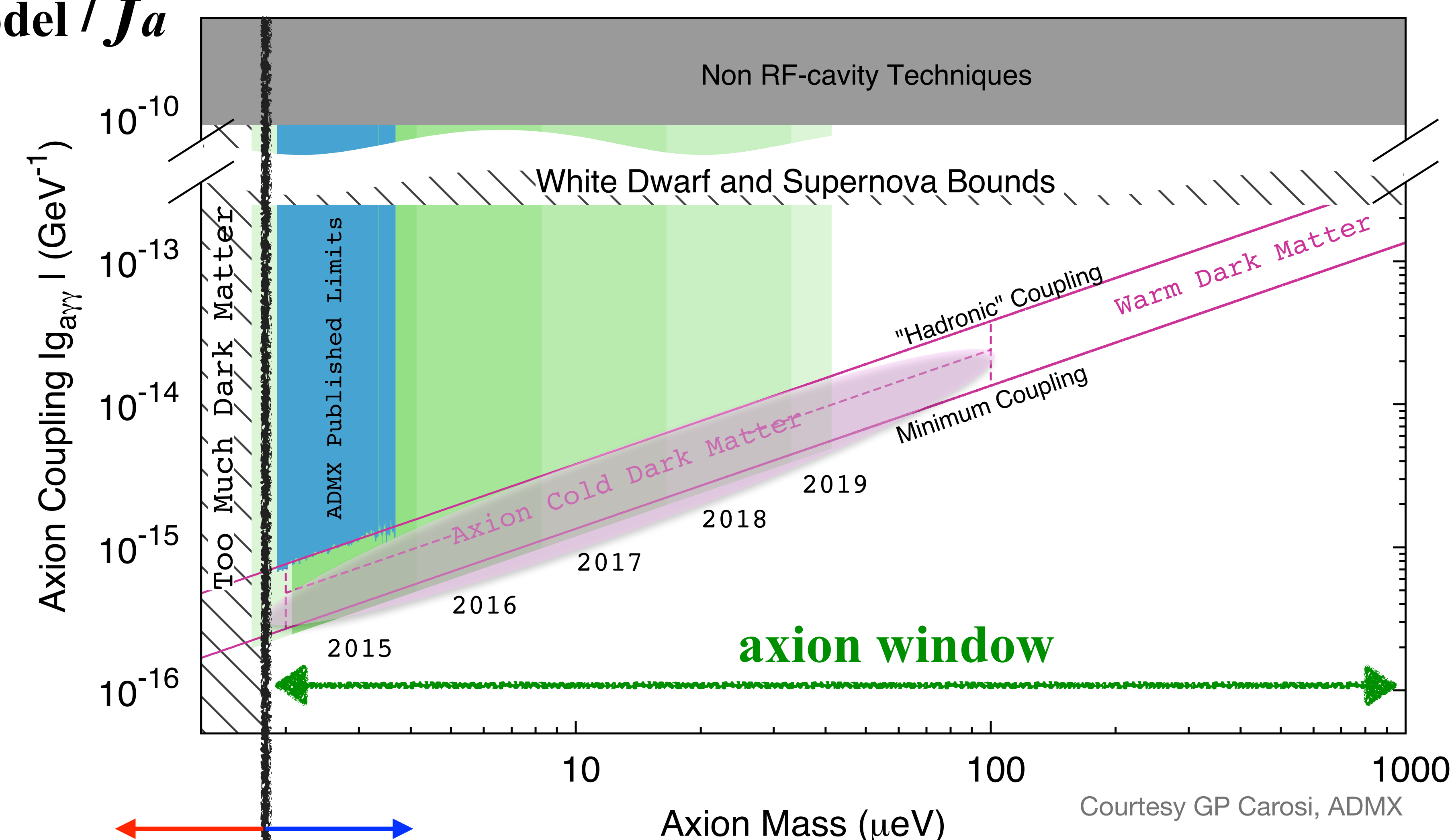
$$\Omega_a \simeq 0.2 \cdot \theta_{\rm ini}^2 \left(\frac{m_a}{10^{-5}\ {\rm eV}}\right)^{-7/6}$$

Over-closure bound

Axion abundance $\leq$ DM abundance

Instanton: $\chi_t \sim T^{-8} \Rightarrow \Omega_a \simeq 0.2 \cdot \theta_{\rm ini}^2 \left(\frac{m_a}{10^{-5}\ {\rm eV}} \right)^{-7/6}$

$k_{\rm model} / f_a$



Mild T-dependence

Steep T-dependence

Over-closure bound

Is DIGA reliable?

At low T ($\sim\Lambda_{\mathrm{QCD}}$), validity is questionable.
The following extreme behaviors are suggested or yet-allowed.

