ブラックホール地平面における粒子と弦の運動のカオス

棚橋典大 [阪大理]

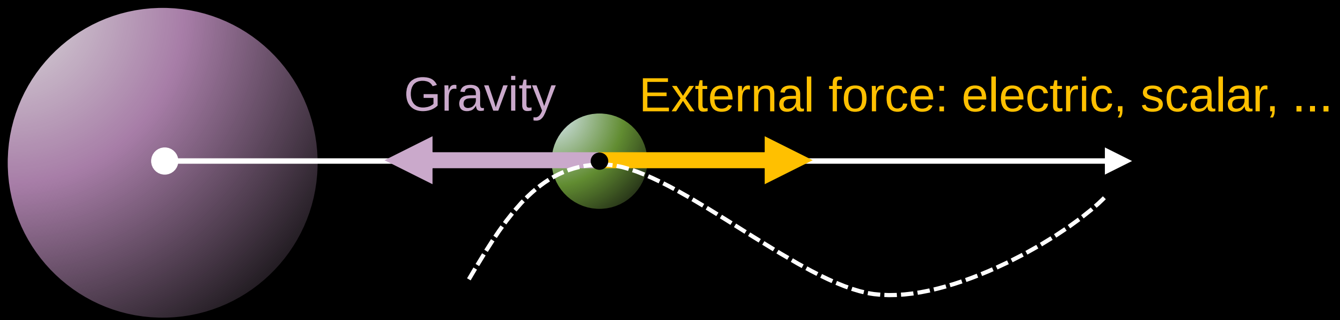
based on

Universality in Chaos of Particle Motion near Black Hole Horizon

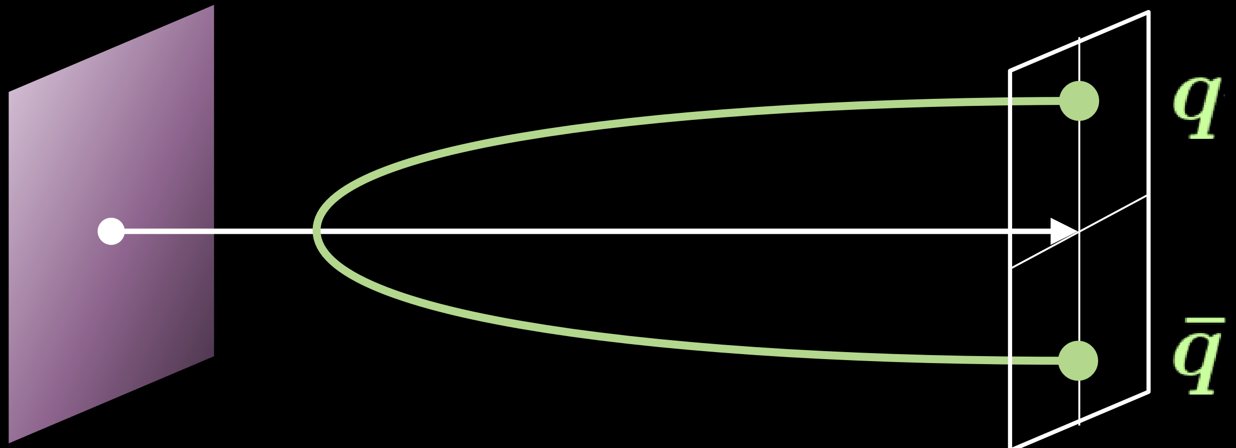
橋本幸士、棚橋典大 [arXiv:1610.06070]

Work in progress (橋本幸士、村田佳樹、棚橋典大)

Classical particle moving near black hole (BH) horizon



String moving near BH horizon in AdS spacetime



- ✓ **Particle & string motion become chaotic** due to BH gravity
- ✓ **Lyapunov exponent** λ of the chaos is bounded by **surface gravity** κ

$$\lambda \leq \kappa = 2\pi T/\hbar$$

- Motivation:

  A bound on chaos in QFT at temperature T :

$$\lambda \leq 2\pi T/\hbar$$

[Maldacena-Shenker-Stanford '15]

- Probing the effect of temperature T to chaos in QFT.

- We study effect of temperature to chaos in classical gravity.

  ↑

  Use BH surface gravity $\kappa = 2\pi T/\hbar$ instead.

- To probe effect of κ, we look at trajectories very close to BH.

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1. Chaos bound for particle near BH

2. Chaos bound for AdS string

3. Summary

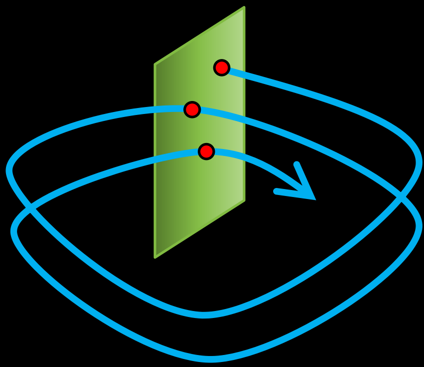
Classical Chaos

Classic chaos in deterministic dynamical systems

= Non-periodic bounded orbits sensitive to initial conditions

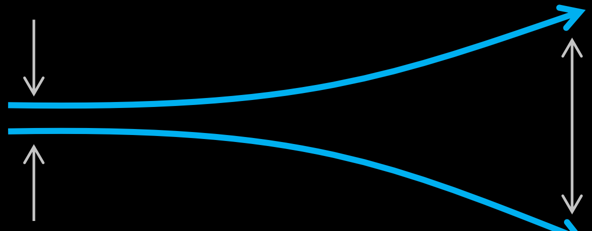
Diagnostics of chaos

Poincaré plot = Section of orbits in phase space



Lyapunov exponent λ = Separation growth rate of nearby orbits

$d(0)$

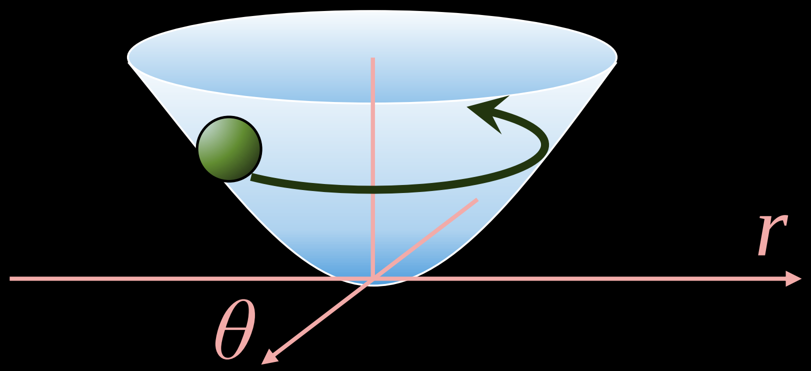


$$d(t) \sim d(0)e^{\lambda t}$$

Chaos Bound for Particle near BH

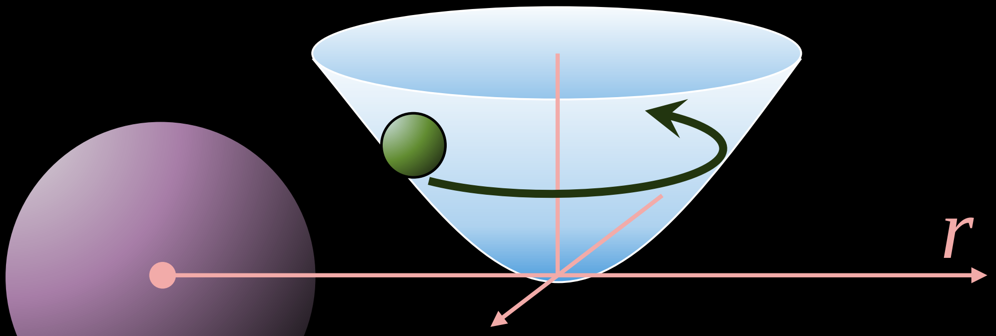
- To realize a particle moving very close to BH horizon,