✓Step function-like behavior

$$\chi_{\mathrm{t}}(T) \sim \chi_{\mathrm{t}}(T{=}0)\ \theta(T_{\mathrm{c}}-T) \quad \text{if } m_{\mathrm{u,d}} < m_{\mathrm{q}}^{\mathrm{crit}}$$

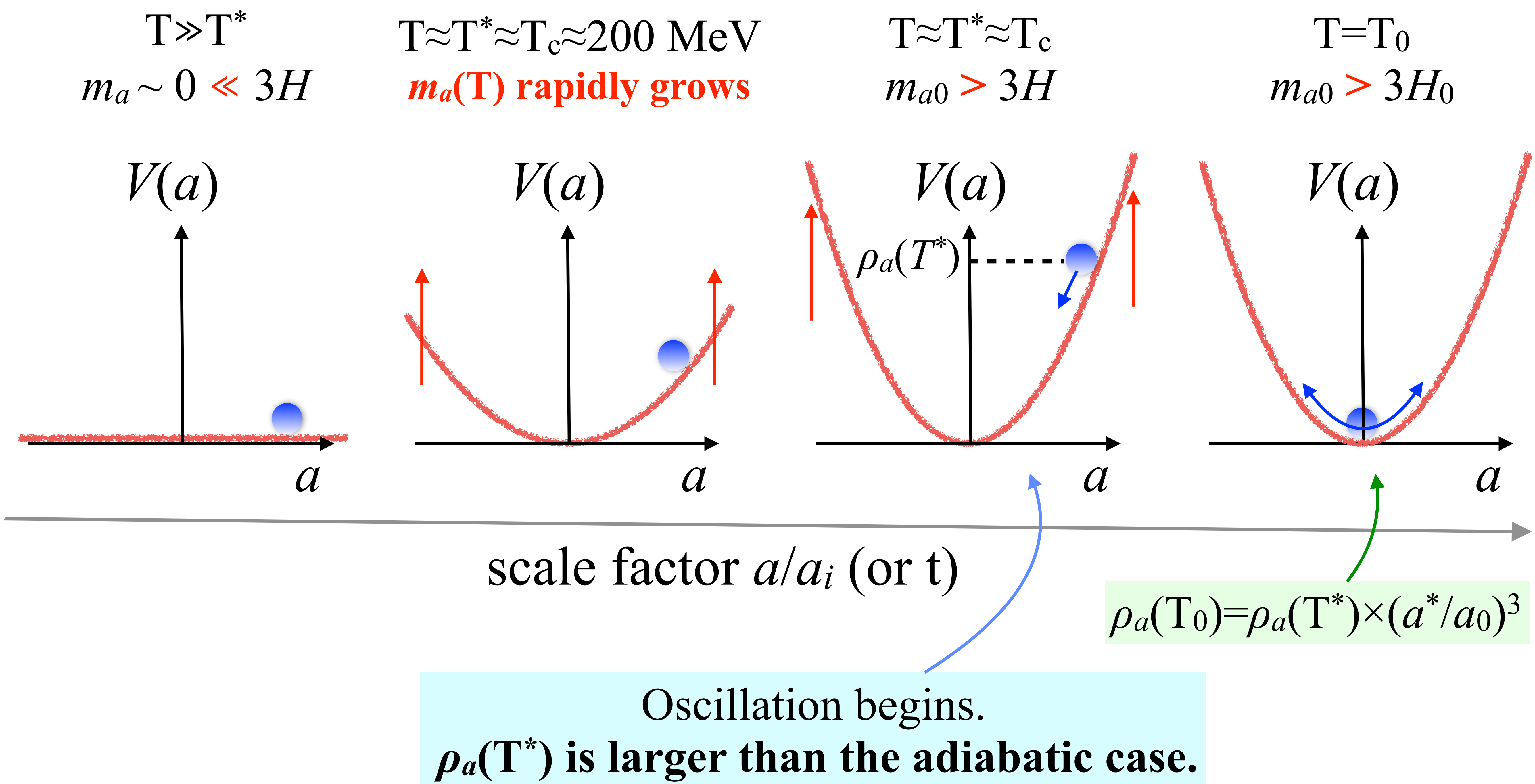
✓A bit milder case

$$\chi_{\mathrm{t}}(T) \sim \chi_{\mathrm{t}}(T{=}0) \qquad \text{for } T \lesssim T_{\mathrm{c}}$$

$$\sim \chi_{\mathrm{t}}(T{=}0)\exp[\,-2\mathrm{c}(m_q)\ T^2/T_{\mathrm{c}}^2\,] \quad \text{for } T > T_{\mathrm{c}}$$

In extreme cases

[Kitano, Yamada (2015)]



Goal

$$\chi_t(T) = \langle Q^2 \rangle / V_4$$

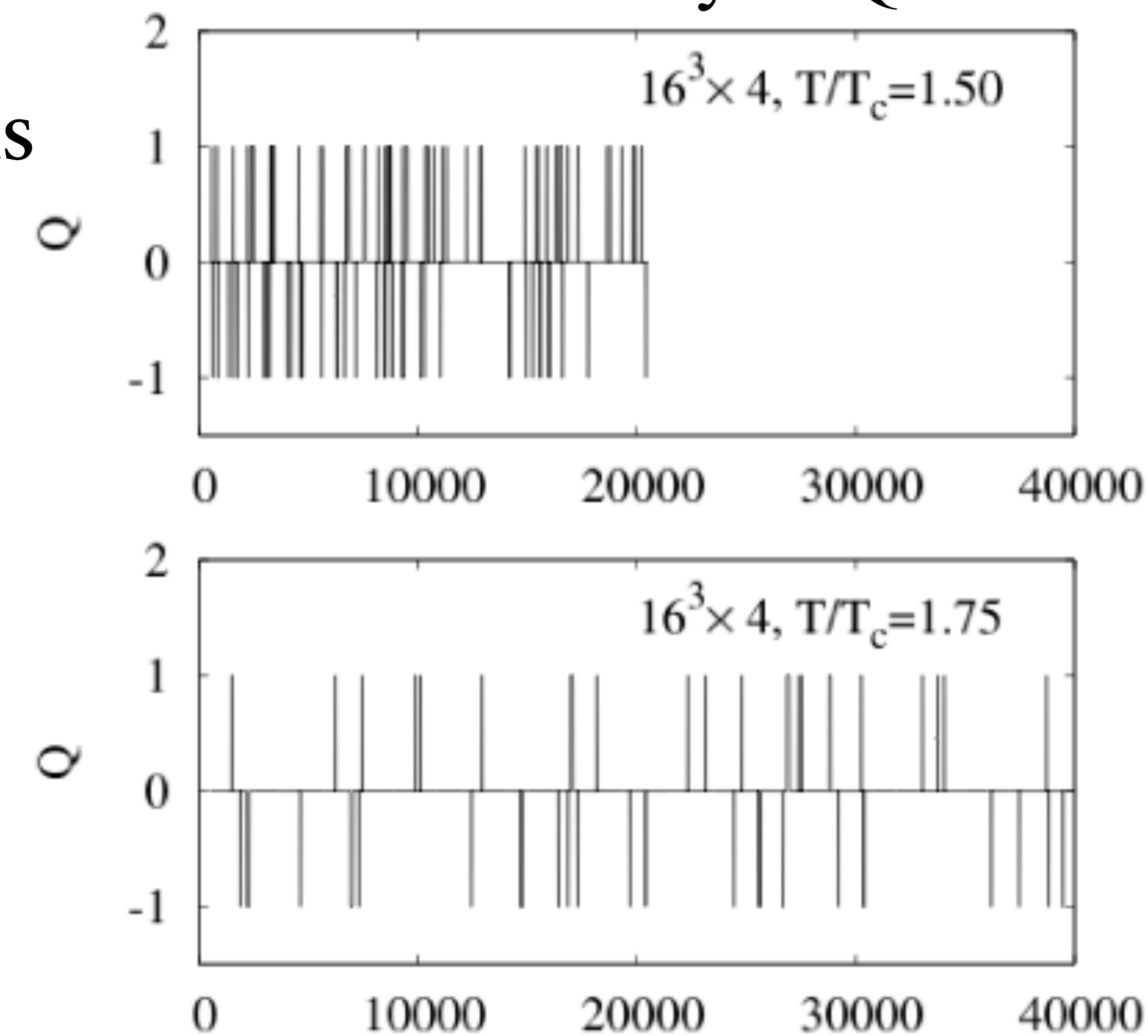
in $T_c < T < O(10 \times T_c)$

Lattice calculations of $\chi_t(T)$

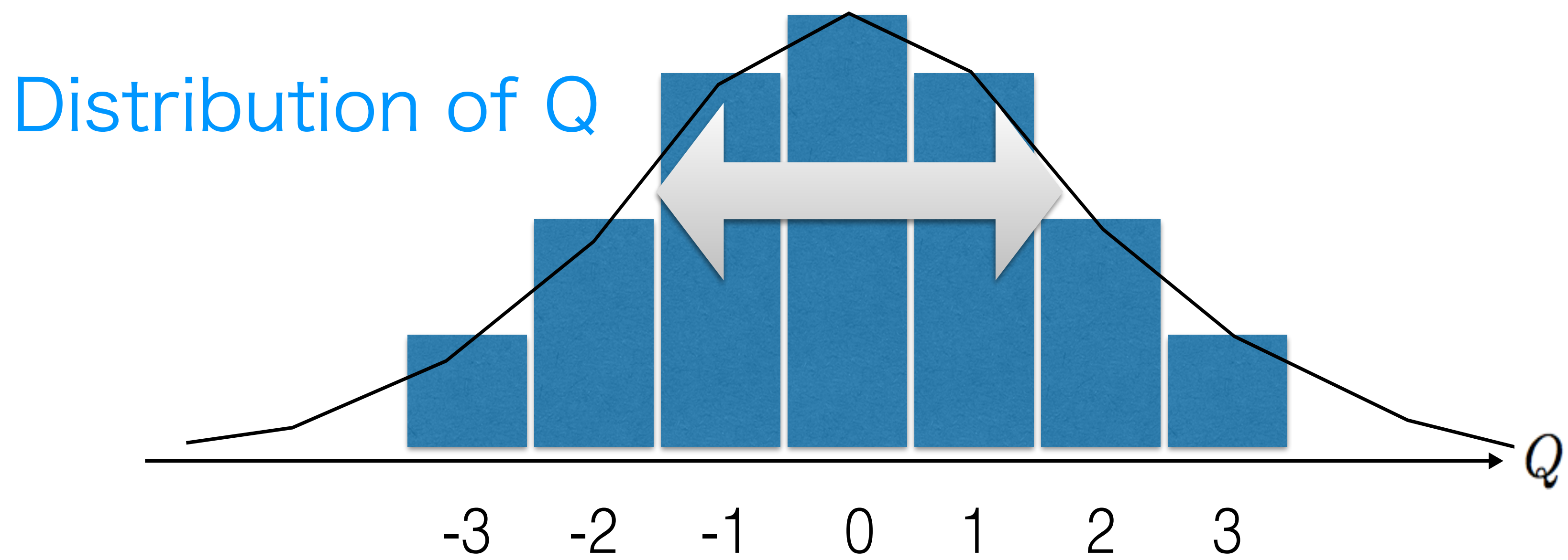
- Generate configurations
- Measure Q at each configuration

$$Q = \int d^4x \frac{1}{32\pi^2} F\tilde{F}$$
$$= \int d^4x \ \mathrm{tr}\gamma_5$$