  1. put a particle in a trapping harmonic potential

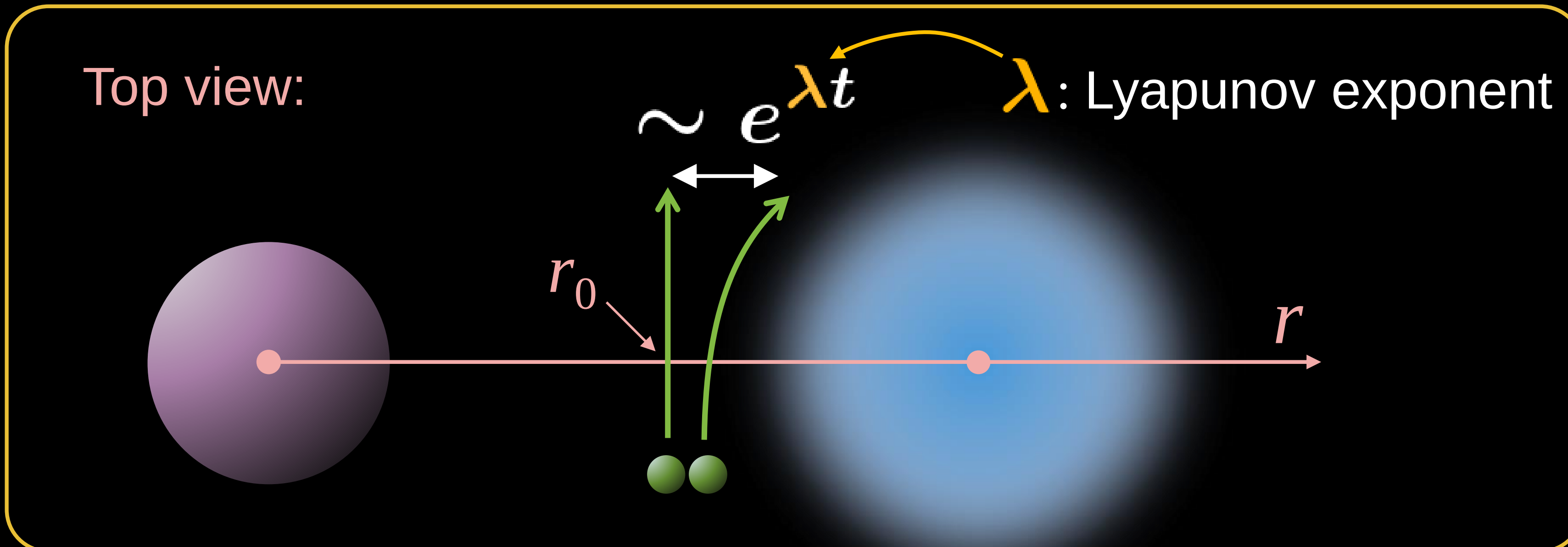


(← no chaos)

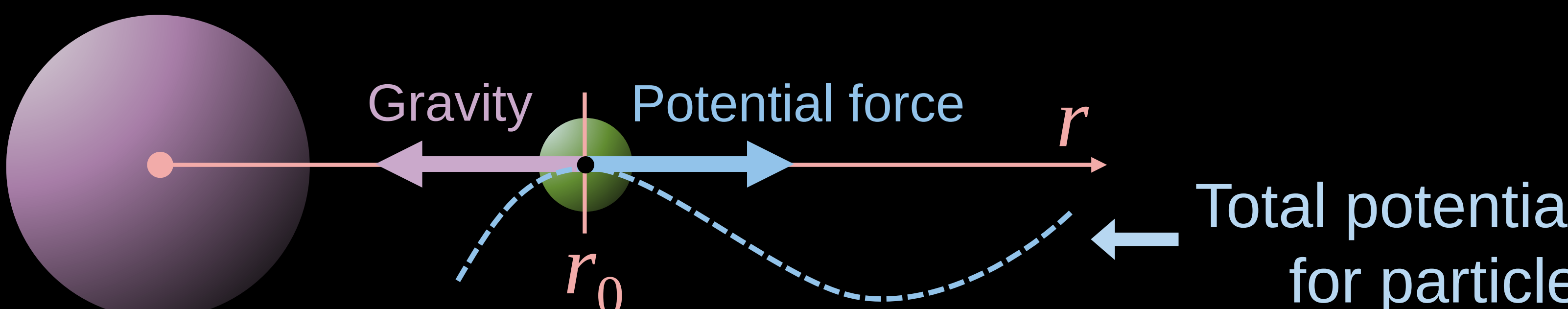
  2. take it close to a BH horizon



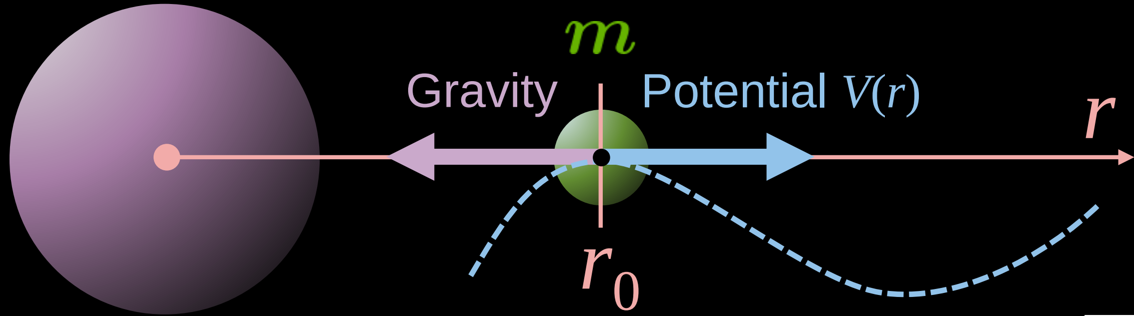
Chaos Bound for Particle near BH



2. take it close to a BH horizon & look at the separatrix



Chaos Bound for Particle near BH



$$\mathcal{L} = -m\sqrt{-g_{\mu\nu}\dot{X}^{\mu}\dot{X}^{\nu}} - V(X) \simeq -m\sqrt{f(r) - \frac{\dot{r}^2}{f(r)}} - V(r)$$

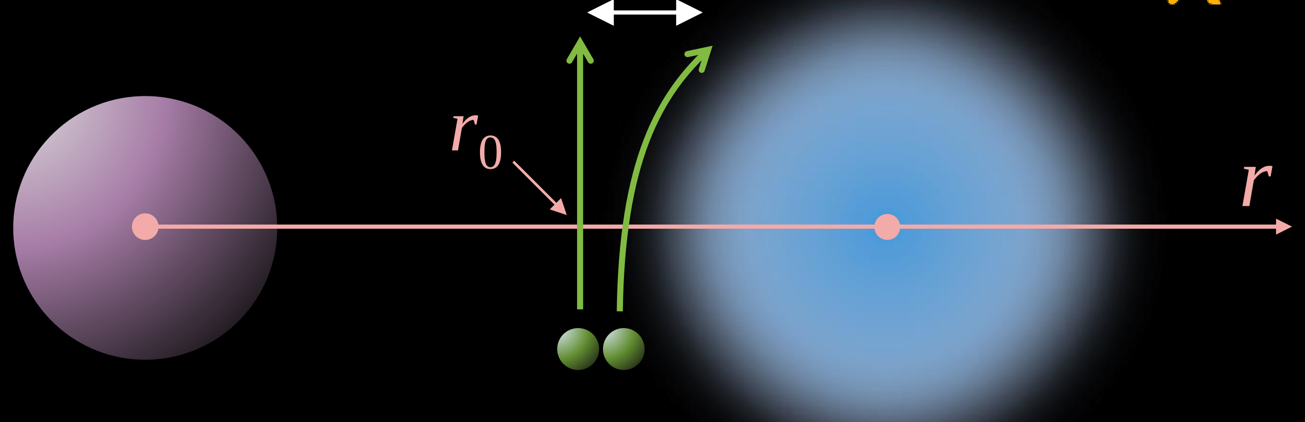
- Slow radial velocity near $r = r_0$
- Linear approximation for $V(r)$ ~ (slope) × $(r - r_{\mathrm{horizon}})$
- Near-horizon limit $r_0 \to r_{\mathrm{horizon}}$

$$\mathcal{L} \simeq C(m, \kappa, \text{slope of } V) \times \left[\dot{r}^2 + \kappa^2 (r - r_0)^2\right]$$