MC history of Q



Lattice calculations of $\chi_t(T)$



width ⇒ $\langle Q^2 \rangle$

$$\chi_t = \frac{\langle Q^2 \rangle}{V}$$

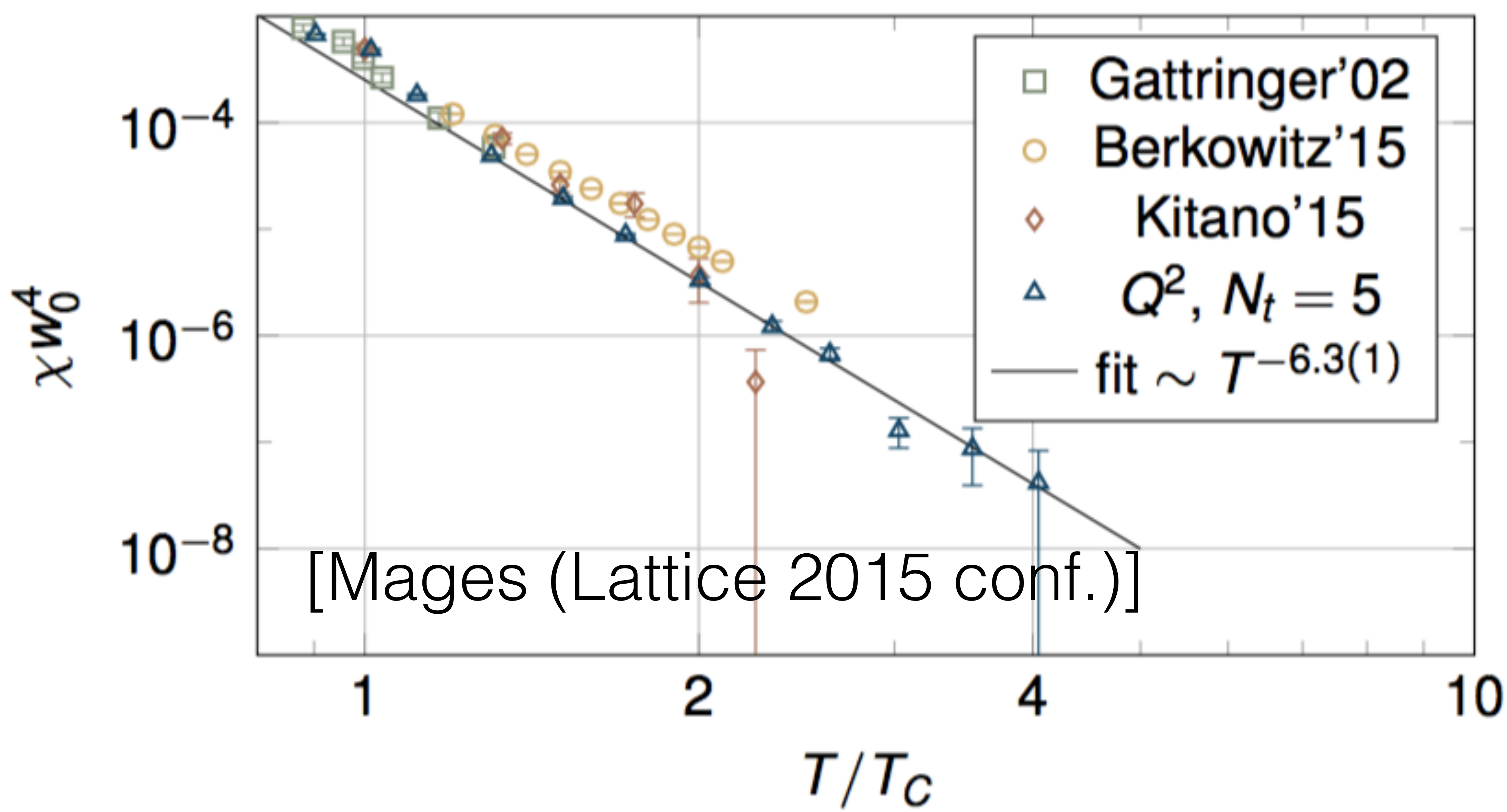
χ_t in pure Yang-Milles

in 2015, three independent calculations appeared.

(in the SU(3) Yang-Milles theory, **no quarks yet**)

- E. Berkowiz, M. Buchoff, E. Rinaldi (LLNL)
  Bosonic (cooling)
- R. Kitano and NY (KEK) Index theorem
- S. Mages et al (BMW) Bosonic (Wilson Flow)

Quenched approximation (no dynamical quark)

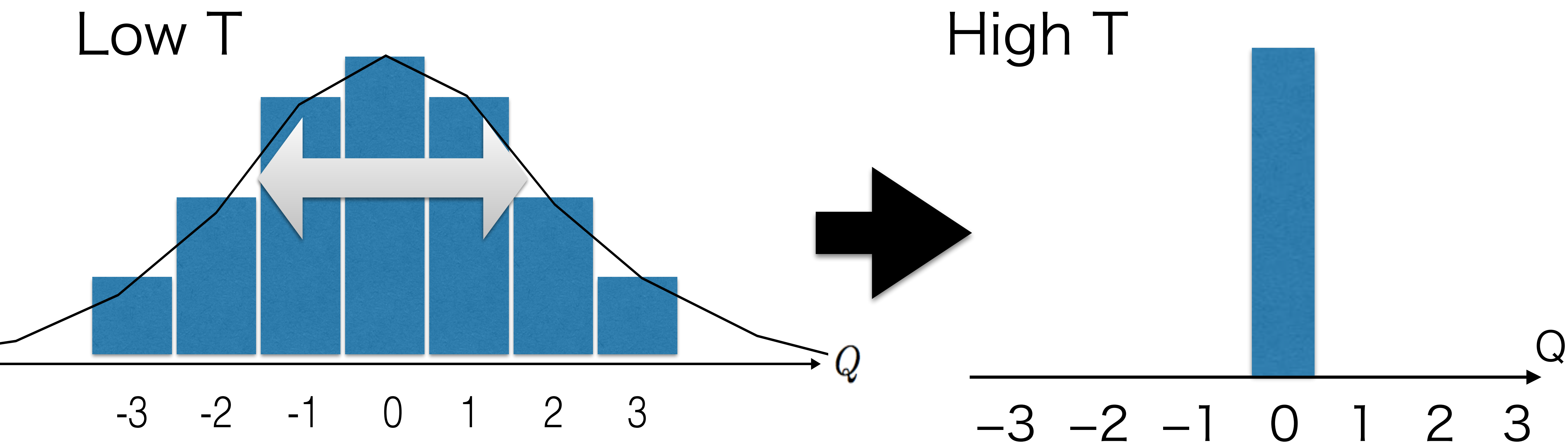


We see a clear power law even at a very low temperature.

Standard method fails at $T > 4\ T_c$.

Problem in the standard method at high T

$\langle Q^2 \rangle$: width of Q-distribution



Above a certain high T, distribution $\Rightarrow \delta(Q)$

Fail to estimate χ_t ($=\langle Q^2 \rangle / V_4$) when $\langle Q^2 \rangle \ll 1$

✓ New Method

J. Frison, R. Kitano, H. Matsufuru, S. Mori, NY, JHEP 1609 (2016) 021 [arXiv:1606.07175 [hep-lat]]

New Method

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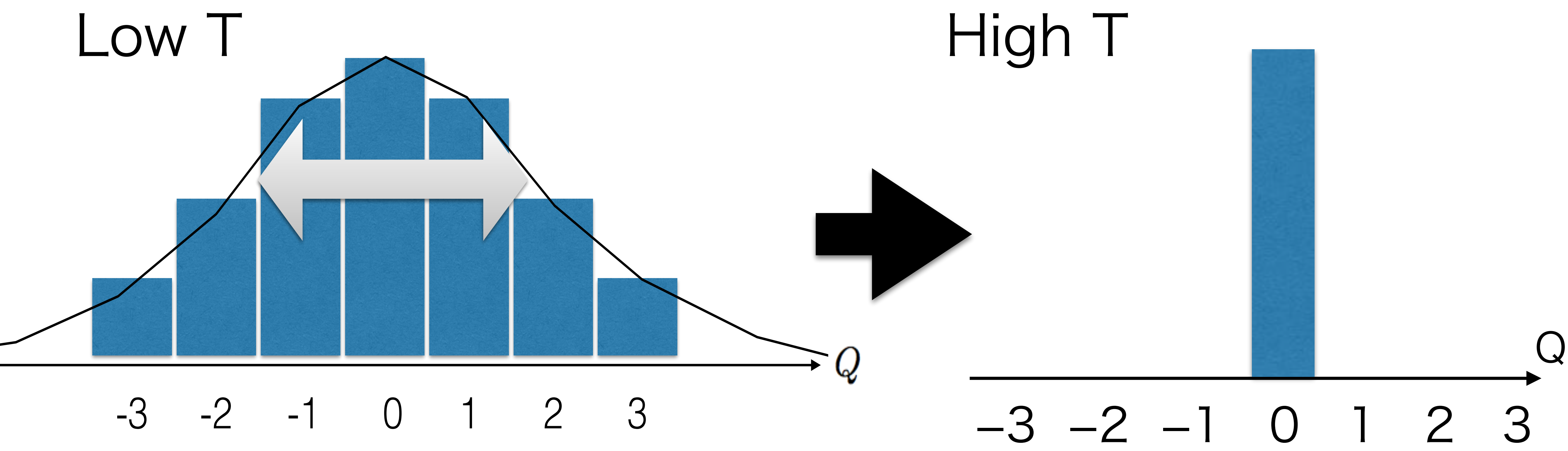


$$\langle Q^2 \rangle = \chi_t V$$

Give up $\chi_t(T)$

Focus on $d \ln \chi_t(T) / d \ln T$

New Method

J. Frison, R. Kitano, H. Matsufuru, S. Mori, NY, JHEP 1609 (2016) 021 [arXiv:1606.07175 [hep-lat]]

$$\left.\frac{d\ln\frac{Z_{Q_2}(T)}{Z_{Q_1}(T)}}{d\ln T}\right|_{N_{\rm site}} = \left.\frac{d\ln w(T)}{d\ln T}\right|_{N_{\rm site}} \left.\frac{d\ln\frac{Z_{Q_2}(w)}{Z_{Q_1}(w)}}{d\ln w}\right|_{N_{\rm site}}$$