Chaos Bound for Particle near BH

Head-on view:

$\sim e^{\kappa t}$ → Lyapunov exponent $\lambda = \kappa$



- Linear approximation for $V(r) \sim (\text{slope}) \times (r - r_{\text{horizon}})$
- Near-horizon limit $r_0 \to r_{\text{horizon}}$

$$\mathcal{L} \simeq C(m, \kappa, \text{slope of } V) \times \left[\dot{r}^2 + \kappa^2 (r - r_0)^2\right]$$

✓ A generic trajectory obeys $\lambda \leq \kappa = 2\pi T/\hbar$

Examples

Charged particle near charged black hole:

$$\mathcal{L} = -m\sqrt{-g_{\mu\nu}(X)\dot{X}^\mu \dot{X}^\nu} - V(X) \quad \text{with} \quad V(X) = e\frac{dX^0}{dt}A_0(X)$$

$$\partial_r\left(\sqrt{-\det g}\, g^{rr} g^{00} \partial_r A_0\right) = 0 \quad \rightarrow \quad V \sim c \times r$$

Particle with scalar force:

$$\mathcal{L} = -\sqrt{-g_{\mu\nu}(X)\dot{X}^\mu \dot{X}^\nu}\,(m + \phi(X))$$

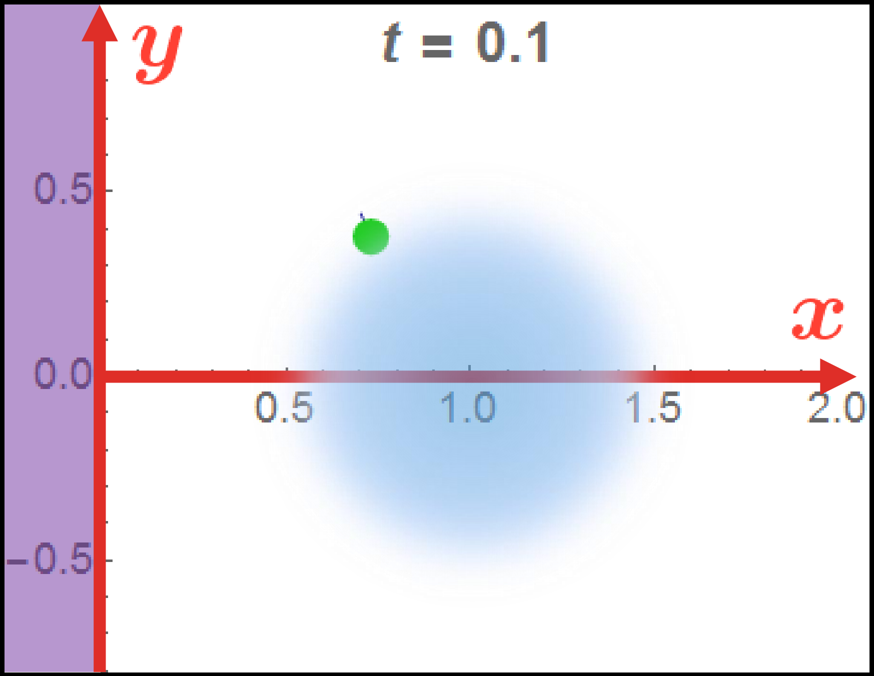
$$\partial_r\left(\sqrt{-\det g}\, g^{rr} \partial_r \phi\right) = 0 \quad \rightarrow \quad V \sim c \times \log r$$

These two examples give $\lambda = \kappa$ for any m and c.

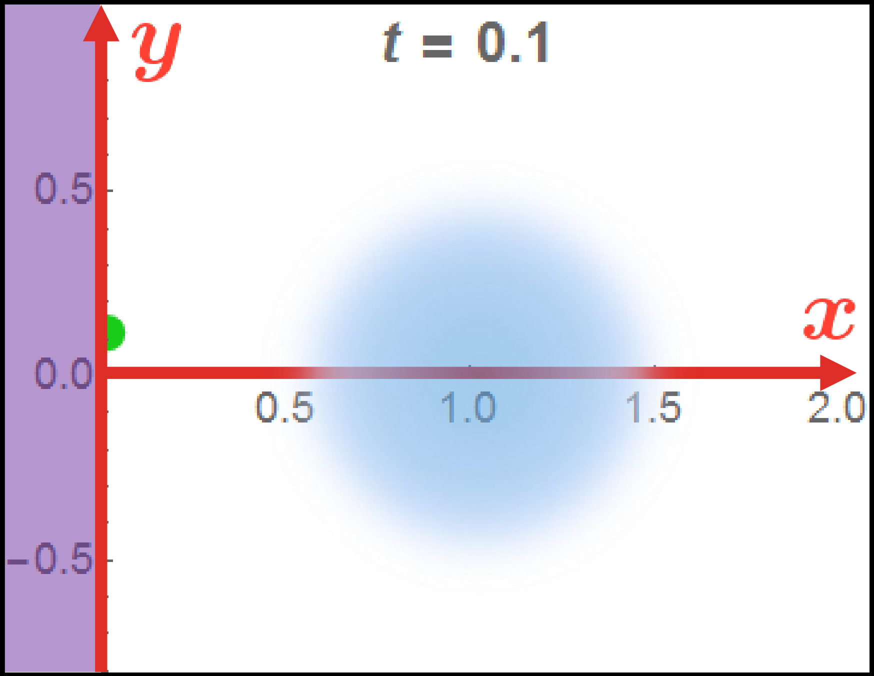
Numerical check

$$\mathcal{L} = -\sqrt{f(x) - \frac{\dot{x}^2}{f(x)} - \dot{y}^2} - \frac{\omega^2}{2}\left[(x - x_c)^2 + y^2\right] \qquad \left(f(x) \equiv 2\kappa x\right)$$

Particle near Potential Minimum



Particle near BH Horizon



Numerical check

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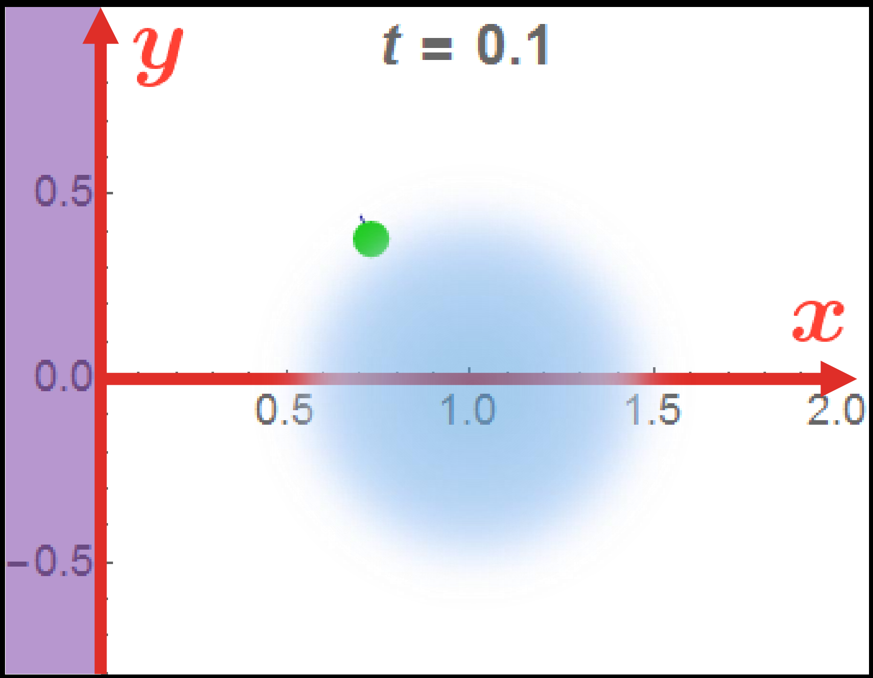
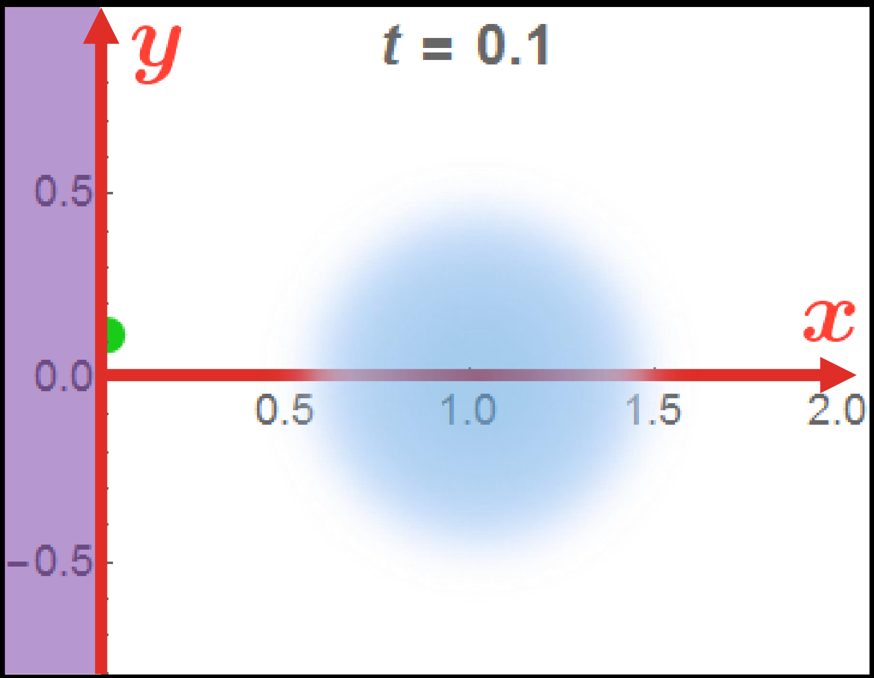
Particle near Potential Minimum



Periodic motion, no chaos

Particle near BH Horizon

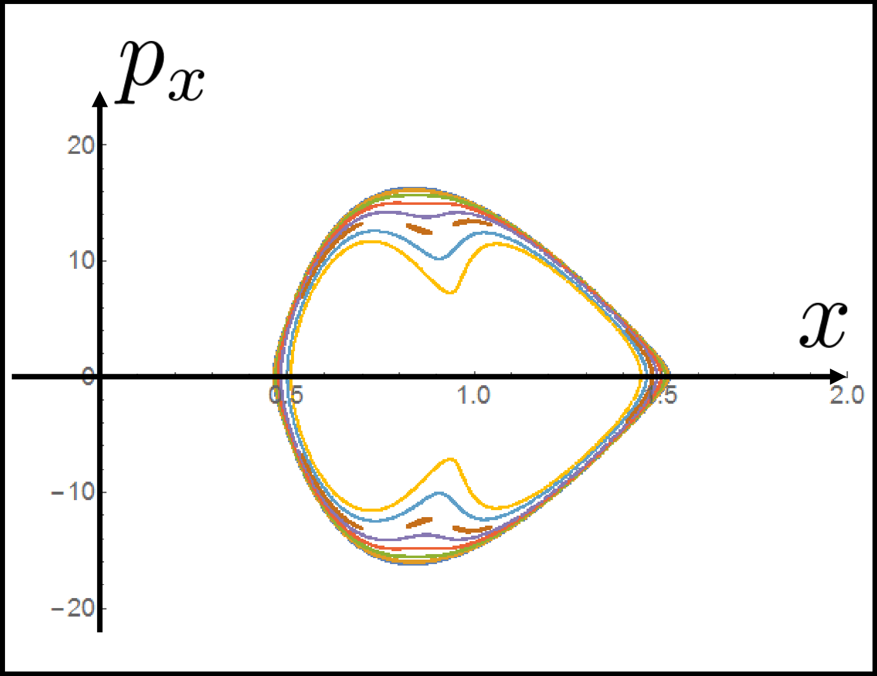


Chaotic motion

Poincaré plot at $y = 0$

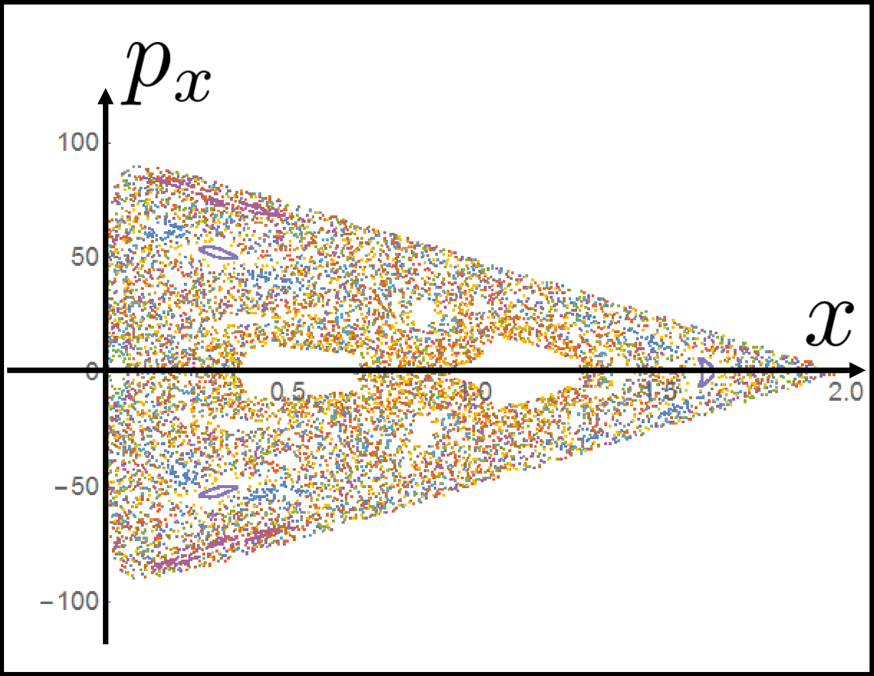
$$\mathcal{L} = -\sqrt{f(x) - \frac{\dot{x}^2}{f(x)} - \dot{y}^2 - \frac{\omega^2}{2}\left[(x - x_c)^2 + y^2\right]} \qquad \left(f(x) \equiv 2\kappa x\right)$$