$\boldsymbol{\omega(T) = \chi_t(T)\, V_4}$ (m_π and $N_{\rm site}$ are fixed)

$$Z(\theta) = \sum_{Q=-\infty}^{\infty} Z_Q\, e^{i\theta Q}$$

$$\left.\frac{d\ln w(T)}{d\ln T}\right|_{N_{\rm site}} = \left.\frac{d\ln\frac{Z_{Q_2}(T)}{Z_{Q_1}(T)}}{d\ln T}\right|_{N_{\rm site}} \times \left(\left.\frac{d\ln\frac{Z_{Q_2}(w)}{Z_{Q_1}(w)}}{d\ln w}\right|_{N_{\rm site}}\right)^{-1}$$

To experts

$$\left.\frac{d\ln w(T)}{d\ln T}\right|_{N_{\rm site}} = \boxed{\left.\frac{d\ln \frac{Z_{Q_2}(T)}{Z_{Q_1}(T)}}{d\ln T}\right|_{N_{\rm site}}} \times \left(\left.\frac{d\ln \frac{Z_{Q_2}(w)}{Z_{Q_1}(w)}}{d\ln w}\right|_{N_{\rm site}}\right)^{-1}$$

$\boldsymbol{\omega(T) = \chi_t(T)\, V_4}$ (m_q and $N_{\rm site}$ are fixed)

$$\boxed{\frac{d\ln \frac{Z_{Q_2}}{Z_{Q_1}}}{d\ln T}} = \frac{\beta^2 \beta_g}{6} \Delta S_g^{(Q_2,Q_1)}(\beta,\bar{m}_q) + N_f\left(1+\frac{d\ln m_q}{d\ln a}\right)\bar{m}_q\left(\langle s_{\bar{q}q}\rangle^{(Q_2)}_{\beta,\bar{m}_q} - \langle s_{\bar{q}q}\rangle^{(Q_1)}_{\beta,\bar{m}_q}\right)$$

where $\Delta S_g^{(Q_2,Q_1)}(\beta,\bar{m}_q) = -\frac{1}{\beta}\left(\langle S_g\rangle^{(Q_2)}_{\beta,\bar{m}_q} - \langle S_g\rangle^{(Q_1)}_{\beta,\bar{m}_q}\right)$

$$\hat{s}_{\bar{q}q} = \sum_x \bar{q}_x q_x$$

To experts

$$\left.\frac{d\ln w(T)}{d\ln T}\right|_{N_{\rm site}} = \left.\frac{d\ln\frac{Z_{Q_2}(T)}{Z_{Q_1}(T)}}{d\ln T}\right|_{N_{\rm site}} \times \left(\boxed{\left.\frac{d\ln\frac{Z_{Q_2}(w)}{Z_{Q_1}(w)}}{d\ln w}\right|_{N_{\rm site}}}\right)^{-1}$$

$\boldsymbol{\omega(T) = \chi_t(T)\, V_4}$ (m_q and $N_{\rm site}$ are fixed)

When $\omega \gg 1$, $$\frac{d\ln\frac{Z_{Q_2}}{Z_{Q_1}}}{d\ln w} = \frac{Q_2^2 - Q_1^2}{2w}(1+\cdots)$$

When $\omega \ll 1$, $$\frac{d\ln\frac{Z_{Q_2}}{Z_{Q_1}}}{d\ln w} = n_{Q_2} - n_{Q_1} + O(w)$$

New Method

J. Frison, R. Kitano, H. Matsufuru, S. Mori, NY, JHEP 1609 (2016) 021 [arXiv:1606.07175 [hep-lat]]

$$\boxed{\begin{aligned}\frac{d\ln\chi_t(T)}{d\ln T} = {} & \left[\frac{\beta^2\,\beta_g}{6}\Delta S_g^{(Q_2,Q_1)}(\beta,\bar{m}_q) + N_f\left(1+\frac{d\ln m_q}{d\ln a}\right)\bar{m}_q\left(\langle s_{\bar{q}q}\rangle^{(Q_2)}_{\beta,\bar{m}_q} - \langle s_{\bar{q}q}\rangle^{(Q_1)}_{\beta,\bar{m}_q}\right)\right] \\ & \times\frac{1}{n_{Q_2}-n_{Q_1}} + 4 + O(w)\,.\end{aligned}}$$

$$\Delta S_g^{(Q_2,Q_1)}(\beta,\bar{m}_q) = -\frac{1}{\beta}\left(\langle S_g\rangle^{(Q_2)}_{\beta,\bar{m}_q} - \langle S_g\rangle^{(Q_1)}_{\beta,\bar{m}_q}\right)$$

Looks complicated, but calculable!

High T Limit ($g^2 \to 0$ limit)

$$\text{Hight } \; T\,\text{Limit} \;\Rightarrow\; S_g^{(Q)}\big|_{\mathrm{BPST}} = \frac{8\pi^2}{g^2}|Q|$$

cf. $S_g = \int d^4x \frac{1}{2} \mathrm{Tr}\,[G_{\mu\nu}G_{\mu\nu}] \geq \int d^4x \frac{1}{2} \left|\mathrm{Tr}\, G_{\mu\nu}\tilde{G}_{\mu\nu}\right|$

$$\frac{d}{d\ln T}\left(\ln\frac{\chi_t(\beta)}{\chi_t(\beta_{\mathrm{ref}})}\right)\Bigg|_{\mathrm{BPST}} \approx -16\pi^2\left(b_0 + b_1\, g^2\right) \quad + 4 \approx -11 + 4$$

With $|Q|=1$, DIGA results $\chi_t(T) \sim T^{-7}$ is reproduced.