Particle near Potential Minimum



Regular KAM tori, no chaos

Particle near BH Horizon



Lyapunov exponent $\lambda \sim 0.2\,\kappa$

Satisfies the bound $\lambda \leq \kappa$

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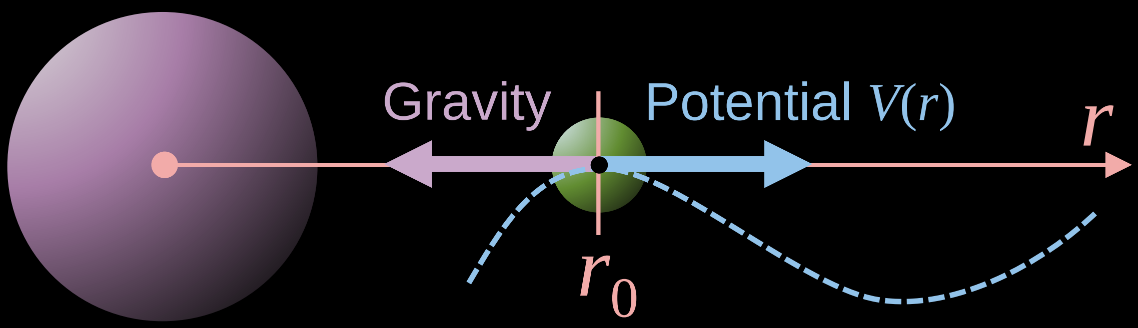
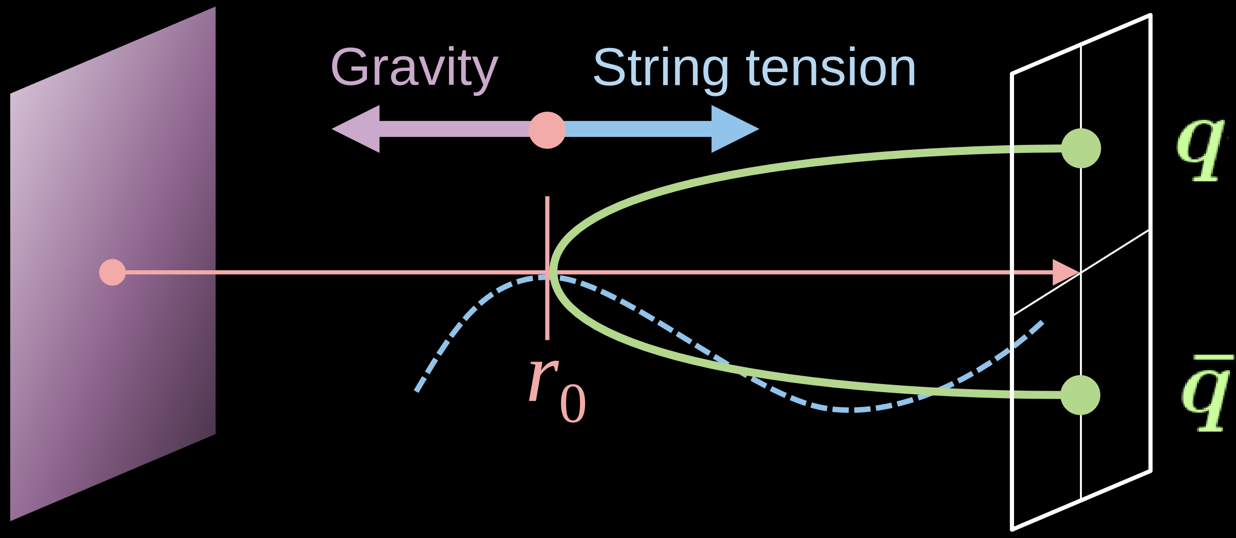
Chaos Bound for AdS String



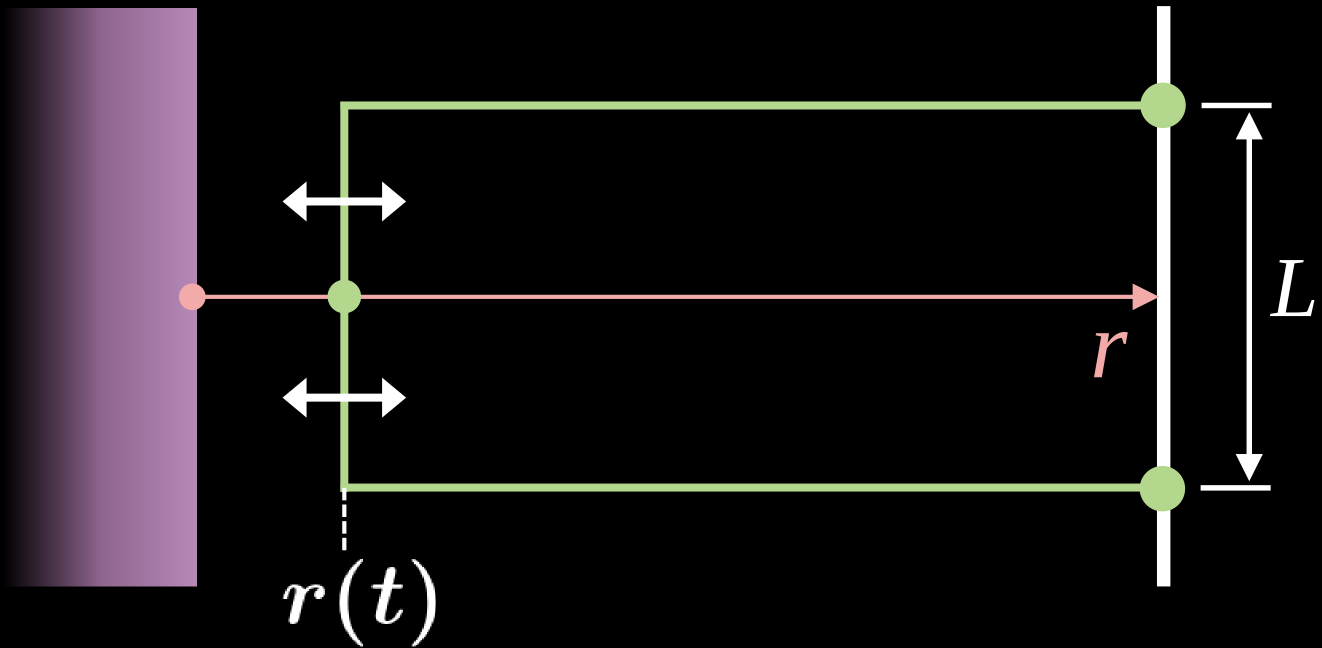


Fundamental string in AdS = “quark-anti quark pair”

Maldacena ‘98
Rey & Yee ‘98

Chaos Bound for AdS String

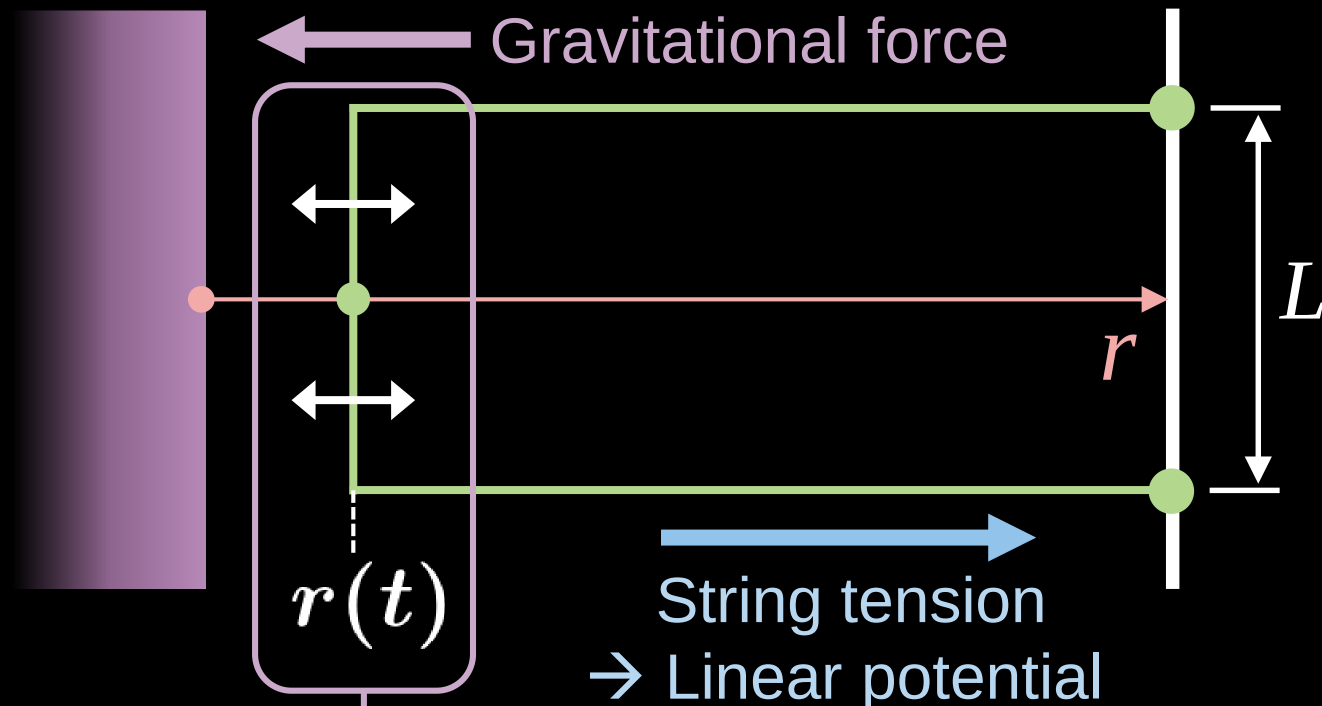
"Square-shaped string" approximation



$$\mathcal{L} \simeq -L\sqrt{r^4(t)f\big(r(t)\big) - \frac{\dot{r}^2(t)}{f\big(r(t)\big)}} + 2\big(r(t) - r_H\big) \quad \left(f(r) = 1 - \frac{r_H^4}{r^4}\right)$$

Chaos Bound for AdS String

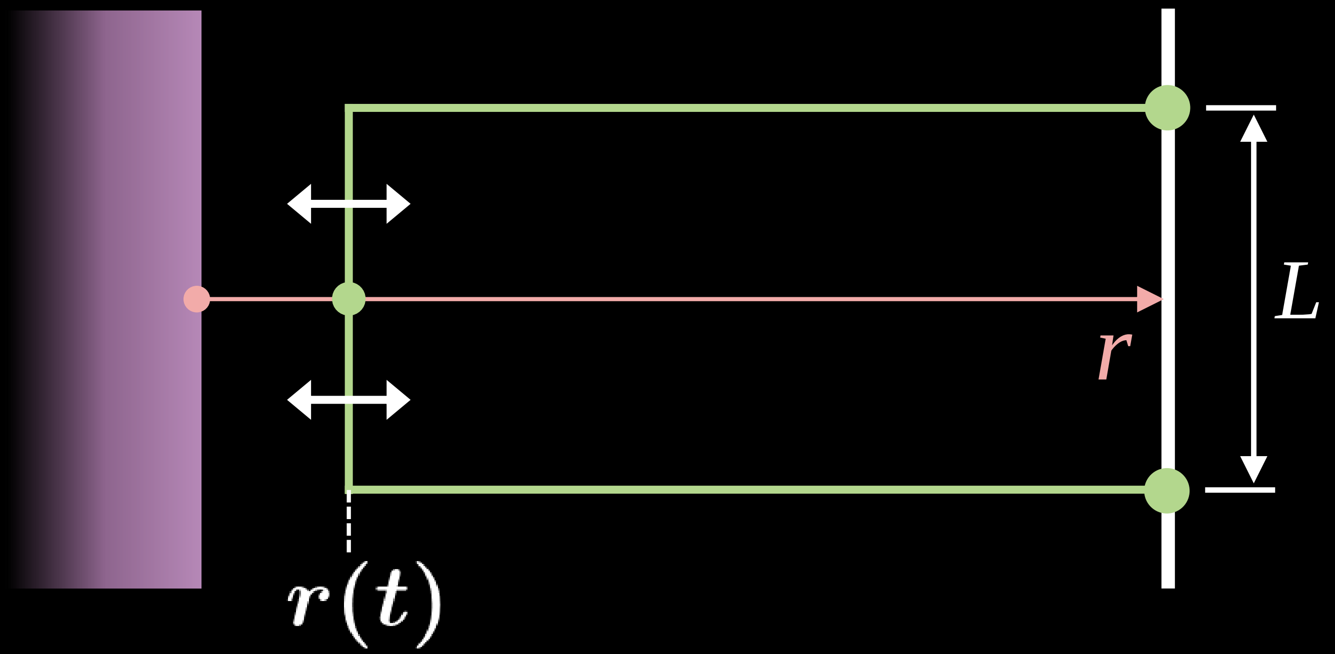
"Square-shaped string" approximation



$$\mathcal{L} \simeq -L\sqrt{r^4(t)f\big(r(t)\big) - \frac{\dot{r}^2(t)}{f\big(r(t)\big)}} + 2\big(r(t) - r_H\big) \qquad \left(f(r) = 1 - \frac{r_H^4}{r^4} \right)$$

Chaos Bound for AdS String

"Square-shaped string" approximation



For a string near horizon,

$$\mathcal{L} \simeq \frac{1}{2r_H^5 L^2}\left[\dot{r}^2 + \kappa^2 \left(r(t) - r_*\right)^2\right] \quad \Rightarrow \quad \lambda \lesssim \kappa$$