The above is the quenched case.

Similar argument in dynamical case.

✓ Test in quench

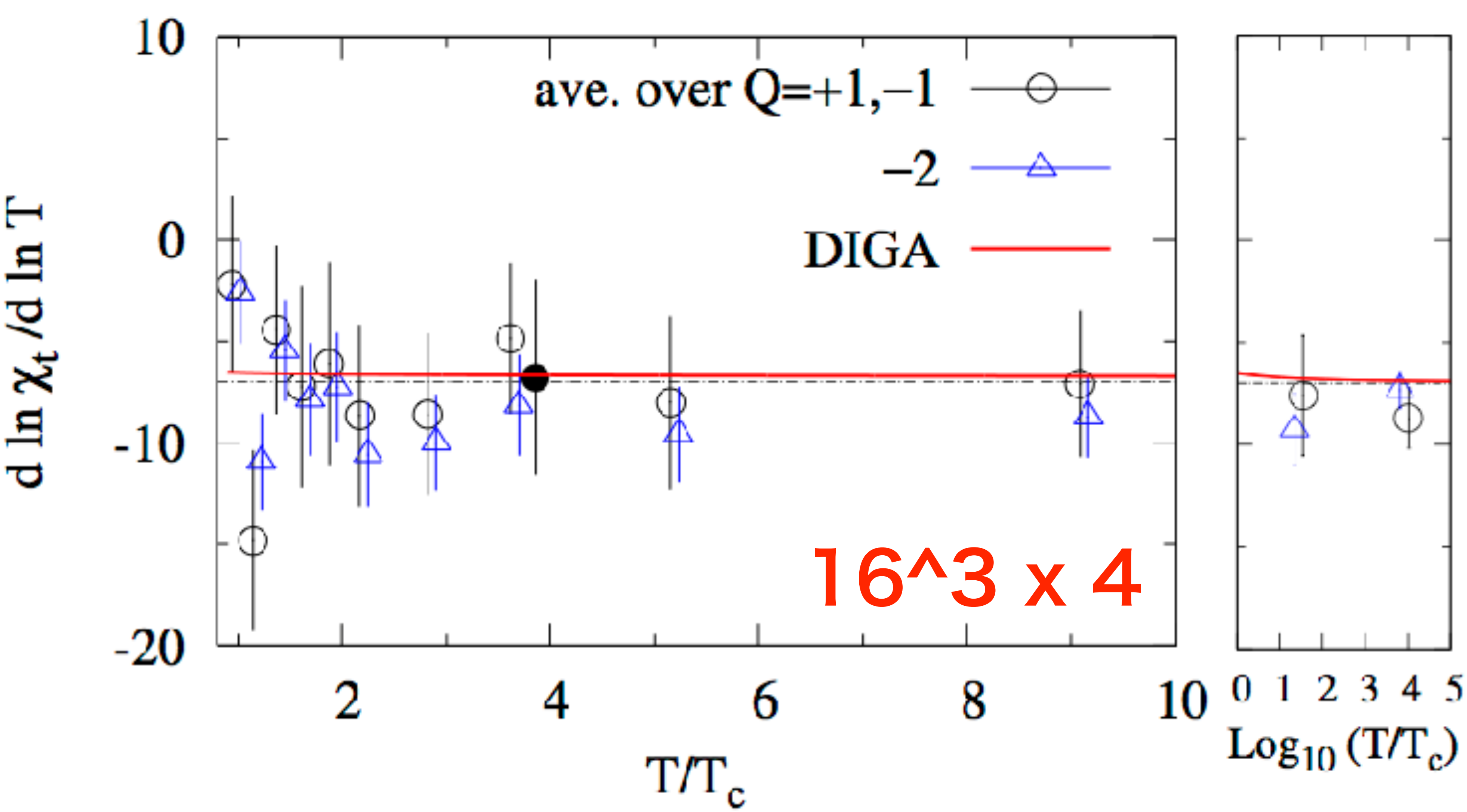
J. Frison, R. Kitano, H. Matsufuru, S. Mori, NY,
JHEP 1609 (2016) 021 [arXiv:1606.07**175** [hep-lat]]

- V=$16^3\times4$ (and $24^3\times4$)
- Iwasaki gauge action w/ Q=0, 1, 2
- $T_\mathrm{c} < T < 10^4\ T_\mathrm{c}$

Test in the quenched approximation

[Kitano, Frison, Matsufuru, Mori, NY (2016)]

$$\frac{d\ln\chi_t(T)}{d\ln T} = \left[\frac{\beta^2\beta_g}{6}\Delta S_g^{(Q_2,Q_1)}(\beta,\bar m_q) + \cancel{N_f\left(1+\frac{d\ln m_q}{d\ln a}\right)\bar m_q\left(\langle s_{\bar q q}\rangle^{(Q_2)}_{\beta,\bar m_q} - \langle s_{\bar q q}\rangle^{(Q_1)}_{\beta,\bar m_q}\right)}\right] \times \frac{1}{n_{Q_2}-n_{Q_1}} + 4 + O(w)\,.$$



Consistent with DIGA, $\chi_t \propto T^{-7}$ at $T > 2\,T_c$

But
poor accuracy
behavior around T_c
unclear.