Chaos Bound for AdS String

Nonlinear dynamics of string

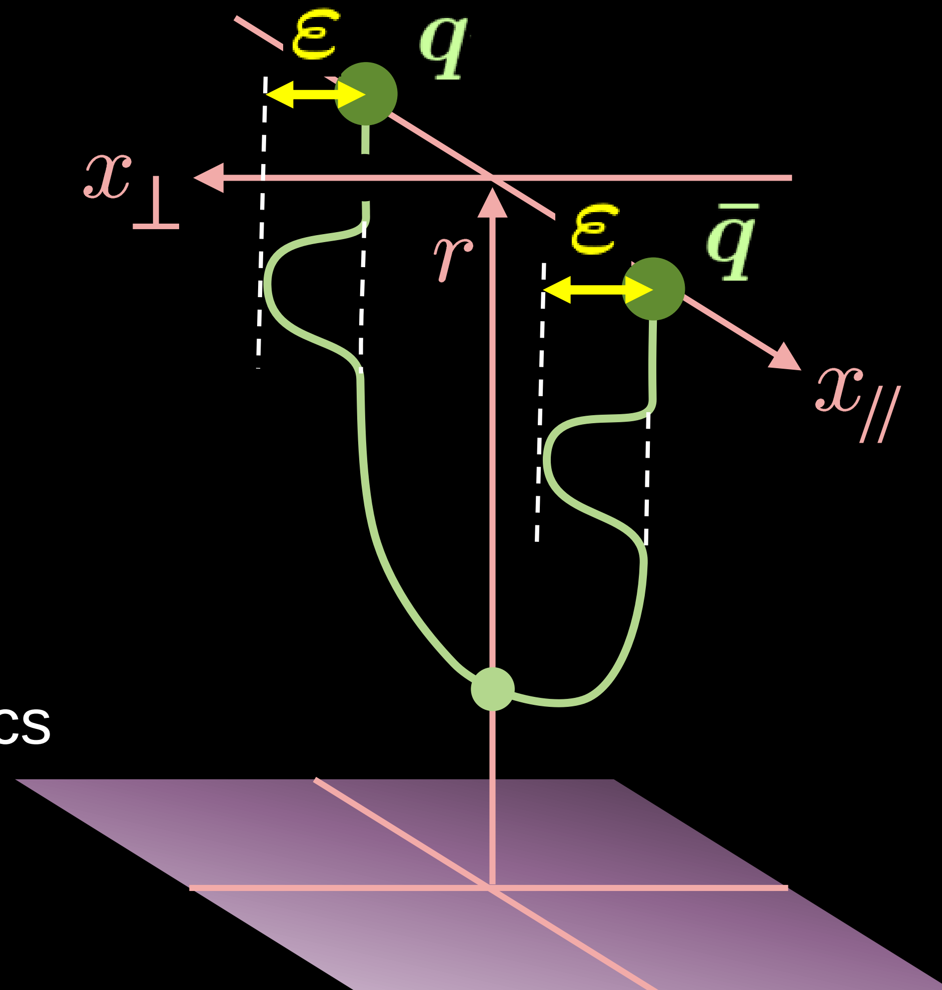
Shake the string end points

by amplitude ε

→ Nonlinear string motion

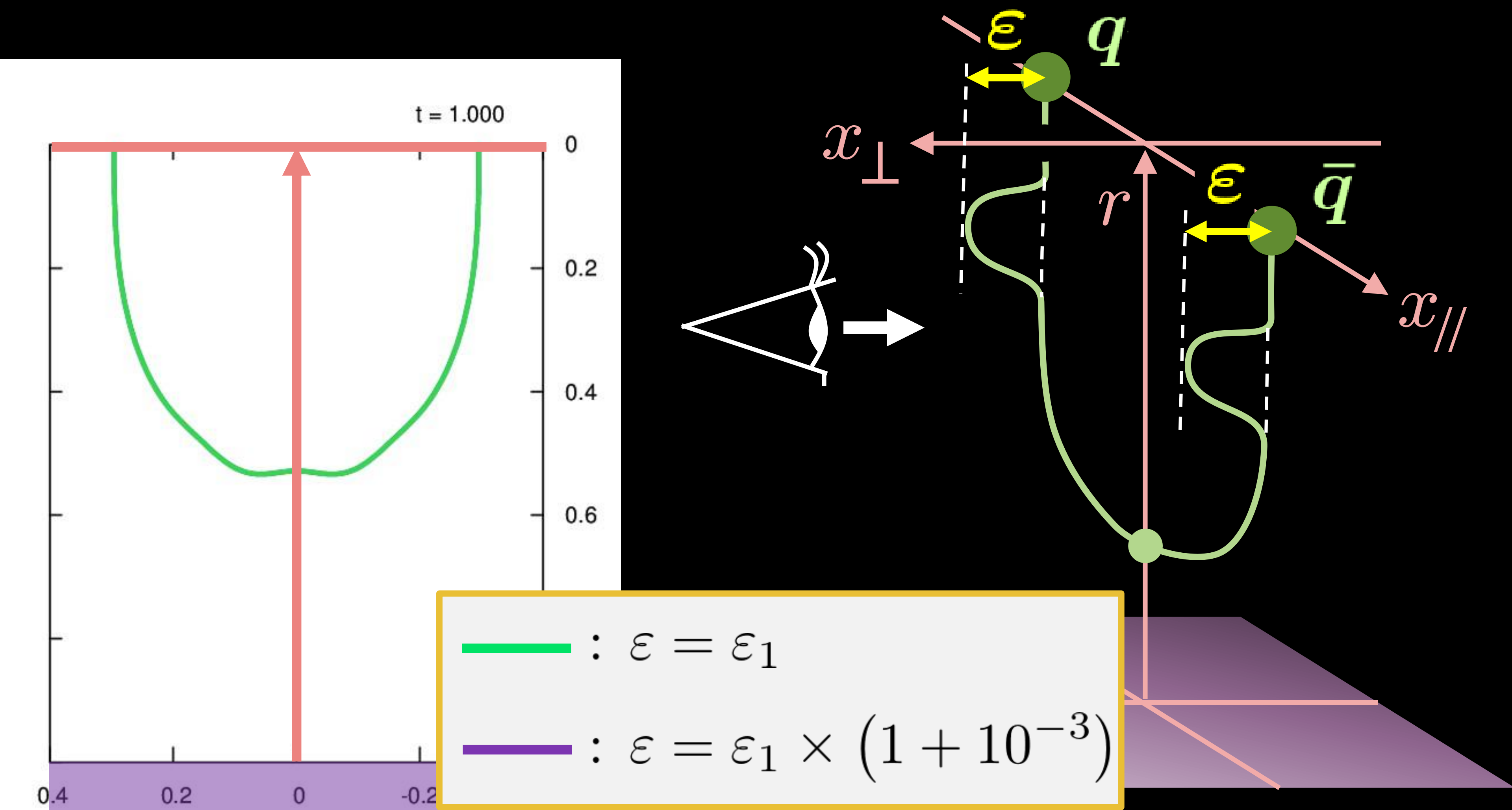
Shake the quark—$\overline{\text{quark}}$ pair

→ Nonlinear flux tube dynamics



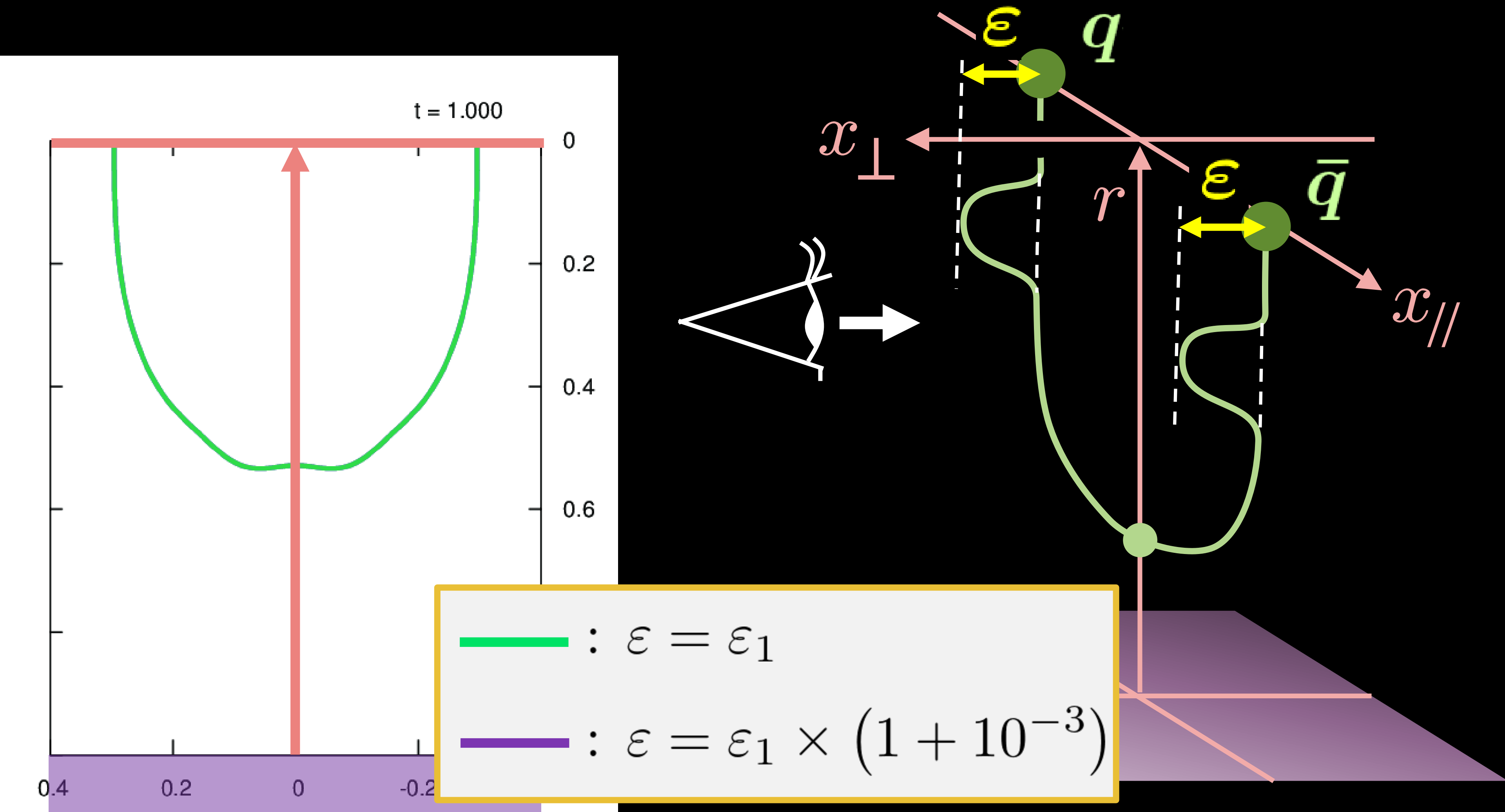
Chaos Bound for AdS String

Nonlinear dynamics of string for slightly different ε



Chaos Bound for AdS String

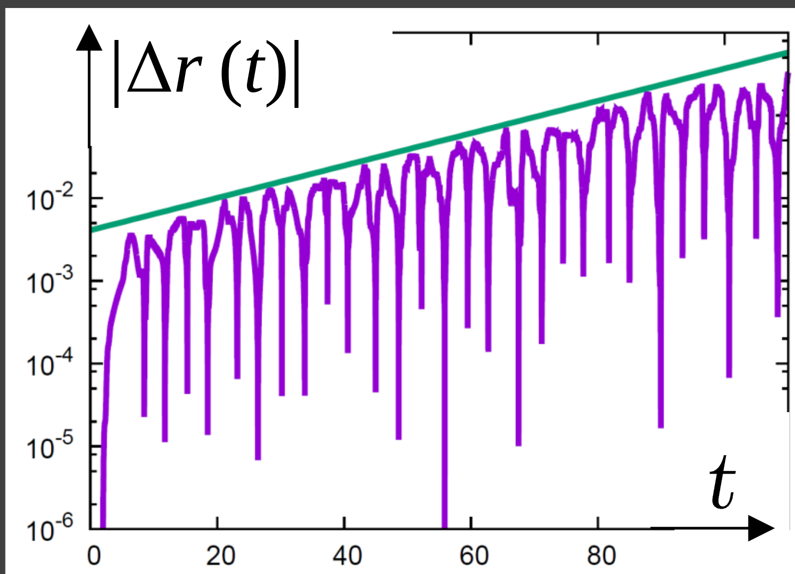
Nonlinear dynamics of string for slightly different ε



Chaos Bound for AdS String

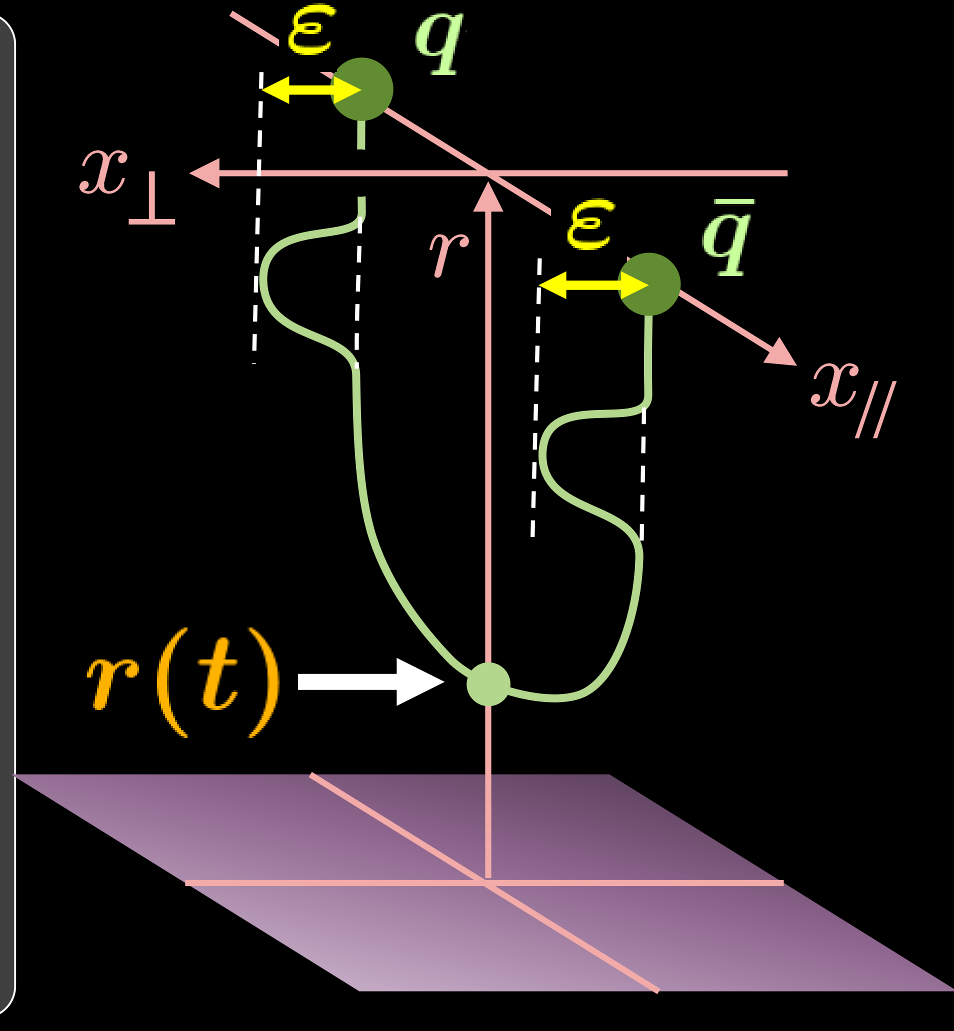
Nonlinear dynamics of string for slightly different ε

String tip position difference $\Delta r(t)$



$$|\Delta r(t)| \sim e^{\lambda t},$$

$$\lambda \sim 0.04\,\kappa < \kappa$$



SUMMARY

- We got a bound on chaos from classical BH-particle system

$$\lambda \leq \kappa = 2\pi T/\hbar$$

which coincides with the bound by Maldacena-Shenker-Stanford.

- Independent of particle mass, external force & metric form.

◆ Extension to string in AdS

✓ Unstable mode similar to the BH-particle system

✓ Instability growth rate: $\lambda \lesssim \kappa = 2\pi T/\hbar$

?: Does this govern chaotic motion of string in AdS?

?: Interpretation as chaos in the gauge theory?