✓ Improved calculation

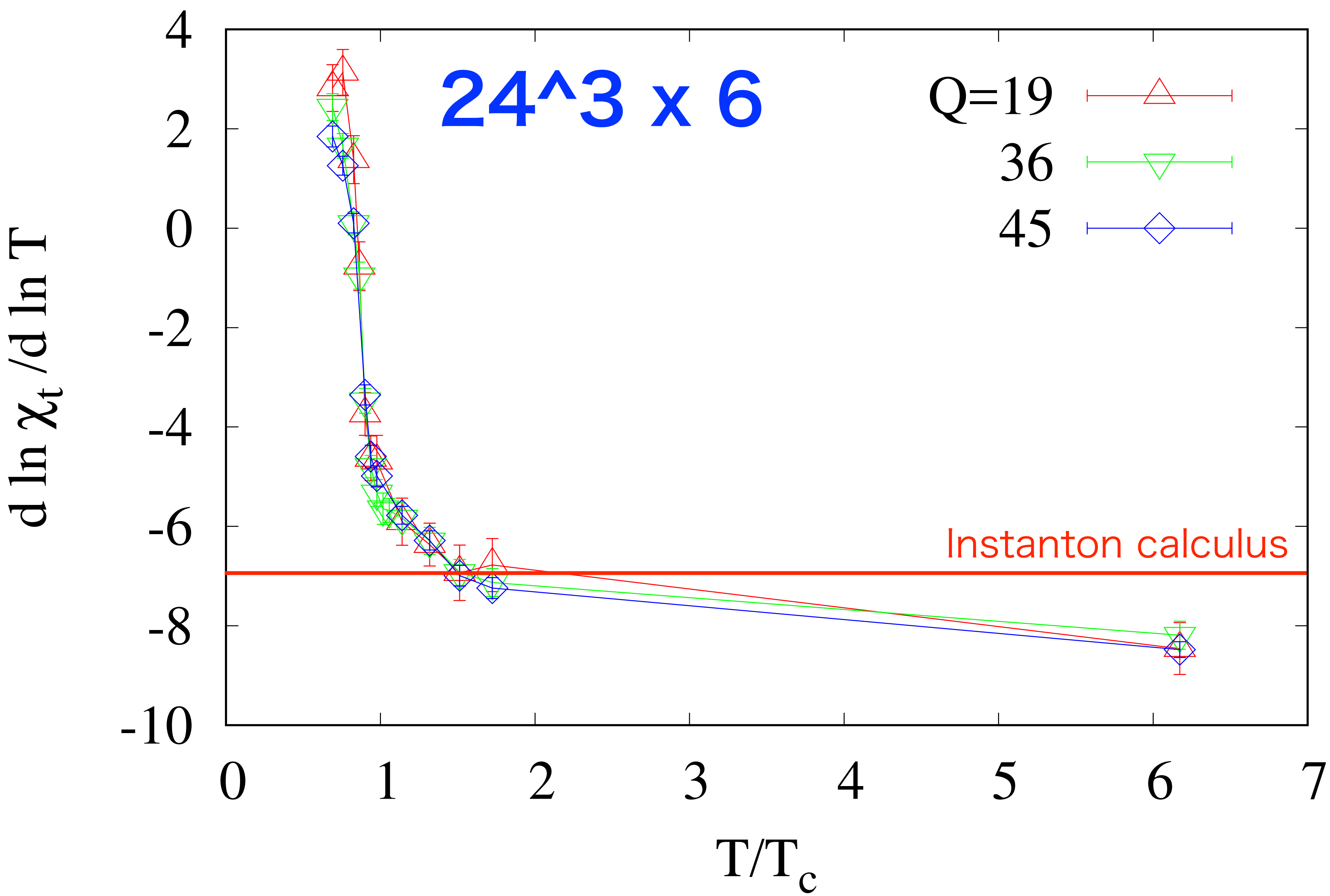
J. Frison, R. Kitano, H. Matsufuru, S. Mori, NY, in preparation

- $V=24^3\times 6$
- Iwasaki gauge action w/ Q=0, 5, 19, 36, 45
- $T_c < T < 6\ T_c$

Test in the quenched approximation

[Kitano, Frison, Matsufuru, Mori, NY, in preparation]

$$\frac{d\ln\chi_t(T)}{d\ln T} = \left[\frac{\beta^2\beta_g}{6}\Delta S_g^{(Q_2,Q_1)}(\beta,\bar{m}_q) + \cancel{N_f\left(1+\frac{d\ln m_q}{d\ln a}\right)\bar{m}_q\left(\langle s_{\bar{q}q}\rangle^{(Q_2)}_{\beta,\bar{m}_q} - \langle s_{\bar{q}q}\rangle^{(Q_1)}_{\beta,\bar{m}_q}\right)}\right] \times \frac{1}{n_{Q_2}-n_{Q_1}} + 4 + O(w)\,.$$



Stat. error significantly reduced !

$\chi_t \propto T^{-8}$ rather than T^{-7} around $T \sim 6\ T_c$

Summary

- ✓ Lattice QCD can explore axion physics!
- ✓ New method allows us to calculate $\chi_t(T)$ at very high T.
- ✓ The instanton calculus will be replaced by lattice determination of $\chi_t(T)$ in the estimate of the axion abundance.